Q5:

Answers

5a: The derivative of the function $x(y) = ye^y$ can be calculated analytically using the product rule of differentiation:

$$x'(y) = (ye^y)' = e^y + y(e^y)' = e^y + ye^y = (1 + y)e^y$$

The derivative $x'(y)$ is positive in the region where $1 + y > 0$, or equivalently, $y > -1$. In this region, $x(y)$ is increasing. The derivative $x'(y)$ is equal to zero when $1 + y = 0$, or $y = -1$. Finally, $x'(y)$ is negative in the region where $1 + y < 0$, or $y < -1$. In this region, $x(y)$ is decreasing.

5b: The function $x(y) = ye^y$ is increasing when $y \geq -1/e$ and decreasing when $y < -1/e$. Hence, the monotonicity regions of $x(y)$ can be labeled as $R_0 = [-1/e, \infty)$ and $R_{-1} = (-\infty, -1/e)$.

Since $x(y)$ is a one-to-one function, it has exactly one inverse for each monotonicity region. Therefore, the real argument Lambert function W has two branches, one for each monotonicity region. The principal branch W_0 is defined as the inverse of $x(y)$ over R_0, while the complementary branch $W_{-1}(x)$ is defined as the inverse of $x(y)$ over R_{-1}.

5c: The minimum value and corresponding argument of $x(y) = ye^y$ can be found by setting the derivative of $x(y)$ to zero and solving for y.

The derivative of $x(y)$ is given by:

$x'(y) = (ye^y)' = (1 + y)e^y$

Setting $x'(y) = 0$, we have:

$$(1 + y)e^y = 0$$

Dividing both sides by e^y, we get:

$$1 + y = 0$$

Solving for y, we find that $y = -1$.

Substituting $y = -1$ into $x(y)$, we find that the minimum value is:

$$x_{\min} = x(-1) = -e^{-1}$$

So the minimum value of x(y) is $x_{min} = -e^{-1}$ and the corresponding argument is $y_{min} = -1$.

5d: Therefore, the function $x(y) = ye^{y}$ is increasing in the interval $(-1, +\infty)$, and is decreasing in the interval $(-\infty, -1)$.

The real argument Lambert function W has two branches: the main branch, $W_0(x)$, which is increasing for $x > -1/e$ and the complementary branch, $W_{-1}(x)$, which is decreasing for $x < -1/e$.

5e:

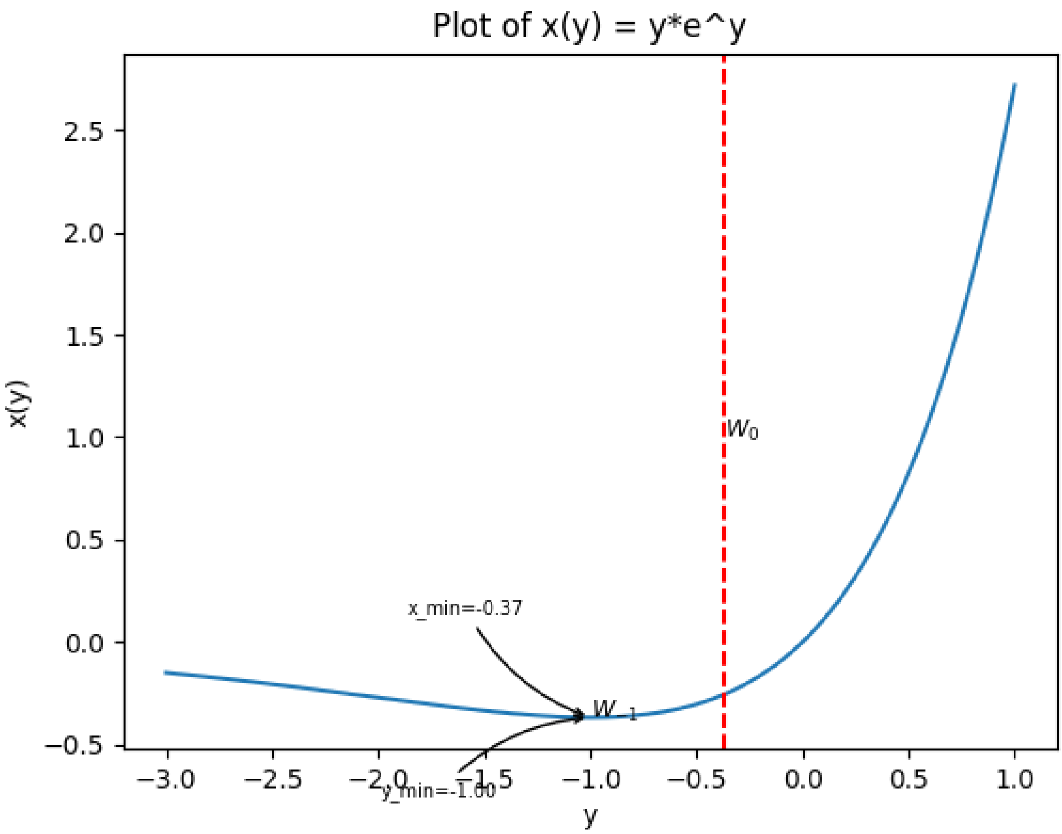

Fig 1: Plot of $x(y) = ye^{y}$ with the characteristic properties shown in the labelling.