Lecture No 02

Title:

TH Design of Fuel Assemblies with Single-Phase Coolant and Constant Material Properties

Temperature Distribution

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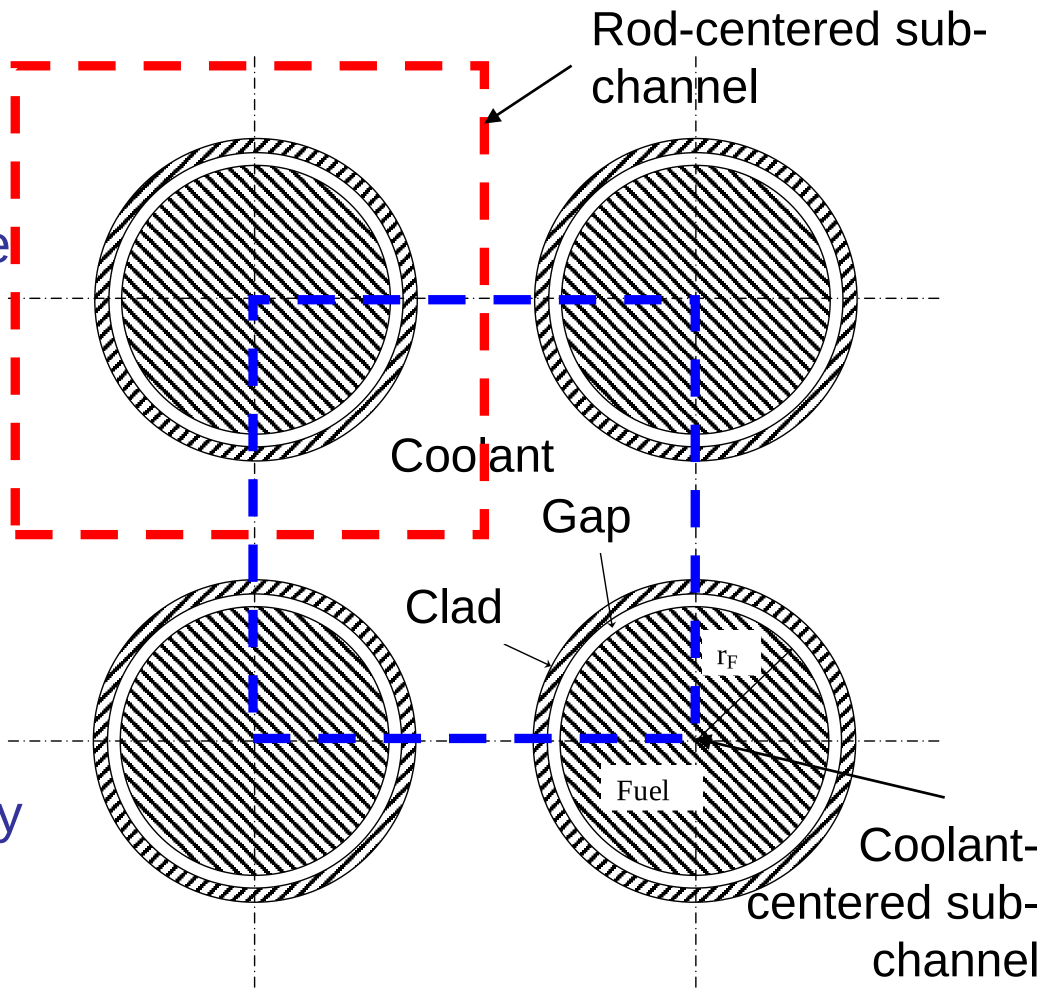
Outline of the Lecture

- Energy balance in fuel assemblies
  - Isolated Sub-channel Model
- Distribution of coolant enthalpy and temperature in single-phase flows
- Distribution of temperature in fuel rods with constant properties and clean surfaces

Isolated Sub-channel Model

- Cross-section over a square lattice with fuel pins
- Heat transfer calculations are performed in an averaged, representative "sub-channel"
- Heat conduction is considered in each rod separately
- Main assumption: no flow of mass, momentum and energy through sub-channel "walls"



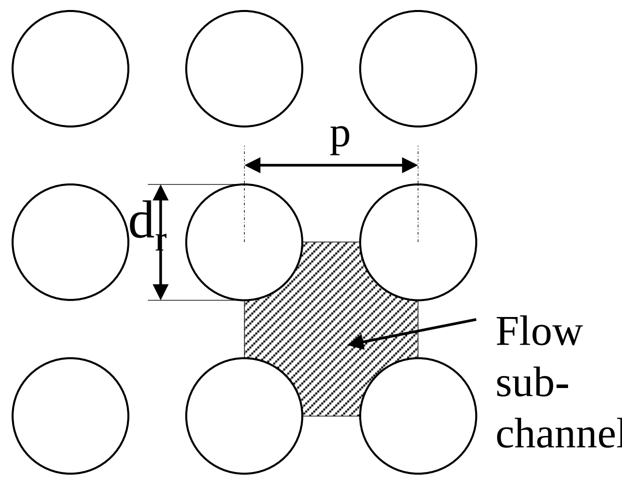
Basic Parameters Describing Isolated Sub-channel (1)

- Hydraulic diameter
  Flow area
  Wetted perimeter

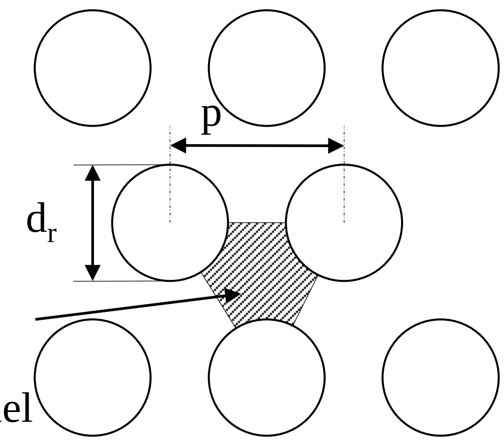
$$D_h = \frac{4A}{P_w}$$

A – channel cross-section area

P_w – channel wetted perimeter



Square lattice



Triangular lattice

$$D_h = \begin{cases} d_r\left[\dfrac{4}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1\right] & \text{for square lattice} \\ d_r\left[\dfrac{2\sqrt{3}}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1\right] & \text{for triangular lattice} \end{cases}$$

p – lattice pitch

d_r – rod diameter

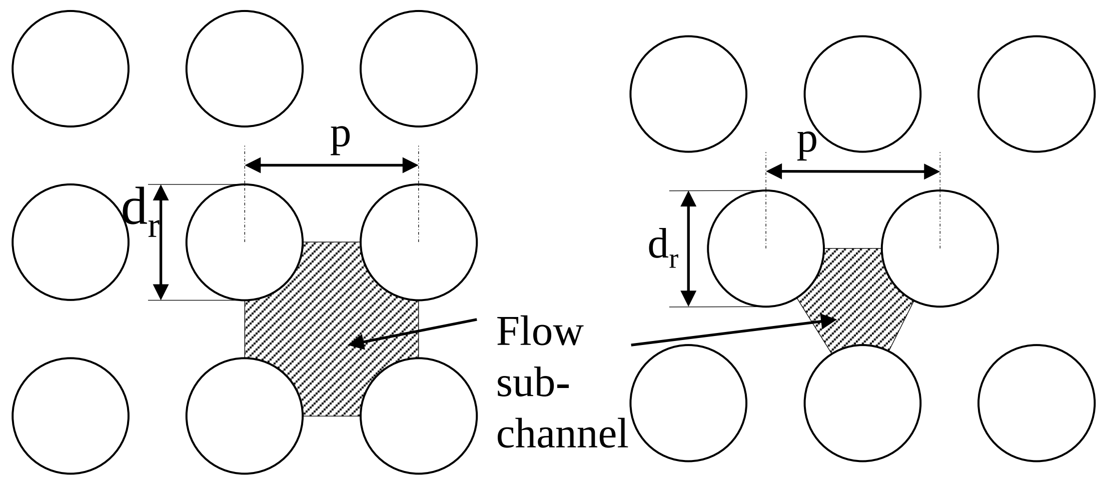
Basic Parameters Describing Isolated Sub-channel (2)

- Heated diameter
  Flow area
  Heated perimeter

$$D_H = \frac{4A}{P_H}$$

A – channel cross-section area

P_H – sub-channel heated perimeter



Square lattice

Triangular lattice

For subchannels with all heated rods we have:

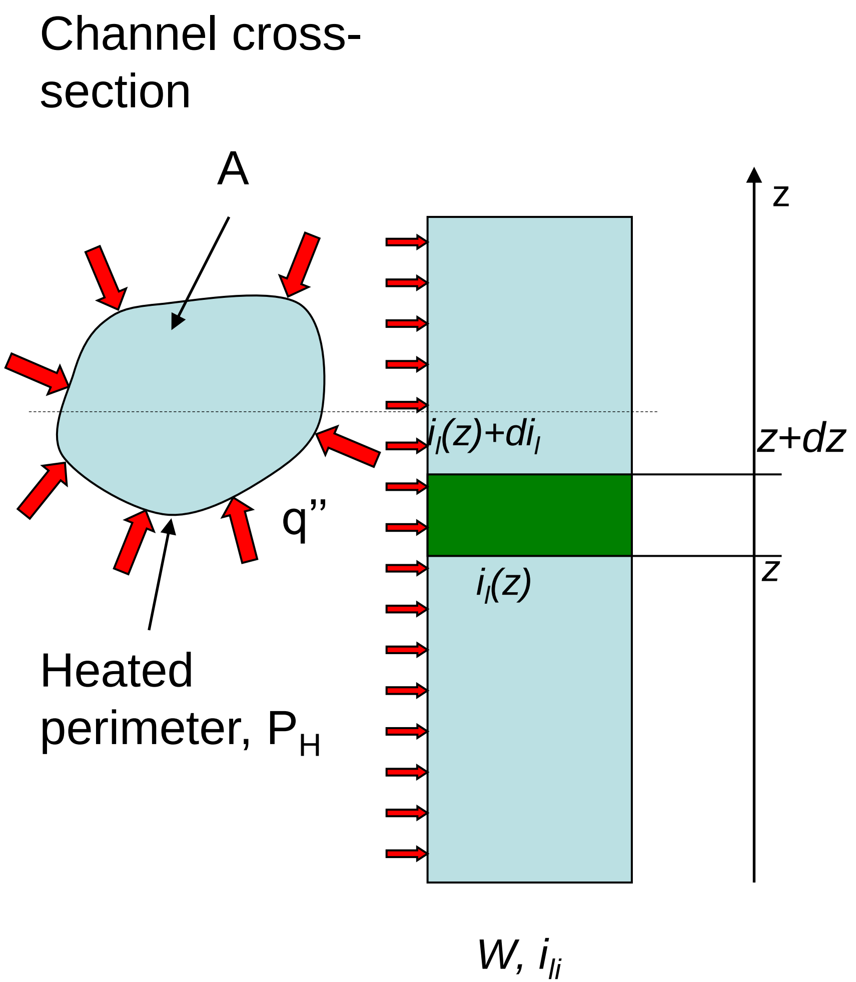
$$D_H = \begin{cases} d_r\left[\frac{4}{\pi}\left(\frac{p}{d_r}\right)^2 - 1\right] & \text{for square lattice} \\ d_r\left[\frac{2\sqrt{3}}{\pi}\left(\frac{p}{d_r}\right)^2 - 1\right] & \text{for triangular lattice} \end{cases}$$

p – lattice pitch

d_r – rod diameter

Coolant Enthalpy Distribution in Heated Channels (1)

- Assume a heated channel as shown in the figure to the right. The channel is uniformly heated along its length with heat flux *q''* [W/m^2], it has a flow cross-section area *A* and heated perimeter P_H.
- The energy balance for a portion of channel *dz* is as follows:



$$W \cdot i_l(z) + q''(z) \cdot P_H(z) \cdot dz = W \cdot \left[i_l(z) + di_l \right]$$

$$\frac{di_l(z)}{dz} = \frac{q''(z) \cdot P_H(z)}{W}$$

W = G*A

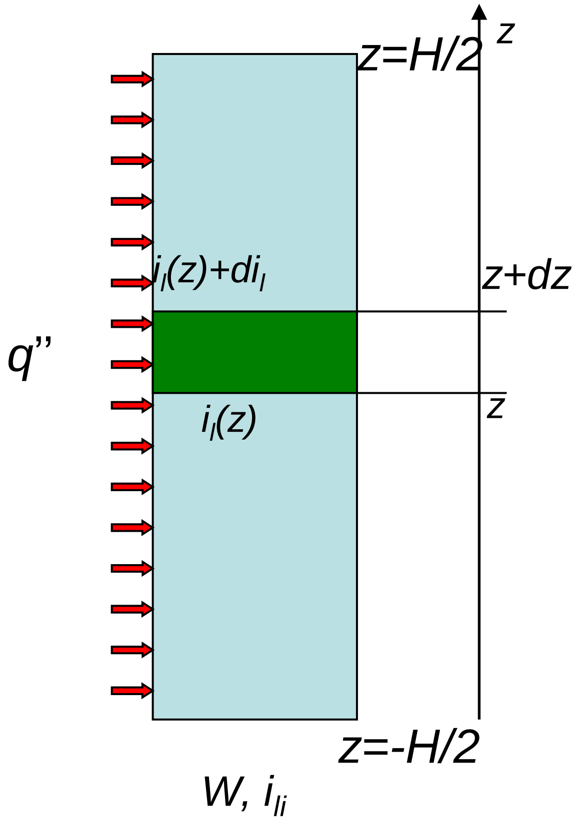
Coolant Enthalpy Distribution in Heated Channels (2)

- Thus, the enthalpy distribution of coolant is described by the following differential equation:

$$\frac{di_l(z)}{dz} = \frac{q''(z) \cdot P_H(z)}{W}$$

- Integration yields

$$i_l(z) = i_{li} + \frac{1}{W}\int_{-H/2}^{z} q''(z) \cdot P_H(z) \cdot dz$$



Coolant Enthalpy Distribution in Heated Channels (3)

- Assuming constant specific heat (calorically perfect fluid) the enthalpy increase can be expressed in terms of the temperature increase as follows:

$di = c_p * dT$

- Using W = G A and assuming a constant channel cross-section area and heat flux distribution, the coolant temperature can be found as,

$$T_{lb}(z) = T_{lbi} + \frac{q'' P_H \left(z + H/2\right)}{c_p G A}$$

$$T_{lb} = \frac{\int_A \rho_l c_{pl} v_l T_l \, dA}{\int_A \rho_l c_{pl} v_l \, dA}$$

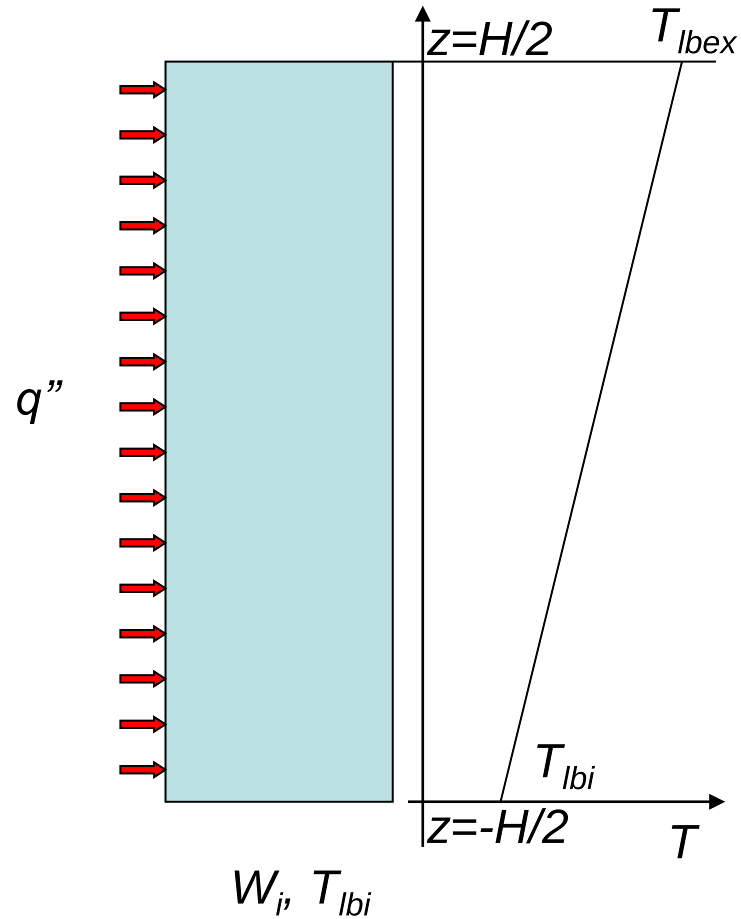
Definition of the bulk liquid temperature

Coolant Enthalpy Distribution in Heated Channels (4)

- The temperature is thus linearly distributed between the inlet and the exit of the assembly

- The exit temperature becomes

$$T_{lbex} = T_{lbi} + \frac{q'' P_H H}{c_p G A}$$

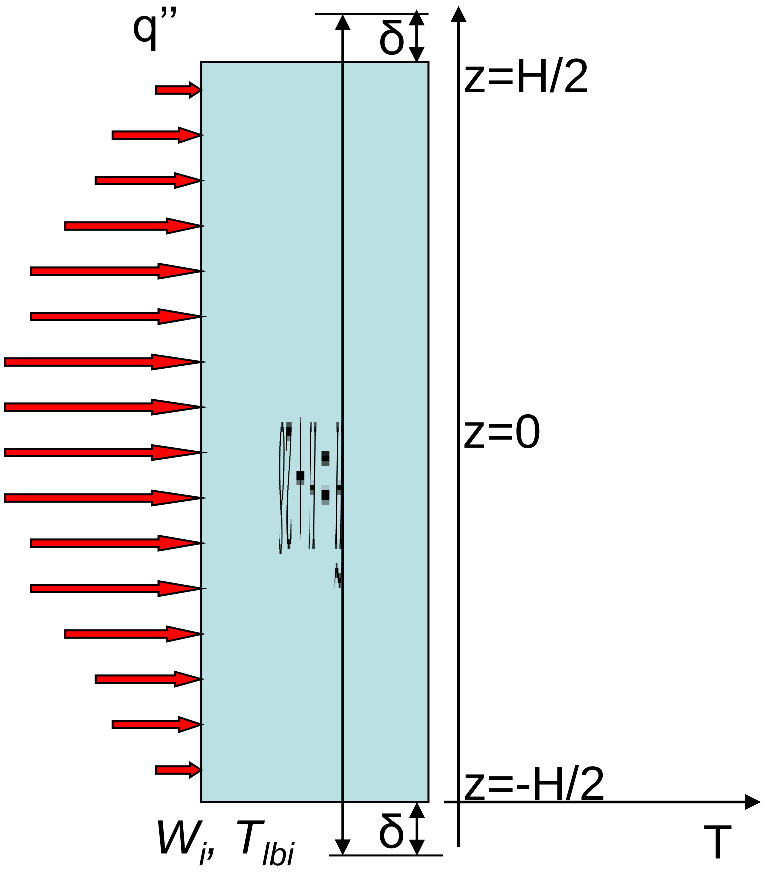


Coolant Enthalpy Distribution in Heated Channels (5)

- Usually the axial power distribution is non-uniform. In a cylindrical reactor the axial power distribution is given by the cosine function:

$$q''(z) = q_0'' \cdot \cos\left(\frac{\pi z}{\tilde{H}}\right)$$

The differential equation for the enthalpy (temperature) distribution is now



$$\frac{di_l(z)}{dz} = \frac{q_0'' \cdot P_H(z)}{W}\cos\left(\frac{\pi z}{\tilde{H}}\right), \quad \text{or} \quad \frac{dT_{lb}(z)}{dz} = \frac{q_0'' \cdot P_H(z)}{W \cdot c_p}\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

Coolant Enthalpy Distribution in Heated Channels (6)

- After integration, (P_H=const) the coolant enthalpy (temperature) distribution is as follows

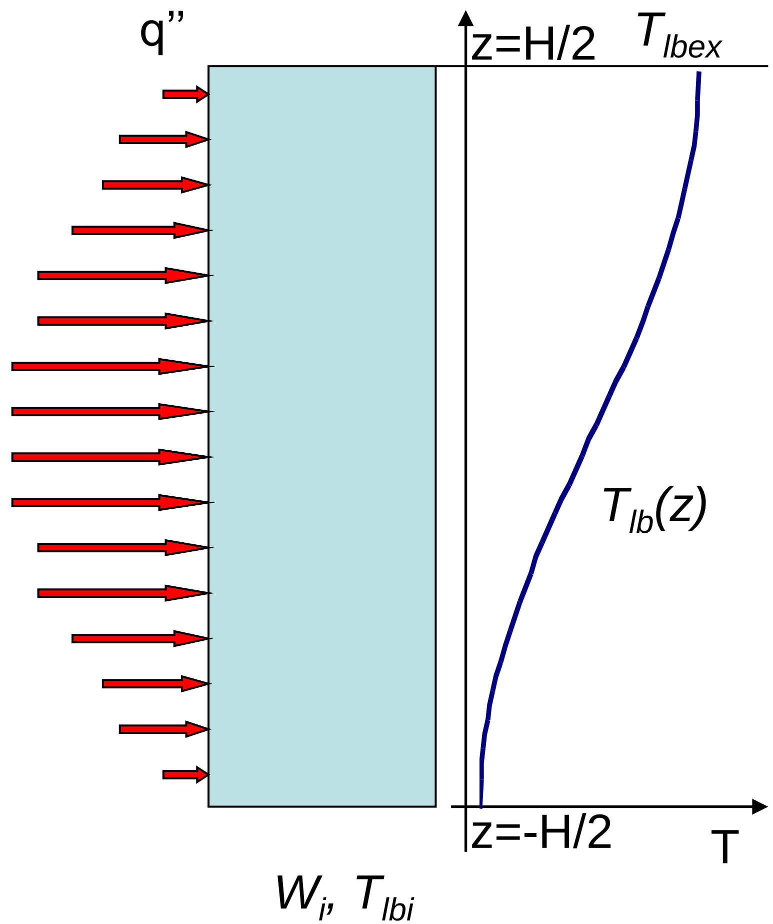
$$i_l(z) = \frac{q_0'' \cdot P_H}{W} \cdot \frac{\tilde{H}}{\pi}\left[\sin\left(\frac{\pi z}{\tilde{H}}\right) + \sin\left(\frac{\pi H}{2\tilde{H}}\right)\right] + i_{li}, \quad or$$

$$T_{lb}(z) = \frac{q_0'' \cdot P_H}{W \cdot c_p} \cdot \frac{\tilde{H}}{\pi}\left[\sin\left(\frac{\pi z}{\tilde{H}}\right) + \sin\left(\frac{\pi H}{2\tilde{H}}\right)\right] + T_{lbi}$$

The exit enthalpy (temperature) can be found as:

$$i_{lex} = i_l(H/2) = \frac{2q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W}\sin\left(\frac{\pi H}{2\tilde{H}}\right) + i_{li}, \quad \text{or}$$

$$T_{lbex} = T_{lb}(H/2) = \frac{2q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}\sin\left(\frac{\pi H}{2\tilde{H}}\right) + T_{lbi}$$



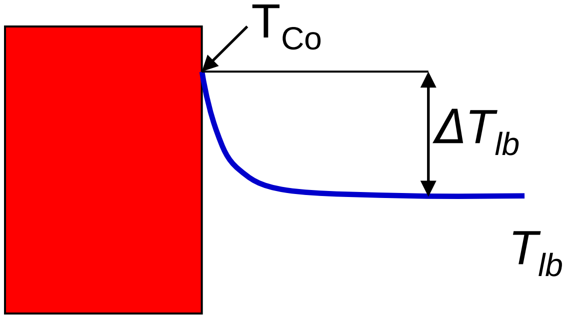
Clad-Coolant Heat Transfer in Channels with Single Phase Flows (1)

- In Light Water Reactors, coolant is sub-cooled at the inlet to the reactor core
- The subcooling is defined as the difference between the saturation temperature and the actual coolant bulk temperature: $\Delta T_{sub} = T_f - T_{lb}$
- For example, if the inlet temperature and pressure of the water coolant are 549 K and 7 MPa, respectively, then the inlet subcooling is equal to 559 K – 549 K = 10 K, since the saturation temperature of water at 7 MPa pressure is equal to 559 K

Clad-Coolant Heat Transfer in Channels with Single Phase Flows (2)

- In the single-phase region, when $z_{in} < z < z_{ONB}$,[*)] the clad surface temperature T_{Co} of the heated wall and the liquid bulk temperature T_{lb} are related to each other as follows,

$$T_{Co} - T_{lb} \equiv \Delta T_{lb} = q''/h$$



- where h is the heat transfer coefficient and ΔT_{lb} is the temperature difference between the surface of the heated wall and the bulk liquid

*) z_{in} – inlet coordinate; ONB – Onset of Nucleate Boiling

Clad-Coolant Heat Transfer in Channels with Single Phase Flows (3)

- The heat transfer coefficient *h* is evaluated from correlations, which, in turn, are based on experimental data and are using the principles of the dimensionless analysis

- The following general relationships are employed

Nu = *f* (Re, Pr, …), where: $\mathrm{Nu} = \frac{hD_h}{\lambda}$ Nusselt number

$\mathrm{Re} = \frac{GD_h}{\mu}$ Reynolds number

, $\mathrm{Pr} = \frac{c_p \mu}{\lambda}, \quad \mathrm{Pr}_w = \left. \frac{c_p \mu}{\lambda} \right|_{T=T_w}$ Prandtl number

Clad-Coolant Heat Transfer in Channels with Single Phase Flows (4)

- For flows in pipes, rectangular channel and annuli, and with $10^4 < Re$, $0.7 < Pr < 160$ and $L/D_h > 60$, the following correlation can be used (Colburn):

$$\mathrm{Nu} = 0.023 \cdot \mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.33}$$

- Another correlation frequently used for heat transfer calculations in pipes was given by Dittus&Boelter:

$$\mathrm{Nu} = 0.023 \cdot \mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}$$

n=0.4 for heating
n=0.3 for cooling

valid for $L/D_h > 60$, $Re > 10^4$ and $0.7 < Pr < 100$

Heat Transfer in Rod Bundles (1)

Heat transfer in the entire bundle is calculated from a single correlation including effects of:

- flow conditions
- fluid properties
- geometry

Typically the correlation is of the form:

$$Nu = F(Re, Pr, D_h/d_r, p/d_r, \dots)$$

Heat Transfer in Rod Bundles (2)

The influence of flow/fluid conditions and geometry factors can be separated:

$Nu = F_1(Re, Pr,...) \times F_2(D_h/d_r, p/d_r,...)$

p – lattice pitch
d_r – rod diameter

Example: the Weisman (1959) correlation:

$$\mathrm{Nu} = A \cdot \mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

$$A = \begin{cases} 0.026\, p/d_r - 0.006 & \text{triangular } 1.1 < p/d_r < 1.5 \\ 0.042\, p/d_r - 0.024 & \text{square } 1.1 < p/d_r < 1.3 \end{cases}$$

Heat Transfer in Rod Bundles (3)

- Subotin et al. (1975) recommended for heat transfer to liquids in bundles

$$Nu = A \cdot \mathrm{Re}^{0.8} \, \mathrm{Pr}^{0.4} \qquad A = 0.0165 + 0.02\left[1 - \frac{0.91}{\left(p/d_r\right)^2}\right]\left(\frac{p}{d_r}\right)^{0.15}$$

Triangular lattice with $1.1<p/d_r<1.8$; $1.0<Pr<20$; $5\ 10^3<Re<5\ 10^5$

- For gas flow in tight rod bundles Ajn and Putjkov (1964) give

$$\frac{Nu_{bundle}}{Nu_{DB}} = 1.184 + 0.351 \cdot \log_{10}\left(p/d_r - 1\right) \qquad 1.03<p/d_r<2.4$$

Nu_{DB} – Dittus-Boelter correlation

p – lattice pitch
d_r – rod diameter

Heat Transfer in Rod Bundles (4)

- In the cited correlations it is assumed that the flow/fluid conditions and the geometry effect are separable

- This, however, seems not to be valid based on an extensive study done by Markoczy (1972)

- He suggested the following form of the correlation

$$Nu_{bundle} = Nu_{pipe} \times F_{geom}(p/d_r, Re, Pr)$$

In other words, the geometry effect is flow/property-dependent

Heat Transfer in Rod Bundles (5)

Markoczy (1972) performed study of experimental data (over 63 bundles of different geometry)

He proposed the following correlation:

$$\frac{\mathrm{Nu}_{bundle}}{\mathrm{Nu}_{DB}} = 1 + 0.91\,\mathrm{Re}^{-0.1}\,\mathrm{Pr}^{0.4}\left(1 - 2e^{-B}\right) \qquad B = \begin{cases} \dfrac{2\sqrt{3}}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1 & \text{triangular} \\ \dfrac{4}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1 & \text{square} \end{cases}$$

Validity region: $3\ 10^3 < \mathrm{Re} < 10^6$; $0.66 < \mathrm{Pr} < 5$; $1.02 < p/d_r < 2.5$

Heat Transfer in Rod Bundles (6)

- In summary, the bundle-wide approach is based on:
  - base correlation, which typically takes into account dependence of the heat transfer coefficient on flow/property conditions
  - geometry factor, which takes into account the dependence on pitch/rod-diameter

$$Nu_{bundle} = F_{geo}(p/d_r,\ldots) \text{ x } Nu_{base}(Re, Pr, ..)$$

Heat Transfer in Rod Bundles (7)

- Occasionally another approach can be encountered in the literature:

- Osmachkin (1974) recommended to use a correlation valid for pipes (e.g. Dittus-Boelter), replacing the hydraulic diameter with the "effective" one:

$$D_{eff} = \frac{2}{(1-\varepsilon)^2}\left(\frac{\varepsilon-3}{2} - \frac{\ln\varepsilon}{1-\varepsilon}\right)D_h$$

ε – fraction of the bundle cross-section occupied by rods:
$\varepsilon = A_r/A_{tot}$; A_r – rod cross-section area, A_{tot} – total (rod+coolant) cross section area

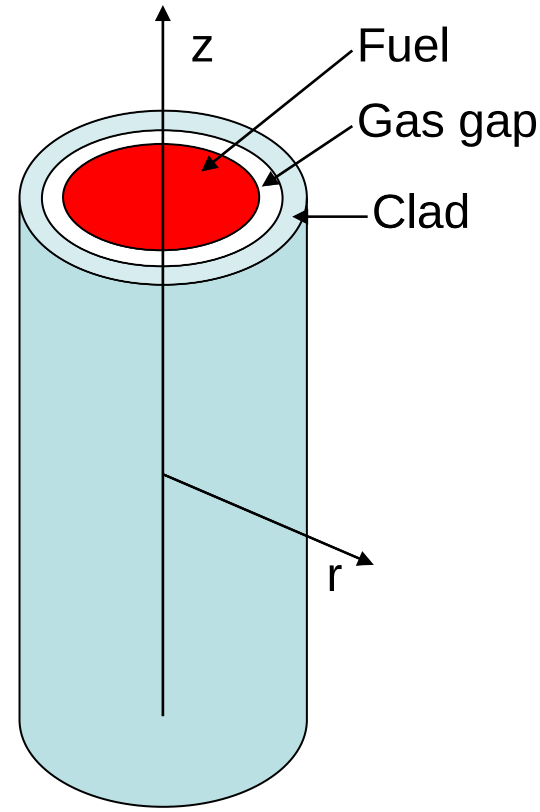
Heat conduction in reactor fuel elements (1)

- In the cylindrical coordinate system, for a fuel rod as shown in figure, the conduction equation can be written as

$$\nabla \cdot \lambda \nabla T = -q'''(\mathbf{r})$$

$$\Downarrow$$

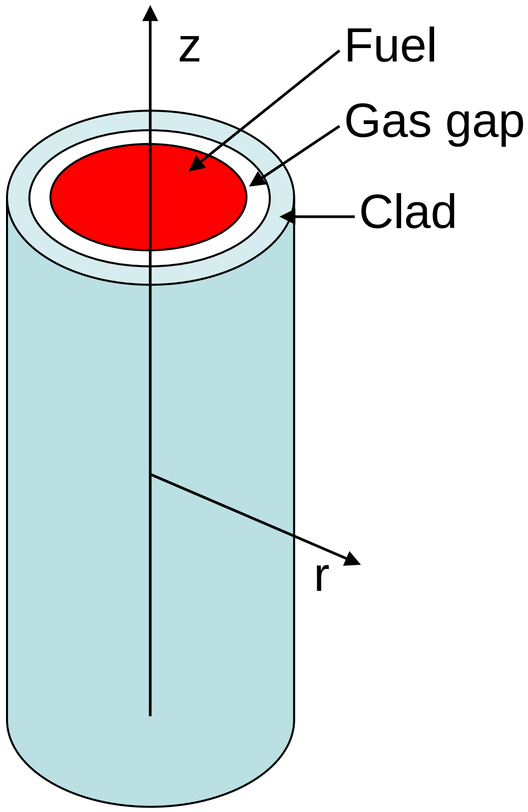
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T(r,z)}{\partial r}\right)+\frac{\partial}{\partial z}\left[\lambda\frac{\partial T(r,z)}{\partial z}\right]=-q'''(r,z)$$



Fuel element

Heat conduction in reactor fuel elements (2)

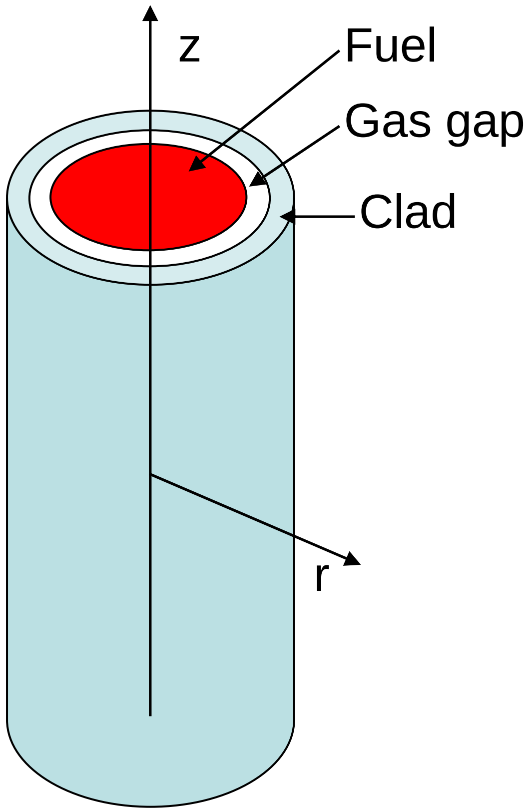
- The conduction equation can be further simplified:
  - Heat conduction in the *z*-direction can be neglected, since temperature gradient dT/dz is much lower than dT/dr
  - In fuel region $q''' = q'''(z)$
  - In gas gap and clad regions $q'''=0$



Fuel element

Heat conduction in reactor fuel elements (3)

- The conduction equation can be thus written for each region separately as:
  - Fuel $$\frac{1}{r}\frac{d}{dr}\left(r\lambda_F \frac{dT_F(r)}{dr}\right) = -q'''(z)$$
  - Gap $$\frac{1}{r}\frac{d}{dr}\left(r\lambda_G \frac{dT_G(r)}{dr}\right) = 0$$
  - Clad $$\frac{1}{r}\frac{d}{dr}\left(r\lambda_C \frac{dT_C(r)}{dr}\right) = 0$$



Fuel element

Heat conduction in reactor fuel elements (4)

- To solve the ordinary differential equations we need boundary conditions:
  - Finite temperature at r = 0
  - 4th kind b.c. at $r = r_{Fo}$

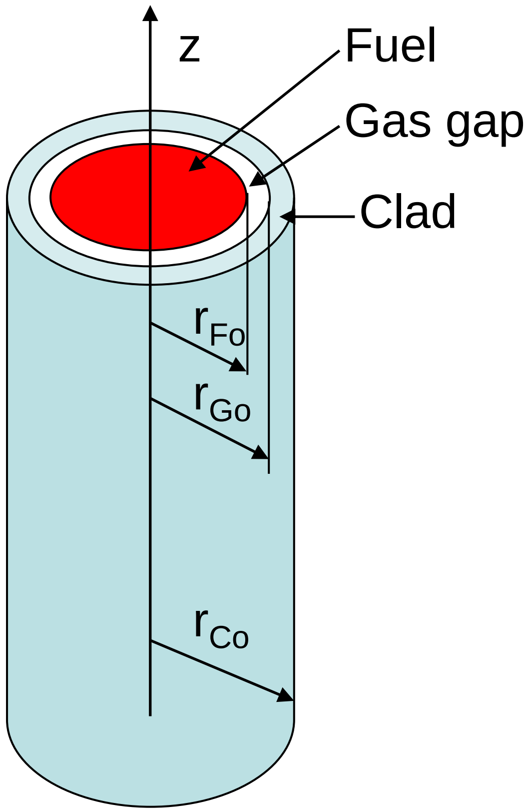
$$T_F\big|_{r=r_{Fo}} = T_G\big|_{r=r_{Fo}} \qquad \lambda_F \frac{dT_F}{dr}\bigg|_{r=r_{Fo}} = \lambda_G \frac{dT_G}{dr}\bigg|_{r=r_{Fo}}$$

  - 4th kind b.c. at $r = r_{Go}$

$$T_G\big|_{r=r_{Go}} = T_C\big|_{r=r_{Go}} \qquad \lambda_G \frac{dT_G}{dr}\bigg|_{r=r_{Go}} = \lambda_C \frac{dT_C}{dr}\bigg|_{r=r_{Go}}$$

  - 3rd kind b.c. at $r = r_{Co}$

$$-\lambda_C \frac{dT_C}{dr}\bigg|_{r=r_{Co}} = h\left(T_{Co} - T_{lb}\right)$$



Fuel element

Heat conduction in reactor fuel elements (5)

- Solution in the fuel region

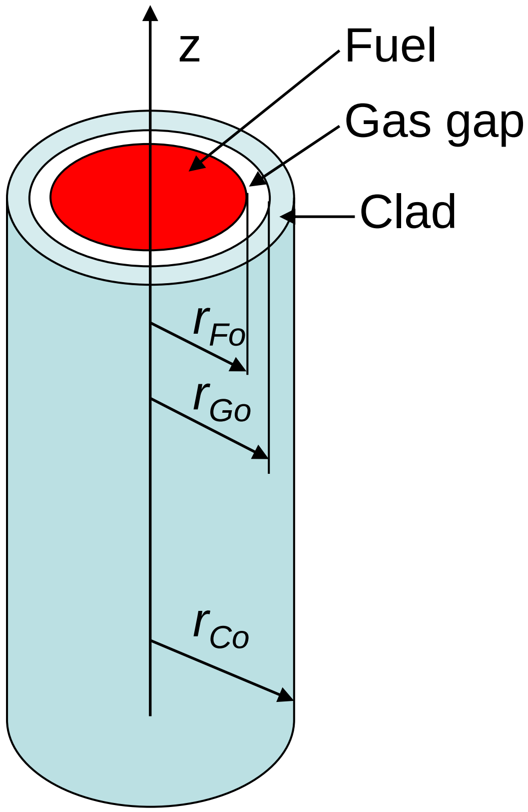
$$\frac{1}{r}\frac{d}{dr}\left(r\lambda_F \frac{dT_F(r)}{dr}\right) = -q'''(z)$$

$$\Downarrow$$

$$\lambda_F \frac{dT_F(r)}{dr} = -\frac{1}{r}\int q'''(z)\cdot r\cdot dr = -\frac{q'''(z)\cdot r}{2} + \frac{C}{r}$$

  - To limit $T_{Fc}=T_F(0)$, the constant C must be equal to zero: C = 0, thus

$$\lambda_F \frac{dT_F(r)}{dr} = -\frac{q'''(z)\cdot r}{2}$$



Fuel element

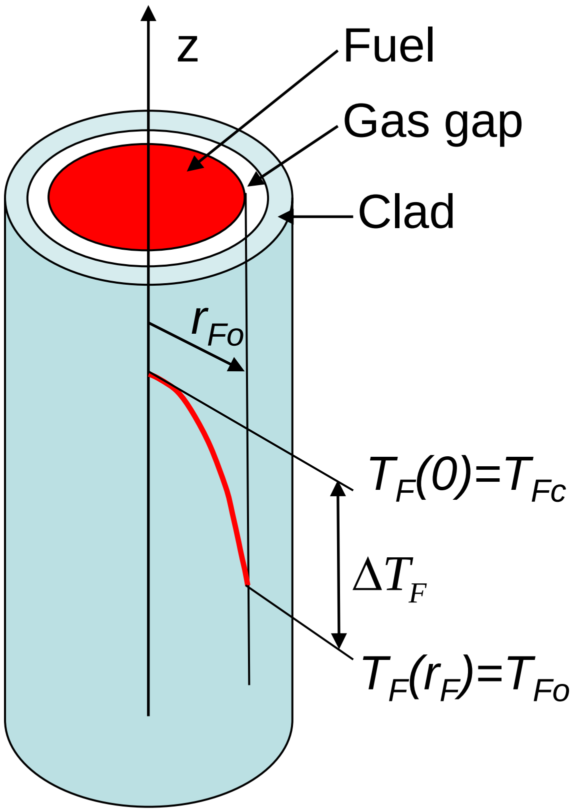
Heat conduction in reactor fuel elements (6)

- If the conductivity of the fuel material is assumed constant, the integration is straightforward as

$$T_F(r_F) - T_F(0) \equiv -\Delta T_F = -\int_0^{r_{Fo}} \frac{q'''(z)\cdot r}{2\cdot\lambda_F} dr$$

or, after integration the temperature rise in fuel region is as follows

$$\Delta T_F(z) \equiv T_F(0) - T_F(r_F) = T_{Fc} - T_{Fo} = \frac{q'''(z)\cdot r_{Fo}^2}{4\cdot\lambda_F}$$



Fuel element

Heat conduction in reactor fuel elements (7)

- Solution in the gas gap

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda_G \frac{dT_G(r)}{dr}\right) = 0$$

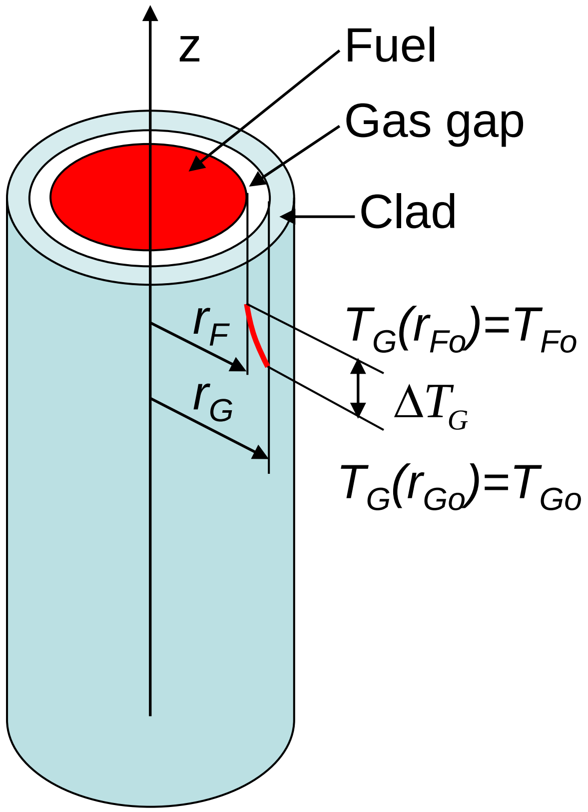
$$\Downarrow$$

$$\lambda_G \frac{dT_G(r)}{dr} = \frac{C'}{r} \Rightarrow T_G(r) = \frac{C'}{\lambda_G}\ln(r) + C''$$

  - Where C' and C'' are constants

- Temperature drop in gap is

$$\Delta T_G \equiv T_G(r_{Fo}) - T_G(r_{Go}) = -\frac{C'}{\lambda_G}\ln\left(\frac{r_{Go}}{r_{Fo}}\right)$$



Fuel element

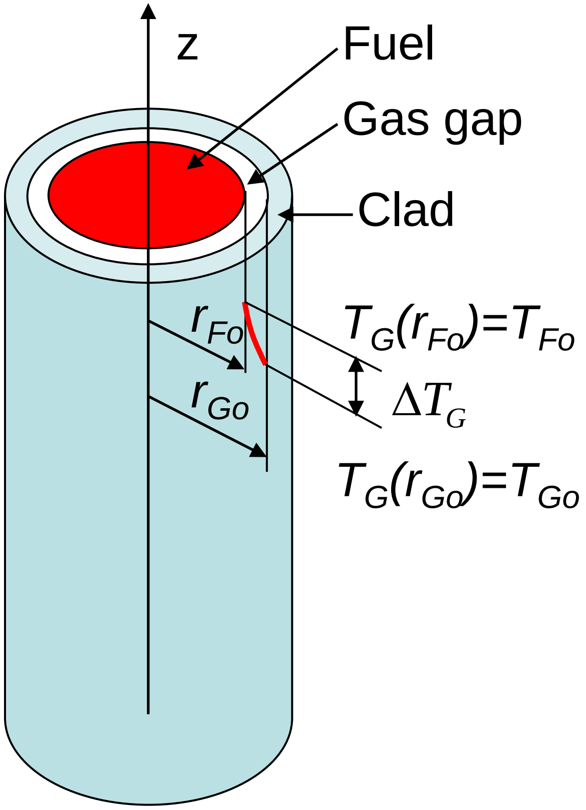
Heat conduction in reactor fuel elements (8)

- The constant C’ can be found from the energy balance at the fuel-gap interface:

$$\left.\begin{aligned} q''\big|_{r_{Fo}} &= -\lambda_G \left.\frac{dT_G(r)}{dr}\right|_{r_{Fo}} = -\frac{C'}{r_{Fo}} \\ q''\big|_{r_{Fo}} \cdot 2\pi r_{Fo} \cdot dz &= q''' \cdot \pi r_{Fo}^2 \cdot dz \end{aligned}\right\} \Rightarrow C' = -\frac{q''' r_{Fo}^2}{2}$$

$$\Downarrow$$

$$\Delta T_G = \frac{q''' r_{Fo}^2}{2\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right)$$



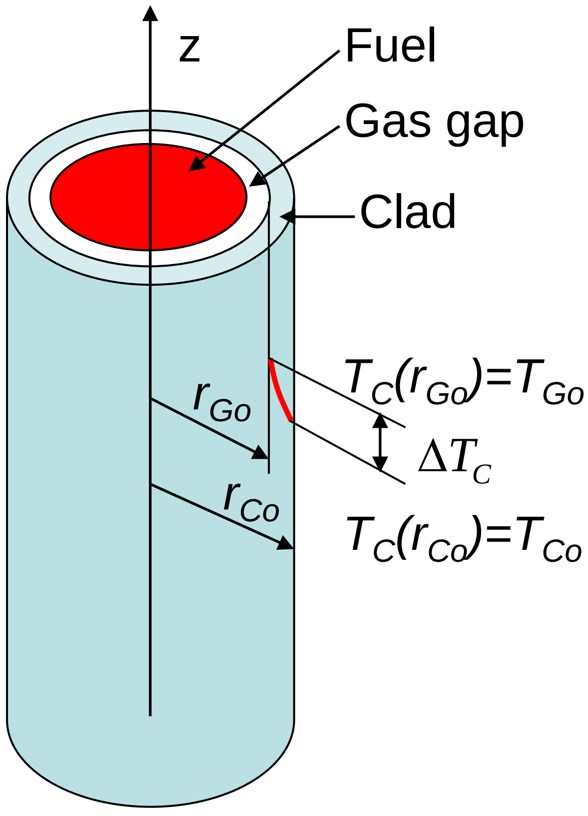
Fuel element

Heat conduction in reactor fuel elements (9)

- Since the conduction equation is the same in the clad region, the temperature rise in the clad is found as

$$\left. \begin{aligned} q''\big|_{r_{Go}} &= -\lambda_C \left.\frac{dT_C(r)}{dr}\right|_{r_{Go}} = -\frac{C'}{r_{Go}} \\ q''\big|_{r_{Go}} \cdot 2\pi r_{Go} \cdot dz &= q''' \cdot \pi r_{Fo}^2 \cdot dz \end{aligned} \right\} \Rightarrow C' = -\frac{q''' r_{Fo}^2}{2}$$

$$\Delta T_C = \frac{q''' r_{Fo}^2}{2\lambda_C} \ln\left(\frac{r_{Co}}{r_{Go}}\right)$$



Fuel element

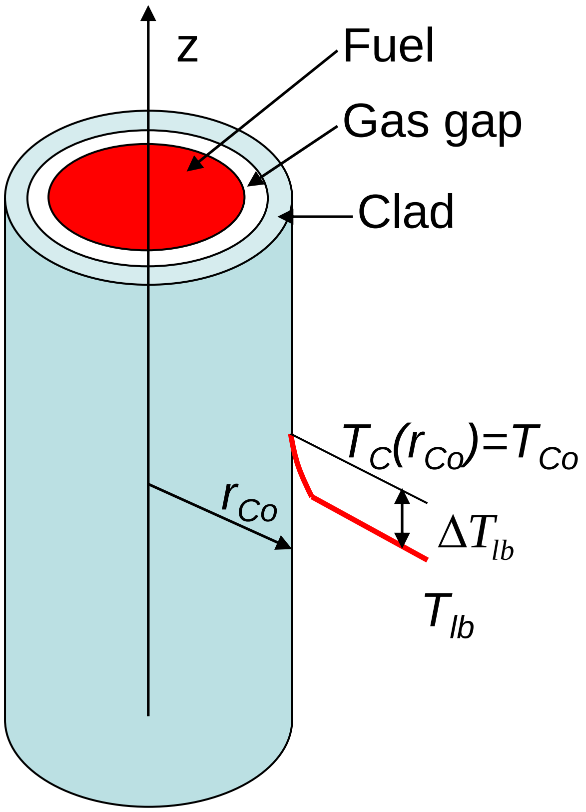
Heat conduction in reactor fuel elements (10)

- Finally, the temperature rise in the thermal boundary layer in coolant can be found from the Newton equation for the convective heat transfer:

$$q''\big|_{r_{Co}} = h \cdot \left(T_{Co} - T_{lb}\right) = h \cdot \Delta T_{lb}$$

since $q''\big|_{r_{Co}} \cdot 2\pi r_{Co} \cdot dz = q''' \cdot \pi r_{Fo}^2 \cdot dz \Rightarrow q''\big|_{r_{Co}} = \dfrac{q''' r_{Fo}^2}{2 r_{Co}}$

thus $$\Delta T_{lb} = \frac{q''' r_{Fo}^2}{2 r_{Co} h}$$



Fuel element

Heat conduction in reactor fuel elements (11)

- The total temperature rise in the fuel element is thus

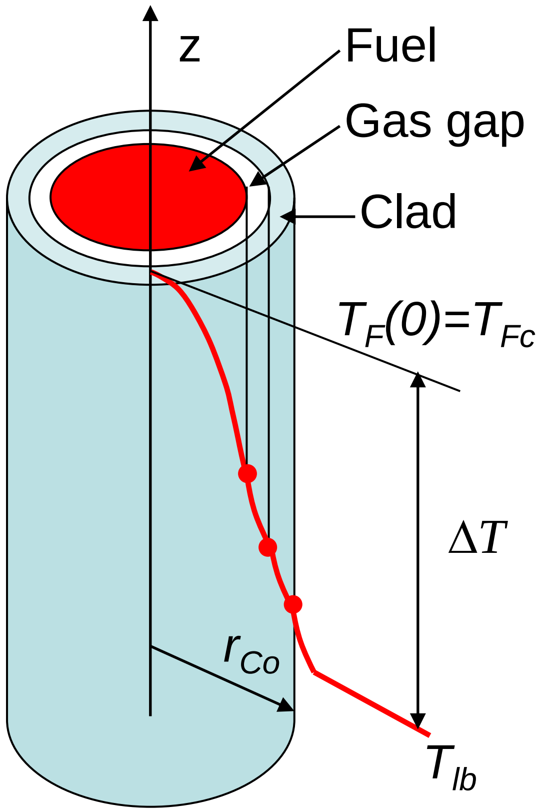
$$\Delta T = \Delta T_F + \Delta T_G + \Delta T_C + \Delta T_{lb} = T_{Fc} - T_{lb}$$

$$\Delta T = \frac{q''' r_{Fo}^2}{4\lambda_F} + \frac{q''' r_{Fo}^2}{2\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{q''' r_{Fo}^2}{2\lambda_C} \ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{q''' r_{Fo}^2}{2 r_{Co} h} =$$

$$\frac{q''' r_{Fo}^2}{4}\left[\frac{1}{\lambda_F} + \frac{2}{\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{2}{\lambda_C} \ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{2}{r_{Co} h}\right]$$

Since $q''' \pi r_{Fo}^2 = q'$ (linear power density)

$$\Delta T = \frac{q'}{4\pi}\left[\frac{1}{\lambda_F} + \frac{2}{\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{2}{\lambda_C} \ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{2}{r_{Co} h}\right]$$



Fuel element

Heat conduction in reactor fuel elements (12)

- For constant fuel conductivity the temperature distribution was obtained as

$$T_F(r) = -\frac{r^2}{4\lambda_F} q''' + C$$

- If the fuel conductivity is considered as a function of a temperature, the integration has to be performed as follows:

$$\lambda_F dT_F = -\frac{r}{2} q''' dr \Rightarrow \int_{T_{Fc}}^{T_{Fo}} \lambda_F dT = -\frac{q'''}{2} \int_0^{r_{Fo}} r dr = -\frac{r_{Fo}^2}{4} q'''$$

Heat conduction in reactor fuel elements (13)

- Introducing the average fuel conductivity given as:

$$\langle \lambda_F \rangle = \frac{1}{T_{Fc} - T_{Fo}} \int_{T_{Fo}}^{T_{Fc}} \lambda_F \, dT$$

- The total temperature drop in the fuel can be found as:

$$\Delta T_F \equiv T_{Fc} - T_{Fo} = \frac{q''' r_{Fo}^2}{4 \langle \lambda_F \rangle}$$

- Or, using the linear power density $q' \equiv \pi r_{Fo}^2 q'''$

$$\Delta T_F = \frac{q'}{4\pi \langle \lambda_F \rangle}$$

We will discuss this case in lecture 3 in more detail

Non-uniform heat flux distribution (1)

- For non-uniform (cosine) heat flux distribution

$$q''(z) = q_0'' \cdot \cos\left(\frac{\pi z}{\tilde{H}}\right) \qquad T_{lb}(z) = \frac{q_0'' \cdot P_H}{W \cdot c_p} \cdot \frac{\tilde{H}}{\pi}\left[\sin\left(\frac{\pi z}{\tilde{H}}\right) + \sin\left(\frac{\pi H}{2\tilde{H}}\right)\right] + T_{lbi}$$

Substituting the above to

$$q'' = h\left(T_{Co} - T_{lb}\right) \Rightarrow T_{Co} = T_{lb} + \frac{q''}{h}$$

yields the following outer clad temperature

$$T_{Co}(z) = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p} \cdot \left[\sin\left(\frac{\pi z}{\tilde{H}}\right) + \sin\left(\frac{\pi H}{2\tilde{H}}\right)\right] + \frac{q_0''}{h} \cdot \cos\left(\frac{\pi z}{\tilde{H}}\right) + T_{lbi}$$

Non-uniform heat flux distribution (2)

- The temperature distribution can be re-written in short as

$$T_{Co}(z) = A + B\sin\left(\frac{\pi z}{\tilde{H}}\right) + C_{Co}\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

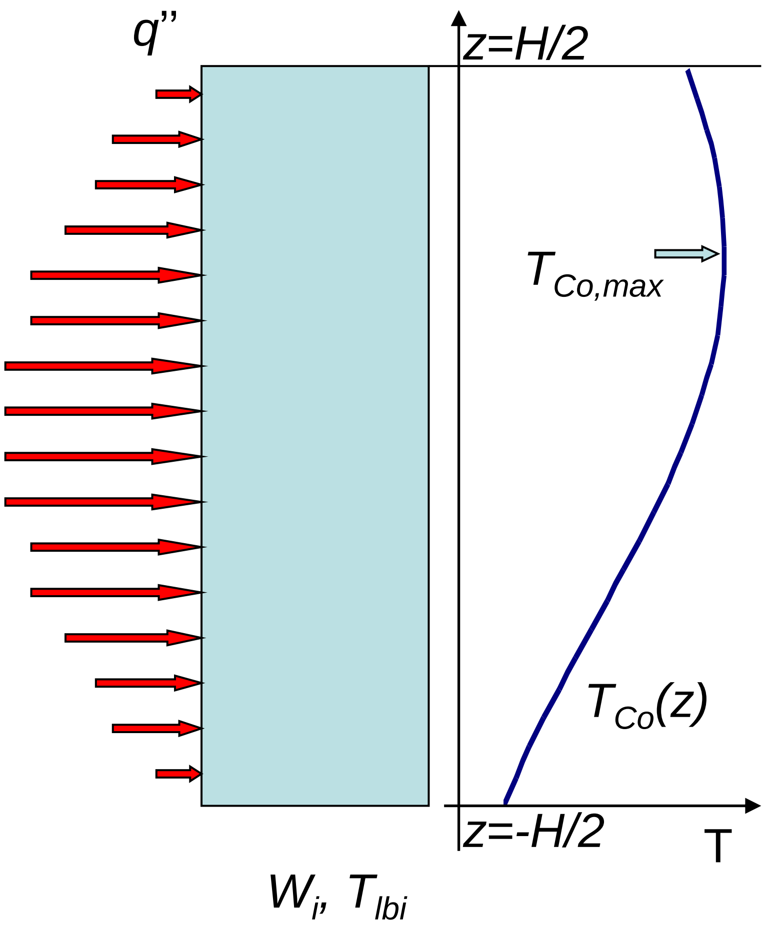
- where

$$A = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}\sin\left(\frac{\pi H}{2\tilde{H}}\right) + T_{lbi}, \quad B = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}, \quad C_{Co} = \frac{q_0''}{h}$$

Non-uniform heat flux distribution (3)

- Figure to the right shows the clad temperature distribution assuming the cosine axial power distribution

- It should be noted that the temperature of the clad outer surface gets its maximum value $T_{Co,max}$ at a certain location $z_{Co,max}$ different from z=0 and $z=H/2$



Non-uniform heat flux distribution (4)

- The location of the maximum clad temperature can be found as:

$$\frac{dT_{Co}(z)}{dz} = 0 \quad \Rightarrow \quad B\cos\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) - C_{Co}\sin\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) = 0$$

$$\tan\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) = \frac{B}{C_{Co}} \quad \Rightarrow \quad z_{Co,\max} = \frac{\tilde{H}}{\pi}\arctan\left(\frac{B}{C_{Co}}\right)$$

- Substituting $z = z_{Co,max}$ in the equation for the clad temperature yields the maximum clad temperature

$$T_{Co,\max} = A + B\sin\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) + C_{Co}\cos\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)$$

Non-uniform heat flux distribution (5)

- Noting that:

$$\sin\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) = \pm\frac{\tan\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)}{\sqrt{1+\tan^2\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)}} = \pm\frac{\frac{B}{C_{Co}}}{\sqrt{1+\left(\frac{B}{C_{Co}}\right)^2}}$$

and

$$\cos\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) = \pm\frac{1}{\sqrt{1+\tan^2\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)}} = \pm\frac{1}{\sqrt{1+\left(\frac{B}{C_{Co}}\right)^2}}$$

- The maximum temperature becomes (taking only + sign above, since $z_{Co,max} > 0$):

$$T_{Co,\max} = A + \sqrt{B^2 + C_{Co}^2}$$

Non-uniform heat flux distribution (6)

- Since the clad maximum temperature is located on the inner surface, it is of interest to find it

$$T_{Ci}(z) = \Delta T_C + T_{Co}(z) =$$

$$\frac{q'}{2\pi\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{q_0''\cdot P_H\cdot\tilde{H}}{\pi\cdot W\cdot c_p}\cdot\left[\sin\left(\frac{\pi z}{\tilde{H}}\right)+\sin\left(\frac{\pi H}{2\tilde{H}}\right)\right]+\frac{q_0''}{h}\cdot\cos\left(\frac{\pi z}{\tilde{H}}\right)+T_{lbi} =$$

$$\frac{q_0''\cdot P_H\cdot\tilde{H}}{\pi\cdot W\cdot c_p}\cdot\left[\sin\left(\frac{\pi z}{\tilde{H}}\right)+\sin\left(\frac{\pi H}{2\tilde{H}}\right)\right]+q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Ci}}+\frac{1}{h}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)+T_{lbi}$$

Non-uniform heat flux distribution (7)

- This temperature can be written again in a short form as

$$T_{Ci}(z) = A + B\sin\left(\frac{\pi z}{\tilde{H}}\right) + C_{Ci}\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

- where

$$A = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}\sin\left(\frac{\pi H}{2\tilde{H}}\right) + T_{lbi}, \quad B = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}, \quad C_{Ci} = q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{1}{h}\right)$$

location and value of the maximum temperature are found in a similar way as for the outer surface:

$$z_{Ci,\max} = \frac{\tilde{H}}{\pi}\arctan\frac{B}{C_{Ci}} \qquad T_{Ci,\max} = A + \sqrt{B^2 + C_{Ci}^2}$$

Non-uniform heat flux distribution (8)

- The **fuel temperature** can be written in short form as

$$T_{Fc}(z) = A + B\sin\left(\frac{\pi z}{\tilde{H}}\right) + C_{Fc}\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

- where

$$C_{Fc} = q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{r_{Co}}{\lambda_G}\ln\frac{r_{Go}}{r_{Gi}} + \frac{r_{Co}}{2\langle\lambda_F\rangle} + \frac{1}{h}\right)$$

and

$$A = \frac{q_0''\cdot P_H\cdot\tilde{H}}{\pi\cdot W\cdot c_p}\sin\left(\frac{\pi H}{2\tilde{H}}\right) + T_{lbi}, \quad B = \frac{q_0''\cdot P_H\cdot\tilde{H}}{\pi\cdot W\cdot c_p}$$

Non-uniform heat flux distribution (9)

- Thus, the location of the maximum fuel temperature and its value are found

$$z_{Fc,\max} = \frac{\tilde{H}}{\pi} \arctan \frac{B}{C_{Fc}} \qquad T_{Fc,\max} = A + \sqrt{B^2 + C_{Fc}^2}$$