Mechanical properties of crystals

Stress and strain

stress = $\frac{\text{force}}{\text{unit area}}$ $\left\{ \begin{array}{l} \text{normal} \\ \text{shear} \end{array} \right.$

$\left(\sigma = \frac{F}{A} \right)$

Strain = fractional extension along x, y, z

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \qquad \begin{pmatrix} \sigma_{12} = \sigma_{21} \\ \sigma_{ij} = \sigma_{ji} \end{pmatrix}$$

6 indep param

$$\epsilon = \begin{pmatrix} \text{similar} \end{pmatrix}$$

Elasticity theory: Hooke's law

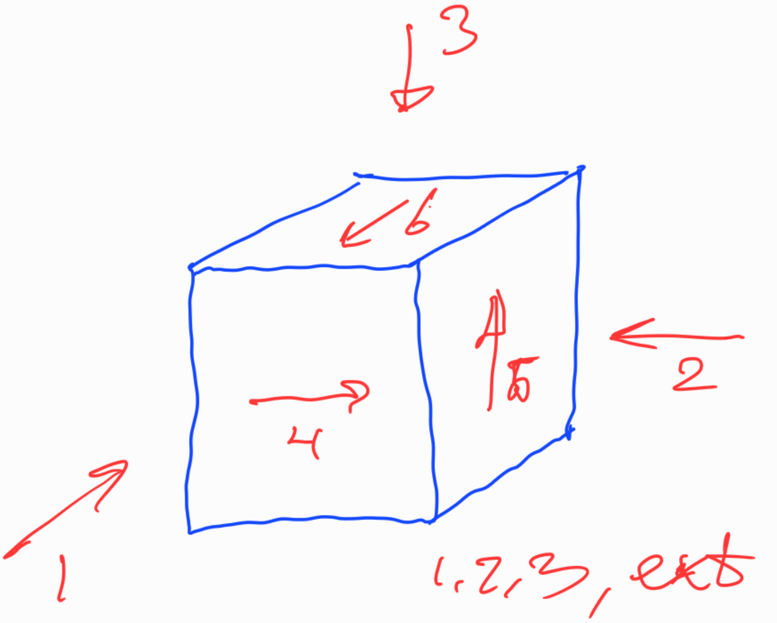
$$\sigma_{ij} = \underline{C_{ijkl}}\, \epsilon_{kl}$$

$81 \to 21 \to \underset{\substack{\uparrow \\ \text{cubic}}}{3}$ indep elements C_{ijkl}

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{12} & & & \\ c_{12} & c_{11} & c_{12} & & 0 & \\ c_{12} & c_{12} & c_{11} & & & \\ & & & & & \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \vdots \end{pmatrix}$$

$$\begin{bmatrix}\sigma_4\\ \sigma_5\\ \sigma_6\end{bmatrix} = \begin{bmatrix} C_{44} & & \\ & C_{44} & \\ & & C_{44}\end{bmatrix}\begin{bmatrix}\varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}$$

$$\begin{bmatrix}\sigma_{11} \to \sigma_1\\ \sigma_{22} \to \sigma_2\\ \sigma_{33} \to \sigma_3\\ \sigma_{12} \to \sigma_4\\ \sigma_{13} \to \sigma_5\\ \sigma_{23} \to \sigma_6\end{bmatrix}$$



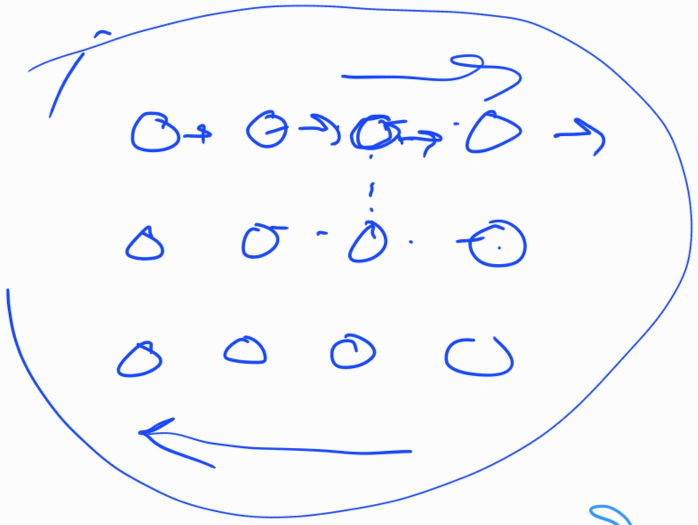
1, 2, 3, ~~ext~~ ext

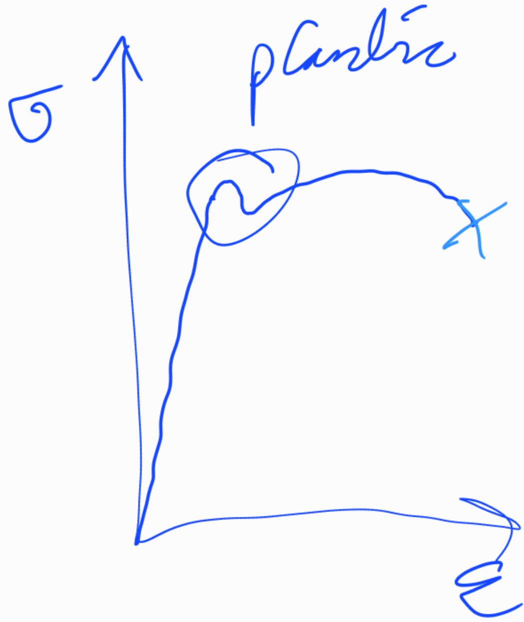
4, 5, 6, shears

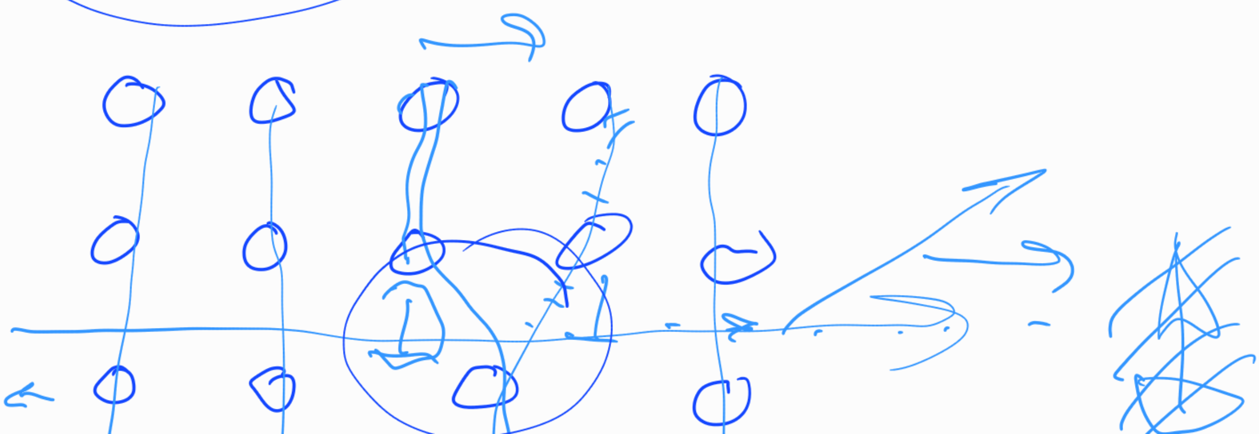
Hooke

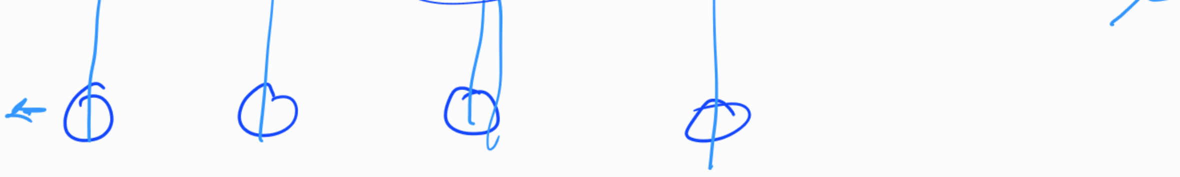
$$\underline{\sigma = C\varepsilon}$$

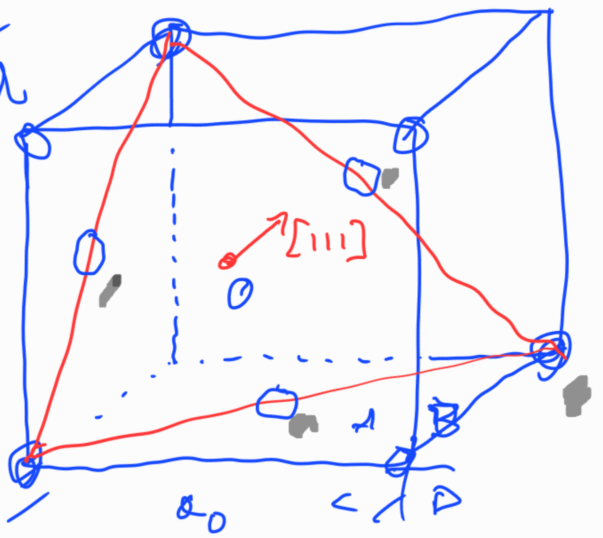
Plastic deformation











[111] plane

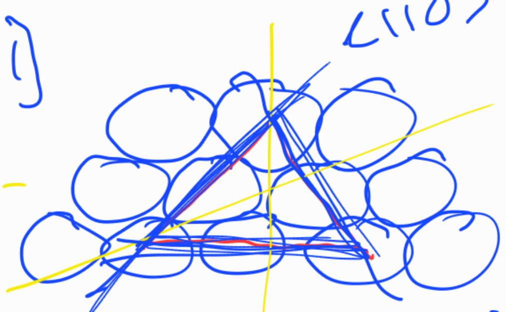
Dislocation glide happens in plane w. highest planar density.

Ex What is the planar density of the [111] and [100] planes in fcc?

$$\rho_{111} = \frac{N_{111}}{A_{111}} = \frac{3\cdot\frac{1}{6} + 3\cdot\frac{1}{2}}{2\cdot\sqrt{3}\,a_0^2/4} = \underline{\frac{4}{\sqrt{3}}\frac{1}{a_0^2}}$$

$$\rho_{100} = \frac{N_{100}}{A_{100}} = \frac{1 + 4\cdot\frac{1}{4}}{a_0^2} = \frac{2}{a_0^2}$$

$\rho_{111} > \rho_{100}$ → Dislocation glide in [111] plane(s)!



Glide direction in that plane given by

$\langle 110 \rangle$ $\langle 110 \rangle$ highest linear density.

$\Rightarrow$ Glide plane + slip direction

$\rightarrow$ glide system

In fcc: 4 planes [111]

3 directions $\langle 110 \rangle$

$\rightarrow$ 12 glide systems

slip

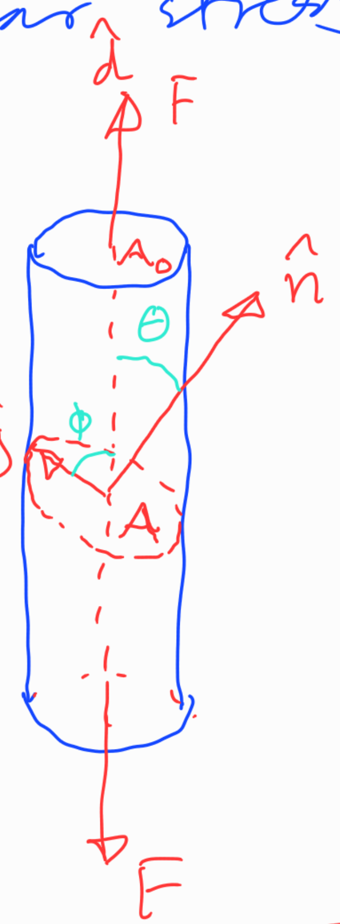
~~$[111]\langle 110 \rangle$~~

$[\bar{1}11]\langle 110 \rangle$

$\vdots$

Critical resolved shear stress

CRSS $\sim \tau$



$$\sigma = \frac{F}{A_0}$$

$$F_{\hat{g}} = F \cos\phi$$

$$\tau^{ss} = \frac{F_{\hat{g}}}{A}$$

$$\Rightarrow \tau^{ss} A = \sigma A_0 \cos\phi$$

$$A = \frac{A_0}{\cos\theta}$$

$$\Rightarrow \boxed{\tau^{ss} = \sigma \cos\phi \cos\theta}$$

Schmid

If $\tau^{ss} \geq \tau^{crit} \rightarrow$ yield

Schmid factor

Activated slip system is the one that maximizes the Schmid factor.