Nuclear Reactor Physics

Reactor Kinetics I

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Contents

Info about the trip to Prague

Info about the trip to Prague

Flight tickets are booked

- The tickets include one checked bag (23 kg) and a cabin luggage (8 kg).
- The return flight to Stockholm on Dec 6 was cancelled and re-booked to 11:50-18:20. Get the new itinerary file from Canvas.
- At the airport (or online), check in yourself using the booking reference (a group of students may have the same code).

Hotel for three nights after arrival is booked for you

- Rooms for two or three students.
- Breakfast included.

The address is:
Castle Residence Praha
Květinářská 755
Praha 8, Libeň, 182 00

Later, I will give you:

- detailed travel instructions (how to arrive to the hotel and the reactor hall),
- lab instructions and report requirements.

Introduction to reactor kinetics

What is reactivity ρ?

- Reactivity ρ is the relative departure of k from unity,

$$\rho = \frac{k-1}{k}$$

- Reactivity ρ can also be interpreted as the relative change in neutron population over one generation.
- Note that:
  - critical reactors have zero reactivity
  - supercritical reactors have positive reactivity
  - subcritical reactors have negative reactivity

Reactor kinetics

Reactor kinetics studies a reactor power response in time to external changes to reactivity, assuming the reactivity is not affected by changes in power. The assumption is valid for zero-power reactors.

Common classification of time problems (kinetics + dynamics)

- **Short time problems** (seconds to tens of minutes).
  - These problems arise during reactor transients when some action disturbs reactor conditions.
  - The total power is assumed to vary with time, but the shape of the power distribution is often assumed to be fixed.
  - The change of nuclide concentrations and depletion of the fuel can be ignored here.
- **Intermediate time problems** (hours to a day or two).
  - These problems account for changing characteristics of fission products, such as ^{135}Xe that has a large absorption cross section. These nuclides have to be tracked.
  - Fuel depletion can be ignored.
  - The reactor is usually assumed to run at a constant total power, but the changes in the power distribution are considered.
- **Long time or depletion problems** (days to months).
  - This class of problems is a major concern for fuel-management calculations.
  - A detail knowledge of the fuel depletion and the distribution of the flux and fuel is required.
  - The reactor is usually assumed to run at a constant total power, but the changes in the power distribution are considered.

What is the effective delayed-neutron fraction, β_{eff}? How does it differ to β?

- β is the probability that a neutron is born as a delayed neutron.
- β_{eff} is the probability that a fission is caused by a delayed neutron.

Which of the two parameters, $\beta_{\text{eff}} or \beta$, is larger in thermal reactors?

- $\beta_{\text{eff}} > \beta$ in thermal reactors.
- As the delayed neutrons are born with a lower energy, they are less likely to be captured during the slowing down.
- Therefore they are more effective than prompt neutrons in causing fission.

What units for reactivity are commonly used?

Reactivity is a dimensionless number; however, is not very practical to use it as such since we need to work with its small deviations from zero. It is common to use the following units for reactivity:

- pcm (percent mille) = one-thousandth of a percent, i.e. $\mathrm{pcm} = 10^{-5}$
- % (percent) $= 10^{-2}$
- \$ (dollar) $= \beta_{\mathrm{eff}}$
- c (cent) $= 10^{-2}\beta_{\mathrm{eff}}$
- β_{eff}

What is the neutron lifetime l?

The neutron lifetime l is the mean time for a fission (prompt) neutron to get absorbed or leak out of the system.

What is the prompt neutron generation time Λ?

- Λ is the effective time it takes to a number of prompt fission neutrons to produce the same number of new prompt fission neutrons.
- Λ is an integral property of the whole reactor.
- The decay time of precursors of delayed neutrons is not reflected in the prompt neutron generation time in any way!
- Generation time depends on the reactor design:
  - Fast reactors have $\Lambda \approx 10^{-8}$s.
  - Thermal reactors have $\Lambda \approx 10^{-3}$s.

Is there a relation between generation time Λ and neutron lifetime l?

$$\Lambda = l/k$$

If the multiplication factor was 2 then the generation time to produce the same number of new fission neutrons (from parent fission neutrons) would be half of the neutron lifetime.

Understanding the fission chain reaction

Understanding the fission chain reaction

Let's consider a critical reactor. What is the probability that fission neutrons produced in a single fission event would cause a single next-generation fission event?

In a critical reactor, the probability must be 1 exactly!

In a critical reactor, what is the probability that the prompt fission neutrons produced in a single fission event cause a single next-generation fission event?

That probability must equal $1 - \beta_{eff}$.

In a critical reactor, what is the probability that two successive fission reactions in the same fission chain are caused by only prompt neutrons?

This probability must be $(1 - \beta_{eff})^2$

Understanding the fission chain reaction

In a critical reactor, what is the probability that the fission chain driven by prompt neutrons would have a length of *n* generations or more?

This probability must be $(1 - \beta_{eff})^n$

What is the probability that the fission chain driven by prompt neutrons would have a length of 100 generations or more?

This number is $(1 - \beta_{eff})^{100}$, which is about 0.5.

So, about half of the "prompt fission chains" are shorter than 100 generations, and half of them are longer than 100 generations.

Understanding the fission chain reaction

Do all prompt fission chains terminate in a critical reactor?

In a critical reactor, each prompt fission chain gets terminated eventually.

If the neutron lifetime in a reactor is about 10^{-4}s then what is the average time during which the prompt fission chains terminate in a critical reactor? (Assume it is the time during which half of the prompt fission chains terminates.)

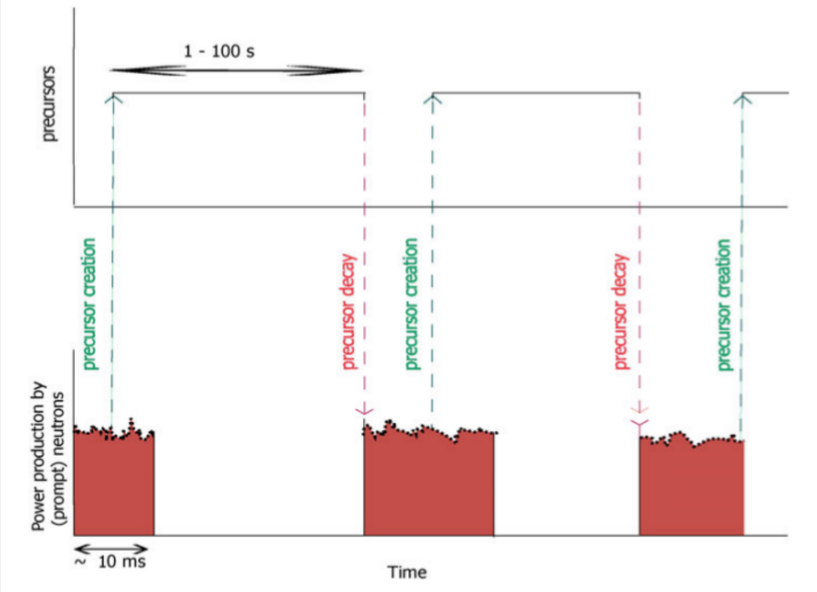
That would be then about 10^{-2}s (10 ms) for about 100 generations.

What happens when the prompt fission chain reaction is terminated? How can the critical reactor continue running?

- At some generation in the prompt fission chain, a precursor of a delayed neutron was born.
- Now, the precursor will take time to decay (this may take a fraction of a second, several seconds, or hundred seconds. . .).
- Once the precursor decays, the fission chain will restart, and a sequence of generations driven by prompt incident neutrons will follow, and so on.
- So, the energy release from the fission chain reaction occurs in **short pulses**.

Understanding the fission chain reaction

Let's have a look at the fission chain reaction



Will the power of the pulses (i.e., the length of the prompt fission chain) change with changes of k?

- Yes. The length of the prompt fission chain (and so the power of the pulses) will increase with increasing k.
- The average number of next-generation fissions caused by prompt neutrons from a **single** previous-generation fission will be, in non-critical systems,

$$(1 - \beta_{eff}) \times k$$

- So, the average number of fissions, n, in a prompt power pulse will be given by equation

$$[(1 - \beta_{eff}) \times k]^n = 0.5$$

  which, e.g. for $k = 1.005$ (corresponding to $\rho \approx 0.7\beta_{eff}$), gives

$$n \approx 340$$

- So, the power will **practically immediately jump up** more than 3-times when reactivity is increased to about 0.7 β_{eff}.

If the length of the prompt fission chain increases three-times how it will affect the number of precursors created within each pulse?

- The number of precursors is **proportional** to the length of the prompt fission chain (level of the power pulse). For systems with $k > 1$, there will be more than one precursors, on average, created at each pulse.
- That means that the fission chain reaction will branch over time, so the rate at which the power pulses occur will not be constant, but it will grow (exponentially) over time.
- That means, that after the increase of k, there will first be a prompt jump of the power of the system (see previous slide), followed by an exponential growth of power in time.

How will the power develop if a critical system is suddenly made subcritical?

In this case,

- power will first promptly drop due to immediate shortening of the prompt fission chains,
- and then an exponential decrease of power over time will follow (since less than one precursor, on average, would be born at each power pulse).

The prompt jump and drop

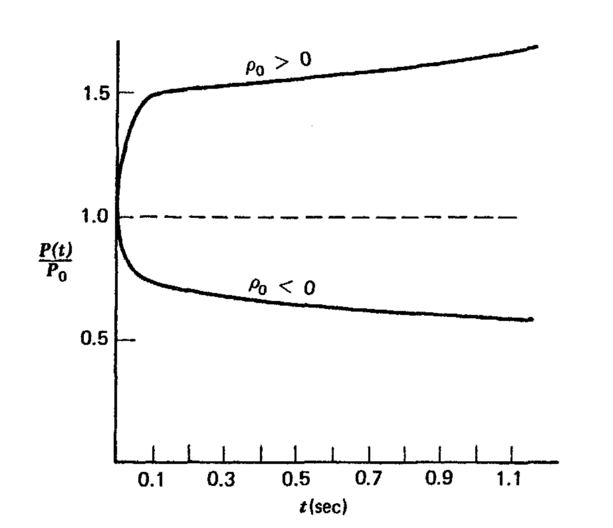


Figure 1: Reactor power level behaviour following a positive step reactivity insertion

Understanding the fission chain reaction

How will the power of a supecritical reactor develop if k is suddenly reduced, but the reactor still remains supercritical?

- If the decrease of k is sudden then the length of the prompt fission chains will also suddenly decrease and so the power will drop at the moment when k is decreased.
- However, since the reactor is still supercritical, the rate of the power pulses will still continue to grow (although at a lower rate than before the reduction of k).
- So, that means that after the initial drop of power, the power will continue to grow.

How will the power of a supecritical reactor develop if k is slowly reduced, but the reactor still remains supercritical?

Depending on the speed och the change, the reduction of the power of each pulse may not cause any decrease of the total power since the growing number of prompt chains could have a larger effect.

Understanding the fission chain reaction

How will the power of a subcritical reactor develop if k is suddenly increased, but the reactor still remains subcritical?

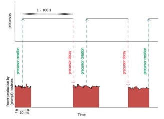
- If the increase of k is sudden then the length of the prompt fission chains will also suddenly increase and so the power will increase at the same moment.
- However, since the reactor is still subcritical, the rate of the power pulses will still continue to decline (although at a lower rate than before the increase of k).
- So, that means that after the initial increase of power, the power will continue to decline over time.

How will the power of a subcritical reactor develop if k is slowly increased, but the reactor still remains subrcritical?

Depending on the speed och the change, the increase of the power of each pulse may not cause any increase of the total power since the decaying number of prompt chains could have a larger effect.

Understanding the fission chain reaction

Is it possible that the gap between the pulses would disappear when reaching a certain k?



For which k this will happen?

This happens when the prompt neutrons from a certain fission reaction cause a single next-generation fission reaction with probability 1., i.e.,

$$(1 - \beta_{eff})k = 1.$$

from where

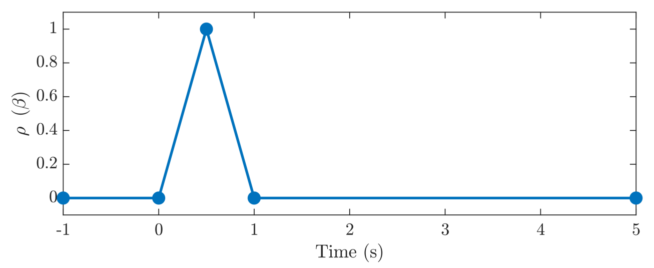
$$k = \frac{1}{1 - \beta_{eff}}$$

which corresponds to reactivity

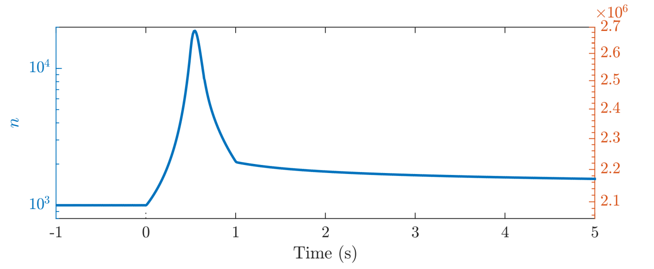
$$\rho = \frac{k-1}{k} = \frac{\frac{1}{1-\beta_{eff}} - 1}{\frac{1}{1-\beta_{eff}}} = \beta_{eff}$$

Understanding the fission chain reaction

What will be reactor response to this fast reactivity change?



Resulting power change



Point kinetic equations

Point kinetic equations

Point kinetic equations

- Point kinetic equations give a solution to reactor power (flux) response in time to changes in reactivity.
- We shall be expressing the total power $P(t)$ in terms of the number of neutrons, $n(t)$, in the system.
- The system of point kinetic equations is created by
  - an equation for $n(t)$, and by
  - six equations for the number of precursors of the i-th group, $c_i(t)$.

How is the space dependence treated in the point kinetic equations?

- Space dependence of the neutron flux is not explicitly considered in the point kinetic equations.
- The shape of the spatial distribution of the flux is assumed to be fixed at the fundamental mode.
- Only the time dependence in the total power (or flux integrated over the volume of the system) is considered.

Point kinetic equations

Kinetic equations specify the rate at which the number of neutrons and precursors change in time.

In general terms, the time change of $n(t)$ equals the sum of:

- the production rate of prompt neutrons,
- the production rate of delayed neutrons,
- the production rate of neutrons coming from an external source, q,
- the neutron loss rate (a negative number).

Point kinetic equations

Let's express the various terms in PK equations

- The neutron loss rate says how many neutrons get lost in a time unit (s).
- We know that on average the neutron lives within the time l.
- That means that n neutrons get lost in time l.

So, what is the **neutron loss rate**?

$$\text{Neutron loss rate} = -n/l$$

The prompt neutron production rate

- For each neutron lost, k new fission neutrons are created.
- Since n/l neutrons are lost every second, kn/l must be created every second.
- This number includes all prompt and delayed neutrons.
- For each neutron produced, $(1-\beta_{\text{eff}})$ accounts for prompt neutrons (which contribute to further fission).

Therefore:

$$\text{Prompt neutron production rate} = (1-\beta_{\text{eff}})kn/l$$

The delayed neutron production rate:

- Delayed neutrons are emitted in decay of precursors.
- There are c_i precursors of ith group in the system, and they decay at the rate $c_i\lambda_i$.

Therefore:

$$\text{Delayed neutron production rate} = \sum_i \lambda_i c_i$$

Let's put all the terms together:

Hence, the equation for $n(t)$ reads as:

$$\frac{dn}{dt} = \frac{(1 - \beta_{\text{eff}})k}{l} n + \sum_i \lambda_i c_i + q - \frac{n}{l}$$

which can be rearranged as

$$\frac{dn}{dt} = \frac{k - 1 - \beta_{\text{eff}} k}{l} n + \sum_i \lambda_i c_i + q$$

And, by dividing the numerator and denominator of the first term on the right side by k, the equation can be written in terms of ρ and Λ as:

$$\frac{dn}{dt} = \frac{\rho - \beta_{\text{eff}}}{\Lambda} n + \sum_i \lambda_i c_i + q$$

Point kinetic equations

In the same fashion, we can also formulate the equation for the number of precursors of *i*-th group, $c_i(t)$

The equation for $c_i(t)$ states that the time change in $c_i(t)$ is given by the sum of:

- the production rate of precursors of *i*-th group from fission,
- the decay rate at which precursors of *i*-th group decay (negative number).

Point kinetic equations

The production rate of precursors of ith group

Since the production rate of prompt neutrons is $(1-\beta_{\text{eff}})kn/l$, the total production of all precursors must be $\beta_{\text{eff}}kn/l$.

Therefore:

$$\text{Production rate of precursors of } i\text{th group} = \beta_{\text{eff}\,i}kn/l$$

The decay rate of precursors of ith group:

The decay rate of precursors of ith group must be equal to the production rate of delayed neutrons of the ith group (that we derived to be $c_i\lambda_i$), just opposite.

Therefore:

$$\text{Decay rate of precursors of } i\text{th group} = -c_i\lambda_i$$

Point kinetic equations

Equation for $c_i(t)$ now reads as

Hence, the equation for $c_i(t)$ reads as:

$$\frac{dc_i}{dt} = \frac{\beta_{\mathrm{eff}\,i} k}{l} n - \lambda_i c_i$$

which is equivalent to

$$\frac{dc_i}{dt} = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n - \lambda_i c_i$$

Note that are six PK equations, $i = 1, \ldots, 6$, for precursors.

Complete point kinetic equations

One equation for $n(t)$:

$$\frac{dn}{dt} = \frac{\rho - \beta_{\mathrm{eff}}}{\Lambda} n + \sum_i \lambda_i c_i + q$$

and six equations for $c_i(t)$

$$\frac{dc_i}{dt} = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n - \lambda_i c_i, \;\; i = 1, \ldots, 6$$

Equilibrium state

Equilibrium state

Finding the equilibrium solution to the kinetic equations for systems with a general reactivity ρ and external source of neutrons can give some interesting results.

We can remove the time derivatives from the point kinetic equations:

$$0 = \frac{\rho - \beta_{\mathrm{eff}}}{\Lambda} n_0 + \sum_i \lambda_i c_i + q$$

$$0 = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n_0 - \lambda_i c_i, \;\; i = 1, \ldots, 6$$

By summing up all seven equations together we can isolate the solution for n as

$$0 = \frac{\rho}{\Lambda} n_0 + q,$$

hence,

$$n_0 = -\frac{\Lambda q}{\rho}$$

Equilibrium state

Having the solution $n_0 = -\frac{\Lambda q}{\rho}$, assume now that the external source of neutrons q has a non-zero intensity. What can we say about reactivity of such an equilibrium system then?

It follows from the equation that only subcritical systems (with negative reactivity) can be at equilibrium with an external source of neutrons.

Can we make some conclusion about the power of systems at equilibrium without an external source of neutrons?

If the external source of neutron s is absent, i.e. $q = 0$, (the case of a running reactor) then we can write the equation in the form

$$n_0 \rho = 0,$$

which can be satisfied for any n as long as $\rho = 0$.

So, nuclear reactors can run at any power (so long as the cooling system is able to be removing heat from the reactor.)

Equilibrium state

The equilibrium number of precursors c_{i0}.

The equilibrium number of precursors c_{i0} that corresponds to n_0 can be obtained from

$$0 = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n_0 - \lambda_i c_i, \;\; i = 1, \ldots, 6$$

as

$$c_i = \frac{\beta_{\mathrm{eff}\,i}}{\lambda_i \Lambda} n_0, \;\; i = 1, \ldots, 6$$

which follows from the PK equation for precursors with the zero time derivative.