Stochastic variability model

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The model assumes the following stochastic differential equation (SDE):

$$dX = \theta(\mu - X)dt + \sigma X dW_t, \tag{0.1}$$

specified by the parameters θ (the inverse of the time scale of the drift term), μ (equilibrium value for X) and σ (coefficient of the stochastic term).

Eq. 0.1 describes a quite simple underlying dynamics: the deterministic (drift) term pushes the system toward an equilibrium value μ, while the evolution is disturbed by a random noise whose amplitude is proportional to the actual value of X. Hence, high states, characterized by large X, will also display the largest fluctuations. See Tavecchio et al. (2020) for the astrophysical justification of this model.

SDE are commonly solved by using standard numerical schemes. For the specific SDE we are considering, the Milstein method is appropriate (see e.g.). Applying this method we obtain the following:

$$X_{i+1} = X_i + \theta(1 - x_i)\Delta t + \sigma W_i X_i \sqrt{\Delta t} + \frac{1}{2}\sigma^2 X_i \Delta t(W_i^2 - 1) \tag{0.2}$$

where W_i is a random variable chosen from a normal distribution with mean 0 and standard deviation 1.

In the plot we report light curves simulated for different set of parameters.

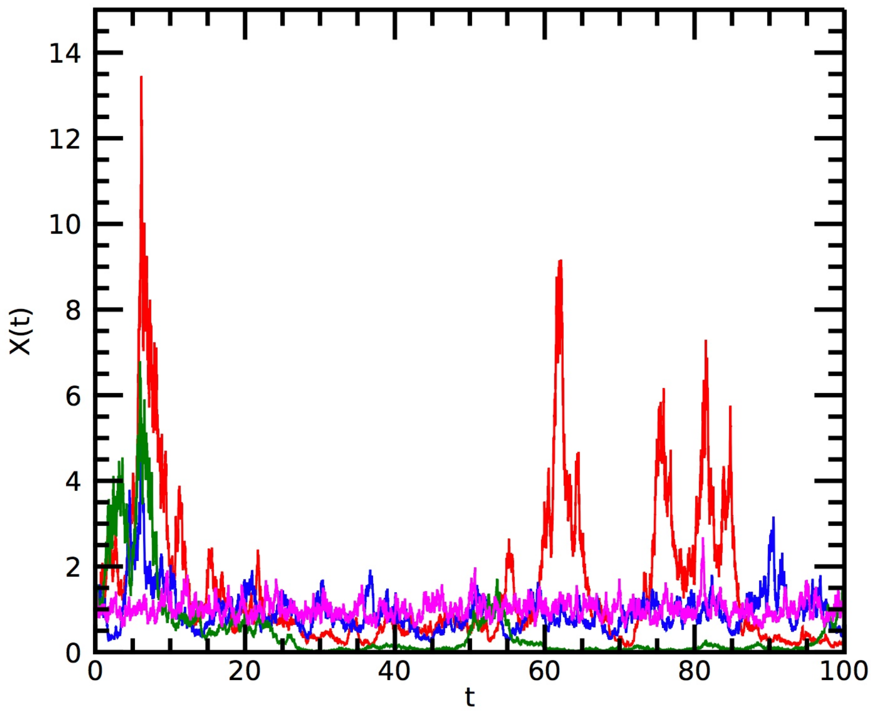

Figure 1: Light curves simulated by numerically solving Eq. 0.1. In all cases we fix $\mu = 1$, $\sigma = 0.5$. The different curves are calculated for $\theta = 0.01, 0.1, 0.5$ and 3 (green, red, blue and magenta). The initial condition is $X_0 = 1$.