Q1: What common coupling schemes are employed in Monte Carlo burnup simulations?

Ans:

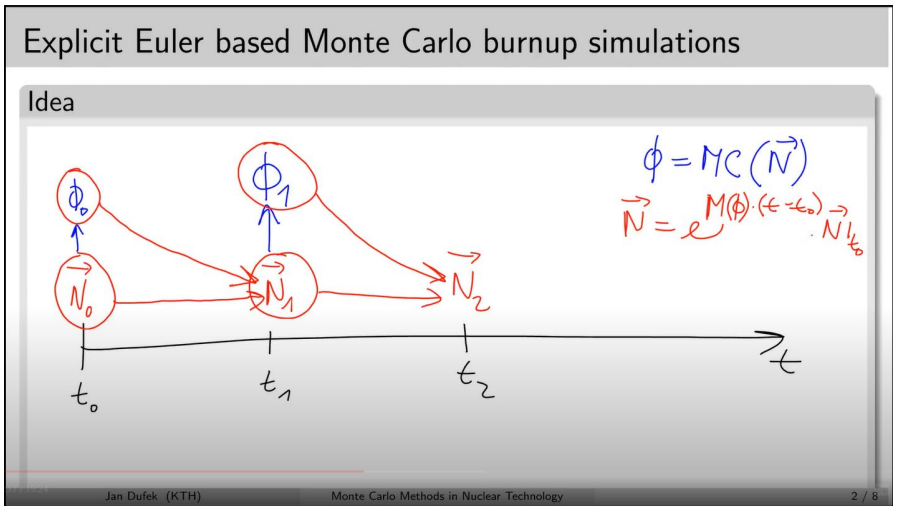
As the burnup equation is an ODE, the coupling schemes are derived from ODE numerical methods, such as:

- Explicit Euler (the beginning-of-step constant flux approximation)
- Predictor-Corrector
- Mid-point method (the middle-of-step constant flux approximation)
- Implicit Euler

Q2: Describe the explicit Euler-based coupling scheme for MC burnup simulations.

Ans:

Also known as "**The beginning-of-step constant flux approximation coupling scheme (MCB, SERPENT)**."



Algorithm:

1. **input:** $\vec{N}_0$
2. for $i \leftarrow 0, 1, \ldots$ do
3. $\quad \phi_i \leftarrow$ **fundamental mode of** $B(\vec{N}_i)$
4. $\quad \vec{N}_{i+1} \leftarrow \exp[\mathbb{M}(\phi_i)\Delta t_i]\vec{N}_i$
5. end for

**The explicit Euler is conditionally stable, i.e., it is stable for sufficiently brief time steps. Numerical instability of this scheme is common in Monte Carlo burnup simulations with strong untreated feedbacks (mainly xenon-135) – the results may contain significant errors.

Instabilities:

- Spatial oscillations of neutron flux
- Spatial oscillations of nuclide field

Can be used in systems:

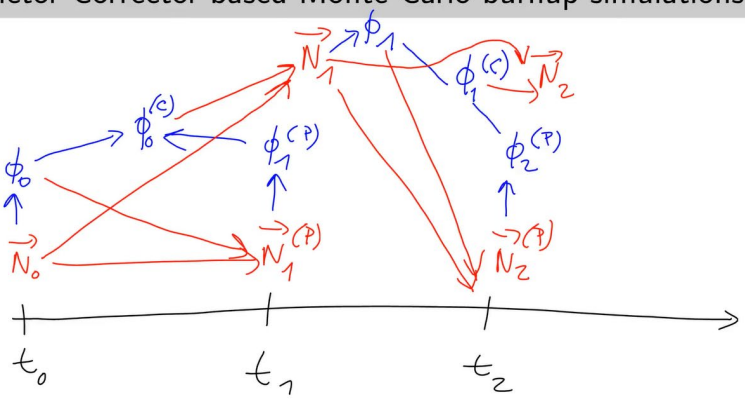
- Xenon oscillations cannot develop, e.g., fast systems, small systems (small dominance ratio), and one fuel material defined (single card for the description of fuel material).

Q3: Describe the predictor-corrector-based coupling scheme for MC burnup simulations.

Ans:

The predictor-corrector scheme (MCODE, SERPENT)

Predictor-Corrector based Monte Carlo burnup simulations



Algorithm:

1. **input:** $\vec{N}_0$
2. for $i \leftarrow 0, 1, \ldots$ do
3. $\quad \phi_i \leftarrow$ **fundamental mode of** $B(\vec{N}_i)$
4. $\quad \vec{N}_{i+1}^{(\mathrm{P})} \leftarrow \exp[\mathbb{M}(\phi_i)\Delta t_i]\vec{N}_i$
5. $\quad \phi_{i+1}^{(\mathrm{P})} \leftarrow$ **fundamental mode of** $B(\vec{N}_{i+1}^{(\mathrm{P})})$
6. $\quad \bar{\phi}_i^{(\mathrm{C})} \leftarrow (\phi_i + \phi_{i+1}^{(\mathrm{P})})/2$
7. $\quad \vec{N}_{i+1} \leftarrow \vec{N}_i \exp[\mathbb{M}(\bar{\phi}_i^{(\mathrm{C})})\Delta t_i]$
8. end for

**The predictor-corrector scheme is also conditionally stable, and similar instabilities as for the EE scheme may also develop for the PC scheme. The numerical instability in the predictor-corrector-based Monte Carlo burnup simulations may introduce more significant errors in the results than the explicit Euler-based simulations. Predictor-corrector-based Monte Carlo burnup simulations may outperform the explicit Euler-based simulations in systems where xenon oscillations cannot develop.

Q4: Describe the mid-point method-based coupling scheme for MC burnup simulations.

Ans:

The middle-of-step constant flux approximation (MONTEBURNS, MCNPX)

Algorithm:

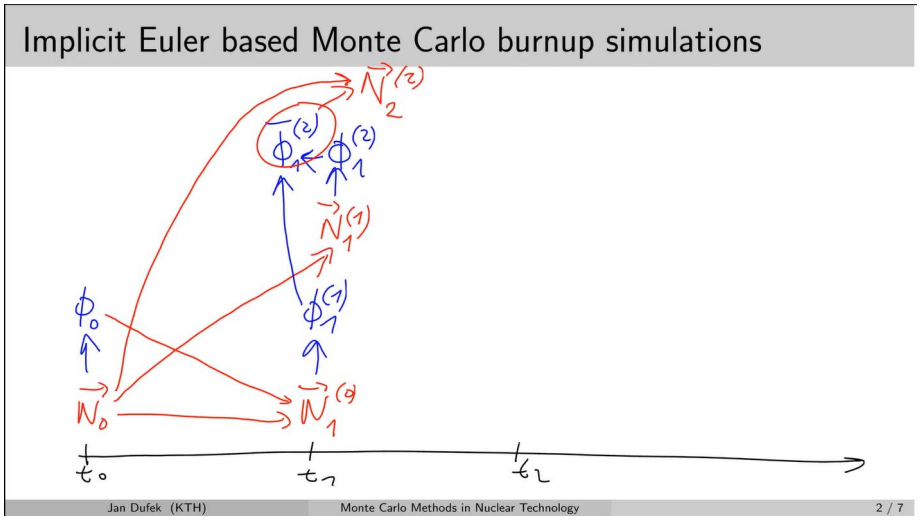
1. **input:** $\vec{N}_0$
2. for $i \leftarrow 0, 1, \ldots$ do
3. $\quad \phi_i \leftarrow$ **fundamental mode of** $B(\vec{N}_i)$
4. $\quad \vec{N}_{i+1/2} \leftarrow \vec{N}_i \exp[\mathbb{M}(\phi_i)\Delta t_i/2]$
5. $\quad \phi_{i+1/2} \leftarrow$ **fundamental mode of** $B(\vec{N}_{i+1/2})$
6. $\quad \vec{N}_{i+1} \leftarrow \vec{N}_i \exp[\mathbb{M}(\phi_{i+1/2})\Delta t_i]$
7. end for

** The mid-point method is also conditionally stable and has similar instabilities as for EE scheme may develop.

Q5: Describe the implicit Euler method-based coupling scheme for MC burnup simulations.

Ans:

Stochastic Implicit Euler (SIE)-based scheme



Algorithm:

SIE-based coupling scheme

1. **input:** $\vec{N}_0$
2. $\phi_0 \leftarrow$ **fundamental mode of** $B(\vec{N}_0)$
3. for $i \leftarrow 0, 1, \ldots$ do
4. $\quad \vec{N}_{i+1}^{(0)} \leftarrow \exp[\mathbb{M}(\phi_i)\Delta t]\vec{N}_i$
5. $\quad$ for $n \leftarrow 1, 2, \ldots$ do
6. $\quad\quad \phi_{i+1}^{(n)} \leftarrow$ **fundamental mode of** $B(\vec{N}_{i+1}^{(n-1)})$
7. $\quad\quad \bar{\phi}_{i+1}^{(n)} \leftarrow \sum_{j=1}^{n} \phi_{i+1}^{(j)}/n$
8. $\quad\quad \vec{N}_{i+1}^{(n)} \leftarrow \vec{N}_i \exp[\mathbb{M}(\bar{\phi}_{i+1}^{(n)})\Delta t]$
9. $\quad$ end for
10. $\quad \vec{N}_{i+1} \leftarrow \vec{N}_{i+1}^{(c)}$
11. $\quad \phi_{i+1} \leftarrow \bar{\phi}_{i+1}^{(c)}$
12. end for

** The SIE method is unconditionally stable (stable for any time step) only when enough inner iterations are set up.

Implicit Euler-based coupling schemes for Monte Carlo burnup simulations are numerically stable. Although the numerical stability is ensured by the implicit Euler-based schemes, setting large time steps is not recommended.

Recent studies suggest that computing efficiency may not be worsened by choosing noticeably short time steps when compensated by reduced neutron history statistics per time step.

Q6: Comment on the numerical stability of various coupling schemes for MC burnup simulations.

Ans:

For

- Explicit Euler (the beginning-of-step constant flux approximation)
  The explicit Euler is conditionally stable, i.e., it is stable for sufficiently brief time steps. Numerical instability of this scheme is common in Monte Carlo burnup simulations with strong untreated feedbacks (mainly xenon-135) – the results may contain significant errors.
  Instabilities: Spatial oscillations of neutron flux & nuclide field

- Predictor-Corrector
  The predictor-corrector scheme is also conditionally stable, and similar instabilities as for the EE scheme may also develop for the PC scheme. The numerical instability in the predictor-corrector-based Monte Carlo burnup simulations may introduce more significant errors in the results than the explicit Euler-based simulations.

- Mid-point method (the middle-of-step constant flux approximation)
  The mid-point method is also conditionally stable and has similar instabilities as for EE scheme may develop.

- Implicit Euler
  The SIE method is unconditionally stable (stable for any time step) only when enough inner iterations are set up.

Q7: What factors affect the numerical stability of MC burnup simulations?

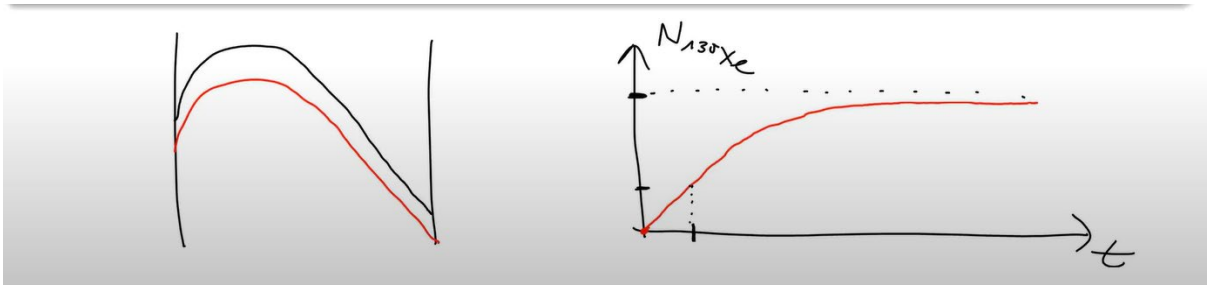
Ans:

Numerical stability is affected by the following:

- **Number of fuel materials set for depletion:** The more fuel materials are defined, the easier for xenon-powered oscillations to develop.
- **Strength of modeled feedback:** The xenon feedback is weak in fast reactors −>no numerical instabilities can develop there.
- **Size of the system (dominance ratio):** The closer the dominance ratio is to one, the more difficult it is to ensure the numerical stability of burnup simulations.
- **Time step size:** The larger the time steps −> the less stable the burnup simulations.
- **Coupling scheme:** A stable coupling scheme can ensure stability.
- **Special treatment of xenon:** Some codes can compute equilibrium xenon concentration during criticality calculations. Activate this feature since it will ensure stability under almost any conditions.

Q8: What is the difference between the saturated and equilibrium xenon calculations?

Ans:



The burn-up solver does not consider the time step length.

The concentration of xenon-135 is calculated for the infinite time step.

Q9: What are the factors affecting the efficiency of MC burnup simulations?

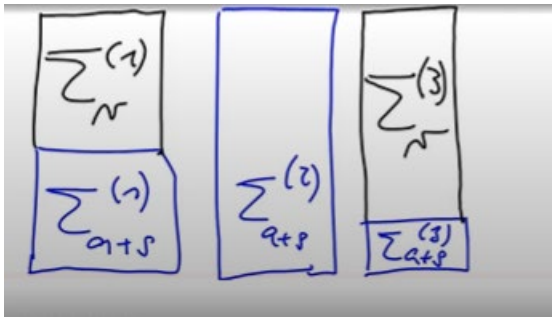
Ans:

The efficiency of MC burnup simulations is affected by the choice of free parameters, such as the time step length, the number of cycles per criticality calculation, and the number of neutrons per criticality cycle.

** Latest research suggests that the optimal burnup process occurs when the burnup solver cost is approximately equal to the MC solver cost in the MC burnup simulation.

Q10: Describe the idea of the Delta tracking method.

Ans:



If the total macroscopic cross-section were equal in all cells, then we would not need to keep track of the distance to the surface, and the cell number since the kernel would be sampled from the same distribution. Only one cross-section would be used to sample the transition kernel. Well, we can do this - we can find what cell has the largest macroscopic total cross-section, and we can increase cross-sections in other cells using the virtual cross-section to match the largest one.

** The virtual collision is not a real collision; it is a collision that does not change the direction, energy, or weight. It is associated with the virtual macroscopic collision cross section Σ_v that characterizes the virtual collision probability per path length traveled in the medium.

Q11: What are the advantages/disadvantages of the Delta tracking method?

Ans:

Advantages of the delta tracking

- Considerable simplification of the MC code
- FOM of the calculation is substantially increased (i.e., the efficiency is improved) if the system doesn't contain strong absorbers.

Disadvantages of the delta tracking

- FOM of the calculation may be decreased if the system contains small cells with strong absorbers. This is because many virtual collisions will be simulated in most cells, taking too much CPU time. (At each virtual collision, the code still needs to figure out what material the cell contains and load the cross sections corresponding to the neutron energy.)
- Surface flux estimators cannot be implemented easily. (This is because the passing of neutrons through the boundaries is not checked)

Q12: What is the difference between the analog and non-analog Monte Carlo simulations?

Ans:

Non-Analog Monte Carlo Methods

These methods try to increase the FOM of the MC transport calculation by optimizing the actual neutron transport. Non-analog methods thus belong to variance reduction techniques.

Principle:

The pdf of some random variables is purposely biased (changed) to increase the occurrence of desired events. This must be compensated by a change in the "statistical weight" w of the simulated neutron so that the obtained results are unbiased.

Analog Monte Carlo Methods

In analog Monte Carlo simulation, neutron transport is simulated according to reality, from neutron "birth" (originating from the source) to "death" (absorption or leakage from the system).

The simulation of all collisions of a single neutron is called a neutron "history." The average behavior of neutrons is estimated via simulating many neutron histories.