Monte Carlo Methods and Simulations in Nuclear Technology

Non-Analog Monte Carlo Simulations

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Non-Analog Monte Carlo Methods

These methods try to increase the FOM of the MC transport calculation by optimising the actual neutron transport. Non-analog methods thus belong to variance reduction techniques.

Principle of Non-Analog Monte Carlo Techniques

The pdf of some random variables are purposely biased (changed) to increase occurrence of desired events. This must be compensated by a change in the "statistical weight" w of the simulated neutron so that the obtained results are unbiased.

Implicit capture (survival biasing)

The implicit capture is useful mainly in so-called shielding calculations where neutrons have to penetrate massive blocks of absorbing materials to collide in the detector. Analog simulations have to simulate extremely large number of neutrons to give statistically credible results. The non-analog transport simulates the capture (and fission) implicitly by reducing the statistical weight of neutrons at each collision, and neutrons are forced to scatter at each collision. The neutron is killed only when the weight drops below the weight cutoff limit w_{limit} (by the Russian Roulette rule - see next slides).

Comment

Other major non-analog (variance reduction) methods will be covered in the next lecture.

Implicit capture - details

- Each neutron starts with the statistical weight of $w = 1$.
- Neutrons are forced to scatter at each collision, thus, reaction type does not need to be sampled.
- The neutron statistical weight is changed after each collision as

$$w = w' \frac{\Sigma_s}{\Sigma_t}.$$

  (w' is the weight before the collision.)
- When w drops below w_{limit} after some collision then Russian Roulette rule is applied, and the neutron is either killed or its weight is increased and the simulation continues.

Russian Roulette rule

If the neutron weight falls below the weight cutoff w_{limit} then a random number u is generated from $U(0,1)$ and compared to $1/d$ where d is a number in a range of [2,10]. (Often $1/d$ is set to w_{limit}) The faith of the neutron is determined according to the following procedure:

- if $u > 1/d$ then the neutron history is terminated,
- if $u \leq 1/d$ then the neutron weight is increased by a factor d and the neutron history continues to be simulated.

Comments

- Setting a small value of w_{limit} allows the neutrons to penetrate deeper in absorbing materials, and makes the shielding calculations possible.
- The statistical weight of neutrons has to be always considered when sampling the result.
- The value of w_{limit} should not be small in non-shielding calculations otherwise the computational time will be wasted on simulating of statistically insignificant neutrons, and FOM will decrease compared to analog simulations.

Non-analog Fission

Non-analog Fission

Simulation of the fission in non-analog simulations

In non-analog simulations the fission reaction is simulated implicitly. The expected number of fission neutrons $\bar{S}(E)$ is sampled at each collision by the collision or track-length estimator.

Collision estimator

At every collision the number of fission neutrons is computed as:

$$\bar{S}(E) = w\nu(E)\frac{\Sigma_f(E)}{\Sigma_t(E)}$$

where $\nu(E)$ is the average number of fission neutrons in an (analog) fission reaction.

Non-analog Fission

Track-length estimator

At every collision the number of fission neutrons is computed as:

$$\bar{S}(E) = wd\nu(E)\Sigma_f(E)$$

where d is the distance between the last two collisions.

Sampling the number of fission neutrons

Knowing $\bar{S}(E)$ we can sample the actual number of fission neutrons $S(E)$ at the collision site using a random number u (generated from $U(0,1)$) as

$$S(E) = \mathrm{INT}(\bar{S}(E) + u)$$

where $\mathrm{INT}(x)$ gives the closest integer number to x that is lower than x.

Tallying Procedures

Tallying Procedures

Estimators

In many situations, we wish to calculate the total flux $\phi(\vec{r}, E)$ (angular flux integrated over all directions) and the reaction rate $R_i(\vec{r})$ (concentration of i-th type of reactions at $\vec{r}$). For this purpose we may employ:

- the collision estimator,
- the path-length estimator,
- or the surface-crossing estimator.

Simplifications

We have to partition (discretize) the spatial and energy domain into I and J bins, respectively, and count neutrons within each bin. (We cannot easily calculate the flux at a specific point in space or energy.)

Tallying Procedures for neutron flux

Collision estimator for neutron flux

The collision estimator for flux uses a counter FC that increases its value as

$$FC(i,j) \leftarrow FC(i,j) + \frac{w}{\Sigma_t(E)}$$

every time when neutron with energy in j-th energy interval collides in space bin i (w is the statistical weight of the neutron).

At the end of the calculation, the total flux is computed as

$$\phi(V_i, E_j) = \frac{FC(i,j)}{h\Delta V_i \Delta E_j}$$

where h is the total number of histories in active cycles.

Tallying Procedures for neutron flux

Path-length estimator for neutron flux

The path-length estimator for flux uses a counter FP that increases its value as

$$FP(i,j) \leftarrow FP(i,j) + w \times p$$

every time when neutron with energy in j-th energy interval travels a distance p in space bin i.

At the end of the calculation, the total flux is computed as

$$\phi(V_i, E_j) = \frac{FP(i,j)}{h \Delta V_i \Delta E_j}$$

where h is the total number of histories in active cycles.

Tallying Procedures for neutron flux

Surface-crossing estimator for neutron flux

The surface-crossing estimator for flux uses a counter FS that increases its value as

$$FS(i,j) \leftarrow FS(i,j) + \frac{w}{|\cos\theta|}$$

every time when neutron with energy in j-th energy interval crosses the monitored surface (θ is the angle between the normal vector of the surface and $\vec{\Omega}$).

At the end of the calculation, the total flux at the surface is computed as

$$\phi(V_i, E_j) = \frac{FS(i,j)}{h\Delta A_i \Delta E_j}$$

where ΔA_i is the area of the monitored surface.

Tallying Procedures for reaction rate

Collision estimator for reaction rate

The collision estimator for reaction rate (for reaction c) uses a counter CC that increases its value as

$$CC(i,j) \leftarrow CC(i,j) + w\frac{\Sigma_c(E)}{\Sigma_t(E)}$$

every time when neutron with energy in j-th energy interval collides in space bin i.

At the end of the calculation, the reaction rate is computed as

$$R(V_{i,j}) = \frac{CC(i,j)}{h\Delta V_i \Delta E_j}$$

Tallying Procedures for reaction rate

Path-length estimator for reaction rates

The path-length estimator for reaction rates uses a counter CP that increases its value as

$$CP(i,j) \leftarrow CP(i,j) + w \times p \times \Sigma_c(E)$$

every time when neutron with energy in j-th energy interval travels a distance p in space bin i.

At the end of the calculation, the reaction rate is computed as

$$R(V_{i,j}) = \frac{CP(i,j)}{h \Delta V_i \Delta E_j}$$

where h is the total number of histories in active cycles.