Homework 1

Fam Cham An
c.fam@innopolis.university

26th September 2020

Description of KUKA KR 10 R1100-2 and kinematic scheme

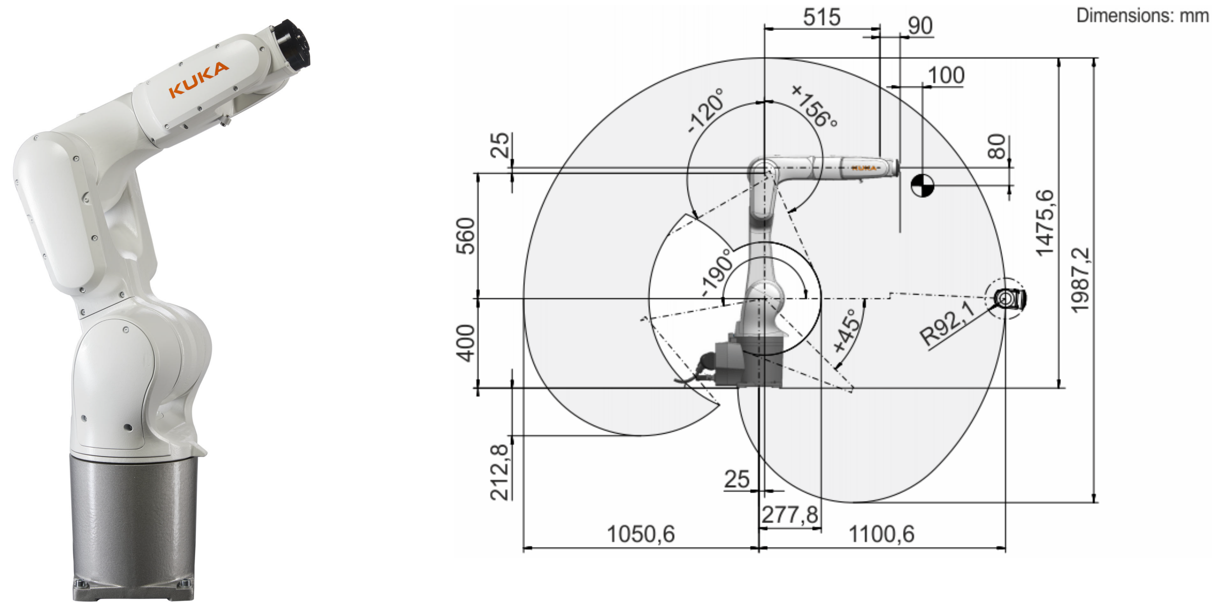

Figure 1: Description of KUKA KR 10 R1100-2

Robot's parameters: $L_1 = 400, L_2 = 560, L_3 = 515, L_4 = 90$.
We have 6 revolute joints with following limits:
J1 = [-170°; +170°]
J2 = [190°; 45°]
J3 = [-120°; 156°]
J4 = [-185°; 185°]
J5 = [-120°; 120°]
J6 = [-350°; 350°]

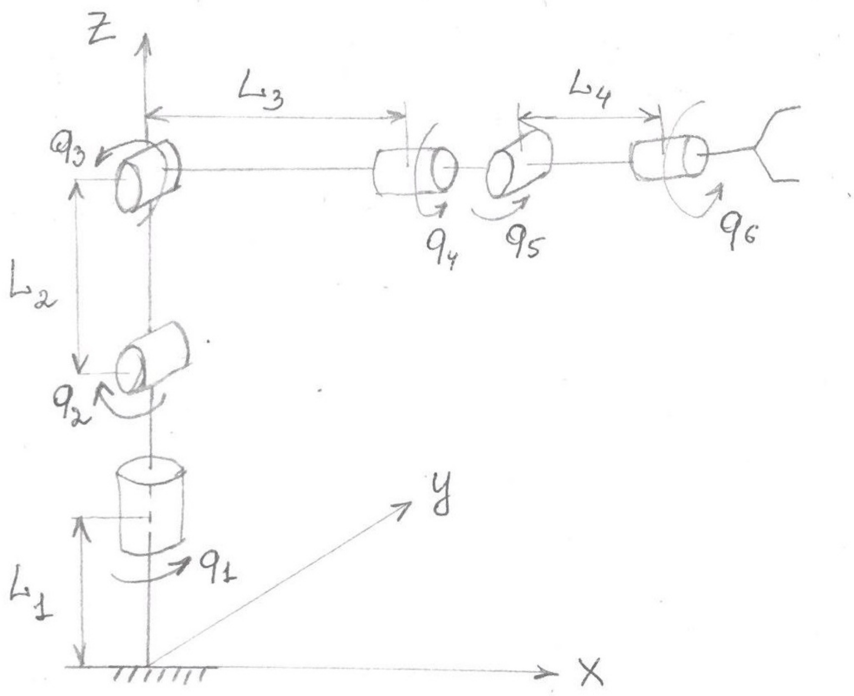

Figure 2: Robot's kinematic scheme

Forward Kinematics

In order to find the position of end effector we need to find forward kinematics solution. This solution can be obtained directly by multiplying rotation and translation matrices (eq.(1)).

$$H = T_z(L_1)R_z(q_1)R_y(q_2)T_z(L_2)R_y(q_3)T_x(L_3)R_x(q_4)R_y(q_5)T_x(L_4)R_x(q_6) \tag{1}$$

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{bmatrix} \tag{2}$$

where

```
TzL1 =

[ 1, 0, 0,  0]
[ 0, 1, 0,  0]
[ 0, 0, 1, L1]
[ 0, 0, 0,  1]
```

```
Rzq1 =

[ cos(q1), -sin(q1), 0, 0]
[ sin(q1),  cos(q1), 0, 0]
[       0,        0, 1, 0]
[       0,        0, 0, 1]
```

```
Ryq2 =

[  cos(q2), 0, sin(q2), 0]
[        0, 1,       0, 0]
[ -sin(q2), 0, cos(q2), 0]
[        0, 0,       0, 1]
```

```
TzL2 =

[ 1, 0, 0,  0]
[ 0, 1, 0,  0]
[ 0, 0, 1, L2]
[ 0, 0, 0,  1]
```

```
Ryq3 =

[  cos(q3), 0, sin(q3), 0]
[        0, 1,       0, 0]
[ -sin(q3), 0, cos(q3), 0]
[        0, 0,       0, 1]
```

```
TxL3 =

[ 1, 0, 0, L3]
[ 0, 1, 0,  0]
[ 0, 0, 1,  0]
[ 0, 0, 0,  1]
```

```
Rxq4 =

[ 1,       0,        0, 0]
[ 0, cos(q4), -sin(q4), 0]
[ 0, sin(q4),  cos(q4), 0]
[ 0,       0,        0, 1]
```

```
Ryq5 =

[  cos(q5), 0, sin(q5), 0]
[        0, 1,       0, 0]
[ -sin(q5), 0, cos(q5), 0]
[        0, 0,       0, 1]
```

```
TxL4 =

[ 1, 0, 0, L4]
[ 0, 1, 0,  0]
[ 0, 0, 1,  0]
[ 0, 0, 0,  1]
```

```
Rxq6 =

[ 1,       0,        0, 0]
[ 0, cos(q6), -sin(q6), 0]
[ 0, sin(q6),  cos(q6), 0]
[ 0,       0,        0, 1]
```

Elements of matrix H can be found in the program Forward_Kinematics.m. There is also test file (Test_FK.txt) with some parameters of q and corresponding matrices H are found. To find the position of end effector we should take elements H_{14}, H_{24}, H_{34}. The position of the end effector with known q parameters can be found following way:

$x = L3 * cos(q2+q3) * cos(q1) + L2 * cos(q1) * sin(q2) - L4 * sin(q1) * sin(q4) * sin(q5) + L4 * cos(q2+q3) * cos(q1) * cos(q5) - L4 * cos(q1) * cos(q2) * cos(q4) * sin(q3) * sin(q5) - L4 * cos(q1) * cos(q3) * cos(q4) * sin(q2) * sin(q5)$

$y = L3 * cos(q2+q3) * sin(q1) + L2 * sin(q1) * sin(q2) + L4 * cos(q2+q3) * cos(q5) * sin(q1) + L4 * cos(q1) * sin(q4) * sin(q5) - L4 * cos(q2) * cos(q4) * sin(q1) * sin(q3) * sin(q5) - L4 * cos(q3) * cos(q4) * sin(q1) * sin(q2) * sin(q5)$

$z = L1 - L3 * sin(q2+q3) + L2 * cos(q2) - (L4 * cos(q2+q3) * sin(q4+q5))/2 - L4 * sin(q2+q3) * cos(q5) + (L4 * sin(q4-q5) * cos(q2+q3))/2$

x_2

Inverse Kinematics

Inverse kinematics allows us to get the configuration of the robot when the position of end effector is known. In order to find joint orientations we need to consider two cases: up and down.

Case "Up"

$$\begin{cases} x = x_2 + l_4 \cos\varphi \\ z = z_2 + l_4 \sin\varphi \end{cases}$$

where φ is the angle between l_4 and positive direction of axis x and shows the orientation of manipulator's end effector.

In triangle ACE we can find the length of AC:

$$AC = \sqrt{x_2^2 + (z_2 - l_1)^2}$$

From triangle ABC using cosine theorem we can get following expression:

$$x_2^2 + (z_2 - l_1)^2 = l_2^2 + l_3^2 + 2l_2l_3cosq_3$$

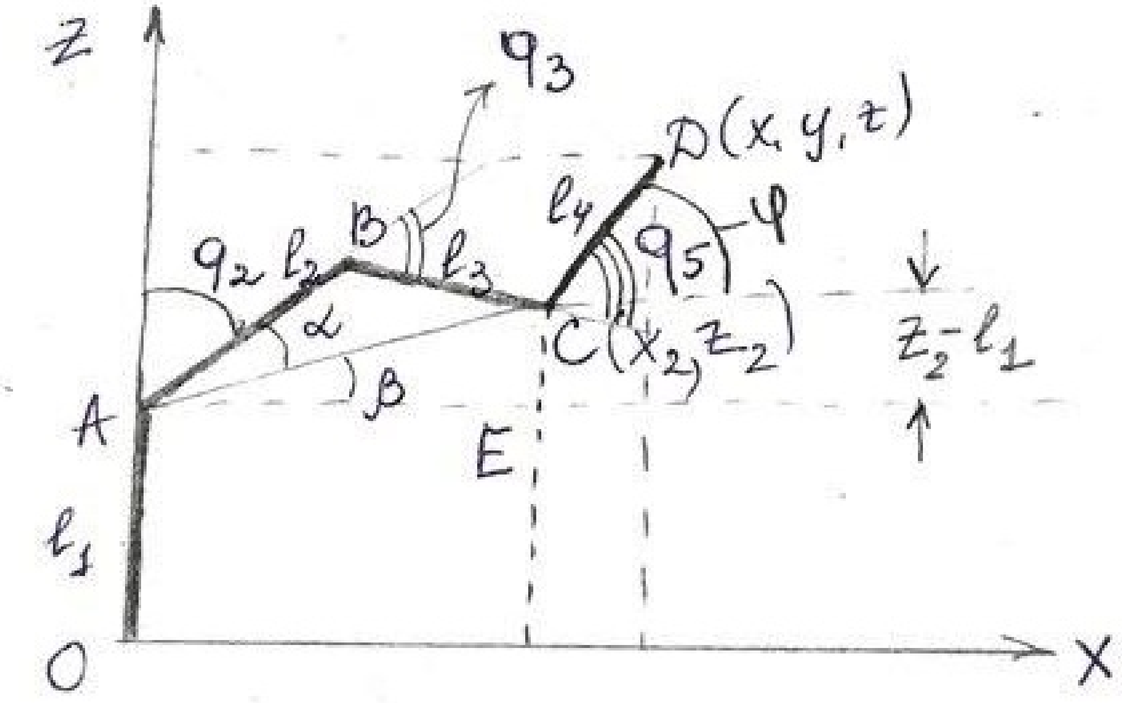

Figure 3: Caption

$$q_3 = arccos(\frac{x_2^2 + (z_2 - l_1)^2 - l_2^2 - l_3^2}{2l_2 l_3}) \tag{3}$$

It's easy to see that $\alpha + \beta = \frac{\pi}{2} - q_2$. So $q_2 = \frac{\pi}{2} - (\alpha + \beta)$
α can be derived from triangle ABC (sine theorem)

$$\frac{l_3}{sin\alpha} = \frac{\sqrt{x_2^2 + (z_2 - l_1)^2}}{sin(\pi - q_3)}$$

$$sin\alpha = \frac{l_3 sinq_3}{\sqrt{x_2^2 + (z_2 - l_1)^2}} \tag{4}$$

$cos\alpha$ can be found using cosine theorem.

$$l_3^2 = l_1^2 + x_2^2 + (z_2 - l_1)^2 - 2l_1\sqrt{x_2^2 + (z_2 - l_1)^2} cos\alpha$$

$$cos\alpha = \frac{l_1^2 + x_2^2 + (z_2 - l_1)^2 - l_3^2}{2l_1\sqrt{x_2^2 + (z_2 - l_1)^2}} \tag{5}$$

Dividing 4 by 5 and using arctan we get α:

$$\alpha = \arctan\frac{2l_1 l_3 sinq_3}{l_1^2 + x_2^2 + (z_2 - l_1)^2 - l_3^2} \tag{6}$$

$$\beta = \arctan\frac{z_2 - l_1}{x_2} \tag{7}$$

From 6 and 7 we get q_2:

$$q_2 = \frac{\pi}{2} - [\arctan\frac{2l_1 l_3 sinq_3}{l_1^2 + x_2^2 + (z_2 - l_1)^2 - l_3^2} + \arctan\frac{z_2 - l_1}{x_2}] \tag{8}$$

q_5 can be found from $\varphi = q_2 + q_3 + q_5$, where $\varphi = \arctan\frac{z - l_1}{x}$

Case "Down"

If we look at picture ?? we can see that

$$\alpha - \beta + \pi - q_2 = \frac{\pi}{2}$$

So

$$q_2 = \frac{\pi}{2} - (-\alpha + \beta) \tag{9}$$

and we can combine two cases in one:

$$q_2 = \frac{\pi}{2} - (m\alpha + \beta) \tag{10}$$

where $m = \pm 1$.

Other parameters

$$q_1 = \arctan \frac{y}{x} \tag{11}$$

$$q_4 = \arctan \frac{z - l_1 - l_2}{y} \tag{12}$$

$$q_5 = \varphi - (\frac{\pi}{2} - q_2) - q_3 \tag{13}$$

Link to Github

https://github.com/fam-ca/HW2_FK_IK.git