Nuclear Reactor Technology, SH2702

Lecture No 6

Title:
Reactor Control and Operation

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Outline of the Lecture

- General principles of reactor control
- Reactor startup
- Control principles of BWR
- Control principles of PWR
- Reactivity changes due to control rods
  - Control rod effectiveness and worth
  - Control rod materials, types and mechanisms
- Inherent reactivity changes in a core
  - Fuel depletion
  - Reactor poisons

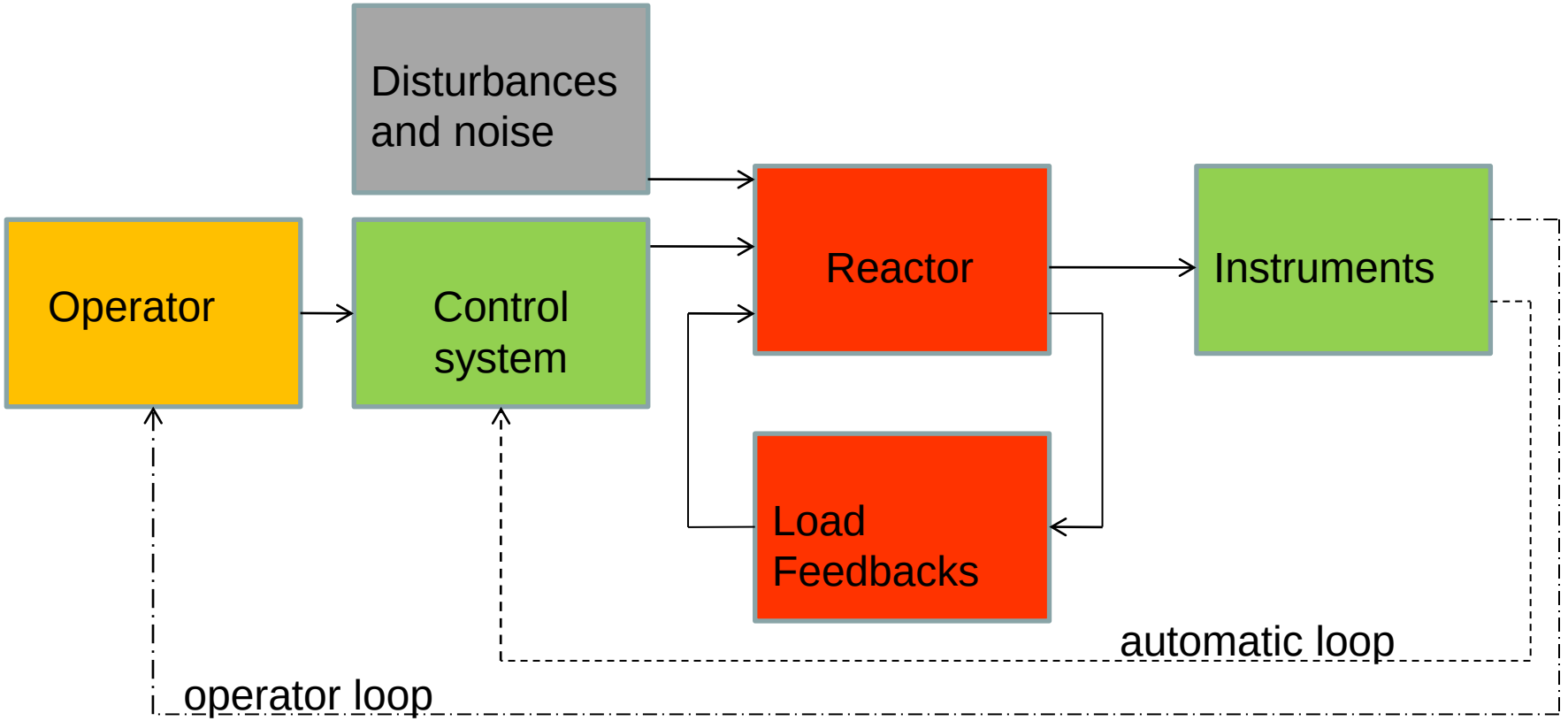
General Features of Reactor Control

- Purpose of reactor control
  - provide means to start the reactor
  - maintain constant power during normal operation
  - provide means to shutdown the reactor
- Protection system
  - automatic system to immediately shutdown the reactor in case of unsafe condition

Four Methods of Reactor Control

- Reactors are controlled by adding/removing of:
  - fuel
  - moderator (used in BWRs by changing flow through the core)
  - reflector
  - neutron absorbers (e.g. control rods in both PWRs and BWRs)

Main Reactor Control Loops



Reactor Startup

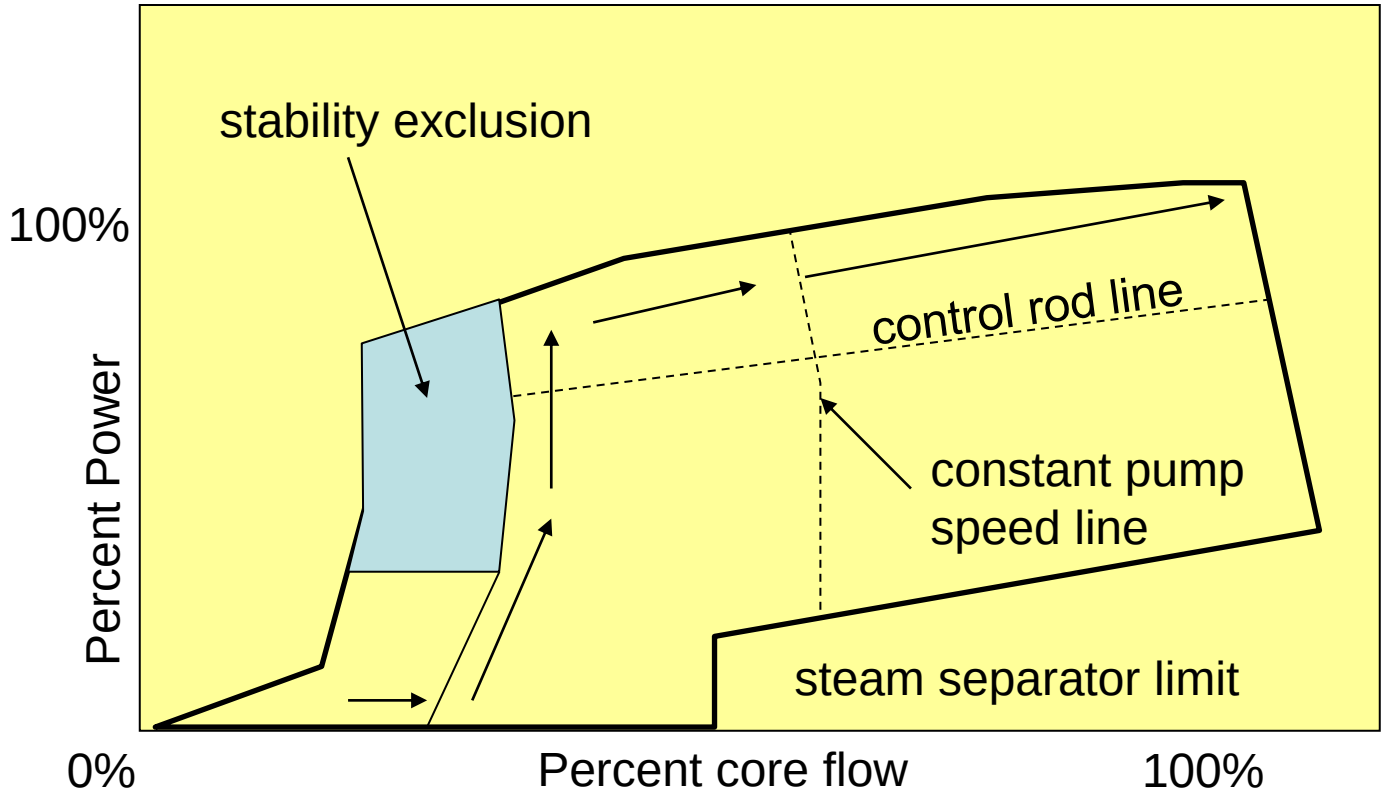
- Initial startup (startup from the cold standby)
  - when starting a new reactor or after refueling
  - hazardous operation since usually neutron flux is very low and difficult to measure
  - peripheral rods are removed in small steps first since they have least worth
- Startup after trip (from the hot standby)
  - much easier operation that initial startup since neutron flux is much higher
  - should be done very soon (minutes) after trip
  - if rapid startup is not possible, reactor can be started only when the concentration of reactor poisons goes down

Startup from Cold Standby (BWR)

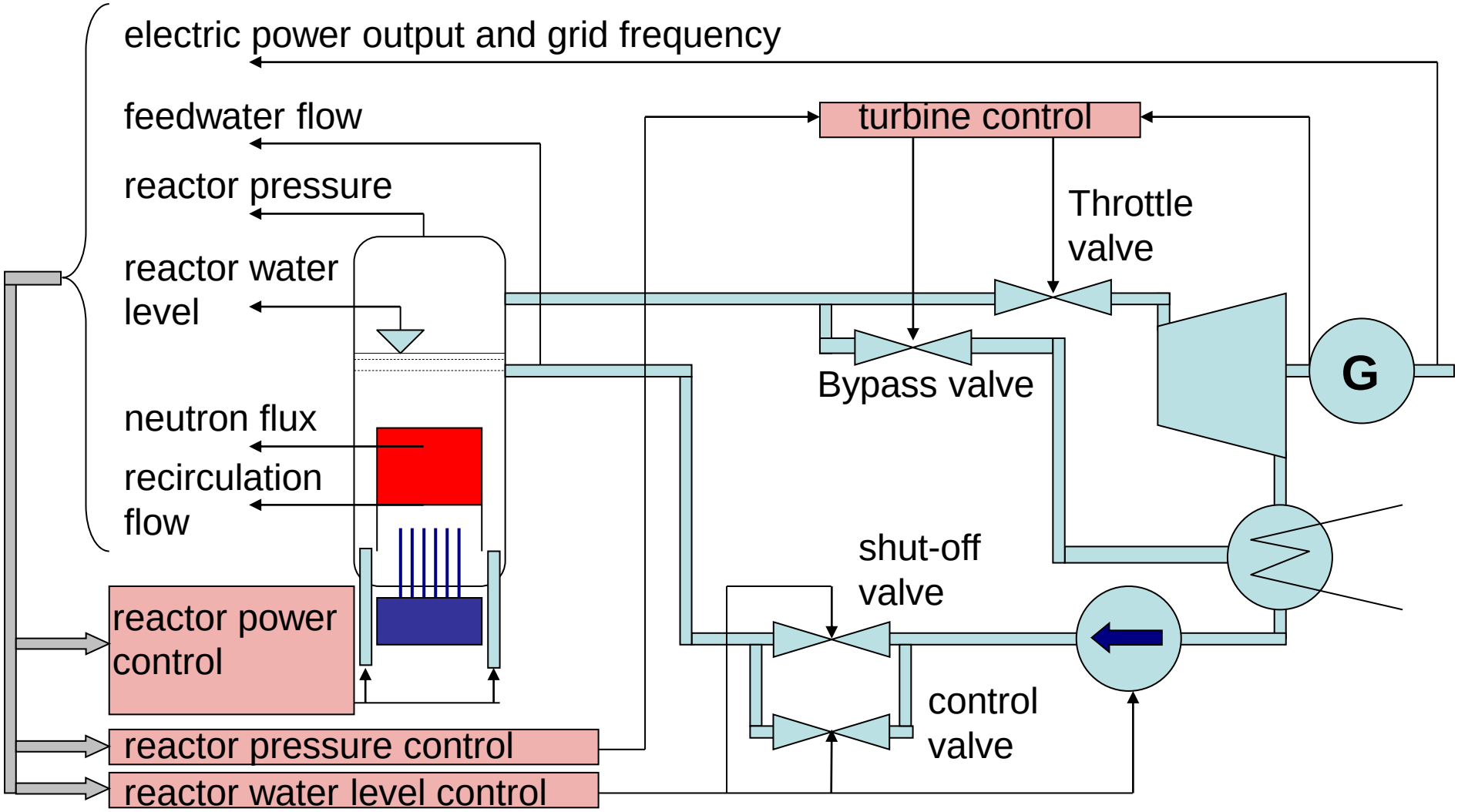
- Startup of the recirculation pumps
- Flow control valves set at ~25% of the rated flow
- Control rods are manually withdrawn according to predetermined schedule to achieve criticality and to bring reactor to ~32% of rated power
- From ~30 to 40% rated power the control of power is through manual control of recirculation flow
- Between ~40 to 75% power control rods are normally used to control power
- Above 75% of rated power change in coolant recirculation flow is used to control power

BWR Control Principles

- Operation of BWRs is often represented by a so-called **operation map**, as shown below



BWR Control Principles



Reactor Power Control

- Reactor **power** control system controls reactor thermal power **through driving the recirculation flow**

- Recirculation flow is changed through controlling the **speed of recirculation pumps** by modifying **current frequency**

Reactor Water Level Control

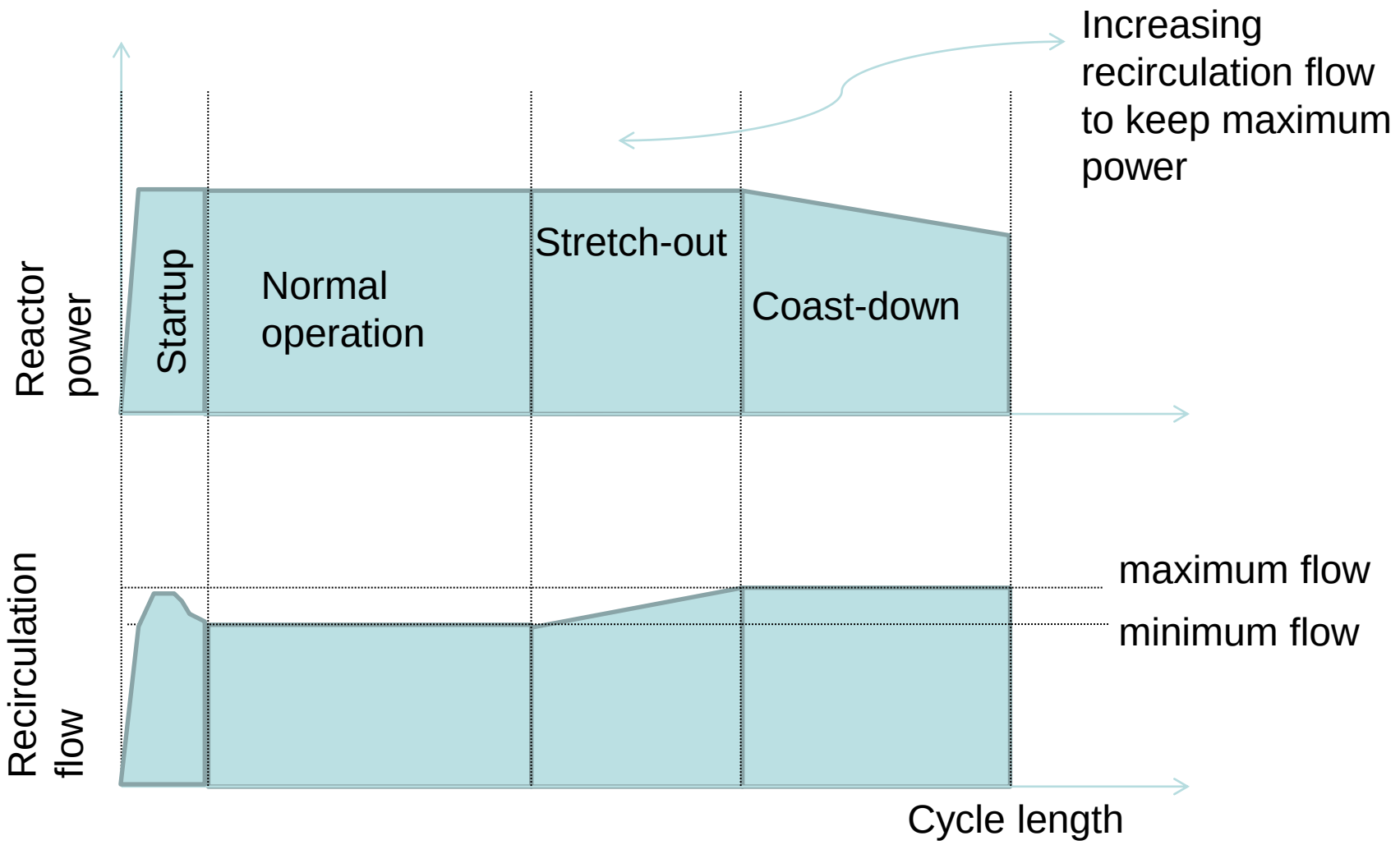
- Reactor water level control is performed by **changing feed water flow**
- Feedwater flow is changed by **modifying current frequency supplying feedwater pumps**
- At **low reactor power** and low feedwater flow the flow is also controlled with **throttling valve** to have more stable behaviour

Turbine Control

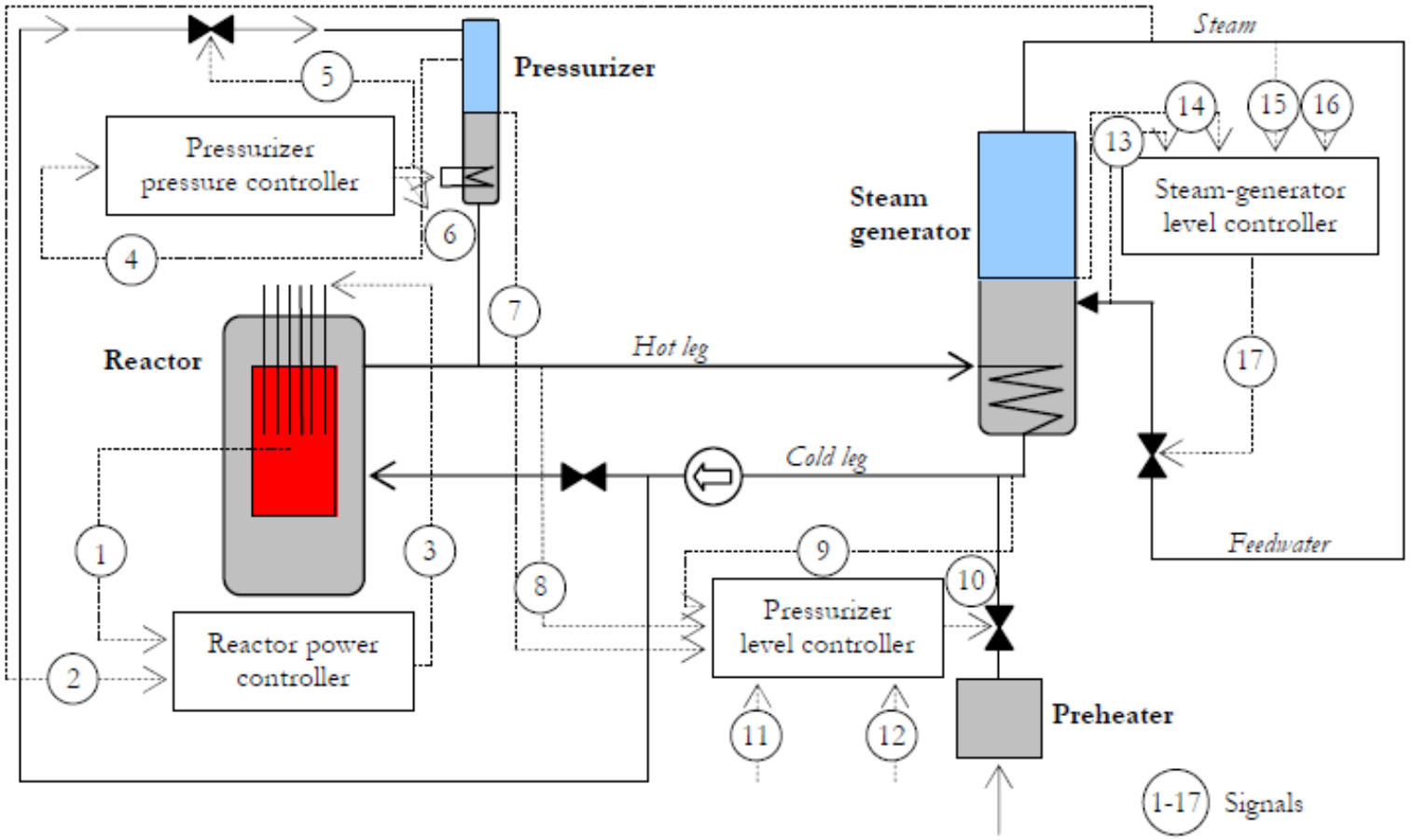
- Turbine control system controls **pressure in reactor pressure vessel**

- Pressure in the reactor pressure vessel is controlled through the **total outflow of steam**, which is distributed between
  - high-pressure control valves
  - control valves of the moisture separator and reheater primary side
  - dump control valves

BWR Operation Cycle



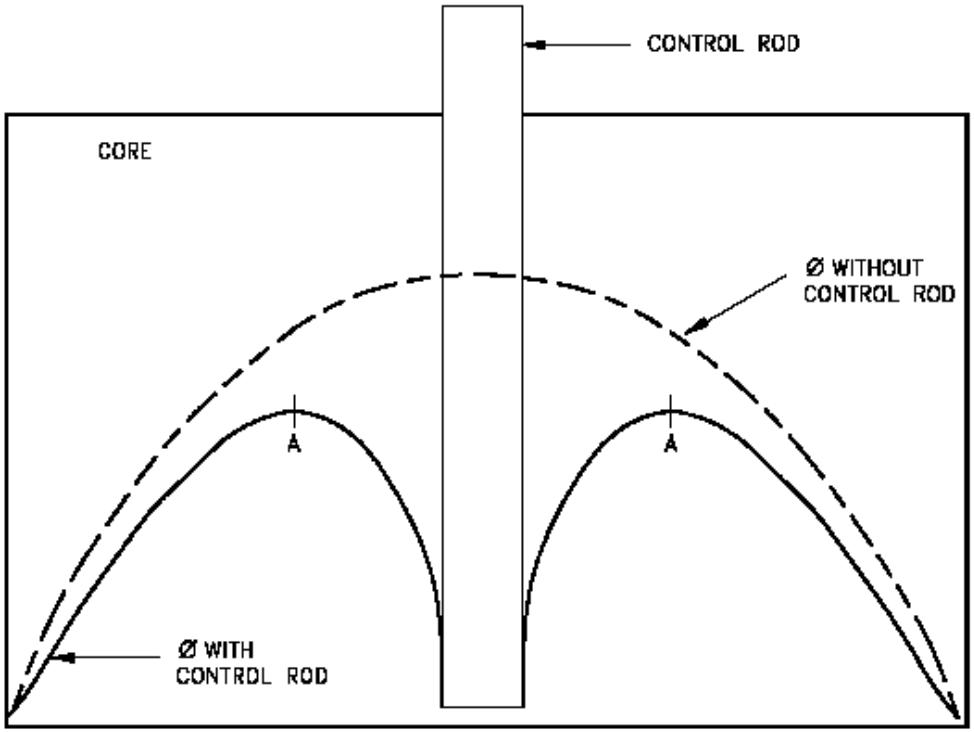
PWR Control Principles



1 - Neutron flux
2 – pressure in steam line,
3 – control rod position,
4 – steam pressure in pressurizer,
5 – valve position,
6 – heating power,
7 – water level in pressurizer,
8 – hot leg temperature,
9 – cold leg temperature,
10 – valve position,
11 – base signal,
12 – correction signal,
13 – feedwater inlet mass flux,
14 – water level in steam generator,
15 – steam mass flow rate,
16 – base signal,
17 – valve position.

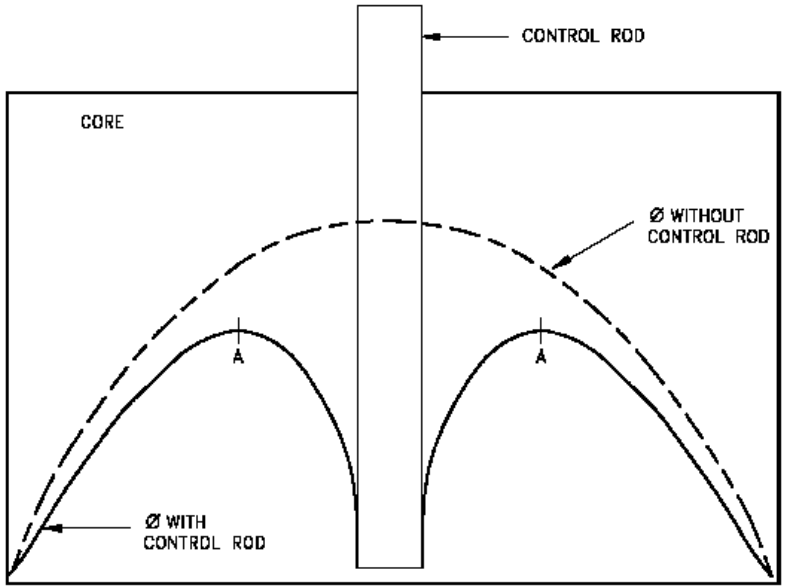
Control Rod Effectiveness (1)

- The effectiveness of a control rod depends largely upon the value of the ratio of the neutron flux at the **location of the rod** to the average neutron flux in the reactor
- The control rod has **maximum effect** (inserts the most negative reactivity) if it is placed in the reactor **where the flux is maximum**
- If a reactor has only **one control rod**, the rod should be placed in the center of the reactor core



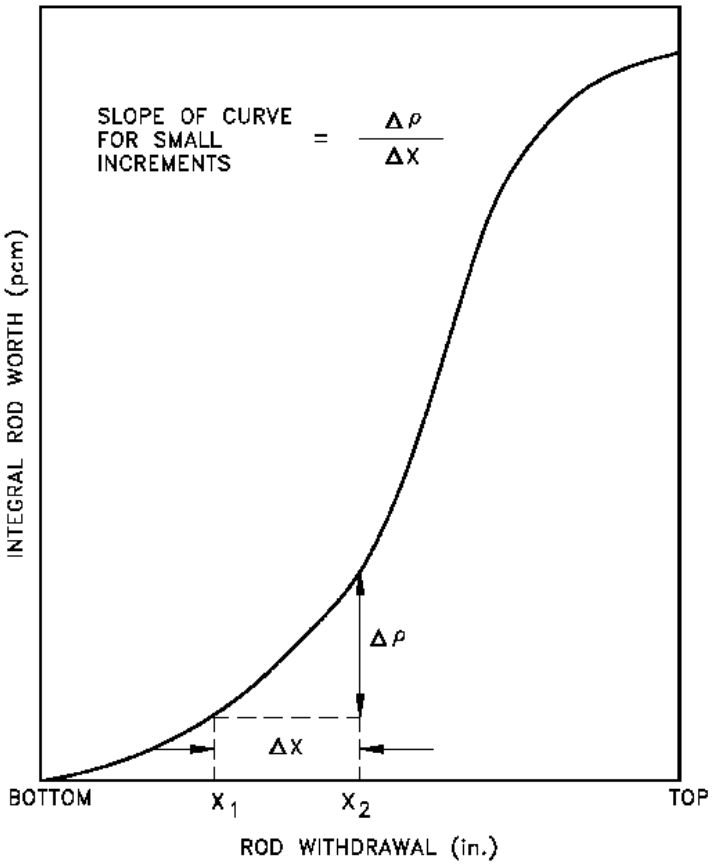
Control Rod Effectiveness (2)

- If additional rods are added to this simple reactor, the most effective location is where the flux is maximum, that is, at **points A**
- Numerous control rods are required for a reactor that has a **large amount of excess reactivity** (that amount of reactivity in excess of that needed to be critical)
- The exact **amount of reactivity** that each control rod inserts depends upon the reactor design
- The change in reactivity caused by control rod motion is referred to as **control rod worth**



Control Rod Worth (1)

- The exact effect of control rods on reactivity can be determined experimentally
- For example, a control rod can be withdrawn in small increments, such as 1 cm, and the change in reactivity can be determined following each increment of withdrawal
- By plotting the resulting reactivity versus the rod position, a graph similar to Figure to the right is obtained
- The graph depicts integral control rod worth over the full range of withdrawal
- The **integral control rod worth** is the total reactivity worth of the rod at that particular degree of withdrawal and is usually defined to be the greatest when the rod is fully withdrawn



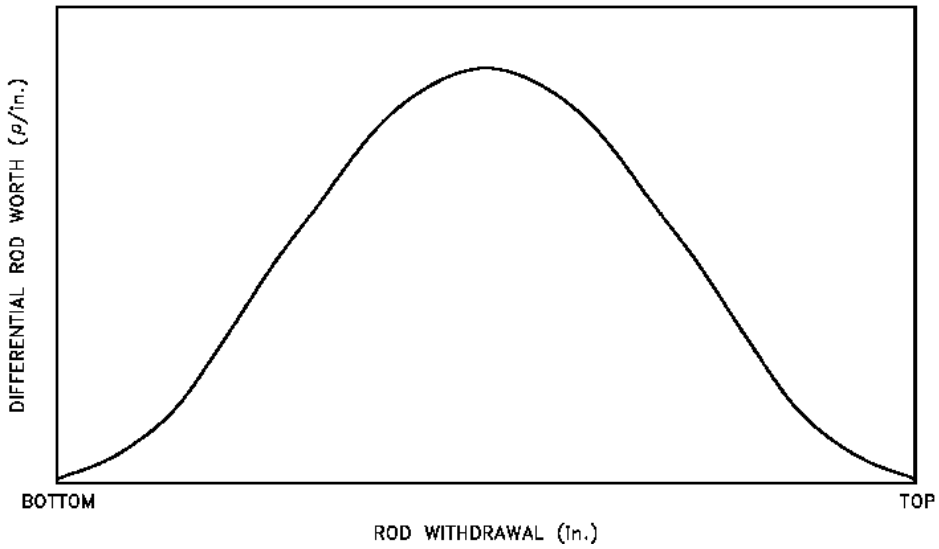
Control Rod Worth (2)

- The slope of the curve ($\Delta\rho/\Delta x$), and therefore the amount of reactivity inserted per unit of withdrawal, is **greatest** when the control rod is **midway** out of the core

- This occurs because the area of **greatest neutron flux** is near the **center of the core**; therefore, the amount of change in neutron absorption is greatest in this area

Control Rod Worth (3)

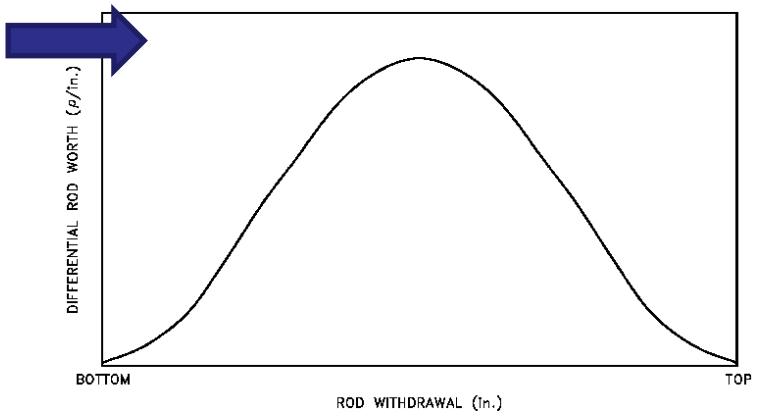
- At the bottom of the core, where there are few neutrons, rod movement has little effect so the change in rod worth per unit length varies little
- As the rod approaches the center of the core its effect becomes greater, and the change in rod worth per unit length is greater
- At the center of the core the differential rod worth is greatest and varies little with rod motion
- From the center of the core to the top, the rod worth per unit length is basically the inverse of the rod worth per unit length from the center to the bottom

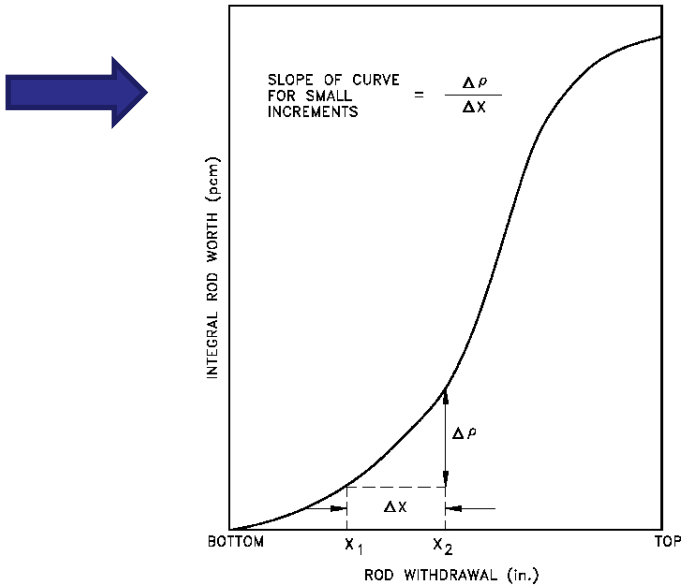


Differential control rod worth

Control Rod Worth (4)

- Differential control rod worth is the **reactivity change per unit movement of a rod**
- The **integral rod worth** at a given withdrawal is merely the summation of all the differential rod worths up to that point of withdrawal
- It is also the **area under the differential rod worth** curve at any given withdrawal position



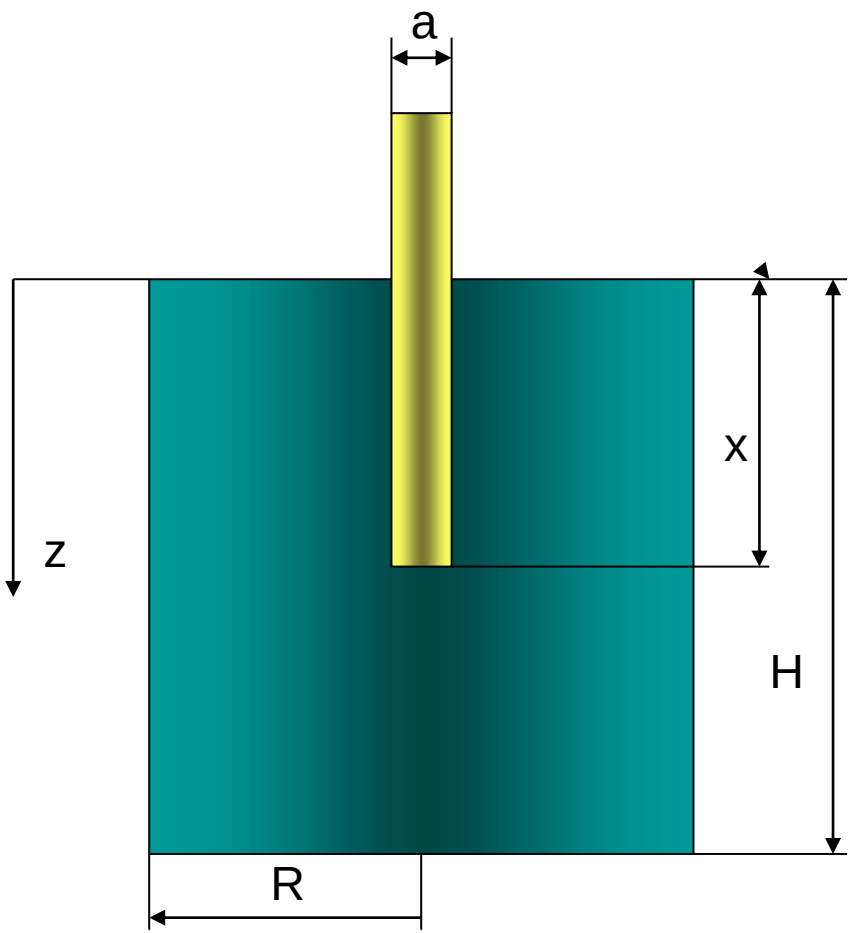


Control Rod Worth (5)

- Consider a reactor with a central partially inserted control rod
- Applying a one-group diffusion approximation and using the perturbation theory it can be shown that the reactivity change *Δρ(x)* depends on the insertion length as follows:

$$\Delta\rho(x) = \Delta\rho(H)\left(\frac{x}{H} - \frac{1}{2\pi}\sin\frac{2\pi x}{H}\right)$$

Where *Δρ(H)* is the reactivity change due to full insertion of the control rod

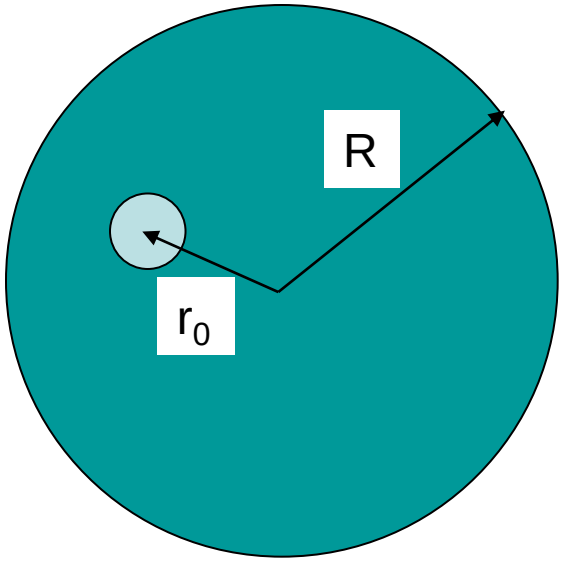


Control Rod Worth (6)

- When the control rod is placed at $r = r_0$ distance from the centerline of the reactor, the reactivity change for such a rod can be estimated as

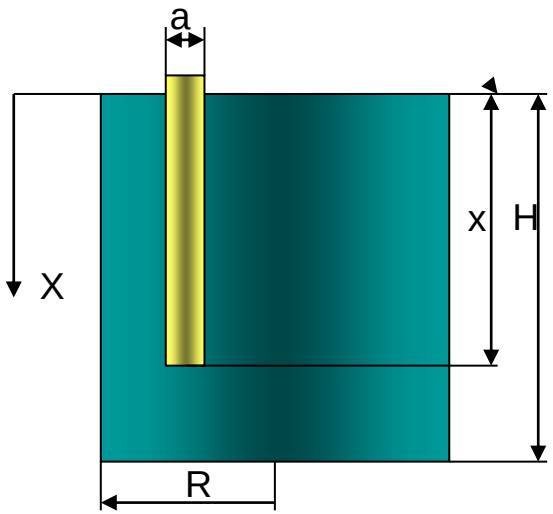
$$\Delta\rho(r_0) = J_0^2\left(\frac{2.4048 r_0}{R}\right)\Delta\rho(0)$$

- Where $\Delta\rho(r_0)$ is the reactivity change for off-centerline rod and $\Delta\rho(0)$ is the reactivity change for a rod inserted at the centerline



Exercise

- A control rod is placed at r_0 = 0.75 m distance from the centerline of a cylindrical reactor with radius R=1.85 m and height H=3.66 m. Calculate the reactivity change caused by a withdrawal of the rod by 2.5 cm, if the rod was initially inserted into the core with 2/3 of its height. The integral worth of the central rod in fully-inserted position is known and equal to 2% (2000 pcm)



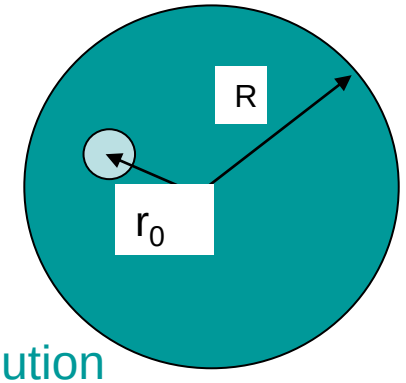
x = 2/3H, dx = -2.5 cm

r_0 = 0.75 m

R = 1.85 m, H = 3.66 m

$$\Delta\rho(x) = \Delta\rho(H)\left(\frac{x}{H} - \frac{1}{2\pi}\sin\frac{2\pi x}{H}\right)$$

$$\Delta\rho(r_0, x) = J_0^2\left(\frac{2.4048 r_0}{R}\right)\Delta\rho(x)$$



Solution

Control Rods

Selection of material

- Rods of neutron-absorbing material are installed in most reactors to provide precise, adjustable control of reactivity
- These rods are able to be moved into or out of the reactor core and typically contain elements such as silver, indium, cadmium, boron, or hafnium
- The material used for the control rods varies depending on reactor design
- Generally, the material selected should have a good absorption cross section for neutrons and have a long lifetime as an absorber (not burn out rapidly)

Control Rods

Selection of material

- The ability of a control rod to absorb neutrons can be adjusted during manufacture
- A control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them
- While it takes **more grey** rods than black rods for a given reactivity effect, the grey rods are often preferred because they cause **smaller depressions** in the neutron flux and power in the vicinity of the rod
- This leads to a **flatter neutron flux** profile and more even power distribution in the core

Control Rods

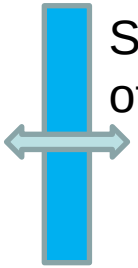
Selection of material

- If grey rods are desired, the amount of material with a high absorption cross section that is loaded in the rod is limited
- Material with a very **high absorption** cross section may not be desired for use in a control rod, because it will **burn out rapidly** due to its high absorption cross section
- The same amount of reactivity worth can be achieved by manufacturing the control rod from material with a slightly lower cross section and by loading more of the material
- This also results in a rod that does not burn out as rapidly

Control Rods

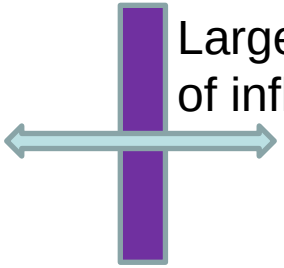
Selection of material

- Another factor in control rod material selection is that materials that **resonantly** absorb neutrons are often preferred to those that merely have high thermal neutron absorption cross sections
- Resonance neutron absorbers absorb neutrons in the **epithermal** energy range
- The path length traveled by the epithermal neutrons in a reactor is greater than the path length traveled by thermal neutrons
- Therefore, a resonance absorber absorbs neutrons that have their last collision farther (on the average) from the control rod than a thermal absorber
- This has the effect of making the area of influence around a resonance absorber larger than around a thermal absorber and is useful in maintaining a flatter flux profile



Small range of influence

Thermal absorber



Large range of influence

Epithermal absorber

Types of Control Rods (1)

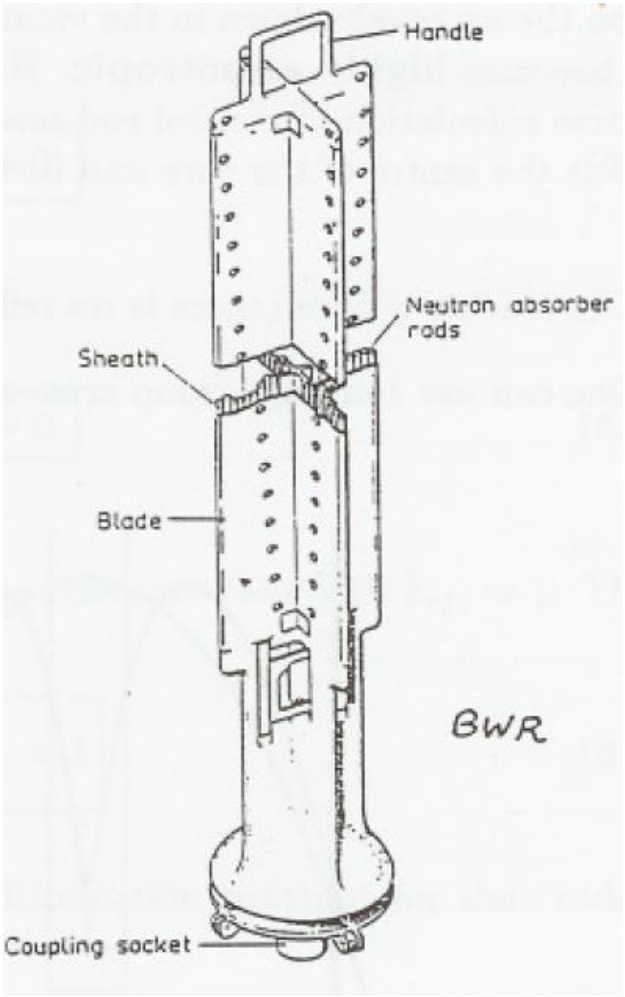
- There are several ways to classify the types of control rods
- One classification method is by the purpose of the control rods
- Three purposes of control rods are as follows:
  - **Shim rods** - used for coarse control and/or to remove reactivity in relatively large amounts
  - **Regulating rods** - used for fine adjustments and to maintain desired power or temperature
  - **Safety rods** - provide a means for very fast shutdown in the event of an unsafe condition. Addition of a large amount of negative reactivity by rapidly inserting the safety rods is referred to as a "scram" or "trip"

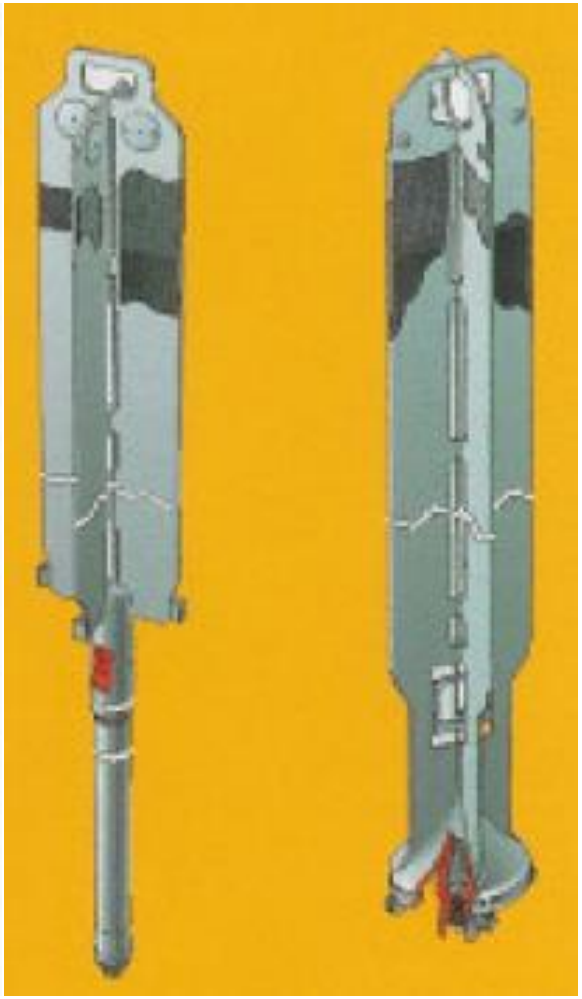
Types of Control Rods (2)

- Not all reactors have different control rods to serve the purposes mentioned above
- Depending upon the type of reactor and the controls necessary, it is possible to use **dual-purpose** or even **triple-purpose** rods
- For example, consider a set of control rods that can insert enough reactivity to be used as **shim rods**
- If the same rods can be operated at slow speeds, they will function as **regulating rods**
- Additionally, these same rods can be designed for rapid insertion, or **scram**
- These rods serve a **triple function** yet meet other specifications such as precise control, range of control, and efficiency

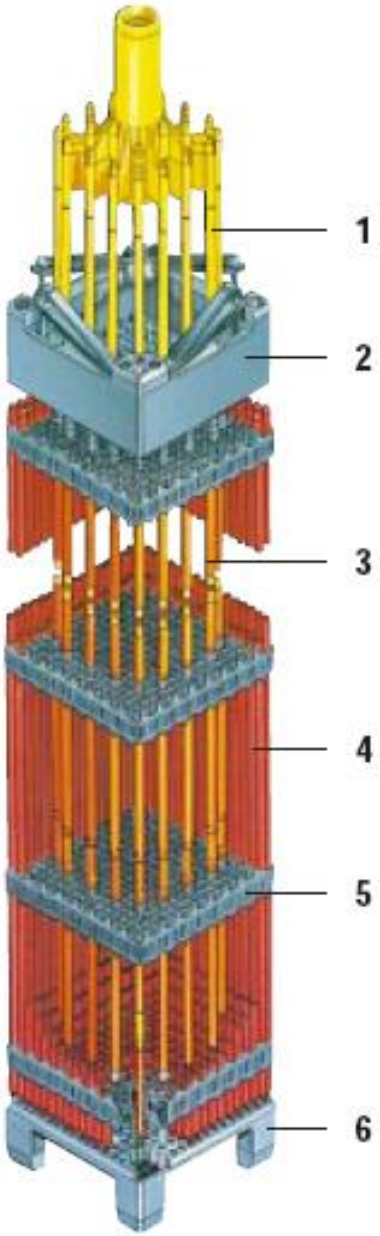
Types of Control Rods (3)

- Examples of BWR control rods





Types of Control Rods (5)



- Example of PWR control rods
  - 1. Control rod, 2. Top plate, 3. Control rod thimble tube, 4. Fuel rods, 5. Spacer grid, 6. Bottom tie plate

Rod Control Mechanisms (1)

- The control rod **insertion rates** on a scram are designed to be sufficient to **protect the reactor** against damage in all transients that are expected to occur during the life of the reactor

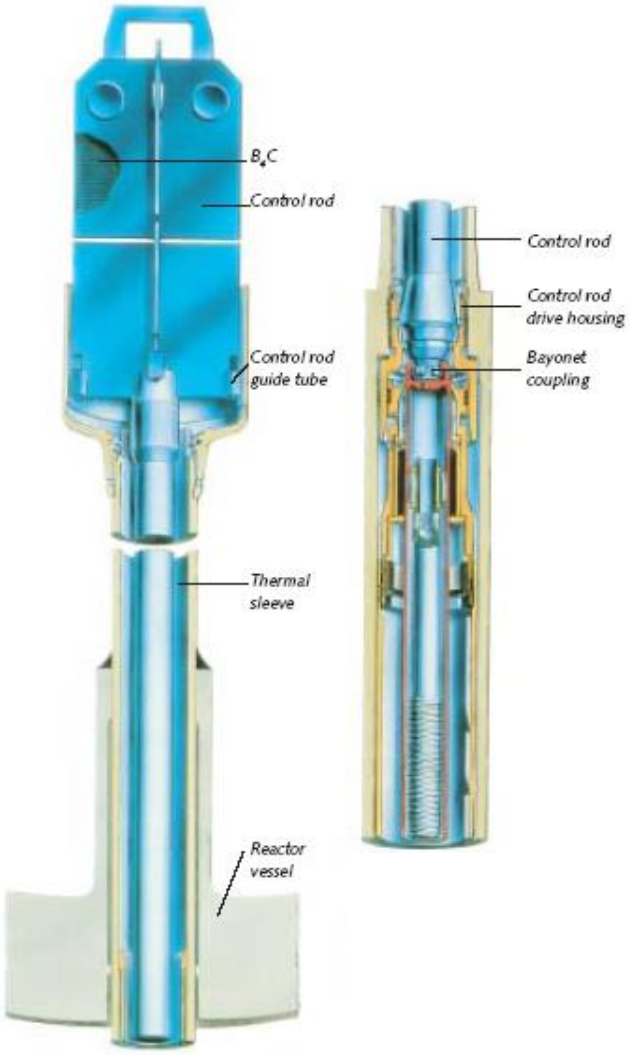
- During normal rod motion, the control rods must be able to move rapidly enough to compensate for the **most rapid rate** at which positive reactivity is expected to build within the reactor in order to provide positive control

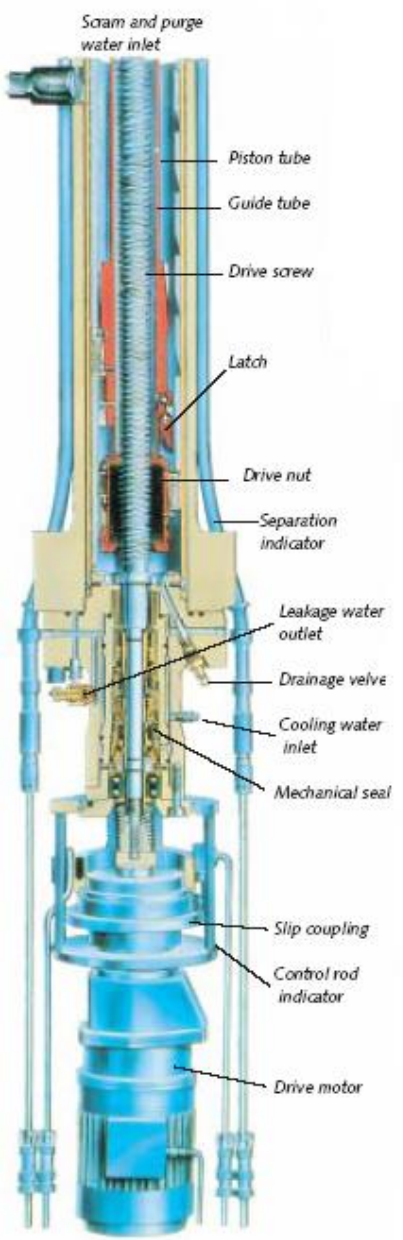
Rod Control Mechanisms (2)

- The transient that is normally considered when setting this minimum rod speed is the burnout of maximum peak xenon while at full power
- Xenon burnout is usually the **most rapid**, non-accident transient expected
- The maximum rod speed is normally limited in order to reduce the **severity of an accident** involving the continuous withdrawal of control rods

Rod Control Mechanisms (3)

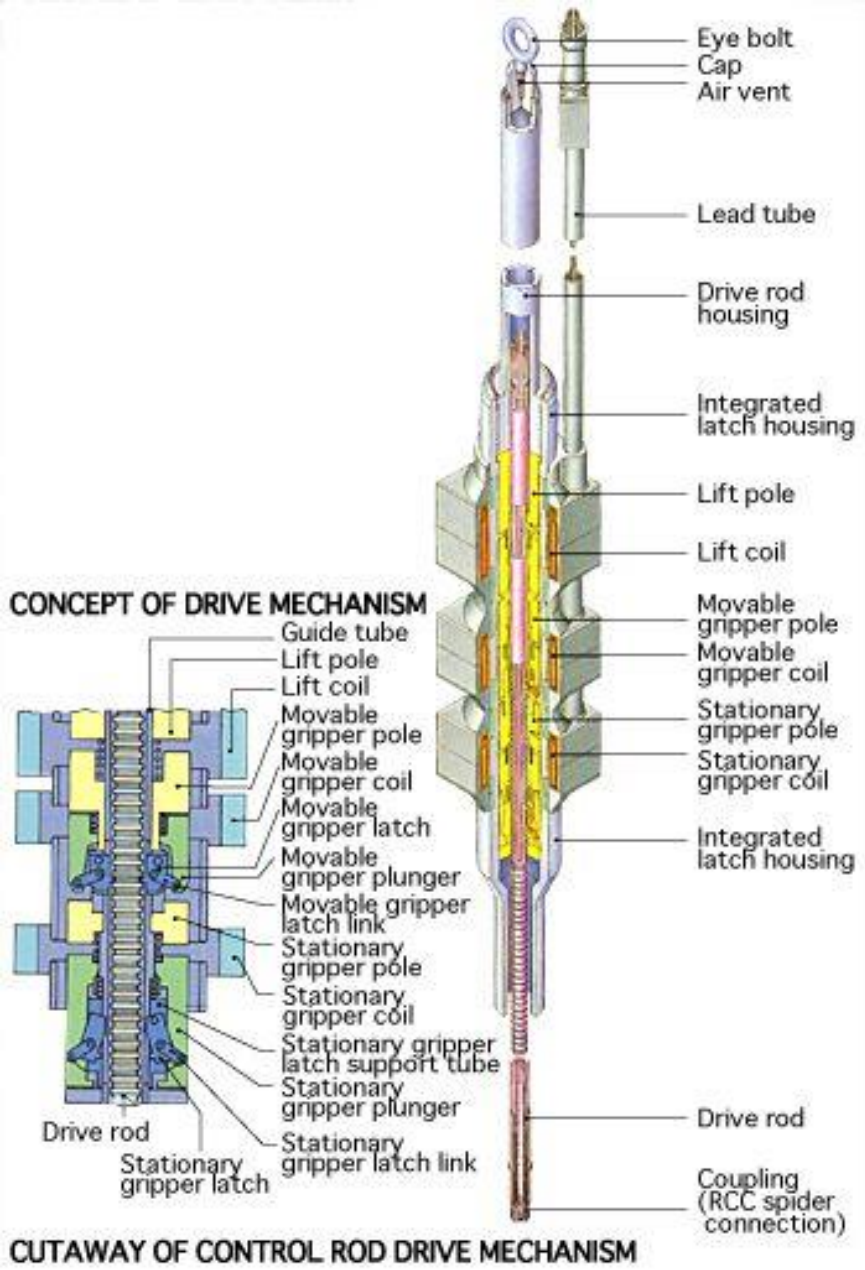
- Example of BWR control rod drive system
  - System allows two independent ways of insertion of control rods:
    - Normal operation with fine motion
    - Hydraulic system for fast scram





Rod Control Mechanisms (4)

- Example of PWR control rod drive system
  - operated with an electromechanical device
  - in an emergency shutdown control rods are inserted by gravity when power is switched off



Fuel Depletion (1)

- During reactor operation ^{235}U in the core undergoes fission and **its amount decreases** while wastes are produced
- This can be compared to burning conventional fuel, e.g. coal – and sometimes one says about burning nuclear fuel in the reactor and term **burnup** is used as a measure of fuel utilization
- Comparison:
  - Energy produced by generating 1 CO_2 molecule is 5 eV
  - Energy produced by fission of one nucleus of U^{235} is 2.10^8 eV
  - Much smaller amount of waste is produced in a nuclear reactor, but the waste created is highly radioactive

Fuel Depletion (2)

- As a reactor is operated, atoms of fuel are constantly consumed, resulting in the **slow depletion** of the fuel frequently referred to as **core burnup**

- There are several major effects of this fuel depletion, most important being change in reactivity

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

- The first, and most obvious, effect of the fuel burnup is that the control rods must be withdrawn or chemical shim concentration reduced to compensate for the negative reactivity effect of this burnup

Fixed Burnable Poisons (1)

- Moveable control rods containing neutron-absorbing materials are one method used to offset the excess fuel
- However, using control rods alone may be impractical
  - E.g. there is physically insufficient room for the control rods and their large mechanisms
- To control large amounts of excess fuel burnable poisons are used
- **Burnable poisons** are materials that have a high neutron absorption cross section that are converted into materials of relatively low absorption cross section as a result of neutron absorption

Fixed Burnable Poisons (2)

- Due to the burnup of the poison material, the negative **reactivity of the poison decreases** over core life
- Ideally, these poisons should decrease their negative reactivity at the **same rate** the fuel's excess positive reactivity is depleted
- Fixed burnable poisons are usually used in the form of compounds of **boron or gadolinium** that are shaped into separate lattice pins or plates, or introduced as additives to the fuel

Soluble Poisons (1)

- **Soluble poisons**, also called **chemical shim**, produce a spatially uniform neutron absorption when dissolved in the water coolant

- The most common soluble poison in PWRs is boric acid (”soluble boron” or ”solbor”)

- The boric acid in the coolant decreases the thermal utilization factor, causing the decrease in reactivity

Soluble Poisons (2)

- By varying the concentration of boric acid in the coolant (a process referred to as boration and dilution) the reactivity of the core can be easily varied

- If the boron concentration is increased (boration), the coolant/ moderator absorbs more neutrons, adding negative reactivity

- If the boron concentration is reduced (dilution), positive reactivity is added

Non-Burnable Poisons (1)

- Non-burnable poison is one that maintains a constant negative reactivity worth over the life of the core
- While no neutron poison is strictly non-burnable, certain materials can be treated as non-burnable poisons under certain conditions – for example hafnium
- The removal – by absorption of neutrons – of one isotope of hafnium leads to the production of another neutron absorber, and continues through a chain of 5 absorbers – resulting in a long-lived burnable poison

Non-Burnable Poisons (2)

- It is possible to make the reactivity of a poison material that is usually a burnable poison more uniform over core life through the use of self-shielding
- In self-shielding the poison material is thick enough that only the outer layer of the poison is exposed to the neutron flux
- The absorptions that take place in the outer layers reduce the number of neutrons that penetrate to the inner material
- As the outer layers of poison absorb neutrons and are converted to non-poison materials, the inner layers begin absorbing more neutrons, and the negative reactivity of the poison is fairly uniform

Fission Product Poisons (1)

- Fission fragments generated at the time of fission decay to produce a variety of fission products
- Fission products are of concern because:
  - they become parasitic absorbers of neutrons
  - Result in long term source of heat
- Xenon-135 and samarium-149 have the most substantial impact on reactor design and operation
- Both these poisons have impact on the thermal utilization factor and thus k_{eff} and reactivity

Production and Removal of Xenon-135 (1)

- The neutron absorption cross section of xenon-135 is equal to 2.6 x 10^6 barns

- It is produced directly by some fissions, but it is more commonly a product of the tellurium-135 decay chain

$$^{135}_{52}\mathrm{Te} \xrightarrow{\beta^-,\ 19\ s} {}^{135}_{53}\mathrm{I} \xrightarrow{\beta^-,\ 6.57\ hr} {}^{135}_{54}\mathrm{Xe} \xrightarrow{\beta^-,\ 9.10\ hr} {}^{135}_{55}\mathrm{Cs} \xrightarrow{\beta^-,\ 2.3\cdot 10^6\ yr} {}^{135}_{56}\mathrm{Ba}\ (\text{stable})$$

- The half-life of Te-135 is so short that it can be assumed that iodine-135 is produced directly from fission

Production and Removal of Xenon-135 (2)

- Iodine-135 is not a strong neutron absorber, but decays to form the neutron poison xenon-135
- 95% of all the xenon-135 comes from the decay of iodine-135
- Therefore, the half-life of iodine-135 plays an important role in the amount of xenon-135 present

Production and Removal of Xenon-135 (3)

- The rate of change of iodine (*dI/dt*; *I* is the concentration of iodine-135) is equal to the rate of production minus the rate of removal

- The rate of production is just equal to yield from fission = $\gamma_I \Sigma_f \varphi$, here $\gamma_I = 0.061$ is the fission yield

- The rate of removal is equal to the decay rate ($\lambda_I I$; λ_I is the decay constant) plus the burnup rate ($\sigma_{a,I} I \varphi$)

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I - \sigma_{a,I} I \phi$$

Production and Removal of Xenon-135 (4)

- Since the microscopic absorption cross section $\sigma_{a,I}$ is quite small, the equation for the iodine-135 concentration can be written as follows

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I$$

- When the rate of production of iodine equals the rate of removal, **equilibrium** exists – the iodine concentration remains then constant and equal to I_0

$$0 = \gamma_I \Sigma_f \phi - \lambda_I I_0 \Rightarrow I_0 = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

Production and Removal of Xenon-135 (5)

- Since the equilibrium iodine concentration is proportional to the neutron flux, φ, it is also proportional to reactor power level

- The rate of change of the xenon-135 concentration (*dX/dt*) is equal to:
  - (+) Xenon-135 production from fission $\gamma_X \Sigma_f \phi$
  - (+) iodine-135 decay $\lambda_I I$
  - (-) xenon-135 decay $\lambda_X X$
  - (-) xenon-135 burnup $\sigma_{a,X} \phi X$

$$\frac{dX}{dt} = \gamma_X \Sigma_f \phi + \lambda_I I - \lambda_X X - \sigma_{a,X} \phi X$$

Production and Removal of Xenon-135 (6)

- The xenon burnup term $\sigma_{a,X}\phi X$ refers to neutron absorption by xenon-135 by the following reaction

$$^{135}_{54}Xe + ^{1}_{0}n \rightarrow ^{136}_{54}Xe + \gamma$$

- Xenon-136 is not a significant neutron absorber – therefore neutron absorption by xenon-135 constitutes removal of poison from the reactor

Production and Removal of Xenon-135 (7)

- At equilibrium:

$$0 = \gamma_X \Sigma_f \phi + \lambda_I I_0 - \lambda_X X_0 - \sigma_{a,X} \phi X_0 \Rightarrow X_0 = \frac{\gamma_X \Sigma_f \phi + \lambda_I I_0}{\lambda_X + \sigma_{a,X} \phi}$$

- Since

$$I_0 = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

- Then the xenon-135 concentration at equilibrium is:

$$X_0 = \frac{(\gamma_I + \gamma_X) \Sigma_f \phi}{\lambda_X + \sigma_{a,X} \phi}$$

Production and Removal of Xenon-135 (8)

- Compare the equilibrium concentrations of iodine-135 and xenon-135:

$$I_0 = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

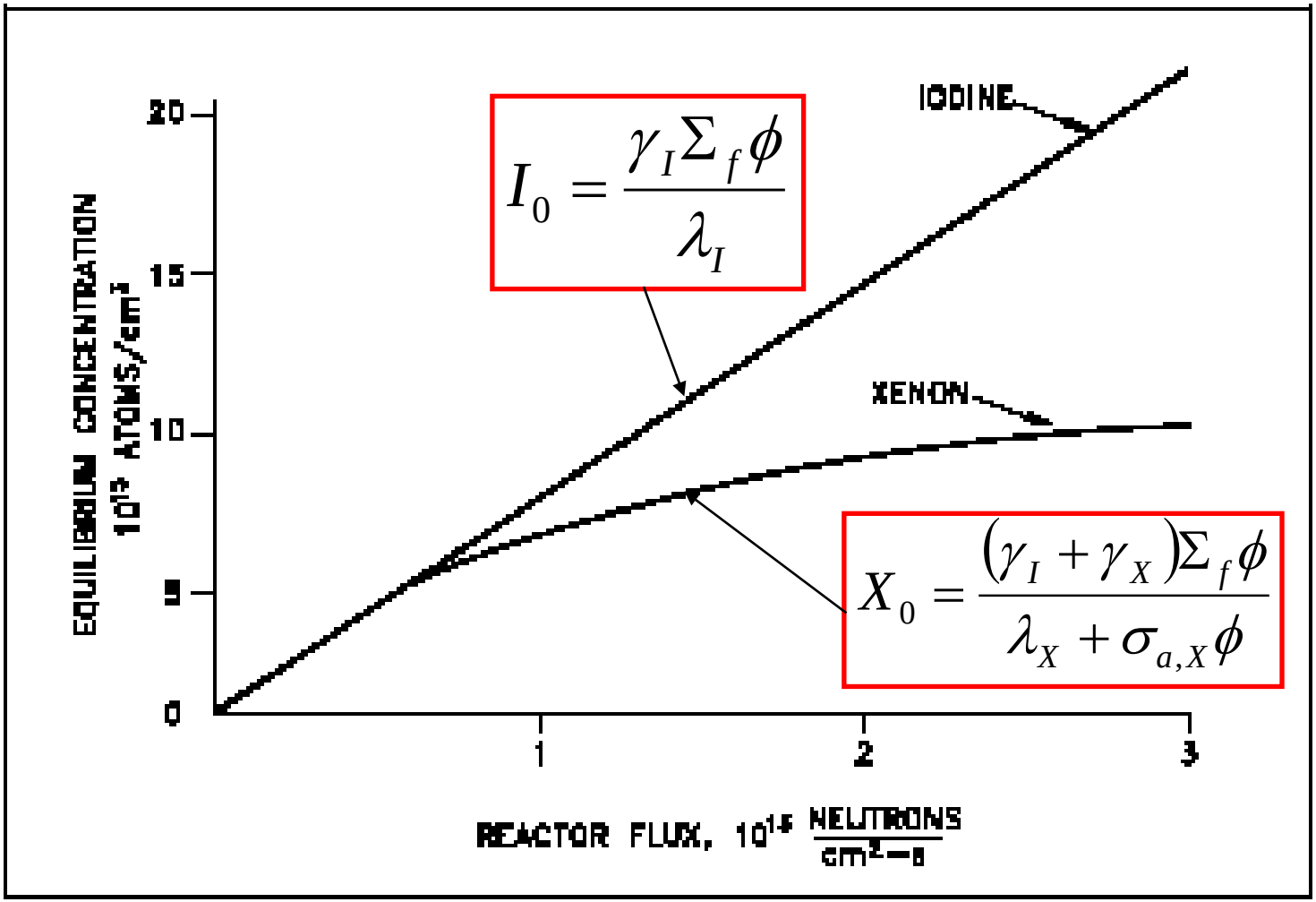
$$X_0 = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi}{\lambda_X + \sigma_{a,X}\phi}$$

- Iodine concentration at equilibrium is linearly proportional to the neutron flux, and thus to the reactor power

- Xenon-135 concentration increases asymptotically to

$$X_0 = \frac{(\gamma_I + \gamma_X)\Sigma_f}{\sigma_{a,X}}$$

Production and Removal of Xenon-135 (9)



Xenon-135 Response to Reactor Power Changes and Shutdown (1)

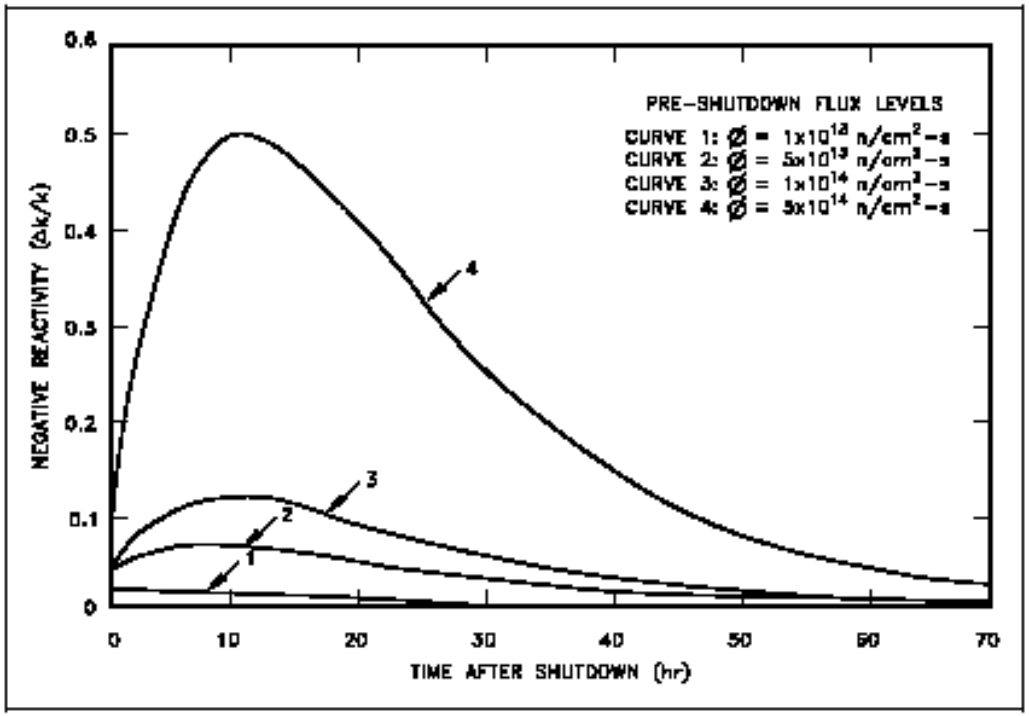
- When a reactor is shutdown, the neutron flux is reduced essentially to zero
- Therefore, after shutdown, xenon-135 is no longer produced by fission and is no longer removed by burnup
- The only remaining production mechanism is the decay of the iodine-135 which was in the core at the time of shutdown
- The only removal mechanism for xenon-135 is decay

Xenon-135 Response to Reactor Power Changes and Shutdown (2)

- Because the decay rate of iodine-135 is faster than the decay rate of xenon-135, the xenon concentration builds to a peak
- The peak is reached when the product of the terms $\lambda_I N_I$ is equal to $\lambda_X N_X$ (in about 10 to 11 hours for large power reactor)
- Subsequently, the production from iodine decay is less than the removal of xenon by decay, and the concentration of xenon-135 decreases

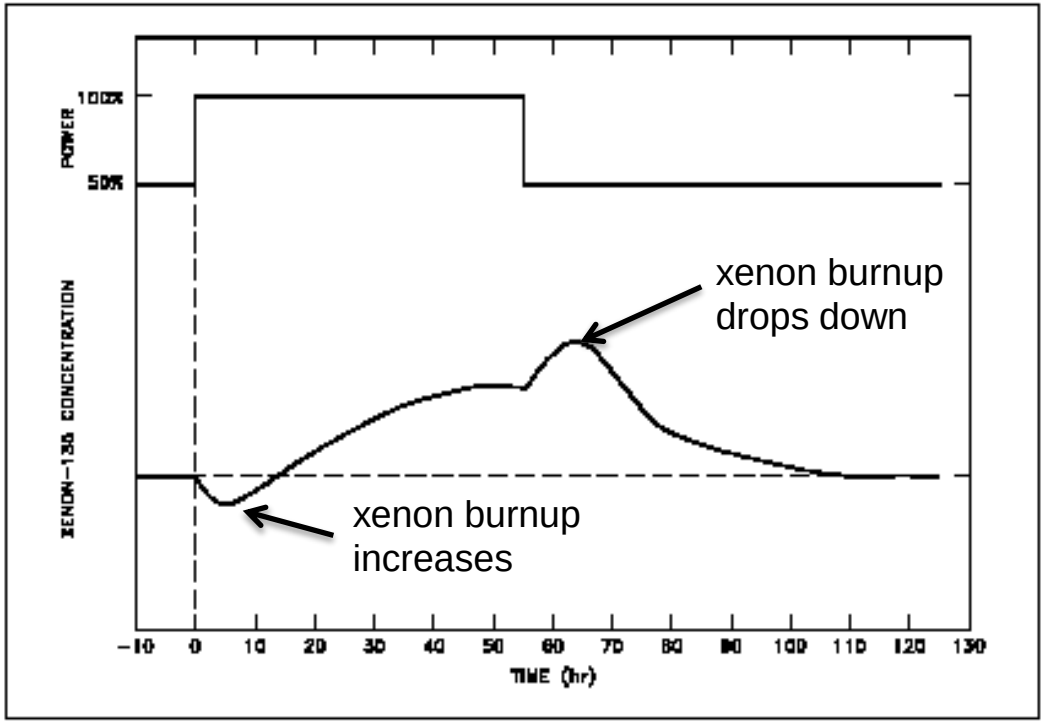
Xenon-135 Response to Reactor Power Changes and Shutdown (3)

- The greater the flux level prior to shutdown, the greater the concentration of iodine-135 at shutdown; therefore, the greater the peak in xenon-135 concentration after shutdown



Xenon-135 Response to Reactor Power Changes and Shutdown (4)

- During periods of **steady state operation**, at a constant neutron flux level, the xenon-135 concentration builds up to its **equilibrium value** within about 40 to 50 hours
- If power level is **increased**, xenon concentration initially **decreases** and then increases to new level
- With power decreases, xenon concentration first increases then decreases



Exercise 2

- EXERCISE: A thermal nuclear reactor was loaded with fresh fuel and started. Assume that the thermal neutron flux step changed from 0 to $2 \cdot 10^{18}$ [m^{-2} s^{-1}] at t = 0.

  Plot the concentration of iodine and xenon as a function of time and calculate the equilibrium concentrations of the two isotopes in the reactor. Given:

$$\lambda_I = 2.9 \times 10^{-5} \left[s^{-1}\right], \lambda_X = 2.1 \times 10^{-5} \left[s^{-1}\right], \gamma_I = 0.061, \gamma_X = 0.003,$$
$$\Sigma_f = 1.8 \cdot 10^{-1} \left[m^{-1}\right], \sigma_{a,X} = 2.6 \times 10^{6} \left[b\right]$$

Solution

Exercise 3

- EXERCISE: The reactor from Exercise 2 was operating a long time at steady-state conditions and then suddenly shut down.

  (a) Plot the concentration of iodine and xenon as a function of time after shut down:

  (b) find the maximum concentration of xenon after shutdown and the time after shutdown when it occurred.

Solution

Exercise 4

- EXERCISE: The reactor from Exercise 2 was operating a long time at stationary conditions and suddenly a partial scram with 50% of power was introduced.

  (a) Plot the concentration of iodine and xenon as a function of time after partial scram:

  (b) find the maximum concentration of xenon after partial scram and the time after partial scram when it occurred.

Solution

Production and Removal of Samarium-149 (1)

- Samarium-149 is the second most important fission-product poison because of its high thermal neutron absorption cross section of 4.1 x 10^4 barns

- Samarium-149 is produced from the decay of the neodymium-149 fission fragment and production of promethium-149, as shown in the decay chain below

$$^{149}_{60}\mathrm{Nd} \xrightarrow[1.72\ hr]{\beta^-} {}^{149}_{61}\mathrm{Pm} \xrightarrow[53.1\ hr]{\beta^-} {}^{149}_{62}\mathrm{Sm}\ (\text{stable})$$

Production and Removal of Samarium-149 (2)

- For the purpose of examining the behavior of samarium-149, the 1.73 hour half-life of neodymium-149 is sufficiently shorter than the 53.1 hour value for promethium-149 that the promethium-149 may be considered as if it was formed directly from fission
- This assumption, and neglecting the small amount of promethium burnup, allows the situation to be described as follows
- Rate of change of ^{149}Pm = yield from fission - decay ^{149}Pm concentration
- therefore:

$$\frac{dP}{dt} = \gamma_P \Sigma_f \phi - \lambda_P P$$

Production and Removal of Samarium-149 (3)

- At equilibrium:

$$0 = \gamma_P \Sigma_f \phi - \lambda_P P_0 \Rightarrow P_0 = \frac{\gamma_P \Sigma_f \phi}{\lambda_P}$$

- As can be seen, the equilibrium concentration of promethium-149 is linearly increasing with the neutron flux and thus with power

Production and Removal of Samarium-149 (4)

- The rate of samarium-149 formation is described as follows:

$$\frac{dS}{dt} = \gamma_S \Sigma_f \phi + \lambda_P P - \sigma_{a,S} \phi S$$

- Since the fission yield of samarium-149 is nearly zero, therefore the equation becomes:

$$\frac{dS}{dt} = \lambda_P P - \sigma_{a,S} \phi S$$

- And at equilibrium:

$$0 = \lambda_P P_0 - \sigma_{a,S} \phi S_0 \Rightarrow S_0 = \frac{\lambda_P P_0}{\sigma_{a,S} \phi} = \frac{\gamma_P \Sigma_f}{\sigma_{a,S}}$$

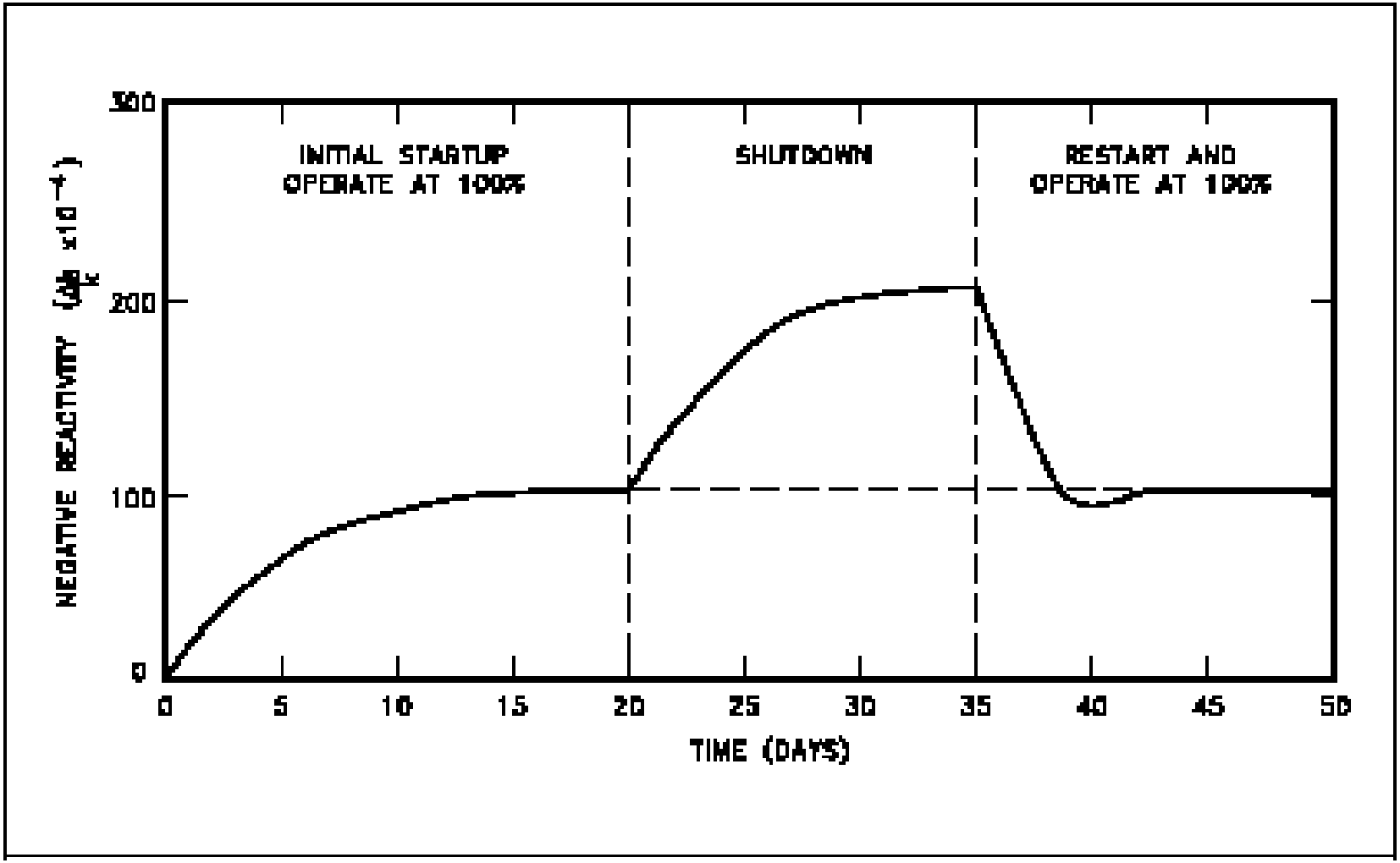
Samarium-149 Response to Reactor Shutdown (1)

- Since the neutron flux drops to essentially zero after reactor shutdown, the rate of samarium-149 production becomes the following:

$$\frac{dS}{dt} = \lambda_P P$$

- Because samarium-149 is not radioactive and is not removed by decay, it presents problems somewhat different from those encountered with xenon-135, as illustrated in Figure on the next slide

Samarium-149 Response to Reactor Shutdown (2)



Samarium-149 Response to Reactor Shutdown (3)

- The equilibrium concentration and the poisoning effect build to an equilibrium value during reactor operation
- This equilibrium is reached in approximately 20 days (~500 hours), and since samarium-149 is stable, the concentration remains essentially constant during reactor operation
- When the reactor is shutdown, the samarium-149 concentration builds up as a result of the decay of the accumulated promethium-149

Samarium-149 Response to Reactor Shutdown (4)

- Samarium-149 does not peak as xenon-135 does, but increases slowly to a maximum value after reactor shutdown
- After shutdown, if the reactor is then operated at power, samarium-149 is burned up and its concentration returns to the equilibrium value
- Samarium poisoning is minor when compared to xenon poisoning
- Although samarium-149 has a constant poisoning effect during long-term sustained operation, its behavior during initial startup and during post-shutdown and restart periods requires special considerations in reactor design

Samarium-149 Response to Reactor Shutdown (5)

- After reactor shutdown, the concentrations of promethium-149 and samarium-149 are described with the following differential equations:

$$\frac{dP}{dt} = -\lambda_P P \qquad \frac{dS}{dt} = \lambda_P P$$

- with initial conditions:

$$P_0 = \frac{\gamma_P \Sigma_f \phi}{\lambda_P} \qquad S_0 = \frac{\gamma_P \Sigma_f}{\sigma_{a,S}}$$

- And the samarium concentratuin as a function of time is obtained as

$$S(t) = S_0 + P_0\left(1 - e^{-\lambda_P t}\right) \xrightarrow[t\to\infty]{} S_0 + P_0$$

Fission Product Poisoning (1)

- To good approximation, the only effect of fission product poisons on the multiplication factor is through the thermal utilization factor
- Thus the reactivity equivalent of poisons is as follows

$$\Delta\rho = \frac{k-1}{k} - \frac{k_0 - 1}{k_0}$$

k_0, k – effective multiplication factor for unpoisoned and poisoned core, respectively

- Assuming critical unpoisoned core (k_0=1), we get

$$\Delta\rho = \frac{k-1}{k} = 1 - \frac{1}{k}$$

Fission Product Poisoning (2)

- Since the poison changes only the thermal utilization factor *f*, we have

$$\frac{k}{k_0} = \frac{f}{f_0} \quad \Rightarrow \quad k = \frac{f}{f_0} \quad \Rightarrow \quad \Delta\rho = 1 - \frac{1}{k} = 1 - \frac{f_0}{f}$$

- since $$f_0 = \frac{\Sigma_{a,F}}{\Sigma_{a,F} + \Sigma_{a,M}} \qquad f = \frac{\Sigma_{a,F}}{\Sigma_{a,F} + \Sigma_{a,M} + \Sigma_{a,P}}$$

- We have

$$\Delta\rho = 1 - \frac{\Sigma_{a,F}\left(\Sigma_{a,F} + \Sigma_{a,M} + \Sigma_{a,P}\right)}{\Sigma_{a,F}\left(\Sigma_{a,F} + \Sigma_{a,M}\right)} = -\frac{\Sigma_{a,P}}{\Sigma_{a,F} + \Sigma_{a,M}}$$

Fission Product Poisoning (3)

- The fission product poisoning will change with the concentration of the poison
- For example, after reactor shutdown, the maximum poisoning will coincide with the maximum concentration of Xe-135
- The maximum reactivity defect can be then found as:

$$\Delta\rho_{\max} \cong -\frac{X_{\max}\sigma_{a,P}}{\Sigma_{a,F}+\Sigma_{a,M}}$$

After reactor shutdown the poisoning can be so high that the reactor restart is not possible

Exercise 5

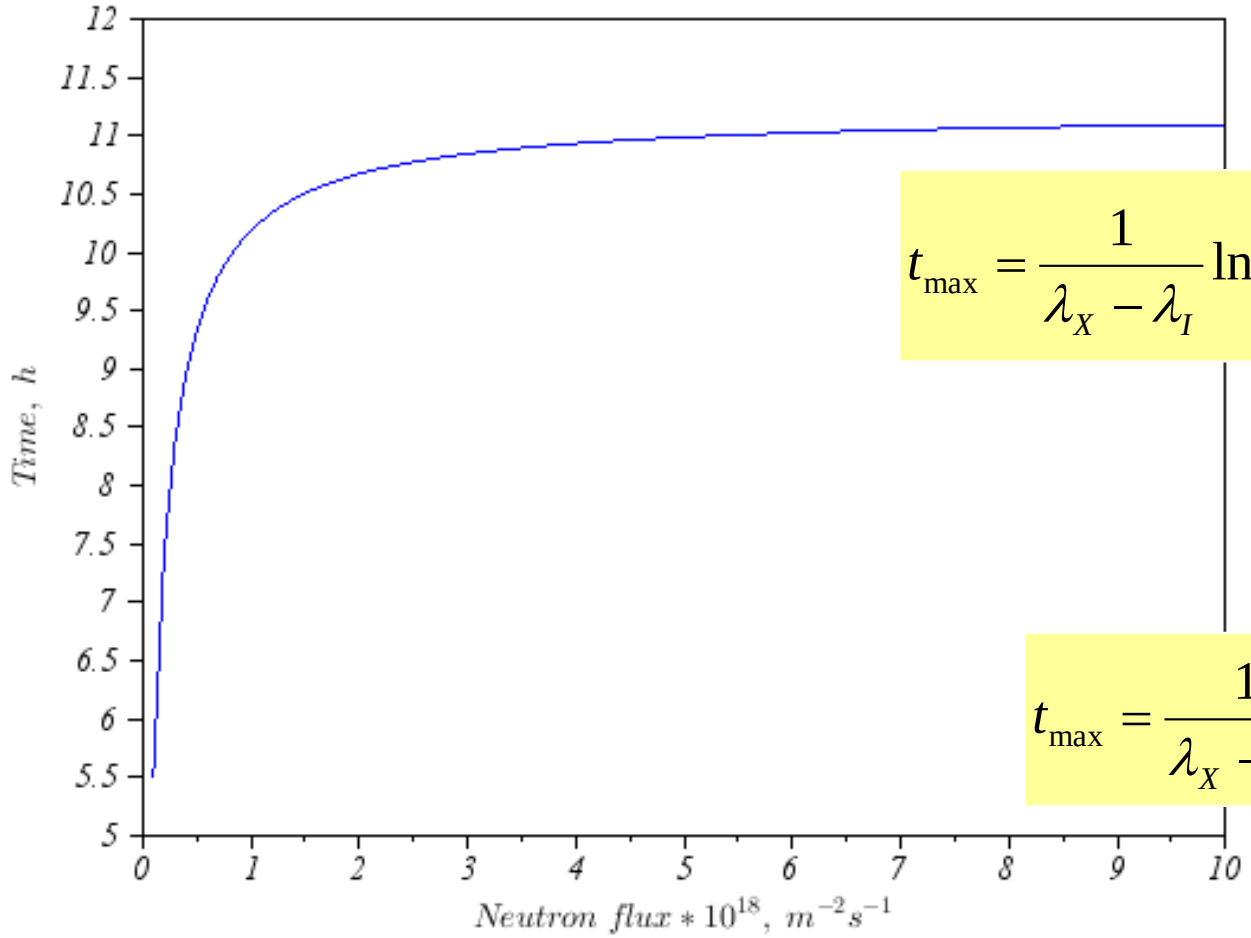
- Derive an expression for the maximum xenon concentration after reactor shutdown and the time when it occurs. Plot the maximum time against the neutron flux.

HINT: find derivative dX(t)/dt and t_{max} such that the derivative at $t=t_{max}$ is equal to zero.

ANS:

$$t_{\max} = \frac{1}{\lambda_X - \lambda_I} \ln\left\{ \frac{\lambda_X}{\lambda_I} \left[1 + \frac{X_0}{I_0} \left(1 - \frac{\lambda_X}{\lambda_I} \right) \right] \right\}$$

Exercise 5



$$t_{\max} = \frac{1}{\lambda_X - \lambda_I} \ln\left\{\frac{\lambda_X}{\lambda_I}\left[1 + \frac{X_0}{I_0}\left(1 - \frac{\lambda_X}{\lambda_I}\right)\right]\right\}$$

For increasing neutron flux (reactor power), the maximum time gets an asymptotic value:

$$t_{\max} = \frac{1}{\lambda_X - \lambda_I} \ln\left(\frac{\lambda_X}{\lambda_I}\right) \approx 11\,\text{h}$$

Exercise 6

- A thermal reactor using a fuel containing 2.5% U-235 has been operating for long time at an average neutron flux of 2 * 10^{18} neutrons/m^2s. What is the maximum poisoning after reactor shutdown? Assume $\Sigma_f / \Sigma_a = 0.6$

HINT: Find X(t_{max}), where then:

$$\Delta\rho_{\max} \cong -\frac{X(t_{\max})\sigma_{a,X}}{\Sigma_a}$$

Given:

$$\lambda_I = 2.9\times10^{-5}\left[s^{-1}\right],\ \lambda_X = 2.1\times10^{-5}\left[s^{-1}\right],$$

$$\gamma_I = 0.061,\ \gamma_X = 0.003,$$

$$\Sigma_f = 1.8\cdot10^{-1}\,[m^{-1}],\ \sigma_{a,X} = 2.6\times10^{6}\,[b]$$

$$t_{\max} = \frac{1}{\lambda_X - \lambda_I}\ln\left\{\frac{\lambda_X}{\lambda_I}\left[1+\frac{X_0}{I_0}\left(1-\frac{\lambda_X}{\lambda_I}\right)\right]\right\}$$

$$I_0 = \frac{\phi\Sigma_f\gamma_I}{\lambda_I};\ X_0 = \frac{\phi\Sigma_f(\gamma_X+\gamma_I)}{\lambda_X+\phi\sigma_{a,X}}$$

$$X(t) = \frac{\phi\Sigma_f(\gamma_X+\gamma_I)}{\lambda_X+\phi\sigma_{a,X}}e^{-\lambda_X t} + \frac{\phi\Sigma_f\gamma_I}{\lambda_I-\lambda_X}\left(e^{-\lambda_X t}-e^{-\lambda_I t}\right)$$