Homework 5

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Report

Task 1: Direct dynamics problem

The purpose of the first task is to find the solution for direct dynamics using Lagrange-Euler approach. For the given torques, forces, initial position and velocities find the joint's trajectory.

We have a 2 DoF manipulator that has one revolute joint (variable q_1) and a prismatic joint (variable q_2). The center of mass of the first link is at the point with coordinates (0,0,0). The center of mass of the second link is at the point $((L+q_2)\cos q_1,\ (L+q_2)\sin q_1,\ 0)$.

Firstly, we compute the Jacobian for each center of mass (for example, using skew theory) in order to find the inertia matrix. Here are Jacobians computed for each center of mass (J_{v1} and J_{v2} - Jacobians for the linear velocity of the first and second links, $J_{\omega1}$ and $J_{\omega2}$ - Jacobians for the angular velocity of the first and second links).

$$J_{v1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{v2} = \begin{bmatrix} -(L+q_2)\sin q_1 & \cos q_1 \\ (L+q_2)\cos q_1 & \sin q_1 \\ 0 & 0 \end{bmatrix}$$

$$J_{\omega1} = J_{\omega2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Kinetic energy: Calculation of Inertia matrix

After obtaining the Jacobians we compute the inertia matrix B_1 and B_2 the following way:

$$B_1 = m_1 J_{v1}^T J_{v1} + J_{\omega1}^T R_1^T I_1 R_1 J_{\omega1}$$

$$B_2 = m_2 J_{v2}^T J_{v2} + J_{\omega2}^T R_2^T I_2 R_2 J_{\omega2}$$

where m_1, m_2 are the masses of the links; R_1, R_2 are the rotation matrices; I_1, I_2 are the inertia tensors (in our case for simplicity they are just some constant values).

The total inertia $B = B_1 + B_2$.

Computed matrices B are:

$$B_1 = \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} m_2L^2 + 2m_2Lq_2^2 + I_2 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$B = \begin{bmatrix} m_2L^2 + 2m_2Lq_2^2 + I_1 + I_2 & 0 \\ 0 & m_2 \end{bmatrix}$$

Potential energy

Since we suppose the the center of mass of the first link is at the point (0,0,0) it means the the height is equal to zero. Thus, the potential energy of the first link is equal to zero and for the second link $U_2 = m_2 g y_{c_2} = m_2 g(q_2 + L)\sin q_1$. Total potential energy $U = U_1 + U_2 = m_2 g(q_2 + L)\sin q_1$. Then taking partial derivatives by q_1 and q_2 we get the gradient of the potential energy:

$$G = \begin{bmatrix} m_2 g(L + q_2)\cos q_1 \\ m_2 g \sin q_1 \end{bmatrix}$$

Coriolis and centrifugal forces

In order to find Coriolis and centrifugal terms we need to calculate Christoffel symbols using the following formula:

$$c_{ijk}(q) = \frac{1}{2}\left(\frac{\partial B_{kj}}{\partial q_i} + \frac{\partial B_{ki}}{\partial q_j} - \frac{\partial B_{ij}}{\partial q_k}\right)$$

Then implementing it in the program we get the following matrix of Coriolis and centrifugal forces:

$$C = \begin{bmatrix} m_2(L + q_2)\dot{q}_2 & m_2(L + q_2)\dot{q}_1 \\ -m_2(L + q_2)\dot{q}_1 & 0 \end{bmatrix}$$

Task2: Equation of motion

The equation of motion in general case looks this way:

$$B(q)\ddot{q} + C(q, \dot{q}) + G(q) = \tau$$

Substituting expressions for matrices B, C and G we get the following expression:

$$\tau = \begin{bmatrix} \ddot{q}_1(m_2L^2 + 2m_2Lq_2 + m_2q_2^2 + I_1 + I_2) + m_2 g \cos q_1(L + q_2) + 2m_2(L + q_2)\dot{q}_1\dot{q}_2) \\ -m_2(L + q_2)\dot{q}_1{}^2 + \ddot{q}_2 m_2 + g m_2 \sin q_1 \end{bmatrix}$$

Example of τ computation

Let

$$q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\dot{q} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\ddot{q} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

Then the results of τ computation will be:

$$\tau = \begin{bmatrix} 7233/250 \\ -11/5 \end{bmatrix}$$

Let the forces and torques have the zero value; $q_1 = \frac{\pi}{2}$, $q_2 = 5$. Then we will get follwing graphs:

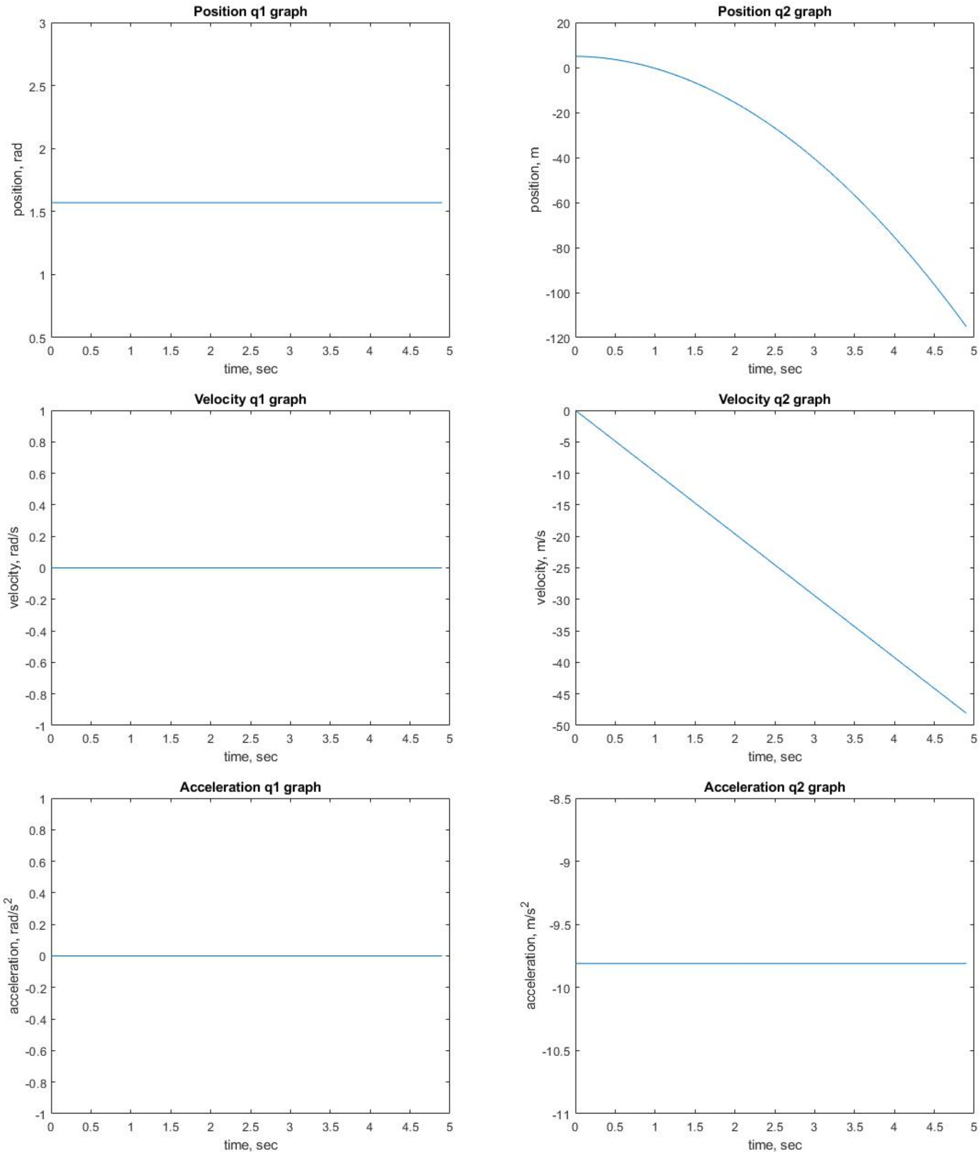

Figure 1: Position, velocity and acceleration graphs

Task 3: Applying new forces (torques)

Let the initial forces and torques be equal to $[\sin 2t; \cos t]$. The initial position is the same as in previous task $(\frac{\pi}{2}, 5)$. Then the obtained graphs will be as in the fig.2.

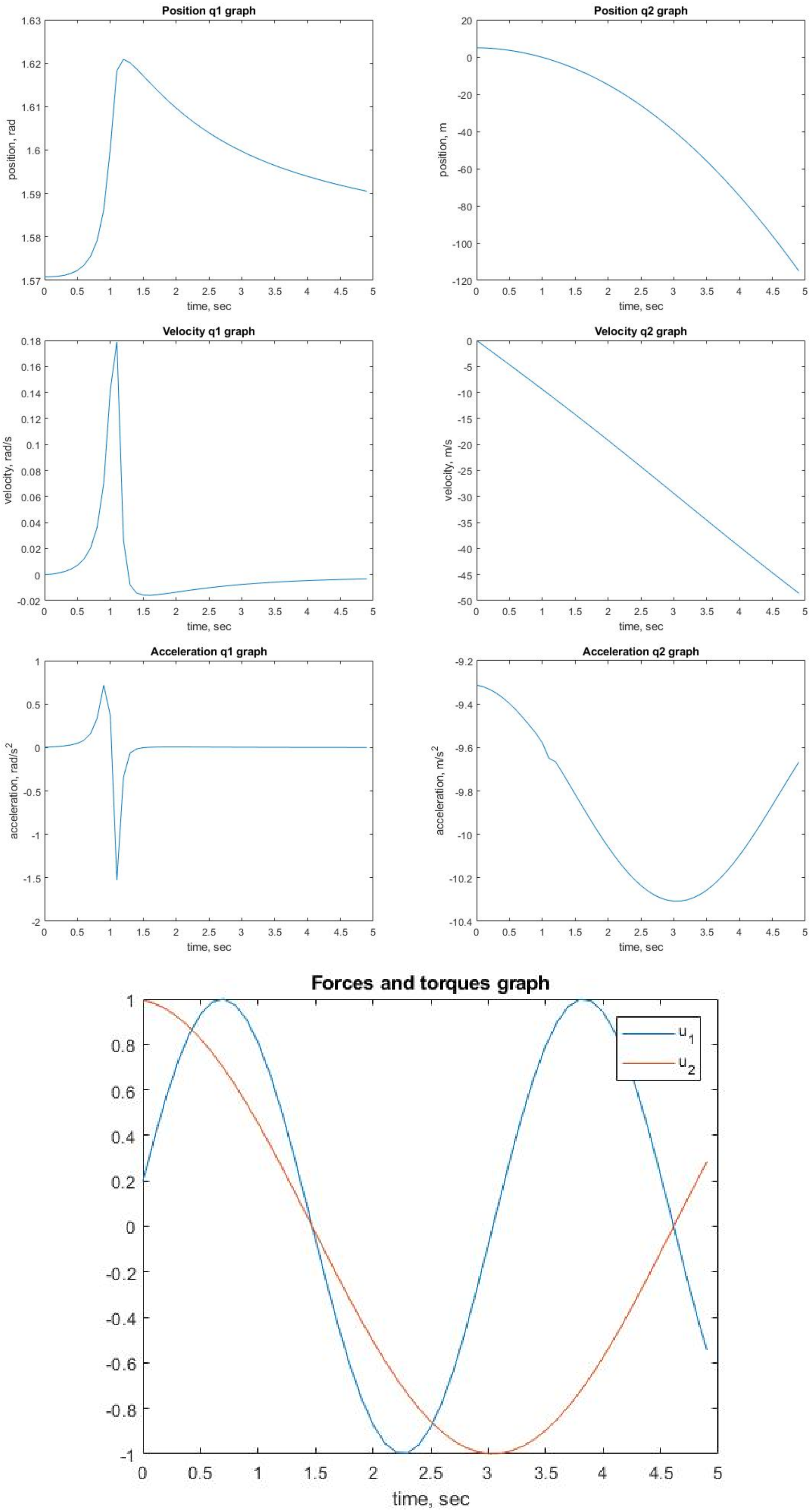

Figure 2: Position, velocity, acceleration and forces(torques) graphs