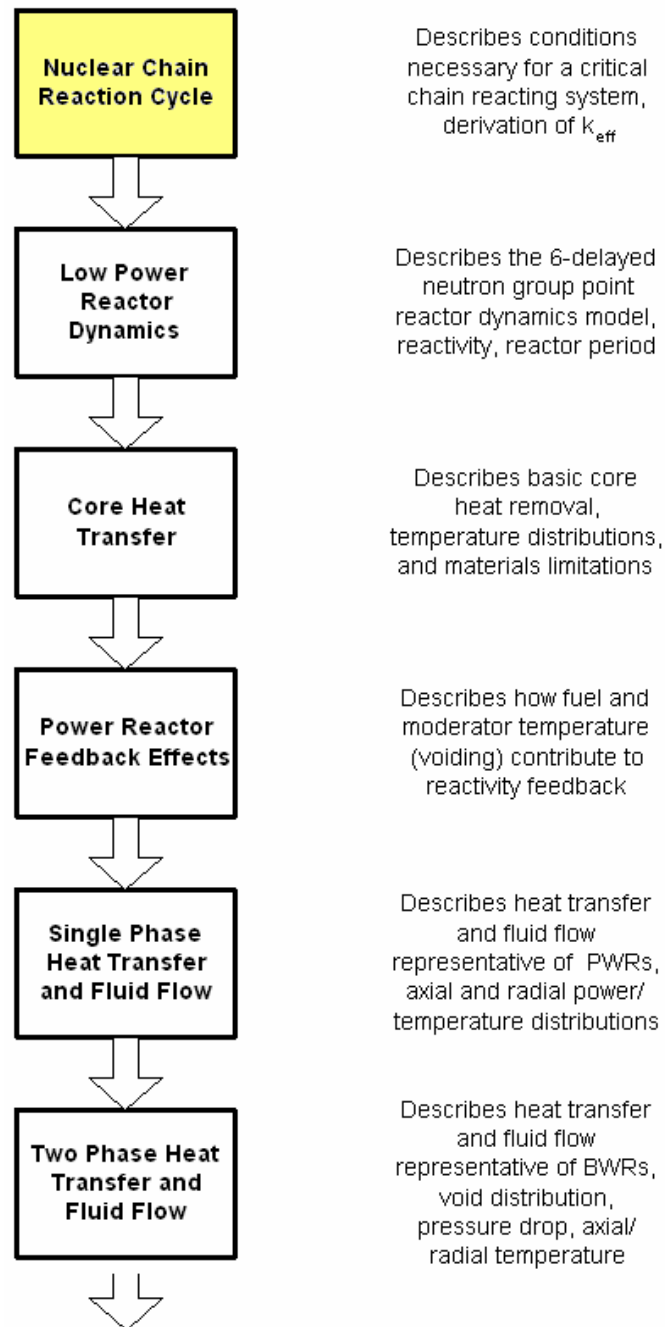


Fundamentals of Nuclear Engineering

Module 7: *Nuclear Chain Reaction Cycle*

Dr. John H. Bickel

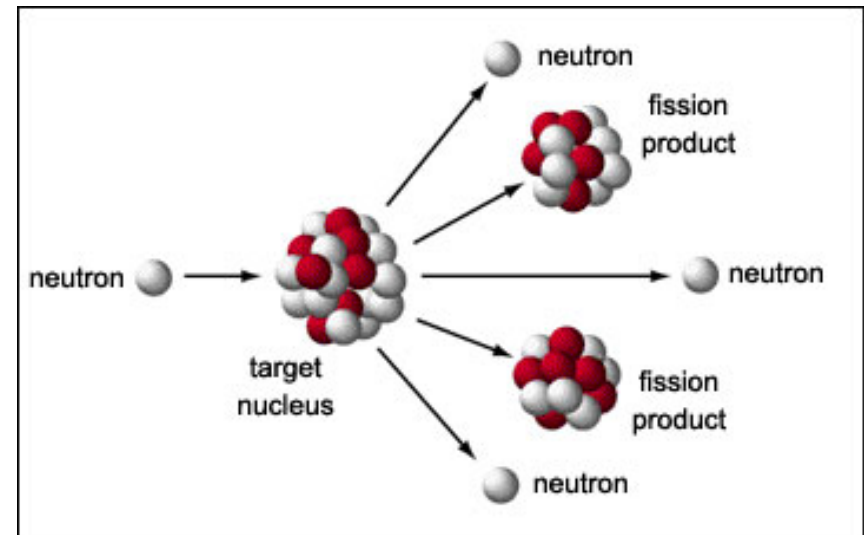


Objectives:

- 1. Define stages of nuclear chain reaction cycle*
- 2. Define multiplication factors of reactor systems:*
 - Subcritical*
 - Critical*
 - Supercritical*
- 3. Define infinite medium system multiplication factor: k_{∞} (four factor formula)*
- 4. Define finite medium system multiplication factor: k_{eff} (six factor formula)*
- 5. Describe differences in: One-Group, Two-Group, Multi-group core physics calculations*

Chain Reacting Systems

*Each Fission
produces
multiple neutrons:*



- Fission yields on average: “ ν ” total neutrons
- Fission yield increases *slightly* with neutron energy
- For U^{235} : $\nu(E) \approx 2.44$
- For U^{233} : $\nu(E) \approx 2.50$
- For Pu^{239} : $\nu(E) \approx 2.90$

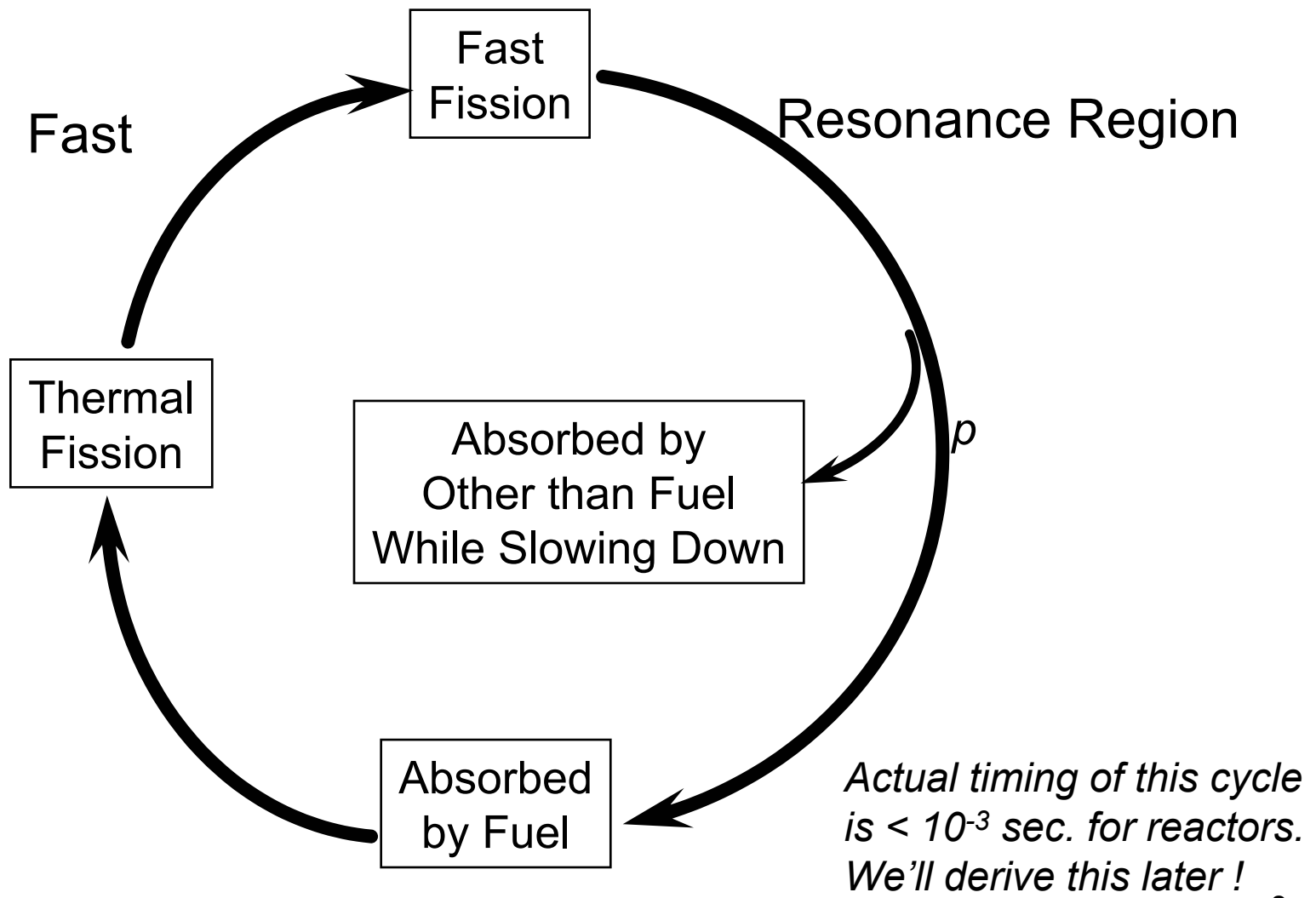
Multiplication Factor

- Multiplication factor: “ k ” is ratio of current neutron population to previous population
- Nuclear system is:
- “*Subcritical*” if $k < 1.0$ - neutron population decreases in successive generations
- “*Critical*” if $k = 1.0$ - neutron population constant in successive generations
- “*Supercritical*” if $k > 1.0$ - neutron population increases in successive generations

Differences Between: Thermal and Fast Reactors

- Thermal reactors *primarily* rely on *thermal neutrons* to initiate fission
- Thermal reactors include a population of *fast*, *epithermal*, and *thermal neutrons*
- Thermal reactors use some relatively *low A-value* moderator/coolant to slow neutrons down to thermal energy
- Fast reactors rely on *fast neutron fission* processes
- Fast reactors must use *high A-value* coolant (liquid metals)
- Criticality is a measure of *net neutron population*, not energy distribution

Infinite Medium Chain Reaction → No Leakage

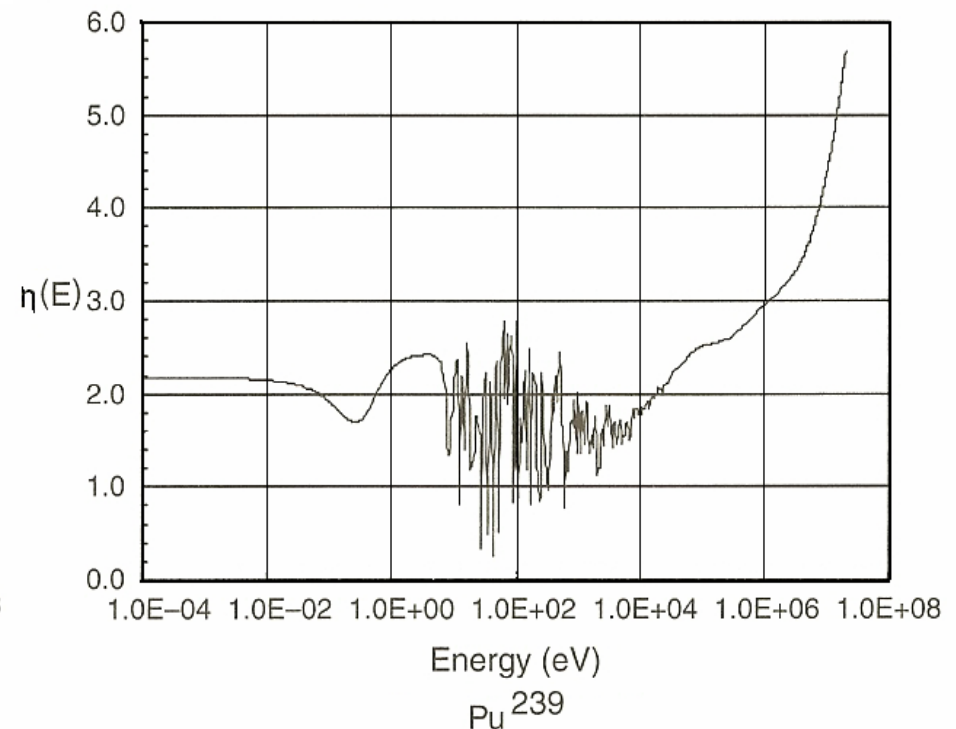
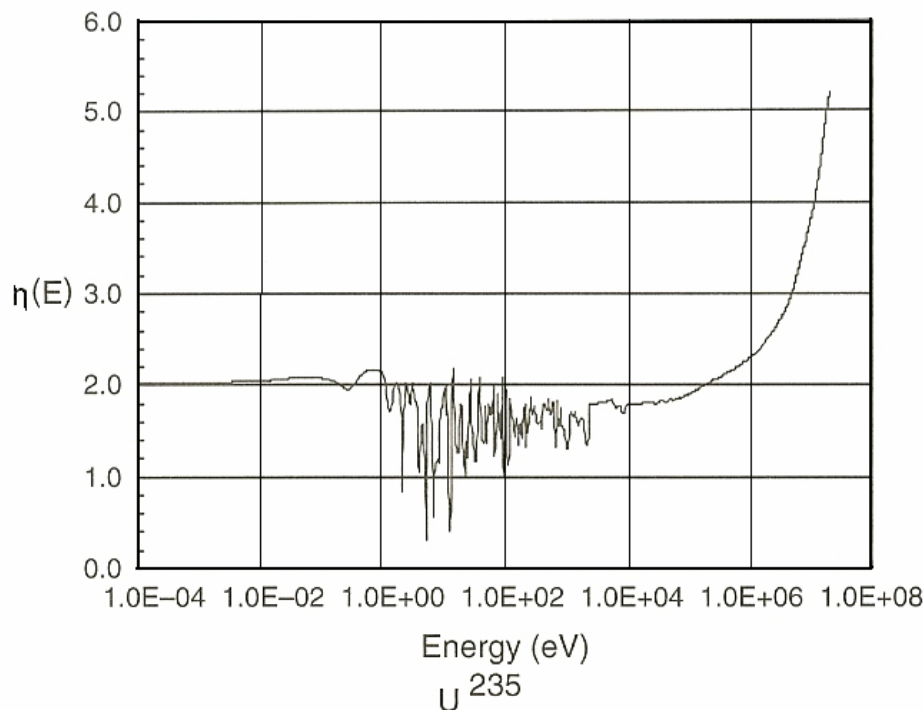


Considering Only Fissile Material

- Ratio of total fission neutrons produced to neutrons absorbed in *infinite medium* is calculated:

$$\eta(E) = \nu(E)\Sigma_f(E) / \Sigma_a(E) = \nu(E)\Sigma_f(E) / (\Sigma_c(E) + \Sigma_f(E))$$

- For one fissile material: $\eta(E) = \nu(E)\sigma_f(E) / (\sigma_c(E) + \sigma_f(E))$
- Examples for pure U^{235} and Pu^{239}



Actual Reactor Physics Considerations

- Neutron yield per neutron absorbed “*simply*” defined:

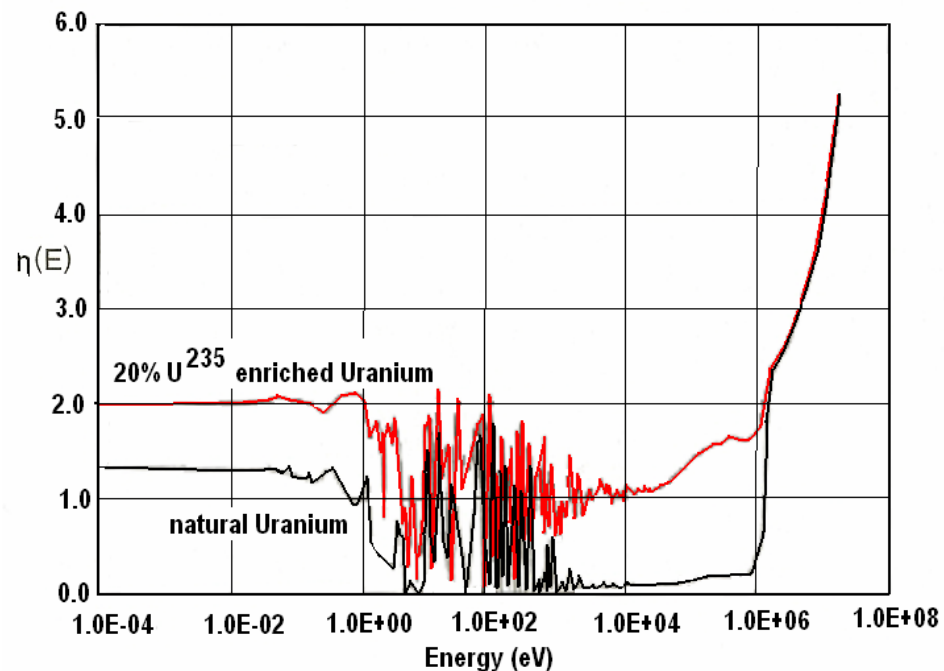
$$\eta(E) = \nu(E)\Sigma_f(E) / \Sigma_a(E) = \nu(E)\Sigma_f(E) / (\Sigma_c(E) + \Sigma_f(E))$$

- Actual core physics calculations must consider:
 - All isotopes which capture neutrons: Xe^{135} , Sm^{149} , B^{10} , etc...
 - All isotopes present in fuel that fission: U^{235} , Pu^{239} , Pu^{241} , etc...
- Fuel supplier’s design would need to consider:
 - Fresh fuel without fission products, Pu^{239} , Pu^{241}
 - Fuel with equilibrium Xe^{135} , Sm^{149} , various buildup of Pu^{239} , Pu^{241} , etc...
- For introductory purposes of these lectures we focus on fresh enriched Uranium fuel

Considering Mixture Fissile Material

- Reactor fuel typically mixture of: 2 - 3% U^{235} , U^{238}
- Define enrichment: $e = N_{U^{235}} / (N_{U^{235}} + N_{U^{238}})$

$$\eta(E) = \frac{[e \nu(E)_{U^{235}} \sigma_f(E)_{U^{235}} + (1-e) \nu(E)_{U^{238}} \sigma_f(E)_{U^{238}}]}{[e(\sigma_f(E)_{U^{235}} + \sigma_c(E)_{U^{235}}) + (1-e)(\sigma_f(E)_{U^{238}} + \sigma_c(E)_{U^{238}})]}$$



*Increasing U^{235} enrichment
increases neutron population*

from: E. E. Lewis,

“Nuclear Reactor Physics”, p. 101

Infinite Medium Multiplication Factor

To generate k_{∞} must consider:

- Materials other than fissile fuel
- Cladding
- Coolant/Moderator
- Control Rods
- Structural Materials
- All cause: scattering, thermalizing, capture
- These impact $\varphi(E)$ distribution by:
- Shifting neutron density towards thermal energies
- Depressing neutron density near resonances

Infinite Medium Multiplication Factor

- To generate k_∞ must weight $\nu(E)$ with $\phi(E)$
- In thermal reactor, cross sections can be *approximated* with thermally averaged values
- This yields:
- k_∞ *approximation* requires corrections for:

$$k_\infty = \frac{\int_0^\infty \nu(E) \Sigma_f(E) \phi(E) dE}{\int_0^\infty (\Sigma_c(E) + \Sigma_f(E)) \phi(E) dE}$$

$$k_\infty \approx \frac{\overline{\nu \Sigma_f}}{\overline{\Sigma_c} + \overline{\Sigma_f}} = \eta$$

Fast fission (adds neutrons)

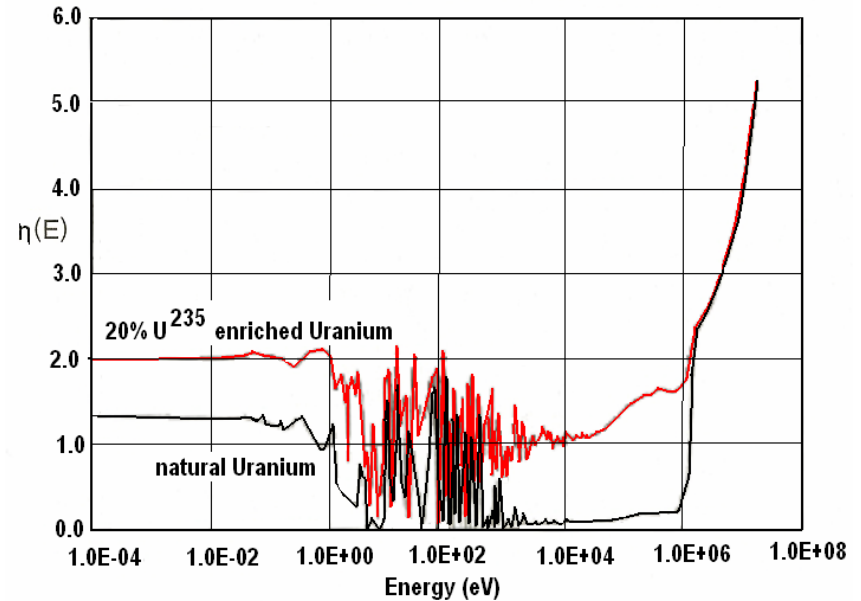
Resonances (remove neutrons)

Fuel vs. Misc. Absorption

(remove neutrons)

Fast Neutron Fission Correction

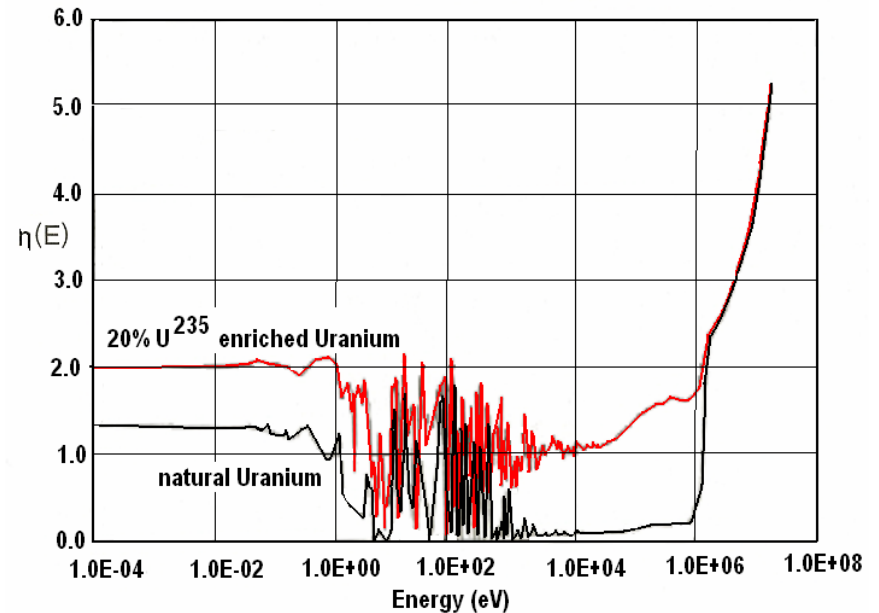
- Given high $\eta(E)$ for fast neutrons, correction factor: ε applied for U^{238}
- ε accounts for additional fissions from fast neutrons
- ε : ratio of total fission neutrons to fission neutrons from thermal neutrons ($E \leq E_t$) only
- Range: $1.0 \leq \varepsilon \leq 1.227$
- $\varepsilon \approx 1.0$ (if no U^{238} present)



$$\varepsilon = \frac{\int_0^{\infty} \nu(E) \Sigma_f(E) \phi(E) dE}{\int_0^{E_t} \nu(E) \Sigma_f(E) \phi(E) dE}$$

Resonance Escape Correction

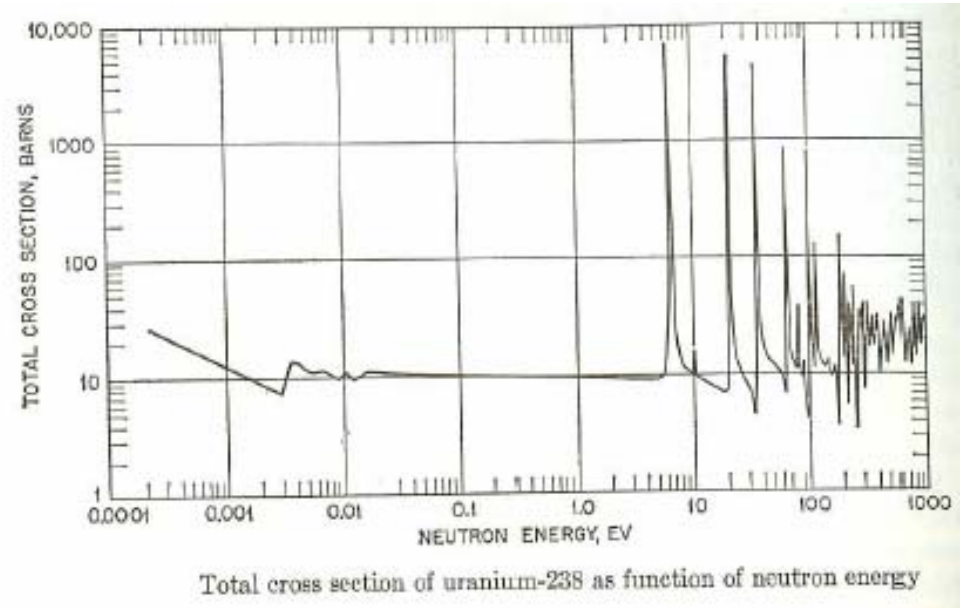
- Resonance capture in 1eV – 10⁴eV range “*depresses*” $\phi(E)$
- Resonance escape probability: “*p*” corrects *thermal approximation* “*v*” for neutron losses during thermalization
- Recall *neutron slowing down model*:
- Resonance escape probability models start from this expression



$$\frac{q(E')}{q(E)} = \exp \left[- \int_{E'}^E \frac{\Sigma c(E) dE}{\xi(E) (\Sigma c(E) + \Sigma s(E)) E} \right]$$

Resonance Escape Correction

- Problem: *Hundreds* of resonances necessitate numerical evaluation or approximation.
- Historical approaches:
- NR - *narrow resonance*
- NRIM - *narrow resonance infinite mass*
- Quasi-experimental p
- Range:
 $p \approx 0.63 - 0.87$ PWR/BWRs
 (current day designs)



$$p = \exp \left[- \frac{2.73}{\bar{\xi}} \left(\frac{N_A}{N_A \sigma_p + N_m \sigma_m} \right)^{1-0.486} \right]$$

from: J. R. Lamarsh,

"Nuclear Reactor Theory", p. 235

Thermal Utilization Correction

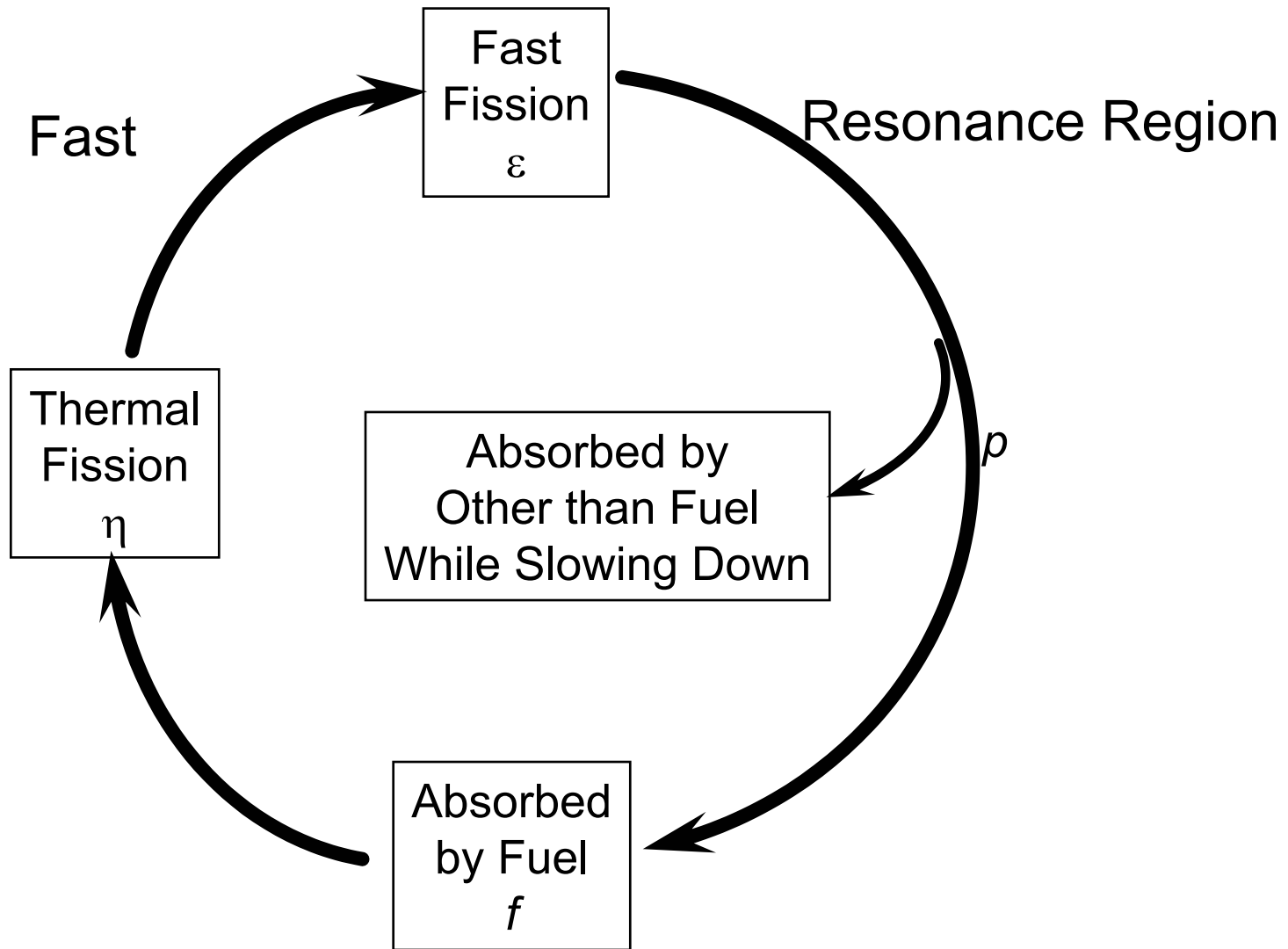
- Thermal neutrons not all absorbed in fuel
- Thermal utilization “ f ” corrects for fraction absorbed in non-fissile materials

$$f = \frac{V_f \int_0^{E_t} (\Sigma_c(E) + \Sigma_f(E)) \phi(E) dE}{V_f \int_0^{E_t} (\Sigma_c(E) + \Sigma_f(E)) \phi(E) dE + V_m \int_0^{E_t} \Sigma_c(E) \phi(E) dE}$$

$$f = \frac{V_f (\bar{\Sigma}_c + \bar{\Sigma}_f) \bar{\phi}_f}{V_f (\bar{\Sigma}_c + \bar{\Sigma}_f) \bar{\phi}_f + V_m \bar{\Sigma}_c \bar{\phi}_m}$$

- Typical value: $f \approx 0.94$ for PWR/BWR (current day designs)

Infinite Medium Chain Reaction → No Leakage

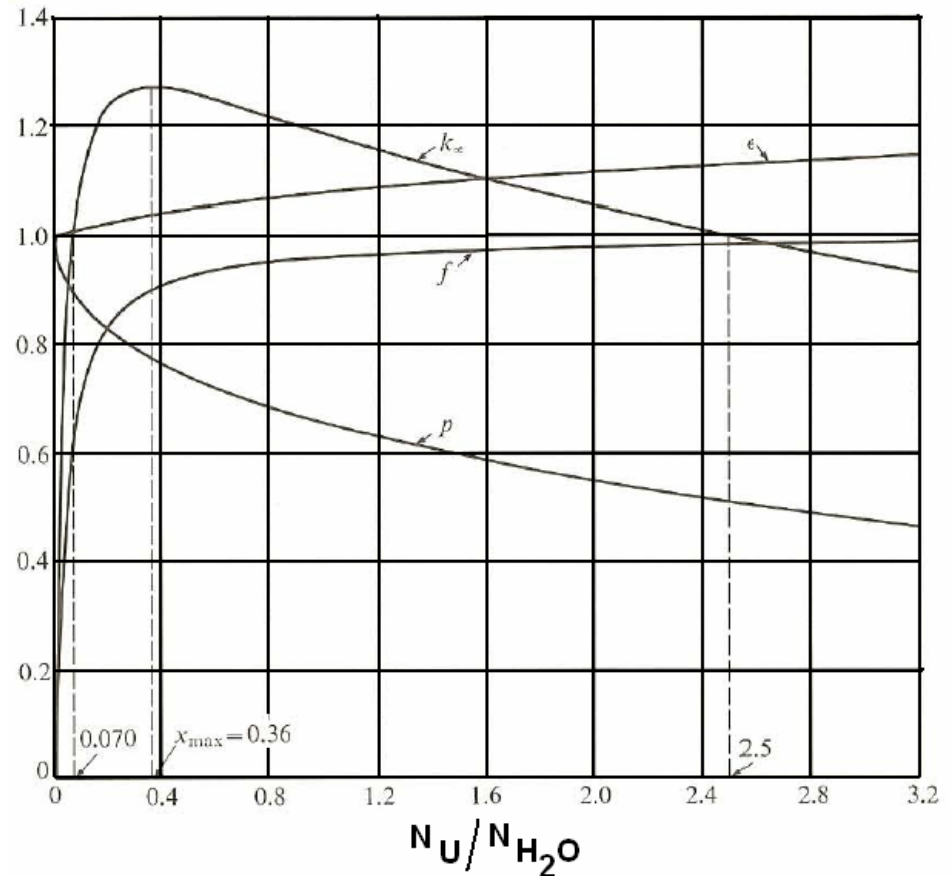


Optimization of Fuel Assembly Design

Effect of Parametrically Varying U-H₂O Ratio

- Assume ~2% Uranium fuel
- Vary Uranium/Water Ratio
- Calculate ϵ , p , f , k_∞ as function of: N_U/N_{H_2O} ratio
- Fast fission, ϵ , increases with more U^{238}
- Resonance escape factor, p , decreases with more U
- Thermal utilization, f , levels off after $N_U/N_{H_2O} = 1.2$
- η is function of Uranium Σ_c , Σ_f
- Maximum k_∞ is for:

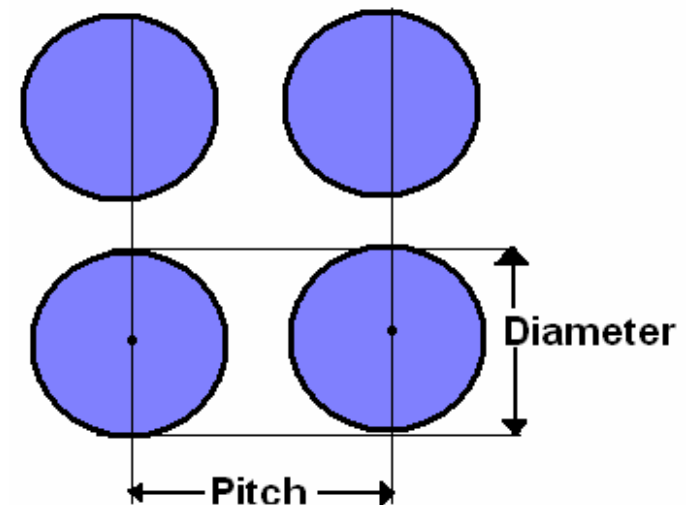
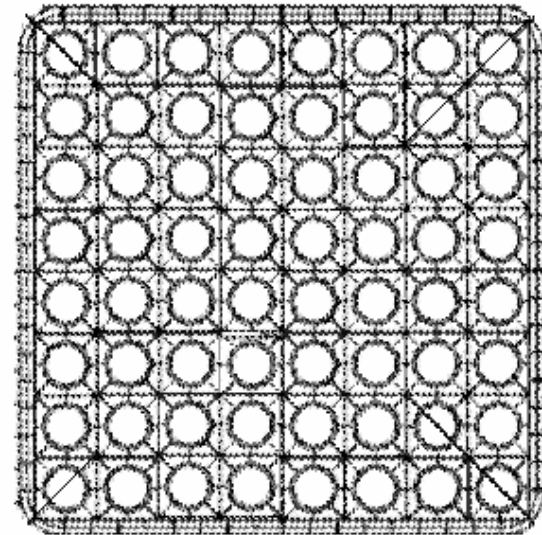
$$N_U/N_{H_2O} = 0.36$$



J. Lamarsh, "Nuclear Reactor Theory", p. 305

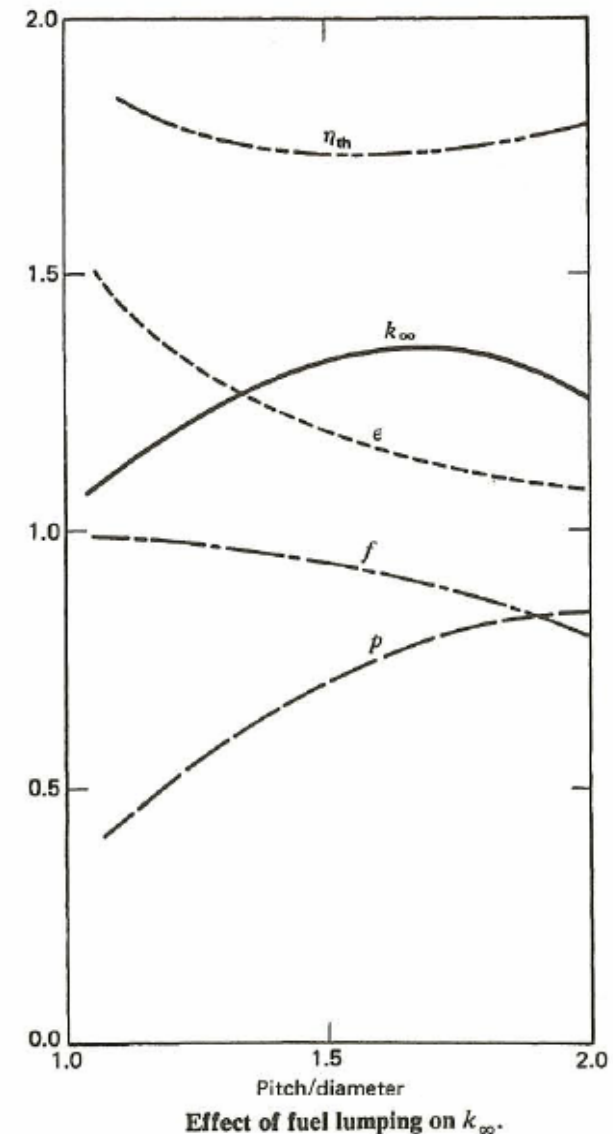
Effect of Core Lattice Geometry on k_{∞}

- Reactors are not designed with homogeneous fuel and moderator mixtures
- Typical BWR 8x8 fuel bundle:
- Ratio of water to Uranium is frequently characterized by:
- Pellet Diameter
- Fuel Rod Pitch (center to center distance of fuel pellets)
- Studies have been performed to optimize water to Uranium mixture and geometry



Effect of Core Lattice Geometry on k_{∞}

- Assume 2-3% Uranium
- Vary fuel pin pitch/diameter ratio
- Calculate η , ϵ , p , f , k_{∞} as function of: pitch/diameter ratio
- Increased pitch increases water:
- Decreases fast fission of U^{238} : ϵ
- Decreases thermal utilization: f
- Increases resonance escape: p
- k_{∞} reaches maximum value at pitch/diameter ≈ 1.65



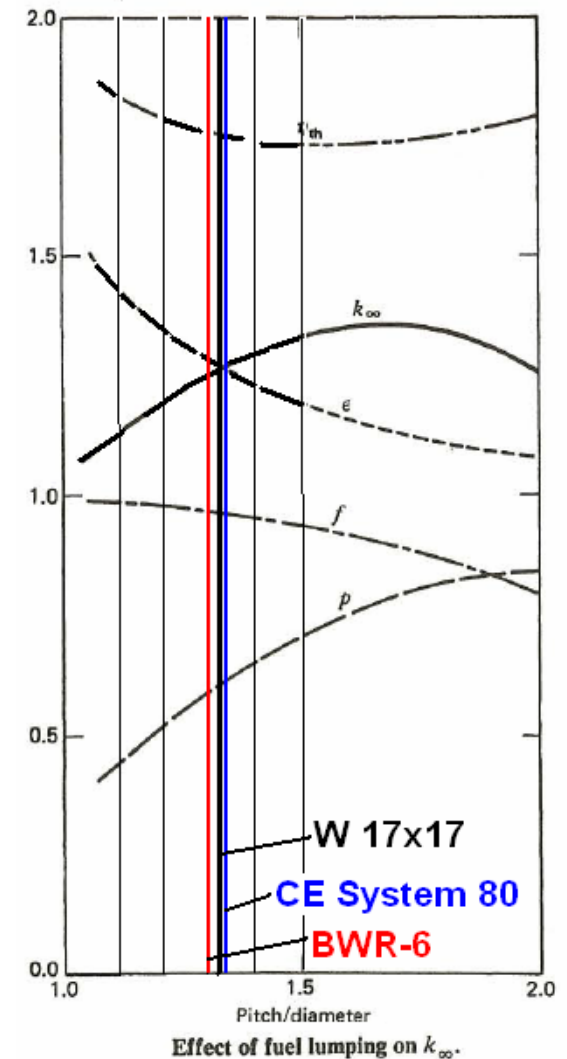
From: J.J.Duderstadt, L.J. Hamilton,
"Nuclear Reactor Analysis, p.405

Homogenous vs. Heterogeneous

- Homogenous reactor system would be uniform mixture of fuel, moderator, absorbers, and poison
- As: p , f factors tend to completely homogenous mixture:
 - $p \rightarrow 1.0$ (due to faster moderation, less resonance capture)
 - But: f decreases (due to parasitic capture in light water)
- Recall:
$$f = \frac{V_f(\bar{\Sigma}_c + \bar{\Sigma}_f)\bar{\varphi}_f}{V_f(\bar{\Sigma}_c + \bar{\Sigma}_f)\bar{\varphi}_f + V_m\bar{\Sigma}_c\bar{\varphi}_m}$$
- Early experiments and calculations showed that separating fuel from moderator allowed minimum critical dimensions to be reduced for light water reactors

Comparisons to Actual Vendor Fuel Designs

Vendor:	GE	W	B&W	CE
Type:	BWR-6	RESAR		System 80
Bundle Array:	8x8	17x17	17x17	16x16
$U^{235}\%$	2.2-2.7	2.1-3.1	2.91	1.9-2.9
Pitch:	1.62cm	1.25cm.	1.27cm.	1.28cm.
Pellet Diameter:	1.25cm	0.94cm.	0.96cm.	0.97cm.
<u>Pitch</u> : Diameter	1.30	1.32	1.32	1.33



Four Factor Formula for: k_{∞}

- Infinite medium multiplication factor
- Using *Thermal Averaged Approximations*:
- $k_{\infty} = \eta \epsilon p f$
- Typical ranges, fresh fuel (no poison/shims):

Parameter	PWR	BWR
η	1.65 - 1.89	1.65 - 1.89
ϵ	1.02 - 1.27	1.02 - 1.28
p	0.63 – 0.87	0.63 – 0.87
f	0.71 - 0.94	0.71 - 0.94
k_{∞}	1.04 - 1.41	1.04 - 1.40

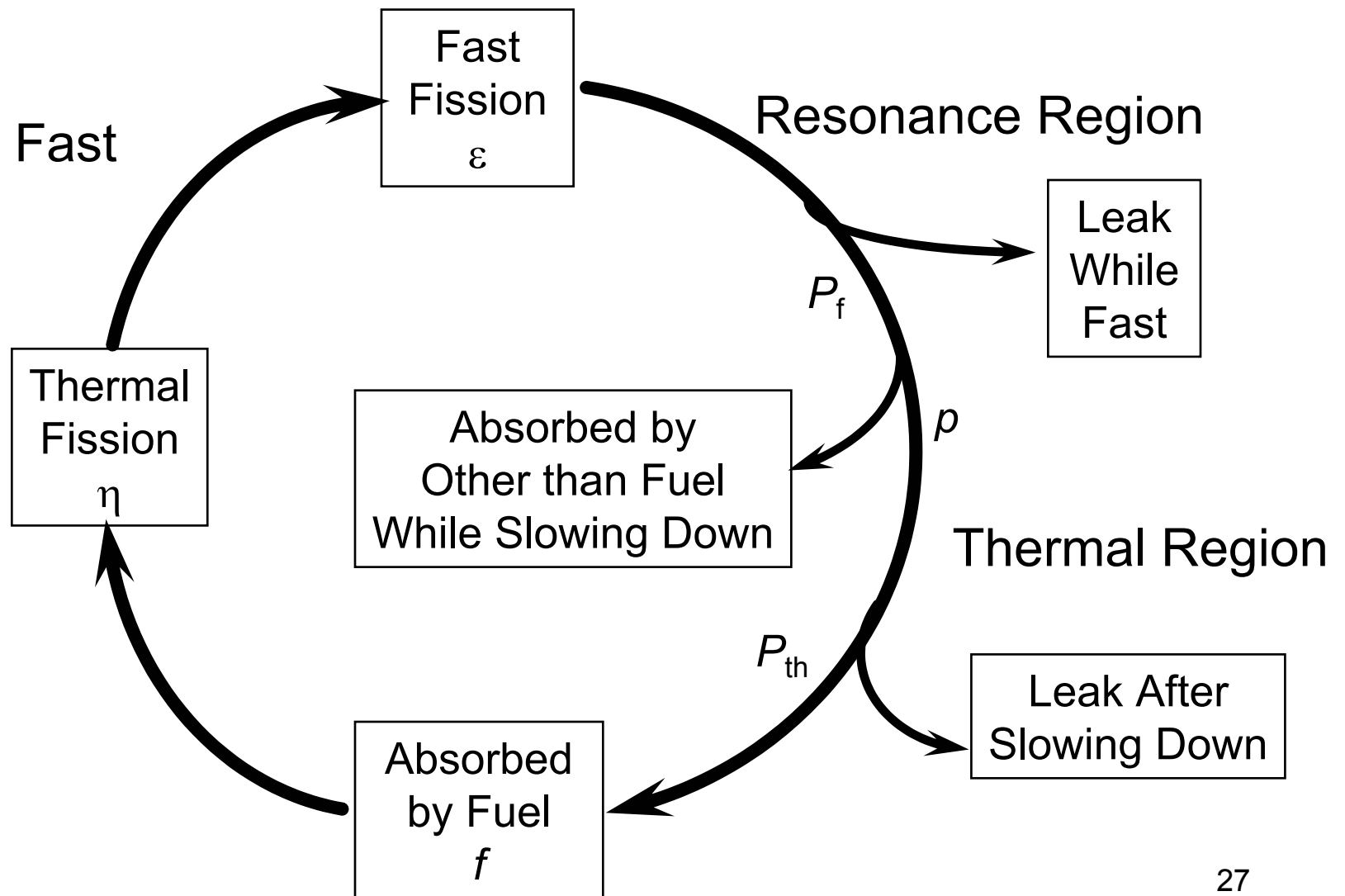
from: E. E. Lewis, “Nuclear Reactor Physics”, p. 101,

J.J. Duderstadt, L.J. Hamilton, “Nuclear Reactor Analysis”, p. 83.

Reactors Not Infinite-Medium Systems

- In ideal infinite medium: *no surface/volume effects*
- Fast and Thermal leakage out of chain reacting region needs to be considered in finite systems
- Leakage effects result in: “ k_{eff} ”
- Effective multiplication factor k_{eff} is derived from k_{∞} via adjustments for leakage effects
- Thus: $k_{eff} = k_{\infty} P_f P_{th}$
- Where:
- P_f corrects k_{∞} for fast neutron leakage
- P_{th} corrects k_{∞} for thermal neutron leakage

Finite Medium Chain Reaction → Leakage



One Group Diffusion Criticality Model

One-Group Diffusion Criticality Model

- Assume that all neutrons in bare (non-reflected) reactor are *thermal* – including fission neutrons
- $P_f \approx 1.0$ – no fast neutron leakage
- $k_{eff} = k_{\infty} P_{th}$
- P_{th} can be determined from One-Group Neutron Diffusion Model and solving for Eigenvalues that yield an assumed Critical condition

One-Group Diffusion Criticality Model

- Assume steady-state “bare” critical reactor system (no reflected neutrons)

$$0 = S(\vec{r}) - \phi(\vec{r})\Sigma_a(\vec{r}) + D\nabla^2\phi(\vec{r})$$

- Assume source is from thermal neutron fission:

$$S(\vec{r}) = \Sigma_a(\vec{r})\phi(\vec{r})k_\infty$$

- Rearrange by dividing out absorption cross section and flux:

$$0 = k_\infty - 1 + \frac{D\nabla^2\phi(\vec{r})}{\Sigma_a(\vec{r})\phi(\vec{r})} = k_\infty - 1 + L^2 \frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})}$$

- Recognize that Geometrical Buckling: B is eigenvalue of:

$$\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})} = -B^2$$

- Given *assumption* of critical system, following constraint exists defining relationship for criticality:

$$0 = k_\infty - 1 - L^2 B^2$$

$$\frac{k_\infty}{1 + L^2 B^2} = 1$$

One-Group Diffusion Criticality Model

- For finite medium, k_{eff} can be defined:

$$k_{eff} = \frac{k_{\infty}}{1 + L^2 B^2}$$

- The thermal non-leakage probability P_{th} is thus:

$$P_{th} = \frac{1}{1 + L^2 B^2}$$

Example: Yankee Rowe – Fresh Fuel

Based upon Yankee Rowe core with SS Clad, 2.7% enriched U²³⁵

$\Sigma_a U^{235} := 0.132$ Average macroscopic neutron absorption cross section in U²³⁵ in cm⁻¹.

$\Sigma_a U^{238} := 0.0192$ Average macroscopic neutron absorption cross section in U²³⁸ in cm⁻¹.

$\Sigma_a H_2O := 0.0131$ Average macroscopic neutron absorption cross section in H₂O in cm⁻¹.

$\Sigma_a Clad := 0.0180$ Average macroscopic neutron absorption cross section in Clad in cm⁻¹.

$\Sigma_f U^{235} := 0.1113$ Average macroscopic fission cross section in U²³⁵ in cm⁻¹.

$\nu := 2.43$ Average number of neutrons generated per U²³⁵ fission

$\Phi_{mu} := 1.12$ Ratio of: $\frac{\phi_m}{\phi_u}$

$\Phi_{cu} := 1.06$ Ratio of: $\frac{\phi_c}{\phi_u}$

$H_o := 700$ Height of Cylindrical Reactor in cm.

$R_o := 150$ Radius of Cylindrical Reactor in cm.

from: S. Glasstone & A. Sesonske, “Nuclear Reactor Engineering” (1967), p. ³²203

Example: Yankee Rowe – Fresh Fuel

$$\eta := \frac{\nu \cdot \Sigma f U235}{\Sigma a U235 + \Sigma a U238}$$

$$\eta = 1.789$$

Neutrons produced per fission
Neutrons absorbed in Uranium

$$\varepsilon := 1.044$$

Fast fission factor

$$p := 0.931$$

Resonance escape probability

$$f := \frac{\Sigma a U235 + \Sigma a U238}{\Sigma a U235 + \Sigma a U238 + \Sigma a H2O \cdot \Phi_{mu} + \Sigma a Clad \cdot \Phi_{cu}}$$

$$f = 0.818$$

Thermal utilization factor

$$\eta \cdot f = 1.462$$

$$L := 2.37$$

Square Root of Diffusion

$$\text{area: } \sqrt{\frac{D}{\Sigma a}}$$

$$B := \sqrt{\left(\frac{2.405}{R_o}\right)^2 + \left(\frac{\pi}{H_o}\right)^2}$$

$$B = 0.017$$

$$B^2 = 2.772 \times 10^{-4}$$

Geometrical Buckling Factor

$$P_{th} := \frac{1}{1 + L^2 \cdot B^2}$$

$$P_{th} = 0.998$$

Calculation of Infinite Medium Multiplication Factor

$$k_{\infty} := \eta \cdot \varepsilon \cdot p \cdot f$$

$$k_{\infty} = 1.421$$

Infinite Medium
Multiplication Factor

Calculation of 1-Group k_{eff} Multiplication Factor

$$k_{eff1G} := \frac{k_{\infty}}{1 + L^2 \cdot B^2}$$

$$k_{eff1G} = 1.419$$

1-Group k_{eff}
Multiplication Factor

from: S. Glasstone & A. Sesonske, "Nuclear Reactor Engineering" (1967), p. 204²³208

Two-Group Diffusion Criticality Model

Two-Group Diffusion Criticality Model

- Assume that all neutrons in bare (non-reflected) reactor are either: *thermal* or *fast*
- P_f calculated instead of being ignored
- $k_{eff} = k_{\infty} P_f P_{th}$
- P_f, P_{th} can be determined from Two-Group Neutron Diffusion Model and solving for Eigenvalues that yield an assumed Critical condition.
- $k_{\infty} = \eta \epsilon p f$ needs to be split up into portions representing *thermal* (ηf) and *fast* (ϵp) neutron contributions.

Two-Group Diffusion Criticality Model

- Assume steady-state “bare” critical reactor system (no reflected neutrons) is represented by system of equations:
- Assume fast neutron source is from thermal neutron fission:
- Assume thermal neutron source is thermalized fission neutrons enhanced by fast fission effect and which escape resonance capture:

$$0 = S_f - \phi_f \Sigma_{a-f} + D_f \nabla^2 \phi_f$$

$$0 = S_{th} - \phi_{th} \Sigma_{a-th} + D_{th} \nabla^2 \phi_{th}$$

$$S_f = \Sigma_{a-th} \phi_{th} \eta f = \frac{D_{th}}{L_{th}^2} \phi_{th} \eta f$$

$$S_{th} = \Sigma_{a-f} \phi_f \epsilon p = \frac{D_f}{L_f^2} \phi_f \epsilon p$$

Two-Group Diffusion Criticality Model

- Making substitutions and rearranging yields:

$$0 = \left(\frac{D_{th}}{D_f} \right) \frac{1}{L_{th}^2} \phi_{th} \eta f - \frac{1}{L_f^2} \phi_f + \nabla^2 \phi_f$$

$$0 = \left(\frac{D_f}{D_{th}} \right) \frac{1}{L_f^2} \phi_f \epsilon p - \frac{1}{L_{th}^2} \phi_{th} + \nabla^2 \phi_{th}$$

- Making substitution for geometric Buckling:

$$0 = \left(\frac{D_{th}}{D_f} \right) \frac{1}{L_{th}^2} \phi_{th} \eta f - \frac{1}{L_f^2} \phi_f - B_f^2 \phi_f$$

$$0 = \left(\frac{D_f}{D_{th}} \right) \frac{1}{L_f^2} \phi_f \epsilon p - \frac{1}{L_{th}^2} \phi_{th} - B_{th}^2 \phi_{th}$$

Two-Group Diffusion Criticality Model

- This is system of linear equations:
- Solving Determinant yields:

$$\begin{bmatrix} -B_f^2 - \frac{1}{L_f^2} & \left(\frac{D_{th}}{D_f}\right) \frac{\eta f}{L_{th}^2} \\ \left(\frac{D_f}{D_{th}}\right) \frac{\epsilon p}{L_f^2} & -B_{th}^2 - \frac{1}{L_{th}^2} \end{bmatrix} \times \begin{bmatrix} \phi_f \\ \phi_{th} \end{bmatrix} = 0$$

$$\begin{vmatrix} -B_f^2 - \frac{1}{L_f^2} & \left(\frac{D_{th}}{D_f}\right) \frac{\eta f}{L_{th}^2} \\ \left(\frac{D_f}{D_{th}}\right) \frac{\epsilon p}{L_f^2} & -B_{th}^2 - \frac{1}{L_{th}^2} \end{vmatrix} = (B_f^2 + \frac{1}{L_f^2})(B_{th}^2 + \frac{1}{L_{th}^2}) - \eta \epsilon p f = 0$$

- Which simplifies to:

$$\frac{k_{\infty}}{(1 + L_f^2 B_f^2)(1 + L_{th}^2 B_{th}^2)} = 1$$

Two-Group Diffusion Criticality Model

- For finite medium, k_{eff} can be defined:

$$k_{eff} = \frac{k_{\infty}}{(1 + L_f^2 B_f^2)(1 + L_{th}^2 B_{th}^2)}$$

- The fast non-leakage probability P_f is thus:

$$P_f = \frac{1}{1 + L_f^2 B_f^2}$$

- The thermal non-leakage probability P_{th} is thus:

$$P_{th} = \frac{1}{1 + L_{th}^2 B_{th}^2}$$

Two-Group Criticality Model – Example

- *Thermal multiplication factor:* $\eta = 1.65$
- *Fast fission factor:* $\varepsilon = 1.02$
- *Resonance escape factor:* $p = 0.87$
- *Thermal utilization factor:* $f = 0.71$

- $k_{\infty} = \eta \varepsilon p f = (1.65)(1.02)(0.87)(0.71) = 1.0396$

- *Fast non-leakage factor:* $P_f = 0.98$
- *Thermal non-leakage factor:* $P_{th} = 0.99$

- $k_{eff} = k_{\infty} P_f P_{th} = (1.0396)(0.97)(0.99) = 1.008$

Geometrical Buckling

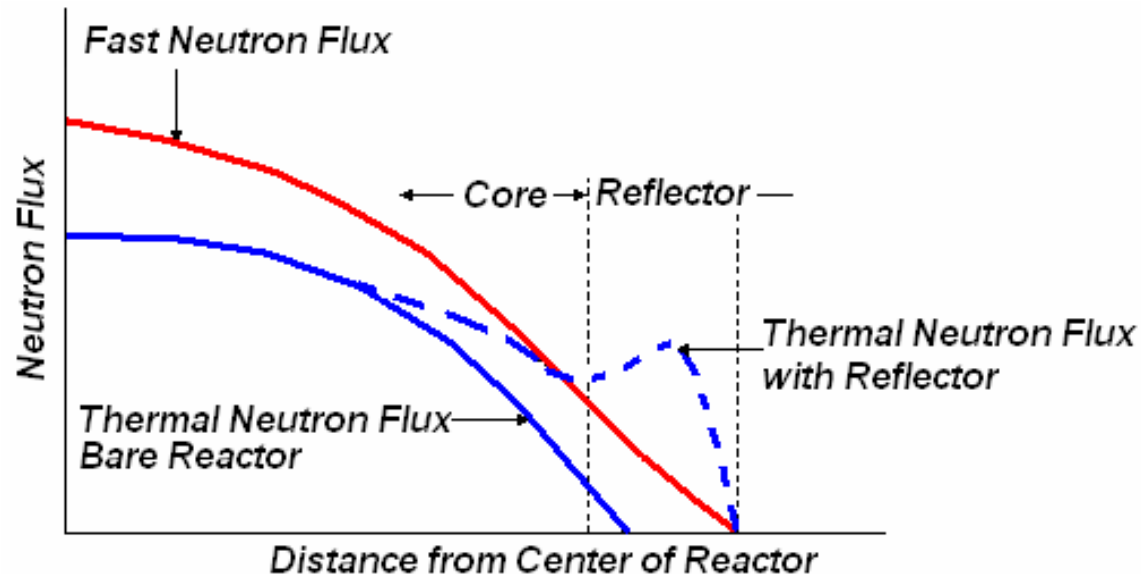
- Geometrical Buckling factor: B^2 is an eigenvalue of Helmholtz type partial differential equation
- Geometrical Buckling factor captures surface to volume effects of different geometries
- Following Buckling factors are for bare, un-reflected core designs:

<i>Geometry:</i>	<i>Dimensions:</i>	<i>Buckling:</i>	<i>Flux Shape:</i>
Rectangular Block	$a \times b \times c$	$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$\phi(x, y, z) = A_0 \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \cos\left(\frac{\pi z}{c}\right)$
Sphere	Radius : R	$B^2 = \left(\frac{\pi}{R}\right)^2$	$\phi(r) = \frac{A_0}{r} \sin\left(\frac{\pi r}{R}\right)$
Cylinder	Radius : R Height : H	$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$	$\phi(r, z) = A_0 J_0\left(\frac{2.405 r}{R}\right) \cos\left(\frac{\pi z}{H}\right)$

Taken from: J. Lamarsh, "Nuclear Reactor Analysis, p.298

Effect of Neutron Reflector on Criticality

- Previous discussion of Two-Group Diffusion model noted impact of water region outside of active core.
- Neutron reflection alters the Buckling coefficients derived for *bare, un-reflected* core geometry



Summary Thoughts on Criticality Evaluation:

- Subcriticality, Criticality, Supercriticality conditions are based upon overall “ k_{eff} ”
- Fuel enrichment, bundle geometry, Uranium to Water ratio directly influences: k_{∞}
- Fresh fuel bundles (neglecting impacts of poisons or control rods) generally have range of $k_{\infty} \sim 1.2$ or higher to provide fuel for multiyear power operation
- Overall geometry of core (height, radius), reflector region impact fast and thermal non-leakage probabilities and thus: k_{eff}
- Classical methods described, reflect correct trends, BUT:
- Actual core design process is computer code intensive