SH2702
Nuclear Reactor Technology

Project work Task 4

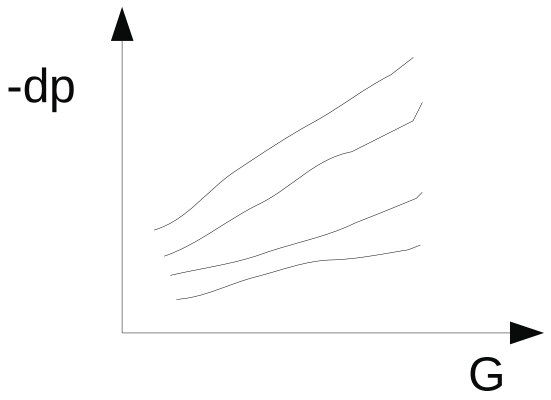
Project work

Topic numbers	Topics
1	Design, operation and safety features of NuSCALE
2	Design, operation and safety features of ABWR
3	Design, operation and safety features of ESBWR
4	Design, operation and safety features of EPR
5	Design, operation and safety features of AP1000

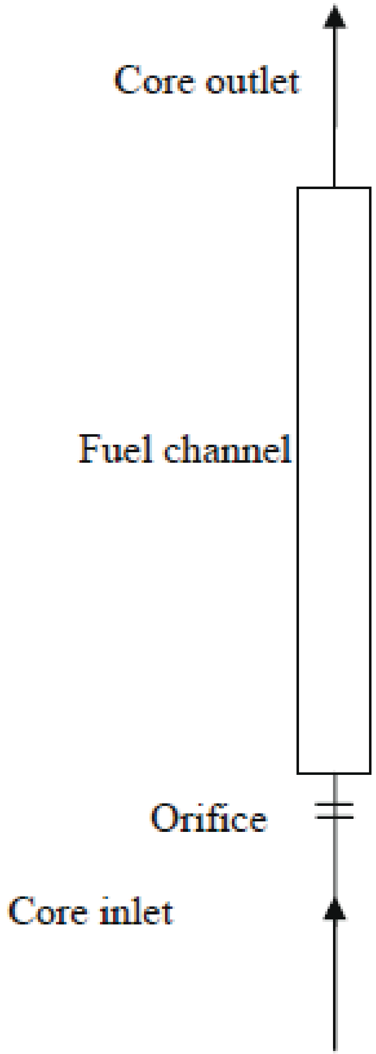
- Task 1 – General design specification of the nuclear power plant with selected reactor type
- Task 2 – Operational principles of the power plant
- Task 3 – Safety features of the power plant
- Task 4 – Calculation of selected core parameters
- Task 5 – Calculation of CHF margins in a hot channel
- Task 6 – Calculation of the maximum cladding and fuel pellet Temperature

Task 4

- 1. Data collection
  - Tables are recommended
- 2. core-averaged thermal-hydraulic calculations
  - Axial enthalpy/temperature distribution
  - Axial void fraction distribution
    - BWRs, from subcooled to saturated
  - Axial pressure distribution
    - Inlet orifices pressure loss, BWRs (50%), PWRs (25%)
  - Flow characteristic of the core (-dp)=f(G)
    - 0%, 50%, 100%, 150% power
    - 1% to 150% flow



Task 4

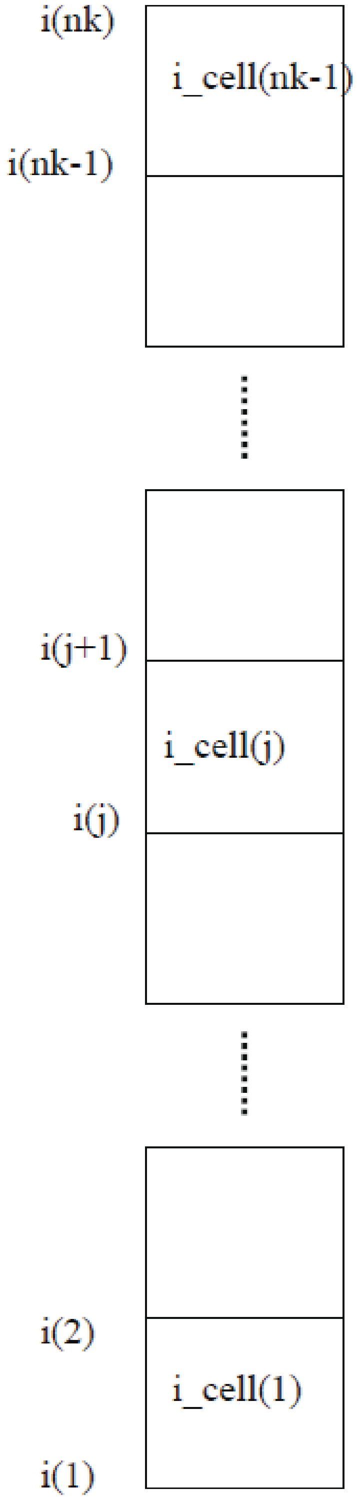


- Inlet orifices pressure loss
  - BWRs (50% at nominal operating conditions)
  - PWRs (25% at nominal operating conditions)

$$\Delta p = p_{out} - p_{in} = \Delta p_{FuelChannel} + \Delta p_{Orifice}$$

$$\left|\Delta p_{Orifice}\right| = \xi_{Orifice} \frac{\rho U^2}{2} = \xi_{Orifice} \frac{G^2}{2\rho}$$

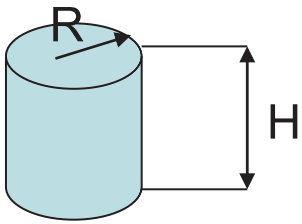
Task 4 Nodalization and numerical solution



- for j = 2 to nk
  - i(j) = i(j-1) + q_cell(j-1) / W (energy balance)
- end for
- while p not converged
  - p(1) = pin + dpInletOrifice
  - for j = 2 to nk
    - xe(j), xa(j), alpha(j) (void fraction model)
    - dpf_cell(j-1), dpg_cell(j-1), dpa_cell(j-1), dpl_cell(j-1)
    - dp_cell(j-1) (pressure drop calculation)
    - p(j) = p(j-1) + dp_cell(j-1)
  - end for
- end while p
- T(j)
  - f(p(j), i(j)) for subcooled water
  - Tsat(j) for saturated water
- Inlet orifices pressure loss coefficient (designed for nominal condition)
- Flow characteristic of the core (-dp)=f(G)

Thermal Power Distribution in Fission Reactors

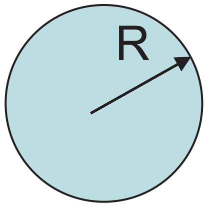
- Distribution of thermal power density in nuclear reactors depends on the shape of the reactor:
  - Finite cylindrical with radius R and height H:



$J_0(x)$ – Bessel function of first kind and zero order. See Compendium, Appendix B

$$q'''(r,z) = q_0''' J_0\left(\frac{2.405r}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

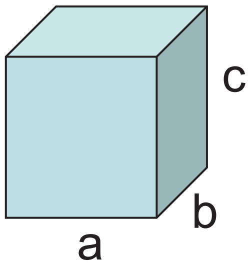
  - Sphere with radius R:



$$q'''(r) = q_0'''\left(\frac{\tilde{R}}{\pi}\right)\frac{\sin\frac{\pi r}{\tilde{R}}}{r}$$

Note: dimensions with tilde are so-called extrapolated dimensions to avoid zero flux at reactor boundary

  - Rectangular parallelepiped with sides a, b, c:

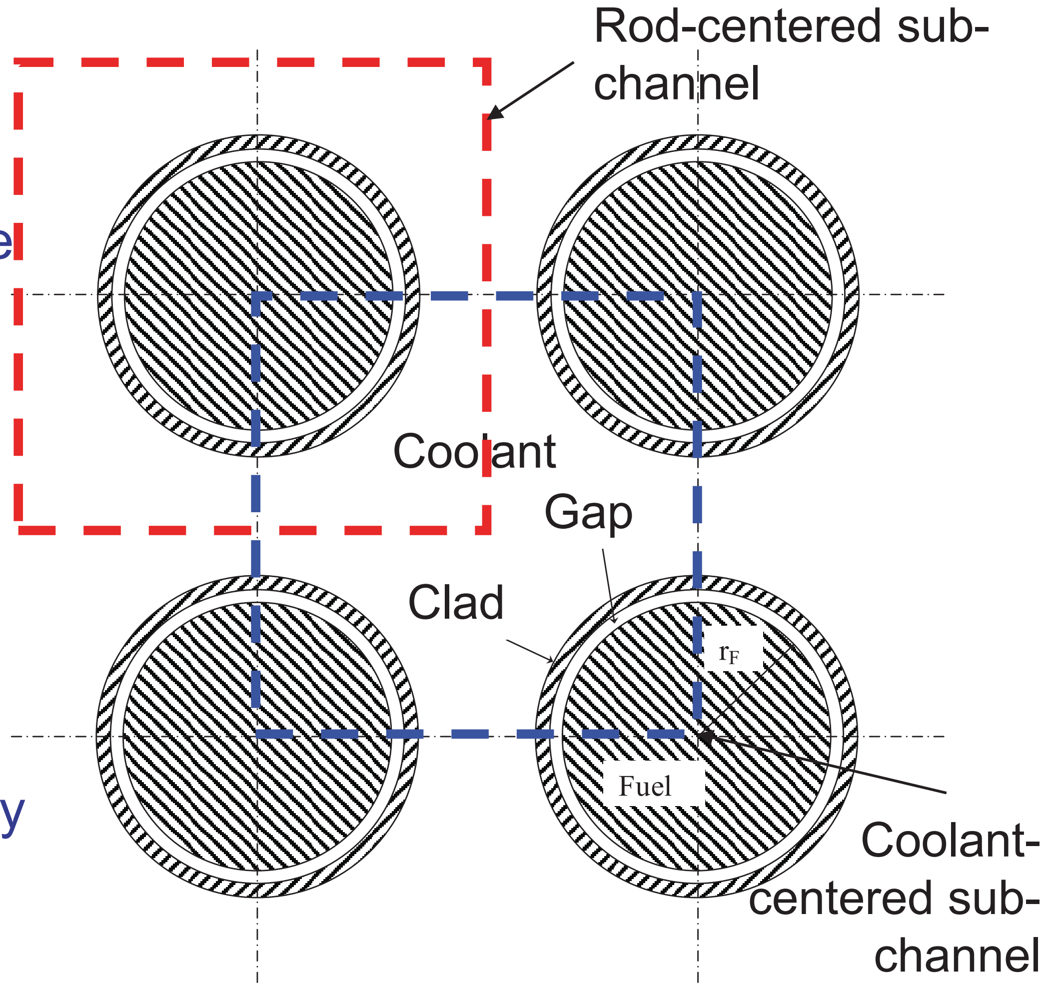


$$q'''(x,y,z) = q_0'''\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{b}}\right)\cos\left(\frac{\pi z}{\tilde{c}}\right)$$

q_0''' - power density at the core centre; r=0, z=0

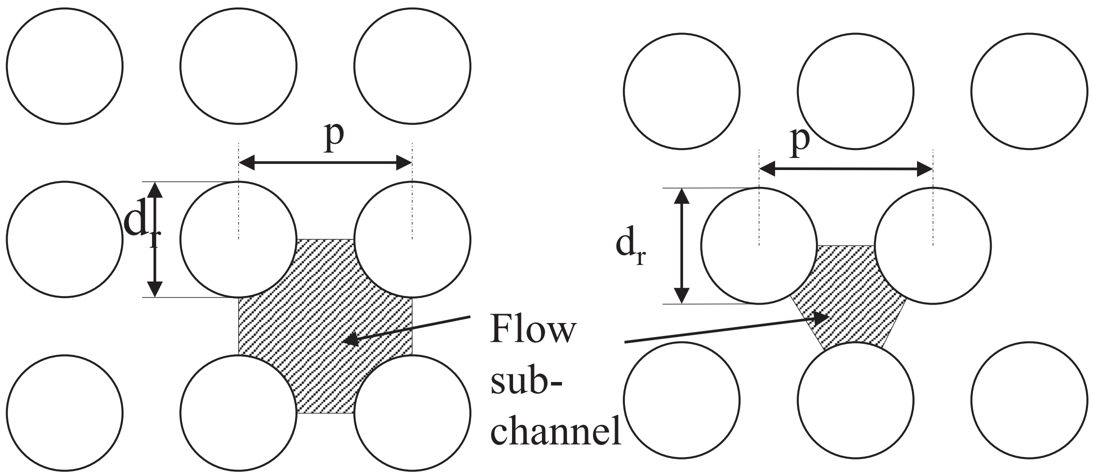
Isolated Sub-channel Model

- Cross-section over a square lattice with fuel pins
- Heat transfer calculations are performed in an averaged, representative “sub-channel”
- Heat conduction is considered in each rod separately
- Main assumption: no flow of mass, momentum and energy through sub-channel ”walls”



Basic Parameters Describing Isolated Sub-channel (1)

- Hydraulic diameter
  Flow area
  Wetted perimeter



Square lattice

Triangular lattice

$$D_h = \frac{4A}{P_w}$$

A – channel cross-section area

P_w – channel wetted perimeter

$$D_h = \begin{cases} d_r\left[\dfrac{4}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1\right] & \text{for square lattice} \\ d_r\left[\dfrac{2\sqrt{3}}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1\right] & \text{for triangular lattice} \end{cases}$$

p – lattice pitch

d_r – rod diameter

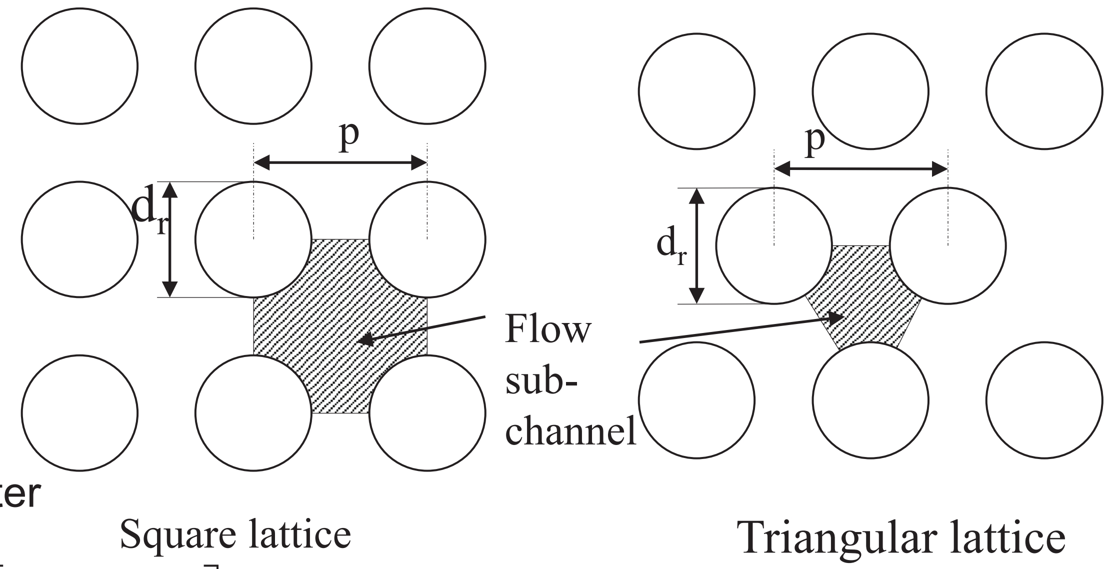
Basic Parameters Describing Isolated Sub-channel (2)

- Heated diameter
  Flow area
  Heated perimeter

$$D_H = \frac{4A}{P_H}$$

A – channel cross-section area

P_H – sub-channel heated perimeter



For subchannels with all heated rods we have:

$$D_H = \begin{cases} d_r\left[\frac{4}{\pi}\left(\frac{p}{d_r}\right)^2 - 1\right] & \text{for square lattice} \\ d_r\left[\frac{2\sqrt{3}}{\pi}\left(\frac{p}{d_r}\right)^2 - 1\right] & \text{for triangular lattice} \end{cases}$$

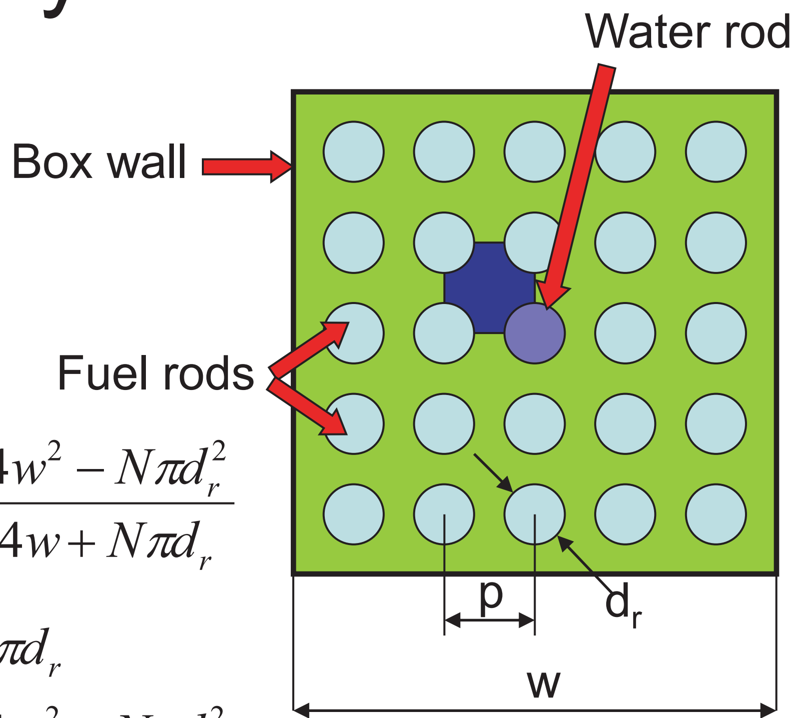
p – lattice pitch

d_r – rod diameter

Whole-Assembly Model

- This model is suitable to BWR fuel assemblies

- Basic parameters:
  - hydraulic diameter D_h $$D_h \equiv \frac{4A}{P_w} = \frac{4w^2 - N\pi d_r^2}{4w + N\pi d_r}$$
  - wetted perimeter P_w $$P_w = 4w + N\pi d_r$$
  - heated diameter D_H $$D_H \equiv \frac{4A}{P_H} = \frac{4w^2 - N\pi d_r^2}{N_{FR}\pi d_r}$$
  - heated perimeter P_H $$P_H = N_{FR}\pi d_r$$
  - N_{FR},N_{WR} – nr of fuel /water rods $$N = N_{FR} + N_{WR}$$



N – total number of rods

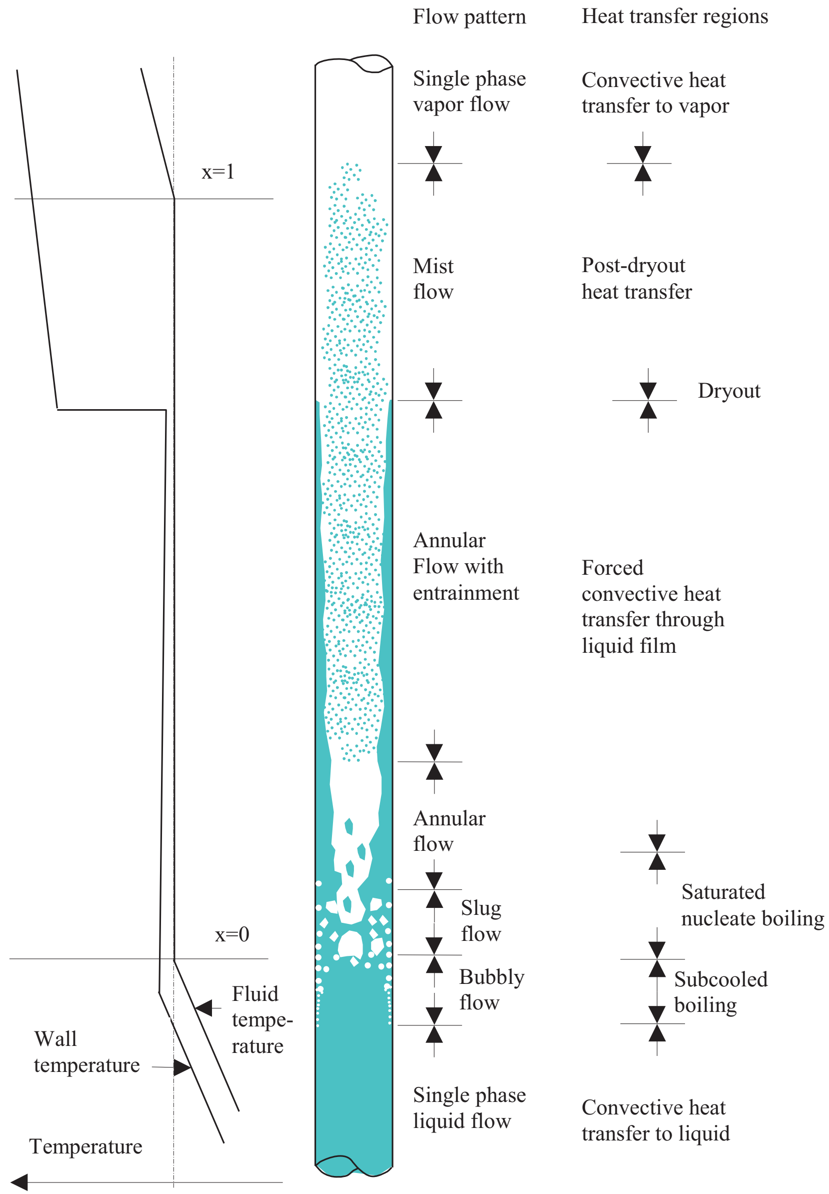
A – cross-section flow area

w – assembly width

d_r – rod diameter

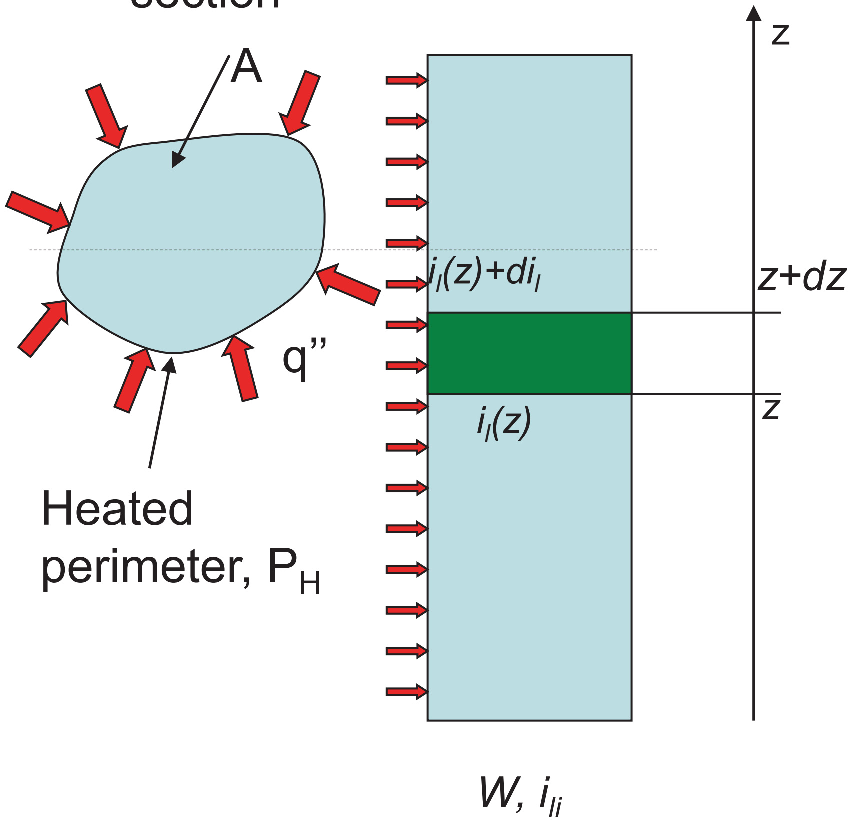
p – lattice pitch

Flow and Heat Transfer Regimes in a Boiling Channel



Coolant Enthalpy Distribution in Heated Channels (1)

- Assume a heated channel as shown in the figure to the right. The channel is uniformly heated along its length with heat flux *q"* [W/m^2], it has a flow cross-section area *A* and heated perimeter P_H.
- The energy balance for a portion of channel *dz* is as follows:



$$W \cdot i_l(z) + q''(z) \cdot P_H(z) \cdot dz = W \cdot \left[i_l(z) + di_l \right]$$

$$\frac{di_l(z)}{dz} = \frac{q''(z) \cdot P_H(z)}{W}$$

W = G*A

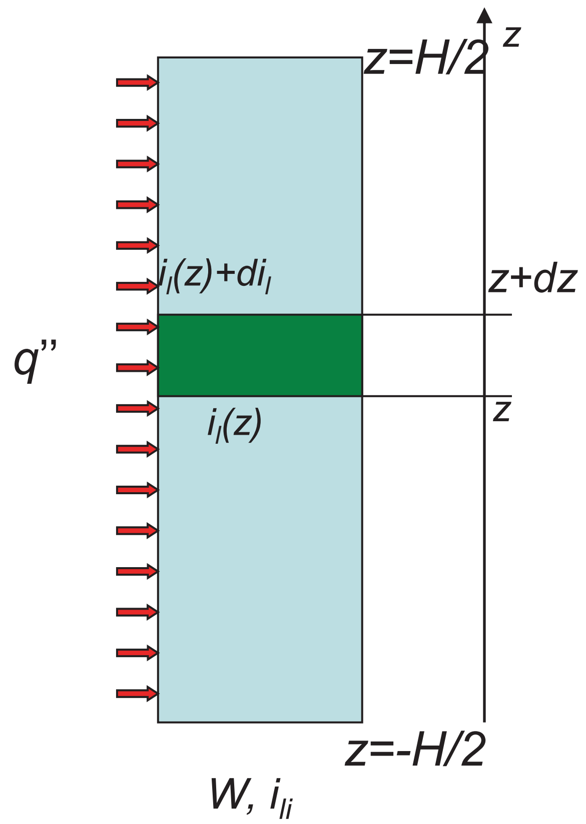
Coolant Enthalpy Distribution in Heated Channels (2)

- Thus, the enthalpy distribution of coolant is described by the following differential equation:

$$\frac{di_l(z)}{dz} = \frac{q''(z)\cdot P_H(z)}{W}$$

- Integration yields

$$i_l(z) = i_{li} + \frac{1}{W}\int_{-H/2}^{z} q''(z)\cdot P_H(z)\cdot dz$$



Coolant Enthalpy Distribution in Heated Channels (3)

- Assuming constant specific heat (calorically perfect fluid) the enthalpy increase can be expressed in terms of the temperature increase as follows:

  $di = c_p * dT$

- Using W = G A and assuming a constant channel cross-section area and heat flux distribution, the coolant temperature can be found as,

$$T_{lb}(z) = T_{lbi} + \frac{q'' P_H (z + H/2)}{c_p G A}$$

$$T_{lb} = \frac{\int_A \rho_l c_{pl} v_l T_l dA}{\int_A \rho_l c_{pl} v_l dA}$$

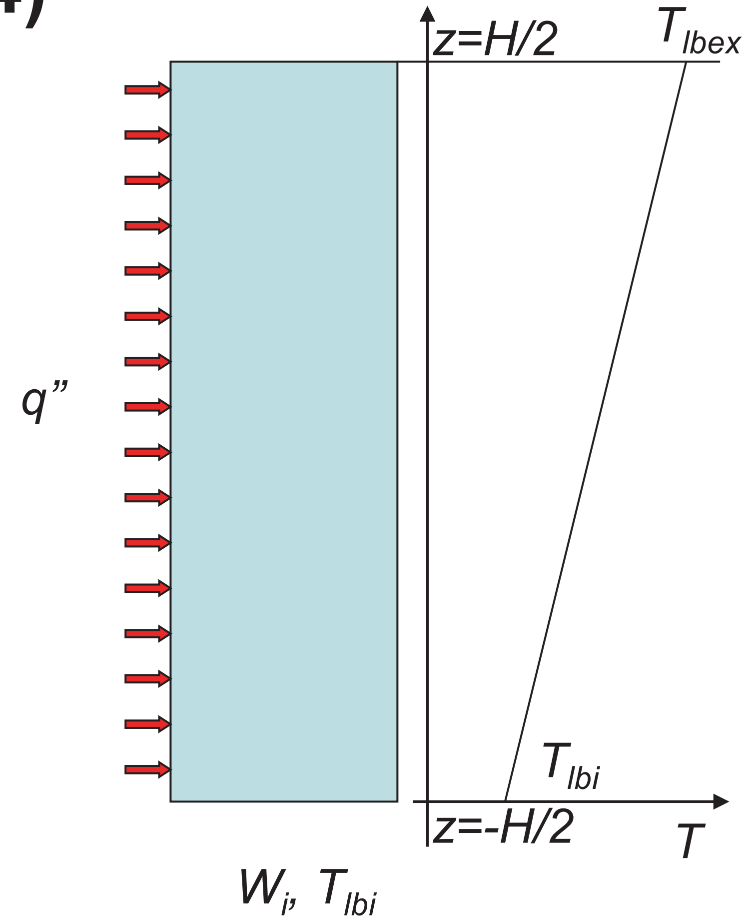
Definition of the bulk liquid temperature

Coolant Enthalpy Distribution in Heated Channels (4)

- The temperature is thus linearly distributed between the inlet and the exit of the assembly

- The exit temperature becomes

$$T_{lbex} = T_{lbi} + \frac{q'' P_H H}{c_p G A}$$

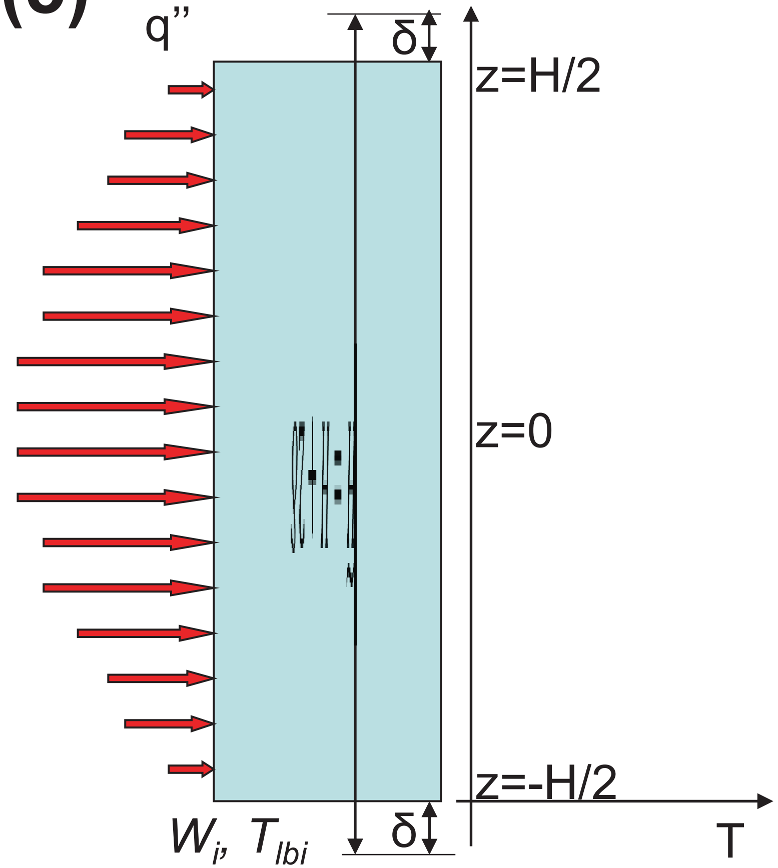


Coolant Enthalpy Distribution in Heated Channels (5)

- Usually the axial power distribution is non-uniform. In a cylindrical reactor the axial power distribution is given by the cosine function:

$$q''(z) = q''_0 \cdot \cos\left(\frac{\pi z}{\widetilde{H}}\right)$$

The differential equation for the enthalpy (temperature) distribution is now



$$\frac{di_l(z)}{dz} = \frac{q''_0 \cdot P_H(z)}{W}\cos\left(\frac{\pi z}{\widetilde{H}}\right), \quad \text{or} \quad \frac{dT_{lb}(z)}{dz} = \frac{q''_0 \cdot P_H(z)}{W \cdot c_p}\cos\left(\frac{\pi z}{\widetilde{H}}\right)$$

Coolant Enthalpy Distribution in Heated Channels (6)

- After integration, (P_H=const) the coolant enthalpy (temperature) distribution is as follows

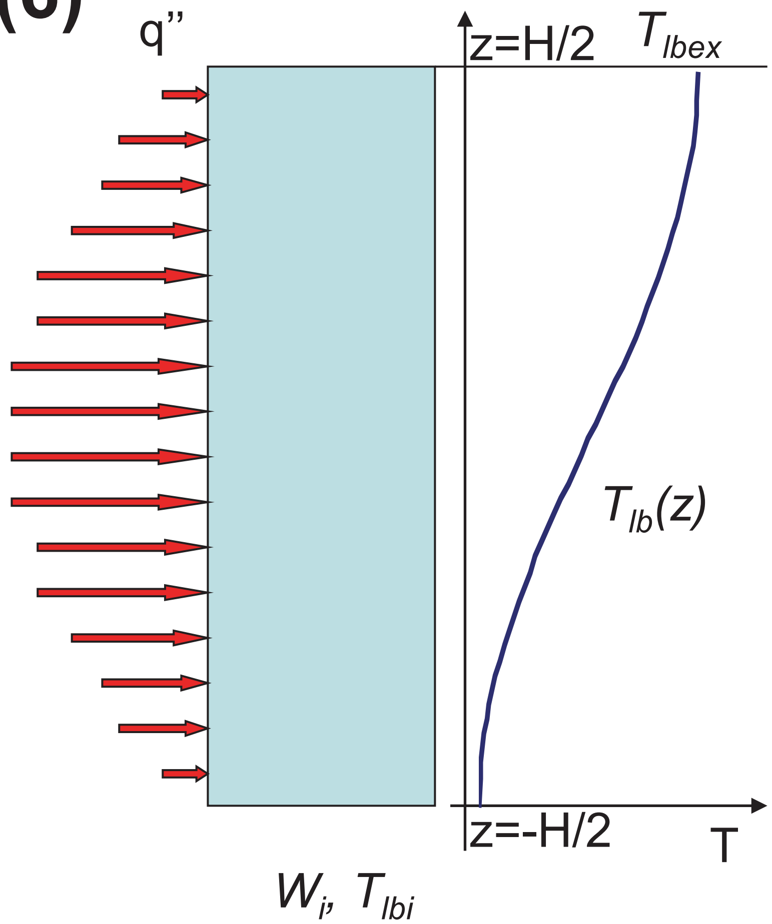
$$i_l(z) = \frac{q_0'' \cdot P_H}{W} \cdot \frac{\widetilde{H}}{\pi}\left[\sin\left(\frac{\pi z}{\widetilde{H}}\right) + \sin\left(\frac{\pi H}{2\widetilde{H}}\right)\right] + i_{li}, \quad or$$

$$T_{lb}(z) = \frac{q_0'' \cdot P_H}{W \cdot c_p} \cdot \frac{\widetilde{H}}{\pi}\left[\sin\left(\frac{\pi z}{\widetilde{H}}\right) + \sin\left(\frac{\pi H}{2\widetilde{H}}\right)\right] + T_{lbi}$$

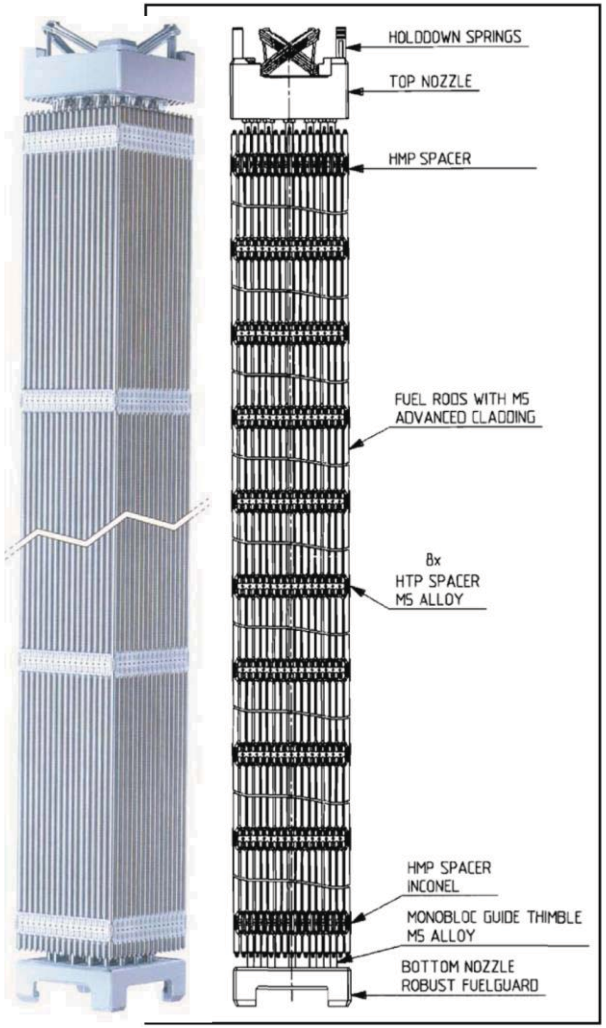
The exit enthalpy (temperature) can be found as:

$$i_{lex} = i_l(H/2) = \frac{2q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W} \sin\left(\frac{\pi H}{2\widetilde{H}}\right) + i_{li}, \quad \text{or}$$

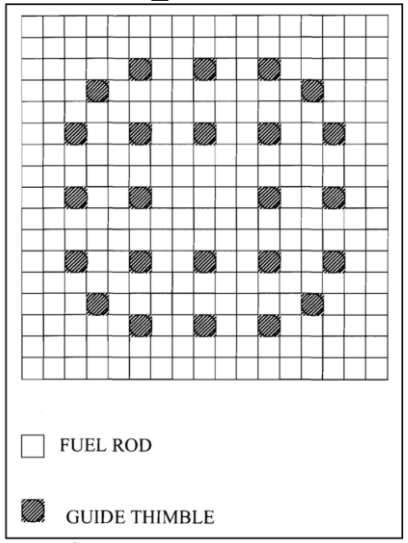
$$T_{lbex} = T_{lb}(H/2) = \frac{2q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p} \sin\left(\frac{\pi H}{2\widetilde{H}}\right) + T_{lbi}$$



PWR Fuel Assembly

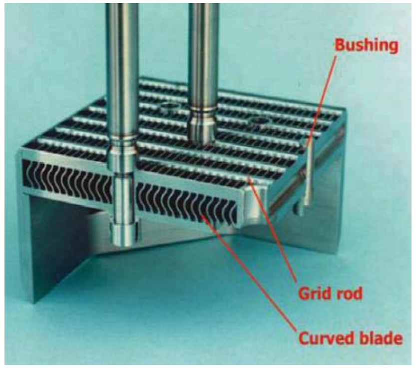


Top nozzle



Cross section

Bottom nozzle with debris filter



Pressure Drop Calculation

- Calculation of pressure drop in single phase flows, including
  - Friction pressure losses
  - Local losses from the spacer grids
  - Local losses at the assembly inlet and exit
  - Local losses due to flow area change
  - Elevation pressure drop

- The total pressure drop in a vertical channel with length *H* and hydraulic diameter D_h can be calculated from the following equation (G = const)

$$-\Delta p_{tot} = -\Delta p_{fric} - \Delta p_{loc} - \Delta p_{elev} = \left(\frac{4C_f H}{D_h} + \sum_i \xi_i \right) \frac{G|G|}{2\rho} + H\rho g$$

Friction Pressure Losses (1)

- Friction pressure losses in a channel with length *H* and hydraulic diameter D_h is calculated as:

$$-\Delta p_{fric} = \frac{4C_f H}{D_h} \frac{G|G|}{2\rho}$$

- where C_f is the (Fanning) friction coefficient, which depends on the Reynolds number and wall roughness, defined as

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

τ_w – wall shear stress,
$U = G/\rho$ – flow velocity

Friction Pressure Losses (2)

- Friction coefficient for pipes
  - Laminar flow (Re < 2300) $$C_f = \frac{16}{\mathrm{Re}}$$
  - Turbulent flow (Blasius formula, $10^4 < \mathrm{Re} < 10^5$) $$C_f = \frac{0.0791}{\mathrm{Re}^{0.25}}$$
  - Turbulent flow in commercial rough tubes (Colebrook formula)

$$\frac{1}{\sqrt{C_f}} = -4.0\log_{10}\left(\frac{k/D_h}{3.7} + \frac{1.255}{\mathrm{Re}\sqrt{C_f}}\right)$$

k – wall roughness [m],

D_h – hydraulic diameter [m]

Friction Pressure Losses (3)

- Friction coefficient for pipes, cont'ed
  - Colebrook formula can be replaced with the Haaland formula (which does not require iterations)

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left[\left(\frac{k / D_h}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

k – wall roughness [m],

D_h – hydraulic diameter [m]

Friction Pressure Losses (4)

- In fuel assemblies, friction coefficients are obtained experimentally and are in general expressed in the following form:

$$C_f = a\,\mathrm{Re}^{-b}$$

a, b > 0 – coefficients that depend on the fuel assembly design

Friction Pressure Losses (5)

- For triangular lattice with $1.0 < p/d_r < 1.5$ the following correlation can be used:

$$C_{f,b} = \frac{0.25\left(0.96\dfrac{p}{d_r}+0.63\right)}{\left(1.82\log_{10}\mathrm{Re}-1.64\right)^2}$$

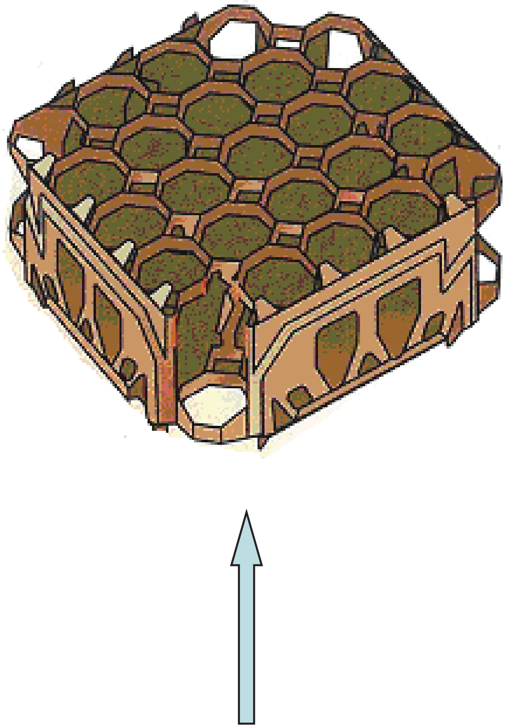
Re > 4000

Local Losses Due to Spacer Grids

- Spacer local pressure loss
  - Geometry-dependent
  - In general, the pressure loss can be calculated as

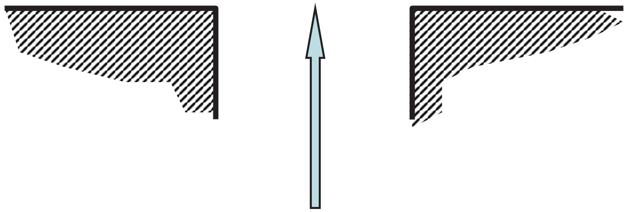
$$\xi_{spac} = a_1 + a_2 \cdot \mathrm{Re}^{-b}$$

  - Constants a_1, a_2 and b are usually obtained from experiments



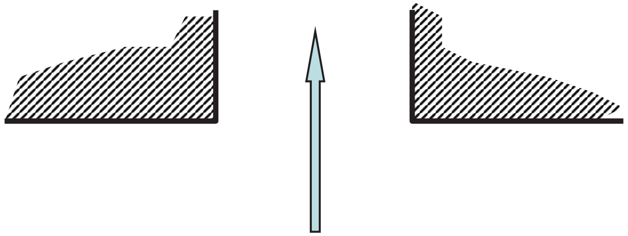
Local Losses Due to Area Changes

Exit from fuel assembly



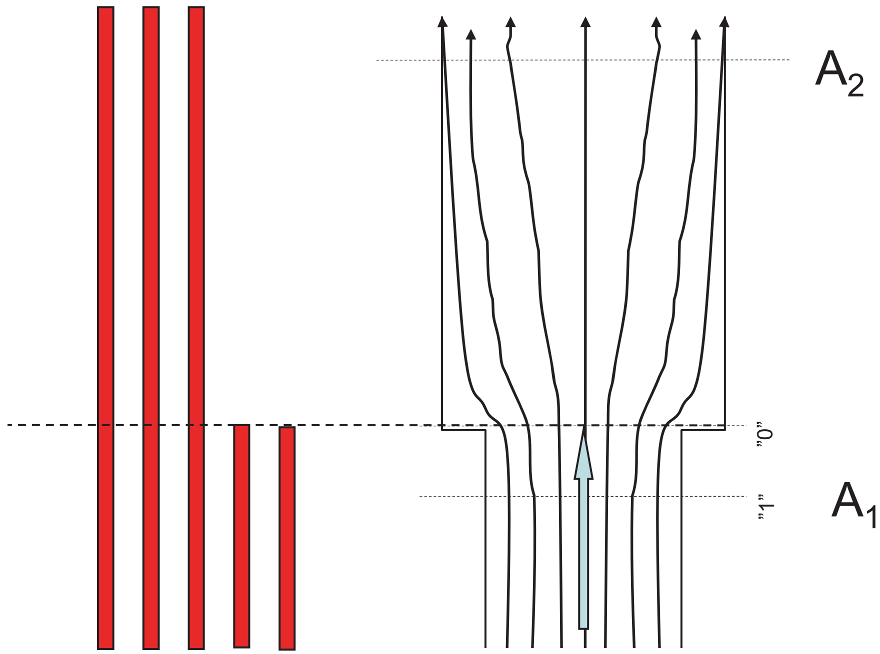
$$-\Delta p_I = \xi_{ex} \cdot \frac{G^2}{2\rho}; \qquad \xi_{ex} = 1.0$$

Inlet to fuel assembly



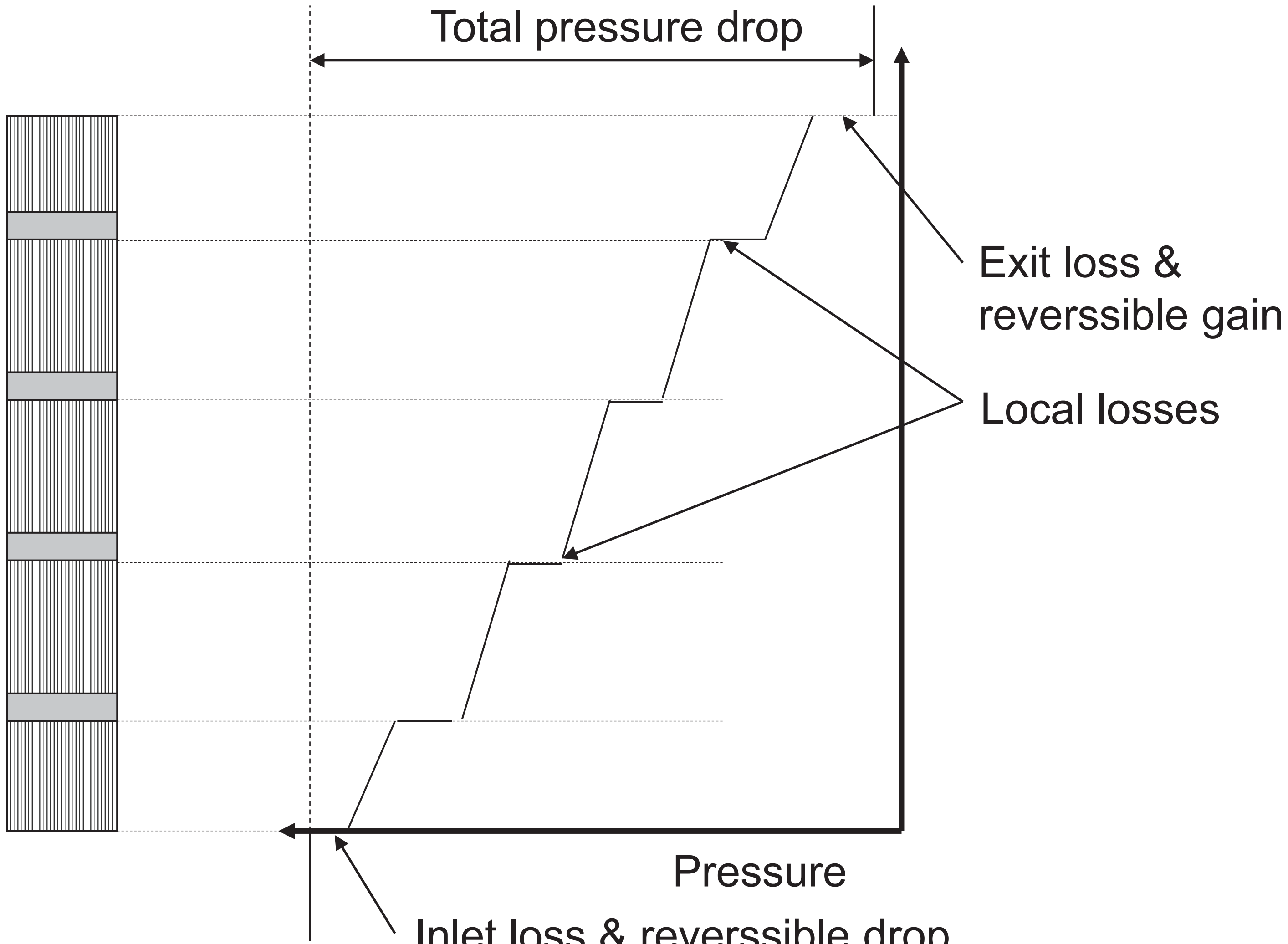
$$-\Delta p_I = \xi_{in} \cdot \frac{G|G|}{2\rho}; \qquad \xi_{in} = 0.5$$

Area change due to part-length rods



$$-\Delta p_I = \left(1 - \frac{A_1}{A_2}\right)^2 \cdot \frac{G_1|G_1|}{2\rho}; \qquad \xi_{enl} = \left(1 - \frac{A_1}{A_2}\right)^2$$

Loss Distribution in Fuel Assembly



Pressure Drop in Two-Phase Flows

- Steady-state momentum equation for a homogeneous two-phase mixture flow in a channel can be written as,

$$\boxed{-\frac{dp}{dz} = \left(\frac{dp}{dz}\right)_w + \rho_m g \sin\varphi + \frac{1}{A}\frac{d}{dz}\left(\frac{G^2 A}{\rho_M}\right)}$$

- Where two definitions of mixture density are introduced:
  - Mixture static density $$\rho_m = \sum_k \rho_k \alpha_k$$
  - Mixture dynamic density $$\rho_M = \left(\sum_k \frac{x_k^2}{\rho_k \alpha_k}\right)^{-1}$$

Local Pressure Loss in Two-Phase Flows

- Local pressure losses in two-phase flows are calculated as:

$$-\Delta p_{loc} = \phi_{lo,d}^2 \xi \frac{G^2}{2\rho_f}$$

Here:

G - total mass flux, kg/m^2.s

ξ - local (single-phase) loss coefficient

$$\phi_{lo,d}^2 = \left[1 + \left(\frac{\rho_f}{\rho_g} - 1\right)x\right]$$ - HEM local two-phase multipl.

Henryk Anglart
Nuclear Reactor Technology Division
Department of Physics, KTH

Friction pressure loss in two-phase flows

- It can be shown that the ratio of two-phase friction loss to single-phase friction loss is as follows,

$$\left(\frac{dp}{dz}\right)_{w,tp} \Bigg/ \left(\frac{dp}{dz}\right)_{w,lo} = \frac{C_{f,tp}}{C_{f,lo}} \frac{\rho_l}{\rho_m}$$

- The above ratio is called a two-phase friction multiplier and is as follows

$$\phi_{lo}^2 = \left(\frac{dp}{dz}\right)_{w,tp} \Bigg/ \left(\frac{dp}{dz}\right)_{w,lo} = \frac{C_{f,tp}}{C_{f,lo}} \frac{\rho_l}{\rho_m}$$

- It should be noted that it is a local variable

Two-Phase Friction Multiplier using HEM

- For Homogeneous Equilibrium Model, it can be shown that the two-phase friction multiplier is the following function of the local equilibrium quality:

$$\phi_{lo}^2 = \left[1+\left(\frac{\mu_f}{\mu_g}-1\right)x\right]^{-0.25}\left[1+\left(\frac{\rho_f}{\rho_g}-1\right)x\right]$$

- where it is assumed that mixture viscosity is given as:

$$\frac{1}{\mu_m} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f}$$

- it should be noted that other models of mixture viscosity are used as well (see Compendium in Thermal-Hydraulics)

Rod Bundle Correlations for ϕ_{lo}^2

- Local two-phase friction multiplier in general depends on local conditions (pressure, mass flux, heat flux) and geometry (pipe, bundle)
- For a rod bundle geometry the following correlation has been obtained (FRIGG)

$$\phi_{lo}^2 = 1 + (2234 - 0.348G)\left(\frac{x}{p}\right)^{0.96}$$

x –quality
p – pressure (bar)
G – mass flux (kg/m^{2}s)

- To capture the effect of heating:

$$\frac{\left(\phi_{lo}^2\right)_{diabatic}}{\left(\phi_{lo}^2\right)_{adiabatic}} = 1 + C\left(\frac{q''}{G}\right)^{0.7}$$

C – constant coefficient
q´´ - heat flux (W/m^2)
G – mass flux (kg/m^{2}s)

EPRI Correlation for ϕ_{lo}^2

$$\phi_{lo}^2 = \left[1 + x\left(\frac{\rho_f}{\rho_g} - 1\right)C\right]$$

$$C = \begin{cases} 1.02x^{-0.175}G_R^{-0.45} & \text{for} \quad p > 4.137\ \text{MPa} \\ 0.357(1 + p_R)x^{-0.175}G_R^{-0.45} & \text{for} \quad 2.068 < p \le 4.137\ \text{MPa} \end{cases}$$

$$p_R = \frac{p}{p_{cr}};\ G_R = \frac{G}{1356.2}$$

x – equilibrium quality
p – pressure (Pa)
G – mass flux (kg/m^{2}s)
p_{cr} – critical pressure (22.1 MPa)

Parameter range: 2.068 < p < 8.963 MPa; 0 < x <1; 475 < G < 4475 kg/m^{2}s; 5.08 < d < 15.24 mm; 127 < L < 2540 mm; geometry: round tubes and vertical upflow; based on 1533 experimental points; RMS error: 9.7%

Mean Value of ϕ_{lo}^2 Over Channel Length

- Integration of ϕ_{lo}^2 along a channel length gives

$$r_3 = \frac{1}{L}\int_0^L \phi_{lo}^2 dz \qquad \phi_{lo}^2 = \left[1+\left(\frac{\mu_f}{\mu_g}-1\right)x\right]^{-0.25}\left[1+\left(\frac{\rho_f}{\rho_g}-1\right)x\right]$$

- The integral to calculate r_3 is thus a function of the quality distribution along the channel.

- In particular, if x = const (unheated channel):

$$r_3 = \phi_{lo}^2$$

Enthalpy and Quality in Heated Channel

- For heated channel, we have:

$$di = \frac{q''(z)P_H dz}{W} \Rightarrow d\left(\frac{i - i_f}{i_{fg}}\right) \equiv dx = \frac{q''(z)P_H dz}{W i_{fg}}$$

thus, assuming z = 0 at the inlet:

$$x(z) - x_{in} = \frac{P_H}{W i_{fg}} \int_0^z q''(z')dz'$$

For uniformly heated channel:

$$x(z) = x_{in} + \frac{P_H q''}{W i_{fg}} z$$

Total Pressure Drop in Boiling Channel

- Integration of the momentum eq. gives the total pressure drop for two-phase flows in channel with length L as:

$$-\Delta p = \underbrace{r_3 C_{f,lo} \frac{4L}{D} \frac{G^2}{2\rho_f}}_{friction} + \underbrace{r_4 L \rho_f g \sin\varphi}_{gravity} + \underbrace{r_2 \frac{G^2}{\rho_f}}_{acceleration} + \underbrace{\left(\sum_{i=1}^{N} \phi_{lo,d,i}^2 \xi_i\right) \frac{G^2}{2\rho_f}}_{local}$$

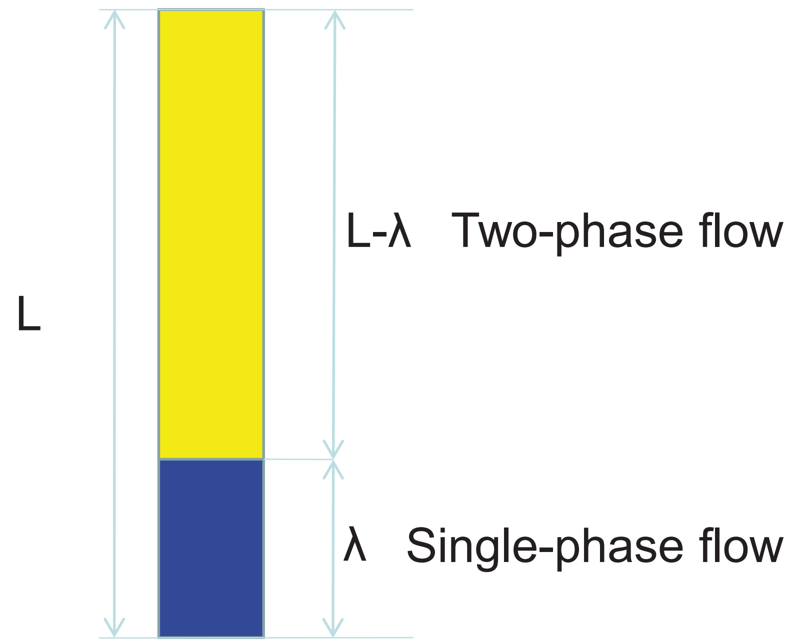
- where:
  - friction multiplier: $r_3 = \frac{1}{L}\int_0^L \phi_{lo}^2 dz$
  - gravity multiplier: $r_4 = \frac{1}{L\rho_f}\int_0^L \left[\alpha\rho_g + (1-\alpha)\rho_f\right] dz$
  - acceleration multiplier: $r_2 \equiv \rho_f \int_0^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right] dz = \left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - \left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{in}$

Friction Loss in BWR Fuel Assembly

- Thus to find friction pressure drop in heated fuel assembly:
  - Find the location of the onset of two-phase flow. If HEM is used, it will be at location where x = 0
  - Let $z = \lambda = z_{SUB}$ where x = 0

$$-\Delta p_{fric} = -\int_0^L \left(\frac{dp}{dz}\right)_{fric} dz =$$

$$-\int_0^\lambda \left(\frac{dp}{dz}\right)_{fric} dz - \int_\lambda^L \left(\frac{dp}{dz}\right)_{fric} dz$$



Friction Loss in BWR Fuel Assembly

- Thus: $$-\Delta p_{fric} = \left(\frac{4C_f \lambda}{D_h} + \frac{4C_{f,lo}}{D_h} \int_{\lambda}^{L} \phi_{lo}^2 dz \right) \frac{G^2}{2\rho_f} = \left[\frac{4C_f \lambda}{D_h} + r_3 \frac{4C_{f,lo}(L-\lambda)}{D_h} \right] \frac{G^2}{2\rho_f}$$
- where $$r_3 = \frac{1}{L-\lambda} \int_{\lambda}^{L} \phi_{lo}^2 dz$$

Assuming uniform power distributions with q''=const

$$r_3 = \int_0^1 \frac{1 + x_{ex}\left(\frac{\rho_f}{\rho_g} - 1\right)\zeta}{\left[1 + x_{ex}\left(\frac{\mu_f}{\mu_g} - 1\right)\zeta\right]^{0.25}} d\zeta$$

where $$x_{ex} = x_{in} + \frac{q'' P_H}{W i_{fg}} L$$

is the exit quality and x_{in} is the inlet quality

Gravity Pressure Drop

- The gravity pressure drop multiplier is given as:

$$r_4 = \frac{1}{L\rho_f}\int_0^L \left[\alpha\rho_g + (1-\alpha)\rho_f\right]dz$$

where using HEM

the local void fraction is obtained as:

$$\alpha = \frac{1}{1 + \frac{\rho_g}{\rho_f}\cdot\left(\frac{1-x}{x}\right)} \quad for \quad 0 < x < 1$$

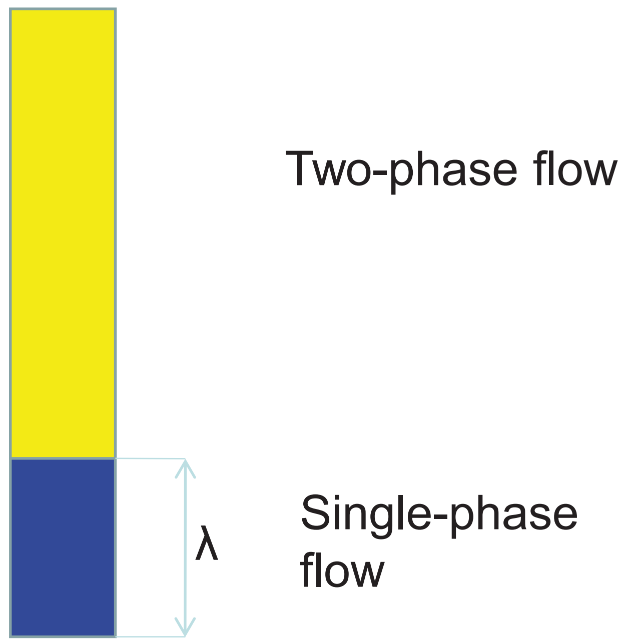
The integral to calculate r_4 is thus a function of the quality distribution along the channel.

Gravity Pressure Drop in BWR Fuel Assembly

- Thus to find the gravity pressure drop in a heated fuel assembly:
  - Find the location of the onset of two-phase flow. If HEM is used, it will be at location where x = 0
  - Let z = λ where x = 0

$$-\Delta p_{grav} = -\int_0^L \left(\frac{dp}{dz}\right)_{grav} dz =$$

$$-\int_0^\lambda \left(\frac{dp}{dz}\right)_{grav} dz - \int_\lambda^L \left(\frac{dp}{dz}\right)_{grav} dz$$



Gravity Pressure Drop in BWR Fuel Assembly

- Thus:

$$-\Delta p_{grav} = \int_0^\lambda \rho_l g \sin\varphi dz + \int_\lambda^L \left[\alpha\rho_g + (1-\alpha)\rho_f\right] g \sin\varphi dz =$$

$$\lambda\rho_l g \sin\varphi + r_4 (L-\lambda)\rho_f \sin\varphi$$

where:

$$r_4 = \frac{1}{(L-\lambda)\rho_f}\int_\lambda^L \left[\alpha\rho_g + (1-\alpha)\rho_f\right] dz$$

assuming uniform power distribution:

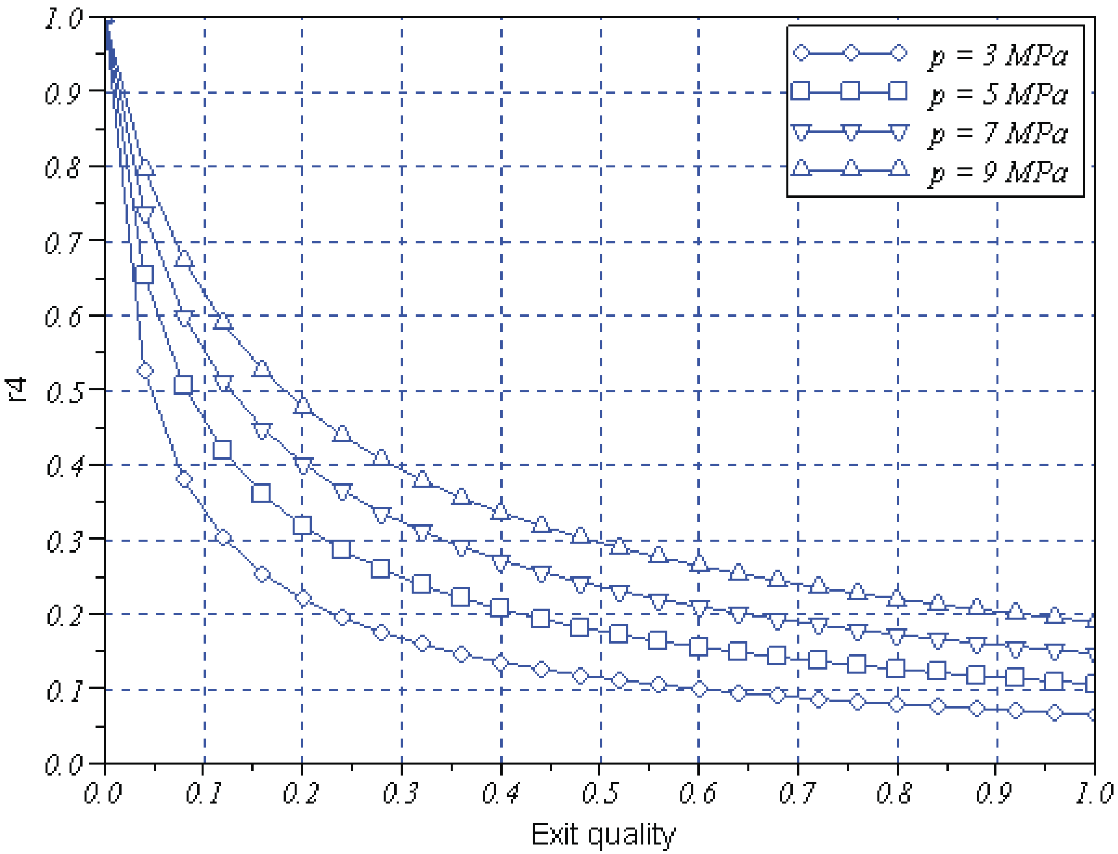
$$r_4 = 1 - x_{ex}\int_0^1 \frac{\zeta}{\rho_g/(\rho_f-\rho_g) + x_{ex}\zeta} d\zeta$$

where x_{ex} is the exit quality

Gravity Pressure Drop Multiplier

$$r_4 = 1 - x_{ex}\int_0^1 \frac{\zeta}{\rho_g/\left(\rho_f-\rho_g\right)+x_{ex}(}$$

This graph can be used to find the value of the r_4 multiplier for known exit quality and system pressure in uniformly heated channel and $x_{in}=0$



The gravity pressure drop is then found as:

$$-\Delta p_{grav} = \lambda\rho_f g\sin\varphi + r_4\left(L-\lambda\right)\rho_f \sin\varphi$$

Acceleration Pressure Drop in Two-Phase Flows

For channel with subcooled water at inlet, the acceleration multiplier can be calculated as:

$$r_2 \equiv \rho_f \int_0^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]dz = \cancelto{0}{\rho_f \int_0^\lambda \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]dz} +$$

$$\rho_f \int_\lambda^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]dz = \left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - \left[\underbrace{\frac{x^2\rho_f}{\alpha\rho_g}}_{0} + \underbrace{\frac{(1-x)^2}{(1-\alpha)}}_{1}\right]_\lambda =$$

$$\left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - 1$$

Thus: $$r_2 = \left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - 1$$

Void Fraction Calculation

- Prediction of void fraction is important because it affects the moderator density, thus, it affects power generation in nuclear reactors

- Two models are widely used in saturated region:
  - Homogeneous Equilibrium Model (HEM)
  - Drift-Flux Model (DFM)

- Void fraction in subcooled region
  - Onset of Nucleate Boiling (ONB)
  - Onset of Significant Void (OSV)
  - Actual quality model

Void Fraction - HEM (1)

- In HEM, it is assumed that both phases are in the thermodynamic equilibrium and flow with the same speed

- Void fraction is calculated in two steps:
  - first the value of the equilibrium quality (x_e) is found as $x_e(z) \equiv \frac{i(z) - i_f}{i_{fg}}$
  - next the value of void fraction is calculated from the following equation:

$$\alpha(z) = \begin{cases} 0 & \text{for} \quad x_e \leq 0 \\ \dfrac{1}{1 + \dfrac{\rho_g}{\rho_f} \cdot \left(\dfrac{1 - x_e(z)}{x_e(z)} \right)} & \text{for} \quad 0 < x_e < 1 \\ 1 & \text{for} \quad x_e \geq 1 \end{cases}$$

Void Fraction - DFM

- Drift flux model allows for:
  - different velocities for both phases
  - thermodynamic equilibrium/non-equilibrium
- The void fraction is calculated from the following relationship:

$$\alpha = \frac{J_v}{C_0 J + U_{vj}}$$

$$J = J_v + J_l$$

$$J_v = \frac{G_v}{\rho_v} = \frac{xG}{\rho_v}$$

$$J_l = \frac{G_l}{\rho_l} = \frac{(1-x)G}{\rho_l}$$

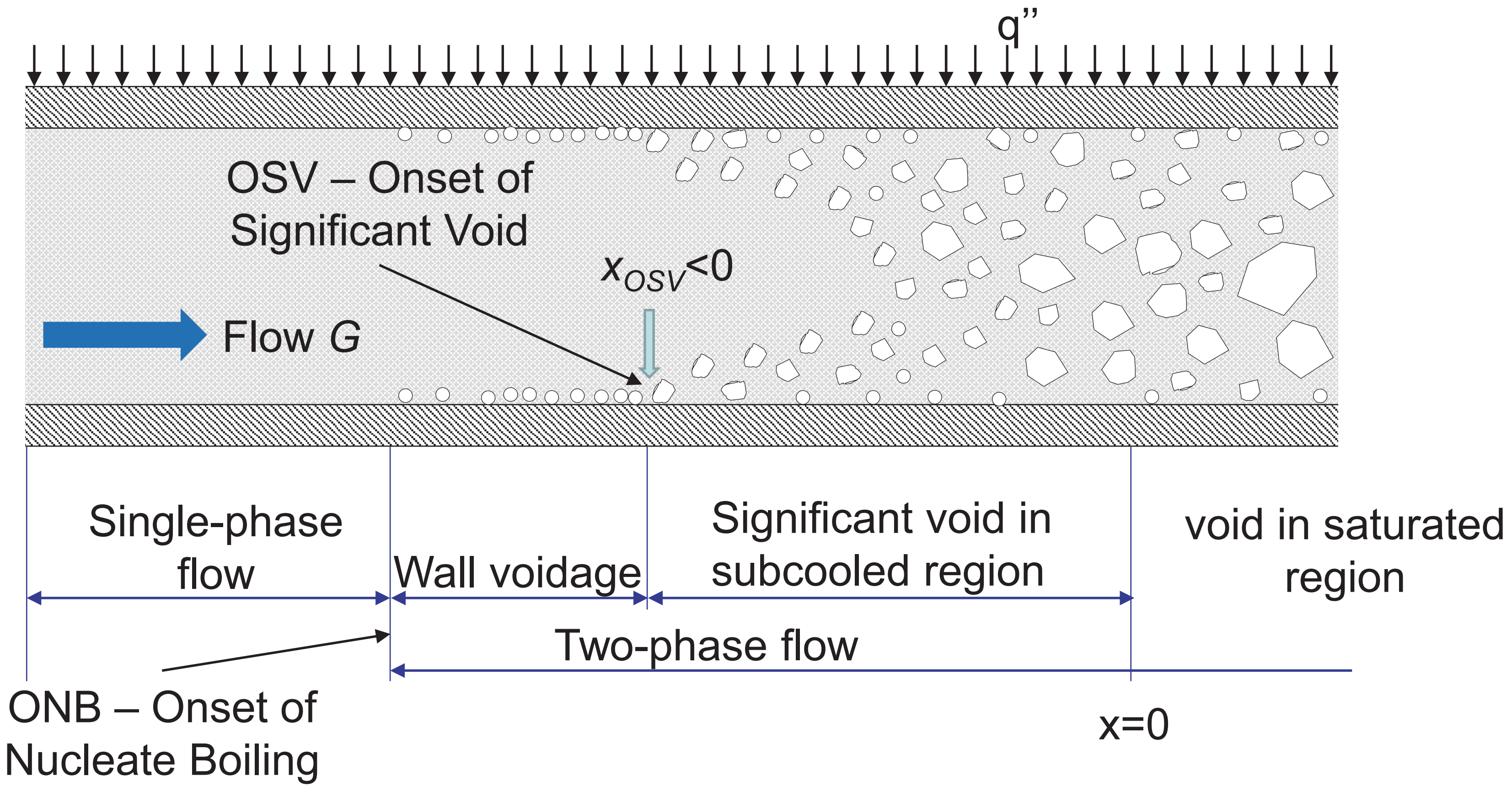
  - here C_0 and U_{vj} are the distribution parameter and the drift velocity, respectively. They are flow-regime dependent.
  - J_v and J are superficial velocities for vapor and for the mixture, respectively

DFM in Thermodynamic Equilibrium

Flow pattern	Distribution parameter	Drift velocity
Bubbly $0 < \alpha \le 0.25$	$C_0 = \begin{cases} 1-0.5p/p_{cr} & & D \ge 0.05m \\ 1.2 & p/p_{cr} < 0.5 & \\ 1.4-0.4p/p_{cr} & p/p_{cr} \ge 0.5 & D < 0.05m \end{cases}$ [1]	$U_{vj} = 1.41\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2}\right)^{0.25}$
Slug/churn $0.25 < \alpha \le 0.75$	$C_0 = 1.15$	$U_{vj} = 0.35\left(\frac{gD(\rho_l - \rho_v)}{\rho_l}\right)^{0.5}$
Annular $0.75 < \alpha \le 0.95$	$C_0 = 1.05$	$U_{vj} = 23\left(\frac{\mu_l j_l}{\rho_v D_h}\right)^{0.5}\frac{(\rho_l - \rho_v)}{\rho_l}$
Mist $0.95 < \alpha < 1$	$C_0 = 1.0$	$U_{vj} = 1.53\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2}\right)^{0.25}$

[1] p_{cr} – critical pressure σ - surface tension $D=D_h$ – hydraulic diameter

Void Fraction in Subcooled Region



Void Fraction – Subcooled Boiling (1)

- It can be assumed that void fraction is negligible up to the Onset of Significant Void (OSV) point.

- This point occurs at the location, where equilibrium quality becomes (Saha-Zuber model):

$$x_{e,OSV} = \begin{cases} -0.0022 \dfrac{q'' \cdot D_h \cdot c_{pf}}{i_{fg} \cdot \lambda_f} & \text{for} \quad \mathrm{Pe} < 70000 \\ -154 \dfrac{q''}{G \cdot i_{fg}} & \text{for} \quad \mathrm{Pe} \geq 70000 \end{cases}$$

  – here Pe is the Peclet number, defined as:

$$\mathrm{Pe} = \mathrm{Re} \cdot \mathrm{Pr} = \frac{G \cdot D_h \cdot c_{pf}}{\lambda_f}$$

q” – heat flux, W/m^2
D_h – hydraulic diameter, m
c_{pf} – fluid spec. heat, J/kgK
λ_f – thermal conduct. W/mK

Void Fraction – Subcooled Boiling (2)

- The actual quality is approximated as (Levy's model):

$$x_a(z) = x_e(z) - x_e(z_{OSV}) \cdot e^{\frac{x_e(z)}{x_e(z_{OSV})} - 1}$$

- The void fraction is then found as:

$$\alpha = \frac{J_v}{C_0 J + U_{vj}}$$

$$J_v = \frac{x_a G}{\rho_g}$$ superficial velocity of vapour

  – where $C_0 = \beta \left[1 + \left(\frac{1}{\beta} \right)^b \right]$

$$J_l = \frac{(1 - x_a) G}{\rho_f}$$ Superficial velocity of liquid

$$\beta = \frac{1}{1 + \frac{\rho_g}{\rho_f} \frac{1 - x_a(z)}{x_a(z)}} \qquad b = \left(\frac{\rho_g}{\rho_f} \right)^{0.1} \qquad U_{vj} = 2.9 \left(\frac{\sigma g (\rho_f - \rho_g)}{\rho_f^2} \right)^{0.25}$$

σ – surface tension, N/m