Task5.12(в)

In[112]:=

```
(* Определим общие данные, x, y, скалярное произведение,
график исходных данных, среднеквадратическую отклонение *)
x = {1910.0, 1920.0, 1930.0, 1940.0, 1950.0, 1960.0, 1970.0, 1980.0, 1990.0, 2000.0};
y = {92 228 496.0, 106 021 537.0, 123 202 624.0, 132 164 569.0, 151 325 798.0,
   179 323 175.0, 203 211 926.0, 226 545 805.0, 248 709 873.0, 281 421 906.0};

points = Transpose[{x, y}];
plotPointList =
  ListPlot[points, PlotStyle → PointSize[Medium], PlotMarkers → {"●", 12},
   AxesLabel → {"x", "y"}, PlotLegends → {"Исходные данные"}];

Y[x_] := y[[(x - 1910.0)/10.0 + 1.0]]
scalarProduct[f1_, f2_] := Sum[f1[x[[k]]] * f2[x[[k]]], {k, 1, Length[x]}]

err2[f1_, f2_] := (0.1 * Sum[(f1[x[[k]]] - f2[x[[k]]])^2, {k, 1, Length[x]}])^(1/2)
;
```

N=2

In[119]:=

```
(* Рассмотрим случай N = 2 *)
g31[x_] := 1
g32[x_] := x - 1955.0
g33[x_] := (x - 1955.0)^2

M3 = {
   {scalarProduct[g31, g31], scalarProduct[g31, g32], scalarProduct[g31, g33]},
   {scalarProduct[g32, g31], scalarProduct[g32, g32], scalarProduct[g32, g33]},
   {scalarProduct[g33, g31], scalarProduct[g33, g32], scalarProduct[g33, g33]}
  };

b3 = {scalarProduct[Y, g31], scalarProduct[Y, g32], scalarProduct[Y, g33]};
p3 = Inverse[M3].b3 (* наши коэффициенты *)

approxFunc3[x_] := p3[[1]] * g31[x] + p3[[2]] * g32[x] + p3[[3]] × g33[x]

plotApproxFunc3 = Plot[approxFunc3[x], {x, 1900, 2010}, PlotStyle → {Red, Thick},
   PlotLegends → {"Аппроксимирующая функция"}, PlotLabel → "N=2"];

(* ошибка *)
err2[Y, approxFunc3]

(* предсказание *)
N[Abs[approxFunc3[2010] - 308 745 538]]

Show[plotApproxFunc3, plotPointList, DisplayFunction → $DisplayFunction]
```

Out[124]=

$$\{1.65888 \times 10^{8}, 2.09661 \times 10^{6}, 10\,336.8\}$$

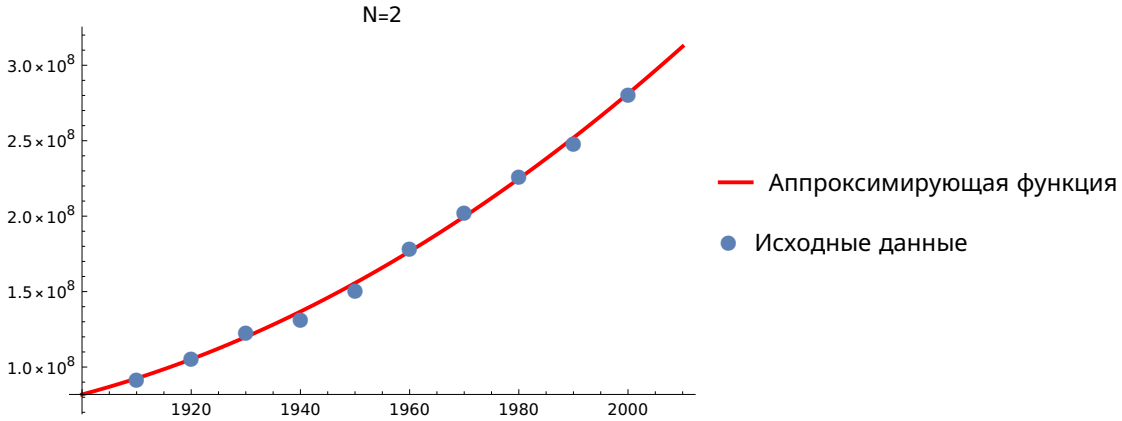
Out[127]=

$$2.91416 \times 10^{6}$$

Out[128]=

$$3.7248 \times 10^{6}$$

Out[129]=


N=2
3.0×10^8
2.5×10^8
2.0×10^8
1.5×10^8
1.0×10^8
1920
1940
1960
1980
2000
Аппроксимирующая функция
Исходные данные

N=3

In[130]:=

```
g41[x_] := 1
g42[x_] := x - 1955.0
g43[x_] := (x - 1955.0)^2
g44[x_] := (x - 1955.0)^3

M4 = {
   {scalarProduct[g41, g41], scalarProduct[g41, g42],
    scalarProduct[g41, g43], scalarProduct[g41, g44]}, {scalarProduct[g42, g41],
    scalarProduct[g42, g42], scalarProduct[g42, g43], scalarProduct[g42, g44]},
   {scalarProduct[g43, g41], scalarProduct[g43, g42],
    scalarProduct[g43, g43], scalarProduct[g43, g44]},
   {scalarProduct[g44, g41], scalarProduct[g44, g42],
    scalarProduct[g44, g43], scalarProduct[g44, g44]}
  };

b4 = {scalarProduct[Y, g41], scalarProduct[Y, g42],
   scalarProduct[Y, g43], scalarProduct[Y, g44]};
p4 = Inverse[M4].b4 (* наши коэффициенты *)

approxFunc4[x_] :=
 p4[[1]] * g41[x] + p4[[2]] * g42[x] + p4[[3]] × g43[x] + p4[[4]] × g44[x]

plotApproxFunc4 = Plot[approxFunc4[x], {x, 1900, 2010}, PlotStyle → {Red, Thick},
   PlotLegends → {"Аппроксимирующая функция"}, PlotLabel → "N=3"];

(* ошибка *)
err2[Y, approxFunc4]

(* предсказание *)
N[Abs[approxFunc4[2010] - 308 745 538]]

Show[plotApproxFunc4, plotPointList, DisplayFunction → $DisplayFunction]
```

Inverse: Result for Inverse of badly conditioned matrix {{10., 0., 8250., 0.}, {0., 8250., 0., 1.20863×10^7}, {8250., 0., 1.20863×10^7, 0.}, {0., 1.20863×10^7, 0., 2.07952×10^10}} may contain significant numerical errors.

Out[136]=

$$\left\{1.65888 \times 10^{8},\ 2.15551 \times 10^{6},\ 10\,336.8,\ -40.2023\right\}$$

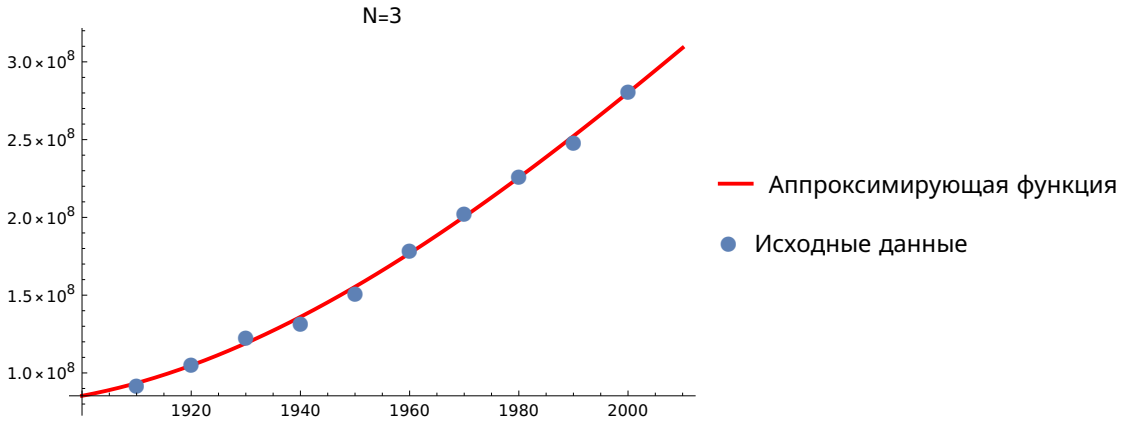
Out[139]=

$$2.82721 \times 10^{6}$$

Out[140]=

$$275\,441.$$

Out[141]=



N=4

In[142]:=

```
g51[x_] := 1
g52[x_] := x - 1955
g53[x_] := (x - 1955)^2
g54[x_] := (x - 1955)^3
g55[x_] := (x - 1955)^4

M5 = {
   {scalarProduct[g51, g51], scalarProduct[g51, g52],
    scalarProduct[g51, g53], scalarProduct[g51, g54], scalarProduct[g51, g55]},
   {scalarProduct[g52, g51], scalarProduct[g52, g52],
    scalarProduct[g52, g53], scalarProduct[g52, g54], scalarProduct[g52, g55]},
   {scalarProduct[g53, g51], scalarProduct[g53, g52],
    scalarProduct[g53, g53], scalarProduct[g53, g54], scalarProduct[g53, g55]},
   {scalarProduct[g54, g51], scalarProduct[g54, g52],
    scalarProduct[g54, g53], scalarProduct[g54, g54], scalarProduct[g54, g55]},
   {scalarProduct[g55, g51], scalarProduct[g55, g52],
    scalarProduct[g55, g53], scalarProduct[g55, g54], scalarProduct[g55, g55]}
  };

b5 = {scalarProduct[Y, g51], scalarProduct[Y, g52],
   scalarProduct[Y, g53], scalarProduct[Y, g54], scalarProduct[Y, g55]};
p5 = Inverse[M5].b5 (* наши коэффициенты *)

approxFunc5[x_] := p5[[1]] * g51[x] + p5[[2]] * g52[x] +
  p5[[3]] × g53[x] + p5[[4]] × g54[x] + p5[[5]] × g55[x]

plotApproxFunc5 = Plot[approxFunc5[x], {x, 1900, 2010}, PlotStyle → {Red, Thick},
   PlotLegends → {"Аппроксимирующая функция"}, PlotLabel → "N=4"];

(* ошибка *)
err2[Y, approxFunc5]

(* предсказание *)
N[Abs[approxFunc5[2010] - 308 745 538]]

Show[plotApproxFunc5, plotPointList, DisplayFunction → $DisplayFunction]
```

Inverse: Result for Inverse of badly conditioned matrix {{10., 0., 8250., 0., 1.20863×10^7}, {0., 8250., 0., 1.20863×10^7, 0.}, {8250., 0., 1.20863×10^7, 0., 2.07952×10^{10}}, {0., 1.20863×10^7, 0., 2.07952×10^{10}, 0.}, {1.20863×10^7, 0., 2.07952×10^{10}, 0., 3.84443×10^{13}}} may contain significant numerical errors.

Out[149]=

$$\{1.65422 \times 10^8,\ 2.15551 \times 10^6,\ 12316.8,\ -40.2023,\ -0.965852\}$$

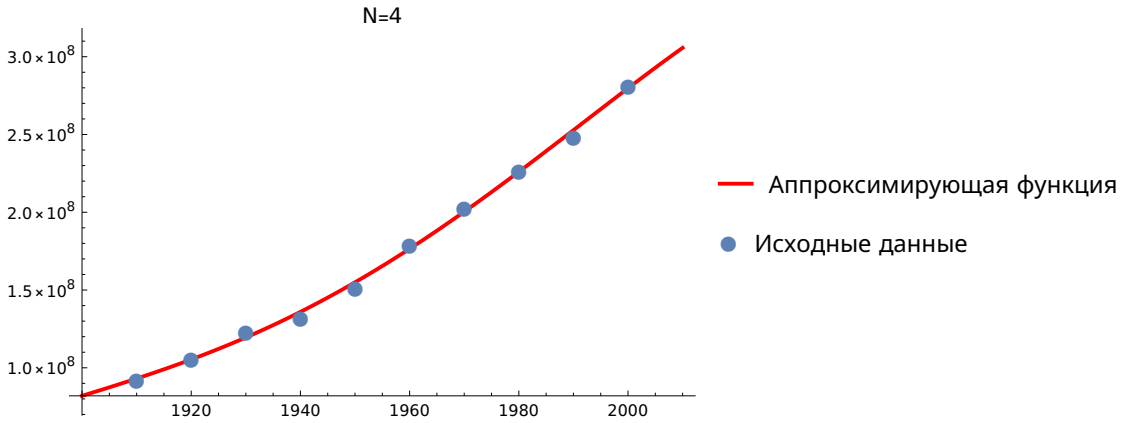
Out[152]=

$$2.7999 \times 10^6$$

Out[153]=

$$3.03936 \times 10^6$$

Out[154]=



N=5

In[267]:=

```
g61[x_] := 1
g62[x_] := x - 1955
g63[x_] := (x - 1955)^2
g64[x_] := (x - 1955)^3
g65[x_] := (x - 1955)^4
g66[x_] := (x - 1955)^5

M6 = {
   {scalarProduct[g61, g61], scalarProduct[g61, g62], scalarProduct[g61, g63],
    scalarProduct[g61, g64], scalarProduct[g61, g65], scalarProduct[g61, g66]},
   {scalarProduct[g62, g61], scalarProduct[g62, g62], scalarProduct[g62, g63],
```

```
    scalarProduct[g62, g64], scalarProduct[g62, g65], scalarProduct[g62, g66]},
   {scalarProduct[g63, g61], scalarProduct[g63, g62], scalarProduct[g63, g63],
    scalarProduct[g63, g64], scalarProduct[g63, g65], scalarProduct[g63, g66]},
   {scalarProduct[g64, g61], scalarProduct[g64, g62], scalarProduct[g64, g63],
    scalarProduct[g64, g64], scalarProduct[g64, g65], scalarProduct[g64, g66]},
   {scalarProduct[g65, g61], scalarProduct[g65, g62], scalarProduct[g65, g63],
    scalarProduct[g65, g64], scalarProduct[g65, g65], scalarProduct[g65, g66]},
   {scalarProduct[g66, g61], scalarProduct[g66, g62],
    scalarProduct[g66, g63], scalarProduct[g66, g64], scalarProduct[g66, g65],
    scalarProduct[g66, g66]}
  };

b6 = {scalarProduct[Y, g61], scalarProduct[Y, g62], scalarProduct[Y, g63],
   scalarProduct[Y, g64], scalarProduct[Y, g65], scalarProduct[Y, g66]};
p6 = Inverse[M6].b6 (* наши коэффициенты *)

approxFunc6[x_] := p6[[1]] * g61[x] + p6[[2]] * g62[x] +
  p6[[3]] × g63[x] + p6[[4]] × g64[x] + p6[[5]] × g65[x] + p6[[6]] × g66[x]

plotApproxFunc6 = Plot[approxFunc6[x], {x, 1900, 2010}, PlotStyle → {Red, Thick},
   PlotLegends → {"Аппроксимирующая функция"}, PlotLabel → "N=6"];

(* ошибка *)
err2[Y, approxFunc6]

(* предсказание *)
N[Abs[approxFunc6[2010] - 308 745 538]]

Show[plotApproxFunc6, plotPointList, DisplayFunction → $DisplayFunction]
```

Inverse: Result for Inverse of badly conditioned matrix {{10., 0., 8250., 0., 1.20863×10^{7}, 0.}, {0., 8250., 0., 1.20863×10^{7}, 0., 2.07952×10^{10}}, {8250., 0., 1.20863×10^{7}, 0., 2.07952×10^{10}, 0.}, {0., ≪4≫, 3.84443×10^{13}}, {1.20863×10^{7}, 0., 2.07952×10^{10}, 0., 3.84443×10^{13}, 0.}, {0., 2.07952×10^{10}, 0., 3.84443×10^{13}, 0., 7.38102×10^{16}}} may contain significant numerical errors.

Out[275]=

$$\{1.65422 \times 10^{8},\ 2.464 \times 10^{6},\ 12\,316.8,\ -670.708,\ -0.965852,\ 0.244067\}$$

Out[278]=

$$1.78692 \times 10^{6}$$

Out[279]=

$$3.18622 \times 10^{7}$$

Out[280]=