Q1: What are all the parameters that need to be set by the user in criticality simulations?

Ans:

• The initial source distribution (for the first cycle).

• The multiplication factor for the first cycle.

• The number of inactive cycles.

• The number of active cycles.

• The number of neutron histories that we simulate at each cycle.

Q2: What is the dominance ratio of a system?

Ans:

The source shape would converge over the cycles even if we didn't normalize it by the eigenvalue:

$$s^{(n)} = \mathbb{H}s^{(n-1)}$$

$$\Rightarrow s^{(n)} = \mathbb{H}^n s^{(0)} \text{----------(i)}$$

Any real function in the domain of $\mathbb{H}$, and so also the initial fission source $s^{(0)}$ (the one that a user must guess to start the MC criticality simulation), can be written as a weighted sum of eigenfunctions s_i :

$$s^{(0)} = \sum_{\mathrm{i}} \gamma_i s_i$$

From (i),

$$s^{(n)} = \mathbb{H}^n \sum_{\mathrm{i}} \gamma_i s_i = \sum_{\mathrm{i}} \gamma_i \mathbb{H}^n s_i$$

which (when combined with the eigenvalue equation) gives

$$s^{(n)} = \sum_{\mathrm{i}} \gamma_i {k_i}^n s_i$$

When the above equation is divided by k_0^n, then it can be written as

$$s^{(n)} \sim \gamma_0 s_0 + \left(\frac{k_1}{k_0}\right)^n \gamma_1 s_1 + \left(\frac{k_2}{k_0}\right)^n \gamma_2 s_2 + \cdots$$

The source $s^{(n)}$ must converge to s_0 as $O\left(\left(\frac{k_1}{k_0}\right)^n\right)$.

The term k_1/k_0 is called the dominance ratio.

**The fundamental mode source and eigenvalue can be computed iteratively by the power iteration (that is the cycles in MC criticality simulation) as

$$s^{(n)} = \frac{\mathbb{H}s^{(\text{n-1})}}{k^{(n-1)}}$$

$$k^{(n)} = \frac{< \mathbb{H}s^{(\text{n-1})} >}{< s^{(\text{n-1})} >} = \frac{k^{(n-1)} < s^{(\text{n})} >}{< s^{(\text{n-1})} >}$$

Q3: How does the convergence rate of criticality simulations relate to the dominance ratio of a system?

Ans:

The convergence rate of the source power iteration is governed by the dominance ratio. The closer the dominance ratio is to unity the slower the source converges, and the more inactive cycles are needed.

Q4: Which systems have a large (and small) dominance ratio?

Ans:

<u>Large dominance ratio</u>

• Typically, large reactors have a dominance ratio very close to unity.

• Coupled systems (systems consisting of similarly reactive components, like a field of containers with nuclear waste)

<u>Small dominance ratio</u>

• Typically, small research reactors.

• Large reactors with a very asymmetrical configuration of control elements.

Q5: Is it possible to decrease the dominance ratio of a system to accelerate the convergence of the fission source?

Ans:

• Nuclear reactor cores usually have a symmetrical configuration of fuel assemblies.

• In such a case, it is possible to model only a small part of the core and apply reflective boundary conditions on the relevant radial surfaces.

• This will remove certain higher modes from the system, which will reduce the dominance ratio.

Q6: What is the bias in the fission source and how it differs to the random noise in the source?

Ans:

Though it is difficult to derive mathematically, it is a fact that the normalisation of the fission source at each cycle introduces a bias (a systematic error) into the computed fission source and the multiplication factor in criticality simulations.

The expected source $E\left(s^{(n)}\right)$ thus converges not to $s0$ but to a biased stationary distribution $s_{0,m}$. The difference between $s_{0,m}$ and s_0 is of order $O(1/m)$.

**The fission source is sampled at a limited number m of points in the system (m is the batch size). Therefore, it always contains a statistical error of the order $O\left(1/\sqrt{(m)}\right)$.

Q7: What effect does the source bias have on the computed multiplication factor in criticality simulations?

Ans:

• The biased fission source causes a bias in the k-eigenvalue of the order $O(1/m)$.

• The biased source is usually flatter than the fundamental source, causing a larger neutron leakage from the system.

• The biased k-eigenvalue is therefore usually smaller than the correct value.

• This creates a risk of under-estimating the reactivity of the system, which is an important safety issue (a super-critical system could be found to be safely sub-critical by the MC calculation).

• The source bias also depends on the size of the system. The source bias is larger in large systems than in small systems (having the same number of neutrons per cycle in both systems).

Q8: How can we lower the bias in the fission source and k-eigenvalue?

Ans:

• The bias can be lowered only be increasing the batch size m.

• For usual systems, the batch size should not be smaller than 500 – 1000 neutrons.

• A bigger batch size may be needed for systems where the fundamental mode source departs considerably from a flat distribution.

Q9: What is the advantage/disadvantage of a small/large neutron batch size in criticality simulations?

Ans:

When m is small then

• many inactive and active cycles can be simulated within the allocated time, so the source can get converged during the inactive cycles in systems with dominance ratio close to unity.

• the bias in source and k may be so large that the accuracy of the result will be poor. The difference between the correct and the calculated value (i.e., accuracy) cannot be estimated, so the problem will be hidden.

When m is large then

• It may not be possible to simulate many inactive and active cycles, so the source may not converge, and results may contain large errors due to the un-converged fission source.

• The source bias (systematic error) will be small, but the source will contain errors of the initial fission source distribution since few cycles were simulated.

Q10: What should we consider when we optimise the batch size in criticality simulations?

Ans:

We must decide on the choice of the number of cycles vs the number of neutrons per cycles considering the computed time allocated for our simulation.

We must look at the system we are modelling, and estimate:

• whether the system may have a dominance ratio close to unity (then many inactive cycles need to be specified (e.g., a thousand) −> that may call for a small batch size [e.g., 1000]),

• whether the fundamental source may differ much to a flat distribution (which would mean the source bias could be large −> a large batch size is needed [e.g. 5000 or more]),

• whether the initial fission source can be sampled from a distribution close to the fundamental mode (then the source is practically converged already, and the number of cycles can be reduced, and the batch size can be increased [e.g., to 50,000 or more]).

Q11: Describe the burnup equation.

Ans:

The burnup equation is an ODE that represents the time change in the nuclide field vector $\vec{N}$. The change is given by the product of the transmutation matrix $\mathbb{M}$ and the nuclide field vector $\vec{N}$. The elements in the matrix represents rates at which specific nuclides are being transmuted into other nuclides. The matrix $\mathbb{M}$ is formally defined by the formula with two matrices: $\mathbb{X}$ (cross section and yield matrix) and $\mathbb{D}$ (decay matrix). The transmutation matrix is dependent on the neutron flux.

The concentration of the fuel isotopes is described by the burnup equation

$$\frac{d\vec{N}(\vec{r},t)}{dt} = \mathbb{M}(\phi,T)\vec{N}(\vec{r},T)$$

Where

$$\mathbb{M}[\phi(\vec{r},E,t),T(\vec{r},t)] = \int_0^\infty \phi(\vec{r},E,t)\,\mathbb{X}(T)\,dE + \mathbb{D}$$

Q12: What is the formal solution of the burnup equation?

Ans:

The burnup equation has a formal solution

$$\vec{N}(\vec{r},T) = e^{[\mathbb{M}(\phi,T)(t-t_0)]}\vec{N}(\vec{r})|_{t_0}$$

For ϕ fixed in time.

There is a special solver called the burn-up solver or fuel deletion solver. Over a specific time period, the solver is provided with the transmutation matrix and the initial nuclide field to solve for the nuclide field in the next time step.