Q1: A thermal research reactor has a thermal neutron flux ϕ= 8E13 neutrons/cm^{2}s and it is fueled by metallic uranium fuel. The volume of the fuel is *V*= 80,000 cm^3 and its density is ρ= 19.1 g/cm^3. The fuel enrichment is *r* = 26.5 w/o in ^{235}U. What is the thermal power *P* of the reactor?
Assume the thermal fission cross-section for ^{235}U is σ_f= 585 barns. Assume a single fission reaction releases 200 MeV of recoverable energy. Also, assume that all fissions are caused by thermal neutrons and occur in ^{235}U, i.e., neglect the fissions caused by fast neutrons and fissions in ^{238}U.

The power *P* =

Ans:

The thermal power of the reactor is

$$P = \Sigma_f \times \phi \times V \times E_{fission}$$

where $E_{fission}$ is the energy recoverable from a single fission reaction,

$$E_{fission} = 200MeV = 200 \times 10^6 \times 1.602 \times 10^{-19}J = 3.204 \times 10^{-11}J$$

and where

$$\Sigma_f = N_{U235} \times \sigma_f$$

where

$$N_{U235} = \rho \times \frac{N_A}{M_{U235}} \times \frac{r}{100}$$

where r is the fuel enrichment in w/o.

Q2: Assume that a MOX fuel (a mix of UO_2 and PuO_2) has a density 10.1 g/cm^3, and the heavy metal in the fuel (i.e., ^{238}U + ^{235}U + ^{239}Pu) contains 17 w/o (weight percent) of ^{235}U and 5 w/o of ^{239}Pu. What is the atomic concentration of ^{239}Pu in the MOX fuel?

The atomic concentration of the ^{239}Pu in the MOX fuel is

Ans:

- ρ: mass density of the MOX fuel
- ρ_0: mass density of oxygen in the fuel
- ρ_5: mass density of U235 in the fuel
- ρ_8: mass density of U238 in the fuel
- ρ_9: mass density of Pu239 in the fuel
- E_5: w/o (weight percent) of U235 in the heavy metal of the fuel
- E_8: w/o (weight percent) of U238 in the heavy metal of the fuel
- E_9: w/o (weight percent) of Pu239 in the heavy metal of the fuel
- M_5: atomic weight of U235
- M_8: atomic weight of U238
- M_9: atomic weight of Pu239
- M_0: atomic weight of oxygen
- N: Avogadro's number
- N_9: atomic concentration of the Pu239 in the fuel

The problem can be described by a set of five equations:

$$N_9 = \frac{N \times \rho_9}{M_9}$$

$$\frac{\rho_5}{\rho_5 + \rho_8 + \rho_9} = \frac{1}{100} E_5$$

$$\frac{\rho_9}{\rho_5 + \rho_8 + \rho_9} = \frac{1}{100} E_9$$

$$\rho_0 + \rho_5 + \rho_8 + \rho_9 = \rho$$

$$\frac{2\rho_5}{M_5} + \frac{2\rho_8}{M_8} + \frac{2\rho_9}{M_9} = \frac{\rho_0}{M_0}$$

The last equation contains terms that are proportional to the atomic concentration of various nuclides, and the equation states that the atomic concentration of oxygen is twice as large as the combined atomic concentrations of the heavy nuclides.

There are five unknowns in the above set of equations: ρ_0, ρ_5, ρ_8, ρ_9 and N_9. The solution may be found directly, or numerically or analytically using a solver such as Sage. The Sage solver gives this solution for N_9:

$$N_9 = \frac{E_9 M_5 M_8 N \rho}{2(E_9 M_0 M_5 M_8 - (E_9 M_0 M_5 + (M_0 M_5 - M_0 M_8) E_5 - 100 M_0 M_5 - 50 M_5 M_8) M_9)}$$

Q3: Consider a bare sub-critical spherical fast reactor coupled with an external source of neutrons. The reactor is filled with a homogeneous mix of ^{235}U and Na. The atomic concentration of sodium is N_S=1.7E22 atoms/cm^3 and the atomic concentration of uranium is N_U=3E21 atoms/cm^3. The reactor runs at the total thermal power P=6 MW, and it leaks 4.6E18 neutrons per second. What is the reactivity ρ of the reactor?

The reactivity is

Assume that:

- the one-group microscopic fission cross section for ^{235}U is 1.4 b,
- the one-group microscopic absorption cross section for ^{235}U is 1.65 b,
- the one-group microscopic absorption cross section for Na is 0.0008 b,
- the average number of fission neutrons from a fission reaction of ^{235}U in this reactor is 2.6,
- the recoverable energy from a single fission reaction is 200 MeV.

Ans:

The multiplication factor of a finite reactor represents the ratio of the production rate of fission neutrons and the rate at which neutrons are being absorbed or leaked out of the system. This can be written formally as

$$k = \frac{\upsilon \times \Sigma_f \times \int_V \phi(\vec{r})dV}{\Sigma_a \int_V \phi(\vec{r})dV + DB^2 \int_V \phi(\vec{r})dV}$$

where

$$DB^2 \int_V \phi(\vec{r})dV \equiv Q$$

represents the given total leakage Q (the number of neutrons that leak out of the reactor per second). The integrated total neutron flux integrated over the volume, $\int_V \phi(\vec{r})dV$, can be obtained from

$$P = E_R \Sigma_f \int_V \phi(\vec{r})dV$$

as

$$\int_V \phi(\vec{r})dV = \frac{P}{E_R \Sigma_f}$$

where E_R

is the recoverable energy from a single fission reaction ($E_R = 200MeV = 3.2E - 11$ J). Hence, it follows that

$$k = \frac{\upsilon \times \Sigma_f \times \frac{P}{E_R \Sigma_f}}{\Sigma_a \frac{P}{E_R \Sigma_f} + Q}$$

where

$$\Sigma_f = \sigma_{fU} \times N_U$$

$$\Sigma_a = \sigma_{aU} \times N_U + \sigma_{aS} \times N_S$$

$$\upsilon = 2.6$$

Finally, reactivity is obtained as

$$\rho = \frac{k-1}{k}$$

Q4: Calculate Σ_a for water of density 0.7 g/cm^3 at 0.1 eV assuming an ideal isotopic composition of the water ^{1}H and ^{16}O, which are known to be 1/v absorber up 100 eV.

Given:

- Density of water, ρ= 0.7 g/cm^3;
- Energy in question, E = 0.1 eV.

Data that might be useful:

- Microscopic capture cross-section at 0.0253 eV for ^{1}H, $\sigma_{c,H1}$ = 0.332128 b;
- Microscopic capture cross-section at 0.0253 eV for ^{16}O, $\sigma_{c,O16}$= 1.900468E-4 b;

Answer is required with 1% accuracy.

Macroscopic absorption cross-section of water, Σ_a =

Ans:

The macroscopic absorption cross-section of the water is

$$\Sigma_{a,H2O} = \sigma_{c,H2O}(E) N_{H2O}$$

The water molar weight and the water molecular density are easily found as

$$M_{H2O} = M_{O16} + 2M_{H1}$$

and

$$N_{H2O} = \frac{\rho_{H2O} N_A}{M_{H2O}}$$

The microscopic capture cross-section of the water molecule is

$$\sigma_{c,H2O}(E) = \sigma_{c,O16}(E) + 2\sigma_{c,H1}(E)$$

The individual microscopic cross-sections are given by

$$\sigma_{c,O16}(E) = \sigma_{c,O16}(E_0)\sqrt{\frac{E_0}{E}}$$

and

$$\sigma_{c,H1}(E) = \sigma_{c,H1}(E_0)\sqrt{\frac{E_0}{E}}$$