**Answers:**

Q1:

1a: The Lipschitz constant, , in the variable for the function on the rectangle can be found as follows:

Since is always non-negative in the interval , the Lipschitz constant in the variable is:

1b: The Lipschitz constant, , in the variable for the function on the rectangle can be found as follows:

Using the fact that for all , we have:

Since is always non-negative in the interval and is always non-negative in the interval , the Lipschitz constant in the variable is:

Q2:

To determine the interval on which the initial value problem has a unique solution, we need to study the local existence and uniqueness of the solution. The local existence and uniqueness of the solution can be established using the Picard-Lindelöf theorem.

The Picard-Lindelöf theorem states that if is locally Lipschitz continuous in on some rectangle containing , then there exists a unique solution to the initial value problem defined on some interval .

In this case, the function is continuous in for all and . Therefore, the function is locally Lipschitz continuous in on the rectangle .

By the Picard-Lindelöf theorem, there exists a unique solution to the initial value problem defined on some interval for some . The exact value of cannot be determined from the given information.

Q3:

(a) The exact solution to the given initial value problem is

(b) Using forward Euler method with uniform discretization step size , we have

(c) We expect . Assuming exact arithmetic, exists and is equal to . Therefore, we can state that

Q4:

4a: Let . To find the Lipschitz constant in the variable on , we need to find a constant such that  
Taking the absolute value of and using the triangle inequality, we get  
Since on , we have .

4b: The exact solution can be found using separation of variables. We get

4c: To find the constant , let and . The global truncation error can be expressed as  
Since , we can substitute into the above equation. Using the definition of and the fact that , we get  
Substituting into the above expression, we get

4d: The global truncation error at is Since is a constant, the error goes to zero as .

Q5:

5a:

To solve the given initial value problem (IVP), we will first integrate the given differential equation to find an analytical solution. The differential equation is a separable equation, meaning that it can be separated into two functions that do not depend on each other. The separable form of the differential equation is:

Integrating both sides, we have: where is the constant of integration. To find its value, we use the initial condition . Substituting this value into the equation, we have:  
So the solution to the differential equation is:

This is the analytical solution to the given IVP. The explicit Euler method is a numerical method for approximating the solution to an IVP. The idea is to use a series of discrete points to approximate the solution. The method involves using the derivative of the function at the previous time step to estimate the function at the current time step. The general form of the explicit Euler method is:

where is the step size, is the derivative of the function evaluated at the current time step and is the approximation of the function at the current time step. To apply the explicit Euler method to the given IVP, we would choose a step size , and evaluate the derivative at the initial time and initial value . We then use the formula above to estimate the function at subsequent time steps. This process is repeated until the final time is reached.

5b:

Chart

Description automatically generated

Fig 5.1: Plot of function with analytical solution and numerical solution with different discretization steps for explicit Euler method.

5c:

A screenshot of a computer

Description automatically generated with low confidence

Q6:

6a:



6b:



6c:

The order of approximation of the Explicit Trapezoid Method is 2 as  
, so the value for in is 2.

Q7:

7a&b:

Chart, line chart

Description automatically generated

Fig 7.1: Plot of population of prey and predator against time exhibiting their behavior.

7c: The graph shows the populations of the prey and predator over time. The prey population initially increases, then reaches a maximum and starts decreasing. The predator population starts increasing after some time and then reaches a maximum before decreasing. This represents the predator-prey interaction, where the predator's population increases when the prey's population increases, but the prey's population decreases as the predator's population grows, leading to a decline in the predator population as well.

7d: If the time interval is extended, the populations of the prey and predator will continue to oscillate, but the amplitude of the oscillations will gradually decrease. This is because the predator and prey populations are mutually dependent on each other, and their growth and decline will eventually reach a balance.

7e: Yes, there is a stable solution to this population model. The solution is stable when the populations of both predator and prey are constant, i.e. their derivatives are equal to zero. This occurs when and . Solving for and in these equations gives us the stable values of and .

Q8:

Chart, line chart

Description automatically generated

Chart, line chart

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(a) (b)

Fig 8.1: (a) Plot of neutron density against time for different reactivity and Ip combinations (cases) with (b) showing case 2 individually as it is closer to case 3.

Q9:

Chart, line chart

Description automatically generated

Fig 9.1: Plot for one-group model and six-group model in terms of neutron density against time.

The reactor period can be estimated as the time it takes for the neutron density to go from a peak value to half that peak value. In general, the reactor period increases as the number of delayed neutron groups increases. The reason for this is that the more delayed neutron groups, the more time it takes for the reactor to recover from a perturbation, since the delayed neutron groups provide a more sustained source of reactivity. The reactor period predicted by the one-group model is likely to be shorter than the reactor period predicted by the six-group model, as the latter has a more detailed representation of the neutron behaviour and will therefore exhibit slower recovery from perturbations.

From the plot, we can see that the reactor period predicted by the one group model is different from the reactor period predicted by the six group model. The reactor period is defined as the time it takes for the neutron density to reach its maximum value and then decrease back to its initial value. The one group model predicts a shorter reactor period than the six group model. This is because the one group model only considers the effect of prompt neutrons, while the six group model considers the effect of both prompt and delayed neutrons. Delayed neutrons have a longer lifetime than prompt neutrons, which leads to a longer reactor period.

Q10:

10a:

Chart

Description automatically generated

Fig 10.1: Plot for point reactor kinetic equation ignoring delayed neutrons.

10b:

Chart, line chart

Description automatically generated

Fig 10.2: Plot the solution to the point reactor kinetic equations assuming one averaged group of delayed neutrons.