**Q9:**

**Answers:**

9a & 9b:

Text

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Table 1 shows that usage of the optimized value for Newton’s method increases efficiency, especially as the value of y increases. However, the optimized value for Halley’s methods shows some fluctuations in the result in terms of error and iteration count. However, in table 2, i.e., in the region, Halley’s method shows fluctuations in terms of error. Still, the number of iterations drops gradually as we decrease the value of y further.

Moreover, Newton’s method showed significant fluctuation regarding the number of iterations needed for certain tolerance to be met. The zero-division error might occur as I have used computer value in consequence of using “if” conditional for choosing “y0” values if specific x is chosen where the computer might have classed the y\_{exact}=-1 less than the -1/e value.

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9c:

Chart

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The intersection point (x = -1/e, y = -1) is where the two branches meet. This can be seen from the fact that for this value of x, both branches have the same y-value of -1. The Lambert function has two branches because it is a multi-valued function, and the two branches meet at the point of intersection where they are equal. The blue branch, W\_0(x), is the principal branch with y >= -1 for x >= -1/e, and the red branch, W\_{-1}(x), is the second branch with y <= -1 for x <= -1/e.