3. Orthonormal Columns in Q Given Q'Q = I

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$$Q = \begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix}$$

$$Q^T Q = I$$

$$\begin{bmatrix} \cdots & q_1^T & \cdots \\ \cdots & q_2^T & \cdots \\ \cdots & q_n^T & \cdots \end{bmatrix} \begin{bmatrix} \vdots & \vdots & & \vdots \\ q_1 & q_2 & \cdots & q_n \\ \vdots & \vdots & & \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

$$\begin{cases} q_1^T q_1 = I \\ q_1^T q_i = 0 \quad (i \neq 1) \end{cases}$$

Also $QQ^T = I$ if Q = square.

so Q is orthogonal matrix

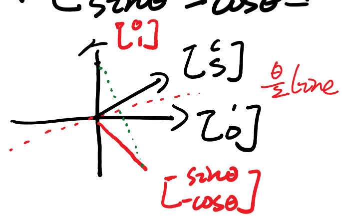
Any x $\|Qx\| = \|x\|$ (the length is same after multiply by Q)

Prove: $\|Qx\|^2 = \|x\|^2$

$$(Qx)^T(Qx) = x^T x$$

$$x^T Q^T Q x = x^T x \quad (\checkmark)$$

$$Q = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \text{ (Reflection matrix)}$$



Householder reflection

start with $u^T u = 1$

$H = I - 2uu^T$ (H is symmetric)

Check $H^T H = I$ (check Orthogonal)

$$= H^T H = I^2 - 4uu^T + 4uu^T uu^T$$

$$= I - 4uu^T + 4u\underbrace{(u^T u)}_{=1}u^T$$

=I