1. General Introduction

1.1 Background of the Study

Hospitals are critical healthcare facilities where patients receive treatment and care. One of the most important resources in hospitals is beds, which must be available when needed to ensure timely patient care. However, managing hospital beds efficiently is challenging due to unpredictable patient arrivals, varying lengths of stay, and limited resources such as staff and equipment. To address these challenges, researchers use mathematical models, particularly queuing theory, to simulate and optimize hospital operations.

This study focuses on modeling hospital bed occupancy using the M/M/c/K finite capacity queuing model, with a specific emphasis on how staffing levels, staff experience, and shift patterns affect bed utilization and patient flow. By integrating human resource variables into the queuing framework, this research aims to provide insights that can support better decision-making in hospital management.

The M/M/c/K model can be represented mathematically as follows:

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & \text{for } 0 \leq n \leq c \\ \frac{(\lambda/\mu)^n}{c! c^{n-c}} P_0 & \text{for } c \leq n \leq K \end{cases} \tag{1.1}$$

where P_n represents the steady-state probability of having exactly n patients in the system at any given time. For $n \leq c$, all patients are being served, while for $c \leq n \leq K$, $(n - c)$ patients are waiting in the queue and c patients are being served. The formula takes different forms in these two regions because in the first case ($n \leq c$), all arriving patients can be served immediately, while in the second case ($c \leq n \leq K$), some patients must wait due to all servers being occupied.

Hospital bed occupancy is a key performance indicator for healthcare systems. Efficient bed management ensures timely patient care, reduces waiting times, and optimizes resource utilization. Poor bed management can lead to overcrowding, delays in admission, or underutilized beds, all of which compromise patient outcomes and operational efficiency.

Queuing theory has been widely applied in healthcare to analyze patient flow and resource allocation. Models like the M/M/c/K queuing model are particularly useful for representing hospital wards with limited capacity, where patients may be turned away if no beds are available [1]. Recent studies have also begun to incorporate staffing variables into queuing models to account for the impact of human resources on system performance [3].

The traffic intensity ρ in this system is given by:

$$\rho = \frac{\lambda}{c\mu} \tag{1.2}$$

where λ is the arrival rate, μ is the service rate per server, and c is the number of servers. When $\rho < 1$, the system is stable in the long run.

The probability of having zero patients in the system (P_0) is calculated as:

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!} \frac{1 - \left(\frac{\lambda}{c\mu}\right)^{K-c+1}}{1 - \frac{\lambda}{c\mu}} \right]^{-1} \tag{1.3}$$

Despite these advancements, few studies have comprehensively integrated both structural factors (e.g., bed capacity, patient arrivals) and human factors (e.g., staffing levels, staff experience) into a single framework. This gap highlights the need for a more holistic approach to hospital bed occupancy modeling.

Most existing studies focus primarily on structural aspects like bed numbers and patient arrivals, often ignoring the critical role of staffing decisions. This limitation prevents a complete understanding of how human resources influence bed usage across different scenarios, especially during high-demand periods.

The probability of a patient being rejected due to all beds being occupied can be calculated as:

$$P_{reject} = P_K = \frac{(\lambda/\mu)^K}{c!c^{K-c}} P_0 \tag{1.4}$$

These questions can be analyzed using key performance metrics from the M/M/c/K model. The average waiting time in the queue (W_q) can be expressed as:

$$W_q = \frac{P_c(\lambda/\mu)^c \rho}{c!c\mu(1-\rho)^2} \frac{1 - (K-c+1)(\frac{\lambda}{c\mu})^{K-c} + (K-c)(\frac{\lambda}{c\mu})^{K-c+1}}{(1 - (\frac{\lambda}{c\mu})^{K-c+1})^2} \tag{1.5}$$

where P_c is the probability of having exactly c patients in the system.

The average number of patients in the system (L_s) is given by:

$$L_s = \sum_{n=0}^{K} n P_n \tag{1.6}$$

where P_n is the probability of having n patients in the system.

1.2 Problem Statement

Efficient management of hospital bed occupancy remains a critical challenge in healthcare systems worldwide. Hospitals frequently encounter issues such as overcrowding, where patient demand surpasses available bed capacity leading to extended waiting times or denial of care [1]. Conversely, underutilization of beds results in wasted resources and operational inefficiencies [3].

A particularly significant challenge lies in managing staff-related variables, where variations in staff experience levels and shift patterns directly impact treatment and discharge processes [7]. While existing research has extensively examined structural aspects such as bed numbers and patient arrivals, there is a notable gap in understanding how human resource factors influence bed utilization across different scenarios, especially during high-demand periods [8]. This research aims to address these challenges by investigating how the M/M/c/K queuing model can be adapted to incorporate staffing variables and analyze their impact on patient flow and bed utilization patterns.

The main goal of this study is to develop and apply the M/M/c/K queuing model to analyze hospital bed occupancy, with a focus on how staffing decisions affect patient flow and bed utilization.

1.3 Research Objectives

1.3.1 General Objective

The main goal of this study is to develop and apply the M/M/c/K queuing model to analyze hospital bed occupancy, with a focus on how staffing decisions affect patient flow and bed utilization.

1.3.2 Specific Objectives

1. To model hospital bed occupancy using the M/M/c/K queuing framework, including patient arrival rates and limited bed capacity.

2. To evaluate how different levels of staff experience (junior vs. senior) affect treatment times and bed turnover.

3. To assess the impact of varying staff-to-patient ratios on key performance indicators,including Average waiting time,blocking probability, bed utilization and overall system throughput

1.4 Scope of the Study

This research focuses on developing a comprehensive mathematical model for inpatient departments, specifically general wards, where the dynamic interaction between staff experience, bed utilization, and patient flow presents complex operational challenges. Following [3], the study employs the M/M/c/K queuing framework, which has proven effective in modeling healthcare systems with finite capacity constraints.

The primary scope encompasses patient arrival patterns, modeled as a Poisson process with rate λ, aligning with established healthcare operations research methodologies [1]. Treatment times are represented through an exponential distribution with service rate μ, which varies based on staff performance and experience levels. This approach enables the investigation of how different staff compositions affect system efficiency, addressing our objective of evaluating experience-based impacts on treatment times and bed turnover.

Central to the study's scope is the analysis of staff-related variables and their influence on system performance. The model incorporates three key staffing dimensions: the number of available staff (c), their experience levels (affecting service rate μ), and shift patterns (morning, afternoon, night). This comprehensive approach to staffing variables, supported by [7]'s findings on experience-based performance variations, allows for detailed examination of staff-to-patient ratio impacts on key performance indicators.

System performance is quantified through the utilization factor (U), calculated as:

$$U = \frac{L_s - L_q}{c} \tag{1.7}$$

where L_q, the average queue length, is given by:

$$L_q = \sum_{n=c}^{K} (n - c) P_n \tag{1.8}$$

These metrics provide quantitative measures for evaluating the effectiveness of different staffing configurations and their impact on bed utilization [8].

While comprehensive within its defined boundaries, this study acknowledges several limitations to maintain focus and feasibility. The research does not address external factors such as ambulance dispatch or outpatient services, as these would introduce additional complexities beyond the core objectives. Similarly, inter-hospital transfers, emergency overflow systems, and multi-department patient movements are excluded from the analysis. Staff absenteeism and training programs, while important, are not considered due to data limitations and the need to maintain model tractability. These boundaries ensure that the research remains focused on its primary objectives while maintaining analytical rigor.

1.5 Significance of the Study

This research contributes significantly to both theoretical advancement and practical application in healthcare operations management. The primary significance lies in the development of an enhanced M/M/c/K queuing model that incorporates staffing variables, addressing a critical gap in current healthcare modeling approaches [3]. By integrating staff experience levels and their impact on treatment times, this study provides hospital administrators with evidence-based tools for optimizing staff composition and resource allocation [7].

The research's comprehensive analysis of staff-to-patient ratios and their impact on key performance indicators offers valuable insights for operational decision-making [1]. The model enables healthcare facilities to evaluate staffing policies and predict system performance under various scenarios, potentially reducing waiting times and improving resource utilization. Furthermore, the findings contribute to the broader field of healthcare operations research, providing a scientific basis for staffing regulations and guidelines that can be adapted across different healthcare settings [8].

1.6 Limitations

This study acknowledges several key limitations in its approach and scope. First, the assumption of Markovian arrivals and service times may not fully capture the complexity of real-world patient flow patterns [3]. While this simplification is necessary for mathematical tractability, it may affect the model's accuracy in certain scenarios. Second, external factors such as seasonal variations, emergency situations, and inter-department dependencies are not incorporated into the model to maintain focus on core staffing variables [1].

The study also faces limitations in its treatment of human factors. Staff fatigue, learning curves, and team dynamics, while important in healthcare delivery, are not explicitly modeled due to the complexity of quantifying these variables [7]. Additionally, the research does not address the impact of staff specialization or the potential variations in care quality across different experience levels. These limitations present opportunities for future research to enhance the model's comprehensiveness and practical applicability.

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