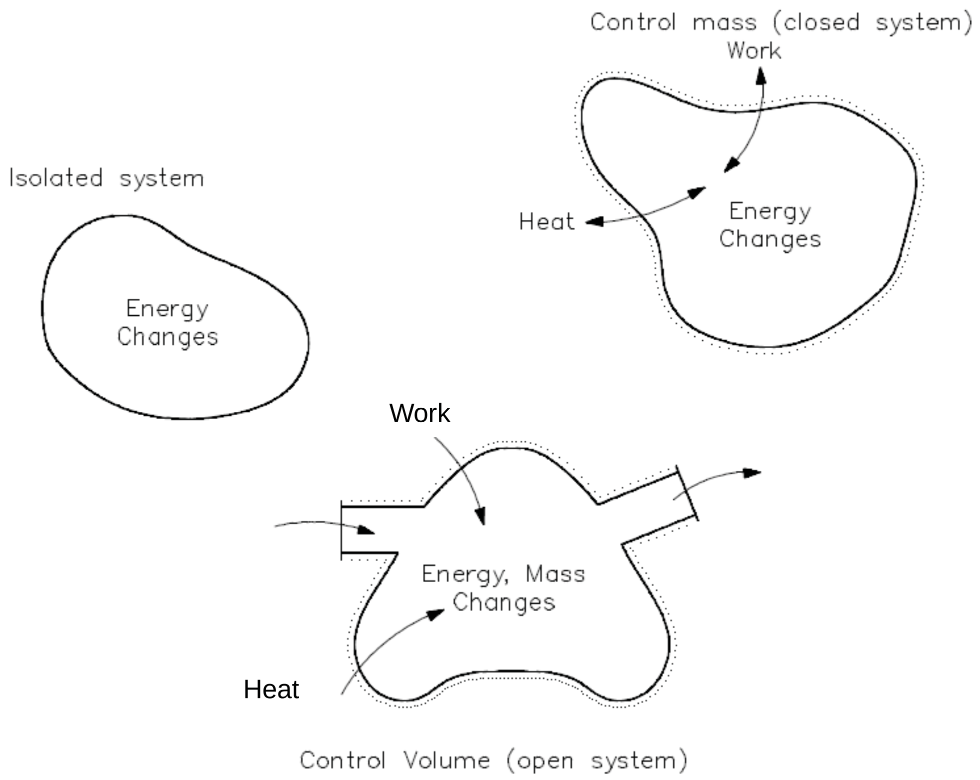
SH2702
Nuclear Reactor Technology

Exercise Session 01

Analysis of nuclear energy systems

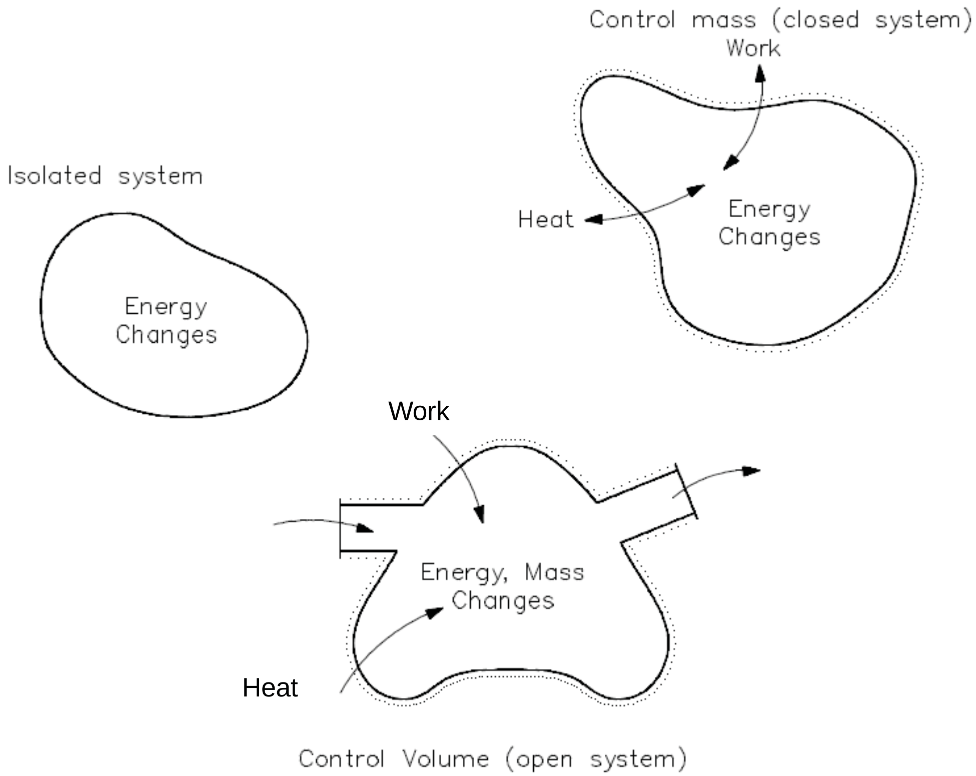
- Thermodynamic systems
  - Isolated system
  - Closed system (control mass)
  - Open system (control volume)
    - For example, heat exchanger, pump, turbine, and nuclear reactor



Open system equations

- Mass conservation

$$\left(\frac{dm}{dt}\right)_{CV} = \sum_{j \in in} W_j - \sum_{k \in out} W_k$$

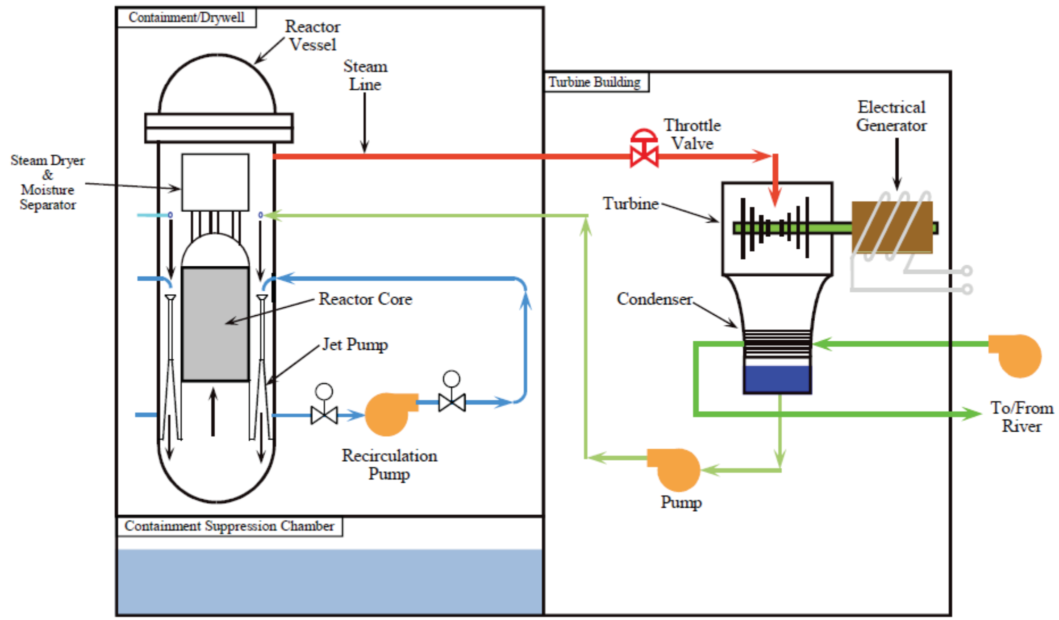


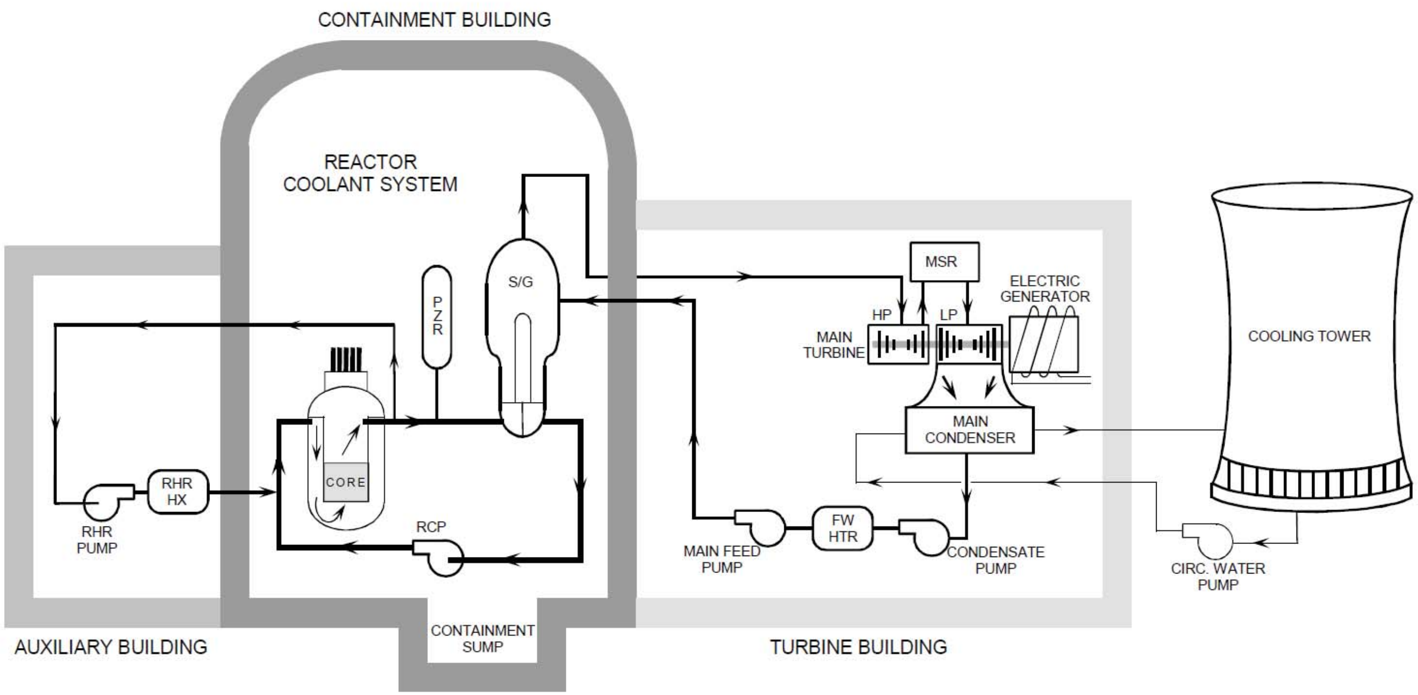
- Energy conservation

$$\left[\frac{d\left(me_T\right)}{dt}\right]_{CV} = q - N_{shaft} - N_{normal} - N_{shear} + \sum_{j \in in} \left(i + e_P + e_K\right)_j W_j - \sum_{k \in out} \left(i + e_P + e_K\right)_k W_k$$

Nuclear power systems

- Nuclear power system analysis
  - Heat exchanger
  - Turbine, pump



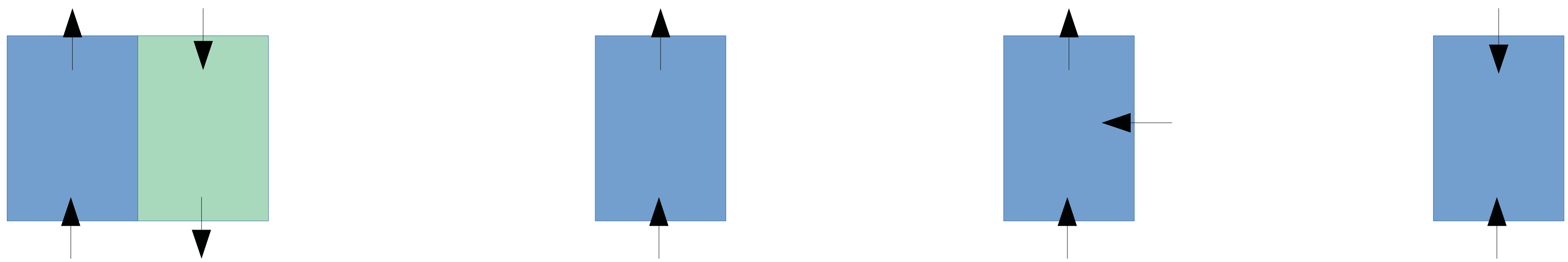


Nuclear power systems

- Nuclear power system analysis
  - Heat exchanger
  - $N_{shaft} = 0$
  - N_{normal}, N_{shear}, e_p, e_K can be neglected sometimes
  - Typically $0 = q + W_{in}{*}i_{in} - W_{out}{*}i_{out}$

$$\left(\frac{dm}{dt}\right)_{CV} = \sum_{j\in in} W_j - \sum_{k\in out} W_k$$

$$\left[\frac{d\left(me_T\right)}{dt}\right]_{CV} = q - N_{shaft} - N_{normal} - N_{shear} + \sum_{j\in in}\left(i + e_P + e_K\right)_j W_j - \sum_{k\in out}\left(i + e_P + e_K\right)_k W_k$$



Heat exchanger
Steam generator
Feedwater heater

Reactor

Feedwater heater

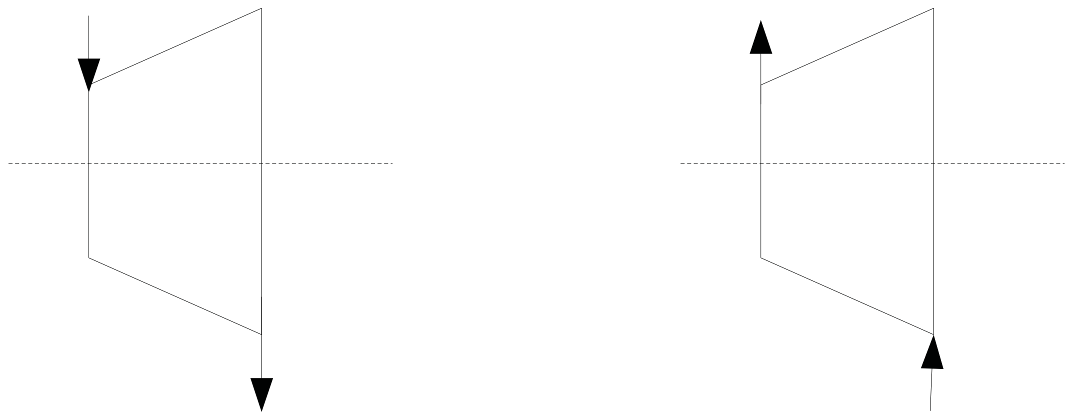
Pressurizer

Nuclear power systems

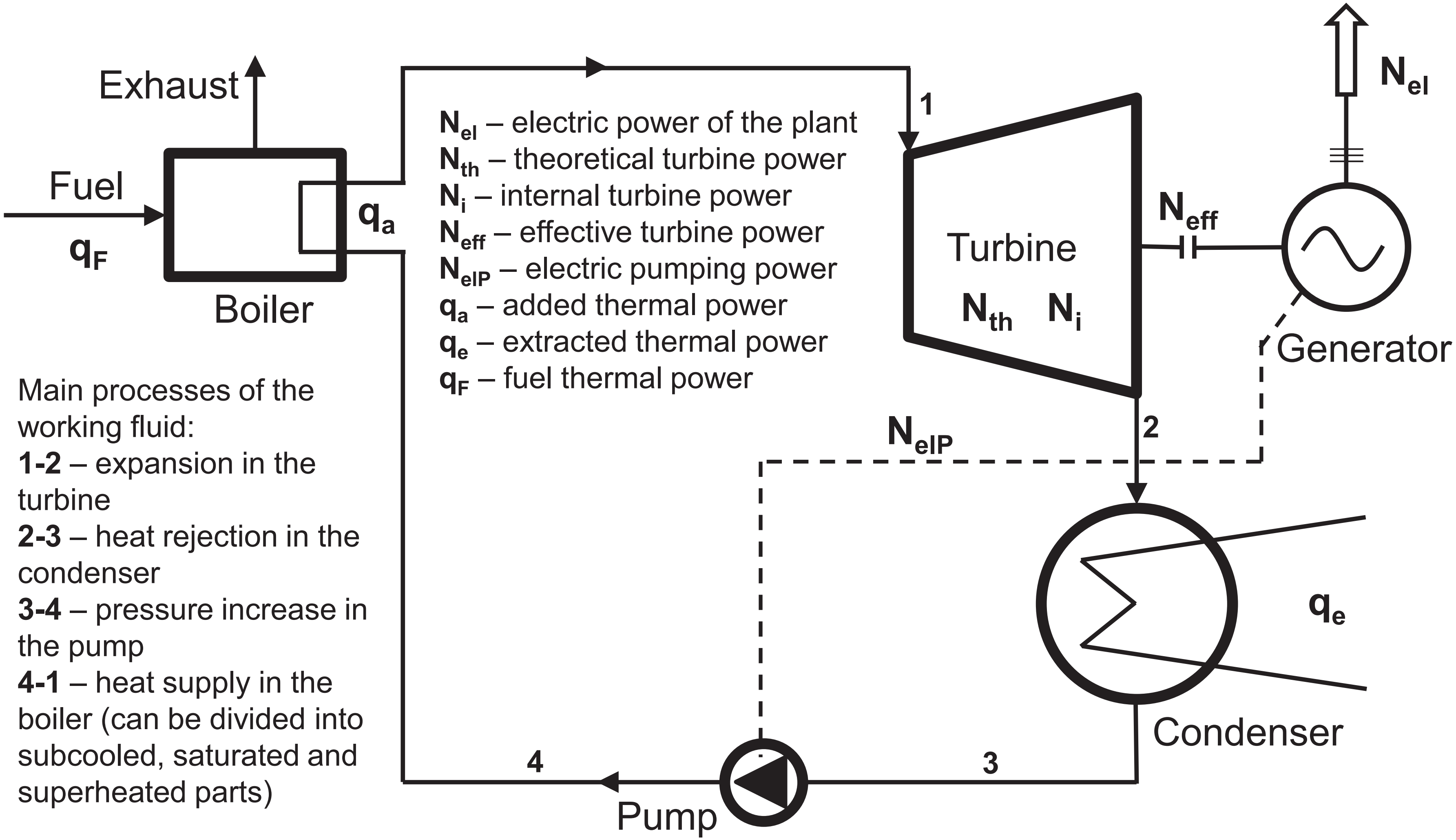
- Nuclear power system analysis
  - Turbine, pump
  - Usually q = 0
  - N_{normal}, N_{shear}, e_p, e_K can be neglected sometimes
  - Typically $0 = - N_{shaft} + W_{in} * i_{in} - W_{out} * i_{out}$

$$\left(\frac{dm}{dt}\right)_{CV} = \sum_{j \in in} W_j - \sum_{k \in out} W_k$$

$$\left[\frac{d\left(me_T\right)}{dt}\right]_{CV} = q - N_{shaft} - N_{normal} - N_{shear} + \sum_{j \in in} \left(i + e_P + e_K\right)_j W_j - \sum_{k \in out} \left(i + e_P + e_K\right)_k W_k$$



Condensing Power Schematic



N_{el} – electric power of the plant
N_{th} – theoretical turbine power
N_i – internal turbine power
N_{eff} – effective turbine power
N_{elP} – electric pumping power
q_a – added thermal power
q_e – extracted thermal power
q_F – fuel thermal power

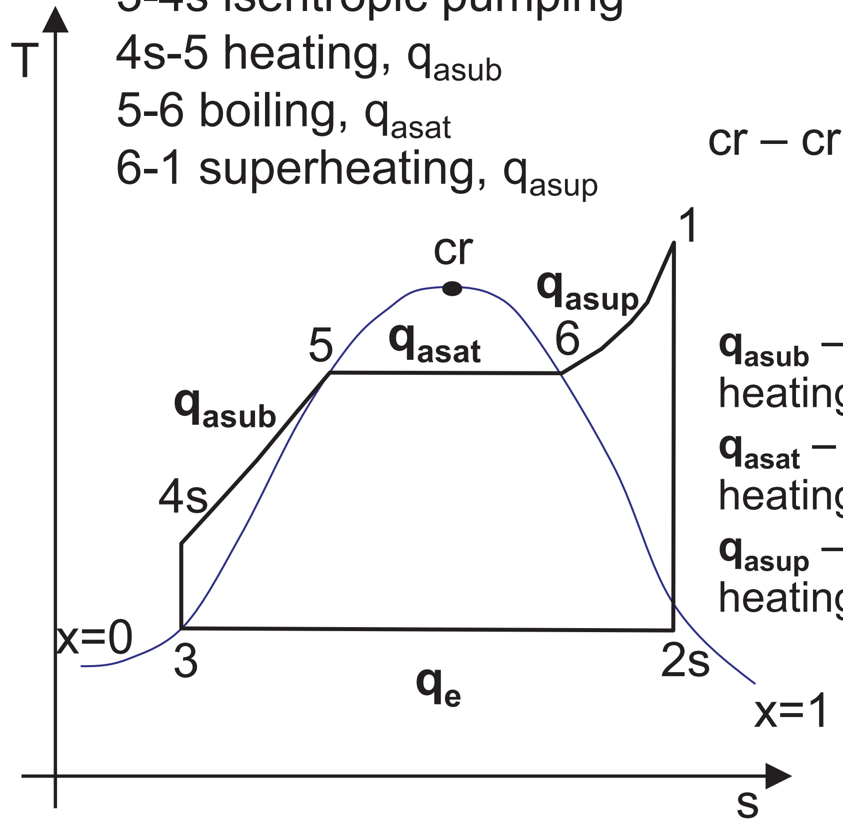
Main processes of the working fluid:
1-2 – expansion in the turbine
2-3 – heat rejection in the condenser
3-4 – pressure increase in the pump
4-1 – heat supply in the boiler (can be divided into subcooled, saturated and superheated parts)

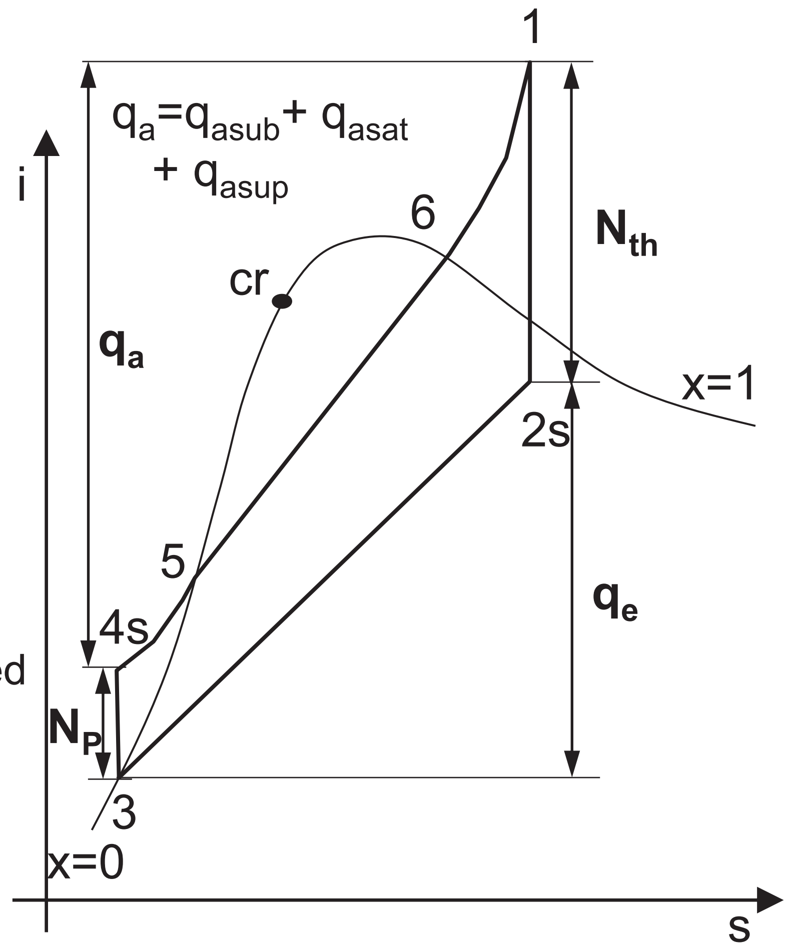
Ideal Sub-critical Rankine Cycle

1-2s isentropic expansion in turbine
2s-3 heat rejection
3-4s isentropic pumping
4s-5 heating, q_{asub}
5-6 boiling, q_{asat}
6-1 superheating, q_{asup}

cr – critical point

q_{asub} – subcooled heating power
q_{asat} – saturated heating power
q_{asup} – superheated heating power





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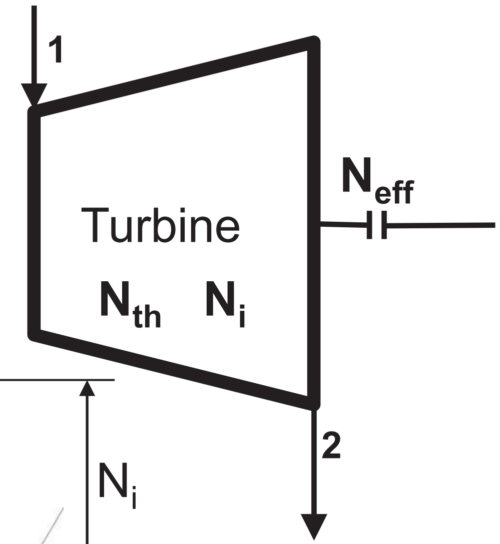
Turbine (1)

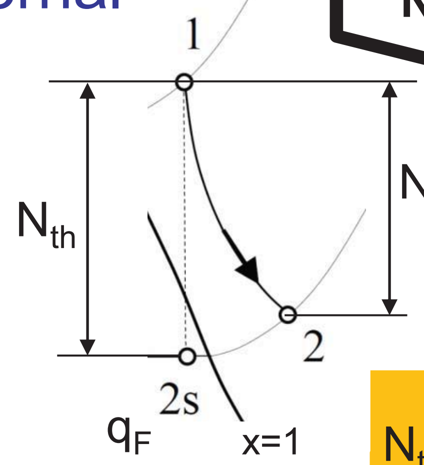
- We define the following turbine powers:
  - the theoretical (isentropic) power $N_{th} = W(i_1 - i_{2s})$
  - the internal turbine power $N_i = W(i_1 - i_2)$
- Based on these powers, the internal turbine efficiency is defined as

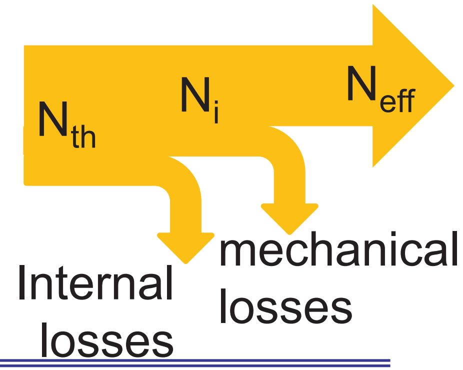
$$\eta_i = \frac{N_i}{N_{th}} \Rightarrow \eta_i = \frac{i_1 - i_2}{i_1 - i_{2s}}$$

- The turbine effective power is based on mechanical efficiency η_m and is given as

$$N_{eff} = \eta_m N_i = \eta_m \eta_i N_{th}$$

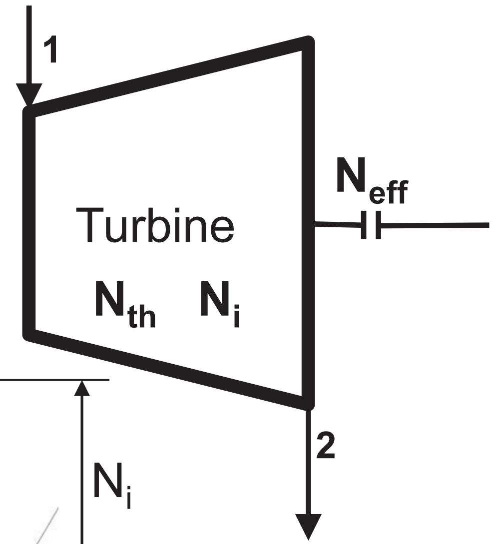


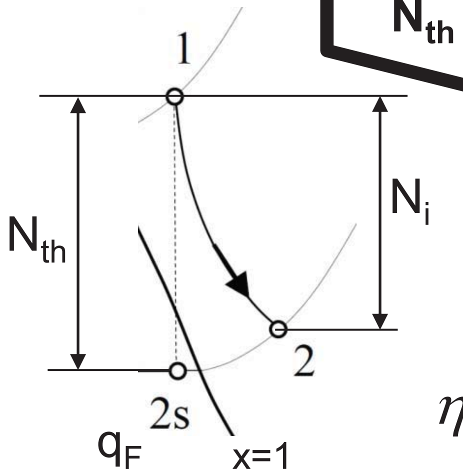




Turbine (2)

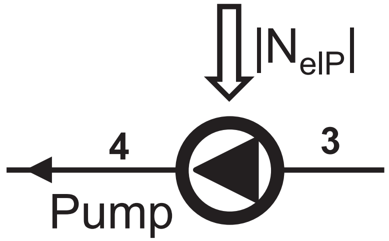
- In modelling a turbine, we usually know the following data:
  - pressure and temperature of steam at the inlet: p_1, T_1
  - pressure of steam at the exit: p_2
  - internal efficiency of turbine η_i
- Our task is to find the specific enthalpy of the steam at the exit, i_2 and N_i. The solution is:
  - 1) we find $i_1(p_1,T_1)$: using XSteam, we have: $i_1 = \text{XSteam}('\text{h\_pT}', p_1, T_1)$
  - 2) we find $s_1(p_1,T_1)$ as $s_1 = \text{XSteam}('\text{s\_ph}', p_1, i_1)$ and next we find i_{2s} as (3) $i_{2s} = \text{XSteam}('\text{h\_ps}', p_2, s_1)$ and finally (4) $i_2 = i_1 - \eta_i\left(i_1 - i_{2s}\right)$





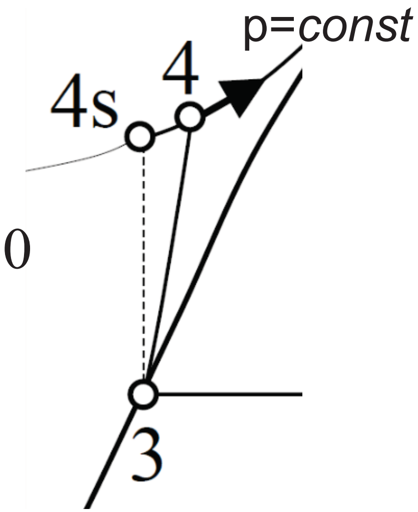
$$\eta_i = \frac{i_1 - i_2}{i_1 - i_{2s}}$$

Pump (1)



- To increase pressure from 3 to 4 pumping power $|N_{iP}|$ has to be supplied
- From the energy conservation principle for steady-state ($dE_T/dt=0$) we have

$$\frac{dE_T}{dt} = q - N_{iP} + W_3\left(i_3 + e_{P3} + e_{K3}\right) - W_4\left(i_4 + e_{P4} + e_{K4}\right) = 0$$

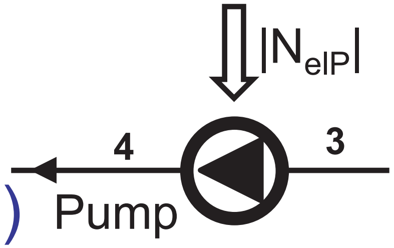


  - here we have to supply power to the system thus $-N_{iP} = |N_{iP}|$, no heat is added thus $q = 0$, we also neglect kinetic and potential energy changes and from mass conservation we have $W_3=W_4=W$

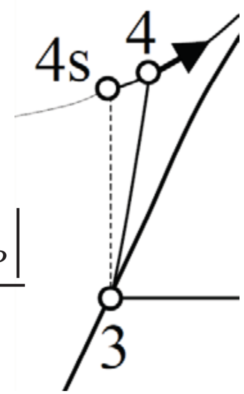
$$|N_{iP}| = W\left(i_4 - i_3\right) = W\left(\underbrace{e_{I4} - e_{I3}}_{\text{internal energy increase}} + \frac{p_4 - p_3}{\rho_e}\right) = \underbrace{W\frac{p_4 - p_3}{\rho_e}}_{N_{uP}=\text{useful pumping power}} + \underset{\text{internal energy increase}}{W\Delta e_I}$$

Here ρ_e is an equivalent fluid density for process 3-4. Typically we assume $\rho_e \approx \rho_3 \approx \rho_4$ (incompressible)

Pump (2)



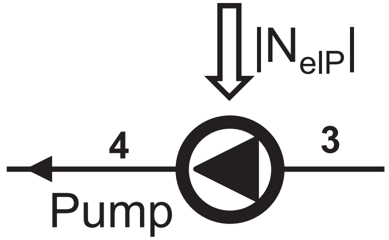
- The pumping power $|N_{thP}|$ is a theoretical pumping power (used in ideal cycle analyses)
- For real process 3-4, we obtain the pumping power $|N_{iP}|$ from energy conservation as $\left|N_{iP}\right| \equiv W\left(i_4 - i_3\right)$ and call this internal power
- Due to internal losses, internal power is: $\left|N_{iP}\right| = \dfrac{\left|N_{uP}\right|}{\eta_{iP}}$
- We also define an effective pumping power, $N_{effP}=N_{mP}$ due to pump mechanical efficiency η_{mP} (= mechanical eff.): $\left|N_{effP}\right| = \dfrac{\left|N_{iP}\right|}{\eta_{mP}}$
- Finally, the electric motor power for pumping is found as: $\left|N_{elP}\right| = \dfrac{\left|N_{effP}\right|}{\eta_{EM}}$



N_{uP} – useful pumping power

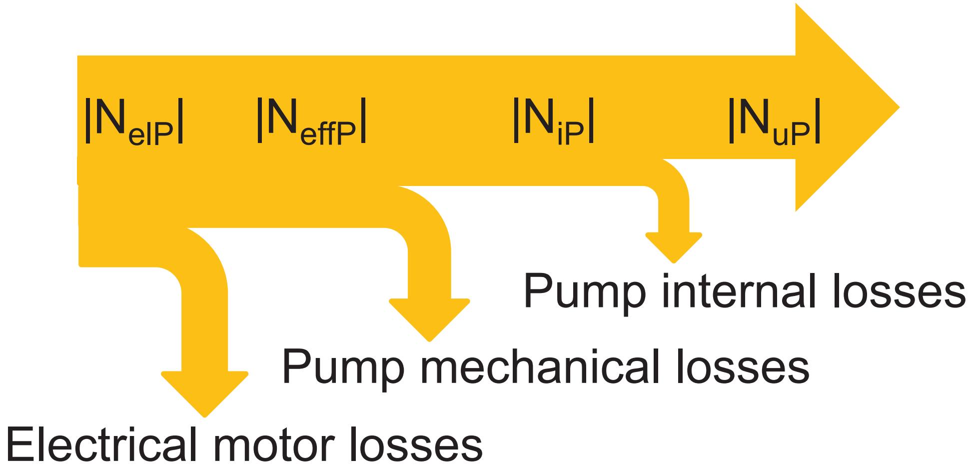
Here η_{EM}: is the electrical motor efficiency

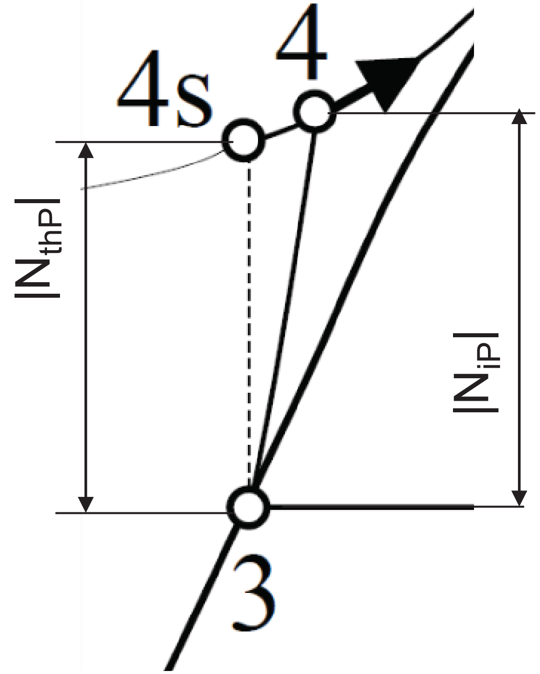
Pump (3)



Pump efficiency:

$$\eta_P \equiv \eta_{iP}\eta_{mP}$$





1) $$\left|N_{elP}\right| = W\frac{p_4 - p_3}{\eta_{iP}\eta_{mP}\eta_{EM}\rho_e}$$

2) $$i_4 - i_3 = \frac{\eta_{mP}\eta_{EM}\left|N_{elP}\right|}{W}$$

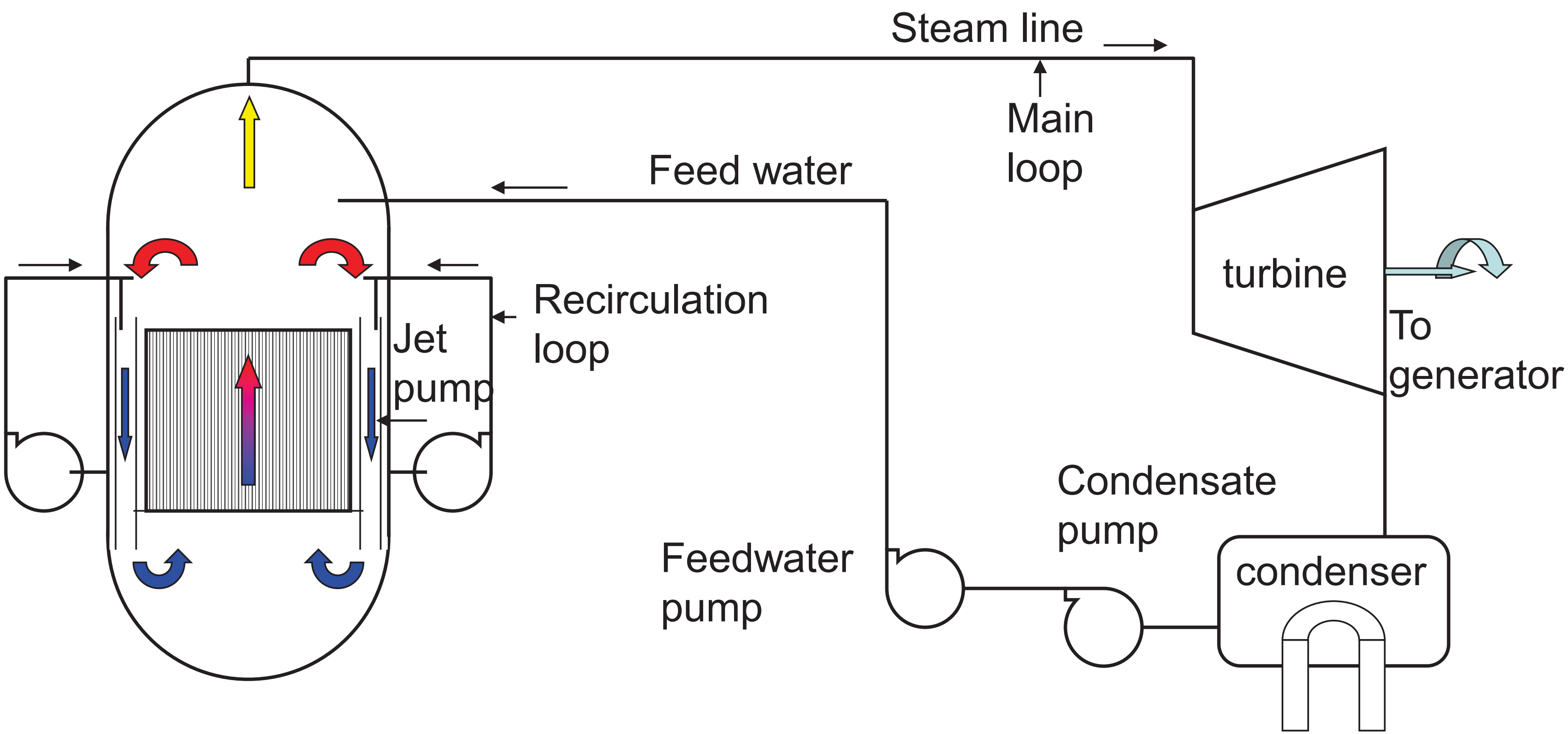
3) $$i_4 - i_3 = \frac{p_4 - p_3}{\rho_e \eta_{iP}}$$

Typical tasks: (**1**) calculate required electrical power to produce given pressure difference; (**2**) calculate specific enthalpy at pump discharge for given electrical power; (**3**) the same as in (2) for given pressure drop

BWR Plant Analysis

- Mass and energy balance can be formulated for the entire reactor pressure vessel (RPV), with separate analysis of:
  - Downcomer
  - Lower plenum
  - Core
  - Steam separators and dryers
- As a result, a consistent flow distribution in the RPV components can be obtained.
- This can be used as an initial state for the further transient analysis.

Schematic of BWR Plant



- Internal flow cannot be calculated with a simple model.

Energy balance in BWR

- We use the Control Volume (CV) approach to formulate the mass and energy conservation for BWR parts
- In general, the equations are as follows:
- Mass conservation

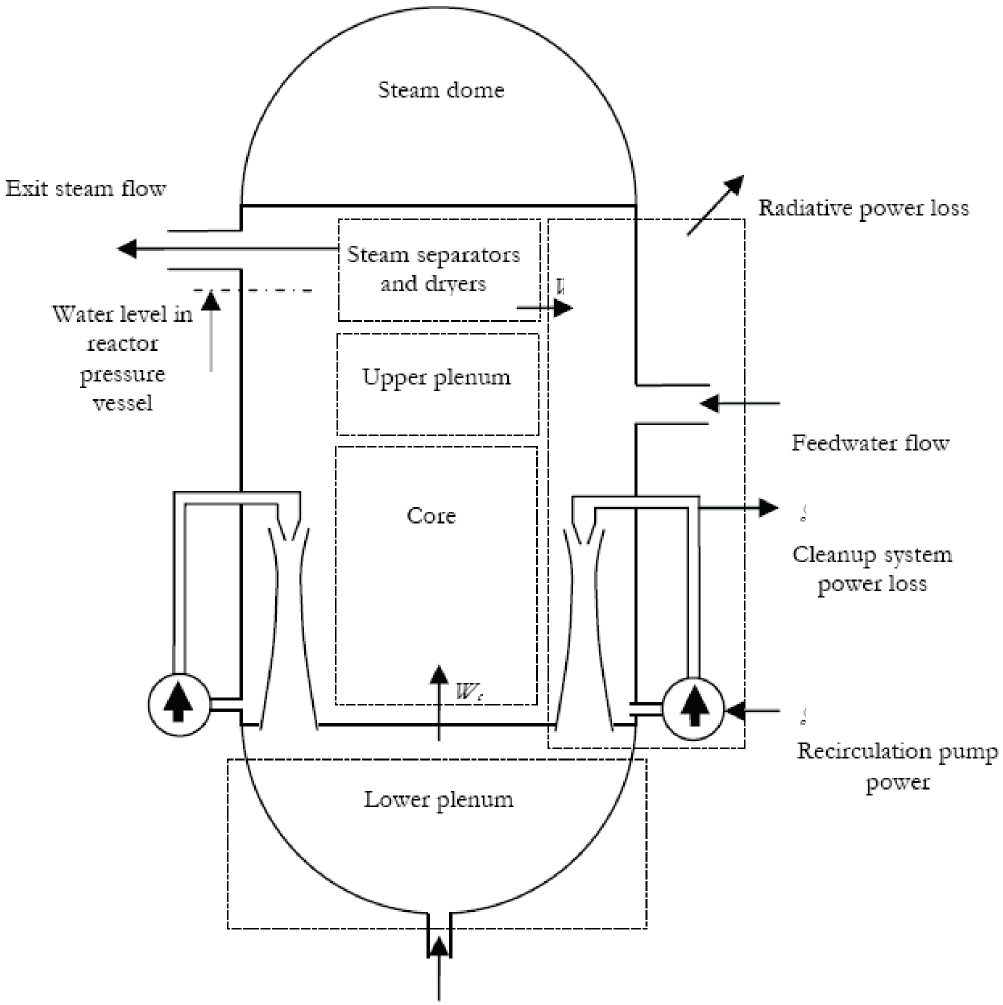
$$\left(\frac{dm}{dt}\right)_{CV} = \sum_{j \in in} W_j - \sum_{k \in out} W_k$$

- where W_j , W_k - mass flow rate through j^{th} inlet and k^{th} outlet.
- Energy conservation

$$\left[\frac{d(me_T)}{dt}\right]_{CV} = q - N_{shaft} - N_{normal} - N_{shear} + \sum_{j \in in} (i + e_P + e_K)_j W_j - \sum_{k \in out} (i + e_P + e_K)_k W_k$$

- where e_T =$e_i + e_P + e_K$ – total specific energy of CV,
- $e_i + e_P + e_K$ – specific internal, potential and kinetic energy, respectively,
- m – mass in CV,
- q – heat/time added,
- N – work/time extracted from CV,
- i – specific enthalpy.

Energy balance in BWR



- A more detailed mass and energy balances can be performed separately in:
  - downcomer (DC)
  - lower plenum (LP)
  - reactor core (RC)
  - separator-dryers (SD)
- see Compendium in Thermal-Hydraulics, Section 5.3.3

General Equations for Steady-State

$$\sum_{j \in in} W_j - \sum_{k \in out} W_k = 0$$

$$q_{th} - q_r - N_{pump} + \sum_{j \in in} i_j W_j - \sum_{k \in out} i_k W_k = 0$$

– Where q_{th} – thermal reactor power,
– q_r – thermal losses due to radiation,
– N_{pump} – pumping power,
– W_j – mass flow rates of inlet streams with specific enthalpy i_j,
– W_k – mass flow rates of outlet streams with specific enthalpy i_k,

- We (usually) neglect kinetic and potential specific energies of inlet/outlet streams.

Energy balance in BWR

- Overall energy balance for the reactor pressure vessel

$$q_{th} - N_P + W_{fw} i_{fw} - W_s i_s + W_{cr} i_{cr} - W_{cl} i_{cl} - q_r = 0$$

q_{th}	– core thermal power,
N_P	– recirculation pump power (negative if added to the system),
q_r	– radiative power loss,
W_{fw}	– feedwater flow rate,
W_s	– steam flow rate,
W_{cr}	– flow rate of the control rod drive system,
W_{cl}	– flow rate to the cleaning system,
i_{fw}	– feedwater specific enthalpy,
i_s	– steam specific enthalpy,
i_{cl}	– specific enthalpy of the cleaning water flow,
i_{cr}	– specific enthalpy of the control rod drive system flow.

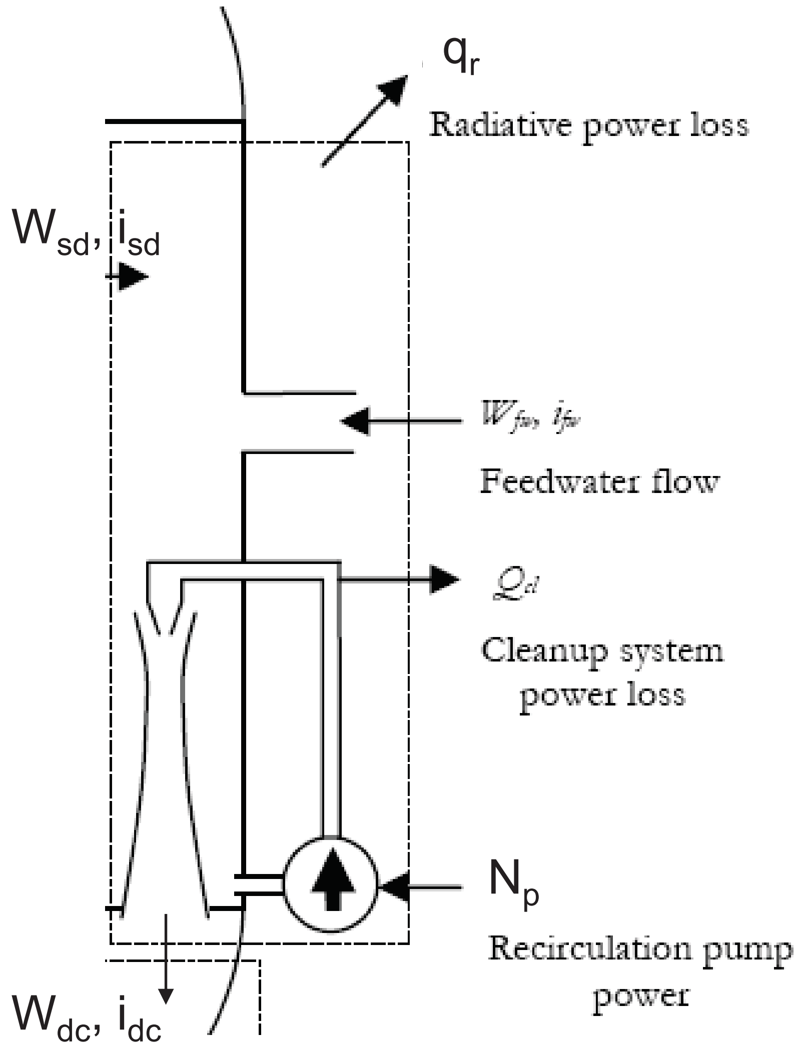
Downcomer

- Mass conservation equation:

$$W_{sd} + W_{fw} - W_{cl} - W_{dc} = 0$$

- Energy conservation equation:

$$W_{sd}i_{sd} + W_{fw}i_{fw} - W_{cl}i_{cl} - W_{dc}i_{dc} - q_r - N_p = 0$$



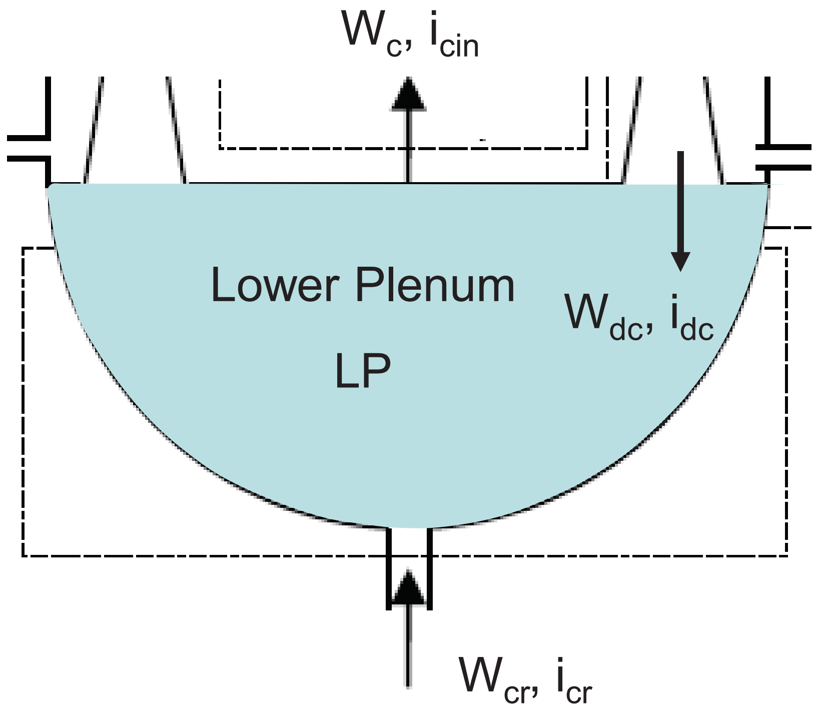
Lower Plenum

- Mass conservation equation:

$$W_{dc} + W_{cr} - W_c = 0$$

- Energy conservation equation:

$$W_{dc} i_{dc} + W_{cr} i_{cr} - W_c i_{cin} = 0$$



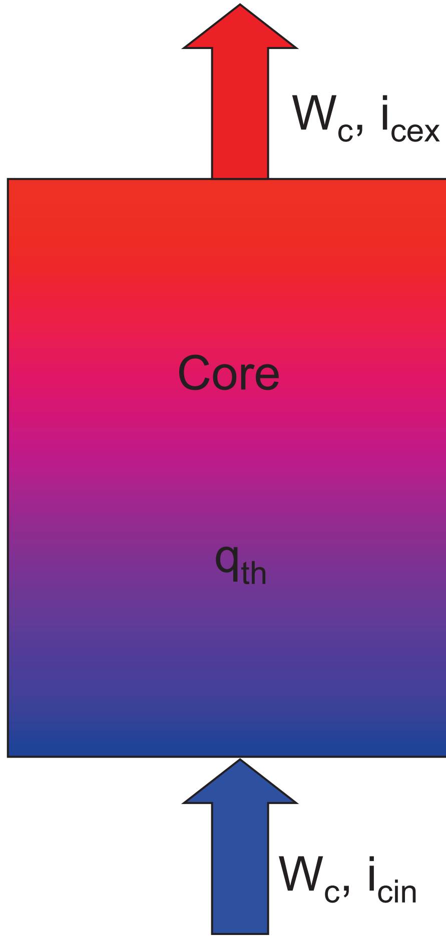
Reactor Core

- Mass conservation equation:

$$W_c - W_c = 0$$

- Energy conservation equation:

$$W_c i_{cin} + q_{th} - W_c i_{cex} = 0$$



Separators and Dryers

- Mass conservation equation:

$$W_c - W_s - W_{sd} = 0$$

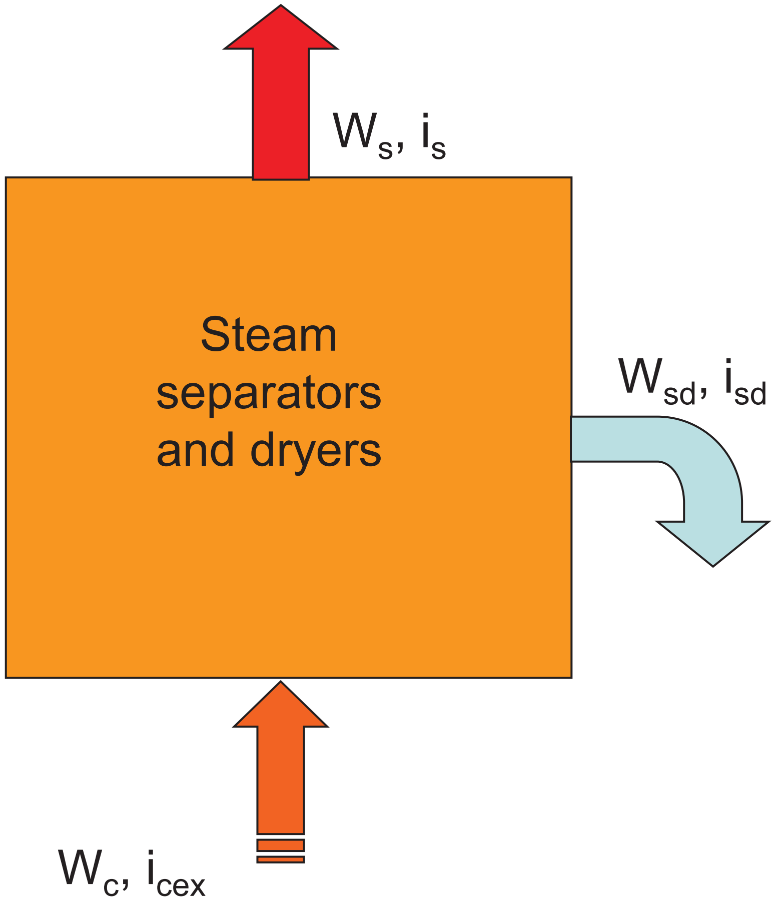
- Energy conservation equation:

$$W_c i_{cex} - W_s i_s - W_{sd} i_{sd} = 0$$

$$i_s = (1 - F_{co}) i_g + F_{co} i_f$$

$$i_{sd} = (1 - F_{cu}) i_f + F_{cu} i_g$$

- Where
  - Carry-over $F_{co} \sim 0.001$
  - Carry-under $F_{cu} \sim 0.0025$



Steam Mass Flow Rate

- The over-all energy balance yields the steam mass flow rate:

$$W_s = \frac{q_{th} - q_r - N_P + W_{cl}(i_{cr} - i_{cl})}{(i_s - i_{fw})}$$

- Neglecting the effect of
  - cleaning and control rod water flow, and
  - assuming that the pumping power is approximately equal to the total heat losses,
- a simplified expression is obtained:

$$W_s \cong \frac{q_{th}}{(i_s - i_{fw})}$$

Steam Mass Flow Rate

- An exact expression for the steam mass flow rate can be obtained from a simultaneous solution of all mass and energy equations in the reactor pressure vessel components:

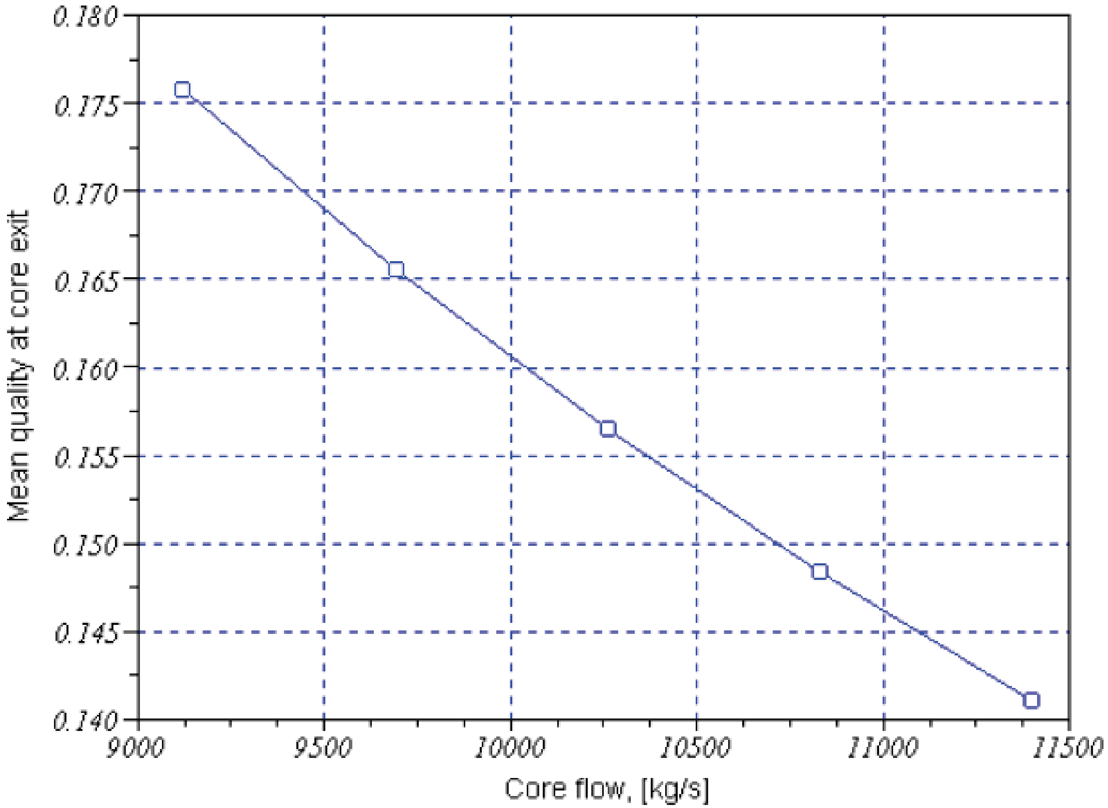
$$W_s = \frac{W_c(q_{th} - q_r) + q_r W_{cl} + W_c(W_{cr} i_{cr} - W_{cl} i_{sd}) - N_p(W_c - W_{cl})}{W_c(i_s - i_{fw}) + W_{cl}(i_{sd} - i_{fw})}$$

q_{th} – core thermal power,
N_P – recirculation pump power (negative if added to the system),
q_r – radiative power loss,
W_{fw} – feedwater flow rate,
W_s – steam flow rate,
W_{cr} – flow rate of the control rod drive system,
W_{cl} – flow rate to the cleaning system,
i_{fw} – feedwater specific enthalpy,
i_s – steam specific enthalpy,
i_{cl} – specific enthalpy of the cleaning water flow,
i_{cr} – specific enthalpy of the control rod drive system flow.

Purpose of Balance Analysis

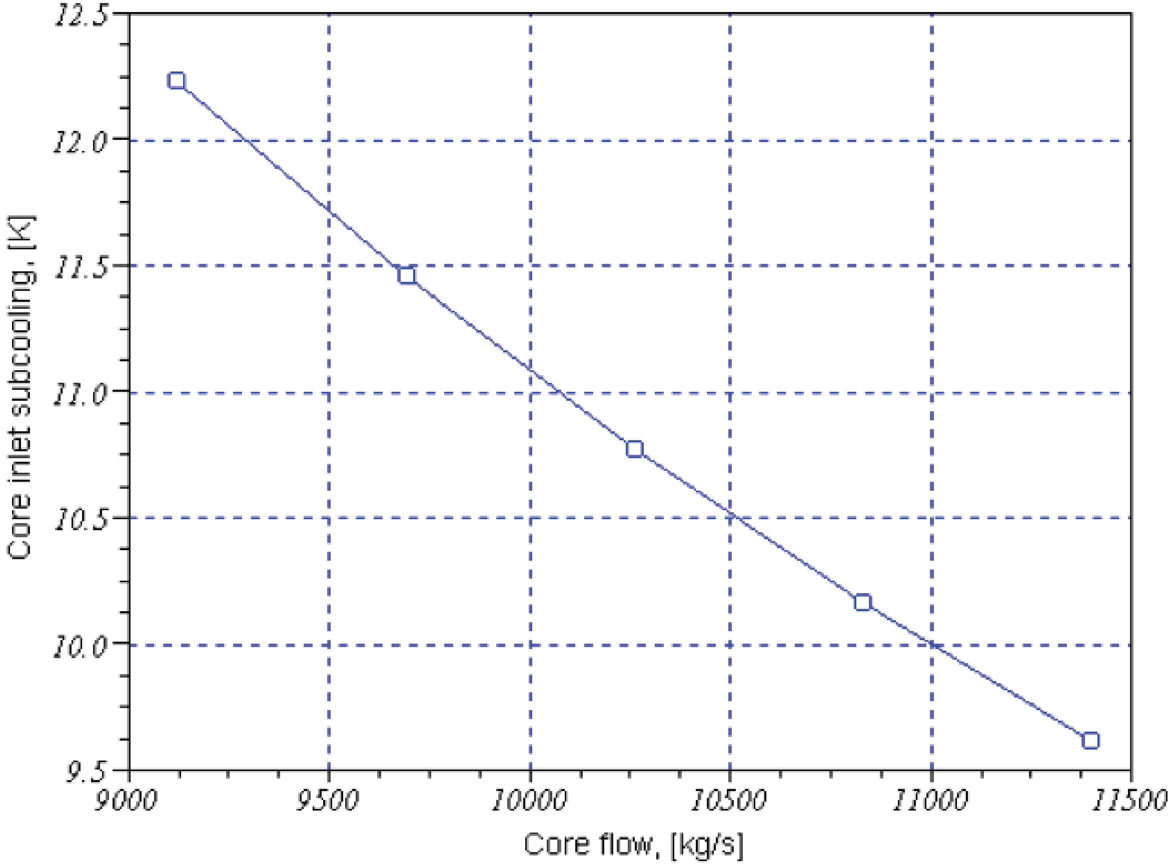
- The purpose of the energy and mass balances is to find a consistent distribution of mass flows and enthalpies in the system.
- For pressure distributions, even the momentum equations have to be solved.
- Such calculations are performed for various power and pressure levels.
- Any transient calculation is then initiated from such a consistent steady-state condition.

Example – Mean quality at core exit versus core flow



- (Keeping all other parameters constant)
- Steam quality $X = \frac{m_{steam}}{m_{steam}+m_{liquid}} = \frac{i-i_l}{i_s-i_l}$

Example – Core inlet subcooling versus core flow



- (Keeping all other parameters constant)

Example – BWR Balance

- Example: Calculate:
  (a) the mean coolant quality at the exit from the BWR core,
  (b) the coolant subcooling at the core inlet
  (c) the steam flow rate from the pressure vessel at steady-state

  assuming the following data:

  reference pressure p = 7 MPa
  core thermal power q_{th} = 3000 MWt
  radiative power loss q_r = 0.1% of q_c
  recirculation pumping power Np = -3.23 MW
  feedwater temperature t_{fw} = 215 °C
  flow rate of the control rod drive system W_{cr} = 65 kg/s
  temperature of water flow to control rod drive system t_{cr} = 60 °C
  carryover fraction F_{co} = 0.001
  carryunder fraction F_{cu} = 0.0025
  coolant flow through the core W_c = 11000 kg/s

Example – BWR Balance

- Solution:

  For pressure p = 7 MPa, the saturation enthalpies for water and steam are:

  i_f = 1267.4 kJ/kg and
  i_g = 2772.6 kJ/kg.

  The entalpy of steam leaving the reactor pressure vessel is thus:

  1267.4 *0.001+ 2772.6*0.999 = 2771.1 kJ/kg.

  Feedwater enthalpy is found from tables (p=7 MPa, T=215 C):

  i_{fw} = 922.2 kJ/kg.

Example – BWR Balance

- Enthalpy of water returning from steam separators and dryers is 1267.4*0.9975+2772.6*0.0025=1271.2 kJ/kg.

- Substituting the data to:

$$W_s = \frac{W_c(q_{th} - q_r) + q_r W_{cl} + W_c(W_{cr} i_{cr} - W_{cl} i_{sd}) - N_p(W_c - W_{cl})}{W_c\left(i_s - i_{fw}\right) + W_{cl}\left(i_{sd} - i_{fw}\right)}$$

- we get $W_s = 1585.4$ kg/s.

- From: $i_{cex} = \frac{W_{sd} i_{sd} + W_s i_s}{W_c}$

- we get $i_{cex} = 1487.4$ kJ/kg, which gives exit quality 0.146.

Example – BWR Balance

- Solution (cont):
- Enthalpy at the core inlet is found from:

$$i_{cin} = i_{cex} - \frac{q_c}{W_c}$$

- $i_{cin} = 1214.4$ kJ/kg.
- From water property tables, the inlet coolant temperature is found as
- T(p=7 MPa, i=1214.4 kJ/kg) = 275.8 °C
- which corresponds to 10 K subcooling.