Q3:

<u>Answers</u>

3a: For $x > 0$, $sgn(x) = 1$ so we have two solutions vis 2 and 4.

For $x \; = \; 0$, $sgn(x) = 0$ so we have one solution i.e., 0.

However, for $x < 0$, $sgn(x) = -1$ so we have $x = -2\frac{W_0 \frac{ln(2)}{2}}{ln(2)}$.

3b: To check the convergence condition, we need to calculate the absolute value of the derivative of the function $f(x) = sgn(x)2^{\left(\frac{x}{2}\right)}$ at the two solutions and determine if the derivative is less than 1 in magnitude. If the magnitude of the derivative is less than 1, it is a sufficient condition for the fixed-point iteration method to converge.

At $x \; = \; 2$, the derivative can be calculated as:

$$df(x)/dx = d\left(sgn(x)2^{\left(\frac{x}{2}\right)}\right)/dx$$

$$= sgn(x)\left(d\left(2^{\left(\frac{x}{2}\right)}\right)/dx\right)$$

$$= sgn(2)\left(2^{(2/2)}(1/2)ln(2)\right)$$

$$= \; ln(2) \; < 1$$

Since the magnitude of the derivative at $x \; = \; 2$ is less than 1, the fixed-point iteration method converges at this solution.

At $x \; = \; 4$, the derivative can be calculated as:

$$df/dx = sgn(x)\left(d\left(2^{\left(\frac{x}{2}\right)}\right)/dx\right)$$

$$= sgn(4)\left(2^{(4/2)}(1/2)ln(2)\right)$$

$$= \; 2ln(2) > 1$$

Since the magnitude of the derivative at $x \; = \; 4$ is greater than 1, the fixed-point iteration method does not converge at this solution.

3c: For negative roots,

$$\frac{d}{dx}\left[-2^{\frac{x}{2}}\right]$$

$$= -\frac{d}{dx}\left[2^{\frac{x}{2}}\right]$$

$$= -\ln(2)\cdot 2^{\frac{x}{2}}\cdot\frac{d}{dx}\left[\frac{x}{2}\right]$$

$$= -\ln(2)\cdot 2^{\frac{x}{2}}\cdot\frac{1}{2}\cdot\frac{d}{dx}[x]$$

$$= -\ln(2)\cdot 1\cdot 2^{\frac{x}{2}-1}$$

$$= -\ln(2)\cdot 2^{\frac{x}{2}-1}.$$

Hence, for the negative values will always give absolute value of derivatives less than 1 which means as shown above it will converge.

3d: At $x_0 = 4.5$,

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    return sgn(x) * (2**(x/2))
                     ~^^~~~~~~
OverflowError: (34, 'Result too large')
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3e: At $x_0 = 4$,

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The result after fixed-point iteration is: 4.0
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3f: At $x_0 = 2.5$,

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The result after fixed-point iteration is: 2.0002178009941285
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3g: At $x_0 = -1.5$,

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The result after fixed-point iteration is: -0.7666811662504153
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