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By M. Ledinegg

Die Wärme, 61, 891-898 (1938)

Translated by R. B. Lees
Argonne National Laboratory

Argonne National Laboratory
P. O. Box 5207
Chicago 80, Illinois

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The flow in parallel-connected heated boiler-tubes lying between two collecting-tubes can become unstable under certain conditions. For forced-circulation - and filter-boilers or for pre-evaporators the course of the tube-friction-losses play the most important role; for cooling mantles on (hot-boxes, heat-chambers with natural circulation the buoyancy-relations are the most important. The conditions for the appearance of instability are studied for both cases. The results are shown in curve-tables.

THE NATURE OF INSTABILITY. One of the most important structural elements of boilers is the packet of evaporating-tubes connected between two collecting-tubes. In boilers with natural circulation it is used above all as cooling-mantel on the heat-chamber, in forced-circulation-boilers to build the whole heating-surface itself. By forced-flow-boilers we understand here not only forced-flow and forced-circulation boilers, but also pre-evaporators. In all cases, water which partly or wholly vaporizes during flow thru the parallel connected boiling-tubes arrives at the inlet connecting-tube. The outflow connecting-tube drains off the vapor-water-mixture. The boiling tubes in a forced-flow-boiler are long and of small diameter, in cooling-mantles short and of large diameter.

The following work concerns itself with the question under what conditions in such systems disturbances can occur in the flow-diagram.

In actual forced-circulation or flow boilers as well as in pre-evporators the observation has been made, as is known, that the quantity flowing

7109-2

thru the various parallel-connected tubes -- despite approximately equal heating -- was very different. The distribution of the flow changed not only spacially but also temporally. The cause of this phenomenon is the temperature of the water at the entrance to the tube-packet being below the saturation vapor temperature. If one tube for any arbitrary reason contains less water than the others more is vaporised, since less latent heat of the liquid need be introduced. The production of more steam however raises the flow-resistance although less water flows thru. Thus less and less water flows in until a new equilibrium state is reached.

At the appearance of this well known phenomenon called "unstable flow", the tube in question can be endangered by too little cooling. In practice therefore in such cases a diaphragm is placed at the tube entrance whose pressure-drop (-gradient) is considerably larger than the gradients to be expected in the tube under all possible operating conditions. It is now important to see under what conditions instability can appear so as to be able to take precautions if necessary.

Not less important is the question of how cooling mantles with natural water circulation basically operate, and whether instability can appear also in them. The study shows that instability is possible in the above sense only in rare cases, above all because of the regulating effect of buoyancy. On the other hand, it turns out that uneven heating of the boiling-tubes has considerable influence on the size of the buoyancy, and this goes as far that the flow in underheated tubes can reverse and they can become down-pipes. Since above all the specific gravity of the various vapor-water-mixture columns, in the boiling-tubes is decisive here for the flow-relations, also the conditions for appearance of instability of this type are different from before. Understanding of them is very important for judging safety in operation of cooling-mantles

7109-3

on heat-chambers.

PRINCIPLES OF FLOW TECHNOLOGY.[1] Instability in forced-flow boilers and cooling-mantles can be studied only together according to the foregoing as far as the general principles of flow technology are concerned. These are: the static pressure gradient in the tube, the pressure drop due to tube friction and to acceleration. Further the knowledge of the specific gravity of a steam-water-mixture is necessary, which might be calculated beforehand.

Specific Gravity. Consider a tube in which steam bubbles may rise with a velocity W_r relative to the water. Let the velocity of the water be W_W, the cross-section of the tube be F. If we imagine the steam bubbles to be separated out and rising in a closed tube, then let the remaining cross-section of the water be F_W. Finally let the weight of water per second at an arbitrary place in the longitudinal direction of the tube be G_W and the weight of steam per second be G_D, as also the corresponding specific gravities, G_W and G_D. The continuity equation for G_W or G_D is:

$$(F-F_W) \cdot \gamma_D \cdot (W_W + W_r) = G_D \qquad (1)$$

$$F_W \cdot \gamma_W \cdot W_W = G_W \qquad (2)$$

After eliminating W_W and smaller transformations the ratio of the water-cross-section to the tube cross-section can be obtained. If we set as abbreviation:

$$\frac{G_D}{F \cdot \gamma_D \cdot W_r} = A \, , \quad \frac{G_W}{F \gamma_W W_r} = B,$$

Then we get:

$$\frac{F_W}{F} = \frac{\substack{\mp\\-} A \substack{\mp\\+} B \substack{+\\+\\+} 1}{2} \left\{ \substack{+\\ \mp\\ \pm} \right. \sqrt{\frac{\left(\substack{\pm\\+\\-} A \substack{+\\+\\+} B \substack{\mp\\+} 1\right)^2}{4} \pm B} \qquad (3)$$

The top row of signs is used when steam and water flow upward in the boiling tube, the middle row when steam and water flow downward, i.e. when we have a heated down-pipe, whereby the water influx velocity must be larger than the relative velocity of the steam bubbles. The third row is used finally when the water flows again downward, however with a velocity smaller than the relative velocity of the steam bubbles, so that the latter can rise. The specific gravity of the steam-water-mixture in an arbitrary cross-section is then according to Seidel [2]:

$$\gamma = \gamma_W \frac{F_W}{F} + \gamma_D \left(1 - \frac{F_W}{F}\right),$$

or, since the second member may be neglected compared to the first,

$$\gamma = \gamma_W \frac{F_W}{F} \tag{4}$$

In horizontal tubes or at high pressures the relative velocity of the steam bubbles $W_r \cong 0$. For this case the expression for γ simplifies and we get again from Equ. 1 and 2 with $G_W + G_D = G$, as well as ν_W or ν_D for the spec. volumes:

$$\gamma = \frac{G}{G\,\nu_W + G_D\,(\nu_D - \nu_W)} \tag{5}$$

<u>Static Pressure Gradient</u>. In order to obtain the static pressure p_{st} which a steam-water-mixture-column in a boiler-tube exerts on its bottom surface, the value of γ must be integrated over the length of tube conducting the steam-water-mixture. If the vaporisation begins at a distance <u>a</u> from the beginning of the tube, and if the tube length is <u>ℓ</u>, the integral is :

41095

$$P_{st} = \int_a^{\ell} \gamma \, dx \tag{6}$$

In order to carry out the integral the variation of γ over the tube length must be considered. If we assume uniform heat inflow over the tube length the amount of steam produced rises linearly from 0 at $x = a$ to a value of G_{D_0} at $x = \ell$. If we put Eq. 5 into Eq. 6 with the above mentioned consideration and integrate, we get:

$$P_{st} = \frac{(\ell - a) \cdot G}{G_{D_0} \, (\nu_D - \nu_W)} \; \ell n \left(1 + \frac{G_{D_0} \, (\nu_D - \nu_W)}{G \, \nu_W} \right) \tag{7}$$

If we develop the logarithm into a series and break off after the first member we obtain with sufficient accuracy:

$$P_{st} = (\ell - a) \cdot \frac{G}{G \nu_W + \frac{G_{D_0}}{2} \, (\nu_o - \nu_W)} \tag{8}$$

The factor of $(\ell - a)$ can be thought of as the mean specific gravity of the steam-water-mixture in a heated boiler-tube. Comparing with Eq. 5 we see that it is calculated simply as if only half the steam production occurred in the tube.

After introducing the mean specific gravity γ_m Eq. 8 becomes:

$$P_{st} = (\ell - a) \, \gamma_m \tag{9}$$

If the case is that of a boiler-tube where the advance-velocity of the steam bubbles is to be considered, the calculation follows directly from Eq. 9; the spec. grav. is now simply calculated from Eq. 3 and 4 where again, in the case of a boiler tube heated along its entire length, half the value is put in for the production of steam.

Friction-losses. If it is assumed that water enters the tube at a temperature below the saturated steam temperature, the vaporization will begin only after a distance a. Accordingly the pressure drop must be calculated separately for the part where only water is and for the part where there is steam-water-mixture. In the first part we get for the pressure drop, e.g., according to "Hütte" [3] (the formula is valid for liquids, gases, and vapors):

$$\Delta p_W = \frac{\beta}{dF^2 \cdot 1000} \cdot \frac{G^2}{\gamma_W} \cdot a \tag{10}$$

Here β is the tube coefficient of friction. The tube length must get increased correspondingly for consideration of the bends.

The pressure-drop of the steam-water-mixture is obtained over the interval (ℓ-a) and must again be obtained by integration, since the specific gravity varies along the tube length. We get:

$$\Delta p_G = \frac{\beta G^2}{d\ 1000\ F^2} \int_a^{\ell} \frac{dx}{\gamma}$$

If we introduce Eq. 5 for the sp. gr. γ and assume again linear steam increase in the tube, we get as a result of the integration:

$$\Delta p_G = \frac{\beta G^2(\ell - a)}{d\ 1000\ F^2} \left[\frac{G\ \nu_W + \frac{G_{D_0}}{2}\ (\nu_D - \nu_W)}{G} \right] \tag{11}$$

7109-7

Comparison of Eq. 10 shows that the expression in brackets represents the mean sp. grav. of the steam-water-mixture valid for the calculation of the pressure drop. If we compare this expression with Eq. 5 we see again that also here the mean sp. gr. is constructed simply, in that half of the weight of steam produced in the tube is to be introduced as weight of steam. Introducing the mean sp. gr. γ_m Equ. 11 becomes:

$$p_0 = \frac{\beta G^2 (\ell - a)}{d F^2 \, 1000} \cdot \frac{1}{\gamma_m} \tag{12}$$

where the latter is again claculated by Eq. 3 or 4 even though the progress-velocity of the steam-bubbles must be considered. Then for vaporization over the whole tube length half the steam-weight, that is $\frac{G_{D_0}}{2}$, is always to be used for G_D. If however the steam content is unchanged over the whole tube length, as in an unheated ascending tube, we must use the whole steam weight.

Acceleration loss. Since the steam-water-mixture in the tube must be accelerated during the progress of the vaporization, there is a pressure drop necessary here. This is given by Seidel [2] after a few transformations as:

$$\Delta P_B = \frac{G \cdot G_{D_0}}{g F^2} \cdot (\nu_D - \nu_W) \tag{13}$$

Inlet loss. With the entrance of the water from the collecting tube into the boiling tube the level of the velocity in the latter must be introduced. For this a pressure drop is required as follows:

$$\Delta P_E = \left(\frac{G}{G}\right)^2 \frac{1}{\gamma_W \, 2 g} \tag{14}$$

7107.3

The pressure Drop in Forced Flow Boilers.

In a heated tube the total pressure drop of the steam-water-mixture is equal to the sum of the partial quantities mentioned in the previous section. Since the total pressure drop is the most important basis for further studies, the accuracy of the derived formulae, in so far as they are useful for the case in question, was tested by means of the results of measurements on a continuous flow vaporiser (evaporator). This was in this case a boiler which consistod of a continuous tube of 25 mm inside width, which forms in order the preheater, evaporator, and superheated. Various operating conditions were tested; the following is given as example:

Measurement:

Amount of water fed in	———	1530 kg/h. or 0.426 kg/s.
Pressure of feed water before boiler	———	85 atm.
" " steam behind "	———	48 "
Superheat temperature at outlet	———	336°

With such large differences between inlet and outlet pressure the static pressure distribution naturally plays no role, and also the entrance loss may be neglected. Merely the friction - and acceleration - losses are considered. The progress-velocity of the steam-bubbles is $W_r \cong 0$.

Next the pressure-drop of the water pre-heated up to the beginning of vaporization and of the superheater after the end of vaporization are calculated in the usual manner. For the former we get 2.1 atm., for the latter 20.4 atm., so that for the true evaporation-part there remains a pressure drop of 37-2.1 - 20.4 = 14.5 atm. The pressure in front of the latter is then 82.9 atm., behind it 68.4 atm. and in the middle 75.6 atm. From the course of the flue-gas temperature the extent of the tube length of the evaporation part could be easily calculated at 249 m. Because of bends an addition had to be made to the tube-length to the amount of

9

62 m. The tube friction-coefficient β was taken from "Hütte" [4] for 1530 kg/h steam-flow-rate and came to 0.97. Now the pressure drop of the mixture in the vaporisation part can be calculated from Eq. 11:

$$\frac{0.97 \times 0.426^2\,(249 + 62)}{0.025 \times 0.000491^2 \times 1000}\left[\frac{0.426 \times 0.00136 + \frac{0.426}{2}\,(0.0258 - 0.00136)}{0.426}\right]$$

$$= 125000 \text{ kg/m}^2 = 12.5 \text{ atm.}$$

The acceleration loss is obtained from Eq. 13:

$$\frac{0.426^2}{10 \times 0.000491^2}\,(0.0349 - 0.00136) = 25000 \text{ kg/m}^2 = 2.5 \text{ atm.}$$

The material constants were taken at the mean pressure of 75.6 atm. In the last equation v_D was calculated at 336°, since an acceleration loss appears also in the superheater part. In the way shown above, the pressure drop in question is calculated to 12.5 + 2.5 = 15 atm. and checks well with the measured value of 14.5 atm. Also under other operating conditions subsequent calculations carried out showed good agreement between measurement and calculation, so that the formulas given for the pressure drop give satisfactorily the true values.

Unstability in the Forced Flow Boiler.

In order to carry out the further tests, we consider a g_a in a heated coil. Let the heat content of the incoming feed water be i_E, its weight per second be G. Let its vaporization occur in part or completely according as a circulating or a continuous flow boiler is in question. Let the quantity of heat which is necessary to heat up 1 kg. of water to the saturated steam temperature be Δi : $\Delta i = i' - i_E$. If the quantity of heat in kcal/sec. entering in all into the coil be called W, the weight of steam evaporation in the latter is:

$$G_{D_0} = \frac{W - G \cdot \Delta i}{r} \qquad (15)$$

If ℓ is the length of the whole coil there occurs a preheating over the length

$$a = \frac{G \cdot \Delta i}{W} \cdot \ell \qquad (16)$$

under the assumption of uniform heat flow. For the flow thru the coil the buoyancy and the entrance loss play only a lesser role, according to what was said in the previous section. For the pressure drop only the friction loss in the preheater part and in the evaporator according to Eq. 10 and 11 and the acceleration loss Eq. 13 are decisive, and we obtain:

$$\Delta p = \frac{\beta}{d \cdot F^2\, 1000} \left\{ \frac{G^2 \cdot a}{\gamma_W} + G^2 (\ell - a) \cdot \left[\frac{G \nu_W + \frac{G_{D_0}}{2} (\nu_0 - \nu_W)}{G} \right] \right\}$$

$$+ \frac{G G_{D_0}}{g F^2} (\nu_D - \nu_W) \qquad (17)$$

For G_{D_0} and a the two Eq. 15 and 16 are now introduced, whereupon we get, if ν_W is neglected relative to ν_D:

$$\Delta p = \frac{\beta}{d \cdot F^2 \cdot 1000} \cdot \ell \nu_W G^2 \left[\frac{W \ell \nu_D}{2 r} \left(G - \frac{2 \Delta i\, G^2}{W} + \frac{\Delta i^2\, G^3}{W^2} \right) \right]$$

$$+ \frac{\nu_D W}{r\, g\, F^2} \left(G - \frac{\Delta i}{W}\, G^2 \right) \qquad (18)$$

0109-11

The pressure drop is now represented as a function of the quantity of heat W introduced per sec., the preheating Δi, and the weight of steam G per sec. This function is shown as a curve in Fig. 1, and is valid for a certain heat flow W, while each curve is drawn for a certain preheating temperature i_E of the water. For higher water inflow temperatures i_E, i.e. smaller Δi, the course along a is always rising. For i_E becoming smaller we reach a limiting value where the curve has an inflexion point (b). For a still lower inflow temperature the course is finally like c, i.e. with a maximum and a minimum value. The latter curve represents the typical picture for an unstable flow, since for a particular given pressure difference there correspond three different water quantities, and they can occur arbitrarily during operation. The condition for appearance of instability is accordingly by Fig. 1 the occurrence of maxima and minima or, for the limiting case, an inflection point; Schuackenberg [5] pointed this out first. Expressed mathematically this is according to Schuackenberg:

$$\frac{d\,\Delta p}{dG} = 0$$

If we carry thru the corresponding differentiation with Eq. 18 and order according to G we get then a quadratic equation corresponding to the two extremes which the curve has. This equation is:

$$G^2 - \frac{\frac{\beta \ell}{d \cdot 1000} \cdot \left(\frac{2 \nu_D \, \Delta i}{r} - 2 \nu_W\right) + \frac{2 \nu_D \, \Delta i}{gr}}{\frac{3 \beta \ell' \, \nu_D \, \Delta i^2}{d \cdot 1000 \cdot 2 \cdot r \cdot W}} \cdot G$$

or when we let the fractions be $\underline{\alpha}$ and β :

$$G^2 - \alpha G + \beta = 0$$

The solution of this quadratic equation is

$$G_{1,2} = \frac{\alpha}{2} \pm \sqrt{\frac{\alpha^2}{4} - \beta} \qquad (19)$$

For a stable flow the root expression is imaginary, in the limiting case is zero, and for unstable flow is real. In the sense of the present question the condition under which a flow is in fact unstable interests us. We get this condition when the root expression in Eq. 19is set equal to zero. We get then the relation:

$$\frac{\alpha^2}{4} = \beta$$

or, after introducing the values of $\underline{\alpha}$ and $\underline{\beta}$ and solving for Δ i, the condition sought is

$$\Delta i_G = \frac{\frac{2\ell\beta}{1000 \cdot d} \cdot \frac{v_w \cdot r}{v_D}}{\frac{2\ell\beta}{1000 \cdot d} + \frac{2}{g} - 2\sqrt{\frac{3\ell\beta}{2 \cdot 1000 \cdot d} \cdot \left(\frac{\ell\beta}{2 \cdot 1000 \cdot d} + \frac{1}{g}\right)}} \qquad (20)$$

In order to show particularly that the calculated value Δi_G represents a boundary value it is written with an index G. The appearance of instability depends accordingly on neither the strength of the heating nor on the mass-flow nor on the absolute value of the pressure difference, but merely on

the value Δi_G and the two expressions $\frac{\ell \beta}{1000 \cdot d}$ and $\frac{\nu_W \cdot r}{\nu_D}$.
The first depends on the tube friction and the geometric properties of the tube, the latter on the pressure.

When operating a steam generator a particular entrance temperature of the water is chosen and therefore also a value Δi_W, which is given the index W as a particular mark; on the other hand, a value Δi_G is calculable from Eq. 20. Then as a condition for the behavior of the flow the following is valid:

$\Delta i_W > i_G$ unstable

$\Delta i_W < i_G$ stable.

In order to facilitate the use of Eq. 20 it is given in Fig. 2 as a family of curves. The abscissae are the values of $\frac{\ell \beta}{1000 \cdot d}$, the ordinates the values of Δi_G. The curves are for the various values of $\frac{\nu_W \cdot r}{\nu_D}$, in place of which however the corresponding pressure itself is written in. We see that the effect of tube friction soon becomes small. But the danger of instability depends strongly on pressure so that it increases with decreasing pressure.

Water Circulation in Cooling Mantles.

Calculation of the water circulation in cooling mantles follows on the basis that the driving force, that is the buoyancy is equal to the impeding force, i.e., the sum of all flow resistances. Since here we deal in general with inclined or with vertical tubes with relatively large diameter, now, contrary to the case of forced flow boilers, the relative velocity of the steam-bubbles must be taken into account in the succeeding calculations.

The methods of operating coolirg mantles are manifold. When it is possible to build it as organically as possible together with the boiler, this not only gives the construction a unified, pleasant appearance, but also the circulation is better due to the shorter plumbing. The fire-heated boiling-tubes

1109 - 14

of the cooling mantle are either wound in between an upper and a lower collecting-chamber, or they are led directly into the evaporating-drum without any upper collecting-chamber. The water is led into the lower collecting-chamber by means of an unheated down-pipe, and the steam-water-mixture is led off from the upper collecting-chamber by the similarly unheated ascending tube. Short-circuit pipes are often used, i.e. external connecting tubes between the upper and lower collecting-chambers to increase the flow of water thru the boiling tube.

Calculation of the water circulation in cooling-mantles is recommended for new operations for which there is no experience, in order on the one hand to determine the quantity of circulation, and on the other hand, to test whether the steam-water-mixture at the exit to the boiling-tube contains enough water to ensure cooling of the pipe material.

The calculation is done separately for down-pipe, ascending tube and boiling-tube. After obtaining the heat inflow to the boiling-tube the quantity of steam produced therein is determined. A certain trial rotation-coefficient [? <u>Umwälzziffer</u>] is now assumed, and the mean specific gravity of the water-steam-mixture is calculated therewith according to Eq. 3 and 4. We must not forget to use half the value for the steam production so long as we deal with a boiler-tube; for the ascending tube we use merely the total amount of steam entering. When the mean sp. gr. has been determined, the friction-losses according to Eq. 12 as well as the static pressure from Eq. 9 may similarly be calculated. Finally we must account for the rest of the losses mentioned in the section "Principles of Flow Technology." After the calculations for ascending tube, downpipe, and boiler-tube have been made, all the flow losses are summed, and we get a certain value which is entered in a diagram over the rotation-coefficient. Finally, the difference in static pressure: ascending - and boiler-tube minus downpipe must be constructed and similarly entered on the graph. If we repeat the calculation for several rotation-coefficients and connect the various points we get two curves which intersect in a point which gives the desired rotation-number.

Instability in Cooling Mantles.

We have just shown that instability can appear also in cooling mantles under certain conditions. Here by instability we mean stopping or reversal of the flow in certain tubes of the cooling mantle, and the following studies are concerned with the conditions under which this is possible.

For this purpose we consider again a normal cooling-mantle which consists of boiler tubes wound in between an upper and a lower collecting tube. The pressure drop of the water-steam-mixture between the two collecting tubes consists of the hydrostatic part, which is obtained from the difference in height ℓ, and of the part from flow-losses. No matter what the flow-velocity in the individual boiler-tubes may be, whether equal or different, or whether it even reverses in some, all parallel-connected tubes must always have the same pressure drop, neglecting pressure differences in the collecting chambers. Starting from here, we therefore see whether more than one flow direction and strength is possible in the boiler-tubes at a certain pressure-drop in the system, and under what conditions this is the case. The hydrostatic pressure of the mixture-column is according to Eq. 9 and 4:

$$p_{st} = \ell \cdot \gamma_m = \ell \gamma_W \frac{F_W}{F}$$

since for cooling-mantles a $\cong 0$. $\frac{F_W}{F}$ is according to Eq. 3 simply a function

0109-11

of $\dfrac{G_D}{F\,\gamma_D\,W_r}$ and $\dfrac{G_W}{F\,\gamma_W\,W_r}$.

Schmidt [6] has made studies of the value W_r, and he studied the dependense of the relative velocity of the steam-bubbles on pressure and specific gravity. Other dependences also existing, such as on tube-diameter, or of tube inclination and above all the concentration of the boiler-water are not considered. The influence of W_r for the following developments is however not large, so that a mean value for it can be used, in particular $W_r = 0.5\ \frac{m.}{sec}$. Then:

$$P_{st} = \ell \cdot \text{Function}\left(\frac{G_D}{F\,\gamma_D}\ ,\ \frac{G_W}{F\,\gamma_W}\right) \tag{21}$$

When calculating the flow losses, we start with friction-loss (Eq. 12) and the other losses are considered to be corresponding increases in the tube coefficient of friction, an approximation which suffices for the present study. If we write the expression $\frac{\beta}{d \cdot 1000}$ by C_1, and the difference in level of the two collecting-chambers by ℓ, we can write for the flow loss using Eq. 12 and 4:

$$\Delta P_G = C_1 \left(\frac{G_W}{F\,\gamma_W}\right)^2 \frac{\ell\,\gamma_W\,F}{F_W} \tag{22}$$

The quantity of water at the entrance G was here set equal to G_W, i.e. to that at the outlet, which is here always admissable. The resistance factor C_1 is found in that for a certain operating condition represented by the index 1 for which a water circulation calculation was carried out, the corresponding values are put in Eq. 22 and the C_1 is calculated therefrom.

9107-17

$$C_1 = \frac{\Delta p_{0,1}}{\ell \cdot \gamma_W} \cdot \left(\frac{F_W}{F}\right)_1 \cdot \left(\frac{F\gamma_W}{G_{W,1}}\right)^2 \tag{23}$$

For the total pressure difference between the two collecting tubes we get then, according as it is a case of a boiling tube with flow forward or backward, the sum or the difference of Eq. 21 and 22:

$$p = \gamma_W \cdot \ell \cdot \left[\frac{F_W}{F} \pm C_1 \left(\frac{G_W}{F\gamma_W}\right)^2 \frac{F}{F_W}\right] \tag{24}$$

If we now write the total pressure difference for the pressure drop of a 1 m. column of water, we get on the left side of the Equation the characteristic number $\frac{\Delta r}{W}$, which can be called P, and which is dependent only on $\frac{G_W}{F\gamma_W}$, $\frac{G_D}{F\gamma_D}$ and C_1.

$$P = \text{Function}\left(\frac{G_W}{F\gamma_W},\ \frac{G_D}{F\gamma_D},\ C_1\right) \tag{25}$$

In order to obtain a clear picture of the dependence of the pressure drop Δ p and its determinant P on its parameters, Eq. 25 is shown in curve form. Figs. 3, 4, 5, and 6 are valid respectively for $C_1 = 0$; 0.02; 0.04; and 0.1. The abscissae are values of $\frac{G_W}{F\gamma_W}$, and the ordinates are values of P. The various curves of each family are for different values of $\frac{G_D}{F\gamma_D}$. Furthermore, the family on the right is for water flowing forward, the lefthand one for backward flow. The value of $\frac{G_W}{F\gamma_W}$ represents the strength of the water circulation, the value of $\frac{G_D}{F\gamma_D}$ the size of the steam production in the boiling-tube.

Diagram 3 for $C_1 = 0$ shows the pressure when there are no flow losses present at all, that is, only the static pressure corresponding to the specific gravity of the column of mixture is effective. Progressively in Figs. 4, 5, and 6 we see how, with rising friction the dynamic pressure overtakes the static, and in particular adds for ascending tubes and subtracts for down-pipes. Correspondingly the curves are drawn upward to the right of the null-point; to the left they fall after having reached a maximum value.

For very small values of $\frac{G_W}{F\gamma_W}$ and P, the curves, because of the approximations used, are only approximately straight. We may mention also that in cooling-mantles the value of $\frac{G_W}{F\gamma_W}$ cannot get completely to zero, since just there according to what was said earlier half the steam production $\frac{G_{D_0}}{2}$ is introduced for G_D. Therefore in the limiting case, that is when the quantity of water fed in is equal to the amount of steam produced, not less than $\frac{G_{D_0}}{2}$ can be put in for G_W.

The friction losses are zero for the expression $\frac{G_W}{F\gamma_W} = 0$, possible only with large approximation, i.e. for a standing water column; the value P in Eq. 25 has then two solutions, one zero and the other:

$$P = 1 - \frac{G_D}{F\gamma_D \times 0.5} \tag{26}$$

The continuation of the ascending-tube curves, not ending in the zero-point, thru $\frac{G_W}{F\gamma_W} = 0$ to the left shows a loop curving back; this is true for rising steam flow, however falling water flow. The region in which this operating condition is possible results when the expression $1 - \frac{G_D}{F\cdot\gamma_D \times 0.5}$ in Eq. 26 becomes zero, i.e. when the two solutions for P mentioned above both converge to zero. Thus for values of $\frac{G_D}{F\gamma_D} \geqq 0.5$ simultaneous falling of water and rising of steam is impossible. Therefore only the curves $\frac{G_D}{F\gamma_D} < 0.5$

9101 - 19

in Figs. 3, 4, 5, and 6 have the loop just mentioned.

By means of the curves we can now easily judge whether instability is possible in a cooling-mantle. We next consider the case that all the boiling-tubes of the cooling mantle are heated uniformly. For a certain steam production the proper $\frac{G_D}{F \gamma_D}$ - curve gives the possible values of the determinant P and thus also the pressure. On the other hand, there is a certain pressure difference between the upper and lower collecting tube and therefore also a determinant P_W, which may take the index W in the following. If we draw a horizontal straight line at the height P_W it cuts the $\frac{G_D}{F \gamma_D}$ - curve in one or more points, which then show the possible operating conditions. If P_W is smaller than the maximum value on the left branch of the curve, two points on the left and one on the right are cut, i.e. there are three possible operating-conditions among which the flow can oscillate. Thus the flow is unstable; any arbitrary pipe of the cooling-mantle can become a downpipe. The limiting case up to which instability is still possible is given when P_W is increased until the horizontal no longer cuts the left branch of the curve, but only touches it. Let this value , as a limiting value, receive the index G instead of W in the following. If P_W is still more increased, instability is no longer possible. Thus:

$$\left.\begin{array}{ll} P_W > P_G & \text{stable} \\ P_W = P_G & \text{boundary value} \\ P_W < P_G & \text{instability possible} \end{array}\right\} \qquad (27)$$

Instability occurs considerably easier naturally, when the heating of the various pipes is different. The mean steam-production of the tube is then shown by a certain $\frac{G_D}{F \gamma_D}$ - curve, and the steam-production of the least heated tube by another lying higher. If we again draw a horizontal line at P_W, it cuts both curves on the right side, except for the case mentioned in the next section, so as to show the mean and the least mass-flow for tubes with foreward flow. On the left side both, only one, or no curve can be cut, according to the value of P_W. In the first case the flow in any arbitrary tube can become unstable or can reverse, in the second case only in less heated tubes, and in the third case in no tube.

If the curve for the least steam production has however the value $\frac{G_D}{F\gamma_D} < 0.5$, then it can easily happen that it is cut first in the part with the loop, i.e. just left of the null-point, while on the right there is no intersection at all. Then reverse flow not only can, but must occur in the less heated tubes.

Instability cannot occur, as was just shown, when the determinant of the pressure-drop P_W has at least the limiting value P_G, which is equal to the maximum value of the appropriate $\frac{G_D}{F\gamma_D}$ - curve for reverse flow of steam and water, or which agrees with the ordinate at $\frac{G_W}{F\gamma_W} = 0$ for opposite flow of these. If therefore we graph over the abscissae $\frac{G_D}{F\gamma_D}$ the value P_G taken from Figs. 3 to 6, we get the limiting conditions for the possibility of instability. This is done in Fig. 7, with foreward flow of steam and reverse flow of water left of the zero-point, and reverse flow of both on the right, and where the various curves are for different tube-friction-coefficients C_1.

In order to use the graph in Fig. 7 the mean water quantity for each tube, γ, must first be calculated and therefore also the determinant $\frac{G_W}{F\gamma_W}$ by means of a water-circulation-calculation, since the latter is required for computing the value C_1 by Eq. 23. Further the water-circulation-calculation gives the pressure difference between the upper and lower collecting tube and thus also the determinant P_W. Finally by postulating equal evaporation in all tubes the value $\frac{G_D}{F \gamma_D}$ can be calculated. Now from the curve family to the right of the zero-point as well as from the line left from the zero-point the values P_G can be read, from which the largest is taken. By Eq. 27 the danger of instability may always be determined. If by assuming uniform heating there is still no danger of instability, we must then test the values for non-uniform heating. For this purpose we must estimate according to the conditions present how high the steam production can be in the least heated tube, from which we get a new value $\frac{G_D}{F\gamma_D}$ and from the graph a new coefficient P_G. Again the danger of instability is estimated from Eq. 27.

We can thus test by means of the method so described how the behavior of a cooling-mantle will be in operation, and the engineer can always ensure that the required stability conditions will be held by the proper choice of pipe diameter and length.

The application for a practical case is now shown by means of an example.

Given:

Difference in level of the two collecting tubes	— l	=	5.3 m.
Cross-section of the boiling tube	— F	=	0.0046 m^2.
Specific gravity of the water	— γ_W	=	835 kg/m^3.
Specific gravity of the steam	— γ_D	=	12.75 kg/m^3.
Mean half steam-production of each tube	— G_D	=	0.0285 kg/sec.

From the water-circulation calculation we get:

Mean sp. grav. of the steam-water-mixture (Eq. 4)	———	$\gamma_m = 585$ kg/m^3.
or ratio of the water-cross-section to the tube-cross-section (Eq. 3)	———	$\frac{F_W}{F} = 0.7$
Static pressure-difference (Eq. 9)	———	$P_{st} = 3110$ kg/m^2.
Flow loss (Eq. 10, 12, 13, 14)	———	$\Delta P_G = 162$ kg/m^2.
Quantity of water per tube	———	$G_W = 2.96$ kg/sec.

Calculation of the determinants:

Tube friction-coefficient (Eq. 23)

$$C_1 = \frac{162}{5.3 \times 835} \times 0.7 \times \left(\frac{0.0046 \times 835}{2.96}\right)^2 = 0.043,$$

Determinant $\frac{G_D}{F\gamma_D}$ for mean steam production

$$\frac{0.0285}{0.0046 \times 12.75} = 0.485$$

Determinant $\frac{G_D}{F\gamma_D}$ for least steam production assumed equal to 30% of the mean = 0.145,

$$\text{Determinant } p_W = \frac{\Delta P}{\gamma_W \ell} = \frac{3110 + 162}{835 \times 5.3} = 0.74$$

From Fig. 7 we get:

Determinant P_G $\left\{ \begin{array}{lll} \text{at mean steam production} & \text{——} & 0.56 \\ \text{at least steam production} & \text{——} & 0.79 \end{array} \right.$

In the present case P_W is thus for uniform heating larger than P_G, however, for an assumed least heating of 30% of the mean it is smaller; in the first case the flow is stable, in the second, unstable. The boundary lies at a lower heating of about 50%.

Conclusions.

The study has shown that the flow thru a heated packet of tubes does not always proceed uniformly. Under conditions which can occur very easily in practice, the flow in the individual tubes becomes unequal or even reverses. In forced flow boilers instability occurs more easily at lower pressure than at higher, since there the volume of steam formed is larger. The span from the latent heat of the liquid i' to the value Δi below it, which indeed comprises the stable region, becomes larger with the pressure according to Fig. 1.

We note also that in a forced flow boiler also non-uniform heating causes a different flow thru the parallel-connected tubes so that the more heated tube contains less water and the less heated tube more, which is the more noticeable the flatter the $\Delta p - G$ - curves go. This case can also lead to tube damage in limiting cases although this is not a case of true instability, since there is always possible a case of flow downward so long as the flow is itself stable. The treatment of this case would fall outside the confines of this work; we note only that using Eq. 18 two relations result from first substituting in the heat flow for the stronger heated group of tubes and second for the weaker heated ones. By setting those equal the ratio of the quantities of water flowing thru the two groups of tubes can be calculated.

Protection from instability in water flow is usually sought by means of stops [diaphragms ?] ; to what extent such are required can be determined easily with Eq. 2.

With cooling-mantles occurrence of instability from the same cause as in forced-flow boilers can hardly appear, since the influence of buoyancy is too large. This is conceivable only in slightly or not at all inclined pipe-systems, i.e., cooling-grates [? Kühlroste], and there only when starting when the pressure is still low, and the water temperature can still lie at a place considerably below saturated steam temperature. On the other hand, instability can easily occur in cooling-mantles as a result of unequal heating of the tubes and different buoyancies in the mixture-columns. Fig. 7 shows the conditions for this; with its help the relations for every practical case can be easily tested. The result of the appearance of instability in cooling-mantles is either percussive flow or indeed a complete reversal of the same. Here also this can lead to tube-damage because of impounding of steam. Here the appearance of instability is rendered probably by high steam pressure as well as uneven heating of the tubes. How far uneven heating is possible we see indeed from the fact that a tube in a corner receives only approximately half the radiation of one in the middle. This can become still worse due to slag accumulations, particularly when they cover the bottom part of the tube. Then there occurs no vaporization in this part of the pipe at all, and the heavy water column opposes buoyancy. We must expect slag crusts particularly on inclined tubes.

With larger flow-losses in the boiler-tubes, i.e. with larger C_1 in Fig. 7, the danger of instability is very much reduced, since on the one hand P_W increases and on the other P_G is decreased.

Short-circuit tubes in cooling-mantles somewhat raise the water-circulation in the latter, whereby the value of P_W is also increased, which is favorable with regard to stability. If, however, the latter value is itself already near 1, then the short-circuit tubes have no more value, since the flow can reverse itself in them.

The object of the present work is achieved, if, by understanding the phenomena in heated parallel-connected tubes, the possibility has been given to the engineer to carry out his design with the greatest possible safety.