ISLR 4.8.7

7. Suppose that we wish to predict whether a given stock will issue a dividend this year ("Yes" or "No") based on X, last year's percent profit. We examine a large number of companies and discover that the mean value of X for companies that issued a dividend was $\bar{X} = 10$, while the mean for those that didn't was $\bar{X} = 0$. In addition, the variance of X for these two sets of companies was $\hat{\sigma}^2 = 36$. Finally, 80% of companies issued dividends. Assuming that X follows a normal distribution, predict the probability that a company will issue a dividend this year given that its percentage profit was $X = 4$ last year.

Hint: Recall that the density function for a normal random variable is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$. *You will need to use Bayes' theorem.*

Answer

Using Bayes Theorem, we know that:

$$p_k(x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)}$$

Here, $\pi_k = 80\% = 0.8$

(k=1 for stock issuing a dividend,

k=0 otherwise)

$\pi_1 = 0.8, \quad \pi_0 = 0.2 \; ; \; \mu_1 = 10$
$\mu_0 = 0$

$x = 4 \; ; \; \sigma^2 = 36 \Rightarrow \sigma = 6$

$$P_k(x) = \frac{\pi_k \cancel{\frac{1}{\sqrt{2\pi\sigma^2}}} e^{-(x-\mu_k)^2/2\sigma^2}}{\cancel{\frac{1}{\sqrt{2\pi\sigma^2}}} \sum_{l=1}^{k} \pi_l e^{-(x-\mu_l)^2/2\sigma^2}}$$

$$\therefore P_1(4) = \frac{0.8 e^{-\frac{1}{72}(4-10)^2}}{0.8 e^{-\frac{(4-10)^2}{72}} + 0.2 e^{-\frac{(4-0)^2}{72}}}$$

$$\therefore P_1(4) = \frac{0.8e^{-\cancel{36}^{1}/\cancel{72}_{2}}}{0.8e^{-\frac{1}{2}} + 0.2e^{-\cancel{16}^{2}/\cancel{72}_{9}}}$$

$$\therefore P_1(4) = \frac{0.8}{e^{\frac{1}{2}}\left(0.8e^{-\frac{1}{2}} + 0.2e^{-\frac{2}{9}}\right)}$$

$$= \frac{0.8}{0.8 + 0.2e^{\frac{1}{2} - \frac{2}{9}}}$$

$$= \frac{0.8}{0.8 + 0.2e^{\frac{9-2}{18}}} = \frac{0.8}{0.8 + 0.2e^{\frac{7}{18}}}$$

$$\therefore P_1(4) \approx \frac{0.8}{1.094} \approx \underline{\underline{0.73}}$$

This is the probability that the company will issue a dividend this year given its percentage profit was 4 last year.