Supplement to "Mergers and Acquisitions with Private Equity Intermediation"

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Abstract

In this supplemental appendix, we provides additional proofs and data sources for the main paper. In Section S.1, we prove Part 2 of Proposition 5 in the main paper on the convergence speed of μ in the fast-search market. In Section S.2, we find formulas for two model statistics: the corporations' and PE funds' average time to sell assets and Public Market Equivalent (PME). Section S.3 provides various existing estimates of the PE funds' average time to sell assets.

S.1 Proof for Part 2 of Proposition 5

We divide the proof into two lemmas.

Lemma S.1. *For every $i \in \mathcal{T}$, if $\mu_i^* = 0$, then $\mu_i^{**} \equiv \lim_{\kappa\to\infty} \kappa\mu_i^\kappa$ exists in $\mathbb{R}$.*

(Proof) The following table summarizes the population limits for some types from Lemma 4 and Lemma 5:

	A. $n_a < n_h$	B. $n_h < n_a < n_h + n_f$	C. $n_h + n_f < n_a$
$\mu^*_{ho} =$	n_a	n_h	n_h
$\mu^*_{fo} =$	0	$n_a - n_h$	$< n_f$
$\mu^*_{lo} =$	0	0	$n_a - n_f - n_h$
$\mu^*_{fe} =$	0	0	> 0

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As μ^*_{ho} and μ^*_{ln} are always strictly positive, we consider other types only:

1. Suppose $\mu^*_{hn} = 0$ (Cases A, B and C): for any κ,

$$\kappa(\lambda_c\mu^\kappa_{lo} + \lambda_f\mu^\kappa_{fo} + \lambda_f\mu^\kappa_{fe})\mu^\kappa_{hn} = -\rho_d\mu^\kappa_{hn} + \rho_u\mu^\kappa_{ln}. \quad \text{(from } (\mu\text{-hn}))$$

By Lemma 1, $\mu^\kappa_i > 0$ for every $i \in \mathcal{T}$,

$$\kappa\mu^\kappa_{hn} = \frac{\rho_u\mu^\kappa_{ln} - \rho_d\mu^\kappa_{hn}}{\lambda_c\mu^\kappa_{lo} + \lambda_f\mu^\kappa_{fo} + \lambda_f\mu^\kappa_{fe}}. \tag{S.1}$$

It follows that

$$\mu^{**}_{hn} \equiv \lim_{\kappa\to\infty}\kappa\mu^\kappa_{hn} = \frac{\rho_u\mu^*_{ln} - \rho_d\mu^*_{hn}}{\lambda_c\mu^*_{lo} + \lambda_f(\mu^*_{fo} + \mu^*_{fe})} = \frac{\rho_u\mu^*_{ln}}{\lambda_c\mu^*_{lo} + \lambda_f(\mu^*_{fo} + \mu^*_{fe})} > 0. \tag{S.2}$$

2. Suppose $\mu^*_{lo} = 0$ (Cases A and B): for every κ,

$$(\lambda_c\mu^\kappa_{hn} + \lambda_f\mu^\kappa_{fn})(\kappa\mu^\kappa_{lo}) = \rho_d\mu^\kappa_{ho} - \rho_u\mu^\kappa_{lo}. \quad \text{(from } (\mu\text{-lo}))$$

It follows that

$$\mu^{**}_{lo} \equiv \lim_{\kappa\to\infty}\kappa\mu^\kappa_{lo} = \frac{\rho_d\mu^*_{ho} - \rho_u\mu^*_{lo}}{\lambda_c\mu^*_{hn} + \lambda_f\mu^*_{fn}} = \frac{\rho_d\mu^*_{ho}}{\lambda_c\mu^*_{hn} + \lambda_f\mu^*_{fn}}.$$

3. Suppose $\mu^*_{fe} = 0$ (Cases A and B): for every κ,

$$(\lambda_f\mu^\kappa_{hn} + \lambda_s\mu^\kappa_{fn})(\kappa\mu^\kappa_{fe}) = \rho_e\mu^\kappa_{fo}. \quad \text{(from } (\mu\text{-fe}))$$

It follows that

$$\mu^{**}_{fe} \equiv \lim_{\kappa\to\infty}\kappa\mu^\kappa_{fe} = \frac{\rho_e\mu^*_{fo}}{\lambda_f\mu^*_{hn} + \lambda_s\mu^*_{fn}}.$$

4. Suppose $\mu^*_{fo} = 0$ (Case A): for every κ,

$$\kappa(\lambda_f\mu^\kappa_{lo} + \lambda_s\mu^\kappa_{fe})\mu^\kappa_{fn} = \kappa\lambda_f\mu^\kappa_{hn}\mu^\kappa_{fo} + \rho_e\mu^\kappa_{fo}. \quad \text{(from } (\mu\text{-fo}))$$

It follows from the convergence of $\kappa\mu^\kappa_{lo}$ and $\kappa\mu^\kappa_{fe}$ in Case A that

$$\mu^{**}_{fo} \equiv \lim_{\kappa\to\infty}\kappa\mu^\kappa_{fo} = \lim_{\kappa\to\infty}\frac{(\lambda_f(\kappa\mu^\kappa_{lo}) + \lambda_s(\kappa\mu^\kappa_{fe}))\mu^\kappa_{fn} - \rho_e\mu^\kappa_{fo}}{\lambda_f\mu^\kappa_{hn}} = \frac{(\lambda_f\mu^{**}_{lo} + \lambda_s\mu^{**}_{fe})\mu^*_{fn}}{\lambda_f\mu^*_{hn}}.$$

5. Suppose $\mu^*_{fn} = 0$ (Case C): for every κ,

$$\mu^\kappa_{hn}\mu^\kappa_{fo} + \mu^\kappa_{hn}\mu^\kappa_{fe} = \mu^\kappa_{lo}\mu^\kappa_{fn}. \quad \text{(from } (\mu\text{-fn}))$$

As $\mu^\kappa_{lo} > 0$ (Lemma 1), we have

$$\kappa\mu^\kappa_{fn} = \frac{\kappa\mu^\kappa_{hn}(\mu^\kappa_{fo} + \mu^\kappa_{fe})}{\mu^\kappa_{lo}}. \tag{S.3}$$

It follows from the convergences of $\kappa\mu^\kappa_{hn}$ that

$$\mu^{**}_{fn} \equiv \lim_{\kappa\to\infty} \kappa\mu^\kappa_{fn} = \lim_{\kappa\to\infty} \frac{\kappa\mu^\kappa_{hn}(\mu^\kappa_{fo} + \mu^\kappa_{fe})}{\mu^\kappa_{lo}} = \frac{\mu^{**}_{hn}(\mu^*_{fo} + \mu^*_{fe})}{\mu^*_{lo}} > 0. \tag{S.4}$$

■

Lemma S.2. *For any $i \in \mathcal{T}$, if $\mu^*_i > 0$, then $\mu^{**}_i \equiv \lim_{\kappa\to\infty} \kappa(\mu^\kappa_i - \mu^*_i)$ exists in $\mathbb{R}$.*

(Proof)

- Case A ($n_a < n_h$): Only $\mu^*_{ho}, \mu^*_{fn}, \mu^*_{ln}$ are strictly positive. As $\kappa \to \infty$,

  $$\kappa(\mu^\kappa_{ho} - \mu^*_{ho}) = \kappa(n_a - \mu^\kappa_{lo} - \mu^\kappa_{fo} - \mu^\kappa_{fe}) - \kappa(n_a - \mu^*_{lo} - \mu^*_{fo} - \mu^*_{fe}) \to -\mu^{**}_{lo} - \mu^{**}_{fo} - \mu^{**}_{fe},$$

  where the convergence of $\kappa\mu^\kappa_{lo}$, $\kappa\mu^\kappa_{fo}$, and $\kappa\mu^\kappa_{fe}$ holds by Lemma S.1.

  We similarly find the convergence speed for μ^κ_{fn} and μ^κ_{ln}:

  $$\begin{aligned} \kappa(\mu^\kappa_{fn} - \mu^*_{fn}) &= \kappa(n_f - \mu^\kappa_{fo} - \mu^\kappa_{fe}) - \kappa(n_f - \mu^*_{fo} - \mu^*_{fe}) \to -\mu^{**}_{fo} - \mu^{**}_{fe}, \quad \text{and} \\ \kappa(\mu^\kappa_{ln} - \mu^*_{ln}) &= \kappa(n_l - \mu^\kappa_{ln}) - \kappa(n_l - \mu^*_{lo}) \to -\mu^{**}_{lo}. \end{aligned}$$

- Case B ($n_h < n_a < n_h + n_f$):

Only $\mu^*_{ho}, \mu^*_{fo}, \mu^*_{ln}$, and μ^*_{fn} are strictly positive. As $\kappa \to \infty$,

$$\begin{aligned}
\kappa(\mu^\kappa_{ho} - \mu^*_{ho}) &= \kappa(n_h - \mu^\kappa_{hn}) - \kappa(n_h - \mu^*_{hn}) \to -\mu^{**}_{hn}, \\
\kappa(\mu^\kappa_{ln} - \mu^*_{ln}) &= \kappa(n_l - \mu^\kappa_{lo}) - \kappa(n_l - \mu^*_{lo}) \to -\mu^{**}_{lo}, \\
\kappa(\mu^\kappa_{fo} - \mu^*_{fo}) &= \kappa(\mu^\kappa_{fo} - (n_a - n_h)) = -\kappa\mu^\kappa_{lo} - \kappa\mu^\kappa_{fe} - \kappa(\mu^\kappa_{ho} - n_h) \\
&\to -\mu^{**}_{lo} - \mu^{**}_{fe} + \mu^{**}_{hn}, \quad \text{and} \\
\kappa(\mu^\kappa_{fn} - \mu^*_{fn}) &= \kappa(\mu^\kappa_{fn} - (n_f - n_a + n_h)) = -\kappa\mu^\kappa_{fe} - \kappa(\mu^\kappa_{fo} - (n_a - n_h)) \\
&\to -\mu^{**}_{fe} + (\mu^{**}_{lo} + \mu^{**}_{fe} - \mu^{**}_{hn}).
\end{aligned}$$

- Case C ($n_h + n_f < n_a$): We have $\mu^*_{ho}, \mu^*_{lo}, \mu^*_{ln}, \mu^*_{fo}$, and μ^*_{fe} that are strictly positive. The proof for the first three types are similar to the previous cases: as $\kappa \to \infty$,

$$\begin{aligned}
\kappa(\mu^\kappa_{ho} - \mu^*_{ho}) &= \kappa(n_h - \mu^\kappa_{hn}) - \kappa(n_h - \mu^*_{hn}) \to -\mu^{**}_{hn}, \\
\kappa(\mu^\kappa_{lo} - \mu^*_{lo}) &= \kappa(\mu^\kappa_{lo} - (n_a - n_h - n_f)) = -\kappa(\mu^\kappa_{fo} + \mu^\kappa_{fe} - n_f) - \kappa(\mu^\kappa_{ho} - n_h) \\
&\to -\mu^{**}_{fn} + \mu^{**}_{hn}, \qquad \text{(S.5)} \\
\kappa(\mu^\kappa_{ln} - \mu^*_{ln}) &= \kappa(n_l - \mu^\kappa_{lo}) - \kappa(n_l - \mu^*_{lo}) \to -\mu^{**}_{lo} = \mu^{**}_{fn} - \mu^{**}_{hn}. \qquad \text{(S.6)}
\end{aligned}$$

It remains to show the convergence speed for μ^κ_{fo} and μ^κ_{fe}. On the one hand, from (μ-fe) and the convergence of μ^κ_{fe}, μ^κ_{fo}, $\kappa\mu^\kappa_{hn}$, and $\kappa\mu^\kappa_{fn}$, we have

$$\kappa(\lambda_f\mu^\kappa_{hn} + \lambda_s\mu^\kappa_{fn})\mu^\kappa_{fe} = \rho_e\mu^\kappa_{fo} \quad \text{and} \quad (\lambda_f\mu^{**}_{hn} + \lambda_s\mu^{**}_{fn})\mu^*_{fe} = \rho_e\mu^*_{fo}.$$

Let

$$\phi^\kappa \equiv \kappa(\lambda_f\mu^\kappa_{hn} + \lambda_s\mu^\kappa_{fn}), \quad \text{and} \quad \phi^{**} \equiv \lambda_f\mu^{**}_{hn} + \lambda_s\mu^{**}_{fn}.$$

Then,

$$\rho_e\kappa(\mu^\kappa_{fo} - \mu^*_{fo}) = \phi^\kappa\kappa\mu^\kappa_{fe} - \phi^{**}\kappa\mu^*_{fe} = \kappa(\phi^\kappa - \phi^{**})\mu^\kappa_{fe} + \phi^{**}\kappa(\mu^\kappa_{fe} - \mu^*_{fe}). \tag{S.7}$$

On the other hand, from $\mu^\kappa_{fn} + \mu^\kappa_{fo} + \mu^\kappa_{fe} = n_f$ and $\mu^*_{fo} + \mu^*_{fe} = n_f$, we have

$$\kappa(\mu^\kappa_{fo} - \mu^*_{fo}) + \kappa(\mu^\kappa_{fe} - \mu^*_{fe}) = -\kappa\mu^\kappa_{fn}. \tag{S.8}$$

By summarizing (S.7) and (S.8), for every κ,

$$\begin{bmatrix} \kappa(\mu^\kappa_{fo} - \mu^*_{fo}) \\ \kappa(\mu^\kappa_{fe} - \mu^*_{fe}) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \rho_e & -\phi^{**} \end{bmatrix}^{-1} \begin{bmatrix} -\kappa\mu^\kappa_{fn} \\ \kappa(\phi^\kappa - \phi^{**})\mu^\kappa_{fe} \end{bmatrix},$$

where the inverse matrix is well-defined because $\phi^{**} > 0$ (see (S.2) and (S.4)). Note that $\kappa\mu^\kappa_{fn}$ and μ^κ_{fe} converge (see (S.4) and Lemma 5). It remains to prove that

$$\kappa(\phi^\kappa - \phi^{**}) = \lambda_f \kappa(\kappa\mu^\kappa_{hn} - \mu^{**}_{hn}) + \lambda_s \kappa(\kappa\mu^\kappa_{fn} - \mu^{**}_{fn}) \quad \text{converges as } \kappa \to \infty.$$

First, from (S.1), (S.2), $\mu^*_{hn} = 0$, and $\mu^*_{fo} + \mu^*_{fe} = n_f$,[1] we have

$$\kappa(\kappa\mu^\kappa_{hn} - \mu^{**}_{hn}) = \frac{\rho_u \kappa\mu^\kappa_{ln} - \rho_d \kappa\mu^\kappa_{hn}}{\lambda_c \mu^\kappa_{lo} + \lambda_f(\mu^\kappa_{fo} + \mu^\kappa_{fe})} - \frac{\rho_u(\kappa\mu^*_{ln})}{\lambda_c \mu^*_{lo} + \lambda_f n_f}.$$

To ease expositions, let A^κ and A^* denote the denominators in the above equation. Then,

$$\begin{aligned} \kappa(\kappa\mu^\kappa_{hn} - \mu^{**}_{hn}) &= \frac{\rho_u \kappa\mu^\kappa_{ln} - \rho_d \kappa\mu^\kappa_{hn}}{A^\kappa} - \frac{\rho_u \kappa\mu^*_{ln}}{A^*} \\ &= \frac{\rho_u \kappa(\mu^\kappa_{ln} - \mu^*_{ln}) - \rho_d \kappa\mu^\kappa_{hn}}{A^\kappa} + \rho_u \mu^*_{ln} \kappa \left(\frac{1}{A^\kappa} - \frac{1}{A^*} \right) \\ &= \frac{\rho_u \kappa(\mu^\kappa_{ln} - \mu^*_{ln}) - \rho_d \kappa\mu^\kappa_{hn}}{A^\kappa} - \rho_u \mu^*_{ln} \frac{\lambda_c \kappa(\mu^\kappa_{lo} - \mu^*_{lo}) - \lambda_f \kappa\mu^\kappa_{fn}}{A^\kappa A^*}, \end{aligned} \tag{S.9}$$

which converges by (S.2), (S.4), (S.5), and (S.6).

Second, from (S.3), (S.4), and $\mu^*_{fo} + \mu^*_{fe} = n_f$, we have

$$\kappa(\kappa\mu^\kappa_{fn} - \mu^{**}_{fn}) = \frac{\kappa\mu^\kappa_{hn} \kappa(\mu^\kappa_{fo} + \mu^\kappa_{fe})}{\mu^\kappa_{lo}} - \frac{\mu^{**}_{hn} \kappa n_f}{\mu^*_{lo}}.$$

[1] Recall that we are considering Case C ($n_h + n_f < n_a$).

Then,

$$\begin{aligned}
\kappa(\kappa\mu^{\kappa}_{fn}-\mu^{**}_{fn}) &= \frac{\kappa\mu^{\kappa}_{hn}\kappa(\mu^{\kappa}_{fo}+\mu^{\kappa}_{fe}-n_f)}{\mu^{\kappa}_{lo}}+\frac{\kappa^2\mu^{\kappa}_{hn}n_f}{\mu^{\kappa}_{lo}}-\frac{\kappa\mu^{**}_{hn}n_f}{\mu^{*}_{lo}}\\
&= -\frac{(\kappa\mu^{\kappa}_{hn})(\kappa\mu^{\kappa}_{fn})}{\mu^{\kappa}_{lo}}+\frac{\kappa(\kappa\mu^{\kappa}_{hn}-\mu^{**}_{hn})n_f}{\mu^{\kappa}_{lo}}+\frac{\kappa\mu^{**}_{hn}n_f}{\mu^{\kappa}_{lo}}-\frac{\kappa\mu^{**}_{hn}n_f}{\mu^{*}_{lo}}\\
&= -\frac{(\kappa\mu^{\kappa}_{hn})(\kappa\mu^{\kappa}_{fn})}{\mu^{\kappa}_{lo}}+\frac{\kappa(\kappa\mu^{\kappa}_{hn}-\mu^{**}_{hn})n_f}{\mu^{\kappa}_{lo}}-\frac{\mu^{**}_{hn}n_f\kappa(\mu^{\kappa}_{lo}-\mu^{*}_{lo})}{\mu^{\kappa}_{lo}\mu^{*}_{lo}},
\end{aligned}$$

which converges by (S.2), (S.4),(S.5), and (S.9). ■

S.2 Model Statistics

We provide closed-form expressions of model statistics: the corporations' and PE funds' average time to sell assets and the PME.

S.2.1 Average time to sell

First, consider the path of a lo-type corporation in a steady-state equilibrium. This corporation can sell its asset upon meeting either a corporate buyer (hn) or a fund buyer (fn). Each kind of meeting arrives with Poisson rate $\lambda_c\mu_{hn}$ or $\lambda_f\mu_{fn}$. The time until the first meeting of each kind, denoted by τ_{lo-hn} and τ_{lo-fn}, follows the exponential distributions. Thus, the time until selling $\tau_{sc}\equiv\min\{\tau_{lo-hn},\tau_{lo-fn}\}$ follows an exponential distribution with parameter $\lambda_c\mu_{hn}+\lambda_f\mu_{fn}$:

$$E[\tau_{sc}]=\frac{1}{\lambda_c\mu_{hn}+\lambda_f\mu_{fn}}.$$

Second, consider the path of a fo-type PE fund in a steady-state equilibrium. The fund sells its asset before receiving a liquidity shock to a corporate buyer (hn), or receives a liquidity shock and enters the exit phase (after which it can sell to either a corporate buyer (hn) or a fund buyer (fn)). We denote by τ_{fo} this period for which a fund maintain its type as fo . The time τ_{fo} follows an exponential distribution with parameter $\lambda_f\mu_{hn}+\rho_e$:

$$E[\tau_{fo}]=\frac{1}{\lambda_f\mu_{hn}+\rho_e}.$$

Finally, we evaluate the path of an fe type fund (an outcome of an fo type fund receiving a liquidity shock before meeting a corporate buyer with probability $\frac{\rho_e}{\lambda_f\mu_{hn}+\rho_e}$). The fe type fund maintains its type until it sells its portfolio asset either to a corporate buyer (hn) or a fund buyer (fn). Thus, the fund maintains its type for the time period τ_{fe}, which follows an exponential distribution with parameter $\lambda_f \mu_{hn} + \lambda_s \mu_{fn}$:

$$E[\tau_{fe}] = \frac{1}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}}.$$

As a result, the overall expected time for a PE fund to sell an asset is:

$$E[\tau_{sf}] = \frac{1}{\lambda_f \mu_{hn} + \rho_e} + \frac{\rho_e}{\lambda_f \mu_{hn} + \rho_e} \left(\frac{1}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}} \right).$$

S.2.2 Public Market Equivalent (PME)

Consider a PE fund that does not hold an asset in a steady-state equilibrium. The fund takes τ_b period of time until purchasing an asset at a price of P_b and takes τ_s period of time (after purchasing) until selling the asset at a price P_s. Let $u(t) \in \{u_f, u_e\}$ denote the payoff flow while holding the asset at $t \in [0, \tau_s]$.

We modify Sorensen and Jagannathan (2015)'s definition of PME in discrete time with a stochastic discount. For our case of continuous time and deterministic discount, we define PME as

$$PME \equiv \frac{\text{Present value of distributions to fund investors}}{\text{Present value of capital calls made by fund investors}} = \frac{PV_{\text{dist}}}{PV_{\text{calls}}},$$

where

$$\begin{aligned} PV_{\text{dist}} &\equiv E\left[e^{-r\tau_b} \int_0^{\tau_s} e^{-rt} u(t) dt + e^{-r\tau_s} P_s\right], \\ PV_{\text{calls}} &\equiv PV_{\text{purchasing price}} + PV_{\text{management fees}} = E\left[P_b e^{-r\tau_b}\right] + E\left[(f P_b) \int_0^{\tau_b + \tau_s} e^{-rt} dt\right]. \end{aligned}$$

The management fees are paid retrospectively, as if the flow of fees which equals a fraction of the fund size (i.e., fP_b) is paid throughout the fund's lifetime. For calibration, we set $f \simeq 2\%$ based on Metrick and Yasuda (2010), which finds that management fees are usually 2% of committed capital and paid from the inception of a fund until its liquidation. For discount rate r, Kaplan and Schoar (2005) use the return on the S&P 500, whereas we use our estimate of the same.

First, we obtain the closed-form expression of PV_{dist}. Since the time to purchase, τ_b , is independent of the time to sell τ_s (post-purchase) and the selling price P_s,

$$PV_{\text{dist}} = E\left[e^{-r\tau_b}\right] E\left[\int_0^{\tau_s} e^{-rt}u(t)dt + e^{-r\tau_s}P_s\right].$$

A purchase of an asset occurs on meeting a corporate seller or a fund at the exit phase, whichever happens first ($\tau_b \equiv \min\{\tau_{lo-fn}, \tau_{fe-fn}\}$). τ_b follows an exponential distribution with parameter $\lambda_f \mu_{lo} + \lambda_s \mu_{fe}$. As such,

$$E\left[e^{-r\tau_b}\right] = \frac{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r}.^{2}$$

The fund can sell either (i) before receiving a liquidity shock to a corporate buyer, or (ii) after receiving a liquidity shock to either a corporate buyer or a fund buyer. The expected continuation payoff, upon receiving a liquidity shock before selling an asset, is

$$V_e \equiv E\left[u_e\left(\int_0^{\tau_e} e^{-rt}dt\right) + e^{-r\tau_e}P_e\right],$$

where τ_e denotes the time that the fund remains as type fe, and P_e denotes the selling price. Note that $\tau_e \equiv \min\{\tau_{fe-hn}, \tau_{fe-fn}\}$ follows an exponential distribution with parameter $\lambda_f \mu_{hn} + \lambda_s \mu_{fn}$. The probability of selling to a corporate buyer $\frac{\lambda_f \mu_{hn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}}$ is independent of the selling time τ_e. Thus

$$\begin{aligned} V_e &= \frac{u_e}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} + \frac{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} \frac{\lambda_f \mu_{hn} P_{fe-hn} + \lambda_s \mu_{fn} P_{fe-fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn}} \\ &= \frac{u_e + \lambda_f \mu_{hn} P_{fe-hn} + \lambda_s \mu_{fn} P_{fe-fn}}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r}. \end{aligned}$$

Similarly, an fo type fund receives a payoff flow u_f during a lifetime spanning $\tau_{fo} \equiv \min\{\tau_{fo-hn}, \tau_e\}$. Eventually, the fund either sells its asset to a corporate buyer at price P_{fo-hn} or receives a liquidity shock and a continuation payoff V_e. Thus,

$$E\left[\int_0^{\tau_s} e^{-rt}u(t)dt + e^{-r\tau_s}P_s\right] = \frac{u_f + \lambda_f \mu_{hn} P_{fo-hn} + \rho_e V_e}{\lambda_f \mu_{hn} + \rho_e + r}.$$

[2] We use (i) $\int_0^{\bar{t}} e^{-rt}dt = -\left.\frac{e^{-rt}}{r}\right|_0^{\bar{t}} = \frac{1-e^{-r\bar{t}}}{r}$, (ii) for $x \sim \exp(\alpha)$, $E[e^{-rx}] = \int_0^{\infty} e^{-rx}\alpha e^{-\alpha x}dx = \frac{\alpha}{\alpha+r}$, and (iii) for $x \sim \exp(\alpha)$, $\int_0^{x} e^{-rt}dt = E\left[\frac{1-e^{-rx}}{r}\right] = \frac{1}{\alpha+r}$.

It follows that

$$PV_{\text{dist}} = \left(\frac{\lambda_f\mu_{lo} + \lambda_s\mu_{fe}}{\lambda_f\mu_{lo} + \lambda_s\mu_{fe} + r}\right)\left(\frac{u_f + \lambda_f\mu_{hn}P_{fo-hn} + \rho_e\left(\frac{u_e+\lambda_f\mu_{hn}P_{fe-hn}+\lambda_s\mu_{fn}P_{fe-fn}}{\lambda_f\mu_{hn}+\lambda_s\mu_{fn}+r}\right)}{\lambda_f\mu_{hn} + \rho_e + r}\right).$$

Second, we find the closed-form expression of PV_{calls}. The time taken to buy τ_b, the time taken to sell τ_s, and the event of purchasing from a low-type corporation, rather than an exiting fund, are all independent from each other. Thus,

$$PV_{\text{calls}} = E\left[P_b\right]E\left[e^{-r\tau_b}\right] + E\left[(fP_b)\right]E\left[\int_0^{\tau_b+\tau_s} e^{-rt}dt\right],$$

where

$$\begin{aligned}
E\left[\int_0^{\tau_b+\tau_s} e^{-rt}dt\right] &= E\left[\int_0^{\tau_b} e^{-rt}dt\right] + E\left[\int_{\tau_b}^{\tau_b+\tau_s} e^{-rt}dt\right] \\
&= E\left[\int_0^{\tau_b} e^{-rt}dt\right] + E\left[e^{-r\tau_b}\right]E\left[\int_0^{\tau_s} e^{-rt}dt\right].
\end{aligned}$$

Note that

$$\begin{aligned}
E\left[P_b\right] &= \frac{\lambda_f\mu_{lo}P_{lo-fn} + \lambda_s\mu_{fe}P_{fe-fn}}{\lambda_f\mu_{lo} + \lambda_s\mu_{fe}}, \\
E\left[e^{-r\tau_b}\right] &= \frac{\lambda_f\mu_{lo} + \lambda_s\mu_{fe}}{\lambda_f\mu_{lo} + \lambda_s\mu_{fe} + r}, \quad \text{and} \\
E\left[\int_0^{\tau_b} e^{-rt}dt\right] &= \frac{1}{\lambda_f\mu_{lo} + \lambda_s\mu_{fe} + r}.
\end{aligned}$$

Last, recall that a fund's type remains fo or fe for the time period $\tau_{fo} \equiv \min\{\tau_{fo-hn}, \tau_e\}$ or $\tau_e \equiv \min\{\tau_{fe-hn}, \tau_{fe-fn}\}$, respectively. Then,

$$\begin{aligned}
E\left[\int_0^{\tau_s} e^{-rt}dt\right] &= E\left[\int_0^{\tau_{fo}} e^{-rt}dt\right] + E\left[\mathbf{1}_{\tau_{fo}=\tau_e}\right]E\left[e^{-r\tau_{fo}}\right]E\left[\int_0^{\tau_e} e^{-rt}dt\right] \\
&= \frac{1}{\lambda_f\mu_{hn} + \rho_e + r} + \frac{\rho_e}{\lambda_f\mu_{hn} + \rho_e}\frac{\lambda_f\mu_{hn} + \rho_e}{\lambda_f\mu_{hn} + \rho_e + r}\frac{1}{\lambda_f\mu_{hn} + \lambda_s\mu_{fn} + r}.
\end{aligned}$$

It follows that

$$PV_{\text{calls}} = \frac{\lambda_f \mu_{lo} P_{lo-fn} + \lambda_s \mu_{fe} P_{fe-fn}}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe} + r} \left(1 + f \left(\frac{1}{\lambda_f \mu_{lo} + \lambda_s \mu_{fe}} + \frac{1 + \rho_e \left(\frac{1}{\lambda_f \mu_{hn} + \lambda_s \mu_{fn} + r} \right)}{\lambda_f \mu_{hn} + \rho_e + r} \right) \right).$$

S.3 Various Estimates of the Time to Sell

A sale of a private firm consists of two major processes: the preparation and the listing-to-sale process. The preparation takes less time if a firm already has high-quality accounting and information systems, which is the case of PE-backed firms (Kaplan and Stromberg (2009)). The preparation for PE-backed firms takes an average of 2 months, while other firms need an average of 6 months (see the upper part of Table S.1). The listing-to-sale process takes about 9 months for various selling agents (see the lower part of Table S.1). We set the total time for selling a firm as 11 months for PE funds and 15 months for corporations.

Ave. Time Taken	Source
For preparations	
1-6 months	https://www.highrockpartners.com/how-long-does-it-take-to-sell-a-company/
12 months	https://www.businessinsider.com/11-stages-of-selling-a-company-2011-4
From listing to sale	
6-9 months	https://www.mabusinessbrokers.com/blog/how-long-does-it-take-to-sell-a-business
9 months	https://www.exitadviser.com/seller-status.aspx?id=long-does-take-sell
9 months	https://www.allbusiness.com/how-long-does-it-take-to-sell-a-business-2-6592268-1.html
12 months	https://www.businessinsider.com/11-stages-of-selling-a-company-2011-4
9 months	https://www.moorestephens.co.uk/msuk/moore-stephens-south/news/july-2017-(1)/how-long-does-it-take-to-sell-a-small-business
9 months	https://www.tvba.co.uk/article/how-long-does-it-take-to-sell-a-company
6-9 months	https://www.simonscottcmc.co.uk/blog/long-take-sell-business/
10 months	https://www.ibgbusiness.com/tips-sell-business-long-take-sell-business/
10 months	https://www.highrockpartners.com/how-long-does-it-take-to-sell-a-company/

Table S.1: Estimated time to sell a firm

References

Kaplan, S. N., and A. Schoar (2005): "Private equity performance: Returns, persistence, and capital flows," *The Journal of Finance*, 60(4), 1791–1823.

Kaplan, S. N., and P. Stromberg (2009): "Leveraged buyouts and private equity," *Journal of Economic Perspectives*, 23(1), 121–46.

Metrick, A., and A. Yasuda (2010): "The economics of private equity funds," *The Review of Financial Studies*, 23(6), 2303–2341.

Sorensen, M., and R. Jagannathan (2015): "The public market equivalent and private equity performance," *Financial Analysts Journal*, 71(4), 43–50.