DoNRS_HW3

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Task 1: Forward kinematics

In order to find the position of end effector we need to find forward kinematics solution. This solution can be obtained directly by multiplying rotation and translation matrices (eq.(1)).

$$H = R_z(q_1)T_z(d_1)R_x(q_2)T_y(a_2)T_y(d_3) \tag{1}$$

Let's assume constant parameters $d_1 = 20, a_2 = 10$. Zero position has links 2 and 3 aligned along y axis.

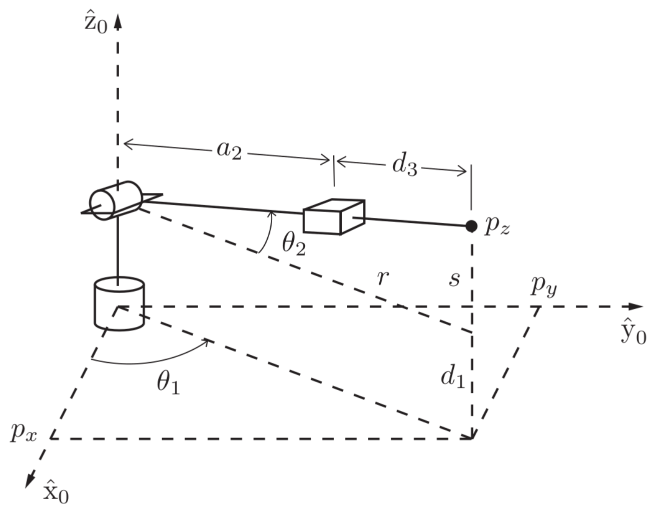

Figure 1: Kinematic scheme ($d_1 = 20, a_2 = 10$)

Task 2: Inverse kinematics

For inverse kinematics we have two solutions for q_1 (this angle is counted from y axis):

$$q_{11} = \pi/2 - atan2(y, x)$$

$$q_{12} = -\pi/2 - atan2(y, x)$$

$$q_2 = atan2(s, r)$$

$$d_3 = \sqrt{r^2 + s^2} - a_2$$

where

$$s = z - d_1$$

$$r = \sqrt{x^2 + y^2}$$

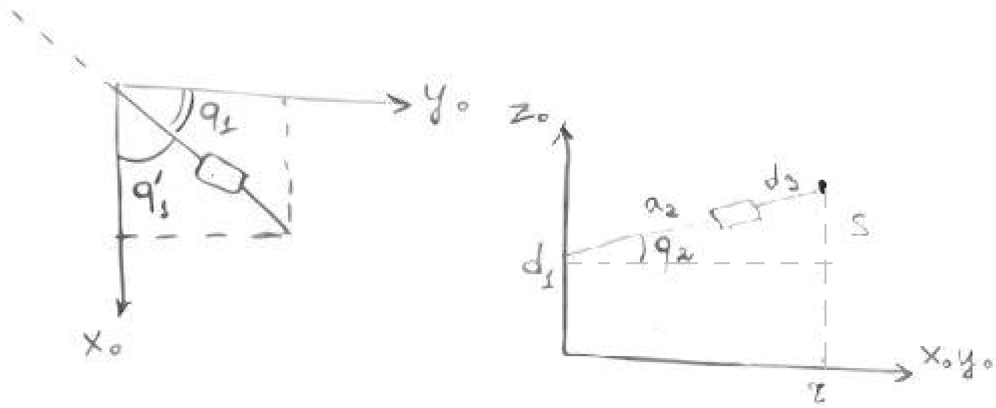

Figure 2: Geometry from top view (left image) and in the plane perpendicular to XY frame (right image).

Task 3: Jacobian solution

In this task three approaches were performed:

1. Classical approach (Jacobian_Classical.m)
2. Skew Theory (Jacobian_SkewTheory.m)
3. Numerical derivatives (Jacobian_NumDerivatives.m)

The results obtained are the same (the comparison is in program main.m) and substracting one Jacobian another gives matrices with zeros. Here is the solution for the Jacobian :

$$J = \begin{bmatrix} -cos(q1)*cos(q2)*(a2+d3) & sin(q1)*sin(q2)*(a2+d3) & -cos(q2)*sin(q1) \\ -cos(q2)*sin(q1)*(a2+d3) & -cos(q1)*sin(q2)*(a2+d3) & cos(q1)*cos(q2) \\ 0 & cos(q2)*(a2+d3) & sin(q2) \\ 0 & cos(q1) & 0 \\ 0 & sin(q1) & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{2}$$

Task 4: Singularities

Jacobian becomes singular when its rank is less than 3 (3 is number of joints).

$$J = \begin{bmatrix} -cos(q1)*cos(q2)*(a2+d3) & sin(q1)*sin(q2)*(a2+d3) & -cos(q2)*sin(q1) \\ -cos(q2)*sin(q1)*(a2+d3) & -cos(q1)*sin(q2)*(a2+d3) & cos(q1)*cos(q2) \\ 0 & cos(q2)*(a2+d3) & sin(q2) \\ 0 & cos(q1) & 0 \\ 0 & sin(q1) & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3}$$

Singularity may occur when the robot is along z axis. In this case $q_2 = \pi/2$, q_1 can take any values (the number of solutions is infinite).

Analyzing Jacobian:

Here the columns 1,2,3 are independent no matter which values of q_1, q_2, d_3 we take.

But if we take d_3 equal to $-a_3$ then the determinant will be equal to 0 and we will have intersection of the robot's parts and this configuration will be a singularity.

Task 5: Velocity of tool frame

The velocity in Cartesian space is calculated using the following equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J * \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{d_3} \end{bmatrix} \tag{4}$$

where $\dot{x}$, $\dot{y}$, $\dot{z}$ are the x, y, z- components of velocities; ω_x, ω_y, ω_z are x, y, z- components of angular velocities; J is Jacobian (6×3). The Jacobian is taken from task 3, $\dot{q_1} = \cos t$, $\dot{q_2} = -2 \cdot \sin 2t$, $\dot{d_3} = 3 \cdot \cos 3t$.

The result is shown in the fig.3. Time duration is 5 sec.

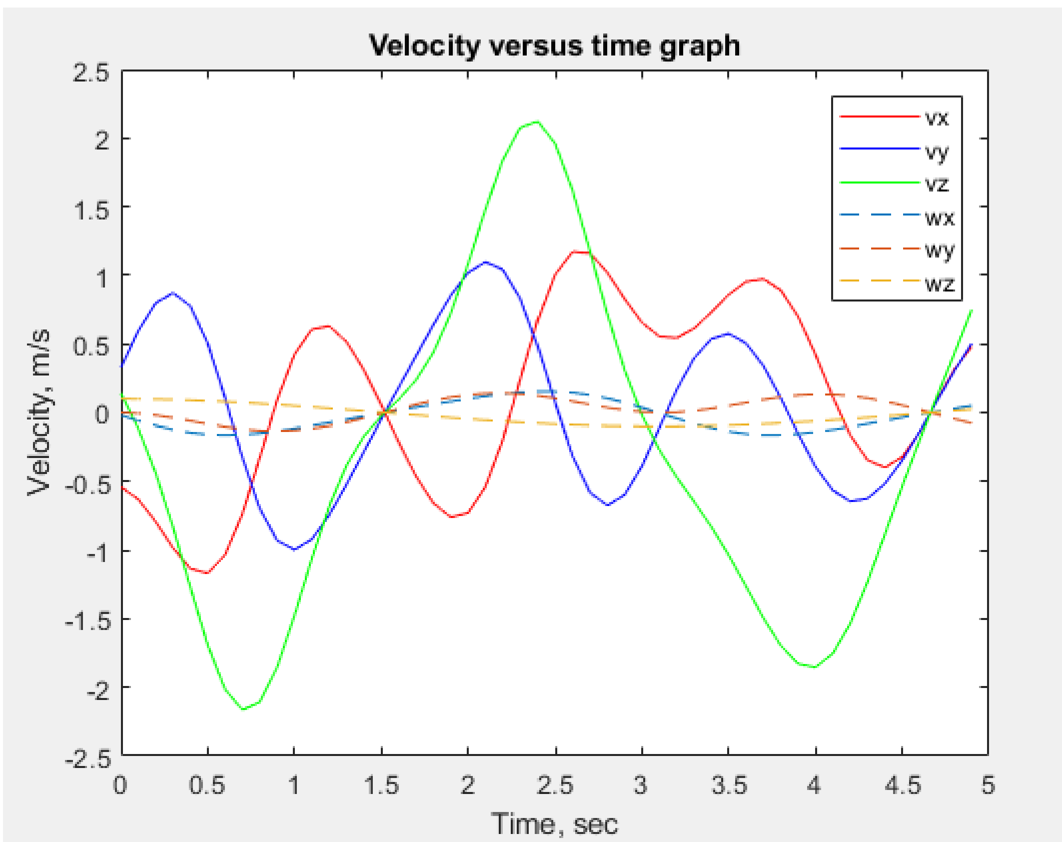

Figure 3: Velocities graph

Link on github

https://github.com/fam-ca/HW3_Jacobian.git