Temperature coefficients of reactivity

We can analyse the reactivity feedbacks through dependence of the multiplication factor on state parameters. For this purpose, we need to express the temperature coefficient in therms of k.

Let's rewrite the temperature coefficient of reactivity α_T in terms of the effective multiplication factor k,

$$\begin{aligned}\alpha_T = \frac{d\rho}{dT} &= \frac{d}{dT}\left(\frac{k-1}{k}\right)\\ &= \frac{k'k - (k-1)k'}{k^2}\\ &= \frac{k'}{k^2} = \frac{1}{k^2}\frac{dk}{dT}\end{aligned}$$

The analysis of temperature feedbacks is much easier if we approximate the temperature coefficient by

$$\alpha_T \cong \frac{1}{k}\frac{dk}{dT}$$

which we can do for k near unity (near critical reactor).

Temperature coefficients of reactivity

In order to analyse dk/dT we need to break down k into a number of factors (via the six-factor formula) and study their own temperature coefficients:

$$k = k_\infty P_t P_f = \eta \epsilon p f P_t P_f$$

where

- η is the average number of fission neutrons emitted per thermal neutron absorbed in fuel,
- ϵ (fast-fission factor) is average number of neutrons produced in all fissions (fast and thermal) per neutron produced in thermal fission alone,
- p is the probability that a fission neutron is not absorbed while slowing down,
- f (thermal utilization) is the average number of thermal neutrons absorbed in fuel per thermal neutron absorbed in the reactor.
- P_t is the non-leakage probability for thermal neutrons
- P_f is the non-leakage probability for fast neutrons

The temperature coefficient of reactivity can be then broken into a number of independent factors describing dependence in different parameters.

E.g., substituting $k = k_\infty P_t P_f$ into

$$\alpha_T = \frac{1}{k}\frac{dk}{dT}$$

gives

$$\begin{aligned}
\alpha_T &= \frac{1}{k_\infty P_t P_f}\frac{d(k_\infty P_t P_f)}{dT} \\
&= \frac{k'_\infty P_t P_f}{k_\infty P_t P_f} + \frac{k_\infty P'_t P_f}{k_\infty P_t P_f} + \frac{k_\infty P_t P'_f}{k_\infty P_t P_f} \\
&= \frac{1}{k_\infty}\frac{dk_\infty}{dT} + \frac{1}{P_t}\frac{dP_t}{dT} + \frac{1}{P_f}\frac{dP_f}{dT}
\end{aligned}$$

That can be rewritten using the notation

$$\alpha_T(x) = \frac{1}{x}\frac{dx}{dT}$$

as

$$\alpha_T = \alpha_T(k) = \alpha_T(k_\infty) + \alpha_T(P_t) + \alpha_T(P_f)$$

Temperature coefficients of reactivity

Similarly, from the equation

$$k_\infty = \eta \epsilon p f$$

we can write for $\alpha_T(k_\infty)$

$$\alpha_T(k_\infty) = \alpha_T(\eta) + \alpha_T(\epsilon) + \alpha_T(p) + \alpha_T(f)$$

Finally, we can state that

$$\alpha_T = \alpha_T(P_t) + \alpha_T(P_f) + \alpha_T(\eta) + \alpha_T(\epsilon) + \alpha_T(p) + \alpha_T(f)$$

Fuel temperature (Doppler) feedback

As the prompt feedback comes primarily from the Doppler effect, we can write

$$\alpha_T^F \cong \alpha_T(p) = \frac{1}{p}\frac{dp}{dT}$$

where p in heterogeneous reactors can be expressed as

$$p = \exp\left[-\frac{N_F V_F I}{\xi_M \Sigma_{sM} V_M}\right]$$

Which terms in the formula may be dependent on the temperature?

- The term $N_F V_F$ represents the total number of fuel atoms, which is not dependent on T.
- The only dependence on T comes from the resonance integral I that is temperature dependent.
- We can therefore write

$$\alpha_T^F = \frac{1}{p}\frac{dp}{dT} = -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M}\frac{dI}{dT}$$

- Now we need to evaluate dI/dT...

Moderator temperature (delayed) feedback

What is the mechanism of the moderator temperature (delayed) feedback? There are two processes...

The moderator temperature feedback comes from the effect of the changing

- **moderator density** on the **moderation process** (expressed by p) and
- neutron **absorption rate in moderator** (on hydrogen) that is expressed by the f factor.

Therefore,

$$\alpha_T^M = \alpha_T^M(p) + \alpha_T^M(f) = \frac{1}{p}\frac{dp}{dT_M} + \frac{1}{f}\frac{df}{dT_M}$$

Moderator temperature (delayed) feedback

What is the sign of the moderator temperature coefficient of resonance escape probability, $\alpha_T^M(p) = \frac{1}{p}\frac{dp}{dT_M}$?

- When the moderator temperature **decreases** then N_M/N_F **increases**. From

$$
\begin{aligned}
p &= e^{-\frac{N_F V_F I}{\xi_M N_M \sigma_{sM} V_M}} \\
&= e^{-\frac{V_F I}{\xi_M \frac{N_M}{N_F} \sigma_{sM} V_M}}
\end{aligned}
$$

  one can see then that p **increases**.

- This is because neutrons get more moderated, and less frequently collide with ^{238}U at resonance energies.
- **Therefore, dp/dT_M and so $\alpha_T^M(p)$ must be negative.**
- **Note that p increases with increasing N_M/N_F.**

Moderator temperature (delayed) feedback

What is the sign of the moderator temperature coefficient of the thermal utilization factor, $\alpha_T^M(f) = \frac{1}{f}\frac{df}{dT_M}$?

- The thermal utilization f in heterogeneous reactors

$$f = \frac{\Sigma_{aF} V_F}{\Sigma_{aM} V_M \zeta + \Sigma_{aF} V_F} = \frac{\sigma_{aF} V_F}{\frac{N_M}{N_F}\sigma_{aM} V_M \zeta + \sigma_{aF} V_F}$$

  decreases when the N_M/N_F ratio **increases** (when the moderator density grows - i.e. when T_M **decreases**). This is because neutrons are more likely to be absorbed in the moderator.

- **Thus, df/dT_M and so $\alpha_T^M(f)$ are positive.**
- **Note that f decreases as the N_M/N_F ratio grows.**

Moderator temperature (delayed) feedback

Knowing there are both positive and negative partial feedbacks forming the total delayed feedback, is the total delayed feedback positive or negative?

- $\alpha_T^M(p)$ is negative.
- $\alpha_T^M(f)$ is positive.

$\Rightarrow \alpha_T^M = \alpha_T^M(p) + \alpha_T^M(f)$ may be negative or positive, depending on the actual conditions.

Is there an optimum N_M/N_F ratio?

Since $k \sim p \times f$ there must be an optimum N_M/N_F ratio for which k reaches a local extreme.

What are the under/over-moderated systems?

- A system is called under-moderated when its N_M/N_F ratio is below the optimum value. α_T^M is negative.
- A system is called over-moderated when its N_M/N_F ratio is above the optimum value. α_T^M is positive.