3) Fast reactor kinetics and reactivity feedbacks

Learning outcomes:

After working through this chapter, you will be able to:

1) Evaluate the impact of fast reactor kinetic parameters on stability of reactors during transients
2) Assess the impact of plutonium on safety parameters of fast reactors
3) Design a fast reactor to make use of leakage for improved safety

Background

In the previous chapter we learnt that a sustainable cycle of breeding nuclear fuel is best achieved in a fast neutron reactor operating on U-Pu fuels. We are now to assess if and how a fast reactor can operate safely using this fuel. By safe operation, we here mean the behavior of the reactor under deviations from nominal operating conditions, induced by perturbations such as reactivity insertions, loss of coolant flow and loss of heat sink. These are so called design basis conditions (DBC) and are expected to occur with a frequency of 10^{-4} - 10^{-2} per year. Any reactor should be designed so that a DBC event does not result in core damage. If a design basis event is combined with the failure of a highly reliable safety system, such as the shut-down system, the event is classified as a design extension condition (DEC). Such events may lead to fuel degradation and should be shown to have a frequency of less than 10^{-4}/year.

Reactivity insertions

Reactivity may be inserted into a fast neutron reactor core in an unplanned fashion, e.g. by inadvertent control rod assembly removal, by gas bubble injection (if the local void coefficient is positive), by fuel assembly bowing or core compaction. In all cases, the power of the reactor will increase, with a rate that is depending on the magnitude $\Delta\rho$ of the insertion, the effective delayed neutron fraction β_{eff} and the effective neutron generation time Λ_{eff}. The latter two are known as "kinetic parameters". In a fast reactor, the neutron generation time is several orders of magnitude shorter than in a thermal spectrum reactor, typically being a fraction of a microsecond. The effective delayed neutron fraction is mostly dependent on the isotopic composition of the fuel. If enriched uranium is used as fuel in a fast reactor, β_{eff} is about the same as in a thermal reactor, but if U-Pu fuel is employed, the delayed neutron fraction is smaller by about a factor of two.

Following the increase in power, the temperature of the fuel, and with a certain delay, the coolant will rise. The reactivity feedback from the fuel will be negative due to the Doppler broadening and axial expansion of the fuel, whereas the feedback from the coolant depends on the balance between spectral hardening and leakage. Once the sum of all reactivity feedbacks is equal (and of opposite sign) to the initial reactivity insertion, power cedes to increase, and a new equilibrium is obtained, where system temperatures approach asymptotic values that are higher than the operational ones.

Loss of flow

The primary coolant mass flow may be reduced as a consequence of failure of pumps or blockage of a coolant channel. In both cases, this will lead to an increase in temperature of the coolant and fuel. If the sum of all reactivity feedbacks is negative, the core becomes sub-critical, power is reduced, and the fuel temperature decreases, which results in a positive contribution to the total reactivity from the fuel. If the latter is sufficiently large, re-criticality may arise during the course of the transient, should shut-down rods not have been inserted. One may observe a damped oscillation of power and temperature leading to a stable asymptotic critical state, where the power produced in the core equals the power that can be removed by natural convection of the primary coolant.

Loss of heat sink

This accident entails that the system for removing heat from the primary system is failing, which could be due to secondary system pump failure or loss of secondary coolant. In difference to the events described above, the first consequence of a loss of heat sink is that the coolant temperature at the inlet of the core increases. Hence, the so-called diagrid, a structure in which the fuel assemblies are positioned, will expand in its radial direction, causing the distance between fuel assemblies to increase. The resulting increase in neutron leakage corresponds to a negative reactivity feedback, inducing sub-criticality and reduction of power. This reduction is enhanced by the increase in fuel temperature. If the coolant temperature coefficient is positive, there is a positive contribution from the coolant to total reactivity while if its negative, the feedback from the coolant enforces the reduction in power. As long as the total reactivity feedback remains negative, core power will approach decay heat levels.

In this case, the asymptotic temperatures of the system depend on how much heat that can be removed by dedicated decay heat removal (DHR) systems. These may be actively actuated by operators, or function in a completely passive mode. An example of the former is a so-called "dip-cooler", connected to a water tank, and the latter may constitute of temperature enhanced radiation of heat from the primary vessel to the environment.

Kinetic parameters and reactivity feedbacks

The final outcome of the above listed DBC/DEC events, in terms of peak and asymptotic system temperatures, will depend on the magnitude of the kinetic parameters and reactivity feedbacks. The main part of this chapter is dedicated to describing the underlying physics of these phenomena, and to understand to what extent core design can be used to affect their magnitude and, in case of the coolant temperature feedback, the sign.

Delayed neutron fraction

"Delayed" neutrons are those that are emitted by unstable fission products having a half-life of up to one minute. The fact that criticality of a reactor includes the contribution of these neutrons, allows ample time for feedbacks and control-mechanisms to act upon a perturbation, including control rod and shut-down rod assembly insertion.

The delayed neutron fraction β is defined as the average number of neutrons emitted by fission products divided by the total number of neutrons released as a consequence of the fission event. The fission products that provide the largest contribution to the delayed neutron fraction are Kr, Br, Rb, I, Xe and Cs. These are elements among the more abundant light and heavy fission products having a relatively low proton number, and therefore a tendency to be neutron rich.

Table 3.1 list the nuclides of the aforementioned elements that feature a beta-decay accompanied by neutron emission, with a half-life in the range of 1.0 to 60 s.

Table 3.1: Fission product precursors of delayed neutrons.

Kr	$t_{1/2}$ [s]	Br	$t_{1/2}$ [s]	Rb	$t_{1/2}$ [s]	I	$t_{1/2}$ [s]	Xe	$t_{1/2}$ [s]	Cs	$t_{1/2}$ [s]
^{92}Kr	1.8	^{86}Br	55.1	^{92}Rb	4.5	^{136m}I	46.9	^{141}Xe	1.7	^{141}Cs	24.5
^{93}Kr	1.3	^{87}Br	55.6	^{93}Rb	5.8	^{137}I	24.5	^{142}Xe	1.2	^{142}Cs	1.7
		^{88}Br	16.3	^{94}Rb	2.7	^{138}I	6.5			^{143}Cs	1.8
		^{89}Br	4.4			^{139}I	2.3			^{144}Cs	1.0
		^{90}Br	1.9								

With a few exceptions, the larger the neutron number, the shorter is the half-life of the neutron emitting fission product.

Figure 3.1 shows the mass distribution of fission products for ^{235}U and ^{239}Pu. As may be observed, the average mass of the of heavy fission products is almost constant, whereas the mass of the light fission product increases when going from the lighter to the heavier mother nuclide. The reason for this is the presence in the fission product spectrum of the so called "doubly magic" nucleus with proton number of 50 and neutron number of 82, yielding an anomalous stability of fission products with mass number 132.

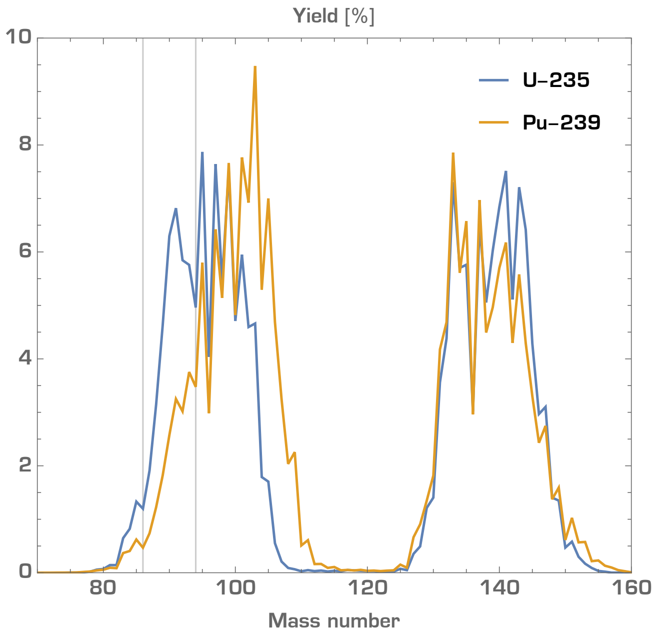

Figure 3.1: Fission product mass yield distribution for fast neutron fission of ^{235}U and ^{239}Pu. Vertical lines indicate the mass range of light fission product delayed neutron emitters

In particular, we may note that, due to the light fission product mass shift, the yield of krypton, bromine and rubidium isotopes in fission of ^{239}Pu is much smaller than for ^{235}U. Hence, we may expect that the delayed neutron fraction should be smaller in reactors operating on U-Pu fuels. This is also the case in reality. Table 3.2 lists the delayed neutron fraction in fission of uranium and plutonium isotopes.

Table 3.2: Delayed neutron fraction in fission of uranium and plutonium nuclides in a fast spectrum

Mother nuclide	ν_{tot}	$\beta = \nu_d/\nu_{tot}$
^{235}U	2.46	0,66 %
^{238}U	2.79	1,89 %
^{239}Pu	2.94	0,22 %
^{240}Pu	3.06	0,31 %
^{241}Pu	2.99	0,52 %
^{242}Pu	3.14	0,70 %

Further, one may note that isotopes of a given element having higher neutron number feature a higher delayed neutron fraction, which may be expected based on their relative neutron surplus.

Effective delayed neutron fraction

β_{eff} is defined as the fraction of fissions induced by delayed neutrons. Figure 3.2 compares the energy spectrum of delayed neutrons emitted from fission products with that of prompt neutrons.

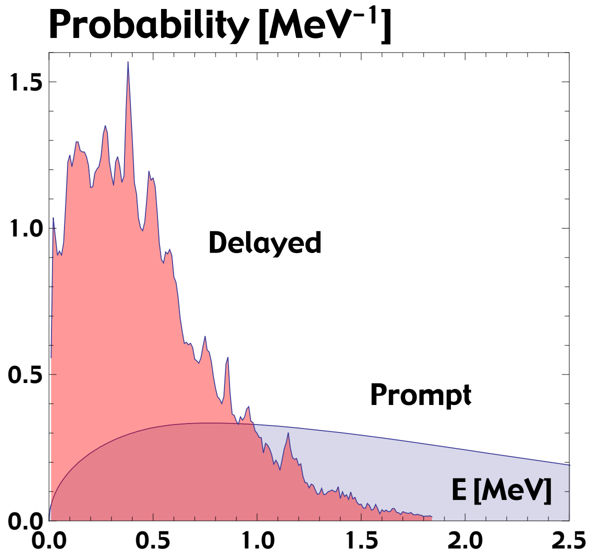

Figure 3.2: Energy spectrum of delayed and prompt fission neutrons

Since the spectrum of delayed neutrons is softer than that of the prompt, they will induce fission with a different probability. In a thermal spectrum reactor, the probability for delayed neutrons to avoid resonance capture in ^{238}U is larger than for prompt neutrons, hence β_{eff} is larger than β. In a fast spectrum the situation is more complex. Since the fission probability of ^{239}Pu is relatively flat in the energy region of both delayed and prompt neutrons, $\beta_{eff} \approx \beta$ for pure U-Pu fuels. However, for Generation-IV fuels containing minor actinides, the probability for delayed neutrons to be captured by americium is larger than that of prompt neutrons, resulting in $\beta_{eff} < \beta$, as will be detailed in the following chapter.

Neutron generation time

The effective neutron generation time Λ_{eff} is defined as the average time from one fission event to the next. In a fast spectrum reactor, Λ_{eff} typically is a fraction of a microsecond, which is three orders of magnitude shorter than in a thermal reactor, since there is much less time spent on neutron slowing down. It should be noted that the neutron generation time may be substantially different from the effective neutron life-time l_{eff}, which is the average time from the birth of a neutron in fission to its absorption. Namely, in reactors with non-absorbing reflectors, neutrons may spend a relatively long time slowing down in the reflector before returning to the core, where they will be captured with a larger probability than neutrons that did not leak out from the core. Such neutrons will contribute to the neutron population growth in a supercritical core to a much lesser extent. Moreover, Λ_{eff} differs slightly form Λ, which is the generation time for prompt neutrons.

Doppler feedback

Doppler broadening of capture resonances in even neutron number actinides is an important safety mechanism in fast neutron reactors. Figure 3.4 shows how the shape of the ^{240}Pu capture resonance at 1 eV changes with temperature. The broadening occurs in the lab system as a result of increasing amplitude of thermal oscillations of atoms in the crystal lattice of the fuel. The magnitude of the broadening of an harmonic oscillator is of the order of the square root of relative increase in thermal energy, which when increasing the fuel temperature from 1000 K to 2000 K is $(k_B \Delta T)^{1/2} \approx 0.01$ eV. We may compare that with the width of the resonance, which is about 0.1 eV for the lowest energy ^{238}U capture resonance (at $E_n = 6.5$ eV) and 2-5 eV for resolved resonances in the fast reactor spectrum range (1 - 20 keV). Hence, Doppler feedback is more effective for resonances located at lower neutron energies. Consequently, the magnitude of the Doppler coefficient is smaller in fast reactors (≈ -0.5 pcm/K) than in thermal spectrum reactors (≈ -2 pcm/K).

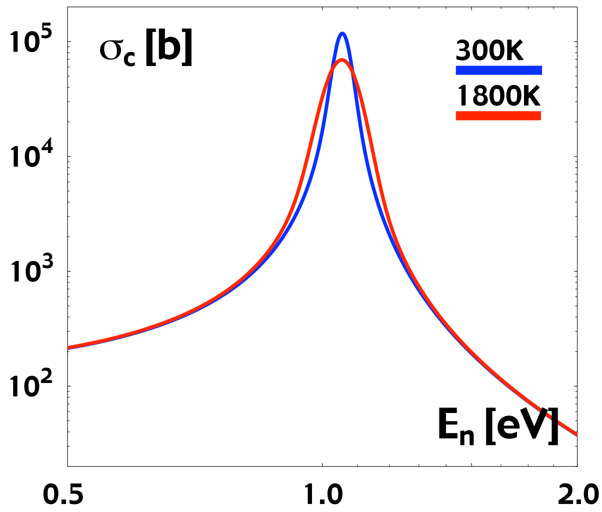

Figure 3.3: Capture cross section of ^{240}Pu as function of fuel temperature

It is to be noted that the area under the resonance cross section remains constant, which is why the Doppler effect can not be calculated by integrating a constant neutron spectrum folded with the capture cross section over the energy range of the spectrum. Rather, when neutrons slow down in a fuel that is heating up, they will encounter a higher cross section for capture on the high energy flank of the resonance. Therefore, the probability of capture increases at this point, leading to fewer neutrons reaching the energy of the resonance peak, where the probability for capture is decreasing. The net effect is a reduction in reactivity.

The Doppler coefficient α_D is defined as the derivative of reactivity with respect to the temperature of the fuel:

$$\alpha_D \equiv \frac{d\rho}{dT_f} = \frac{1}{k^2}\frac{dk}{dT_f}.$$

Figure 3.4 displays the temperature dependence of the k_{eff} in a fast reactor.

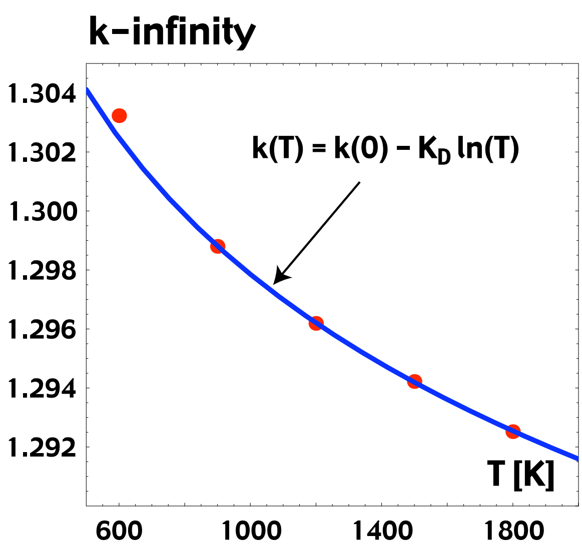

Figure 3.4: Temperature dependence of k_{eff} in a fast reactor.

The calculation shows that in-between 900 and 1800 Kelvin, reactivity dops logarithmically with temperature and we may write

$$\rho(T) = \rho(0) + K_D \, ln(T_f),$$

where K_D is the "Doppler constant". Thus, we have for K_D:

$$K_D = T \frac{d\rho}{dT},$$

The Doppler constant is usually expressed in the unit of *pcm*. For the Doppler coefficient one obtains:

$$\alpha_D = \frac{K_D}{T_f}.$$

One should note that the logarithmic dependence of reactivity with respect to temperature in a fast reactor is incidental. E.g., in a thermal spectrum reactor, the dependence is approximately proportional to the square root of temperature. However, within the range of typical fast reactor designs, the logarithmic dependence is a good approximation, even for reactors with harder neutron spectra, such as gas-cooled fast reactors or reactors using metallic alloy fuels.

Fuel axial expansion

When a nuclear fuel column heats up, thermal expansion will lead to a reduction in its density. In the same time, the height of the fuel column increases. The diameter of the fuel pellets or slugs may also increase, if the fuel cladding does not restrain the differential thermal expansion of the fuel and the clad. The axial expansion of the fuel leads to an increase in neutron leakage from the core in the radial direction, since the height of the active zone increases. On the other hand, the radial expansion of the fuel does not lead to an increase of the axial leakage, since the diameter of the active zone remains unchanged. Therefore, the reduction of reactivity associated with reduced density of the fuel is entirely due to the axial expansion.

This phenomenon is negligible in commercial thermal spectrum reactors, due to their larger size, in conjunction with a much smaller neutron mean free path. In fast reactors, the contribution of leakage to total reactivity is proportional to the ratio between surface and volume, that is inversely proportional to radius. Hence, in large cores, the axial expansion reactivity feedback is of second order, whereas in small cores, it can be comparable to the fuel Doppler feedback and the coolant temperature feedback. The effect is accentuated in cores with height to diameter ratio H/D > 0.5, since the radial leakage is larger than axial leakage when the height of the core is larger than its radius.

The response time of the axial expansion is determined by the velocity of sound in the solid, which is of the order of 5000 m/s. Considering that the length of a fuel column in a fast reactor is about 1 m, the time constant for axial expansion is about 0.1 milliseconds. This is two orders of magnitude slower than the time constant for the Doppler effect, but still considerably faster than any other reactivity feedback. For practical purposes, we many consider fuel axial expansion to be a prompt reactivity feedback.

Coolant temperature coefficient

In fast rectors, the coolant slows downs neutrons by elastic scattering on sodium or by inelastic scattering on lead. A reduction in density leads to a harder spectrum, which reduces the amount of captures in even neutron number actinides, and hence results in an increase in reactivity. On the other hand, a reduced density also increases leakage of neutrons and a reduction of capture in the coolant. As discussed in the context of fuel axial expansion, the leakage effect is inversely proportional to the radius of the core. Hence, for large cores with U-Pu fuels having conversion ratio equal to or larger than unity, the coolant temperature coefficient tends to be positive, whereas for smaller cores it may be negative. Large cores with a conversion ratio below unity may also have a negative coolant temperature coefficient. Such cores are obviously burning plutonium, and are therefore not qualified to meet Generation IV criterion number one.

In the literature, it is sometimes argued that designing the core in the shape of a ”pancake”, i.e. with a small H/D ratio is beneficial for reducing the magnitude of the coolant temperature coefficient. However, this is mainly an effect of having to increase the fissile fraction of nuclides in the fuel when the surface to volume ratio of the core increases. Keeping the conversion ratio constant when reducing the height to diameter ratio, one would have to increase the total volume of the core to retain criticality, effectively eliminating any effect from enhanced leakage.

The response time for thermal expansion of the coolant is governed by the speed of sound which is about 1800 m/s in liquid lead, and 2500 m/s in liquid sodium.

One should keep in mind that the coolant temperature coefficient is always negative outside of the active zone of the core.

Diagrid radial expansion

In a liquid metal cooled fast reactor, the feet of the fuel assemblies are fixed by inserting them into a ”diagrid”, which consists of two vertically separated circular steel plates with holes for insertion of fuel assembly feet. When heating up the coolant in the diagrid, it will expand in the radial direction, leading to an increasing size of the the gap between the fuel assemblies. Consequently, axial neutron leakage from the core will increase. This also means that, the magnitude of the diagrid radial expansion coefficient may be raised by reducing the height to diameter ratio of the core.

It is to be noted that in the cases of a reactivity insertion or a loss of flow accident, this feedback is delayed with respect to the other mechanisms already discussed, since it requires that the increase in active zone coolant temperature is transplanted to the inlet region of the core. Depending on the coolant inventory (i.e. heat capacity) in hot and cold legs, this may take 100s of seconds. Thus, TOP and LOF transient peak temperatures are rarely affected by the magnitude of the radial expansion coefficient. Nevertheless, since the radial expansion coefficient often is of the same magnitude, and sometimes is even larger than other reactivity coefficients, it plays an important role for the asymptotic system temperatures in these transient scenarios.

In the case of loss of heat sink, diagrid radial expansion will be the first reactivity feedback to act, since the increase in coolant temperature at the outlet of the heat exchanger or steam generator is directly transplanted to the inlet region of the core.

The point kinetic approximation

Having defined the kinetic parameters and the reactivity feedbacks operating in a fast reactor, we may write down the point kinetic equations for the power evolution:

$$\frac{d\dot{Q}}{dt} = \frac{dn(t)}{dt} = \frac{\rho(t) - \beta_{eff}}{\Lambda_{eff}} n(t) + \sum_{i=1}^{k} \lambda_i C_i(t)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda_{eff}} n(t) - \lambda_i C_i(t), \; \beta_{eff} = \sum_{i=1}^{k} \beta_i$$

where n is the neutron population in the core, C_i is the concentration of the delayed neutron precursors listed in Table 3.1, λ_i is their decay rate and β_i their respective contribution to the effective delayed neutron fraction. Often, precursors with similar half-life are grouped together, and computer codes solving the point kinetic equation use six or eight precursor groups.

The point kinetic equation relies on the approximation of having a constant shape of the neutron flux in the core during the transient. This approximation is better valid in fast reactor than in a thermal reactor, due to the larger neutron mean free path, is of the order of *cm* in the former, whereas that of a thermal neutron is of the order of *mm*.

The reactivity feedbacks may be coupled to the point kinetic equation through the following expression for the change in reactivity:

$$\delta\rho = K_D \, ln\left(\frac{\bar{T}_{fuel}(t)}{\bar{T}_{fuel}(0)}\right) + \alpha_{axial}\delta\bar{T}_{fuel}(t) + \alpha_{coolant}\delta\bar{T}_{coolant}(t) + \alpha_{radial}\delta\bar{T}_{diagrid}(t)$$

where $\bar{T}_{fuel}$ and $\bar{T}_{coolant}$ are averaged over the active zone of the core. Contributions to reactivity from e.g. a temperature change of the coolant outside of the active zone may be treated by adding specific terms.

Reactivity coefficients and core design

In the design of Generation IV reactors, we aim at achieving a conversion ratio for U-Pu fuels in a fast spectrum larger than unity, in a core that can sustain a burn-up at least equal to the LWR industry standard of 5%. The design shall be such that passive safety mechanisms shall be sufficient to avoid core damage in case of DBC transients and to ensure limited fuel degradation in case of DEC transients. Here, we intend to explore how core design may be advanced for the purpose of optimizing reactivity coefficients.

In Table 3.3, we display the effective delayed neutron fraction, Doppler constant and reactivity coefficients calculated for lead-cooled reactors with $(^{238}U_{0.875},Pu_{0.125})^{15}N$ fuel, having k_{eff} = 1.0 and an in-pile conversion ratio equal to 1.43 at BoL. This conversion ratio allows to reach a core averaged fuel burn-up of 5% with a reactivity swing of less than 300 pcm. Since the swing is less than the effective delayed neutron fraction, the cores can not be taken to prompt criticality even if ejecting the entire control rod bank worth. The two cores here studied differ in terms of fuel column height and the number of fuel assemblies, which is translated into an effective active zone diameter. Cores with an H/D ratio closer to unity have a smaller critical mass, and consequently a lower power.

Table 3.3: Effective delayed neutron fraction, Doppler constant and reactivity coefficients as function of height to diameter ratio, for LFRs with $(^{238}U_{0.875},Pu_{0.125})^{15}N$ fuel and an in-pile conversion ratio at BoL equal to 1.43.

Dimensions	H/D = 0.25	H/D = 0.70
Parameter	H = 1.00 m	H = 1.60 m
β_{eff}	400 pcm	400 pcm
K_D	−820 pcm	−790 pcm
α_{axial}	−0.12 pcm/K	−0.16 pcm/K
α_{Pb}	+0.41 pcm/K	+0.33 pcm/K
$\alpha_{diagrid}$	−0.49 pcm/K	−0.42 pcm/K

Consistent with our previous discourse, the fuel axial expansion coefficient becomes more negative for the core with a larger H/D ratio, whereas the diagrid radial expansion coefficient becomes less negative. Moreover, the coolant temperature coefficient is less positive, underlining the fact that adopting a "pancake" core design does not mitigate the positive coolant temperature coefficient, unless the U/Pu ratio is adjusted to yield a lower conversion ratio. Let us now investigate this option.

Table 3.4 lists the safety parameters of critical lead-cooled reactors with a $(^{238}U_{0.833},Pu_{0.167})^{15}N$ fuel yielding an in-pile conversion ratio of 1.15. In this case, the reactor looses about 2000 pcm reactivity per percent burn-up.

Hence control rod assemblies with a total worth of 10 000 pcm have to be inserted into the core at BoL, in order to reach a final burn-up of 5%. Based on lessons learnt in the SL-1 accident, it is required that withdrawal of the most efficient control rod assembly should not result in prompt criticality of a reactor. In the present case, in order for a single control rod assembly to be worth less than a dollar, one needs to design the core with at least 25 control rod assemblies.

Table 3.4: Doppler constant and reactivity coefficients as function of height to diameter ratio, for LFRs with $(^{238}U_{0.833},Pu_{0.167})^{15}N$ fuel and an in-pile conversion ratio at BoL equal to 1.15.

Dimensions	H/D = 0.16	H/D = 0.50
Parameter	H = 0.62 m	H = 1.15 m
β_{eff}	400 pcm	400 pcm
K_D	-720 pcm	-590 pcm
α_{axial}	-0.12 pcm/K	-0.20 pcm/K
α_{Pb}	+0.28 pcm/K	+0.11 pcm/K
$\alpha_{diagrid}$	-0.53 pcm/K	-0.44 pcm/K

As we can see, the coolant temperature coefficient is less positive for fuels with a lower conversion ratio and approaches zero when increasing the H/D ratio of the core. The Doppler constant is slightly smaller in magnitude than for the higher conversion ratio cores, due to the lower rate of neutron capture in ^{238}U.

Questions:

- Explain why the delayed neutron fraction is small for plutonium bearing fuels.
- Why is the Doppler feedback smaller in a fast reactor than in a thermal spectrum reactor?
- Explain why the coolant temperature coefficient can be either positive or negative in a fast reactor.
- Why does radial expansion of the fuel not affect reactivity?
- How can one design a fast neutron core to optimize feedback from fuel axial expansion and diagrid radial expansion?
- From the perspectives of safety, what are the pros and cons for designing a fast reactor core with high or low conversion ratio?

Exercises:

- Calculate kinetic parameters and reactivity feedback coefficients in sodium cooled reactors with oxide, nitride and metal alloy fuels, using a Monte Carlo code, such as Serpent.
- Assess the minimum number of control rod assemblies required to compensate for reactivity loss, as function of conversion ratio, considering a burn-up target of at least 50 GWd/ton.