Sustainable Energy Transformation Technologies, SH2706

Lecture No 16

Title:
Design and Operation of Nuclear Power Plants

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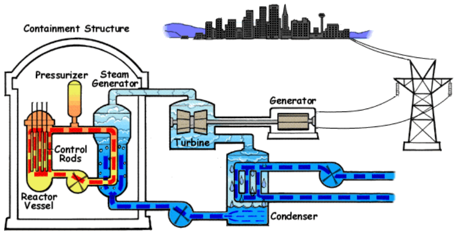
Outline

- Overview of major existing designs
- Principles of operation
  - neutron life cycle
  - six-factor formula
  - reactor kinetics and dynamics
- Fuel conversion and breeding
- Future perspectives

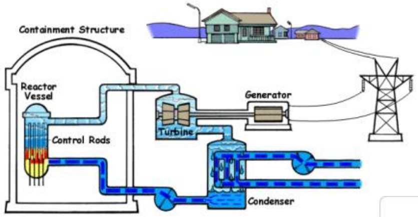
Introduction

Main Reactor Types

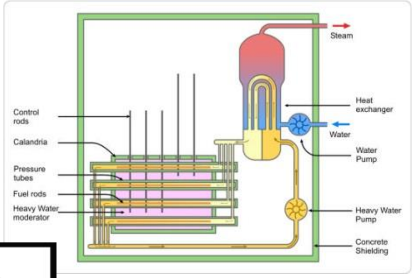
- Main reactor types
  - Pressurized Water Reactor (PWR)
    (Steam generated in steam generator)



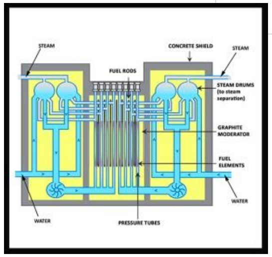
  - Boiling Water Reactor (BWR)
    (Steam generated in reactor)



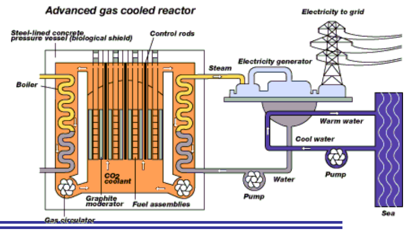
  - Pressurized Heavy Water Reactor (PHWR)
    (Heavy water used as moderator)



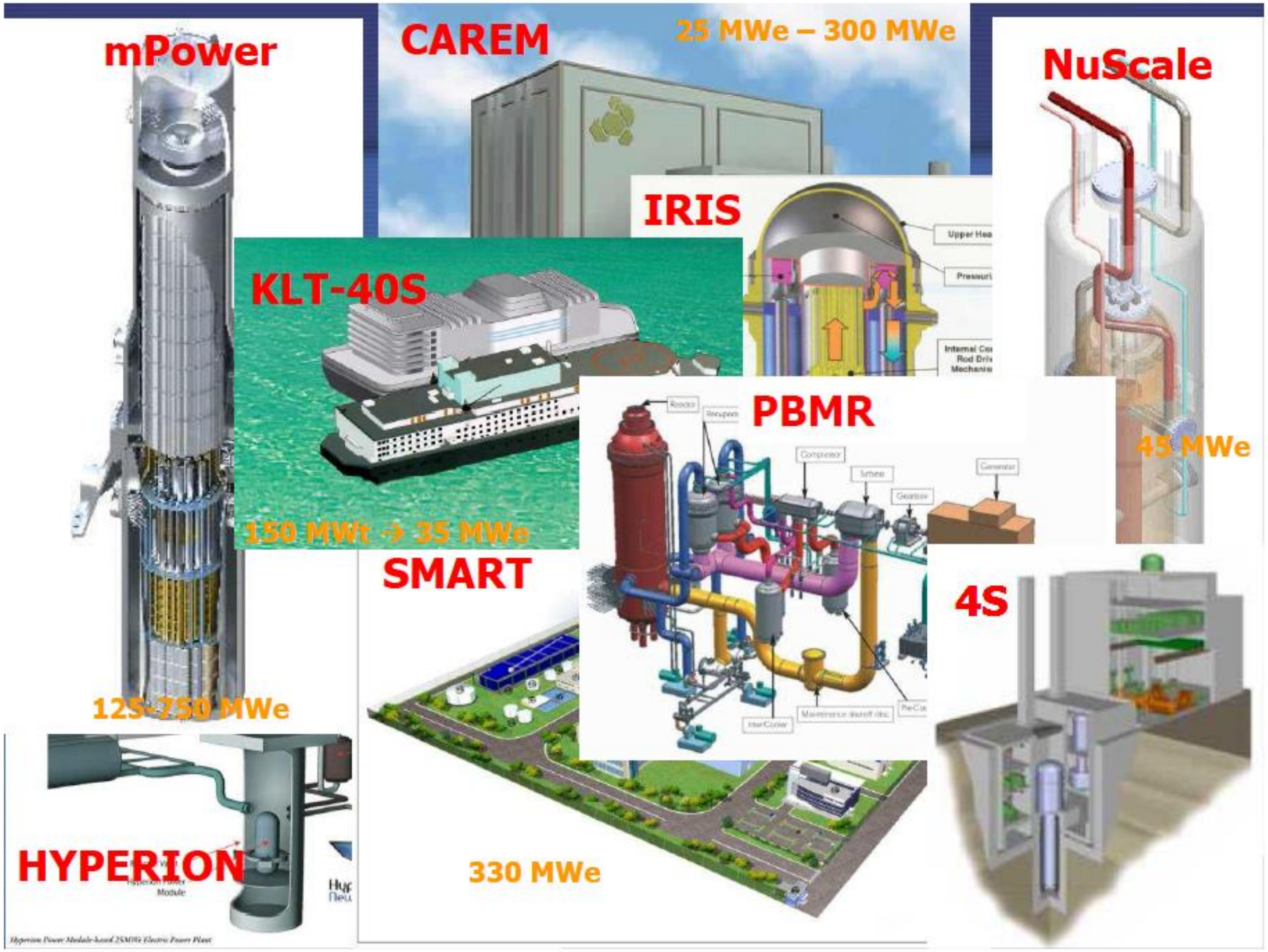
  - Light Water Graphite Reactor (LWGR)
    (Graphite used as moderator)



  - Advanced Gas Reactor (AGR)
    (Gas used as coolant)



SMRs - Small Modular Reactors



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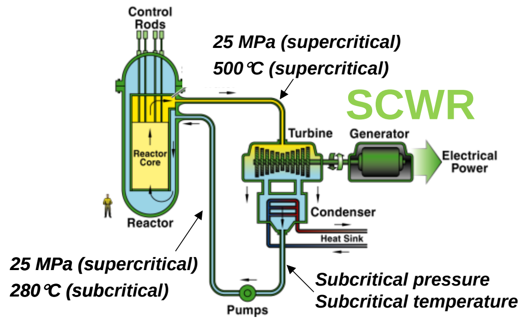
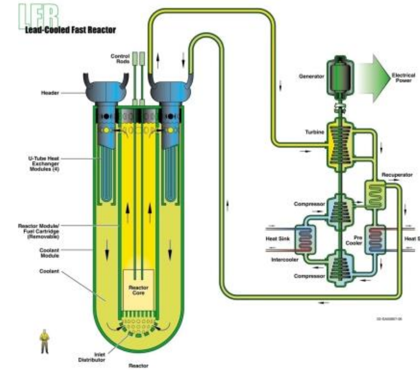
Generation IV Reactor Types

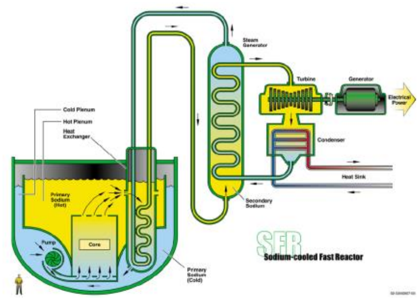
The Generation IV initiative established in 2001 by 12 countries and EU

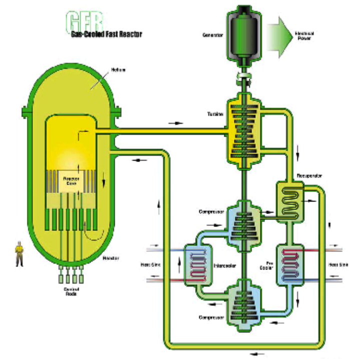
Goals for Gen-IV:

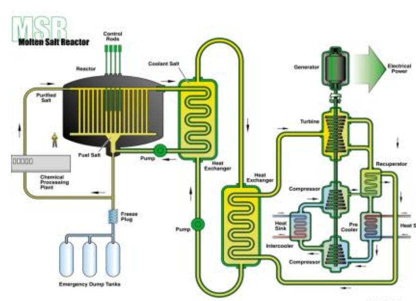
- Safe, competitive and reliable production of electricity and/or heat
- Meet stringent safety requirements
- Maximum use of fuel and produce minimum waste
- Proliferation resistant
- Meet the public requirements on energy production units

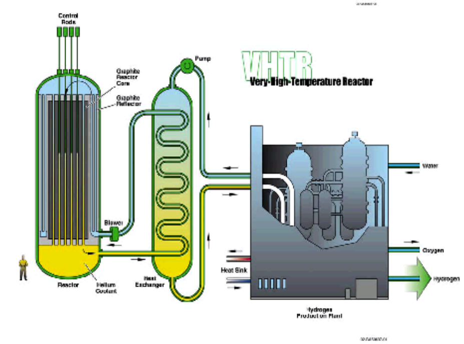








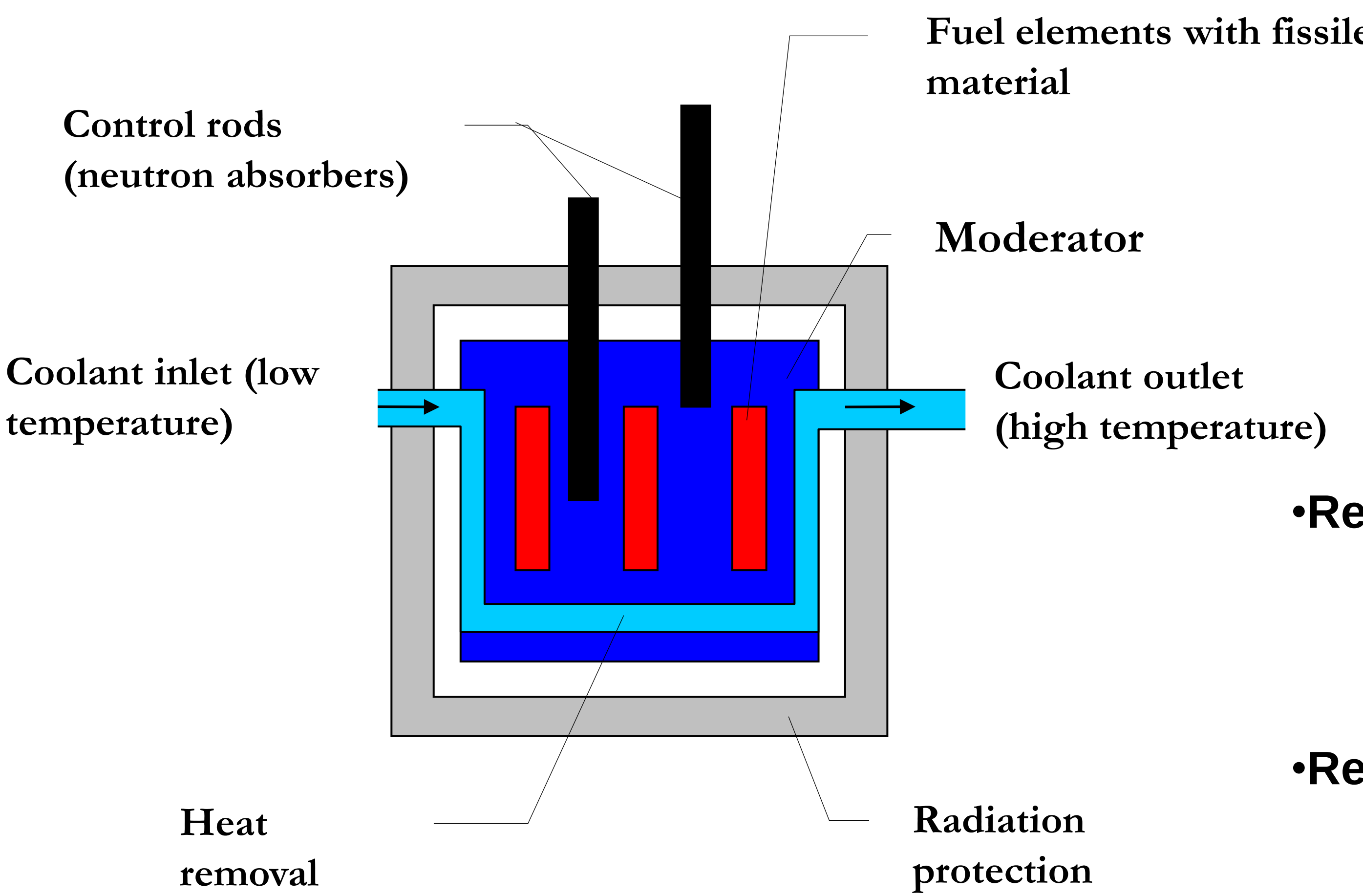




Characteristics of Gen IV designs

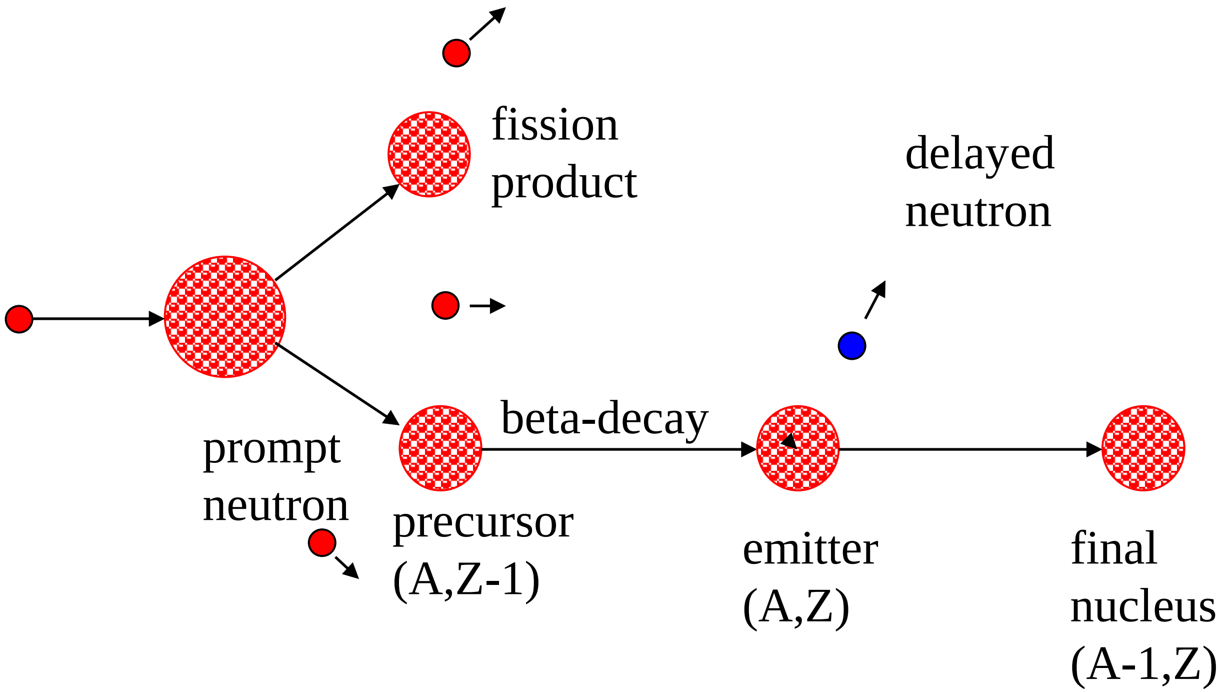
	Neutron Spectrum	*Fuel Cycle*	*Size*	*Applications*	*R&D*
Sodium Cooled Fast Reactor (SFR)	**Fast**	**Closed**	**Med to Large**	**Electricity, Actinide Mgmt. (AM)**	**Advanced Recycle**
Lead-alloy Cooled Fast Reactor (LFR)	**Fast**	**Closed**	**Small to Large**	**Electricity, Hydrogen Production**	**Fuels, Materials compatibility**
Gas-Cooled Fast Reactor (GFR)	**Fast**	**Closed**	**Med**	**Electricity, Hydrogen, AM**	**Fuels, Materials, Safety**
Very High Temp. Gas Reactor (VHTR)	**Thermal**	**Open**	**Med**	**Electricity, Hydrogen, Process Heat**	**Fuels, Materials, H_2 production**
Supercritical Water Reactor (SCWR)	**Thermal, Fast**	**Open, Closed**	**Large**	**Electricity**	**Materials, Safety**
Molten Salt Reactor (MSR)	**Epithermal**	**Closed**	**Large**	**Electricity, Hydrogen, AM**	**Fuel, Fuel treatment, Materials, Safety and Reliability**

Schematic of Nuclear Reactor



- **Reactor kinetics**
  - ➢reactivity is function of time only
- **Reactor dynamics**
  - ➢reactivity is a function of reactor power

Fission and delayed neutrons



Precursors of delayed neutrons

- Almost all neutrons are emitted immediately after fission
  - These neutrons are termed as **prompt neutrons**
  - Prompt neutrons are emitted immediately after fission ($<10^{-14}$s)
  - Some of ca 500 possible fission products first beta-decay into daughter product and then immediately emit a neutron
  - There are about 40 such fission products and they are called **precursors of delayed neutrons**, or short: **precursors**

Delayed neutrons

- Since delayed neutrons are important for reactor kinetics and dynamics, one has to trace them
- It is difficult to treat separately all 40 precursors
- Therefore it has been customary to represent precursors with six groups (families)
- Their yields and decay constants are measured experimentally

Six groups of delayed neutrons

Approximate half-life [s]	Number of fission neutrons delayed per fission			Energy (MeV)
	U-233	U-235	Pu-239	
55	5.7×10^{-4}	5.2×10^{-4}	2.1×10^{-4}	0.25
23	19.7×10^{-4}	34.6×10^{-4}	18.2×10^{-4}	0.46
6.2	16.6×10^{-4}	31.0×10^{-4}	12.9×10^{-4}	0.41
2.3	18.4×10^{-4}	62.4×10^{-4}	19.9×10^{-4}	0.45
0.61	3.4×10^{-4}	18.2×10^{-4}	5.2×10^{-4}	0.41
0.23	2.2×10^{-4}	6.6×10^{-4}	2.7×10^{-4}	-
Total delayed	0.0066	0.0158	0.00607	
Total fission neutrons	2.49	2.42	2.93	
Fraction delayed	0.00265	0.00653	0.00207	

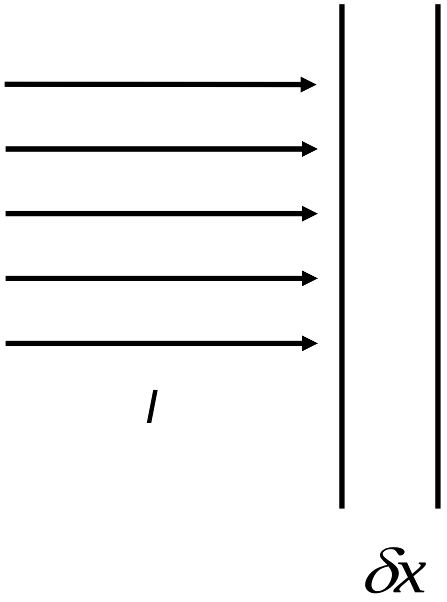
Cross-section for neutron reactions (1)

- To quantify the probability of a certain reaction of a neutron with matter it is convenient to utilize the concept of ***cross-sections***

- The cross-section of a target nucleus for any given reaction is thus a measure of the probability of a particular neutron-nucleus interaction and is a property of the nucleus and of the energy of the incident neutron

Cross-section for neutron reactions (2)

- Suppose a uniform, parallel beam of *I* mono-energetic neutrons per m^2 impinges perpendicularly, for a given time, on a thin layer δx m in thickness, of a target material containing *N* atoms per m^3, so that $N\delta x$ is the number of target nuclei per m^2 (see figure to the right)



- The nuclear microscopic cross section for a specified reaction is then defined as

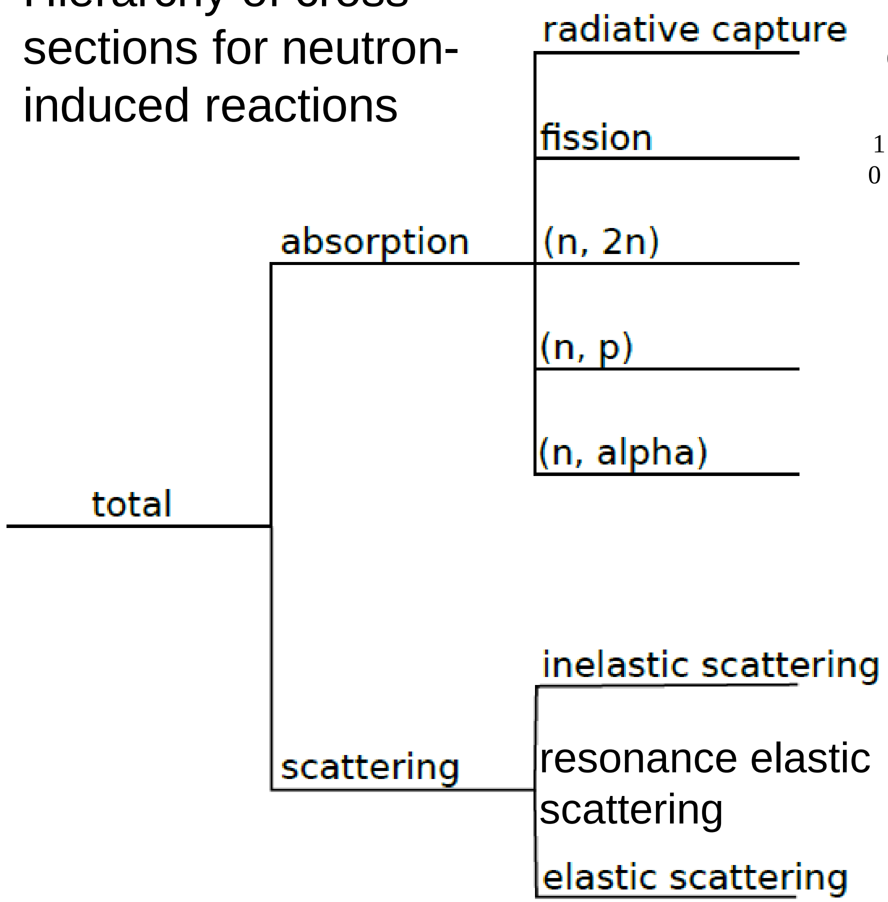
$$\sigma = \frac{N_R}{(N\delta x)I} \; m^2 / \text{nucleus}$$

Microscopic cross-section unit: 1 b (barn) = 10^{-28} m^2/nucleus

- Where N_R is the number of reactions /m^2

Cross-section for neutron reactions (3)

Hierarchy of cross sections for neutron-induced reactions



$$ {}_0^1n + {}_Z^AX \rightarrow \left({}_Z^{A+1}X \right)^* \rightarrow {}_Z^{A+1}X + \gamma $$

$$ {}_0^1n + {}_Z^AX \rightarrow \left({}_Z^{A+1}X \right)^* \rightarrow {}_{Z_1}^{A_1}X + {}_{Z_2}^{A_2}X + 2 \div 3\,{}_0^1n $$

$$ {}_0^1n + {}_Z^AX \rightarrow \left({}_Z^{A+1}X \right)^* \rightarrow \left({}_Z^AX \right)^* + {}_0^1n $$

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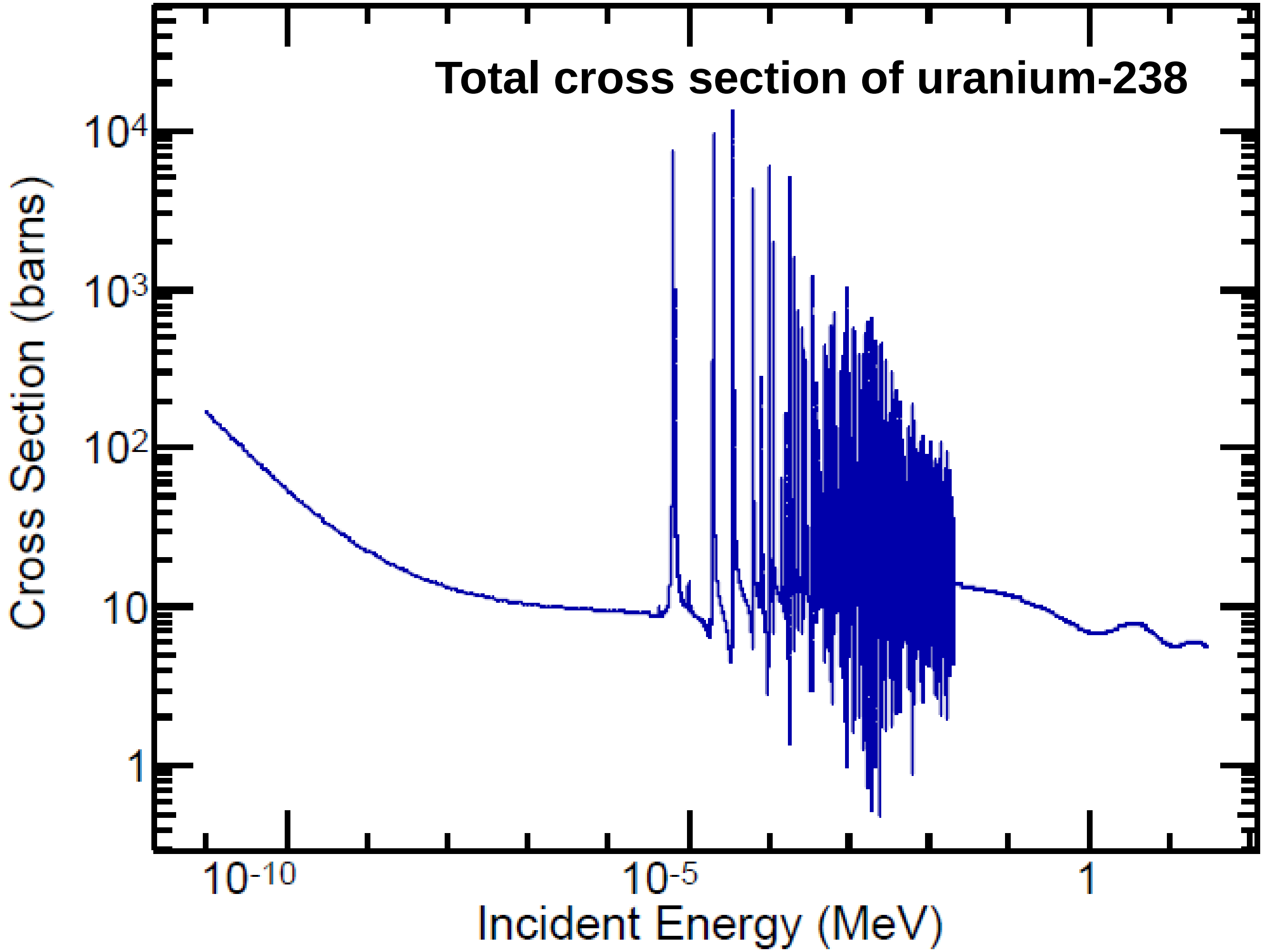
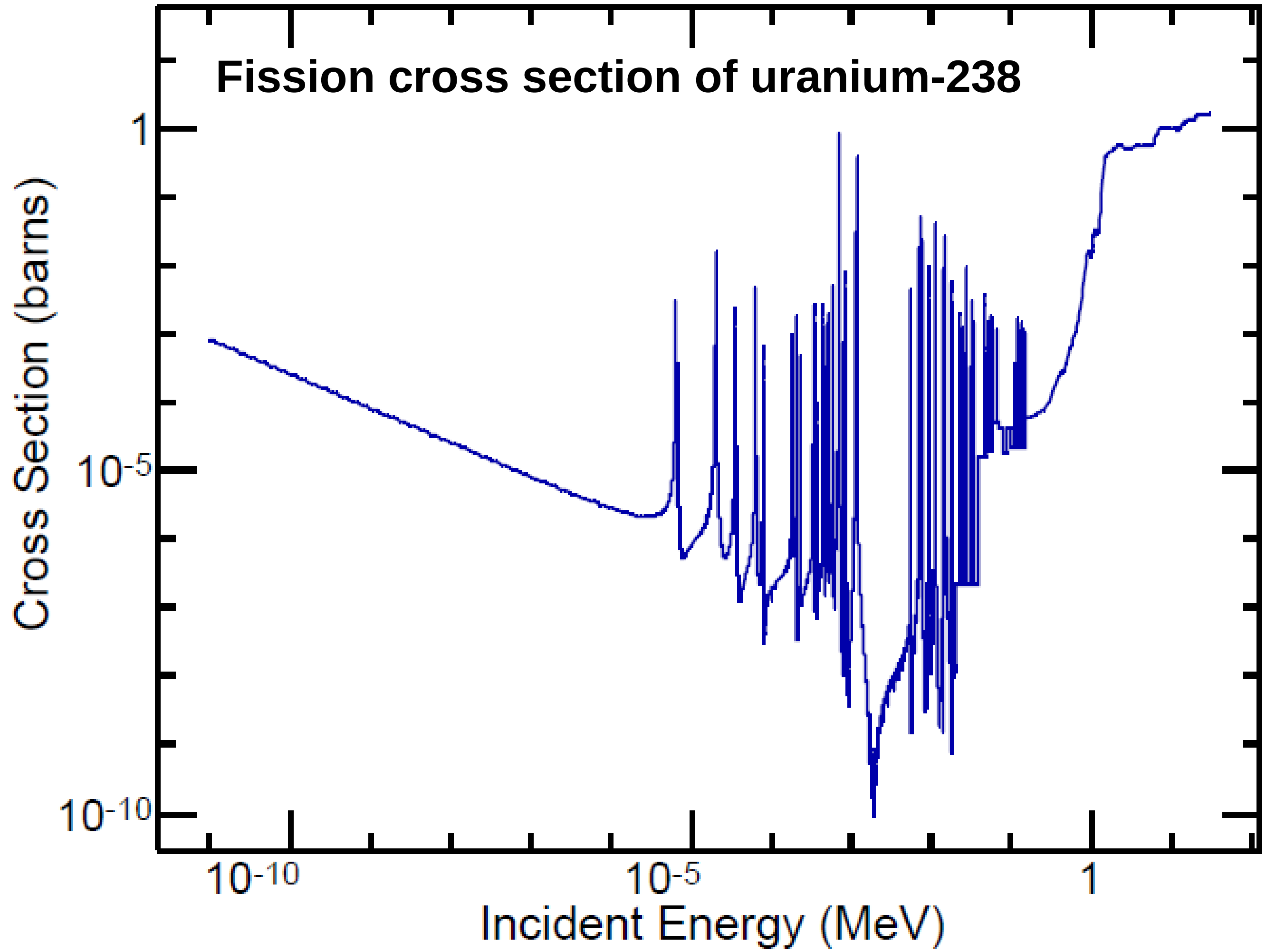
$$ {}_0^1n + {}_Z^AX \rightarrow {}_0^1n + {}_Z^AX $$

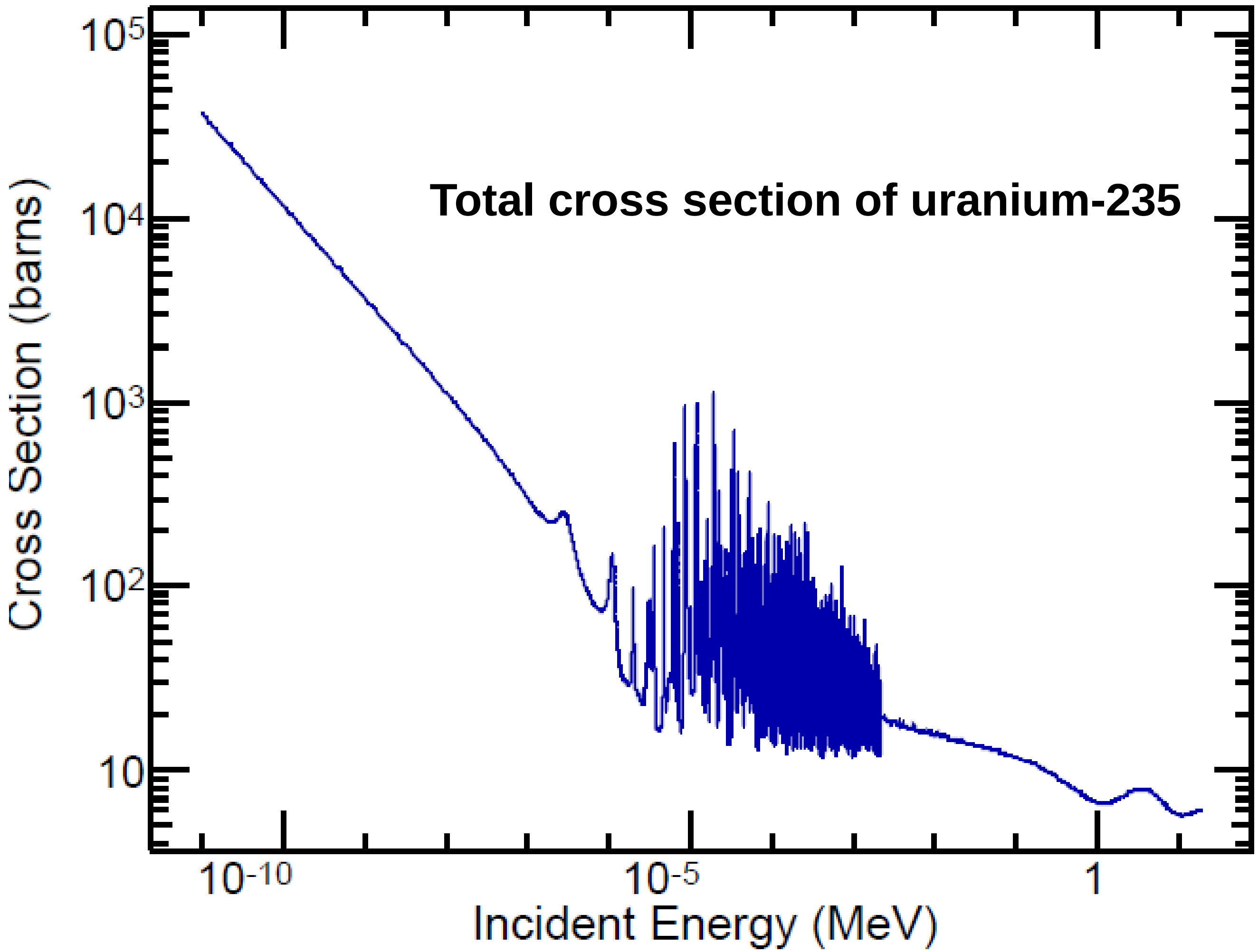
Cross-section for neutron reactions (4)



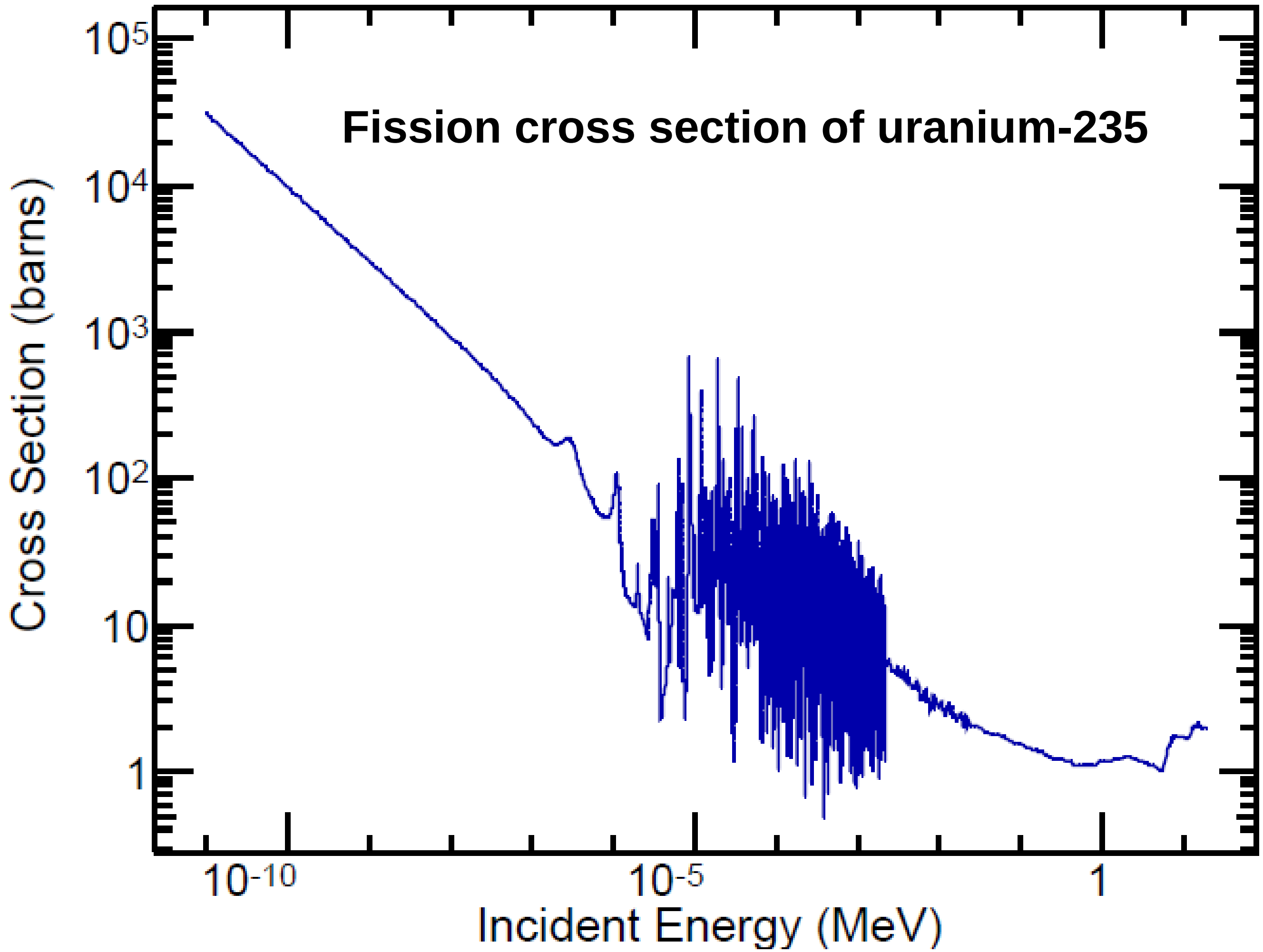
Cross-section for neutron reactions (5)



Cross-section for neutron reactions (6)



Cross-section for neutron reactions (7)



Macroscopic Cross Section (1)

- The cross section σ for a given reaction applies to a single nucleus and is thus called the ***microscopic cross section***.

- Since *N* is the number of target nuclei per m^3, the product $N\sigma$ represents the total cross section of the nuclei per m^3

- Thus, the ***macroscopic cross section*** Σ is introduced as

$$\Sigma = N\sigma \ \mathrm{m}^{-1}$$

Macroscopic Cross Section (2)

- If a target material is an element of atomic weight *A*, 1 mole has a mass of 10^{-3} *A* kg and contains the **Avogadro number** ($N_A = 6.02 \cdot 10^{23}$) of atoms. If the element density is ρ kg/m^3, the number of atoms per m^3 *N* is given as

$$N = \frac{\rho\left[\frac{\text{kg}}{\text{m}^3}\right]}{\underbrace{10^{-3} A\left[\frac{\text{kg}}{\text{mol}}\right]}_{\text{mol/m}^3}} N_A\left[\frac{\text{atoms}}{\text{mol}}\right] = \frac{10^3 \rho N_A}{A}\left[\frac{\text{atoms}}{\text{m}^3}\right]$$

- The macroscopic cross section is thus

$$\Sigma = \frac{10^3 \rho N_A}{A}\sigma$$

Macroscopic Cross Section (3)

- For a compound of molecular weight M and density ρ kg/m^3, the number N_i of atoms of the i_{th} kind per m^3 is given by the following equation

$$N_i = \frac{10^3 \rho N_A}{M} \nu_i$$

- where ν_i is the number of atoms of the kind i in a molecule of the compound. The macroscopic cross section for this element in the given target material is then … and for compound …

$$\Sigma_i = N_i \sigma_i = \frac{10^3 \rho N_A}{M} \nu_i \sigma_i \quad \Rightarrow \quad \Sigma = \frac{10^3 \rho N_A}{M} \left(\nu_1 \sigma_1 + \nu_2 \sigma_2 + \cdots \right)$$

Macroscopic Cross Section (4)

- Example:
  - The microscopic cross section for the capture of thermal neutrons by hydrogen is 0.33 b and for oxygen 2 • 10^{-4} b Calculate the macroscopic capture cross section of water, with density 10^3 kg/m^3, for thermal neutrons
- Solution:
  - The molecular weight *M* of water is 18 and the density is 1000 kg/m^3. The molecule contains 2 atoms of hydrogen and 1 of oxygen. Equation for compound capture cross section yields

$$\Sigma_{c,H_2O} = \frac{10^3 1000 N_A}{18}\left(2\cdot 0.33 + 1\cdot 2\cdot 10^{-4}\right)\cdot 10^{-28} \approx 2.2\ \mathrm{m}^{-1}$$

Average logarithmic energy loss (1)

- A useful quantity in the study of the slowing down of neutrons is the average value of the decrease in the natural logarithm of the neutron energy per collision, or the **average logarithmic energy loss** per collision

- It is the average of all collisions of $\ln E_1 - \ln E_2 = \ln(E_1/E_2)$, where E_1 is the energy of the neutron before and E_2 is that after collision

$$\xi \equiv \overline{\ln \frac{E_1}{E_2}} = \frac{\int_{-1}^{1} \ln \frac{E_1}{E_2} d(\cos\theta_C)}{\int_{-1}^{1} d(\cos\theta_C)}$$

Here θ_C is the collision angle in the center-of-mass system; integration means averaging over all possible collision angles

Average logarithmic energy loss (2)

- We get, taking $\alpha = \frac{A-1}{A+1}$, $\mu_C = \cos\theta_C$, the following:

$$\xi \equiv \overline{\ln\frac{E_1}{E_2}} = \frac{-\int_{-1}^{1}\ln\left[\frac{(1+\alpha)}{2}+\frac{(1-\alpha)\mu_C}{2}\right]d\mu_C}{\int_{-1}^{1}d\mu_C} = 1+\frac{\alpha\ln\alpha}{1-\alpha}$$

- Or

$$\xi = 1+\frac{(A-1)^2}{2A}\ln\frac{A-1}{A+1}$$

- It can be seen that for hydrogen (A=1) ξ=1 and for uranium-238 ξ=0.0083

Mean weighted logarithmic energy decrement in chemical compounds

- If the moderator is not a single isotope, but a compound containing *n* different nuclei, the effective or mean (weighted) value of log. energy decr. ξ is given by

$$\langle \xi \rangle = \frac{\sigma_{s1}\xi_1 + \sigma_{s2}\xi_2 + \ldots + \sigma_{sn}\xi_n}{\sigma_{s1} + \sigma_{s2} + \ldots + \sigma_{sn}} \qquad \xi = 1 + \frac{(A-1)^2}{2A} \ln \frac{A-1}{A+1}$$

- For example for H_2O we get

$$\langle \xi \rangle_{H_2O} = \frac{2\sigma_{s(H)}\xi_H + \sigma_{s(O)}\xi_O}{2\sigma_{s(H)} + \sigma_{s(O)}}$$

$$\xi_H = 1$$

$$\xi_O = 1 + \frac{(16-1)^2}{2 \cdot 16} \ln \frac{16-1}{16+1} \cong 0.12$$

Moderating power and moderating ratio

- An interesting application of the logarithmic energy decrement per collision is to compute the average number of collisions necessary to thermalize a fission neutron

- It can be shown that this number is = 14.4/ξ, when slowing down neutrons from energy 45 keV to 0.025 eV.
- One also defines the **moderating** or **slowing down power** of a material as: $\boldsymbol{\xi\Sigma_s}$

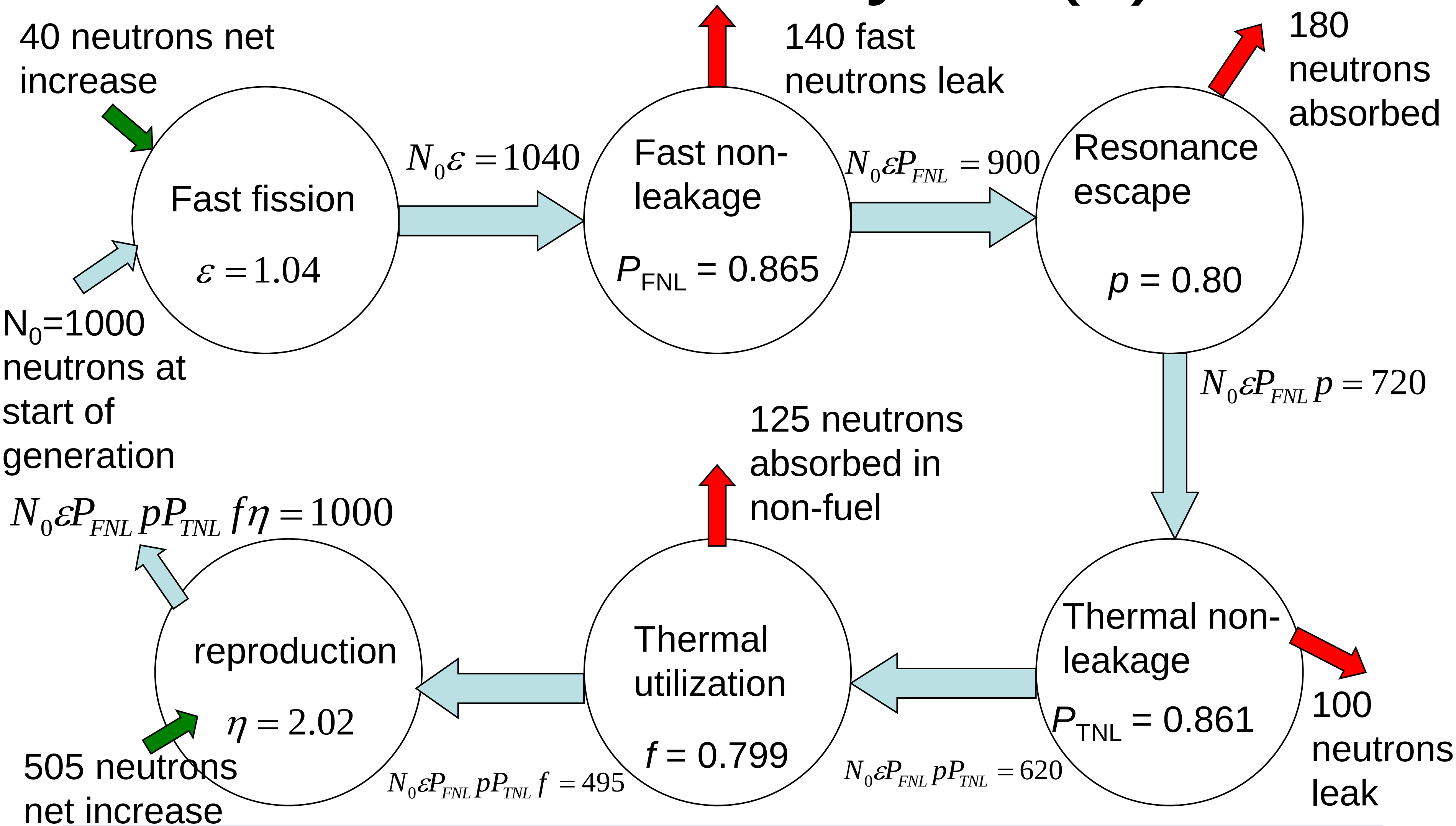
- However, this parameter is not sufficient to describe how good a given material is as a moderator, since one also wishes the moderator to be a week absorber of neutrons

- That's why one uses **moderating ratio** = $\boldsymbol{\xi\Sigma_s / \Sigma_a}$ as a figure of merit. Example: $MR(H_2O)=71$; $MR(D_2O)=5670$; $MR(C)=192$

Neutron Life Cycle (1)

- Neutron life cycle is a sequence of events between a neutron birth in a fission and neutron disappearance from the reactor
- Not all neutrons produced by fission will cause new fission:
  - Some will be **absorbed by non-fissionable material**
  - Some will be **absorbed parasitically in fissionable material**
  - Others will **leak out of the reactor**
- For the maintenance of a self-sustaining chain reaction it is enough that, on the average, at least one neutron produced in fission that causes fission of another nucleus

Neutron Life Cycle (2)



Infinite Multiplication Factor

- The **infinite multiplication factor** is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation:

$$\boxed{k_\infty = \frac{\text{Neutron production from fission in one generation}}{\text{Neutron absorption in the preceding generation}}}$$

or shortly

$$\boxed{k_\infty = \frac{\text{Rate of neutron production}}{\text{Rate of neutron absorption}}}$$

Four Factor Formula (1)

- For some thermal reactors, the infinite multiplication factor k_∞ can be evaluated with a fair degree of accuracy by means of the **four factor formula**

- The basis of this formula is the assumed division of the neutrons into three categories:
  - Fission neutrons with energies in excess of about 1MeV which can cause fission in uranium-238 as well as in uranium-235
  - Neutrons in the resonance region which may be captured by uranium-238
  - Thermal neutrons which cause nearly all the fission in uranium-235 and thereby generate fission neutrons

Four Factor Formula (2)

- A group of fast neutrons can enter into several reactions
- Some of these reactions reduce the size of the neutron group while other reactions allow the group to increase in size or produce a second generation
- There are four factors that give the inherent multiplication ability of the fuel and moderator materials:

$$\boxed{k_\infty = \varepsilon \cdot p \cdot f \cdot \eta}$$

$$\begin{aligned} \text{where: } \varepsilon &= \textbf{Fast fission factor} \\ p &= \textbf{Resonance escape probability} \\ f &= \textbf{Thermal utilization factor} \\ \eta &= \textbf{Reproduction factor} \end{aligned}$$

Fast Fission Factor

- The **fast fission factor** is defined as the ratio of the *net number of fast neutrons produced by all fissions* to the *number of fast neutrons produced by thermal fissions*

$$\boxed{\varepsilon = \frac{\text{Number of fast neutrons produced by all fissions}}{\text{Number of fast neutrons produced by thermal fissions}}}$$

- In order for a neutron to be absorbed by a fuel nucleus as a fast neutron, it must pass close to a fuel nucleus while it is a fast neutron

Resonance Escape Probability

- the **resonance escape probability**, *p* is defined as the ratio of the *number of neutrons that reach thermal energies* to the *number of fast neutrons that start to slow down*

$$p = \frac{\text{Number of neutrons that reach thermal energy}}{\text{Number of fast neutrons that start to slow down}}$$

- The value of resonance escape probability is determined largely by the fuel-moderator arrangement and the amount of enrichment of uranium-235. It can be found as

$$p = \exp\left[-\frac{2.73}{\langle \xi \rangle}\left(\frac{N_8}{\Sigma_s}\right)^{0.514}\right]$$

N_8 – number density of ^{238}U, <ξ> - logarithmic energy decrement for the mixture moderator-fuel, Σ_s – macroscopic scattering cross section for the moderator plus fuel

p increases with <ξ> - better moderation, and with Σ_s – more encounters with moderator, but decreases with N_8 – higher density of absorbers

Thermal Utilization Factor

- Once thermalized, the neutrons continue to diffuse throughout the reactor and are subject to absorption by other materials in the reactor as well as the fuel
- The **thermal utilization factor** *f* is defined as the ratio of the *number of thermal neutrons absorbed in the fuel* to the *number of thermal neutrons absorbed in all reactor material*:

$$f = \frac{\text{Number of thermal neutrons absorbed in the fuel}}{\text{Number of thermal neutrons absorbed in all reactor materials}}$$

$$f = \frac{\Sigma_{a,FUEL}}{\Sigma_{a,FUEL} + \Sigma_{a,MODERATOR}}$$

$\Sigma_{a,FUEL}$, $\Sigma_{a,MODERATOR}$ – macroscopic cross section for fuel and moderator, respectively. For fixed moderator-to-fuel ratio, *f* increases with enrichment, but it decreases with increasing moderator-to-fuel ratio, due to increasing chance to absorb a neutron in the moderator

Reproduction Factor

- Most of the neutrons absorbed in the fuel cause fission, but some do not
- The **reproduction factor** is defined as the ratio of the *fast neutrons produced by thermal fission* to the *number of thermal neutrons absorbed in the fuel*

$$\boxed{\eta = \frac{\text{Number of fast neutrons produced by thermal fission}}{\text{Number of thermal neutrons absorbed in the fuel}}}$$

It can be found from:

$$\eta = 2.42 \frac{e\sigma_{f,5}}{(1-e)\sigma_{a,8} + e\sigma_{a,5}}$$

e – enrichment of fuel = $N_5 * A_5/(N_5 * A_5 + N_8 * A_8)$
$\sigma_{a,8}$ – microscopic cross section for absorption in U-238
$\sigma_{a,5}$ – same as above for U-235
$\sigma_{f,5}$ – same as above for fission

Effective Multiplication Factor (1)

- The infinite multiplication factor can fully represent only a reactor that is infinitely large
- To completely describe the neutron life cycle in a real, finite reactor, it is necessary to account for neutrons that leak out
- The multiplication factor that takes leakage into account is the effective multiplication factor k_{eff}

Effective Multiplication Factor (2)

- For **critical** reactor the neutron population is neither increasing nor decreasing and $k_{eff} = 1$
- If the neutron production is grater than the absorption and leakage, the reactor is called **supercritical**; $k_{eff} > 1$
- If the neutron production is less than the absorption and leakage, the reactor is called **subcritical**; $k_{eff} < 1$
- $k_{eff} = k_{\infty} \times P_{FNL} \times P_{TNL}$, where P_{FNL} is the **fast non-leakage probability** and P_{FNL} is the **thermal non-leakage probability**

Fast Non-Leakage Probability

- In a realistic reactor of finite size some of the fast neutrons leak out of the boundaries of the reactor core before they begin the slowing down process

- **The fast non-leakage probability** P_{FNL} is defined as the ratio of the number of fast neutrons that do not leak from the reactor to the number of fast neutrons produced by all fissions

$$\boxed{P_{FNL} = \frac{\text{Number of fast neutrons that do not leak from reactor}}{\text{Number of fast neutrons produced by all fissions}}}$$

Thermal Non-Leakage Probability

- Neutrons can also leak out of a finite reactor after they reach thermal energies

- The **thermal non-leakage probability** is defined as the ratio of the *number of thermal neutrons that do not leak from the reactor core* to the *number of neutrons that reach thermal energies*

$$P_{TNL} = \frac{\text{Number of thermal neutrons that do not leak from reactor}}{\text{Number of neutrons that reach the thermal energies}}$$

Six Factor Formula

- Six-factor formula takes into account leakage of neutrons
- $k_{eff} = k_{\infty} \times P_{FNL} \times P_{TNL}$
- Inclusion of expression for k_{∞} (four-factor formula) yields

$$\boxed{k_{eff} = \varepsilon \cdot P_{FNL} \cdot p \cdot P_{TNL} \cdot f \cdot \eta}$$

Reactivity

- The fractional departure of a system from criticality is called **reactivity** and is defined as:

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

- Reactivity is an important parameter since it determines the time behavior of a reactor:
  - $\rho < 0$ reactor is subcritical and the power is decreasing
  - $\rho = 0$ reactor is critical and the power is constant
  - $\rho > 0$ reactor is supercritical and the power is increasing

Reactivity Coefficients

- **Reactivity coefficients** are used to quantify the effect of variation in various parameters on the reactivity of the core
- The amount of reactivity change per unit change in given parameter is the reactivity coefficient in respect to this parameter
- For instance the increase in moderator temperature will (in most cases) cause a decrease in the reactivity of the core

Moderator Temperature Coefficient (1)

- The change in reactivity per degree change in temperature is called the temperature coefficient of reactivity

- Because different materials in the reactor have different temperatures during reactor operation, several different temperature coefficients are used

- Usually the two dominant temperature coefficients are the **moderator temperature coefficient** and the **fuel temperature coefficient**

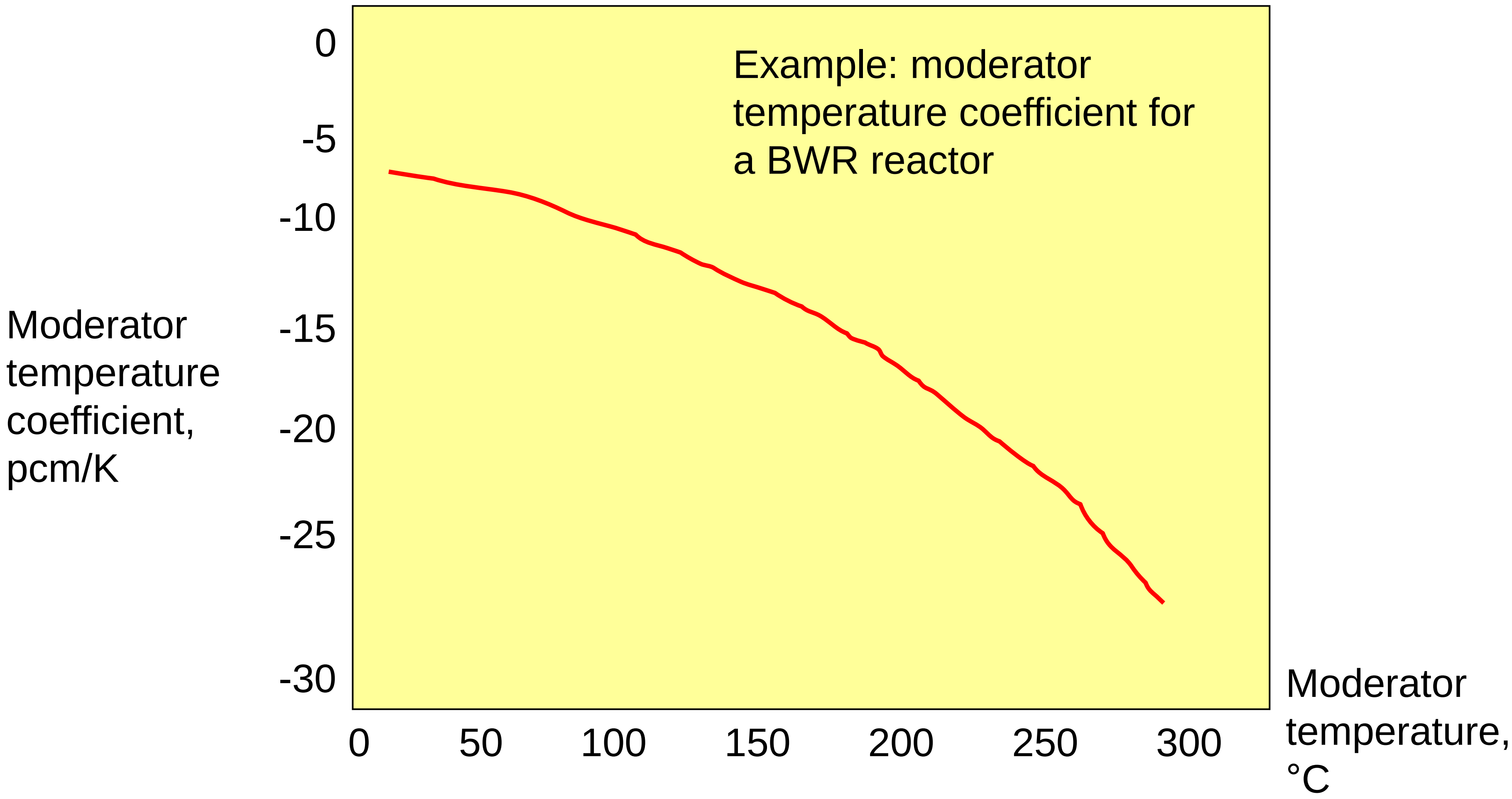
Moderator Temperature Coefficient (2)

- the change in reactivity per degree change in moderator temperature is called the **moderator temperature coefficient** (also **delayed temperature coefficient**)

- The magnitude and sign (+ or -) of the moderator temperature coefficient is primarily a function of moderator-to-fuel ratio:
  - If a reactor is under moderated it will have a negative moderator temperature coefficient

- Negative moderator temperature coefficient is desirable because of it self-regulating effect

Moderator Temperature Coefficient (3)



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Moderator Temperature Coefficient (4)

- The moderator temperature coefficient is about -5 to -10 pcm/K at room temperature, but it decreases to ca -25 pcm/K for operating temperature (286 C)

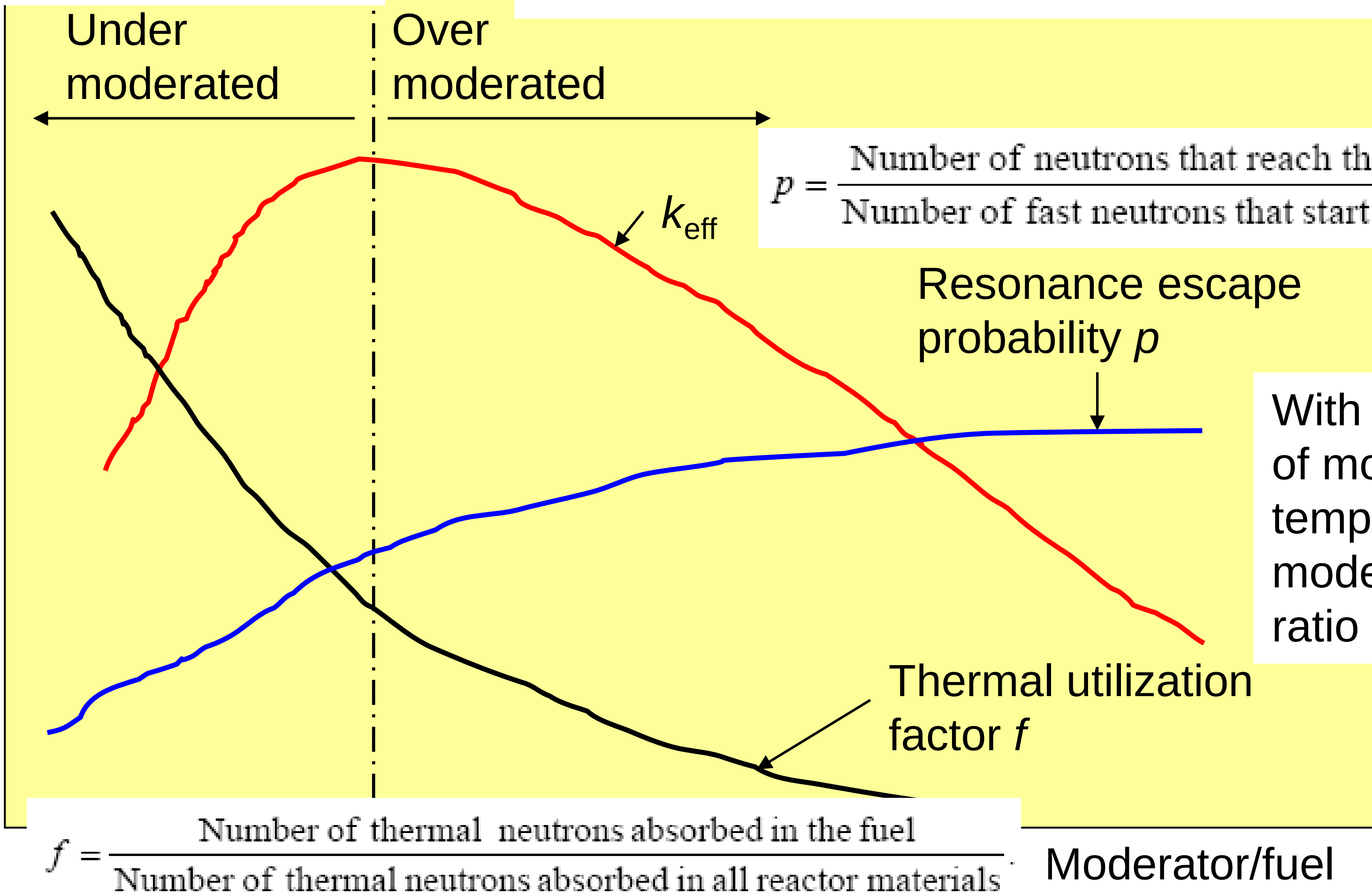
- In BWRs it can be up to -50 pcm/K

- At the end of cycle the coefficient can be slightly positive, about +5 pcm/K

Under and over moderation

Here k_{eff} decreases when moderator temperature increases

Here k_{eff} increases when moderator temperature increases!



$$p = \frac{\text{Number of neutrons that reach thermal energy}}{\text{Number of fast neutrons that start to slow down}}$$

With increase of moderator temperature, moderator/fuel ratio decreases

$$f = \frac{\text{Number of thermal neutrons absorbed in the fuel}}{\text{Number of thermal neutrons absorbed in all reactor materials}}$$

Fuel Temperature Coefficient (1)

- Another temperature coefficient – the fuel temperature coefficient – has a greater effect than the moderator temperature coefficient for some reactors

- The **fuel temperature coefficient** is the change in reactivity per degree change in fuel temperature

- The coefficient is also called the **prompt temperature coefficient** because an increase in reactor power causes an immediate change in fuel temperature

Fuel Temperature Coefficient (2)

- A negative fuel temperature coefficient is more important than a negative moderator temperature coefficient
  - fuel temperature immediately increases following an increase in reactor power
  - The time for heat to be transferred to moderator is measured in seconds

- When large positive reactivity is inserted,
  - moderator temperature cannot prevent rise for several seconds,
  - fuel temperature coefficient starts adding negative reactivity immediately

Fuel Temperature Coefficient (3)

- Another name applied to the fuel temperature coefficient of reactivity is the **fuel Doppler reactivity coefficient (**or just **Doppler coefficient)**

- The name is applied because in typical low enrichment, light water moderated, thermal reactors the fuel temperature coefficient of reactivity is negative and is the result of the Doppler effect, also called Doppler broadening – that is increased absorption of neutrons in U-238

Typical values of reactivity coefficients

Type of coefficient	BWR	PWR	HTGR	LMFBR
Fuel Doppler (pcm/K)	-4 to -1	-4 to -1	-7	-0.6 to -2.5
Coolant void (pcm/%void)	-200 to -100	0	0	-12 to +20
Moderator (pcm/K)	-50 to -8	-50 to -8	+1.0	
Expansion (pcm/K)	~0	~0	~0	-0.92

Reactor Kinetics

- Reactor kinetics
  - Reactivity is only a function of time (affected by control system, noise or disturbances)
  - Reactivity will change from zero if:
    - boron concentration in coolant is changed
    - control rod position is changed
    - density of moderator is changed
  - Reactivity is changing steadily during reactor operation due to:
    - fuel burn-up (less and less fissile materials are in the core)
    - burnable absorbers (poisons)
    - reactor poisoning (accumulation of Xenon-135 and Samarium-149)

Point Reactor Kinetics

Point Reactor Kinetics Model

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \sum_{i=1}^{6} \lambda_i C_i + S$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i C_i, \quad i = 1, \ldots, 6$$

Neutron balance equation: *n* – neutron concentration in reactor

Balance equation for concentration of precursors of delayed neutrons: C_i – precursor-*i* concentration

ρ – reactivity, β_i – yield of precursor *i*, λ_i – decay constant of precursor *i*
Λ – average neutron generation time,
S – neutron sources

$$\beta = \sum_{i=1}^{6} \beta_i$$

$$\frac{\beta}{\lambda} = \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} \Rightarrow \lambda = \frac{\beta}{\sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}}$$

One group approximation

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda C + S$$

$$\frac{dC}{dt} = \frac{\beta}{\Lambda} n - \lambda C$$

One Group Approximation

One group approximation has an exact analytical solution for a step change of reactivity!

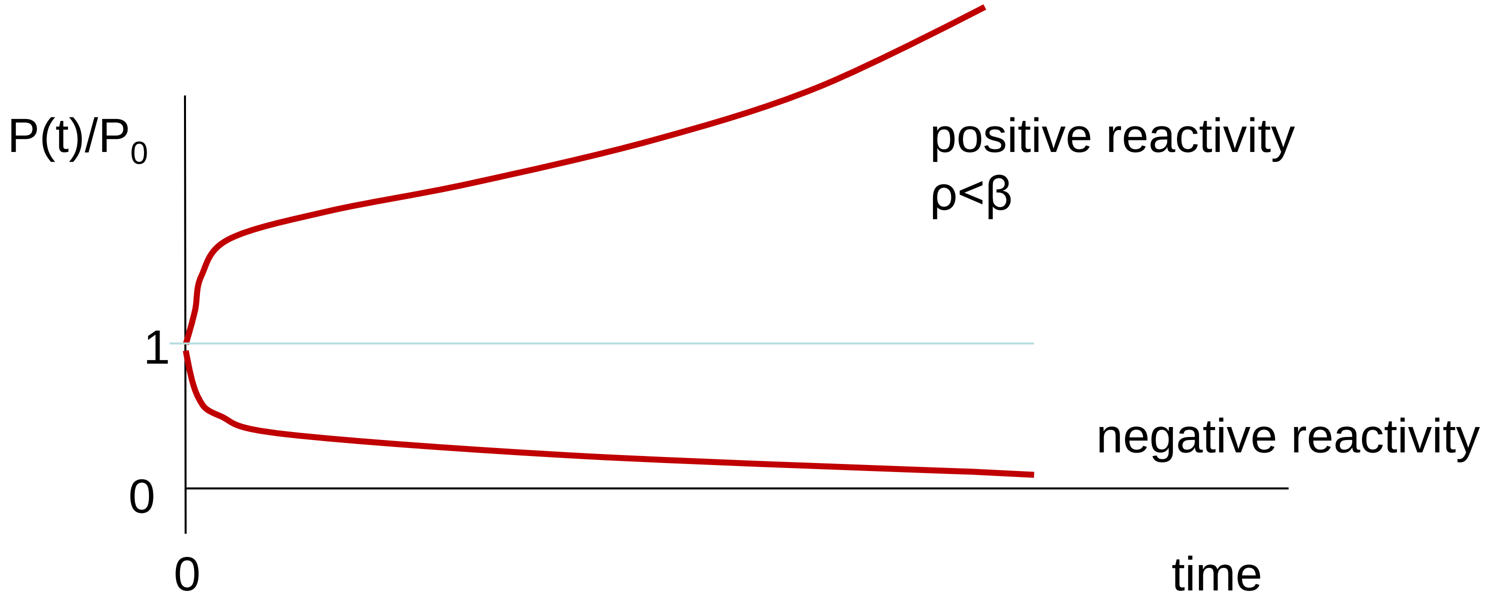
$$x(t) \equiv \frac{n(t)-n_0}{n_0} = \frac{\rho_0}{\Lambda}\left[\frac{\lambda}{s_1 s_2} + \frac{s_1+\lambda}{s_1(s_1-s_2)}e^{s_1 t} + \frac{s_2+\lambda}{s_2(s_2-s_1)}e^{s_2 t}\right]$$

where $$s_{1,2} = \frac{-\left(\frac{\beta}{\Lambda} - \frac{\rho_0}{\Lambda} + \lambda\right) \pm \sqrt{\left(\frac{\beta}{\Lambda} - \frac{\rho_0}{\Lambda} + \lambda\right)^2 + 4\frac{\lambda\rho_0}{\Lambda}}}{2}$$

n_0 – neutron concentration at time t = 0, ρ_0 – step change of reactivity at time 0 ; *x(t)* – relative change of neutron concentration (or power) in reactor

Reactor Power Change

- Reactor power change:
  - Reactor power change P(t) after step change of reactivity is as shown below (P_0 – initial power)



Reactor Dynamics – Reactivity Feedbacks

- The total reactivity, including feedbacks, can be written as:

Total reactivity

Fuel feedback

Coolant temperature perturbation

$$\rho(t) = \rho_C(t) - \rho_F(t) - \rho_{CL}(t) + \rho_u(t) + \rho_w(t)$$

External perturbation

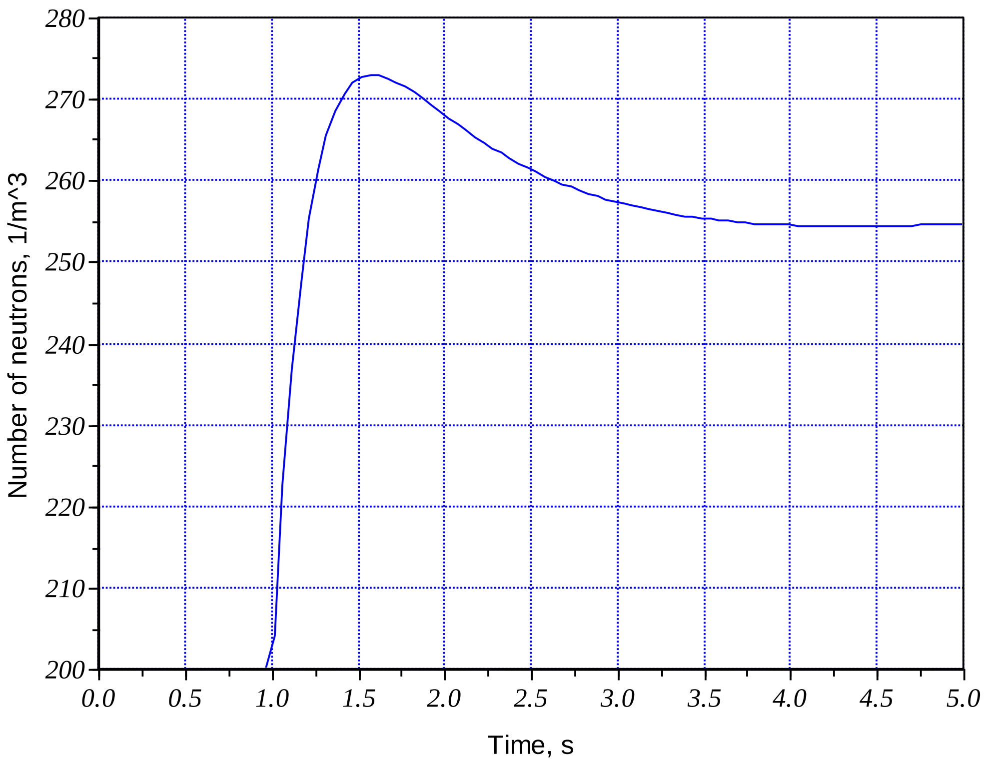
Coolant feedback

Coolant mass flow rate perturbation

NOTE: we follow here the convention that feedbacks have a minus sign

Positive step change of reactivity

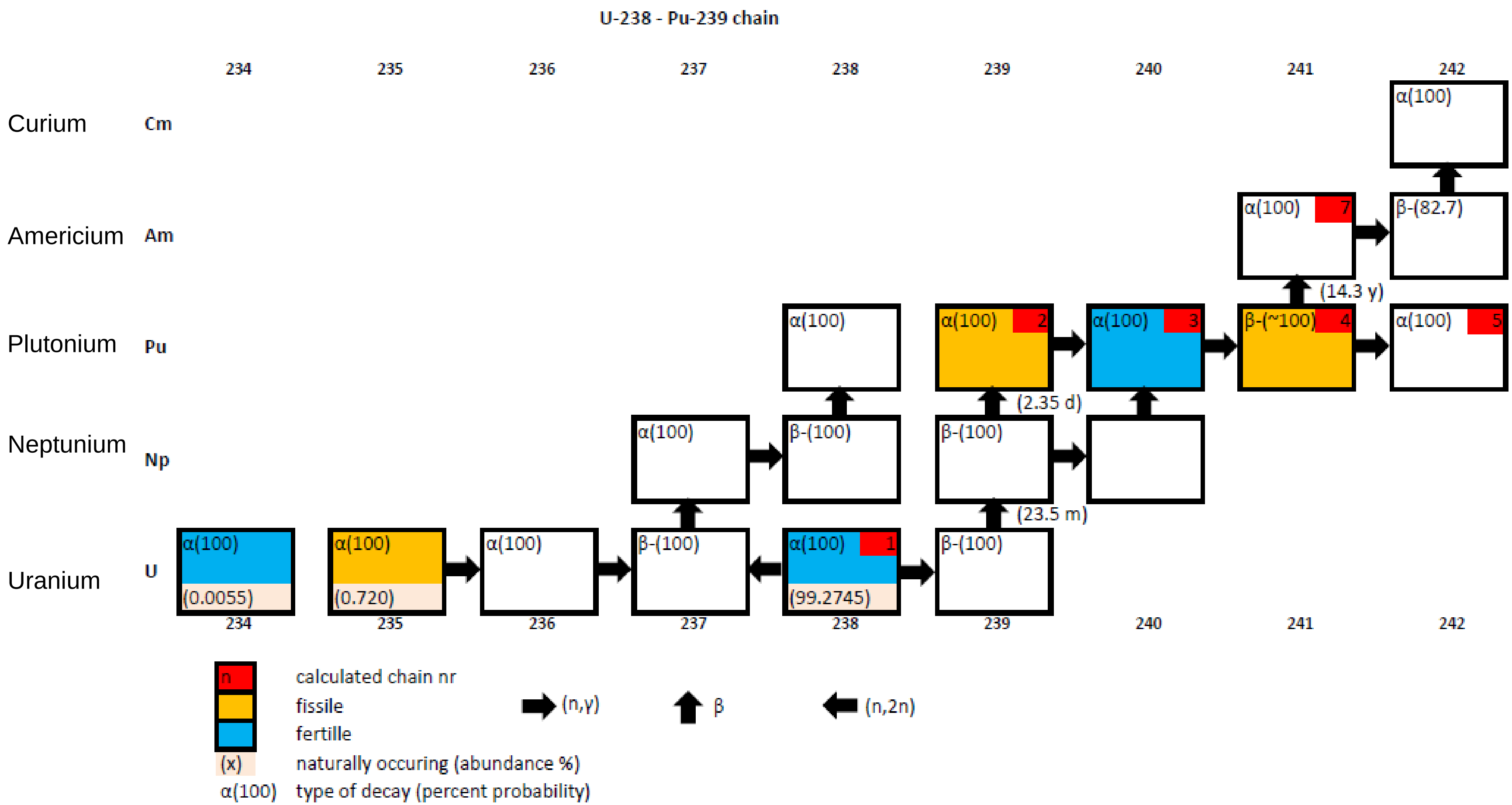
- Time-domain solution: the number of neutrons increases from initial value of 200 to maximum value of 273 at time t ~= 1.6 s and then drops down to ~255.
- The drop is caused by the negative reactivity feedback



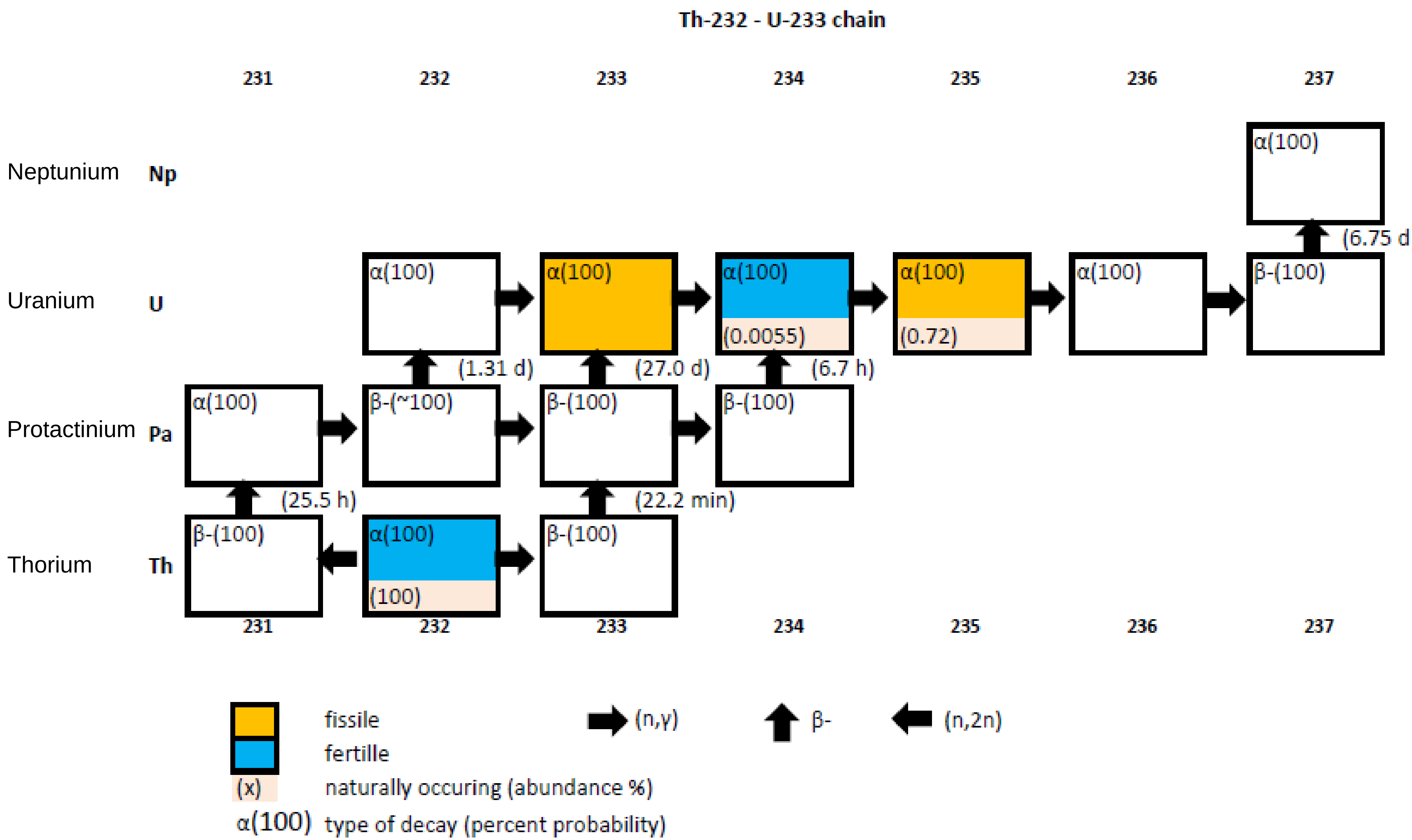
Fuel Conversion and Breeding

- One of the most important features of nuclear reactors is their inherent ability to produce fuel during operation
- This is because some fertile material (e.g. uranium 238) is transmuted into fissile material (here plutonium 239)
- This feature can be deliberately utilized in specially designed reactors which can produce more fuel than they are using
- Such reactors are called breeders

Conversion Chain U-238→Pu-239



Conversion Chain Th-232→U-233



Conversion Ratio

- The degree of conversion that occurs in a reactor is denoted as **conversion ratio** CR, defined as:

$$CR = \frac{\text{Fissile material produced}}{\text{Fissile material destroyed}} = \frac{FP}{FD}$$

or

$$\overline{CR} = \frac{\int_0^{T_f} dt \int_{V_c} RR_c^{(FP)}(\vec{r},t)\,d\mathrm{v}}{\int_0^{T_f} dt \int_{V_c} RR_a^{(FD)}(\vec{r},t)\,d\mathrm{v}}$$

$RR_c^{(FP)}$	capture reaction rate
$RR_a^{(FD)}$	absorption reaction rate
T_f	fuel cycle length
V_c	core volume

Conversion Ratio

- The conversion ratio is applicable to thermal reactors with natural or slightly enriched uranium
- In a thermal reactor ^{239}Pu is produced due to capture of thermal and resonance neutrons by ^{238}U
- Let's make an estimate of the reaction rates:
  - The rate of production of fission neutrons is

$$\phi N \sigma_a \eta \varepsilon$$

where

ϕ - thermal neutron flux

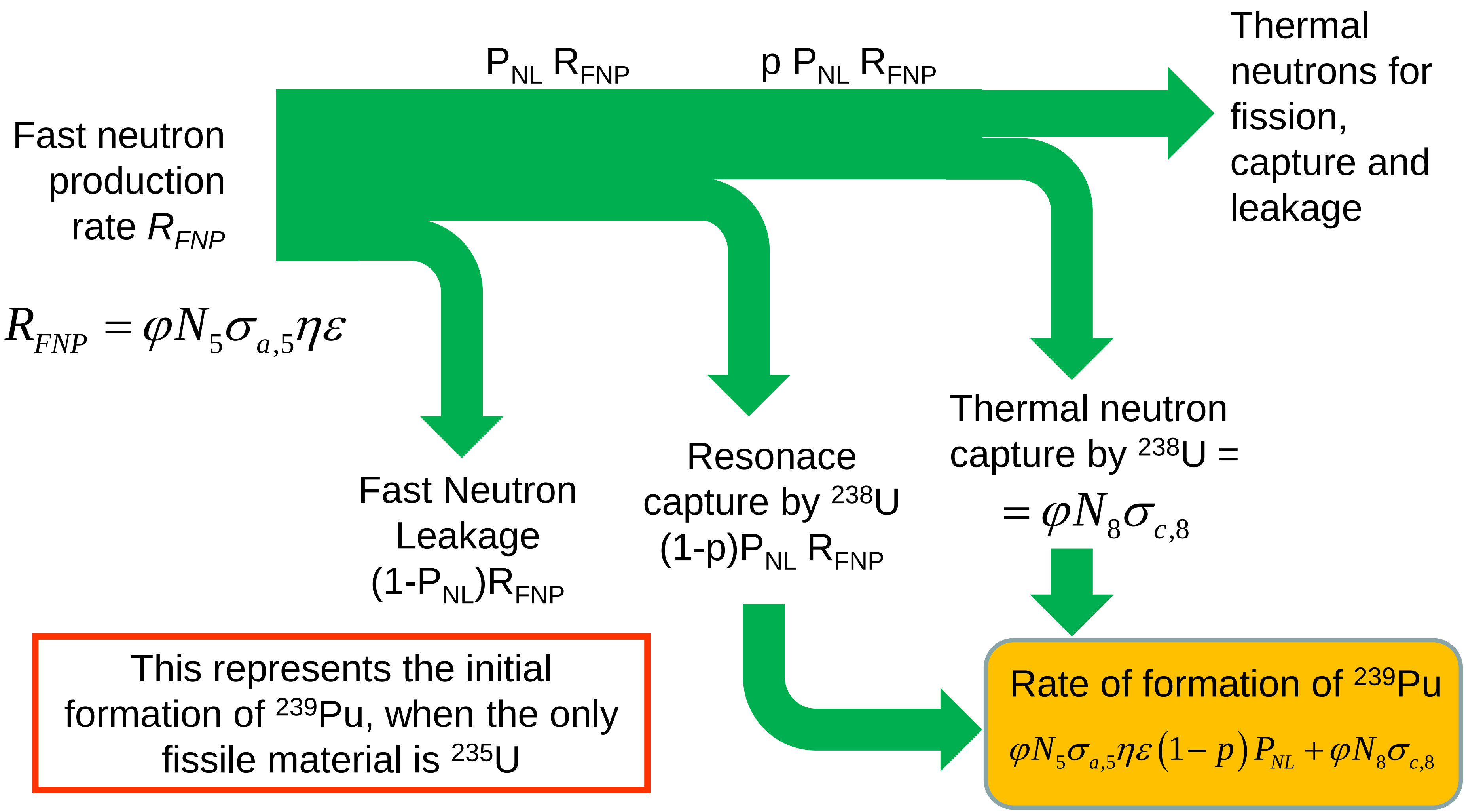
N – concentration of fissile nuclei

σ_a - thermal-neutron absorption cross section

η - number of fast neutrons produced per absorbed neutron

ε - fast-fission factor

Conversion Ratio



This represents the initial formation of ^{239}Pu, when the only fissile material is ^{235}U

Conversion Ratio

- Thus the initial conversion ratio is

$$CR(initial) = \frac{\varphi N_5 \sigma_{a,5} \eta \varepsilon \left(1 - p\right) P_{NL} + \varphi N_8 \sigma_{c,8}}{\varphi N_5 \sigma_{a,5}} = \frac{N_8 \sigma_{c,8}}{N_5 \sigma_{a,5}} + \eta \varepsilon \left(1 - p\right) P_{NL}$$

- After some operation period fissions and captures occur in ^{239}Pu and ^{235}U; this tends to decrease CR (^{239}Pu has larger cross-section for capture of thermal neutrons)

- Also fission products and heavy nuclides capture the resonance neutrons, decreasing CR as well

Conversion Ratio (Example)

- **EXAMPLE:** In a critical reactor with natural uranium as fuel for each 1000 neutrons absorbed in ^{235}U, 250 neutrons are absorbed in resonances of ^{238}U and 645 neutrons are absorbed by ^{238}U at thermal energies. Assume no neutron leakage from the reactor.

- Calculate the conversion ratio in the reactor.

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Conversion Ratio (Example)

- **SOLUTION:** Each absorption of a neutron (irrespective if thermal or resonance) produces an atom of ^{239}Pu via the following reaction:

$$^{238}\mathrm{U}(\mathrm{n},\gamma)^{239}\mathrm{U} \xrightarrow{\beta^-(23.5\mathrm{m})} {}^{239}\mathrm{Np} \xrightarrow{\beta^-(2.35\mathrm{d})} {}^{239}\mathrm{Pu}$$

  So fissile material produced (FMP) is 250 + 645 = 895

- Since the fissile material destroyed (FMD) is 1000, from the definition, the conversion ratio is:

  CR = FMP/FMD = 895/1000 = 0.895

Breeding Ratio

- If conversion ratio is greater than 1, it is called a **breeding ratio** BR

- A reactor with the conversion ratio CR less than 1 is called a **converter**
- A reactor with the conversion ratio CR greater than 1 is called a **breeder**

- Present LWRs are converters

- Fast reactors have potential to be breeders

Breeding Ratio

- Breeding ratio has the same definition as the conversion ratio

$$BR = \frac{\text{Fissile material produced}}{\text{Fissile material destroyed}} = \frac{FP}{FD}$$

- However, breeding can occur in core and in the blanket, thus, at any time *t*, we have

$$BR(t) = \frac{\int_{core} \phi(t)\Sigma_c^{fertile}(t)dV + \int_{blanket} \phi(t)\Sigma_c^{fertile}(t)dV}{\int_{core} \phi(t)\left(\Sigma_f(t) + \Sigma_c(t)\right)^{fissile} dV}$$

a blanket surrounds the core and contains a fertile material

Breeding Ratio

- It can be assumed that all the neutrons leaking from the core lead to breeding
- This is a reasonable assumption, since some fissions occur in the blanket compensating for the leakage
- We divide BR into two parts:
  - external (blanket)
  - internal (core)
- Here

$$\text{Ext. breeding ratio} = \frac{\text{core leakage(= breeding in blanket)}}{\text{Destruction of fissile mat. in core}}$$

Breeding Ratio

- The **core leakage** can be found from the following neutron balance in the core:

Core leakage = neutrons produced in core (= $\int_{core} \phi \nu \Sigma_f^{core} dV$)

- loss of neutrons for fission (= $\int_{core} \phi \Sigma_f^{core} dV$)

- loss of neutrons by capture (= $\int_{core} \phi \Sigma_c^{core} dV$)

$$= \int_{core} \phi \left(\nu \Sigma_f - \Sigma_f - \Sigma_c \right)^{core} dV$$

here

ν - the number of neutrons produced per fission

Σ_f, Σ_c - include contributions of all materials in the core

Breeding Ratio

- Omitting time *t* in notation and neglecting any spatial variations in the core leakage and the destruction rate, we get

$$BR_{external} \approx \frac{\left(\nu\Sigma_f - \Sigma_f - \Sigma_c\right)^{core}}{\left(\Sigma_f + \Sigma_c\right)^{fissile}} = \frac{\Sigma_f^{core}}{\Sigma_f^{fissile}} \cdot \frac{\nu - \left(1 + \alpha^*\right)}{1 + \alpha}$$

where

$\alpha^* = \Sigma_c / \Sigma_f$ - for the core

$\alpha = \Sigma_c / \Sigma_f$ - for the fissile species

if $\alpha^* = \alpha$

$$BR_{external} \approx \frac{\Sigma_f^{core}}{\Sigma_f^{fissile}} \left(\eta - 1\right)$$

$$\eta = \nu / \left(1 + \alpha\right)$$

Breeding Ratio

- The **internal breeding ratio** can be found as:

$$BR_{internal} = \frac{\int_{core} \phi \Sigma_c^{fertile} dV}{\int_{core} \phi \left(\Sigma_f + \Sigma_c \right)^{fissile} dV} \approx \frac{\Sigma_c^{fertile}}{\Sigma_f^{fissile}} \cdot \frac{1}{1+\alpha} = \frac{\Sigma_c^{fertile}}{\Sigma_f^{fissile}} \cdot \frac{\eta}{\nu}$$

Typical cross section and fission neutron data

	Thermal neutrons						Fast neutrons	
Material	σ_f	σ_c	σ_a	α	ν	η	ν	η
^{233}U	531	48.0	579	0.090	2.49	2.29	2.58	2.40
^{235}U	582	99.0	681	0.170	2.42	2.07	2.51	2.35
^{239}Pu	747	269	1012	0.362	2.93	2.15	3.04	2.90
^{238}U	-	2.70	2.70	-				
U-nat	4.20	3.40	7.60	0.81				

Breeding Ratio

- The sodium cooled fast reactors, with $^{239}PuO_2$ and $^{238}UO_2$ are expected to have the total (external+internal) breeding ratio of about 1.2

- With carbide (UC and PuC) fuel and blanket material the breeding ratio should be even larger due to higher heavy-metal atom densities than oxides

Breeding Gain

- In a breeder reactor it is possible that the conversion ratio for the core only, is less than 1, while the breeding ratio for the entire reactor, core and blanket, is greater than 1
- Breeding gain is defined as: $G = BR - 1$
- This is equivalent to:

$$G = \frac{F_{EOC} - F_{BOC}}{FD} = \frac{FG}{FD}$$

FG – Fissile material gained

FD – Fissile material destroyed

here: F_{EOC}, F_{BOC} – fuel inventory @ EOC and BOC, resp.

BOC – beginning of cycle **EOC** – end of cycle

Breeder Reactor

- It is possible for a nuclear reactor to breed over a wide neutron energy spectrum

- Adequate breeding ratios can be achieved for a given energy spectrum only by carefully selecting the appropriate fissile isotopes for that spectrum

- It can be shown that a **high breeding gain can be obtained only with a fast neutron spectrum**

- **Thus fast-spectrum reactors can serve as breeder reactors**

Breeding Potential

- The highest possible breeding ratio (**breeding potential**) is

$$\overline{BR}_{\max} = \overline{\eta} - 1$$

where $$\overline{\eta} = \frac{\overline{\nu}_f \overline{\sigma}_f}{\overline{\sigma}_a} = \frac{\overline{\nu}_f}{1 + \dfrac{\overline{\sigma}_c}{\overline{\sigma}_f}} = \frac{\overline{\nu}_f}{1 + \overline{\alpha}}$$

$$\overline{\sigma}_a = \overline{\sigma}_c + \overline{\sigma}_f$$

absorption = capture + fission

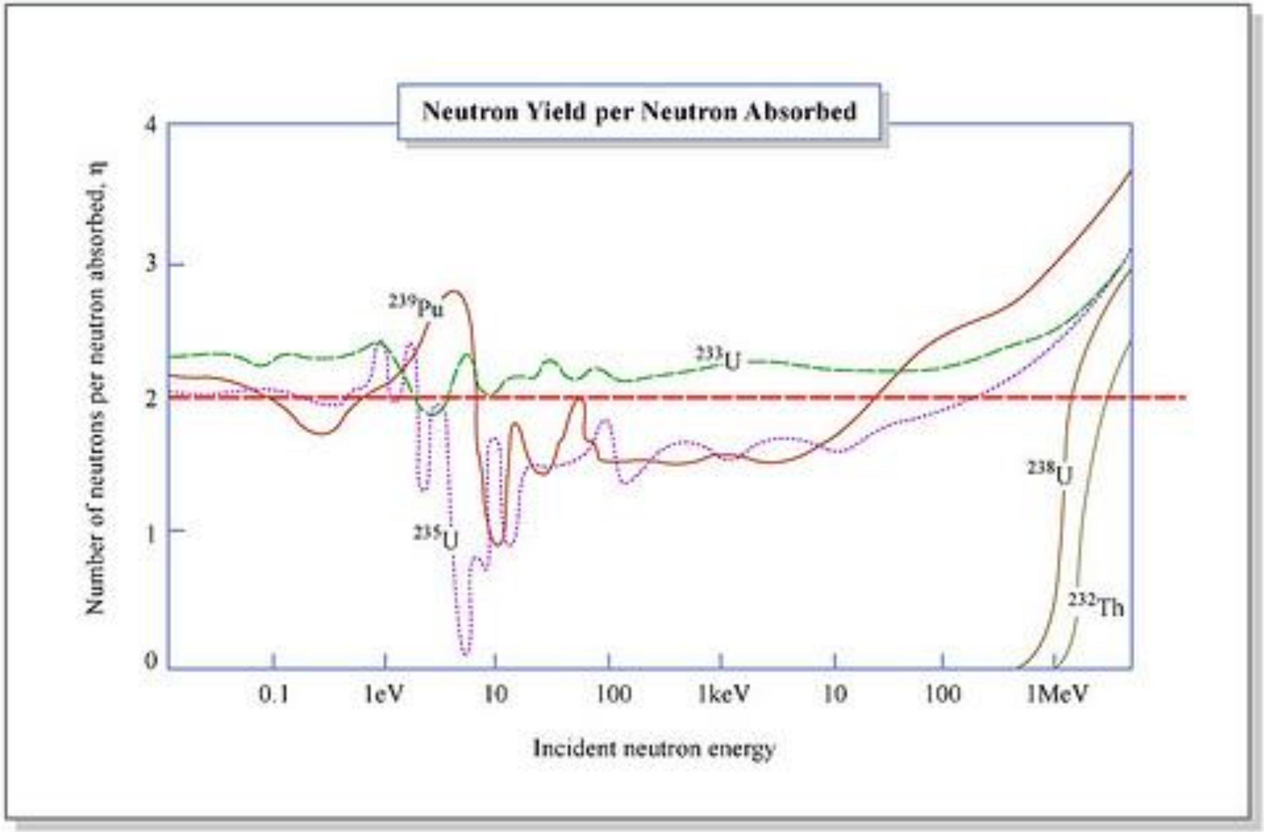
$\overline{\eta}$ Number of neutrons produced per neutron absorbed

$\overline{\nu}_f$ Number of neutrons per fission

$\overline{\alpha}$ Capture-to-fission ratio

All quantities are averaged over the energy spectrum (as indicated by the overbar)

Neutron Yield per Absorption



Minimum η for Breeder

- Balance of neutrons for breeding:
  - one neutron must be absorbed in fissile material to continue the chain reaction
  - L neutrons are lost unproductively (absorption + leakage)
  - $\eta - (1 + L)$ neutrons are captured in fertile material

- Since for breeding at least one neutron must be captured in the fertile material, we have

$$\overline{\eta} \geq 2 + L$$

Averaged Values of η

$\bar{\eta}$ averaging spectrum type	Pu-239	U-235	U-233
Averaged over LWR spectrum	2.04	2.06	2.26
Averaged over oxide-fueled FSR	2.45	2.10	2.31

Doubling Time (1)

- The Reactor Doubling Time (RDT) can be expressed as:

$$RDT = M_0/M_g$$

M_0 – initial fissile inventory in a reactor (kg)

M_g – the fissile material gained during a year (kg/y)

- Accurate computation of M_g is complex and requires computer codes

Doubling Time (2)

- It is instructive to consider an approximate calculation of M_g – the fission material gained during a year:

- It can be expressed in terms of breeding gain, *G*, rated power *P*, fraction of time at rated power *F* and $\overline{\alpha}$ - capture-to-fission ratio. First, we express M_g as:

M_g = G*(fissile mass destroyed/y) ~= G*(1+ $\overline{\alpha}$) *(fissile mass fissioned/y)

Next we expressed “fissile mass fissioned” in terms of *P*

Doubling Time (3)

- "fissile mass fissioned/y" = (*P*[MWth]x10^6) x *power in W*
  (2.93 x 10^{10} fissions/W·s) x *number of fissions per J*
  (3.15 x 10^7s/y) x *number of seconds in a year*
  (*F*) x *fraction of time at rated power*
  (239 kg/kmol) / *molar mass of fuel*
  (6.02 x 10^{26} atoms/kmol) = *Avogadro's number*
  P x *F* / 2.73

- Thus:

$$M_g = \frac{G \cdot P \cdot F \cdot \left(1 + \bar{\alpha}\right)}{2.73}$$

Doubling Time (4)

- Finally, the doubling time is found as:

$$RDT \cong \frac{2.73 M_0}{G \cdot P \cdot F \cdot \left(1 + \bar{\alpha}\right)}$$

- Here, P is in MWth, M_0 in kg and *RDT* in years

- We see that the *RDT* is proportional to the fissile specific inventory, M_0/P, and inversely proportional to breeding gain G
  - for FSR with oxide fuel M_0/P ~= 1 ÷ 2 kg/MWth

Future Perspectives of Nuclear Power

- Currently, nuclear power is mainly increasing in such countries as China, India and Russia
  - nuclear power is needed to provide emission-free electricity

- In western countries, the development is focused on
  - Small Modular Reactors
  - Generation IV reactors

- Use of fast-breeder reactors could expand the nuclear fuel resources by factor 50 – 60
  - this could significantly contribute to sustainable development of the energy sector