Q9:

<u>Answers:</u>

9a & 9b:

Table 1. Comparison of Newton's and Halley's methods for evaluating W_0(x).

	Exact:	y = -1		y = -1+2^-10		y = -1/2		y = 8	
Method		Err	Iter	Err		Err		Err	
Newton	y_0 = 1	6.85403323608113E-06	20	2.6406619413332777e-07	16	9.81057742477364E-06	7	0.9997669027670781	100
	y_0 = opt	6.85403323608113E-06	20	4.419464361138381e-07	14	1.9162112323001246e-06	5	1.850441471162867e-07	4
Halley	y_0 = 1	5.272811389526445e-06	10	5.272835717664775e-06	10	4.389252938294686e-06	10	1.49569440921482E-06	12
	y_0 = opt	zero division error		4.651583286588906e-06	5	6.26816346419639E-06	9	7.48312286447117E-06	7

Table 2. Comparison of Newton's and Halley's methods for evaluating W_{-1}(x).

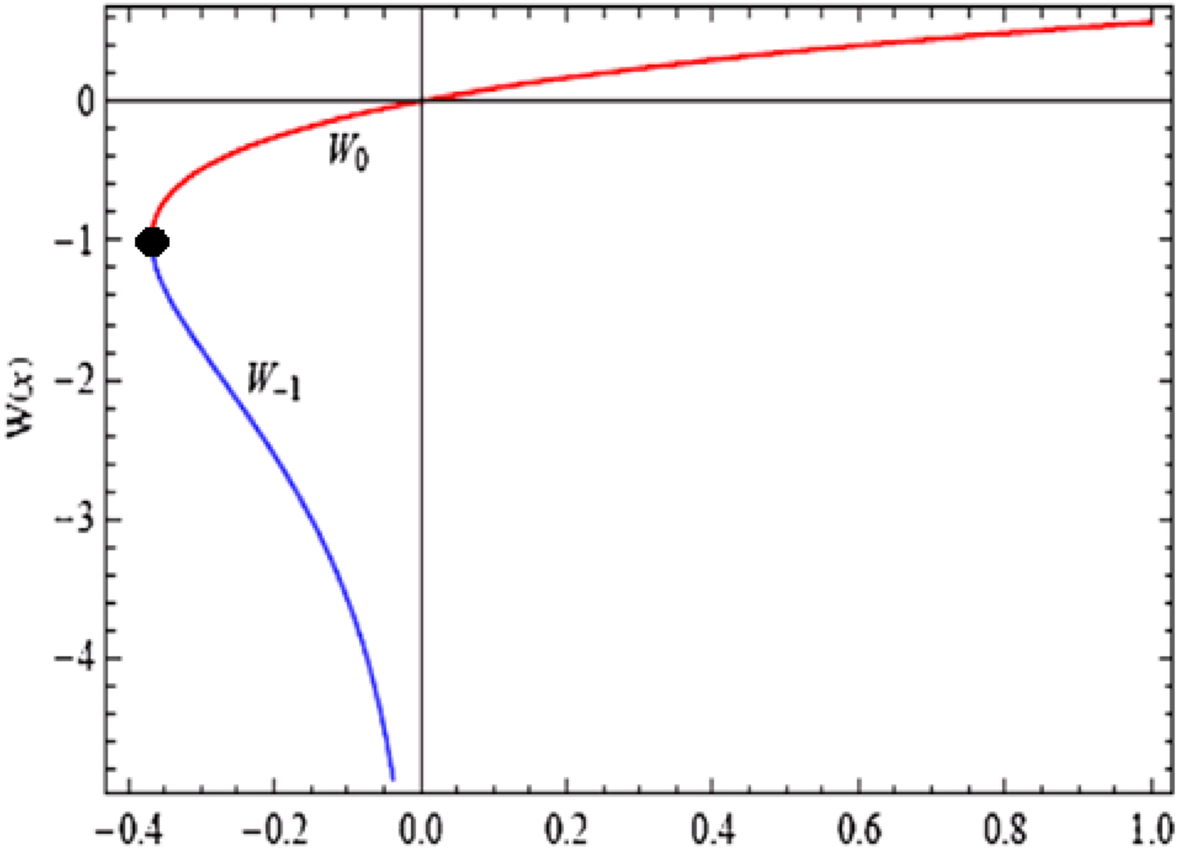
	Exact:	y = -1		y = -1-2^-10		y = -1.5		y = -8	
Method		Err	Iter	Err		Err		Err	
Newton	y_0 = -2	7.099431064139239e-06	16	3.619090791495694e-07	12	5.587808349361012e-08	4	2.861924919272951 2e-09	9
	y_0 = opt	7.099431064139239e-06	16	3.178784027113579e-07	1	6.338421268958783e-06	3	1.2281669636848846e-07	4
Halley	y_0 = -2	6.17934179682159E-06	24	6.917772354589452e-06	24	3.9312309428995e-06	20	1.73401522976181e-07	13
	y_0 = opt	zero division error		4.3133640301703555e-06	52	5.950298098493807e-06	18	2.06528825444302E-07	15

Table 1 shows that usage of the optimized value for Newton's method increases efficiency, especially as the value of y increases. However, the optimized value for Halley's methods shows some fluctuations in the result in terms of error and iteration count. However, in table 2, i.e., in the $W\_\{-1\}$ region, Halley's method shows fluctuations in terms of error. Still, the number of iterations drops gradually as we decrease the value of y further.

Moreover, Newton's method showed significant fluctuation regarding the number of iterations needed for certain tolerance to be met. The zero-division error might occur as I have used computer value in consequence of using "if" conditional for choosing "y0" values if specific x is chosen where the computer might have classed the y_{exact}=-1 less than the -1/e value.

$$\begin{cases} y = \varnothing & a < -1/e \\ y = W_{-1}(-1/e) = W_0(-1/e) = -1 & a = -1/e \\ y_1 = W_{-1}(a) < y_2 = W_0(a) & -1/e < a < 0 \\ y = W_0(a) & a \geq 0 \end{cases}$$

9c:



The intersection point (x = -1/e, y = -1) is where the two branches meet. This can be seen from the fact that for this value of x, both branches have the same y-value of -1. The Lambert function has two branches because it is a multi-valued function, and the two branches meet at the point of intersection where they are equal. The blue branch, W_0(x), is the principal branch with y >= -1 for x >= -1/e, and the red branch, W_{-1}(x), is the second branch with y <= -1 for x <= -1/e.