Lecture No 11

Title:
Stability of Boiling Channels

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Outline of the Lecture

- Classification of instabilities
- Principles of perturbation analysis
- Frequency-domain method and the stability criterion
- Boiling channel stability
  - Stability map of a boiling channel

Classification of Instabilities

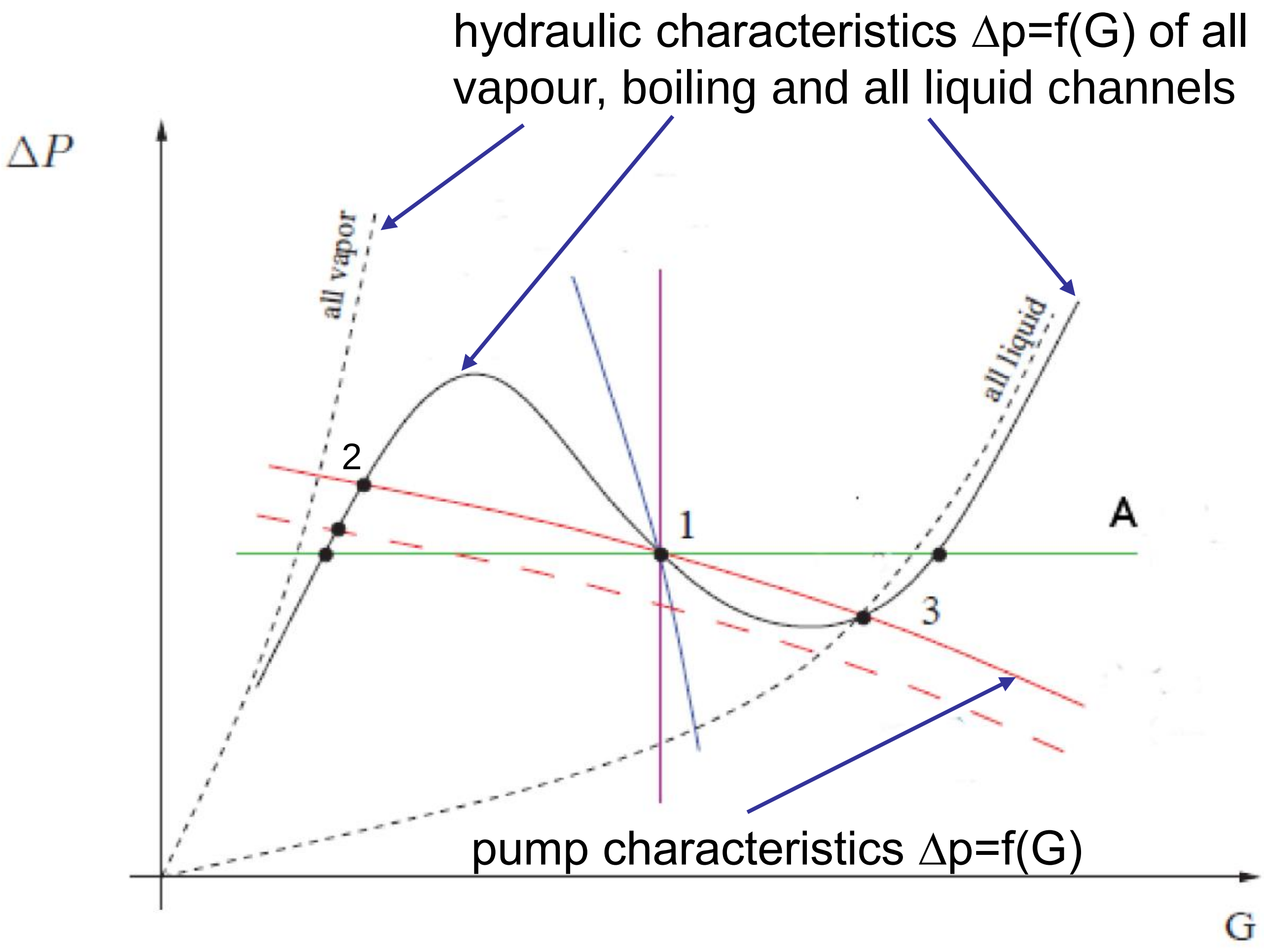
- Static instabilities
  - Excursive (Ledinegg) instability
  - Flow regime relaxation instability
  - Nucleation instabilities

- Dynamic instabilities
  - Density-wave oscillations
  - Pressure drop oscillations
  - Acoustic instabilities
  - Condensation induced instabilities

Static Instabilities - Ledinegg Instability

- ***Excursive (i.e., Ledinegg) instabilities*** are non-periodic flow transients
- Instabilities of this type plagued early low-pressure fossil boilers, since flow excursions could lead to burn-out of the boiler tubes
- Ledinegg instability may occur in heated channels with low system pressure and low inlet loss coefficient, where pressure drop may decrease with increasing flow

Ledinegg Instability (2)



If system operates around point 1, the flow will not be stable.

If pressure drop (flow resistance in channel) Δp slightly decreases, pump will provide higher G until equilibrium is reached at point 3.
Similarly for slight pressure increase at point 1, the flow will decrease until an equilibrium is reached at point 2

Flow Regime Relaxation Instability

- ***Flow regime relaxation instabilities*** are caused by the pressure-drop characteristics of the different flow regimes
- For example, pressure drop in slug flows is less then in bubbly flow with the same flow rates of gas and liquid
  - If a system is operating in the bubbly flow regime near the flow regime boundary, a small negative perturbation in liquid flow rate may cause a transition to slug flow
  - As a result, pressure drop in the channel will decrease
  - If the channel operates at a constant pressure drop condition (as in case of a large number of parallel channels), more liquid will enter the channel to satisfy the boundary conditions
  - This, in turn, may cause the system to return to bubbly flow regime

Nucleation Instabilities

- ***Nucleation instabilities*** include bumping and geysering phenomena

- These instabilities are characterized by a periodic relaxation of the meta-stable condition that builds up due to insufficient nucleation sites

- In particular, if the liquid superheat builds up until the existing nucleation sites are activated, rapid boiling and expulsion of the resultant two-phase mixture may occur

Dynamic Instabilities – Density Wave Oscillations

- ***Density-wave oscillations*** can occur in both diabatic and adiabatic two-phase systems and in diabatic single-phase systems
- Generally speaking, density wave oscillations are caused by the lag introduced into the thermal-hydraulic system by the finite speed of propagation of density perturbations

- This type of instability is one of the most important and of practical concern in BWRs and will be discussed in more detail in the following part of this lecture

Pressure-Drop Oscillations

- ***Pressure-drop oscillations*** can occur in loops having a negative slope (similar to the situation described for the excursive instability) and containing a compressible volume (e.g. an accumulator)
- In such systems excursions may occur periodically

- Flow regime excited instabilities can occur when a particular flow regime, normally slug flow, induces a periodic disturbance in the system operating state
- If this disturbance is at a frequency that is close to the natural frequency of the two-phase system, a resonance can occur

Acoustic Instabilities (1)

- ***Acoustic instabilities*** may occur in two-phase system having a specific combination of geometric characteristics and sonic speed
- As in single-phase gas flows, organ-pipe-type standing waves can be set up when a pressure pulse propagates through two-phase mixtures flowing in a conduit
- On reaching an area change or obstruction, the change in acoustic impedance causes a pressure pulse of opposite polarity to propagate in the opposite direction

Acoustic Instabilities (2)

- If the excitation frequency and geometry of the conduit is such that an integral number of one-quarter wavelengths can fit within it, the standing waves may appear

- Such acoustic-induced channel pressure drop oscillations of large amplitude have been observed for subcooled systems operating in the negative-slope region of the system pressure-drop versus flow curve

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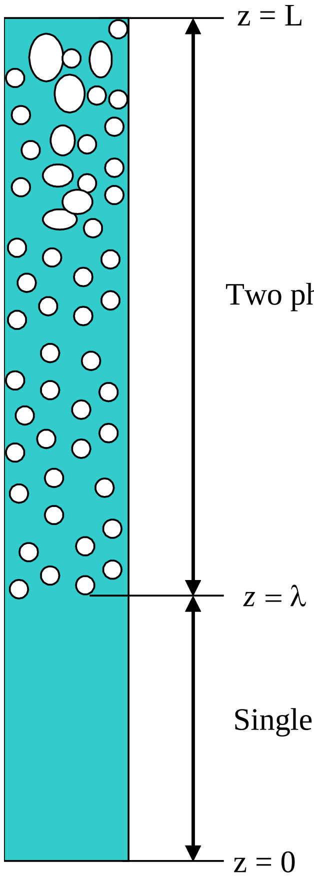
Condensation-Induced Instabilities (1)

- ***Condensation-induced instabilities*** are known to lead to large water-hammer-type loads, however, their nature is not fully understood
- A typical example is the so-called **chugging phenomena** that has been observed in the vent pipes of steam relief valves which are submerged in a liquid pool
- When the steam first exits into the subcooled pool of liquid it is normally at a high enough velocity to form a jet within the pool

Condensation-Induced Instabilities (2)

- However, as the steam flow rate drops off, the condensation rate in the pool may be large enough to completely collapse the steam jet, and cause a liquid slug to surge up into the discharge line
- Subsequently, the steam can heat up the interface of the liquid slug to saturation, allowing the pressure of the discharging steam to increase such that it blows the slug back into the liquid pool
- A cyclic process can occur with large inertial loads associated with the liquid slug motion being transmitted to the walls of the vessel containing the pool

Boiling Channel Dynamics



The HEM conservation equations for two-phase flow in a boiling channel are as follows:

mass $$\frac{\partial \rho_m}{\partial t} + \frac{1}{A}\frac{\partial (GA)}{\partial z} = 0$$

energy $$\frac{\partial \left[\left(\rho_m i_M - p \right) A \right]}{\partial t} + \frac{\partial \left(G i_m A \right)}{\partial z} = q'' P_H$$

momentum $$\frac{\partial G}{\partial t} + \frac{1}{A}\frac{\partial}{\partial z}\left(\frac{G^2 A}{\rho_m} \right) + \frac{\partial p}{\partial z} + \left[\varphi_{lo}^2 \frac{4 C_{f,lo}}{D_h} + \sum_{i=1}^{n} \xi_i \varphi_{lo,di}^2 \delta\left(z - z_i \right) \right] \frac{G^2}{2\rho_f} + \rho_m g \sin\phi = 0$$

where

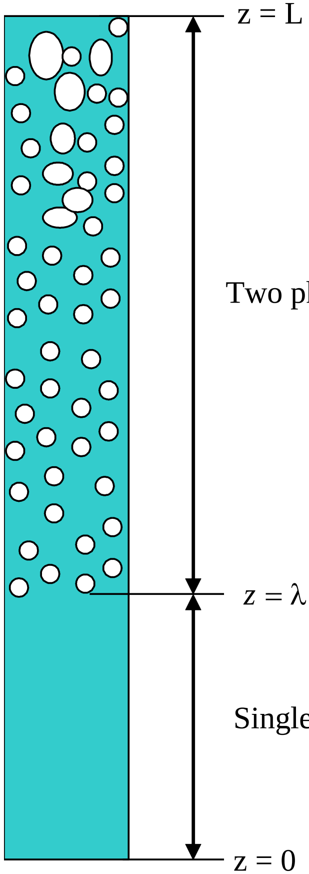
$$\rho_m = (1-\alpha)\rho_f + \alpha \rho_g$$

$$i_m = i_f \left(1 - x \right) + i_g x$$

$$i_M = \frac{\rho_f i_f \left(1 - \alpha \right) + \rho_g i_g \alpha}{\rho_m}$$

G – mass flux, *i* – enthalpy, *p* – pressure, P_H – heated perimeter, D_h – hydraulic diameter, *A*- cross-section area, *q"* – heat flux, *z* – distance, *t* – time, $C_{f,lo}$ – Fanning friction loss coefficient for liquid-only flow, ξ_i – local loss coefficient at location i, $\varphi_{lo,di}^2$ - local obstacle loss multiplier, φ_{lo}^2 - friction multiplier, $\delta(z-z_i)$ – Dirac's delta

Total Pressure Drop



Integration of momentum equation yields

$$\Delta p_{2\varphi} = p(\lambda) - p_{ex} = \int_{\lambda}^{L} \left\{ \frac{\partial G}{\partial t} + \frac{\partial}{\partial z}\left(\frac{G^2}{\rho_m}\right) + \varphi_{lo}^2 \frac{4C_{f,lo}}{D_h}\frac{G^2}{2\rho_f} + \rho_m g \right\} dz +$$

$$\sum_{i \in L_{2\varphi}} \varphi_{lo,di}^2 \left(z_i\right) \xi_i \frac{G^2(z_i)}{2\rho_f}$$

$$\Delta p_{1\varphi} = p_{in} - p(\lambda) = \int_{0}^{\lambda} \left\{ \frac{\partial G}{\partial t} + \frac{1}{\rho_f}\frac{\partial G^2}{\partial z} + \frac{4C_f}{D_h}\frac{G^2}{2\rho_f} + \rho_f g \right\} dz +$$

$$\sum_{i \in L_{1\varphi}} \xi_i \frac{G^2(z_i)}{2\rho_f}$$

The total pressure drop is: $\Delta p_{ch} = \Delta p_{1\phi} + \Delta p_{2\phi} = const$

Constant Pressure Drop

- When boiling channels operate in parallel, such as in a reactor core, then pressure drop in all channels is the same and is determined by the pressure drop in the whole core
- Thus, any perturbations in single- and two-phase regions of the channel will be coupled by the following constraint:

$$\delta\Delta p_{ch} = \delta\Delta p_{1\phi} + \delta\Delta p_{2\phi} = 0$$

- This equation says that any perturbation of pressure drop in the single-phase region is accompanied with an equal (but with an opposite sign) perturbation in the two-phase region (and vice versa)

Constant Pressure Drop (2)

- With a proper timing, the oscillations can be self-sustained
- Assume that the inlet velocity perturbation δJ_{in} results from a perturbation of the pressure drop in the single-phase region. We can describe this with the following transfer function: $$G = \frac{\delta J_{in}}{\delta \Delta p_{1\phi}}$$ G is the *single-phase-pressure-drop-to-inlet velocity* transfer function
- Further, the perturbation of inlet velocity causes pressure drop perturbation in the two-phase region, as given by the following transfer function: $$H = \frac{\delta \Delta p_{2\phi}}{\delta J_{in}}$$ H is the *inlet-velocity-to-two-phase-pressure-drop* transfer function

Constant Pressure Drop (3)

- The total perturbation in the channel can be given as

$$\delta\Delta p_{ch} = \left(\frac{\delta\Delta p_{1\phi}}{\delta J_{in}} + \frac{\delta\Delta p_{2\phi}}{\delta J_{in}} \right) \delta J_{in} = \left(\frac{1}{G} + H \right) \delta J_{in}$$

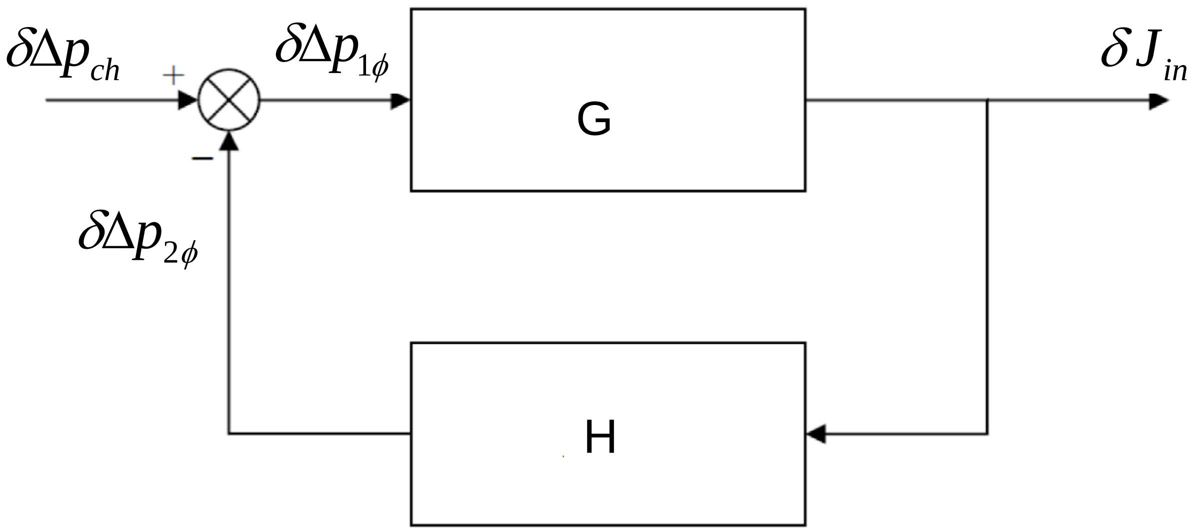
- Thus

$$\frac{\delta J_{in}}{\delta\Delta p_{ch}} = \left(\frac{1}{G} + H \right)^{-1} = \frac{G}{1+GH}$$

- This equation represents a transfer function of a boiling channel and describes the pressure-drop-perturbation to inlet-velocity-perturbation effect

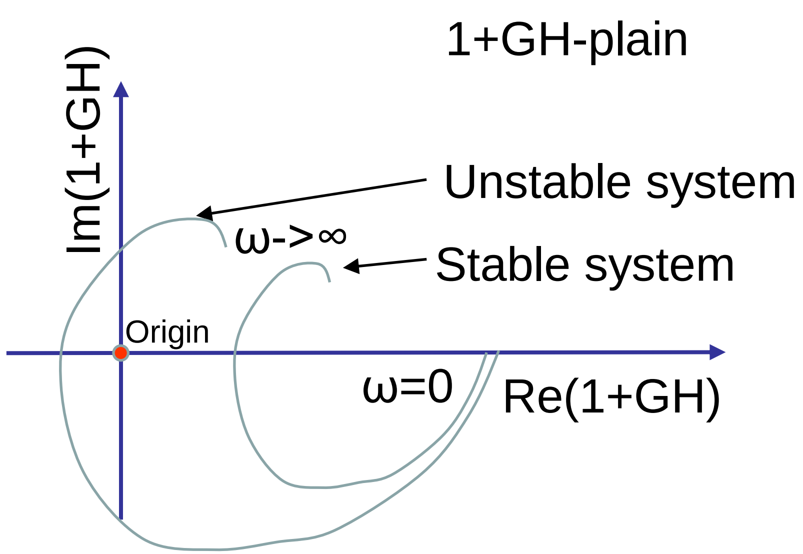
Constant Pressure Drop (4)

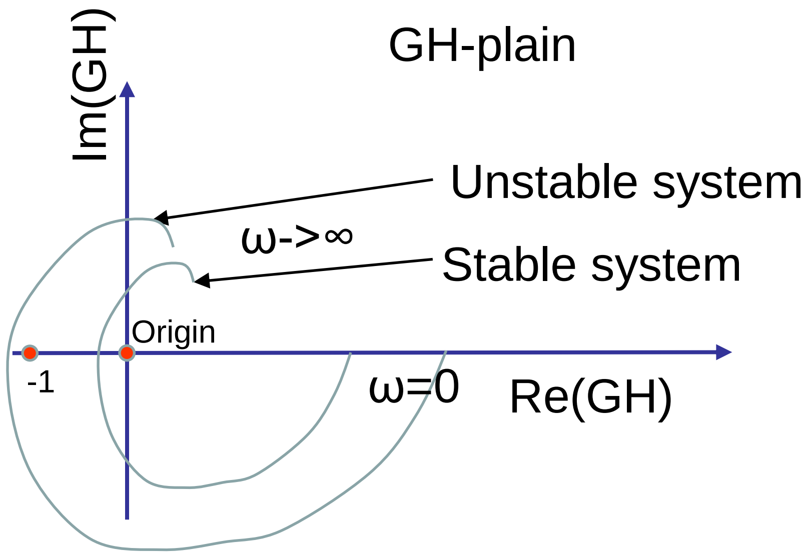
- The obtained transfer function corresponds to a closed loop transfer function known from the dynamic analysis of control systems with feedback, as shown in figure



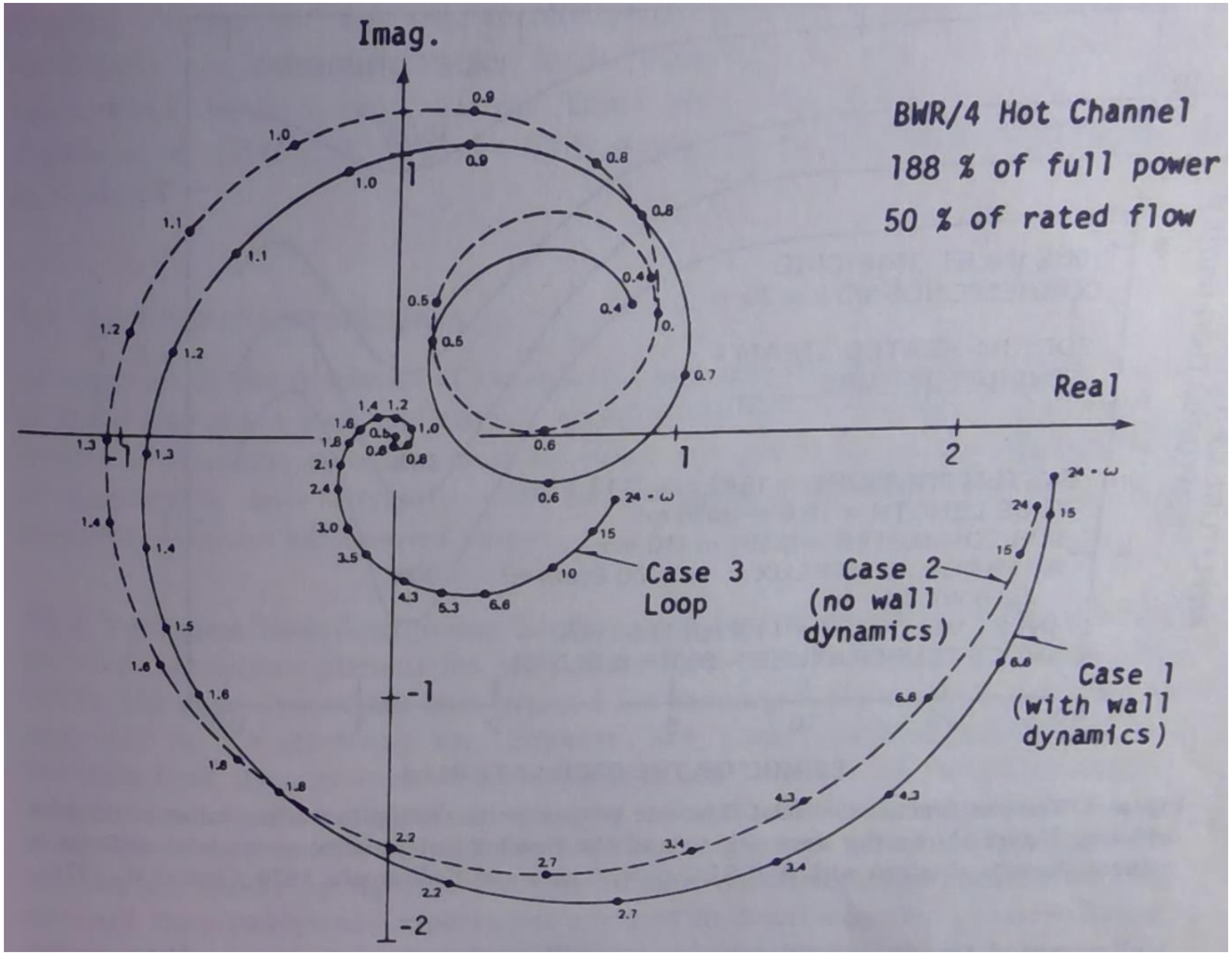
Closed Loop Stability

- Stability of the closed loop is determined by roots of the characteristic equation 1+GH=0
- According to the Nyquist criterion, system is unstable if the polar plot of Im(1+GH) vs Re(1+GH) encircles the origin (0,0); it's equiv. to encircling (-1,0) in the GH-plane





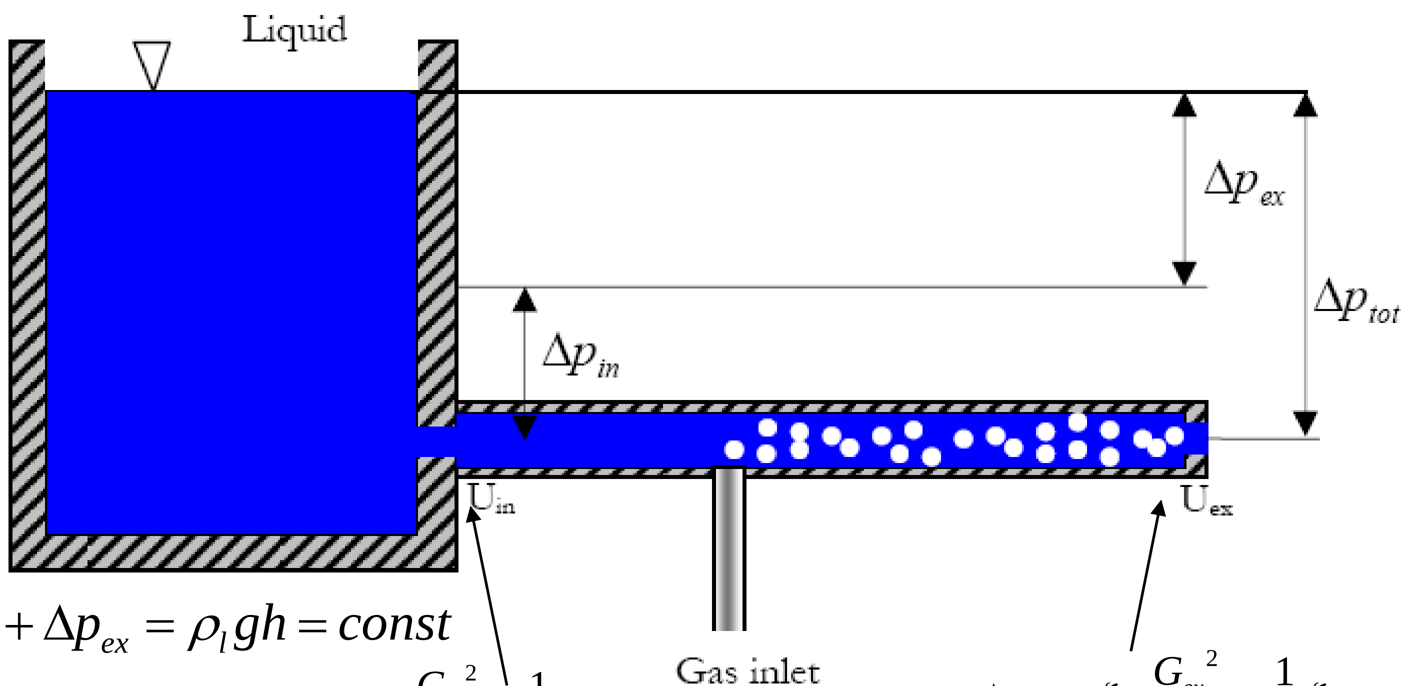
Example- Hot Channel in BWR/4



Nyquist plots for BWR channel (Case 1,2) and entire primary circulation loop (Lahey and Yadigaroglu, 1974).

In Case 2, the channel becomes unstable with a period of approximately 1.3 s

Density-Wave Instabilities (1)



$$\Delta p_{tot} = \Delta p_{in} + \Delta p_{ex} = \rho_l g h = const$$

$$\Delta p_{in} = \xi_{in} \frac{G_{in}^{\ 2}}{2\rho_l} = \frac{1}{2} \xi_{in} \rho_l J_{in}^2$$

$$\Delta p_{ex} = \xi_{ex} \frac{G_{ex}^{\ 2}}{2\rho_{ex}} = \frac{1}{2} \xi_{ex} \rho_{ex} J_{ex}^2$$

J_{in} ↑->immediately->J_{ex}↑->immediately-> Δp_{ex}↑->Δp_{in}↓-> J_{in} ↓ (Fast process)
J_{in}↑->delay->ρ_{ex}↑->immediately->Δp_{ex}↑->immediately->Δp_{in}↓->J_{in} ↓ (Slow process)

Density-Wave Instabilities (2)

- Oscillation mechanisms:
  - opening the inlet orifice for a short time induces a density wave that propagates through the pipe and passes the exit orifice after a time τ
  - During the passage, the corresponding pressure drop at the exit increases temporarily. Since the total pressure drop always remains constant, the increased pressure drop over the exit will bring a corresponding pressure decrease at the inlet
  - This means that less water enters into the channel creating a subsequent decrease in density, which induces the wave that propagates through the channel. This dynamic process causes density wave propagation in the channel

Density-Wave Instabilities (3)

- If the amplitude of the waves is converging the system is said to be stable and if the amplitude is diverging it is said to be unstable

- These finite propagation times, τ, induce time-lag effects and phase-angle shifts between the channel pressure drop and the inlet flow, which may cause self-exciting oscillations

Density-Wave Instabilities (4)

- Density-wave instabilities can further be classified as follows:
  - Loop instabilities
  - Parallel-channel instabilities
  - Channel-to-channel instabilities
  - Neutronically-coupled instabilities

Density-Wave Instabilities (5)

- The most important modes of density-wave instabilities are **loop** and **parallel-channel** instabilities.
  - The parallel-channel mode corresponds to a system of a big number of channels connected in parallel, in which a constant pressure condition governs flow through each of the channels
  - The principles of density-wave instability that are described here correspond just to this mode of instability
  - The loop instability is very similar, however, the boundary condition of zero pressure drop in the loop is imposed: $\Delta p_{loop}=0$

Stability Map of a Boiling Channel (1)

- Stability behavior of a boiling channel can be represented on a so-called **stability map**
- One such map, proposed by Ishii and Zuber, uses two parameters to describe the channel stability:

Subcooling number

$$N_{sub} = \frac{\left(\rho_f - \rho_g\right)}{\rho_g} \frac{\left(i_f - i_{in}\right)}{i_{fg}}$$

Phase change number

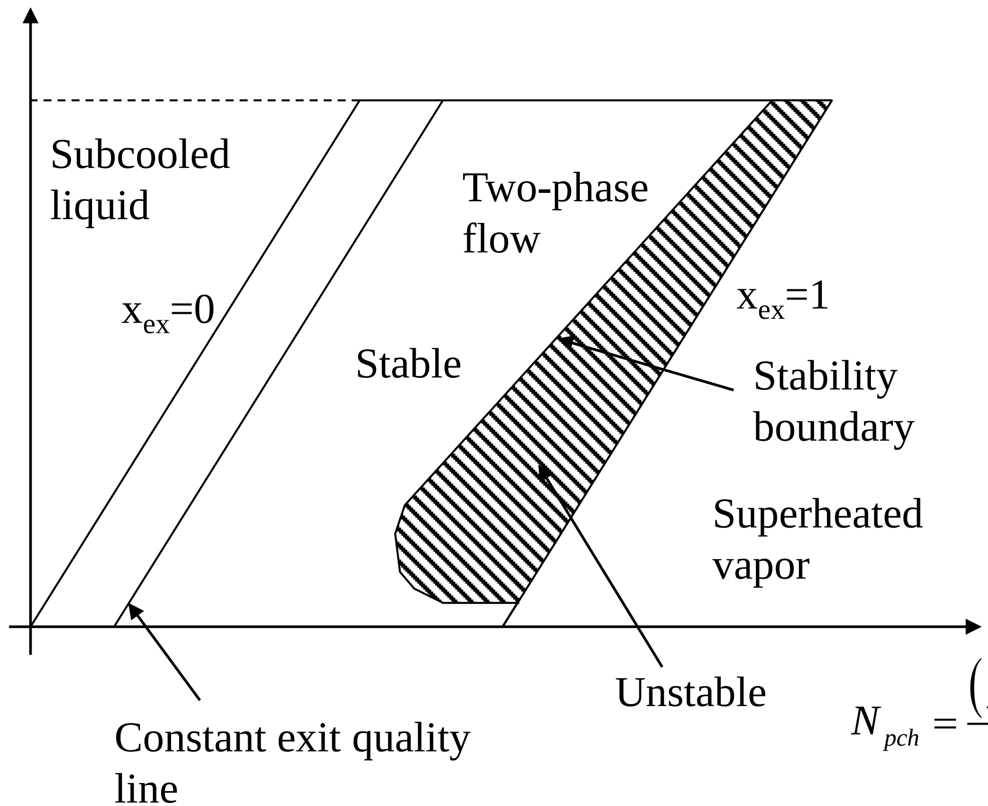
$$N_{pch} = \frac{\left(\rho_f - \rho_g\right)}{\rho_g} \frac{q_0'' P_H L_H}{J_{in} A \rho_f i_{fg}} = \frac{\left(\rho_f - \rho_g\right)}{\rho_g} \frac{q}{W i_{fg}}$$

i- specific enthalpy, i_{fg} – latent heat, *q”* – heat flux, J_{in} – inlet velocity, *A* – flow cross-section area, *W* – mass flow rate, *q* – total heat, P_H – heated perimeter, L_H – heated length; indices: *f* – saturated liquid, *g* – saturated vapour, *in* -inlet

Stability Map of a Boiling Channel (2)

- Ishii-Zuber stability map

$$N_{sub} = \frac{(\rho_f - \rho_g)(i_f - i_{in})}{\rho_g i_{fg}}$$



$$N_{pch} = \frac{(\rho_f - \rho_g) q_0'' P_H L_H}{J_{in} A \rho_f \rho_g i_{fg}}$$

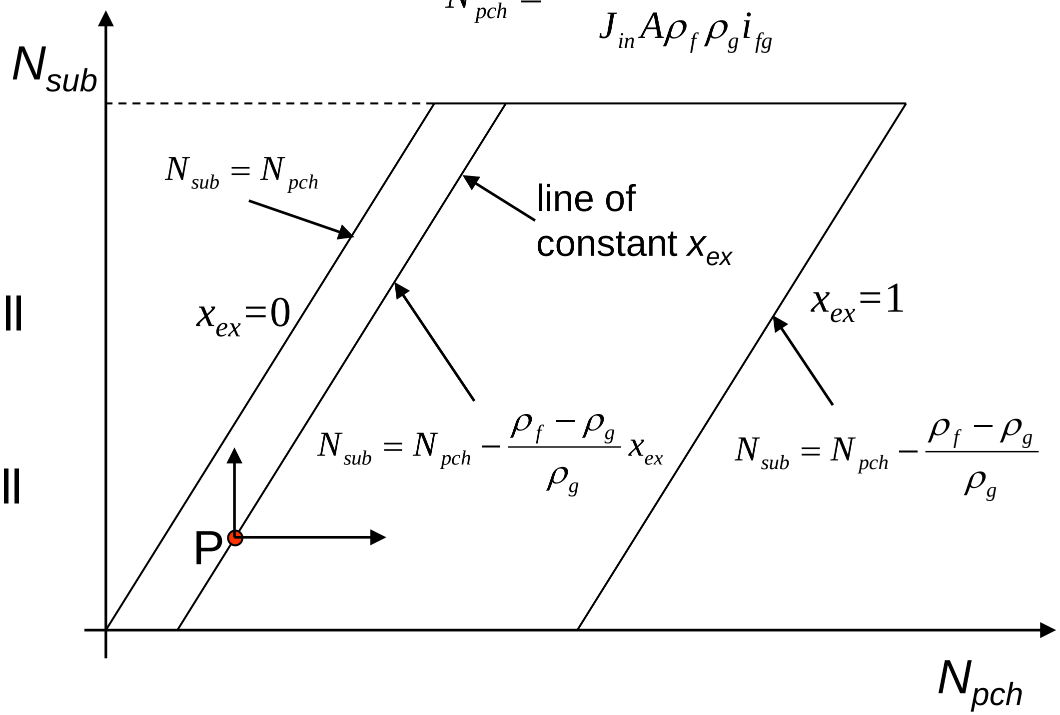
Stability Map of a Boiling Channel (3)

From energy balance: $x_{ex} = \dfrac{q_0'' P_H L_H}{G A i_{fg}} + x_{in}$

$$N_{sub} = \frac{(\rho_f - \rho_g)(i_f - i_{in})}{\rho_g i_{fg}}$$

$$N_{pch} = \frac{(\rho_f - \rho_g) q_0'' P_H L_H}{J_{in} A \rho_f \rho_g i_{fg}}$$

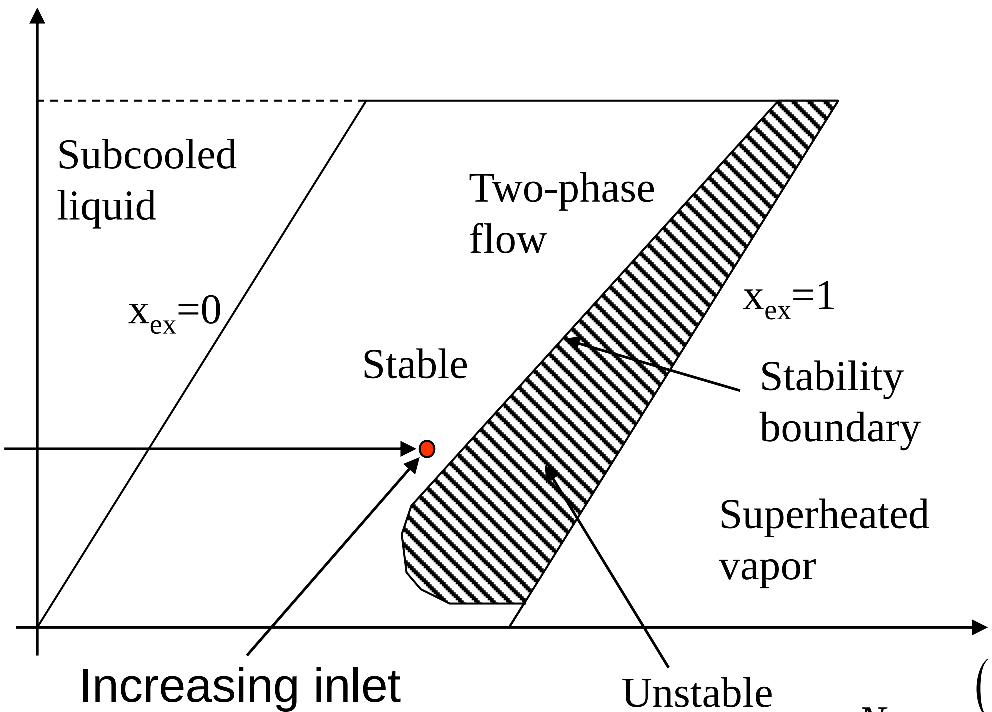
Assume that a channel operates at point P.
-Increasing inlet subcoling will increase N_{sub} only, and point P will move vertically up
-Increasing power or decreasing mass flux will increase N_{pch} only, and point P will move to the right



Stability Map of a Boiling Channel (4)

- Influence of power, inlet subcooling and mass flux

$$N_{sub} = \frac{(\rho_f - \rho_g)(i_f - i_{in})}{\rho_g i_{fg}}$$



increasing power or decreasing mass flux will destabilize the channel

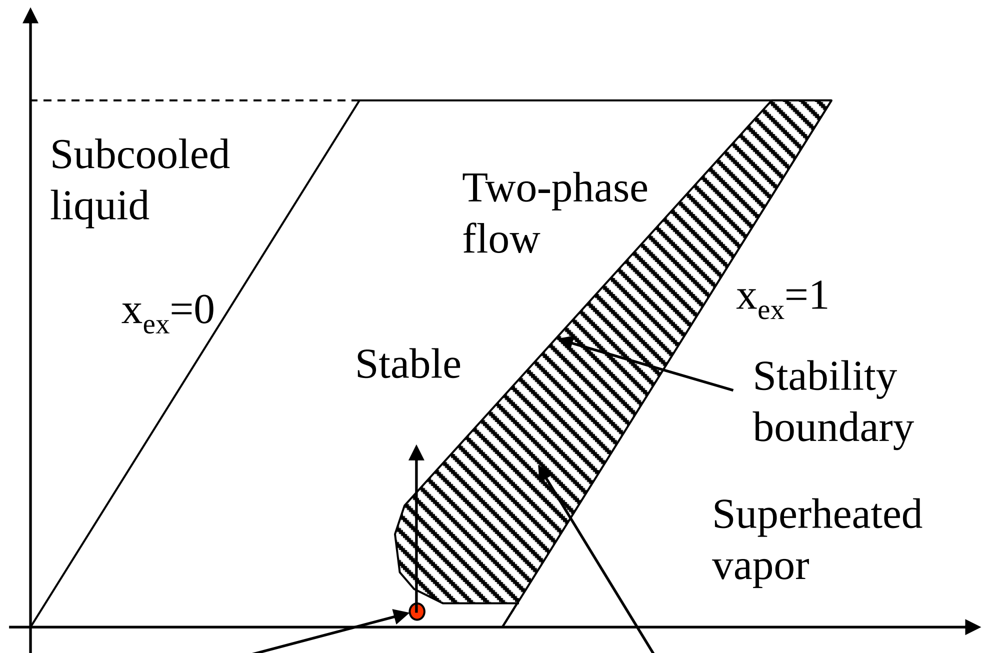
Increasing inlet subcooling will stabilize the channel

$$N_{pch} = \frac{(\rho_f - \rho_g) q_0'' P_H L_H}{J_{in} A \rho_f \rho_g i_{fg}}$$

Stability Map of a Boiling Channel (5)

- Behavior for small N_{sub} and high N_{pch}

$$N_{sub} = \frac{(\rho_f - \rho_g)(i_f - i_{in})}{\rho_g i_{fg}}$$



Increasing inlet subcooling will first destabilize the channel!

$$N_{pch} = \frac{(\rho_f - \rho_g) q_0'' P_H L_H}{J_{in} A \rho_f \rho_g i_{fg}}$$

Stability Map of a Boiling Channel (6)

- Simplified stability criterion (Ishii)
  - It can be seen in Ishii-Zuber stability map that stability boundary curve for higher inlet subcoolings is nearly parallel with the line of a constant exit quality x_{ex}, given by

$$N_{sub} = N_{pch} - \frac{\rho_f - \rho_g}{\rho_g} x_{ex}$$

  - Ishii used this observation and derived a simple stability criterion for the high inlet subcoolings as follows

$$x_{ex} \le \left[\frac{\left(\xi_{in} + \phi + \xi_{ex}\right)}{1 + \phi/4 + \xi_{ex}/2} \right] \frac{\rho_g}{\rho_f - \rho_g} \qquad \phi = \frac{4 r_3 C_f L_H}{D}$$

ξ_{in}, ξ_{ex} – inlet and outlet loss coefficients, r_3 – integral two-phase friction multiplier

  - According to the criterion, the channel is stable as long as the above inequality is satisfied (but remember what happens for small inlet subcooling!)