Advanced Robotics
Homework 1

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Report

1 MSA model of the Tripteron

In order to make the MSA model of the robot, we need to understand what links and joints are included. So in this work the length of the links are equal to 1 m. All the links are considered as flexible cylindrical beams. All the joints are also considered as flexible. The size of the workspace is $1 \times 1 \times 1$.

The parameters for the stiffness model:

1. The area $A = \frac{\pi d^2}{4}$, where $d = 0.15$m
2. Young's module $E = 70 \cdot 10^9$
3. Coulomb's module $G = 25.2 \cdot 10^9$
4. Principle moment inertia components $I_x = I_y = I_z = \frac{\pi d^4}{64}$
5. Torsional moment of inertia $I_\rho = \frac{\pi d^4}{32}$
6. Also we should consider the rigid platform, so the distance from each corner to the center = 0.1 m.

Taking into account equations 16-37 from stiffness paper [1], we can aggregate all the equations in such way to keep the equality valid. This operation was done in the **A_agg**, **B_agg**, **C_agg**, **D_agg** functions. On the next page you can see how the aggregated matrix looks like (fig. 1). The stiffness element matrices $K^{11}_{i,j}$, $K^{12}_{i,j}$, $K^{21}_{i,j}$, $K^{22}_{i,j}$ should be taken in the global frame which means that we should use the following equation:

$$K^{\alpha\beta}_{i,j\,global} = Q \cdot K^{\alpha\beta}_{i,j\,local} \cdot Q^T \qquad \alpha, \beta = 1, 2$$

After that we used equation 38 from [1] in order to obtain Cartesian stiffness matrix (function **Kc_leg**):

$$K_c = D - C \cdot A^{-1} \cdot B$$

The next step is to use Hook's law (equation 15 from [1]) to find the deflection (**deflection_ee_tripetron** function).

The above algorithm is for computing only one deflection for one end-effector (EE) position. Now we need to find the rest deflections for all the EE positions in the cube of size $1 \times 1 \times 1$.

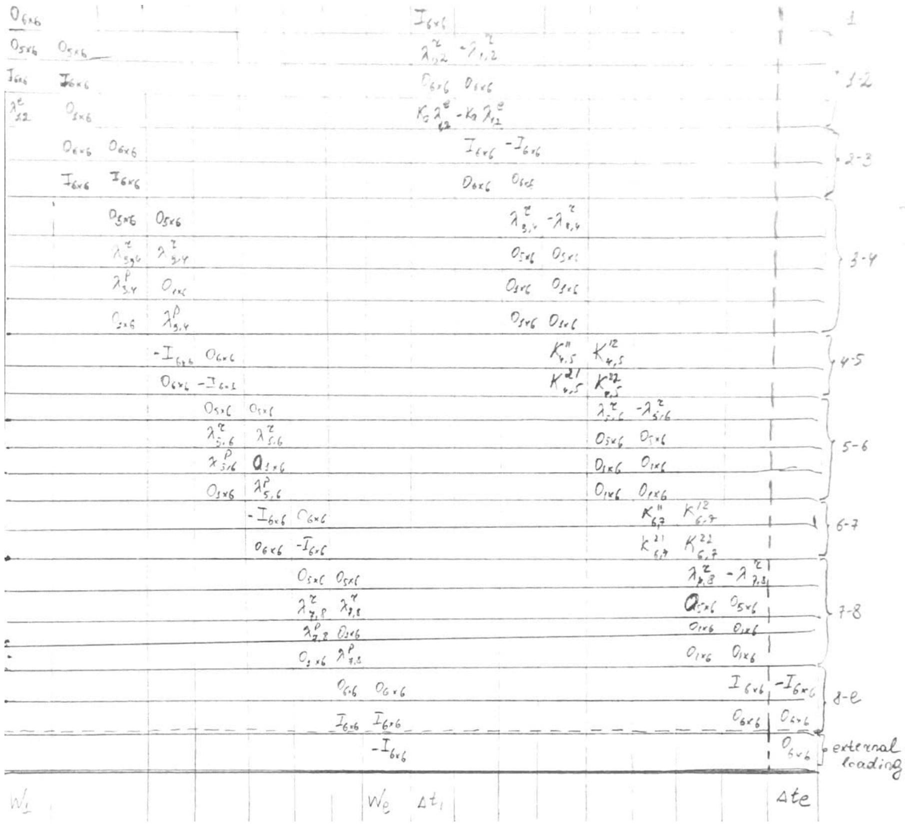


Figure 1: Aggregated Matrix

2 Deflection map plot

In the equation 19 of paper [1] the matrix $D_{8,e}$ was presented as:

$$D_{8,e} = \begin{bmatrix} I_{3\times3} & [d_{8,e}\times]^T \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}$$

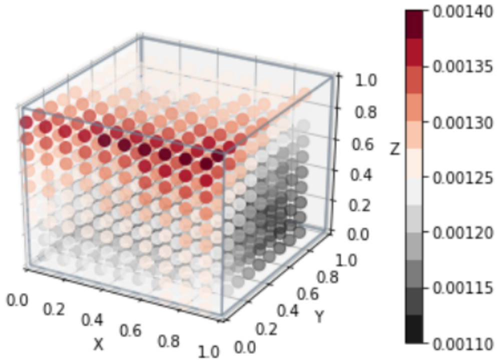
And as we have the rigid platform so we should present the matrix $[d_{8,e}\times]^T = -[d_{8,e}\times]$ the following way:

$$[d_{8,e}\times]^T = \begin{bmatrix} 0 & 0.1 & -0.1 \\ -0.1 & 0 & 0.1 \\ 0.1 & -0.1 & 0 \end{bmatrix}$$

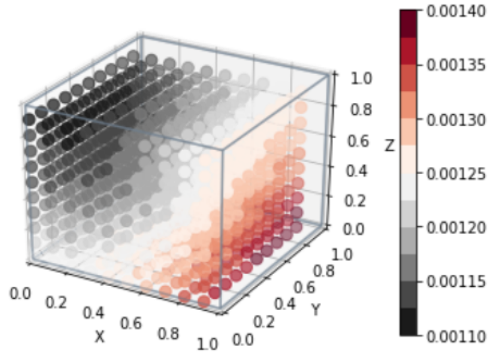
The deflection map obtained is presented on the figure 2 (functions **get_wrench_deflection_plot_data** and **plotDeflection** in the program).

3 Discussion of results

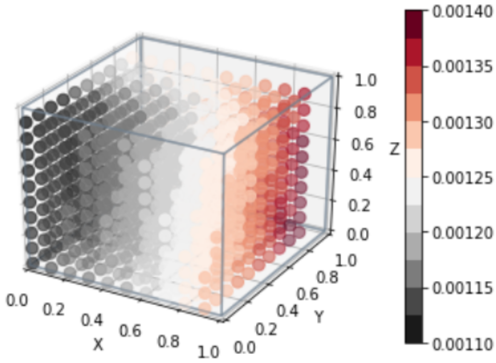
Comparing the results with the results obtained in [1] we can see that the maps look similar but the values of deflection are slightly different. This can be explained by the small value of the platform's size that was included in MSA model and the different force applied. The force applied is equal to 100N. Three deflection maps with different wrenches were plotted. We can see that if we apply force along x axis the maximum value of deflections will be located along the x axis but diagonally on the opposite side. Also the time that was contributed to compute matrices and plot all graphs is 34 seconds in average.



(a) $W_e = [100, 0, 0, 0, 0, 0]$



(b) $W_e = [0, 100, 0, 0, 0, 0]$



(c) $W_e = [0, 0, 100, 0, 0, 0]$

Figure 2: Deflection maps with different forces applied

References

[1] Kirsanov D., Sevostianov I., Rodionov O., Ostanin M. Stiffness analisys of the Tripteron parallel manipulator, 2020.

[2] Popov D., Skvortsova V., Klimchik A. Stiffness modeling of 3RRR Parallel Spherical Manipulator, 2019.