3.

7. The table below provides a training data set containing six observations, three predictors, and one qualitative response variable.

Obs.	X_1	X_2	X_3	Y
1	0	3	0	Red
2	2	0	0	Red
3	0	1	3	Red
4	0	1	2	Green
5	−1	0	1	Green
6	1	1	1	Red

Suppose we wish to use this data set to make a prediction for Y when $X_1 = X_2 = X_3 = 0$ using K-nearest neighbors.

(a) Compute the Euclidean distance between each observation and the test point, $X_1 = X_2 = X_3 = 0$.

(b) What is our prediction with $K = 1$? Why?

(c) What is our prediction with $K = 3$? Why?

(d) If the Bayes decision boundary in this problem is highly nonlinear, then would we expect the *best* value for K to be large or small? Why?

a) Euclidean Distances

1- 0, 3, 0

$$d = \sqrt{(0-0)^2 + (3-0)^2 + (0-0)^2}$$

$$= \sqrt{3^2} = 3$$

2. 2, 0, 0

$$d = \sqrt{2^2 + 0^2 + 0^2} = \sqrt{2^2} = 2$$

3. 0, 1, 3

$$d = \sqrt{0^2 + 1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$$

$$\therefore d = 3.16$$

4. 0, 1, 2

$$d = \sqrt{0^2 + 1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$$

$$\therefore d = 2.24$$

5. -1, 0, 1

$$d = \sqrt{(-1)^2 + 0^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$\therefore d = 1.414$$

6. 1, 1, 1

$$d = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\therefore d = 1.732$$

b) When $K = 1$, our prediction will be Green since the point $(-1, 0, 1)$ is the closest to the test point $(0, 0, 0)$; and the point $(-1, 0, 1)$ belongs to class 'Green'.

c) When $K = 3$, the points that we'll take into consideration will be:

$(-1, 0, 1)$ Green

$(1, 1, 1)$ Red

$(2, 0, 0)$ Red

Since 2/3 of the closest points belong to class 'Red', our prediction for (0,0,0) with $K=3$ will be 'Red'.

d) If the Bayes decision boundary is highly non-linear, we can expect the best K value to be small as a model with a smaller K value would only look at few of the nearest neighbours of a test point for prediction, thus allowing the model to estimate a highly non-linear Bayes decision boundary.