

SH2702
Nuclear Reactor Technology
Exercise Session 03

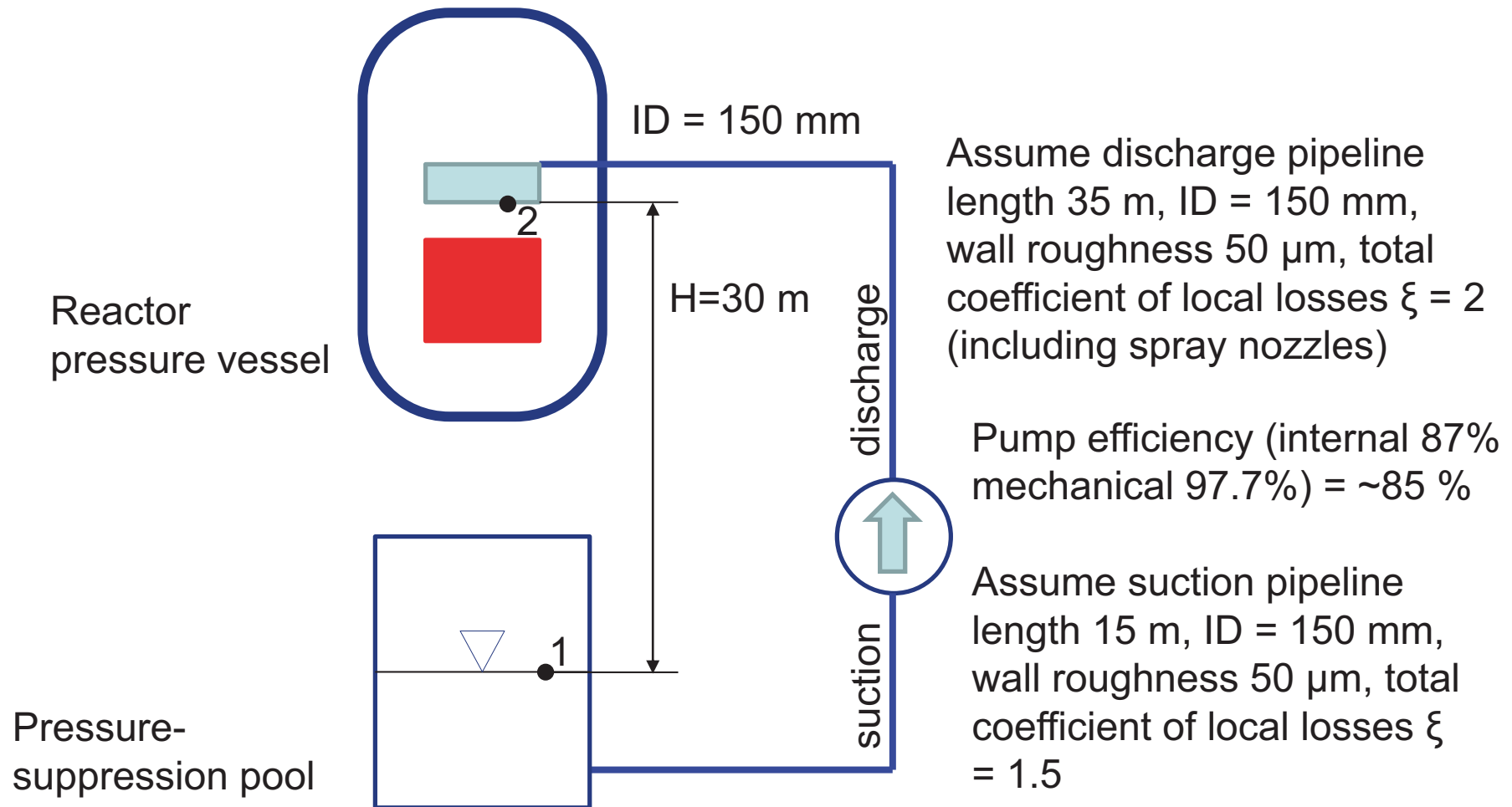
Emergency Core Cooling System (ECCS) in BWR

- BWR has a core spray system to provide water to the core in case the coolant level in the vessel is so low that there is a high risk that the core will be uncovered.
- The spray system will then prevent over-heating of the core by spraying water over fuel elements and eventually refill the core with water.
- The water is pumped from the suppression pools to a water box with nozzles placed just above the core.
- The system has usually at least two completely independent pumping loops, with connected fittings and instrumentation.

ECCS in BWR - Exercise

- Each of the loops has 100% capacity needed to provide safe cooling of the core.
- **EXERCISE:** Assume that one line of the Emergency Core Cooling System (ECCS) provides 150 kg/s of water from pressure suppression pool to the reactor pressure vessel (RPV). The pressure in RPV is 10 bars (abs). Assume atmospheric pressure and temperature $T = 25^\circ\text{C}$ in the pressure-suppression pool. Calculate:
 - (a) the required (mechanical) pumping power
 - (b) the specific enthalpy of water leaving the reactor at steady-state (long operation of ECCS with all parameters constant)

ECCS in BWR - Exercise



ECSS in BWR - Exercise

- **Assume:**
 - in the whole system, the same liquid properties as in the suppression pool; discharge spec. enthalpy= $i(p=1 \text{ MPa}, T=25^\circ\text{C})$
 - 500 nozzles with diameter 15 mm placed above the core
 - infinitely large water surface area in the suppression pool
 - core thermal power after shutdown (decay heat) 60 MW
- **Hint:** use the following equation to find the total pressure change from point 1 to 2 ($\Delta p_{\text{pump}} = p_{\text{dis}} - p_{\text{suc}}$)

$$\underbrace{p_1 - p_2}_{\text{total pressure drop}} = \frac{W^2}{2\rho} \left[\underbrace{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right)}_{\text{reversible velocity head}} + \underbrace{\sum_k \left(\frac{4L_k}{D_{h,k}} C_{f,k} \right) \frac{1}{A_k^2}}_{\text{irreversible friction loss}} + \underbrace{\sum_j \xi_j \frac{1}{A_{j,\min}^2}}_{\text{irreversible local loss}} \right] + \underbrace{\rho g (H_2 - H_1)}_{\text{reversible gravity head}} - \Delta p_{\text{pump}}$$

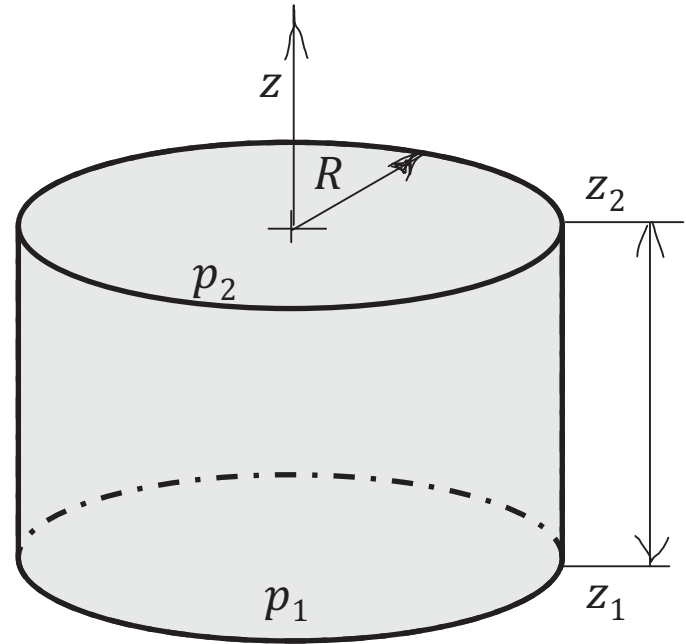
W – mass flow rate, A – cross-section area, D_h – hydraulic diameter, H – vertical height, L – pipe length, C_f – fanning friction factor, ρ – density, p – pressure

Friction Factors

- For turbulent flows, τ_w , can be obtained only from correlations
- The correlations are frequently expressed in terms of friction factors
- Two different definitions for friction factors are used:

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

Fanning friction factor



$$\lambda \equiv \frac{4\tau_w}{\frac{1}{2}\rho U^2}$$

Darcy-Weisbach friction factor

Haaland's Correlation

- To avoid iteration, Haaland proposed the following **explicit** correlation for the Darcy-Weisbach friction factor

$$\frac{1}{\sqrt{\lambda}} = -1.8 \log_{10} \left[\left(\frac{k/D}{3.7} \right)^{1.11} + \frac{6.9}{\text{Re}} \right]$$

- This correlation agrees within 1% with the Colebrook's correlation
 - k , D and Re have the same meaning as in the Colebrook's correlation

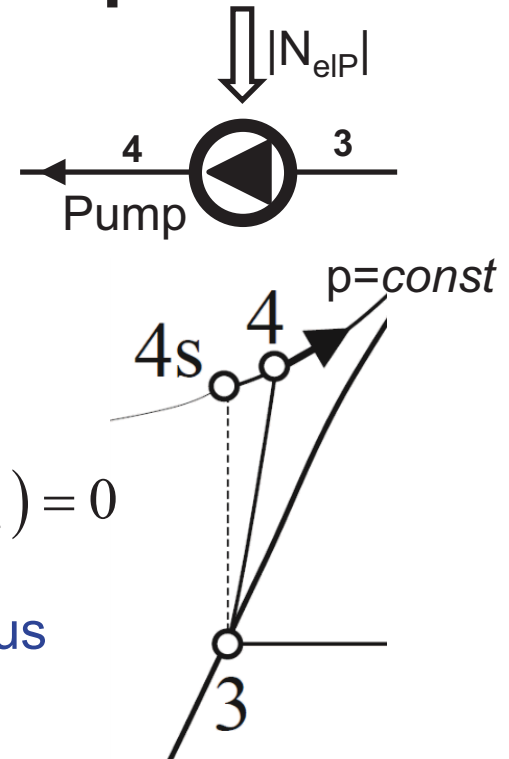
ECCS in BWR - Pump

- To increase pressure from 3 to 4 (see fig.) pumping power $|N_{iP}|$ has to be supplied
- From the energy conservation principle for steady-state ($dE_T/dt=0$) we have

$$\frac{dE_T}{dt} = q - N_{iP} + W_3 (i_3 + e_{P3} + e_{K3}) - W_4 (i_4 + e_{P4} + e_{K4}) = 0$$

- here we have to supply power to the system thus $N_{iP} < 0$; no heat is added thus $q = 0$; we also neglect kinetic and potential energy changes and from mass conservation we have $W_3 = W_4 = W$

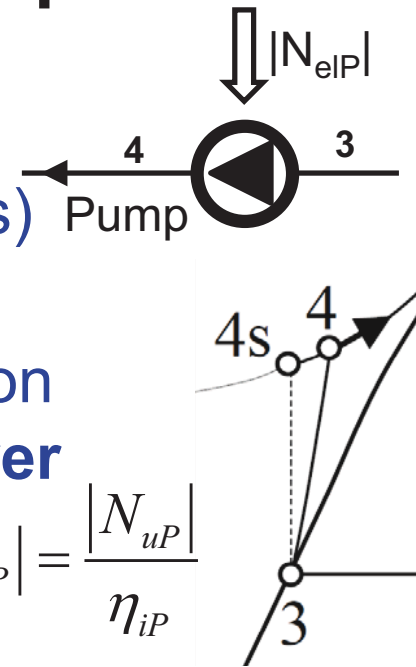
$$|N_{iP}| = W (i_4 - i_3) = W \left(\underbrace{e_{I4} - e_{I3}}_{\text{internal energy increase}} + \frac{p_4 - p_3}{\rho_e} \right) = \underbrace{W \frac{p_4 - p_3}{\rho_e}}_{\substack{N_{up} = \text{useful pumping} \\ \text{power}}} + \underbrace{W \Delta e_I}_{\text{internal energy increase}}$$



Here ρ_e is an equivalent fluid density for process 3-4. Typically we assume $\rho_e \approx \rho_3 \approx \rho_4$ (incompressible)

ECCS in BWR - Pump

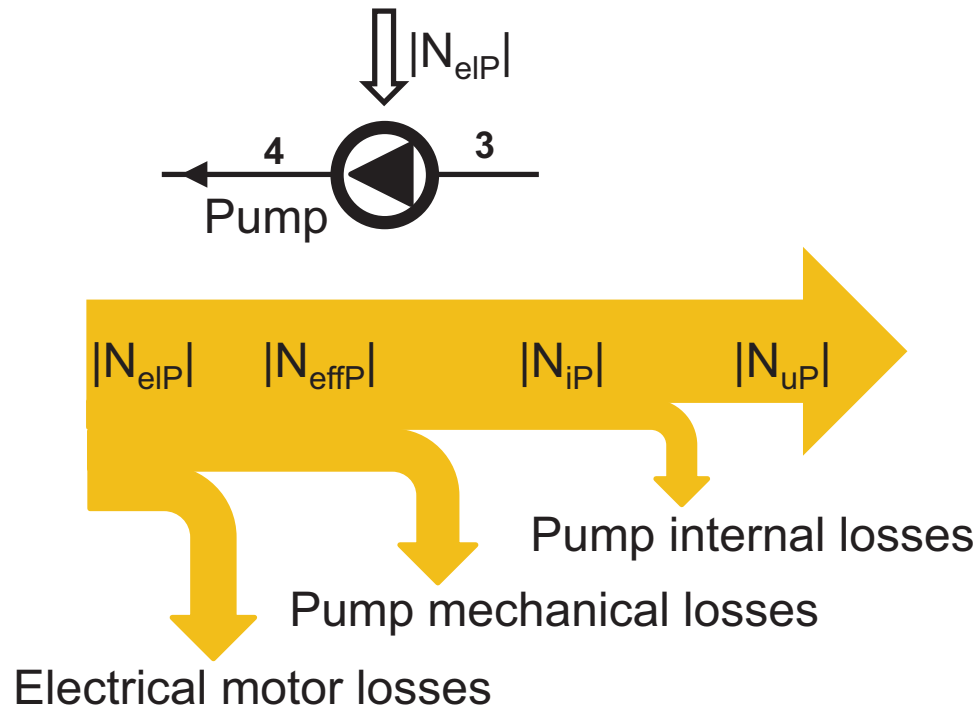
- The pumping power $|N_{thP}|$ is a theoretical pumping power (used in ideal cycle analyses)
- For real process 3-4, we obtain the pumping power $|N_{iP}|$ from energy conservation as $|N_{iP}| \equiv W(i_4 - i_3)$ and call this **internal power**
- Due to internal losses, internal power is: $|N_{iP}| = \frac{|N_{uP}|}{\eta_{iP}}$
- We also define an effective (or mechanical) pumping power, N_{effP} , due to pump mechanical efficiency η_{mP} :
- Finally, the electric motor power for pumping is found as:



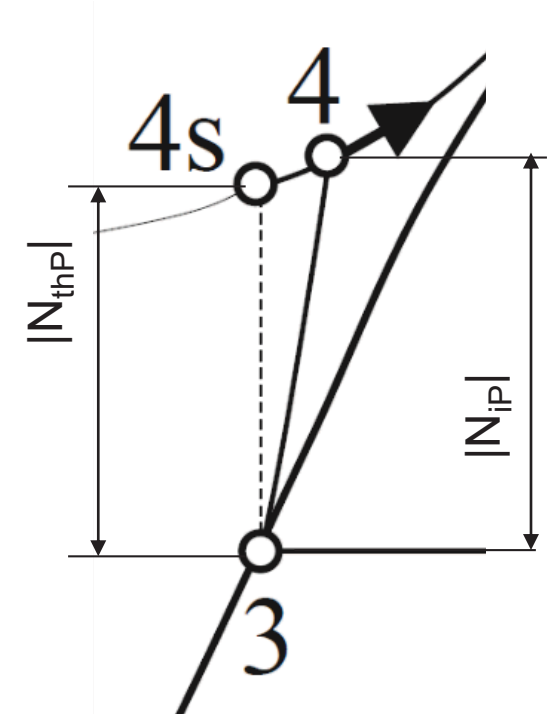
$$|N_{effP}| = \frac{|N_{iP}|}{\eta_{mP}} \quad N_{uP} - \text{useful pumping power}$$

$$|N_{elP}| = \frac{|N_{effP}|}{\eta_{EM}} \quad \text{Here } \eta_{EM} \text{ is the electrical motor efficiency}$$

ECCS in BWR - Pump



Typical tasks: **(1)** calculate required electrical power to produce given pressure difference; **(2)** calculate specific enthalpy at pump discharge for given electrical power; **(3)** the same as in (2) for given pressure drop



$$\begin{aligned} 1) \quad |N_{elP}| &= W \frac{p_4 - p_3}{\eta_{iP} \eta_{mP} \eta_{EM} \rho_e} \\ 2) \quad i_4 - i_3 &= \frac{\eta_{mP} \eta_{EM} |N_{elP}|}{W} \\ 3) \quad i_4 - i_3 &= \frac{p_4 - p_3}{\rho_e \eta_{iP}} \end{aligned}$$

ECSS in BWR - Exercise

- **Hint (cont.)** Here we have case 1, but we rather have to find the effective (mechanical power) that is needed:

$$\left| N_{effP} \right| = \eta_{EM} \left| N_{elP} \right| = W \frac{\Delta p}{\eta_{iP} \eta_{mP} \rho_e}$$

we can note that this mechanical power is related to torque and angular speed of the pump as:

$$\left| N_{effP} \right| = \omega T$$

T – torque,
 ω – angular speed

ECSS in BWR - Exercise

Solution: we find water density and dynamic viscosity:

- density $\rho = f(p=1.01 \text{ bar}, T=25 \text{ }^{\circ}\text{C}) = 997.05 \text{ kg/m}^3$
- viscosity $\mu = f(p=1.01 \text{ bar}, T=25 \text{ }^{\circ}\text{C}) = 8.9 \cdot 10^{-4} \text{ Pa s}$

We find Reynolds number in pipes:

$$\text{Re} = W \cdot D_h / (A \cdot \mu) = 150 \cdot 0.15 / (\pi \cdot 0.15^2 / 4 \cdot 8.9 \cdot 10^{-4}) = 1.43 \cdot 10^6$$

Now we can find the Fanning friction factor C_f from the Haaland correlation as $C_f = 0.004$.

ECCS in BWR - Exercise

We calculate the required pressure head that should be provided by the pump:

$$\Delta p_{pump} = \underbrace{p_2 - p_1}_{\text{pressure drop}} + \frac{W^2}{2\rho} \left[\frac{1}{A_2^2} + \underbrace{\frac{4L_s C_{f,s}}{D_s A_s^2} + \frac{\xi_s}{A_s^2}}_{\text{suction}} + \underbrace{\frac{4L_d C_{f,d}}{D_d A_d^2} + \frac{\xi_d}{A_d^2}}_{\text{discharge}} \right] + \underbrace{\rho g H}_{\text{gravity head}} =$$

$$1 \times 10^6 - 1.01 \times 10^5 + \frac{150^2}{2 \times 997.05} \left[\frac{1}{0.088^2} + \underbrace{\frac{4 \times 15 \times 0.004}{0.15 \times 0.018^2} + \frac{1.5}{0.018^2}}_{\text{suction}} + \underbrace{\frac{4 \times 35 \times 0.004}{0.15 \times 0.018^2} + \frac{2}{0.018^2}}_{\text{discharge}} \right] +$$

$$997.05 \times 9.81 \times 30 = 1.51 \times 10^6 \text{ Pa}$$

Thus, the pressure change $\Delta p = 1.51 \text{ MPa}$

ECCS in BWR - Exercise

The useful (hydraulic) pumping power is

$$|N_{uP}| = \frac{W \Delta p}{\rho} = \frac{150 \times 1.51 \cdot 10^6}{997.05} = 2.27 \cdot 10^5 \quad \text{W}$$

Note: as specified, we used the density of water the same as in the suppression pool

The mechanical pumping power (85% total pump efficiency):

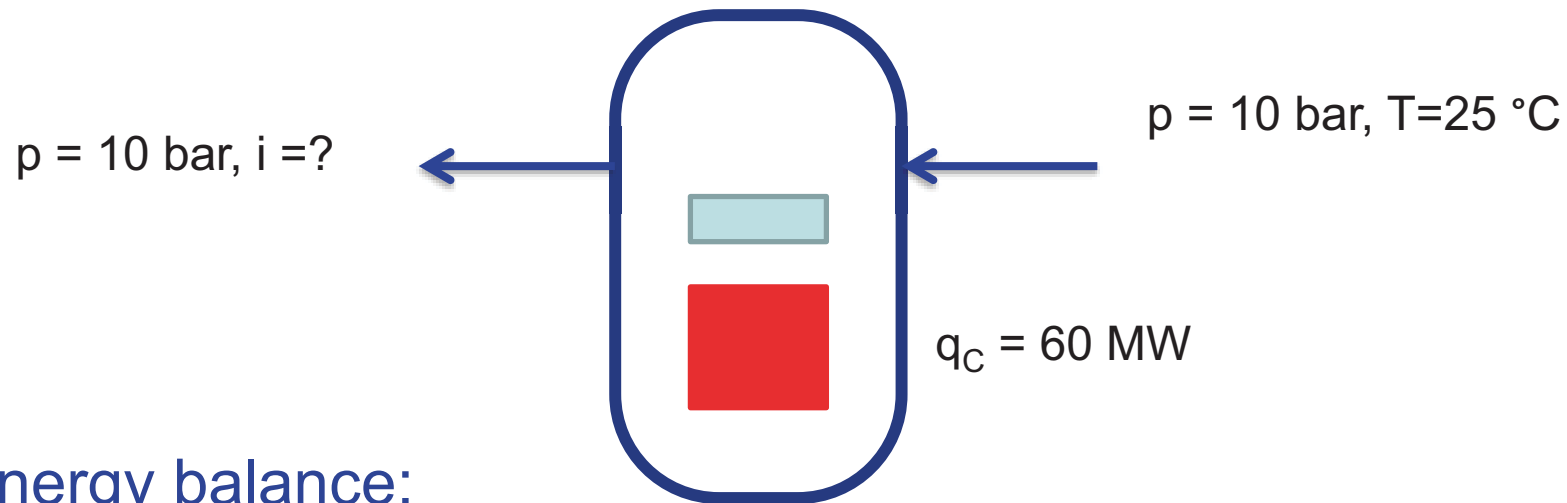
$$|N_{effP}| = \frac{|N_{uP}|}{\eta_p} = \frac{2.27 \cdot 10^5}{0.85} = 2.67 \cdot 10^5 \quad \text{W}$$

$$\eta_p = \eta_{iP} \eta_{mP}$$

Answer: the mechanical pumping power that has to be supplied is 267 kW

ECCS in BWR - Exercise

The exit enthalpy of the water is found from the energy balance:



Energy balance:

$$W(i_{ex} - i_{in}) = q_C \Rightarrow i_{ex} = i_{in} + \frac{q_C}{W} \quad i_{in} = f(p=10 \text{ bar}, T = 25 \text{ }^{\circ}\text{C}) = 105.8 \text{ kJ/kg}$$

ECCS in BWR - Exercise

- We find the specific enthalpy of water at exit as:

$$i_{ex} = i_{in} + \frac{q_C}{W} = 105800 + \frac{60 \cdot 10^6 \text{ J/s}}{150 \text{ kg/s}} = 505.8 \frac{\text{kJ}}{\text{kg}}$$

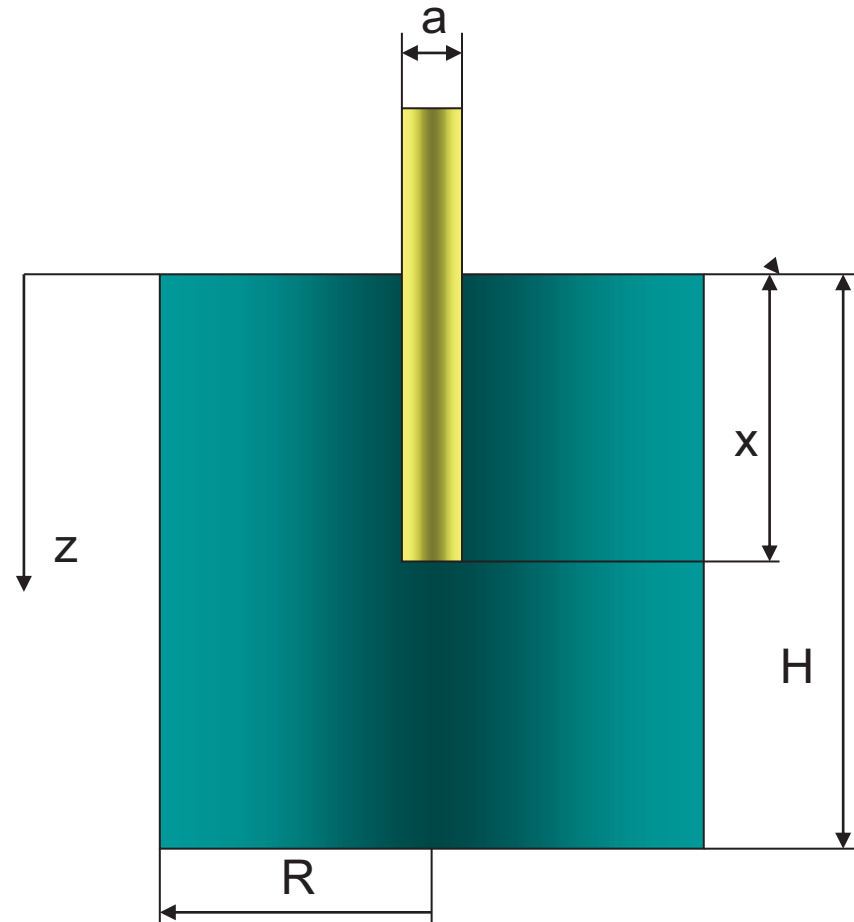
- We can check, that this is a subcooled liquid, since the saturated liquid enthalpy at 10 bars is $i_f(10 \text{ bars}) = 762.7 \text{ kJ/kg}$. We find its temperature $T = 120.3 \text{ °C} < T_f(10 \text{ bar}) = 179.9 \text{ °C}$.
- However, when this water escapes to open atmosphere with atmospheric pressure, it will evaporate (flashing).

Control Rod Worth

- Consider a reactor with a central partially inserted control rod
- Applying a one-group diffusion approximation and using the perturbation theory it can be shown that the reactivity change $\Delta\rho(x)$ depends on the insertion length as follows:

$$\Delta\rho(x) = \Delta\rho(H) \left(\frac{x}{H} - \frac{1}{2\pi} \sin \frac{2\pi x}{H} \right)$$

Where $\Delta\rho(H)$ is the reactivity change due to full insertion of the control rod

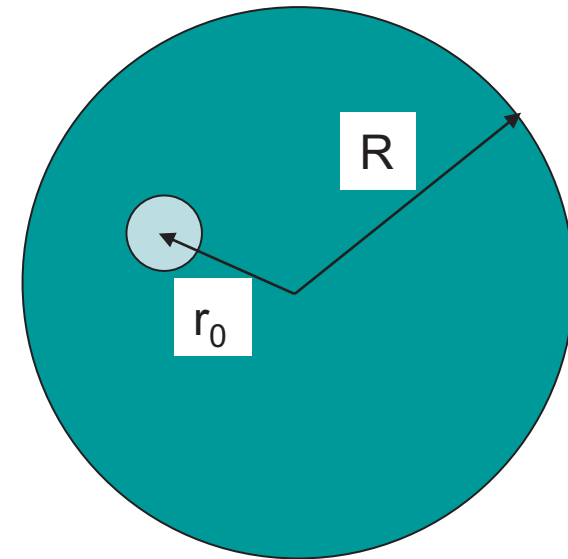


Control Rod Worth

- When the control rod is placed at $r = r_0$ distance from the centerline of the reactor, the reactivity change for such a rod can be estimated as

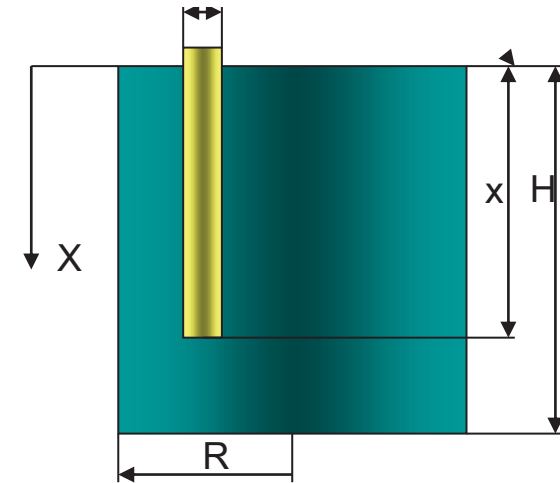
$$\Delta\rho(r_0) = J_0^2 \left(\frac{2.4048 r_0}{R} \right) \Delta\rho(0)$$

- Where $\Delta\rho(r_0)$ is the reactivity change for off-centerline rod and $\Delta\rho(0)$ is the reactivity change for a rod inserted at the centerline



Control Rod Worth - Exercise

- A control rod is placed at $r_0 = 0.75$ m distance from the centerline of a cylindrical reactor with radius $R=1.85$ m and height $H=3.66$ m. Calculate the reactivity change caused by a withdrawal of the rod by 2.5 cm, if the rod was initially inserted into the core with $2/3$ of its height. The integral worth of the central rod in fully-inserted position is known and equal to 2% (2000 pcm)



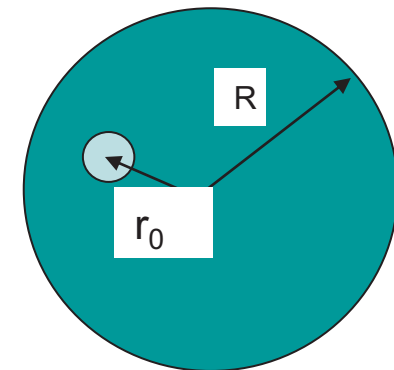
$$x = 2/3H, dx = -2.5 \text{ cm}$$

$$r_0 = 0.75 \text{ m}$$

$$R = 1.85 \text{ m}, H = 3.66 \text{ m}$$

$$\Delta\rho(x) = \Delta\rho(H) \left(\frac{x}{H} - \frac{1}{2\pi} \sin \frac{2\pi x}{H} \right)$$

$$\Delta\rho(r_0, x) = J_0^2 \left(\frac{2.4048 r_0}{R} \right) \Delta\rho(x)$$



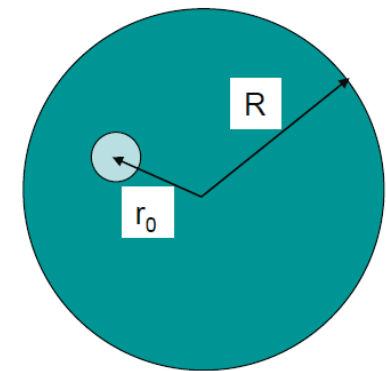
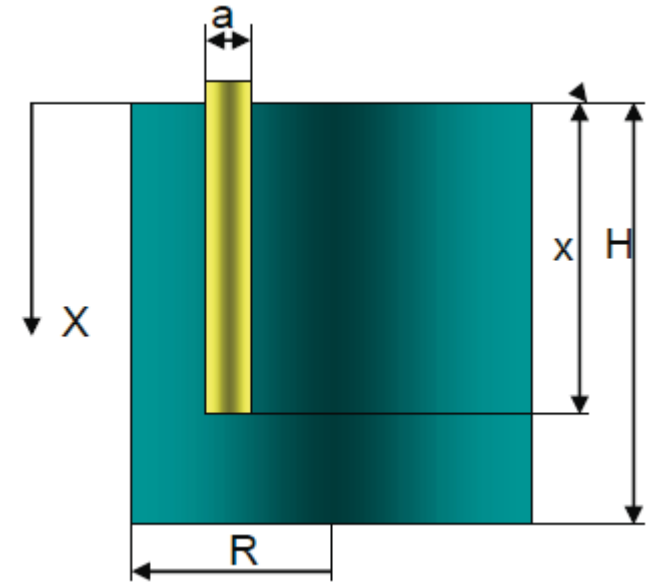
- Solution

$$\Delta\rho(x) = \Delta\rho(H) \left(\frac{x}{H} - \frac{1}{2\pi} \sin \frac{2\pi x}{H} \right)$$

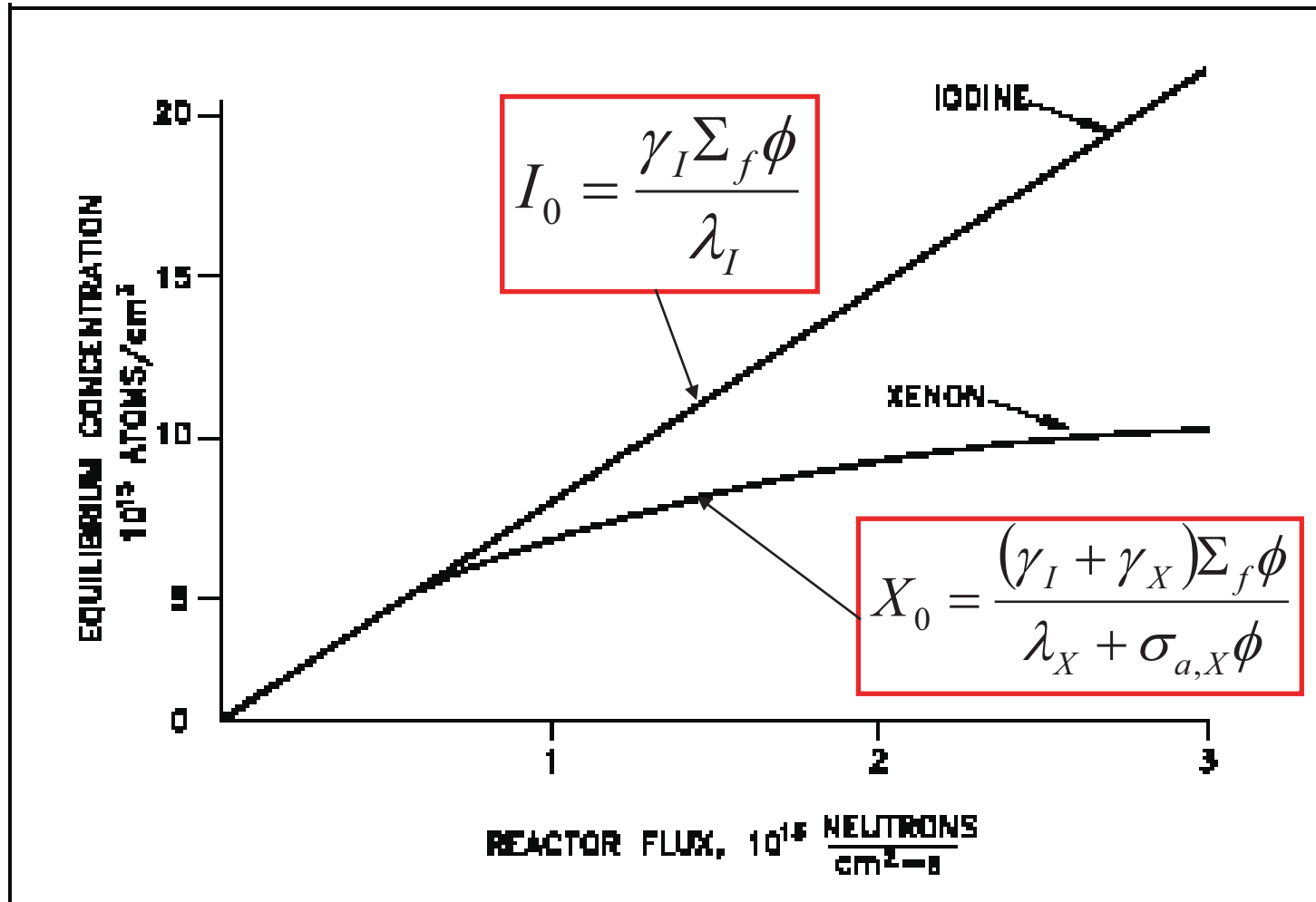
- $\text{drhoH} = -2000 \text{ pcm}$
- $\text{drho0} = \text{drhoH} * (x_0/H - \sin(2*\pi*x_0/H)/(2*\pi))$
- $\text{drho1} = \text{drhoH} * (x_1/H - \sin(2*\pi*x_1/H)/(2*\pi))$

$$\Delta\rho(r_0, x) = J_0^2 \left(\frac{2.4048 r_0}{R} \right) \Delta\rho(x)$$

- $\text{RadialFactor} = \text{besselj}(0, 2.4048*r/R)^2$
- $\text{drho} = (\text{drho1} - \text{drho0}) * \text{RadialFactor} = 12.5 \text{ pcm}$



Production and Removal of Xenon-135



Production and Removal of Xenon-135 - Exercise

- EXERCISE: A thermal nuclear reactor was loaded with fresh fuel and started. Assume that the thermal neutron flux step changed from 0 to $2 \cdot 10^{18} \text{ [m}^{-2} \text{ s}^{-1}]$ at $t = 0$.

Plot the concentration of iodine and xenon as a function of time and calculate the equilibrium concentrations of the two isotopes in the reactor. Given:

$$\lambda_I = 2.9 \times 10^{-5} \text{ [s}^{-1}], \lambda_X = 2.1 \times 10^{-5} \text{ [s}^{-1}], \gamma_I = 0.061, \gamma_X = 0.003, \\ \Sigma_f = 1.8 \cdot 10^{-1} \text{ [m}^{-1}], \sigma_{a,X} = 2.6 \times 10^6 \text{ [b]}$$

- Solution

- $I_0 = \gamma_I \Sigma_f \phi / \lambda_I = 7.57 \times 10^{20} \text{ m}^{-3}$
- $X_0 = (\gamma_I + \gamma_X) \Sigma_f \phi / (\lambda_X + \sigma_{a,X} \phi) = 4.26 \times 10^{19} \text{ m}^{-3}$

