Nuclear Reactor Physics

Reactor Dynamics I

Jan Dufek

2022

KTH Royal Institute of Technology

Contents

Reactivity feedbacks

Temperature coefficients of reactivity

Fuel temperature (Doppler) feedback

Moderator temperature (delayed) feedback

Reactivity feedbacks

Reactivity feedbacks

What is the mechanism by which the reactivity feedback impacts the reactor power during a reactivity change?

When reactivity is changed, the following loop starts:

1. The change in reactivity will trigger changes in power.
2. Changes in power will change the temperature field.
3. Changes in the temperature field affect the macroscopic cross sections either due to changes in atomic concentrations or microscopic cross sections (that are also temperature dependent).
4. The change in cross sections cause changes in reactivity.
5. Go to Step (1).

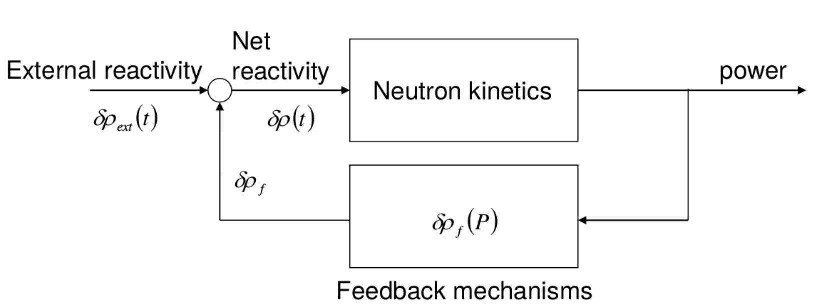
Note

Depending on the nature of the reactivity feedbacks, the above feedback loop may cause the power to:

- converge to a new steady-state (lower or higher than the previous level), or
- diverge (either to zero or infinity).

Reactivity feedbacks

A nuclear reactor with feedback can be sketched on a block diagram:



In general, a reactivity change can be represented as a sum of:

- $\delta\rho_{\mathrm{ext}}(t)$ - externally controlled reactivity changes, for instance due to control rod movement,
- $\delta\rho_{\mathrm{f}}(t)$ - reactivity changes due to reactivity feedbacks,

$$\delta\rho(t) = \delta\rho_{\mathrm{ext}}(t) + \delta\rho_{\mathrm{f}}(t)$$

Reactivity feedbacks

Specific reactivity feedbacks can be identified due to changes in specific state parameters, such as:

- fuel temperature T_{F} (also called the Doppler feedback or the prompt feedback),
- coolant/moderator temperature T_{M} (also called the delayed feedback),
- fuel depletion,
- pressure p,
- void fraction α,
- concentration of poisons, etc.

What is a coefficient of reactivity?

We can express $\delta\rho_\mathrm{f}$ in a simplified linear form as a function of various parameters:

$$\begin{aligned}\delta\rho_\mathrm{f}(T_\mathrm{F}, T_\mathrm{M}, p, \alpha, \ldots) &\approx \frac{\partial\rho}{\partial T_\mathrm{F}}\delta T_\mathrm{F} + \frac{\partial\rho}{\partial T_\mathrm{M}}\delta T_\mathrm{M} + \frac{\partial\rho}{\partial p}\delta p + \frac{\partial\rho}{\partial\alpha}\delta\alpha + \ldots \\ &= \alpha_T^F\delta T_\mathrm{F} + \alpha_T^M\delta T_\mathrm{M} + \alpha_p\delta p + \alpha_\mathrm{VF}\delta\alpha + \ldots\end{aligned}$$

where α_T^F, α_T^M, α_p, α_VF are the fuel temperature, moderator temperature, pressure and void fraction coefficients of reactivity, respectively, **evaluated at specific conditions**.

- **A reactivity coefficient corresponds to the reactivity change caused by a unit change in the specific state parameter.**
- Reactivity coefficients are used to quantify the effect of condition variations on reactivity.

Can you state some typical values of reactivity coefficients?

Type of coefficient	BWR	PWR	HTGR	LMFBR
Fuel Doppler (pcm/K)	-4 to -1	-4 to -1	-7	-0.6 to -2.5
Coolant void (pcm/%void)	-200 to -100		0	-12 to +20
Moderator (pcm/K)	-50 to -8	-50 to -8	+1.0	
Expansion (pcm/K)	~0	~0	~0	-0.92

Temperature coefficients of reactivity

Temperature coefficients of reactivity

Assume the temperature coefficient of reactivity α_T (and so $\partial\rho/\partial T$) is negative. How will the power develop in time when you increase the temperature?

An increase in T gives a decrease in ρ, so the increase in reactor temperature leads to a decrease in power, which tends to decrease the temperature and establish a new steady-state close to the original state.

Assume the temperature coefficient of reactivity α_T (and so $\partial\rho/\partial T$) is negative. How will the power develop in time when you decrease the temperature?

A decrease in T gives an increase in ρ, so the decrease in reactor temperature leads to an increase in power, which tends to increase the temperature and establish a new steady-state close to the original state.

Temperature coefficients of reactivity

Assume the temperature coefficient of reactivity α_T (and so $\partial\rho/\partial T$) is positive. How will the power develop in time when you increase the temperature?

An increase in the temperature of the reactor leads to an increase in the value of ρ, which, in turn, leads to an increase in the power level of the reactor, giving rise to a further increase in the temperature, another increase in ρ, and so on. Thus, an increase in temperature leads to ever increasing temperature and power until the reactor is either shut down by outside intervention or its fuel melts down.

Assume the temperature coefficient of reactivity α_T (and so $\partial\rho/\partial T$) is positive. How will the power develop in time when you decrease the temperature?

A decrease in T leads to a decrease in ρ. This reduces the reactor power, which reduces the temperature, giving a further decrease in ρ, and so on, until the reactor eventually shuts down.

Temperature coefficients of reactivity

We can analyse the reactivity feedbacks through dependence of the multiplication factor on state parameters. For this purpose, we need to express the temperature coefficient in therms of k.

Let's rewrite the temperature coefficient of reactivity α_T in terms of the effective multiplication factor k,

$$\begin{aligned} \alpha_T = \frac{d\rho}{dT} &= \frac{d}{dT}\left(\frac{k-1}{k}\right) \\ &= \frac{k'k-(k-1)k'}{k^2} \\ &= \frac{k'}{k^2} = \frac{1}{k^2}\frac{dk}{dT} \end{aligned}$$

The analysis of temperature feedbacks is much easier if we approximate the temperature coefficient by

$$\alpha_T \cong \frac{1}{k}\frac{dk}{dT}$$

which we can do for k near unity (near critical reactor).

Temperature coefficients of reactivity

In order to analyse dk/dT we need to break down k into a number of factors (via the six-factor formula) and study their own temperature coefficients:

$$k = k_\infty P_t P_f = \eta \epsilon p f P_t P_f$$

where

- η is the average number of fission neutrons emitted per thermal neutron absorbed in fuel,
- ϵ (fast-fission factor) is average number of neutrons produced in all fissions (fast and thermal) per neutron produced in thermal fission alone,
- p is the probability that a fission neutron is not absorbed while slowing down,
- f (thermal utilization) is the average number of thermal neutrons absorbed in fuel per thermal neutron absorbed in the reactor.
- P_t is the non-leakage probability for thermal neutrons
- P_f is the non-leakage probability for fast neutrons

Temperature coefficients of reactivity

The temperature coefficient of reactivity can be then broken into a number of independent factors describing dependence in different parameters.

E.g., substituting $k = k_\infty P_t P_f$ into

$$\alpha_T = \frac{1}{k}\frac{dk}{dT}$$

gives

$$\begin{aligned}
\alpha_T &= \frac{1}{k_\infty P_t P_f}\frac{d(k_\infty P_t P_f)}{dT} \\
&= \frac{k'_\infty P_t P_f}{k_\infty P_t P_f} + \frac{k_\infty P'_t P_f}{k_\infty P_t P_f} + \frac{k_\infty P_t P'_f}{k_\infty P_t P_f} \\
&= \frac{1}{k_\infty}\frac{dk_\infty}{dT} + \frac{1}{P_t}\frac{dP_t}{dT} + \frac{1}{P_f}\frac{dP_f}{dT}
\end{aligned}$$

That can be rewritten using the notation

$$\alpha_T(x) = \frac{1}{x}\frac{dx}{dT}$$

as

$$\alpha_T = \alpha_T(k) = \alpha_T(k_\infty) + \alpha_T(P_t) + \alpha_T(P_f)$$

Temperature coefficients of reactivity

Similarly, from the equation

$$k_\infty = \eta \epsilon p f$$

we can write for $\alpha_T(k_\infty)$

$$\alpha_T(k_\infty) = \alpha_T(\eta) + \alpha_T(\epsilon) + \alpha_T(p) + \alpha_T(f)$$

Finally, we can state that

$$\alpha_T = \alpha_T(P_t) + \alpha_T(P_f) + \alpha_T(\eta) + \alpha_T(\epsilon) + \alpha_T(p) + \alpha_T(f)$$

Fuel temperature (Doppler) feedback

What is the mechanism of the fuel temperature (Doppler) feedback?

- Fuel temperature feedback is a **prompt** feedback coming from an increased neutron resonance capture when the fuel temperature grows.
- The effect is due to broadening of resonances in microscopic cross section for neutron capture on ^{238}U.
- This feedback is also called the **Doppler feedback**.

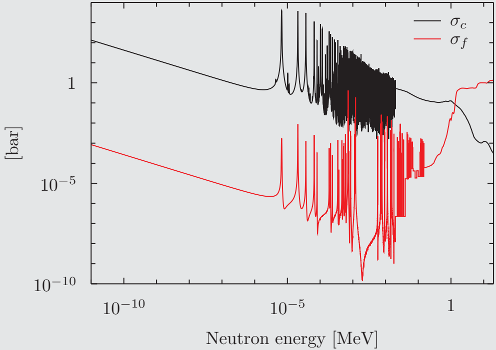


Figure 1: Microscopic cross sections of ^{238}U

Fuel temperature (Doppler) feedback

Why resonances in XS of ^{238}U broaden when fuel temperature grows?

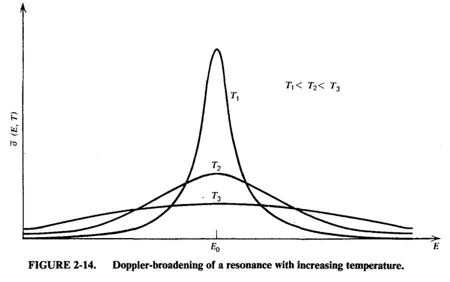


Figure 2: Doppler-broadening of a resonance peak in XS of ^{238}U

- The broadening is caused by the increased thermal motion of ^{238}U atoms.
- As the resonances widen it becomes more probable for neutrons to fall in the resonances.
- While the resonances widen, they also decrease. Nevertheless, the decrease cannot compensate the increased risk.

Fuel temperature (Doppler) feedback

As the prompt feedback comes primarily from the Doppler effect, we can write

$$\alpha_T^F \cong \alpha_T(p) = \frac{1}{p}\frac{dp}{dT}$$

where p in heterogeneous reactors can be expressed as

$$p = \exp\left[-\frac{N_F V_F I}{\xi_M \Sigma_{sM} V_M}\right]$$

Which terms in the formula may be dependent on the temperature?

- The term $N_F V_F$ represents the total number of fuel atoms, which is not dependent on T.
- The only dependence on T comes from the resonance integral I that is temperature dependent.
- We can therefore write

$$\alpha_T^F = \frac{1}{p}\frac{dp}{dT} = -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M}\frac{dI}{dT}$$

- Now we need to evaluate dI/dT...

Fuel temperature (Doppler) feedback

How does the resonance integral I depend on the temperature?

The temperature dependence of I has been measured and approximated by an empirical formula

$$I(T) = I(300\mathrm{K})[1 + \beta_I(\sqrt{T} - \sqrt{300\mathrm{K}})]$$

where β_I is given approximately by

$$\beta_I = A + C/a\rho$$

where a is the rod radius in cm, and ρ is the fuel density in g/cm^3. For $^{238}UO_2$ fuel: $A \cong 61 \times 10^{-4}$ and $C \cong 0.94 \times 10^{-2}$. From that, we can write

$$\frac{dI}{dT} = \frac{I(300\mathrm{K})\beta_I}{2\sqrt{T}}$$

and so

$$\alpha_T^F = -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M}\frac{dI}{dT} = -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M}\frac{I(300\mathrm{K})\beta_I}{2\sqrt{T}}$$

which can be written as

$$\alpha_T^F = \frac{\beta_I}{2\sqrt{T}} \ln\left[p(300\mathrm{K})\right]$$

Fuel temperature (Doppler) feedback

A negative prompt feedback is more important for ensuring the reactor stability than other feedbacks, e.g. one that comes from the moderator temperature.

- This is because an increase in reactor power causes an immediate change in fuel temperature (hence the feedback from fuel is prompt) while it takes a certain time (seconds) for the heat to get transferred into the moderator.
- The moderator feedback is delayed, which may lead to reactor instability even if the feedback is negative!

Moderator temperature (delayed) feedback

Moderator temperature (delayed) feedback

What is the mechanism of the moderator temperature (delayed) feedback? There are two processes. . .

The moderator temperature feedback comes from the effect of the changing

- **moderator density** on the **moderation process** (expressed by p) and
- neutron **absorption rate in moderator** (on hydrogen) that is expressed by the f factor.

Therefore,

$$\alpha_T^M = \alpha_T^M(p) + \alpha_T^M(f) = \frac{1}{p}\frac{dp}{dT_M} + \frac{1}{f}\frac{df}{dT_M}$$

Moderator temperature (delayed) feedback

What is the sign of the moderator temperature coefficient of resonance escape probability, $\alpha_T^M(p) = \frac{1}{p}\frac{dp}{dT_M}$?

- When the moderator temperature **decreases** then N_M/N_F **increases**. From

$$p = e^{-\frac{N_F V_F I}{\xi_M N_M \sigma_{sM} V_M}}$$
$$= e^{-\frac{V_F I}{\xi_M \frac{N_M}{N_F} \sigma_{sM} V_M}}$$

  one can see then that p **increases**.

- This is because neutrons get more moderated, and less frequently collide with ^{238}U at resonance energies.
- **Therefore, dp/dT_M and so $\alpha_T^M(p)$ must be negative.**
- **Note that p increases with increasing N_M/N_F.**

Moderator temperature (delayed) feedback

What is the sign of the moderator temperature coefficient of the thermal utilization factor, $\alpha_T^M(f) = \frac{1}{f}\frac{df}{dT_M}$?

- The thermal utilization f in heterogeneous reactors

$$
\begin{aligned}
f &= \frac{\Sigma_{aF} V_F}{\Sigma_{aM} V_M \zeta + \Sigma_{aF} V_F} \\
&= \frac{\sigma_{aF} V_F}{\frac{N_M}{N_F}\sigma_{aM} V_M \zeta + \sigma_{aF} V_F}
\end{aligned}
$$

  decreases when the N_M/N_F ratio **increases** (when the moderator density grows - i.e. when T_M **decreases**). This is because neutrons are more likely to be absorbed in the moderator.

- **Thus, df/dT_M and so $\alpha_T^M(f)$ are positive.**
- **Note that f decreases as the N_M/N_F ratio grows.**

Moderator temperature (delayed) feedback

Knowing there are both positive and negative partial feedbacks forming the total delayed feedback, is the total delayed feedback positive or negative?

- $\alpha_T^M(p)$ is negative.
- $\alpha_T^M(f)$ is positive.

$\Rightarrow \alpha_T^M = \alpha_T^M(p) + \alpha_T^M(f)$ may be negative or positive, depending on the actual conditions.

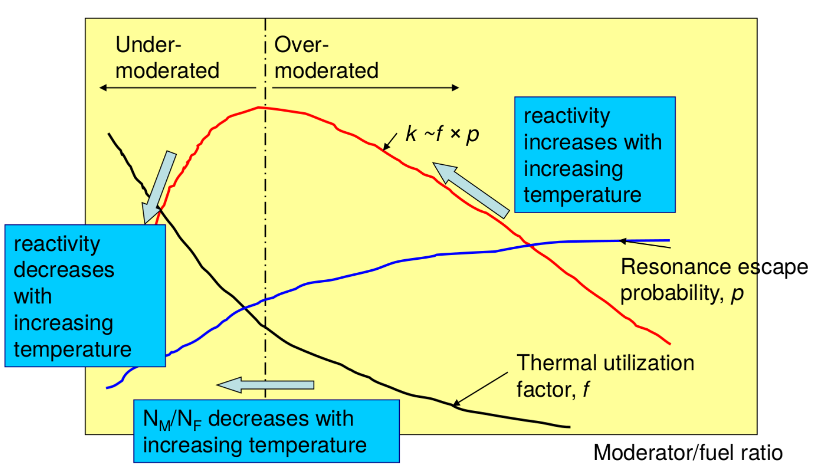
Is there an optimum N_M/N_F ratio?

Since $k \sim p \times f$ there must be an optimum N_M/N_F ratio for which k reaches a local extreme.

What are the under/over-moderated systems?

- A system is called under-moderated when its N_M/N_F ratio is below the optimum value. α_T^M is negative.
- A system is called over-moderated when its N_M/N_F ratio is above the optimum value. α_T^M is positive.

Moderator temperature (delayed) feedback



Over-moderated systems are unstable. Reactors must be designed to be under-moderated.