Advanced Robotics
Homework 2

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Report

1 Robot design

The parameters for the stiffness model:

1. The area $A = \frac{\pi d^2}{4}$, where $d = 0.15$m
2. Young's module $E = 70 \cdot 10^9$
3. Coulomb's module $G = 25.2 \cdot 10^9$
4. Principle moment inertia components $I_x = I_y = I_z = \frac{\pi d^4}{64}$
5. Torsional moment of inertia $I_\rho = \frac{\pi d^4}{32}$
6. Also we should consider the rigid platform, so the distance from each corner to the center = 0.1 m.

2 VJM model of the Tripteron

In VJM model we need to use extended model for forward kinematics. After the actuated joint we add 1 DoF virtual spring, after each rigid link we add 6 DoF virtual spring (fig.1). In this work the length of the links are equal to 1 m. All the links are considered as flexible cylindrical beams. All the joints are also considered as flexible. The size of the workspace is $1 \times 1 \times 1$. The forward kinematics can be written the following way [1]:

$$T = T_{base_i} T_z(d_i) T_z(\theta_{i,1}) R_z(q_{i,1}) T_x(l_{i,1}) T_{3D}(\theta_{i,2-7}) R_z(q_{i,2}) T_x(l_{i,2}) T_{3D}(\theta_{i,8-13}) R_z(q_{i,3}) T_{tool_i}$$

After writing forward kinematics function we need to compute the Jacobians for virtual joint and passive joint variables. This procedure was performed in **Jacobian_virtual** and **Jacobian_passive** functions.

The next step is to compute K_θ (of size 13×13) as an aggregated matrix, which diagonal blocks are equal to K_{22} and then using equations 11-14 find total Cartesian sriffness matrix K_c (**KTheta** and **Kc_total** functions in the program).

The next step is to use Hook's law (equation 15 from [1]) to find the deflection (**deflection_total** function).

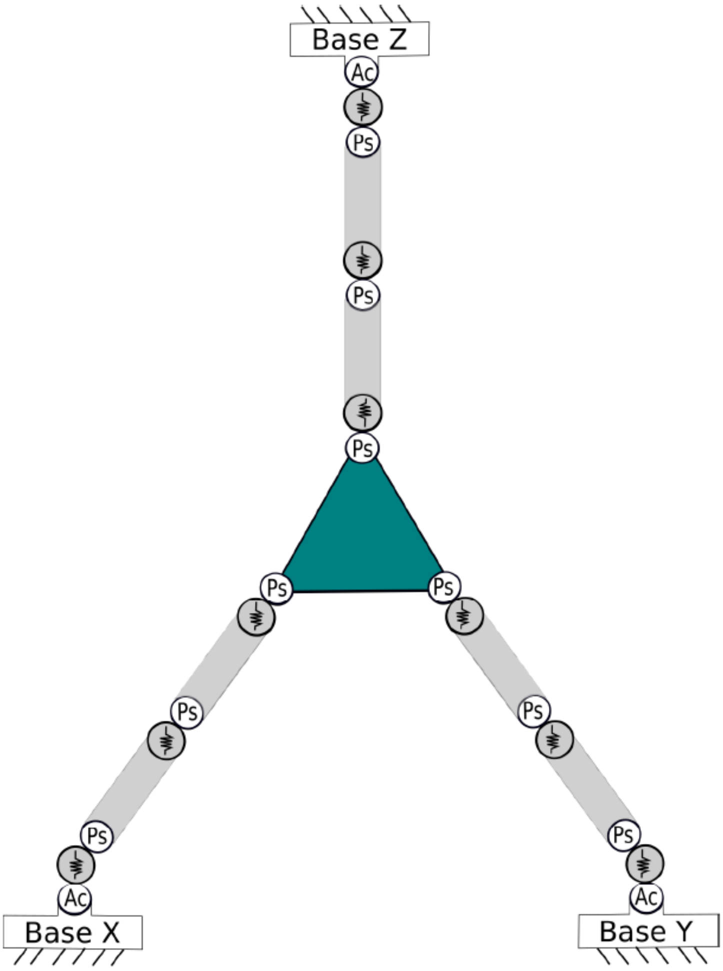
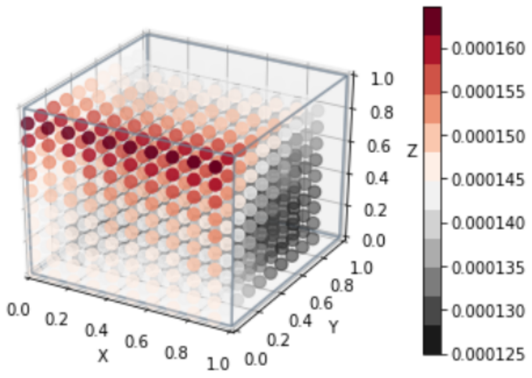


Figure 1: VJM model of Tripteron

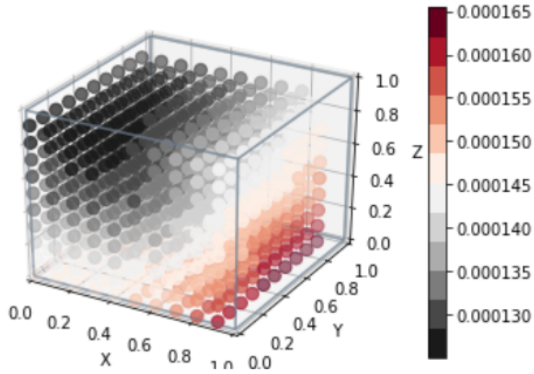
3 Deflection map plot

The above algorithm is for computing only one deflection for one end-effector (EE) position. Now we need to find the rest deflections for all the EE positions in the cube of size $1 \times 1 \times 1$. (**get_wrench_deflection_plot_data** function)

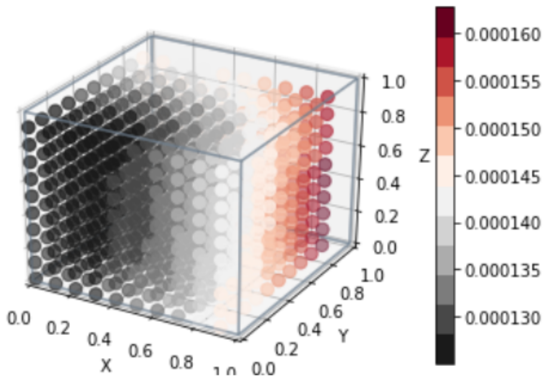
The deflection map obtained is presented on the figure 2 (**plotDeflection** function in the program).



(a) $W_e = [100, 0, 0, 0, 0, 0]$



(b) $W_e = [0, 100, 0, 0, 0, 0]$



(c) $W_e = [0, 0, 100, 0, 0, 0]$

Figure 2: Deflection maps with different forces applied

4 MSA vs VJM

The differences of the deflections in each point obtained by MSA and VJM models are presented in the fig. 3

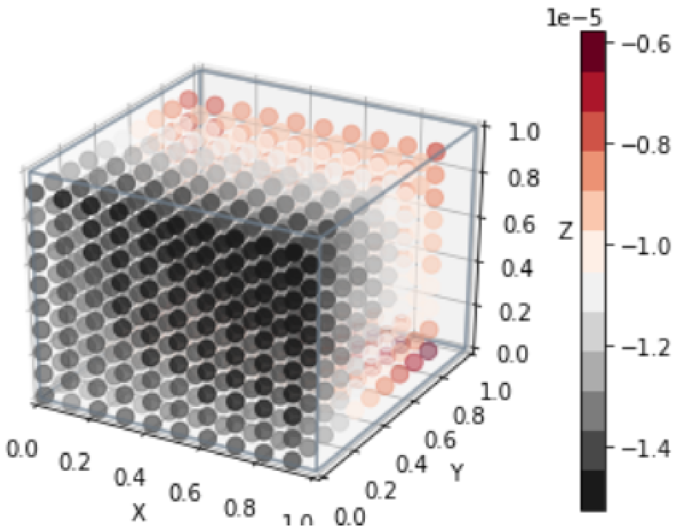


Figure 3: Difference between MSA and VJM deflection map

5 Discussion of results

Comparing the results of simulation MSA and VJM models we see that although the difference are not high (1e-6) there is some gradient in the plot while in the paper the authors show that the difference between the models is homogeneous except for some other random points. This can be explained by the error while we were modelling the rigid platform.

As for the complexity we see that VJM is easier to implement but computationally it takes more time (1 minute 24 seconds vs 34 seconds).

Three deflection maps with different wrenches were plotted. We can see that if we apply force along x axis the maximum value of deflections will be located along the x axis but diagonally on the opposite side.

6 Link to Github

https://github.com/fam-ca/VJM

References

[1] Kirsanov D., Sevostianov I., Rodionov O., Ostanin M. Stiffness analisys of the Tripteron parallel manipulator, 2020.

[2] Popov D., Skvortsova V., Klimchik A. Stiffness modeling of 3RRR Parallel Spherical Manipulator, 2019.