Written Exam, Radiation damage in materials (SH2605) – VT 2014
08.00 – 13.00, March 11, 2014, FD41, AlbaNova, KTH, Stockholm

With solutions!

Problem 1 [2p]

Grain size is sometimes classified according to the "ASTM grain size number" G. With n equal to the number of grains per square inch at a length magnification of 100, we have $n = 2^{G-1}$. A coarse grain structure means 1 <= G <= 5 and a fine grain structure corresponds to 5 <= G <= 8.
a) What is the mean grain size when $G = 2$ (coarse GS), and when $G = 7$ (fine GS). [1p]
b) What is the approximate number of atom layers across a grain in each case? [1p]

Solution:
a) The mean grain size can be characterized by it's length: $L(G)=\frac{2.54\cdot 10^{-4}}{\sqrt{2^{G-1}}}$ in meters. For G=2 this becomes 0.18 mm, for G=7 it becomes 32 µm. In terms of area, one can simply square these numbers on the assuption that an average grain is square.
b) The number of layers can then be deduced using a typical atomic plane spacing of 2Å to be: For G=2: 0.9 million layers, G=7: 0.16 million layers.

Problem 2 [1p]

Sheet steel is produced by rolling steel in a number of steps, thereby deforming the material, which introduces dislocations. If the starting product is a 1 m^3 block, and the final product has a dislocation density of 10^{16} m^{-2}, what is the total length of dislocations in the material? [1p]
Half an extra point if you use a convenient unit. [+0.5 p]

Solution:
The length of all dislocations added together is easiest to construct by assuming they are all straight and going through the metal cube from one face to another. Thus the total length becomes $l = V/\rho = 10^{16}$ m. This is very close to a light year!

Problem 3 [3p]

a) Derive the expression for the maximal energy transfer from an elastic collision, as a function of the kinetic energy of the incoming particle (T_{inc}). [2p]
b) What is the maximal kinetic energy that a W atom in the divertor of a fusion reactor receives from 14 MeV neutrons coming from the d+t reactions in the plasma? [1p]

Solution:
a) Conserve momentum and kinetic energy, assuming that the knocked-on particle (mass M) is at rest and that the incoming particle (mass m) is perfectly back-scattered.

$$\begin{matrix} p_b = p_a \\ T_b = T_a \end{matrix} \rightarrow \begin{matrix} p_b^m + p_b^M = p_a^m + p_a^M \\ T_b^m + T_b^M = T_a^m + T_a^M \end{matrix} \rightarrow \begin{matrix} mv_b + Mu_b = mv_a + Mu_a \\ \frac{1}{2}(mv_b^2 + Mu_b^2) = \frac{1}{2}(mv_a^2 + Mu_a^2) \end{matrix} \rightarrow \begin{matrix} mv_b = mv_a + Mu_a \\ mv_b^2 = mv_a^2 + Mu_a^2 \end{matrix} \rightarrow$$

$$v_a = v_b - \frac{M}{m} u_a \rightarrow mv_b^2 = m\left(v_b - u_a \frac{M}{m}\right)^2 + Mu_a^2 = mv_b^2 - 2 v_b u_a M + u_a^2 \frac{M^2}{m} + M u_a^2 ,$$

strike out mv_b^2 from both sides and divide by *M*, then we get $u_a = 2\frac{mv_b}{M+m}$, thus rendering $T_a^M = \frac{1}{2}Mu_a^2 = \frac{1}{2}mv_b^2 \frac{4\text{mM}}{(M+m)^2} = \frac{4\text{mM}}{(M+m)^2} T_{\text{inc}} = \gamma\, T_{\text{inc}}$

b) m=1, M=184, T_{inc}=14MeV → $T_a^M = 4\frac{mM}{(M+m)^2} T_{\text{inc}} = 300\, keV$, i.e. γ = 0.022

Problem 4 [3p]

The critical size of a void nucleus is determined by the change in Gibbs free energy due to a void of *n* vacancies, $\Delta G_n = -n k_B T ln S_v + (36\pi\Omega^2)^{1/3}\gamma n^{2/3}$

a) Determine an expression for the critical void embryo radius! [2p]
b) What is the critical void embryo radius in bcc iron where the irradiation at 300°C is such that the vacancy number concentration is 10^{-14} ? [1p]

Solution:
a) The critical radius will be determined by the critical number of vacancies. Differentiating the above expression with respect to *n* gives $n_{crit} = \frac{32\pi\Omega^2\gamma^3}{3(kT\ln(S_v))^3}$

Since the radius is related to the number by an assumed perfect packing of vacancies, then the radius corresponding to this number of vacancies is given by the simple expression $R_{crit} = \sqrt[3]{\frac{3 n_{crit}\Omega}{4\pi}} = \frac{2\Omega\gamma}{kTln(S_v)}$

b) All data is in the table except the supersaturation factor. This one can be calculated by taking the Boltzmann factor for the equilibrium vacancy concentration. Thus, we get $R_{crit} = 6\,\text{Å}$ (or 69 vacancies).

Problem 5 [3p]

Assume that the steady state concentration of vacancies in bcc iron, in a 1 MeV neutron flux of 10^{15} $cm^{-2}s^{-1}$ is given by $C_v = \frac{K_0}{K_{vV} C_V}$ and that the average void radius is 5 nm, the void density is 10^{17} m^{-3} and the total scattering cross section is 3 barns.

a) Determine the temperature at which the supersaturation factor of vacancies becomes unity, i.e. the temperature at which thermal effects start to dominate over irradiation ones. [2p]
b) What is the main assumption for this expression of the steady state vacancy concentration? [1p]

Solution:
a) Expanding the different coefficients properly gives us an expression from which we can break out the temperature: $T_{crit} = \frac{E_f + E_m}{k\ln\left(\frac{16 S_v E_d \pi R_V \alpha a_0^2 \nu e^{S_f/k}}{\xi\gamma T \sigma N \varphi}\right)} = 1271\,K \approx 1000^o C$

b) The main assumption is that we are working at high enough temperature for sink reactions to dominate over mutual recombination.

Problem 6 [4p]

a) Which plane has the highest planar density in a bcc crystal? [1p]
b) What are the slip systems in a bcc crystal, with that plane as slip plane? [1p]
c) Which slip system will activate first during plastic deformation of the crystal in the direction n_T = [0.6899, 0.6132, 0.3847] ? [2p]

Solution:
a) The planar density is highest in the {110} planes.
b) The <111> directions have the highest linear density and thus the slip systems are the twelve {110}<111> combinations.
c) The (101)$[11\bar{1}]$ system has the highest Schmid factor (0.4) and will be activated first.

Data sheet:

Various properties of selected metals:

	a_0	E_d	E_f^v	S_f^v	E_m^v	S_m^v	E_f^{sia}	S_f^{sia}	γ
Iron	2.86 Å	40 eV	2.15 eV	2.4 k_B	0.71 eV	0	4.0 eV	0.7 k_B	1.8 J/m²
Tungsten	3.17 Å	60 eV	3.55 eV	1.8 k_B	0.9 eV	0	9.5 eV	0.4 k_B	2.8 J/m²

Models:

The Kinchin-Pease model:
$$n(T)=\begin{cases} 0\ , & T<E_d \\ 1\ , & E_d<T<2E_d \\ \dfrac{T}{2E_d}\ , & 2E_d<T<E_c \\ \dfrac{E_c}{2E_d}\ , & T>E_c \end{cases}$$

Rate theory defect generation term: $K_0=\xi n(T)\sigma_s N\phi$

Neutral void-vacancy reaction rate: $K_{vV}=4\pi R_V D_v$

Constants:
Boltzmann's constant: $k_B=1.38\cdot10^{-23}\,J/K$
One inch = 2.54 cm