### Medium Range Projectile Motion Trials

Mean Range

$$R_{avg} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$R_{avg} = \frac{1}{2} \sum_{i=1}^{2} x_i = \frac{1}{2} * (147.61 \text{ cm} + 148.81 \text{ cm})$$

$$R_{avg} = 148.21 \text{ cm}$$

Range Uncertainty

$$\delta R = \frac{\sigma}{\sqrt{N}} \qquad \sigma = \sqrt{\frac{\sum_{i=1}^{N} (x - x_i)^2}{N - 1}} = \sqrt{\frac{(148.21 \text{ cm} - 147.61 \text{cm})^2 + (148.21 \text{ cm} - 148.81 \text{cm})^2}{2 - 1}} = 0.84852 \text{ cm}$$

$$\delta R = \frac{0.84852 \, cm}{\sqrt{2}}$$

$$\delta R = 0.60000 \text{ cm}$$

**Initial Speed** 

$$v_0 = \frac{R_{avg}}{\sqrt{\frac{2\Delta y}{a}}}$$

$$v_0 = \frac{148.21 \, cm}{\sqrt{\frac{2(83.50 \, cm)}{980 \, cm/s^2}}}$$

$$v_0 = 359 \text{ cm/s}$$

**Initial Speed Uncertainty** 

$$\delta v_0 = f(x,y)^* \sqrt{(\frac{n^* \delta x}{x})^2 + (\frac{m^* \delta y}{y})^2}$$

$$\delta v_0 = 359 \text{ cm/s} * \sqrt{\left(\frac{1*0.60000 \text{ cm}}{148.21 \text{ cm}}\right)^2 + \left(\frac{\frac{1}{2}*0.05}{0.413}\right)^2}$$

$$\delta v_0 = 21.8 \text{ cm/s}$$

#### Medium Range Pendulum Arm with No Masses Attached Trials

**Initial Kinetic Energy** 

$$K_0 = \frac{1}{2} \mathbf{m} v_0^2$$

$$K_0 = \frac{1}{2} * 0.207 \text{ kg } * (3.59 \text{ m/s})^2$$

$$K_0 = 0.372 \,\mathrm{J}$$

Initial Kinetic Energy Uncertainty

$$\delta K_0 = f(x,y) * \sqrt{\left(\frac{n*\delta x}{x}\right)^2 + \left(\frac{m*\delta y}{y}\right)^2}$$

$$\delta K_0 = 0.372 \,\mathrm{J} * \sqrt{\left(\frac{1*0.0001 \,kg}{0.207 \,kg}\right)^2 + \left(\frac{2*0.218 \,m/s}{3.59 \,m/s}\right)^2}$$

$$\delta K_0 = 0.0452 \,\mathrm{J}$$

Final Gravitational Potential Energy

$$U_g = \text{mgy}$$
 
$$U_g = 0.2069 \text{ kg} * 9.80 \text{ m/s}^2 * (0.271 \text{ m} * \sin(44.9^\circ))$$
 
$$U_g = 0.489 \text{ J}$$

Final Gravitational Potential Energy Uncertainty

$$\delta U_g = \sqrt{\left(\frac{\partial f(x,y)}{\partial x}\right)^2 \left(\delta x\right)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \left(\delta y\right)^2}$$

$$\delta U_g = \sqrt{\left(\frac{0.489 J}{0.2069 \, kg}\right)^2 \left(0.0001 \, kg\right)^2 + \left(\frac{0.489 J}{0.191 \, m}\right)^2 \left(0.0005 \, m\right)^2}$$

$$\delta U_g = 0.00130 \, J$$

**Energy Percent Difference** 

$$PD = \left| \frac{E_1 - E_2}{\frac{E_1 + E_2}{2}} \right| *100\%$$

$$PD = \left| \frac{K_0 - U_g}{\frac{K_0 + U_g}{2}} \right| *100\%$$

$$PD = \left| \frac{0.372J - 0.489J}{\frac{0.372J + 0.489J}{2}} \right| *100\% = 27.2\%$$

## **TABLE OF RESULTS**

PROJECTILE LAUNCHER SETTING	MEAN RANGE $R_{avg}\pm\delta R$ $(cm)$	INITIAL SPEED $v_0 \pm \delta v_0$ $\left(\frac{m}{s}\right)$	
MEDIUM	148.21±0.60000	3.59±0.218	
LONG	201.66 <u>±</u> 0.28008	4.89 <u>±</u> 0.295	

		CHANGE IN ENERGY		
ATTACHED BRASS MASSES	PROJECTILE LAUNCHER SETTING	KINETIC ENERGY $ K_0 \pm \delta K_0 $ $ (J) $	GRAVIATIONAL POTENTIAL ENERGY $U_g \pm \delta U_g$ $(J)$	PERCENT DIFFERENCE PD (%)
NONE	MEDIUM	0.372±0.0452	0.489± 0.00130	27.2
	LONG	0.506 <u>±</u> 0.0611	0.490 <u>±</u> 0.00104	3.21
вотн	MEDIUM	0.562 ± 0.0683	0.397±0.00154	34.4
	LONG	0.766 <u>±</u> 0.0924	0.533±0.00154	35.9