## Solutions, exam Oct. 26, 2009

### Problem 1

Consider one dislocation enclosed in a $l \times l$ area. From the figure, we have $\frac{l-b}{R} = \frac{l}{R+l}$. A little algebra, and ignoring the term $lb$, we have $l^2 = Rb$. We have $l^2 = 1/\rho$, which leads to $\rho = 1/(Rb)$.

Insering numbers, we have, $R \approx 0.1$ cm, $b = 2.5 \times 10^{-8}$ cm lead to $\rho = 4 \times 10^8/\text{cm}^2$, or $\rho = 4 \times 10^{12}/\text{m}^2$.

### Problem 2

Total length of dislocations $l = 10^{16}$ m, which is close to one light year!

### Problem 3

For neutrons in Fe, we have $\gamma = 0.069$. Then, $\frac{\gamma E_i}{4E_D} = 430$ displacements per neutron. Rate of displacements $R = N\sigma\frac{\gamma E_i}{4E_D}\phi = 1 \times 10^{17}$ displacements per cm$^3$ and second, or $1.3 \times 10^{-6}$ dpa/s, or about 30 dpa/year.

### Problem 4

i) The distribution of recoil energies is not changed (there is no $T$ dependence in Eq. 2). Therefore, the KP prediction of the number of displaced atoms is unchanged.

ii) The cross section increases with decreasing energy $E_i$.

ii) The mean-free path correspondignly increases ($\lambda = 1/(\sigma\rho)$, where $\rho$ is the density).

### Problem 5

i) Direct integration yields

$$\bar{T} = \frac{\check{T}\ln(\hat{T}/\check{T})}{1-\check{T}/\hat{T}} \tag{1}$$

For high energies $E_i$ of the incoming particle, and using $\check{T} = E_D$ we can write

$$\bar{T} = E_D \ln(\gamma E_i/E_D). \tag{2}$$

ii) By integration

$$\sigma(E_i) = \frac{\pi b_0^2}{4}\left(\frac{\hat{T}}{E_D} - 1\right), \tag{3}$$

which for high energies simplifies to

$$\sigma(E_i) = \frac{\pi b_0^2}{4}\frac{\gamma E_i}{E_D}. \tag{4}$$

**Problem 6**

i) The maximum (head-on) energy transferred will be $\gamma E_i$, which is $0.138 \times 2$ MeV $= 276$ keV.

ii) The mean energy transfer is $\bar{T} \approx E_D \ln\left(\frac{\gamma E_i}{E_D}\right) = 232$ eV (Note, much smaller than $\gamma E_i/2$!).

iii) We have $b_0 = \frac{13^2 e}{4\pi\varepsilon_0}\frac{1}{\eta E_i}$, with $E_i$ in eV. I obtain $b_0 = 9.71 \times 10^{-15}$ in SI units.

Plugging into Eq. 4 gives

$$\sigma(E_i) = \frac{\pi 9.71 \times 10^{-15}}{4}\frac{0.138 \times 2 \times 10^6}{25} = 8 \times 10^{-25}\ \mathrm{m}^2 = 8 \times 10^{-21}\ \mathrm{cm}^2. \tag{5}$$

The density in fcc Al is $\rho = 6 \times 10^{22}$ atoms/cm$^3$. Then, the mean free path is $\lambda = 1/(\rho\sigma) = 0.002$ cm or 20 $\mu$m.

**Problem 7**

For a dislocation segment of lenght $l$ subject to a shear stress $\tau$ we have a force $F_s = \tau l b$.

For the line tension we have $F_T = Gb^2/2$.

For Orowan looping, the critical situation is when the dislocation drag acts in the forward direction, i.e., when the angle between the dislocation at the pinning point, and the line connecting the pinning point, is $\pi/2$. Then, we have $F_s = F_T$ leads to $\tau = \frac{Gb}{2d}$ where $d$ is the distance between particles.

Inserting numbers, one finds

$\tau = 84 \times 10^9 \times 2.5 \times 10^{-10}/0.1 \times 10^-6 = 210$ MPa.