Lectures on Thermal-Hydraulics in Nuclear Energy Engineering

Lecture No 05

Title:

Boiling Channel – Part I: Subcooled Boiling Heat Transfer

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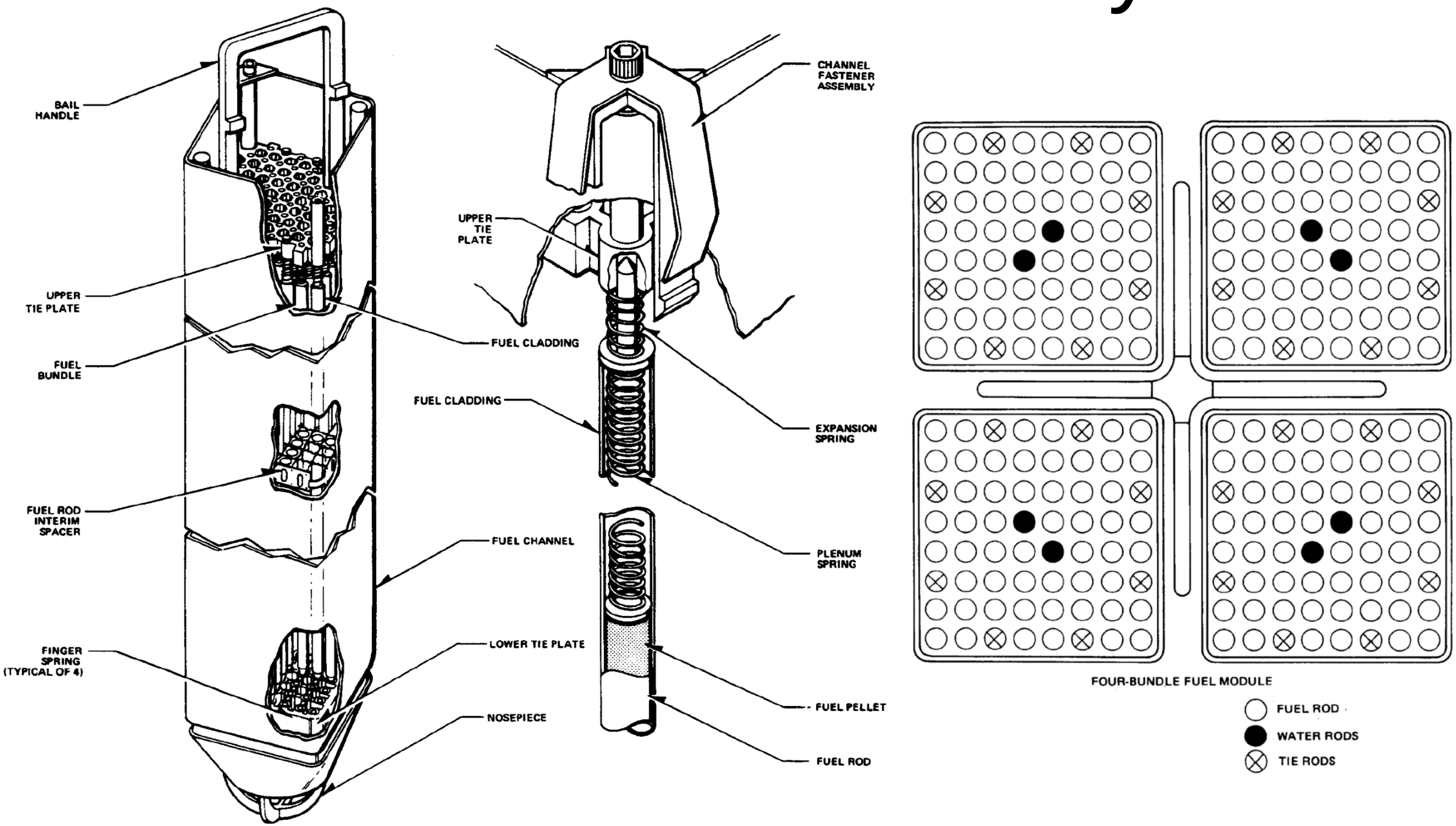
Outline of the Lecture

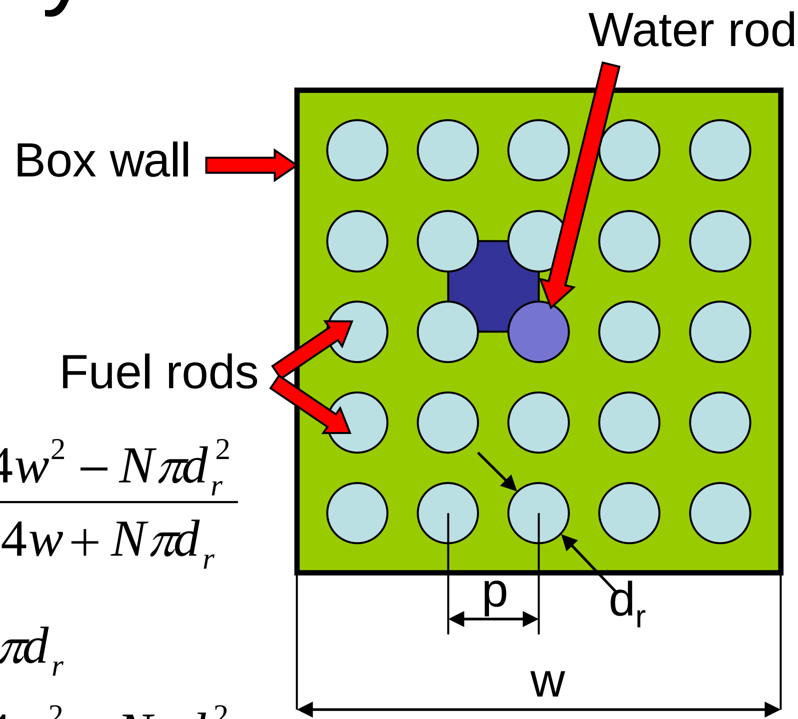
- Energy balance in BWR fuel
  - Whole-Assembly Model
- Heat transfer regimes
- Onset of nucleate boiling
- Subcooled flow boiling
  - Partial subcooled nucleate boiling
  - Fully-developed subcooled nucleate boiling

BWR Fuel Assembly



Whole-Assembly Model

- This model is suitable to BWR fuel assemblies



- Basic parameters:
  - hydraulic diameter D_h $$D_h \equiv \frac{4A}{P_w} = \frac{4w^2 - N\pi d_r^2}{4w + N\pi d_r}$$
  - wetted perimeter P_w $$P_w = 4w + N\pi d_r$$
  - heated diameter D_H $$D_H \equiv \frac{4A}{P_H} = \frac{4w^2 - N\pi d_r^2}{N_{FR}\pi d_r}$$
  - heated perimeter P_H $$P_H = N_{FR}\pi d_r$$
  - N_{FR},N_{WR} – nr of fuel /water rods $$N = N_{FR} + N_{WR}$$

N – total number of rods

A – cross-section flow area

w – assembly width

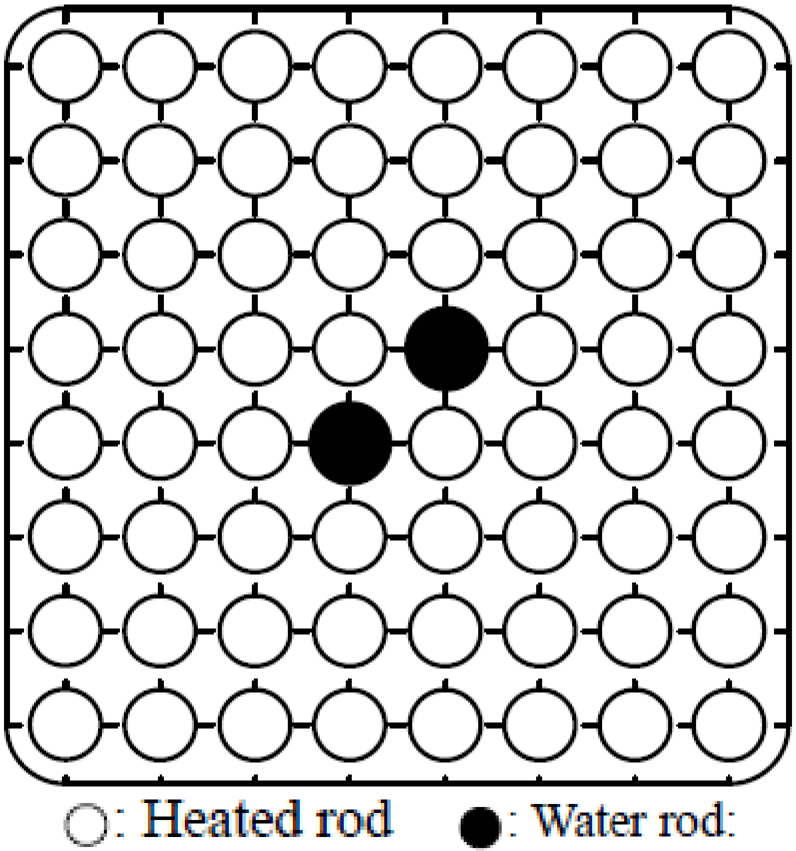
d_r – rod diameter

p – lattice pitch

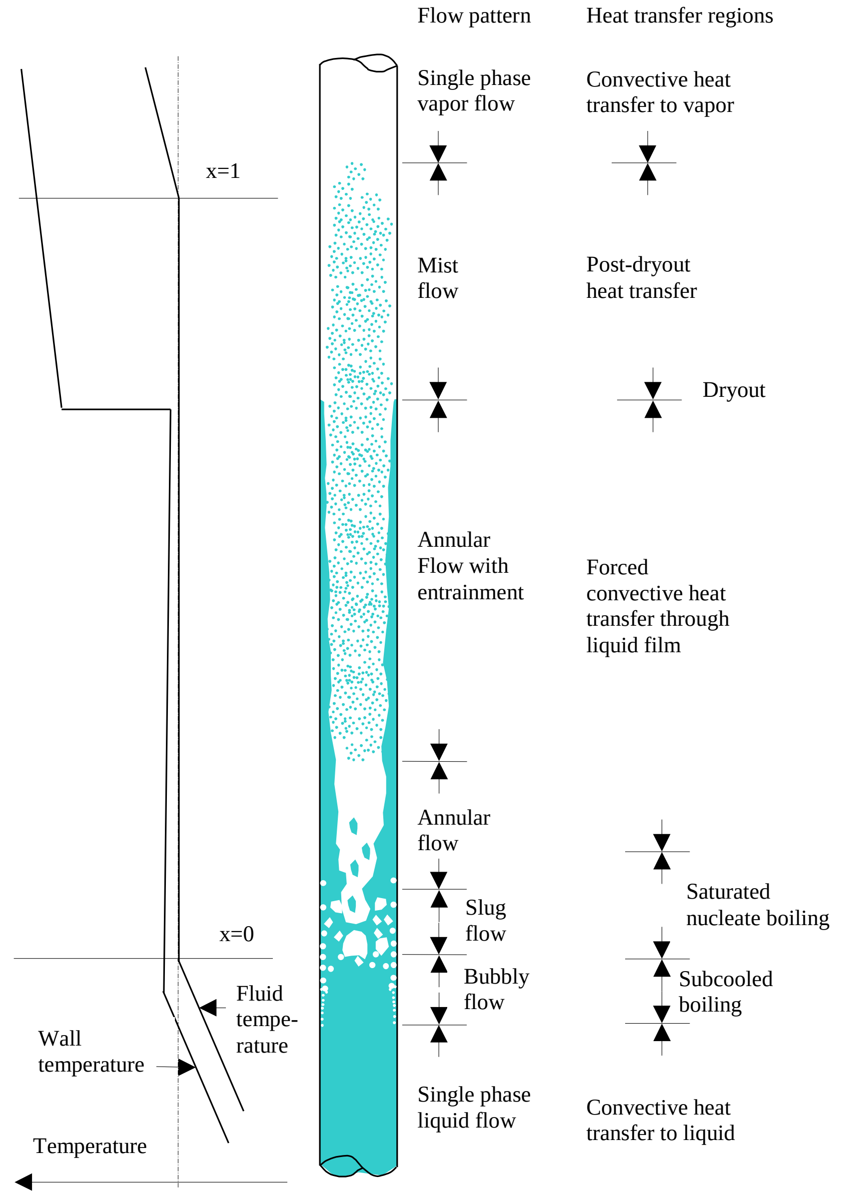
Whole-Assembly Model –Exercise

For the BWR fuel assembly shown in the figure, calculate (1) flow area, (2) wetted perimeter, (3) hydraulic diameter (4) heated perimeter, (5) heated diameter. Assume Whole-Assembly Model. Neglect corner radius in the channel box.

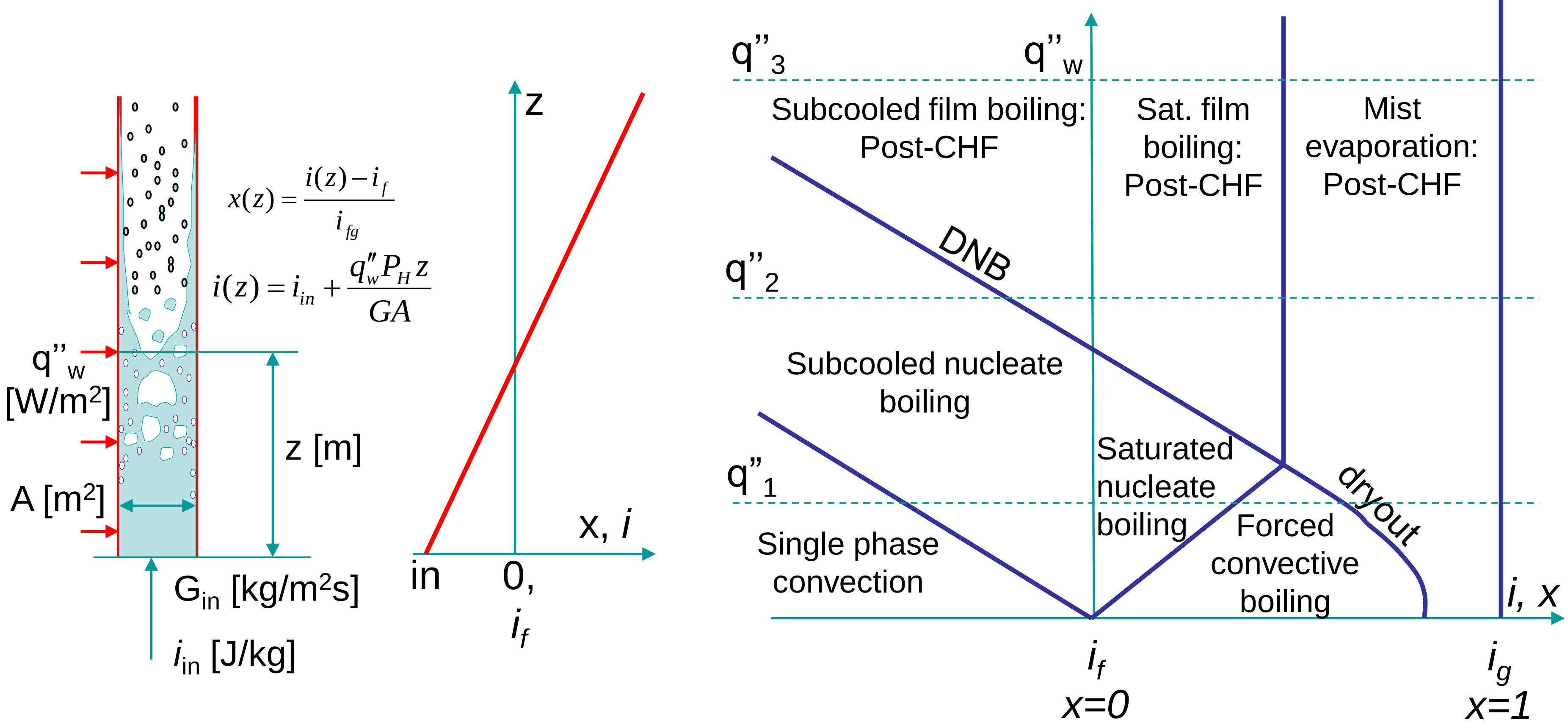
Given:
Number of heated rods: 8X8-2
Number of water rods: 2
Heated rod outer diameter: 12.3 mm
Heated rod pitch: 16.2 mm
Water rod outside diameter: 15 mm
Channel box inner width: 132.5 mm



Flow and Heat Transfer Regimes in a Boiling Channel



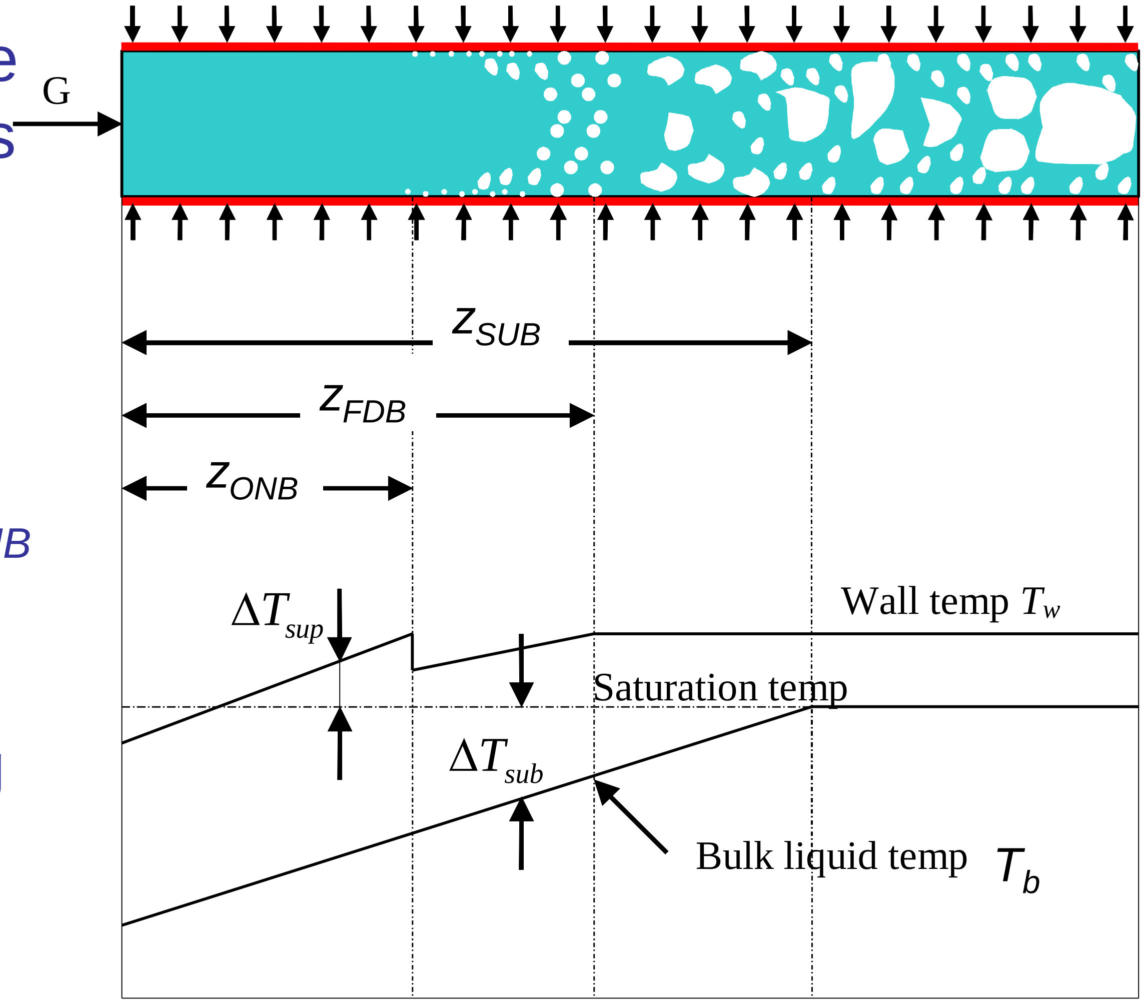
Heat transfer regimes with constant wall heat flux



Boiling channel Enthalpy distribution Channel heat transfer regime map

Subcooled-Boiling Region

- Onset of Nucleate Boiling (ONB) – is a point where boiling first appears in the channel. It is located at $z = z_{ONB}$ from the inlet
- z_{FDB} – fully developed boiling
- z_{SUB} – subcooled region flow



Subcooled Region Length

- The coolant temperature in a uniformly heated channel is a linear function of the axial distance

$$T_b(z) = T_{in} + \frac{q'' P_H z}{c_p G A}$$

T_{in} – inlet fluid temperature

- From which, the length of the subcooled region can be readily obtained as (a point at which $T_b(z)=T_{sat}$):

$$z_{SUB} = \frac{c_p G A}{q'' P_H}\left(T_{sat} - T_{in}\right) = \frac{c_p G A}{q'' P_H}\Delta T_{subi}$$

ΔT_{subi} Inlet subcooling

T_{sat} – saturation temperature

Wall Superheat

- The Newton equation of cooling is as follows

$$T_w - T_b = q''/h$$

T_w wall surface temperature

- Thus, the wall surface temperature becomes

$$T_w(z) = T_b(z) + \frac{q''}{h} = T_{in} + q''\left(\frac{P_H z}{c_p G A} + \frac{1}{h}\right)$$

- Or, introducing so-called **wall superheat** *$\Delta T_{sup}(z)$*, it can be expressed as a function of *z*-coordinate as follows:

$$\Delta T_{\sup}(z) \equiv T_w(z) - T_{sat} = -\Delta T_{subi} + q''\left(\frac{P_H z}{c_p G A} + \frac{1}{h}\right)$$

Bowring's Model (1)

- Clearly, there is no boiling when the wall superheat is less than zero

- Bowring suggested that at the onset-of-nucleate-boiling point the wall superheat is equal to that which results from a subcooled boiling correlation

- Experiments indicate that in subcooled boiling the wall superheat and the applied heat flux are related as

$$\Delta T_{\mathrm{sup}} = \psi \cdot \left(q''\right)^{n}$$

n and ψ - parameters

Bowring's Model (2)

- Thus Bowring's expression for the local superheat for onset of nucleate boiling is

$$\Delta T_{\sup}(z)\Big|_{ONB} \equiv T_w(z_{ONB}) - T_{sat} = -\Delta T_{subi} + q''\left(\frac{P_H z_{ONB}}{c_p G A} + \frac{1}{h}\right) = \psi \cdot \left(q''\right)^n$$

- From which, the coordinate of onset of nucleate boiling z_{ONB} is found as

$$\boxed{z_{ONB} = \left[\frac{\Delta T_{subi} + \psi \cdot \left(q''\right)^n - \dfrac{q''}{h}}{q'' P_H}\right] c_p G A}$$

Here $\Delta T_{\sup} = \psi \cdot \left(q''\right)^n$ has to be found from a suitable correlation for subcooled boiling heat transfer

Subcooled Boiling Correlations

- Examples of subcooled boiling heat transfer correlations: With general form $\Delta T_{sup} = f(q'', p, ...)$
  - Jens-Lottes

$$\Delta T_{sup} = 25\left(\frac{q''}{10^6}\right)^{0.25} e^{-p/62}$$

  - Thom et al.

$$\Delta T_{sup} = 22.65\left(\frac{q''}{10^6}\right)^{0.5} e^{-p/87}$$

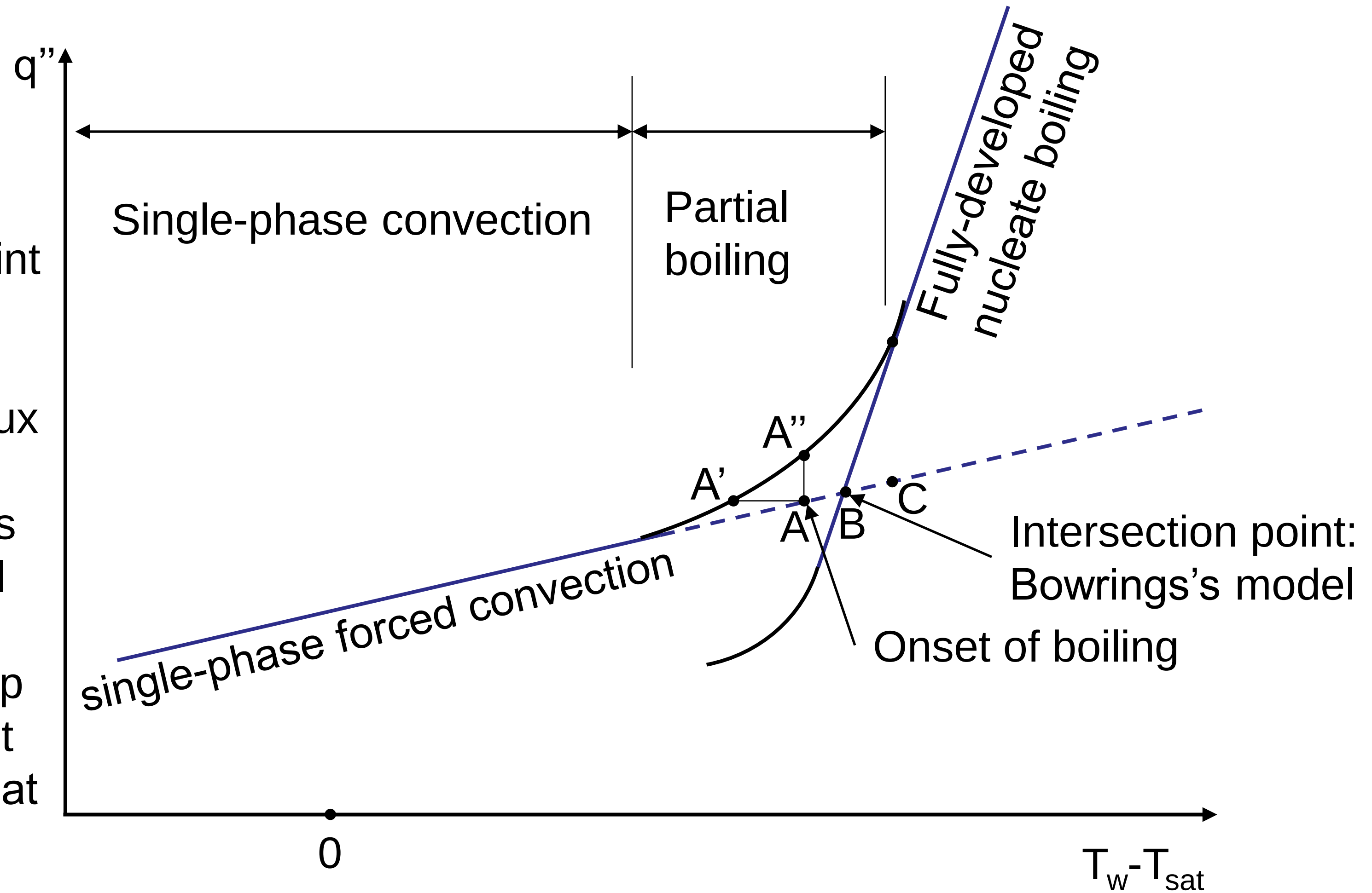
ΔT_{sup} – wall superheat, K

p – pressure, bar

q'' – heat flux, W/m^2

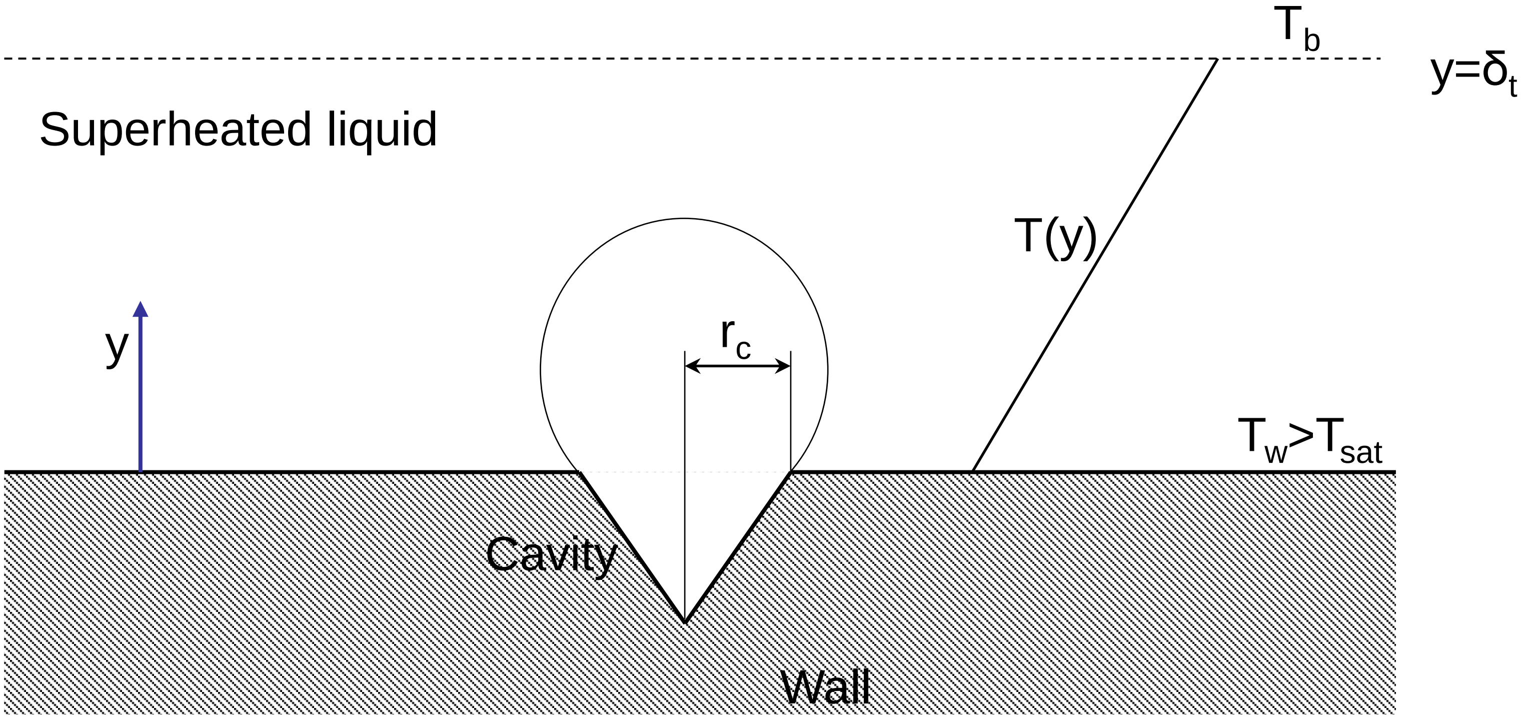
Heat Flux vs Wall Superheat

The onset may occur at any point A, B or C (for example). With constant heat flux condition, when the onset occurs at pt A, the local boiling parameters jump horizontally to pt A' (wall superheat decreases)



Hsu's Model (1962)

- Hsu postulated, that nucleate boiling is possible in the thermal boundary layer only when the wall superheat is high enough to allow grow of bubbles at the wall



Hsu's Model (1962)

- He derived the following criterion for the wall superheat at the onset of nucleate boiling:

$$\Delta T_{\text{sup}} = -\Delta T_{\text{sub}} + \frac{\theta + \sqrt{4 \cdot \Delta T_{\text{sub}} \cdot \theta + \theta^2}}{2} \qquad \theta = \frac{12.8 \cdot \sigma \cdot T_{sat}}{\rho_g \cdot i_{fg} \cdot \delta_t} \qquad \delta_t \cong \frac{\lambda_f}{h_{1\phi}}$$

λ_f – saturated liquid thermal conductivity, W/mK
$h_{1\phi}$ – single-phase heat transfer coefficient, W/m^{2}K

ΔT_{sub} – local subcooling, K
σ – surface tension, N/m
T_{sat} – saturation temperature, K
ρ_g – vapour saturated density, kg/m^3
i_{fg} – latent heat, J/kg
δ_t – thermal boundary thickness, m

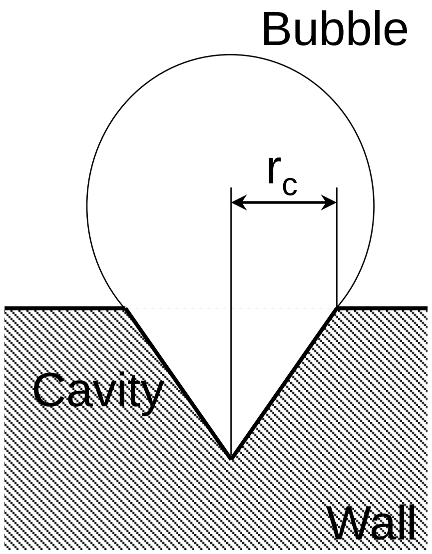
Hsu's Model (1962)

- Hsu's model predicts also which cavity sizes at the wall surface will allow the bubble to grow
- The minimum cavity size is given as

$$r_{c,\min} = \frac{\delta_t}{4}\left[\frac{\Delta T_{\text{sup}}}{\Delta T_{\text{sub}} + \Delta T_{\text{sup}}} - \sqrt{\left(\frac{\Delta T_{\text{sup}}}{\Delta T_{\text{sub}} + \Delta T_{\text{sup}}}\right)^2 - \frac{\theta}{\Delta T_{\text{sub}} + \Delta T_{\text{sup}}}}\right]$$

- and the maximum:

$$r_{c,\max} = \frac{\delta_t}{4}\left[\frac{\Delta T_{\text{sup}}}{\Delta T_{\text{sub}} + \Delta T_{\text{sup}}} + \sqrt{\left(\frac{\Delta T_{\text{sup}}}{\Delta T_{\text{sub}} + \Delta T_{\text{sup}}}\right)^2 - \frac{\theta}{\Delta T_{\text{sub}} + \Delta T_{\text{sup}}}}\right]$$



Example 1

- Find the wall superheat at which the nucleate boiling starts for water at atmospheric pressure and 0 K subcooling. Heat transfer coefficient is $h_{1\varphi}$=11 kW/m^{2}K
- SOLUTION: using XSteam we find water properties: σ=0.059 N/m; λ_f=0.678 W/mK; T_{sat}=373.15 K; ρ_g=0.598 kg/m^3; i_{fg}=2.257·10^6 J/kg.

  We find $\delta_t=\lambda_f/\, h_{1\varphi}$ =6.16·10^{-5} m. And: $\theta = \dfrac{12.8 \cdot \sigma \cdot T_{sat}}{\rho_g \cdot i_{fg} \cdot \delta_t} = 3.4\ \mathrm{K}$

$$\Delta T_{\mathrm{sup}} = -\Delta T_{\mathrm{sub}} + \frac{\theta + \sqrt{4 \cdot \Delta T_{\mathrm{sub}} \cdot \theta + \theta^2}}{2} = \theta = 3.4\ \mathrm{K}$$

Thus, the boiling will start when the wall superheat exceeds 3.4 K

Example 2

- For conditions as in Example 1, assume that wall superheat is 5 K. calculate the cavity size range for which nucleate boiling is possible. Use the same δ_t.
- SOLUTION: we have now : $\Delta T_{\mathrm{sup}} = 5\,\mathrm{K}$ and $\theta = \dfrac{12.8 \cdot \sigma \cdot T_{sat}}{\rho_g \cdot i_{fg} \cdot \delta_t} = 3.4\,\mathrm{K}$
  Thus:

$$r_{c,\min} = \frac{\delta_t}{4}\left[\frac{\Delta T_{\mathrm{sup}}}{\Delta T_{\mathrm{sub}} + \Delta T_{\mathrm{sup}}} - \sqrt{\left(\frac{\Delta T_{\mathrm{sup}}}{\Delta T_{\mathrm{sub}} + \Delta T_{\mathrm{sup}}}\right)^2 - \frac{\theta}{\Delta T_{\mathrm{sub}} + \Delta T_{\mathrm{sup}}}}\right] = 6.65 \cdot 10^{-6}\ \mathrm{m}$$

and:

$$r_{c,\max} = \frac{\delta_t}{4}\left[\frac{\Delta T_{\mathrm{sup}}}{\Delta T_{\mathrm{sub}} + \Delta T_{\mathrm{sup}}} + \sqrt{\left(\frac{\Delta T_{\mathrm{sup}}}{\Delta T_{\mathrm{sub}} + \Delta T_{\mathrm{sup}}}\right)^2 - \frac{\theta}{\Delta T_{\mathrm{sub}} + \Delta T_{\mathrm{sup}}}}\right] = 2.42 \cdot 10^{-5}\ \mathrm{m}$$

Thus only cavities with size from 6.65 μm to 24.2 μm will have potential to be active and generate vapour bubbles

Implications of Hsu's Model

- The model predicts a certain **minimum wall superheat** that must be attained before subcooled nucleate boiling can occur
- When the subcooled nucleate boiling is predicted to be possible, the model provides a **range of nucleation site sizes** that have potential to be active
- This nucleation size range depends on wall superheat, fluid properties and thermal layer thickness
- The significance of the model is in **providing insight** into the mechanisms observed in experiments, even though its predictive capacity is limited

Sato and Matsumura (1964)

- Sato and Matsumura derived their model from similar assumptions as adopted by Hsu
- They showed that the following condition should be satisfied at ONB:

$$q''_w = \frac{\lambda_f \cdot i_{fg} \cdot \rho_g \cdot \Delta T_{\mathrm{sup}}^2}{8\sigma T_{\mathrm{sat}}}$$

where:

λ_f – saturated liquid thermal conductivity, W/mK
ΔT_{sup} – local wall superheat, K
σ – surface tension, N/m
T_{sat} – saturation temperature, K
ρ_g – vapour saturated density, kg/m^3
i_{fg} – latent heat, J/kg
q''_w – wall heat flux, W/m^2

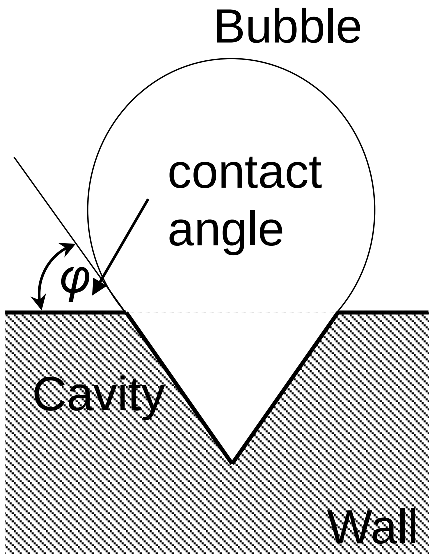
Davis and Anderson (1966)

- Davis and Anderson extended the model by Sato and Matsumura to include the effect of the contact angle:
- They showed that the following dependence of the contact angle exists:

$$q''_w = \frac{\lambda_f \cdot i_{fg} \cdot \rho_g \cdot \Delta T_{\sup}^2}{8\left(1+\cos\varphi\right)\sigma T_{\mathrm{sat}}}$$

where:

λ_f – saturated liquid thermal conductivity, W/mK
ΔT_{sup} – local wall superheat, K
σ – surface tension, N/m
T_{sat} – saturation temperature, K
ρ_g – vapour saturated density, kg/m^3
i_{fg} – latent heat, J/kg
q''_w – wall heat flux, W/m^2



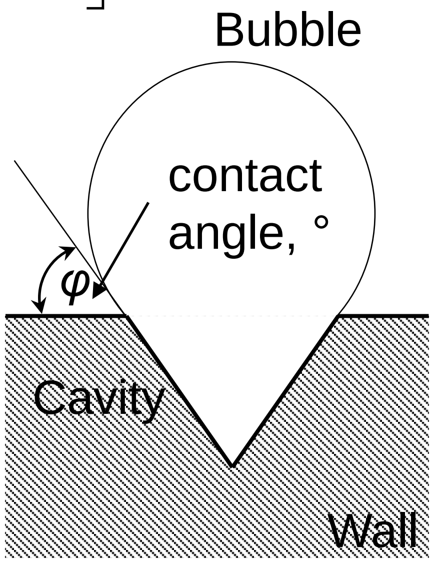
Basu *et al*. (2002)

- Basu *et al*. considered that not all cavities will remain active in subcooled boiling and some of them will be flooded, especially when surface is hydrophilic. They proposed:

$$q''_w = \frac{F^2 \lambda_f \cdot i_{fg} \cdot \rho_g \cdot \Delta T_{\mathrm{sup}}^2}{2\sigma T_{\mathrm{sat}}}$$

$$F = 1 - \exp\left[-\varphi_{rad}^3 - 0.5\varphi_{rad}\right]$$

$$\varphi_{rad} = \pi\varphi/180$$



λ_f – saturated liquid thermal conductivity, W/mK
ΔT_{sup} – local wall superheat, K
σ – surface tension, N/m
T_{sat} – saturation temperature, K
ρ_g – vapour saturated density, kg/m^3
i_{fg} – latent heat, J/kg
q''_w – wall heat flux, W/m^2

Example 3

Water flows upward in a vertical tube with inside diameter of D=10 mm. The pressure along the tube is constant and equal 6124 kPa. Subcooled water enters the pipe and its mass flux is $G=9000\ kg/m^2s$. The wall is held at uniform temperature of 281 °C. Estimate the inlet subcooling knowing that ONB point is located 100 mm downstream of the inlet. Use Sato&Matsumura correlation.

SOLUTION: from XSteam we find: $T_{sat} = 550$ K, $\rho_f=755.7\ kg/m^3$, $\rho_g=31.5\ kg/m^3$, $i_{fg}=1.563\cdot10^6$ J/kg, $c_{pf}=5231$ J/kg K, $\mu_f=9.47\cdot10^{-5}$ Pa.s, $Pr_f=0.851$; $\lambda_f=0.582$ W/mK, $\sigma=0.0197$ N/m

Example 3

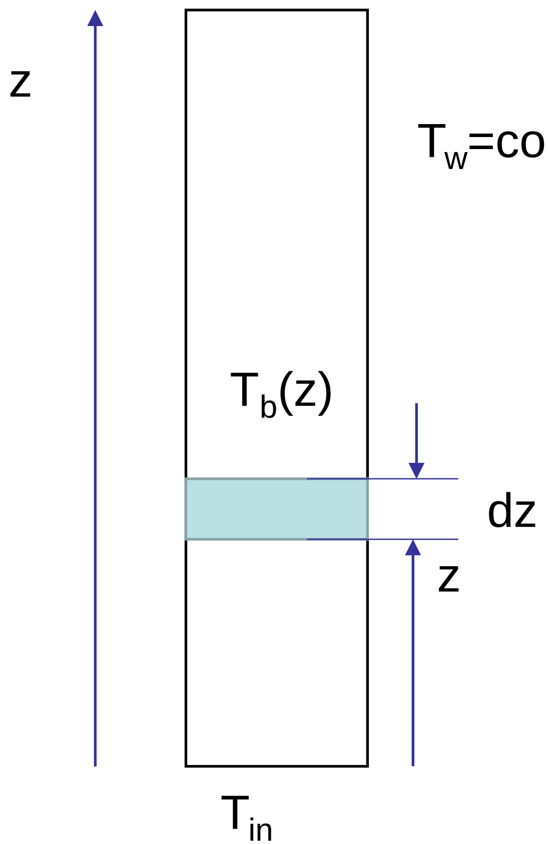
- We need to find heat flux distribution as

$$q''_w = h \cdot \left[T_w - T_b \left(z \right) \right]$$

- where *h* is the single-phase heat transfer coefficient (we will use the Dittus-Boelter correlation to find it) and $T_b(z)$ is the liquid bulk temperature (we will use energy balance to find it)

$$h = 0.023 \cdot \frac{\lambda_f}{D} \left(\frac{G \cdot D}{\mu_f} \right)^{0.8} \mathrm{Pr}_f^{0.4} = 76.02 \frac{\mathrm{kW}}{\mathrm{m^2 K}}$$

Example 3



Energy conservation for differential channel length *dz*:

$$c_p \cdot dT_b \cdot G \frac{\pi D^2}{4} = q''_w \pi D dz =$$

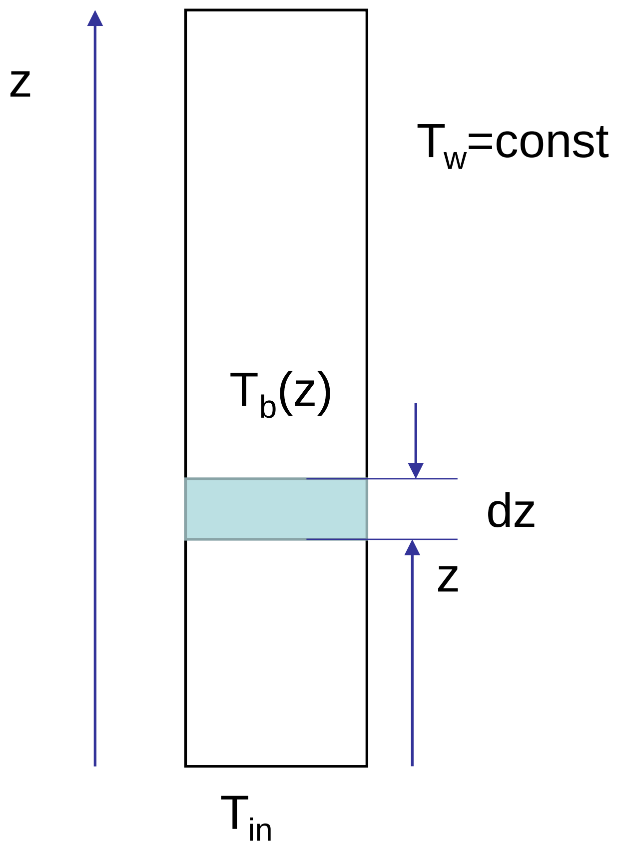
$$h \cdot \left(T_w - T_b\right) \pi D dz$$

$$\frac{dT_b}{T_w - T_b} = \frac{4h}{c_p \cdot G \cdot D} dz$$

After integration:

$$T_b(z) = T_w - \left(T_w - T_{in}\right) e^{-\frac{4h \cdot z}{c_p \cdot G \cdot D}}$$

Example 3



We have the following condition:

$$q''_{ONB} = q''(z_{ONB}) = h\left[T_w - T_b(z_{ONB})\right] =$$

$$h(T_w - T_{in})e^{-\frac{4h \cdot z_{ONB}}{c_p \cdot G \cdot D}} = \frac{\lambda_f i_{fg} \rho_g (T_w - T_{sat})^2}{8\sigma T_{sat}}$$

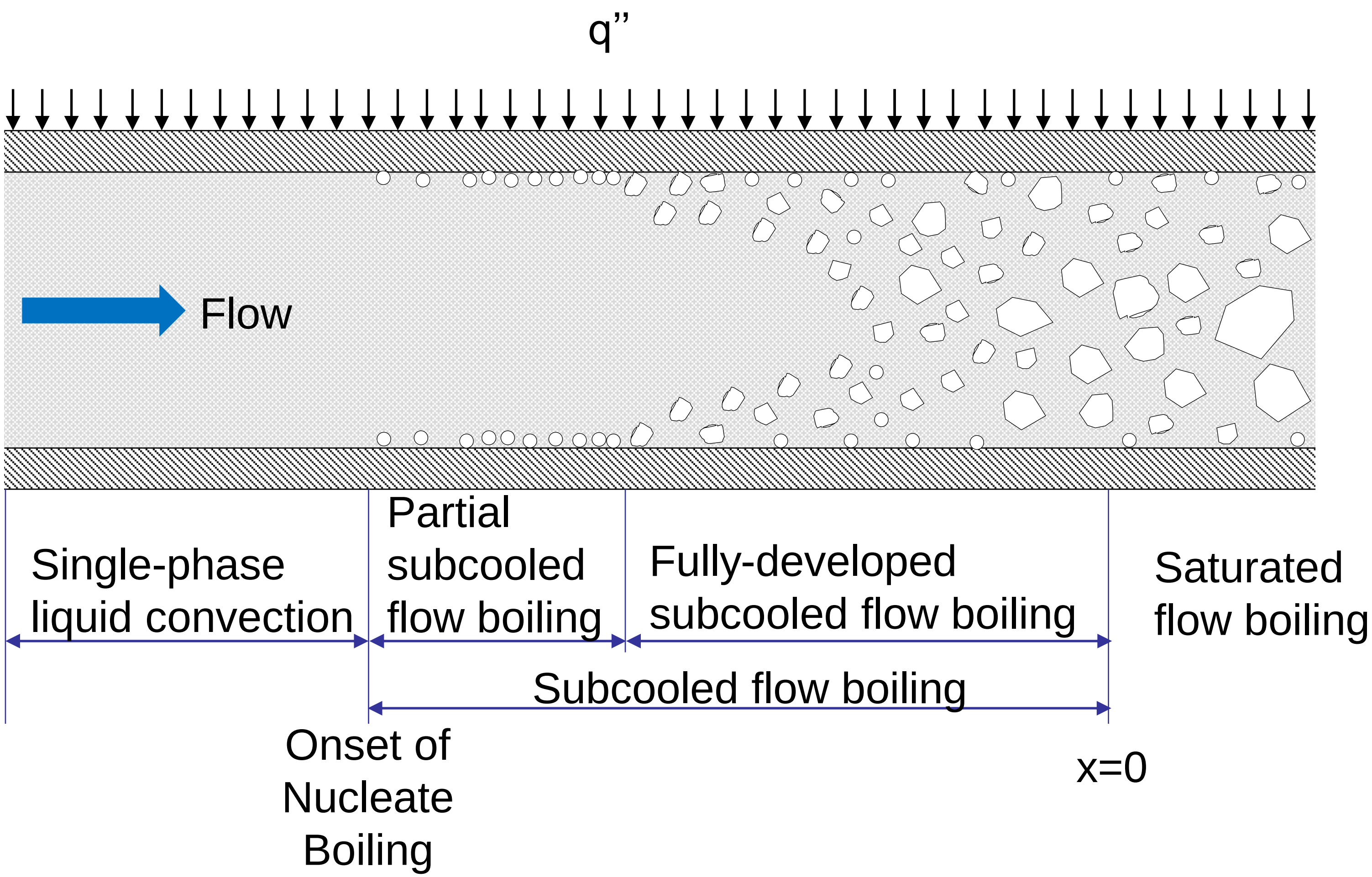
From this equation we find T_{in} as:

$$T_{in} = T_w - \frac{q''_{ONB}}{h} e^{\frac{4h \cdot z_{ONB}}{c_p \cdot G \cdot D}} = 475.64 \text{ K}$$

$$\Delta T_{subi} = T_{sat} - T_{in} = 74.35 \text{ K}$$

Thus the inlet subcooling is 74.35 K

Regimes of Subcooled Boiling



Partial Subcooled Boiling

- Partial subcooled boiling can be treated as a superposition of single-phase liquid (1φ) contribution and subcooled-nucleate boiling (2φ) contribution

- It is plausible to partition the total heat flux in these two contributions as

$$q''_{tot} = q''_{1\phi} + q''_{2\phi}$$

- where $q''_{1\phi} = h_{1\phi}\left(T_w - T_b\right)$ $\quad q''_{2\phi} = h_{2\phi}\left(T_w - T_{sat}\right)^m$

- For example, Rohsenow's model is

$$\frac{q''_{2\phi}}{\mu_f i_{fg}}\left[\frac{\sigma}{g\left(\rho_f - \rho_g\right)}\right]^{1/2} = \left(\frac{1}{C_{sf}}\right)^{1/r} \mathrm{Pr}_f^{-s/r}\left[\frac{c_{pf}\left(T_w - T_{sat}\right)}{i_{fg}}\right]^{1/r}$$

For water s=1, r=1/3. C_{sf} depends on liquid-surface combination. For water on polished stainless steel C_{sf}=0.0132

Partial Subcooled Boiling

- Thus using the Rohsenow correlation we have

$$q''_{2\phi} = \frac{\mu_f i_{fg}}{\left[\frac{\sigma}{g\left(\rho_f - \rho_g\right)}\right]^{1/2}} \left(\frac{1}{C_{sf}}\right)^{1/r} \mathrm{Pr}_f^{-s/r} \left[\frac{c_{pf}\left(T_w - T_{sat}\right)}{i_{fg}}\right]^{1/r} =$$

$$\frac{\mu_f i_{fg}}{\left[\frac{\sigma}{g\left(\rho_f - \rho_g\right)}\right]^{1/2}} \left(\frac{c_{pf}}{C_{sf} i_{fg}}\right)^{1/r} \mathrm{Pr}_f^{-s/r} \left(T_w - T_{sat}\right)^{1/r} = h_{2\phi}\left(T_w - T_{sat}\right)^m$$

- where $h_{2\phi} = \frac{\mu_f i_{fg}}{\left[\frac{\sigma}{g\left(\rho_f - \rho_g\right)}\right]^{1/2}} \left(\frac{c_{pf}}{C_{sf} i_{fg}}\right)^{1/r} \mathrm{Pr}_f^{-s/r}$ and $m = 1/r$

Partial Subcooled Boiling

- We can now solve the following equation for unknown T_w (assuming that all other parameters do not depend on T_w):

$$q''_{tot} - h_{1\phi}\left(T_w - T_b\right) - h_{2\phi}\left(T_w - T_{sat}\right)^m = 0 \quad \text{or} \quad F\left(T_w\right) = 0$$

- Since the equation is nonlinear, we can use iterative Newton approach to find T_w that satisfies equation F=0
- First we guess $T_w = T_w > T_{sat}$ for which we have $F\left(T_w\right) = \varepsilon \neq 0$
- Here ε is the error that must be reduced to 0.
- Let us seek such δT_w for which $F\left(T_w + \delta T_w\right) = 0$. Expanding this function around T_w yields:

$$F\left(T_w + \delta T_w\right) = F\left(T_w\right) + \left.\frac{\partial F}{\partial T}\right|_{T_w} \delta T_w = 0$$

Partial Subcooled Boiling

- Thus we find δT_w as:

$$\delta T_w = -\frac{F\left(T_w\right)}{\left.\frac{\partial F}{\partial T_w}\right|_{T_w}} = -\varepsilon\left(\left.\frac{\partial F}{\partial T_w}\right|_{T_w}\right)^{-1}$$

- Substituting function *F* into this expression gives:

$$\delta T_w = \frac{-q''_{tot} + h_{1\phi}\left(T_w - T_b\right) + h_{2\phi}\left(T_w - T_{sat}\right)^m}{h_{1\phi} + m h_{2\phi}\left(T_w - T_{sat}\right)^{m-1}}$$

Partial Subcooled Boiling

- Usually several iterations is needed until the temperature correction will be less than a specified allowable error:

$$\delta T_w < error$$

- The found wall temperature T_w will be the temperature that will prevail in the partial boiling region

- It should be mentioned that this temperature will be a function of many parameters:

$$T_w = f\left(G, p, q''_{tot}, T_b\right)$$

Example 4

- Calculate bulk temperature and wall temperature in a PWR subchannel. Find the ONB temperature using, e.g., Sato-Matsumura model, and wall temperature in fully-developed subcooled boiling using, e.g., the Thom et al. correlation
- Given:
  pressure 15.5 MPa (everywhere the same), inlet temperature T_{in} = 300 °C, heat flux 685 kW/m^2, rod diameter d_r=9.4 mm, fuel rod pitch 12.5 mm, length of fuel assembly H=3.67 m, total core flow W_c = 17222 kg/s number of fuel rods in the core N_{FR} = 50952. Assume the same coolant flow in all subchannels.

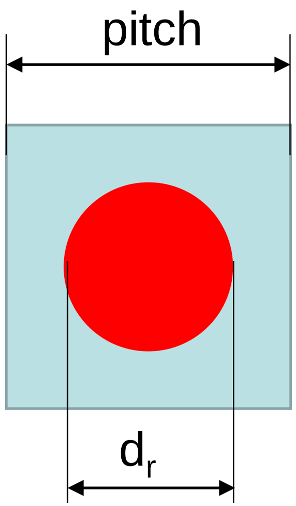
Example 4

- We calculate wall temperature from Dittus-Boelter correlation in single-phase convection region, until:

$$T_w > T_{ONB} = T_{sat} + \sqrt{\frac{8\sigma T_{\mathrm{sat}} q''_w}{\lambda_f \cdot i_{fg} \cdot \rho_g}}$$ (Sato-Matsumura correlation)

- Beyond this value of T_w the partial subcooled flow boiling prevails
- To find temperature distributions along the subchannel, calculations are performed at several axial locations *z*, for which energy balance can be formulated
- We start with subchannel geometry

Example 4



- The flow area of a subchannel is:

$$A_{sch} = pitch^2 - \pi d_r^2 / 4 = 8.69 \cdot 10^{-5}\ \mathrm{m}^2$$

- The wetted perimeter is $P_w = \pi d_r = 0.0295\,\mathrm{m}$
- The heated perimeter is the same: $P_H = P_w$
- The hydraulic diameter is: $D_h = 4 A_{sch} / P_w = 0.0118\,\mathrm{m}$
- The inlet spec. enthalpy is found as i_{in} = XSteam('h_pT',p,T_{in})*1000
- Spec. enthalpy at *z* is found from the energy balance:

$$i(z) = i_{in} + \frac{q'' P_H z}{G A_{sch}}$$

and bulk temperature at *z* is found as

T_b(z)= XSteam('T_ph',p,i(z)/1000)

Example 4

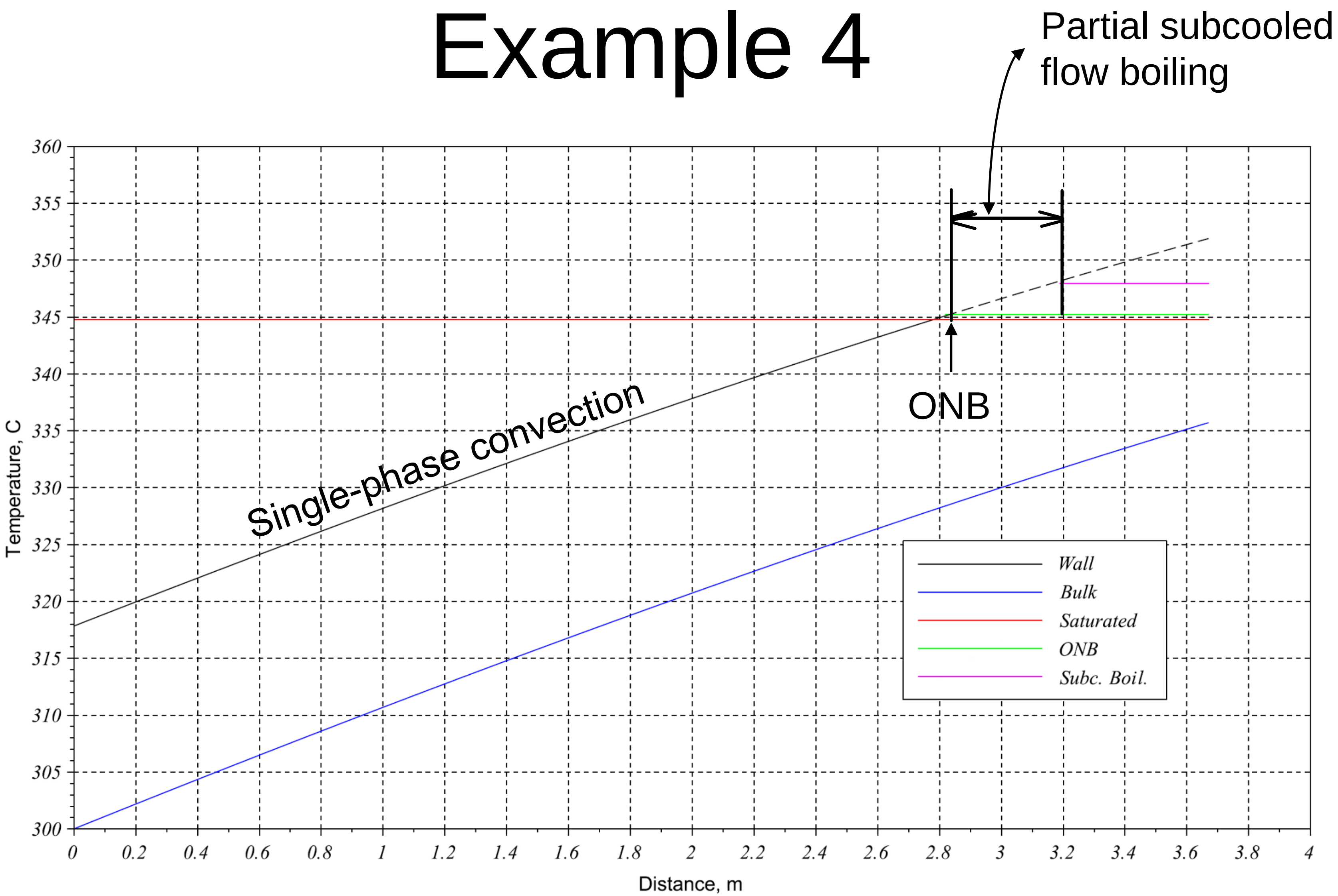
- When bulk temperature is found, the wall temperature is calculated at each location as:

$$T_w(z) = T_b(z) + \frac{q''}{h}$$

- where *h* is found at each location from the Dittus-Boelter correlation. Note that all water properties should be calculated at the local bulk temperature

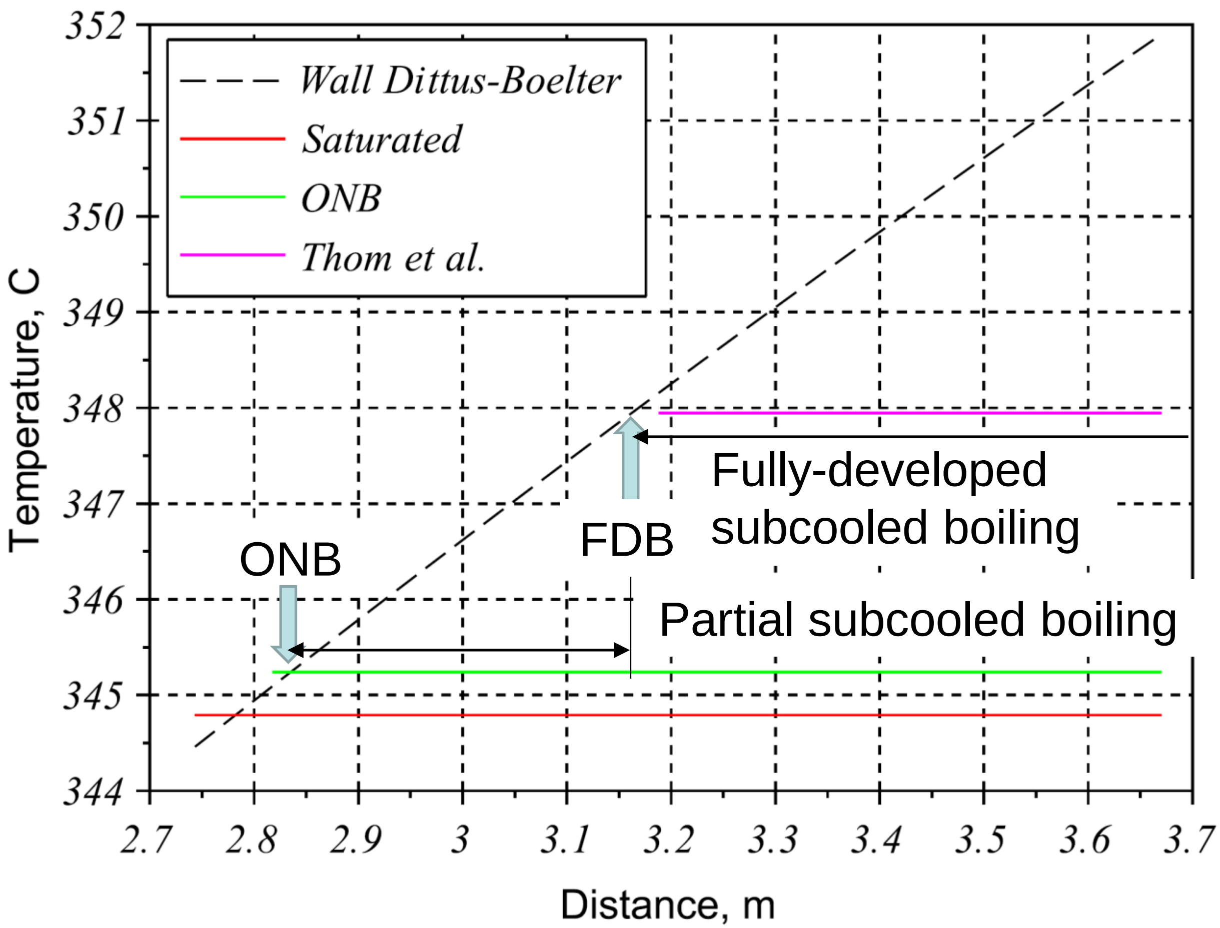
- ONB is at the location where $T_w = T_{ONB}$

Example 4



Partial subcooled flow boiling
ONB
Single-phase convection
Wall
Bulk
Saturated
ONB
Subc. Boil.
Temperature, C
Distance, m
360
355
350
345
340
335
330
325
320
315
310
305
300
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4

Example 4



Up to the ONB point the wall temperature is found from single-phase correlations (e.g. Dittus-Boelter)

Downstream of FDB point the Thom et al. correlation can be used, for example

In-between there is partial boiling region where some modelling is needed.