**Q5:**

**Answers**

5a: The derivative of the function can be calculated analytically using the product rule of differentiation:

The derivative is positive in the region where , or equivalently, . In this region, is increasing. The derivative is equal to zero when , or . Finally, is negative in the region where , or . In this region, is decreasing.

5b: The function is increasing when and decreasing when . Hence, the monotonicity regions of can be labeled as and .

Since is a one-to-one function, it has exactly one inverse for each monotonicity region. Therefore, the real argument Lambert function has two branches, one for each monotonicity region. The principal branch is defined as the inverse of over , while the complementary branch is defined as the inverse of over .

5c: The minimum value and corresponding argument of can be found by setting the derivative of to zero and solving for .

The derivative of is given by:

Setting , we have:

Dividing both sides by , we get:

Solving for , we find that .

Substituting into , we find that the minimum value is:

So the minimum value of x(y) is and the corresponding argument is .

Top of Form

5d: Therefore, the function is increasing in the interval , and is decreasing in the interval .

The real argument Lambert function has two branches: the main branch, , which is increasing for and the complementary branch, , which is decreasing for .

5e:

Chart, line chart, histogram

Description automatically generated

Fig 1: Plot of with the characteristic properties shown in the labelling.