Q10:

Answers

Seven years later, in 1900, Max Planck derived Planck's Law, which describes the spectral density of electromagnetic radiation from a black body, formulated as:

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

--- (1)

Planck's Law produces a continuous function unique to each black body temperature. Wien's Law determines the wavelength of peak emission, so deriving Wien's Law involves finding the maximum value of Planck's Law as a function of temperature.

The first step is to take the partial derivative of Planck's Law (1) with respect to wavelength, λ.

$$\frac{\partial E}{\partial \lambda} = \frac{2hc^2}{\lambda^6 \left(e^{\frac{hc}{\lambda k_B T}} - 1\right)} \left(\frac{e^{\frac{hc}{\lambda k_B T}}}{e^{\frac{hc}{\lambda k_B T}} - 1} - 5 \right)$$

---(2)

Next, setting (2) equal to zero and simplifying:

$$\frac{hc}{\lambda k_B T} \left(\frac{e^{\frac{hc}{\lambda k_B T}}}{e^{\frac{hc}{\lambda k_B T}} - 1} - 5 \right) = 0$$

---(3)

Defining $x \equiv \frac{hc}{\lambda k_B T}$, equation (3) becomes:

$$\frac{xe^x}{e^x - 1} - 5 = 0$$

---(4)

Rearranging equation (4) gives:

$$e^x(x-5)+5=0$$

---(5)

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LambertW(-5*exp(-5)) + 5
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This is the said compact transcendental equation from which Wien's displacement constant b, can be calculated as given in question

$$\lambda_{max}=\frac{b}{T}$$

Since, $x \equiv \frac{hc}{\lambda k_B T}$, we can get the value of b from numerically calculated value of x, by

$$b=\lambda_{max}T=\frac{hc}{xk_B}$$

NIST database value for the following constants:

$h=6.62607015\times10^{-34}\,Js$

$c=\ 299792458\,ms^{-1}$

$k_B\ =1.380649\times10^{-23}\,JK^{-1}$

Newton-Raphson's method is used to solve (5) and use that value of x to obtain value of b to 10 decimal places:

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The solution is: x = 4.965114231744276
The value of b is: b = 0.0028977720
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