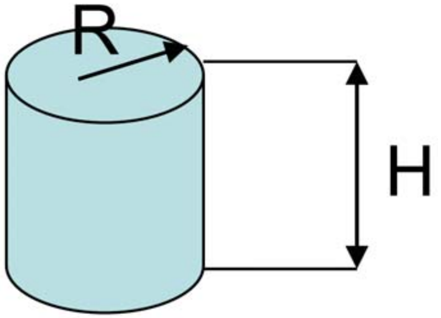
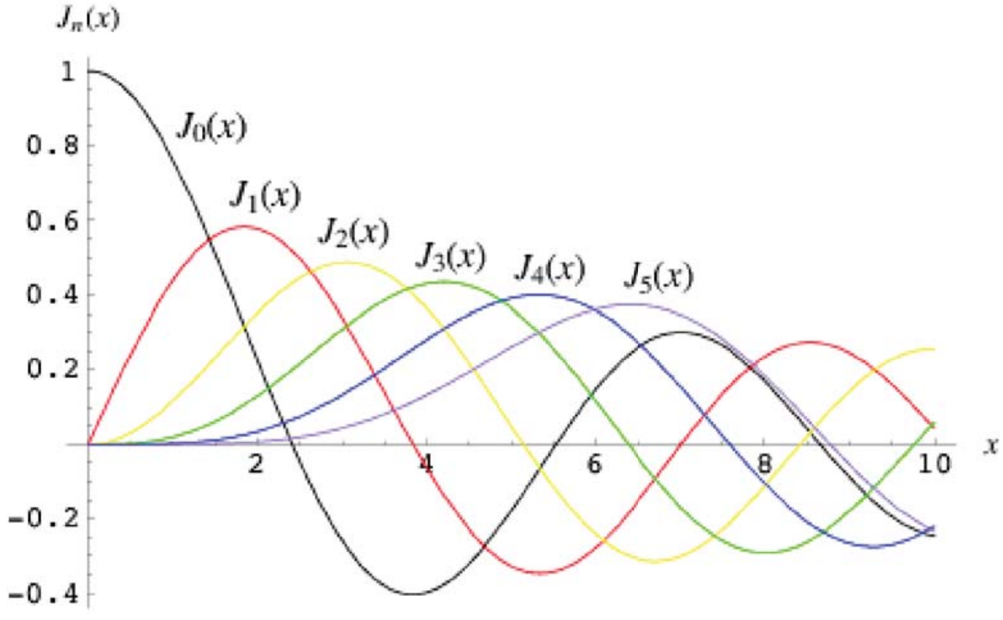
SH2701
Thermal-Hydraulics in Nuclear Energy Engineering

Exercise Session 01

E01_P01

- Calculate the peaking factor of a cylindrical reactor core.
  - Assume the reactor is reflected with $\frac{R}{\tilde{R}} \cong \frac{H}{\tilde{H}} \cong \frac{5}{6}$
  - Bessel fuction values:
    - J0(0) = 1
    - J0(2.0042) = 0.2215
    - J0(2.405) = -9.056e-5
    - J1(0) = 0
    - J1(2.0042) = 0.5765
    - J1(2.405) = 0.5191





Peaking Factors (1)

- Peaking factor is a ratio of the maximum to average power densities in a reactor core
- Peaking factor can be calculated for the whole core volume: $$f_V = \frac{q_0'''}{\overline{q}'''} = \frac{q'''(0,0)}{\frac{1}{V}\int_V q''' dV}$$
- In a cylindrical core, we have in addition radial and axial peaking factors: $$f_R(z_P) = \frac{q'''(0,z_P)}{\frac{1}{\pi R^2}\int_0^R q'''(r,z_P)2\pi r dr} \qquad f_A(r_P) = \frac{q'''(r_P,0)}{\frac{1}{H}\int_{-H/2}^{H/2} q'''(r_P,z)dz}$$
- Here z_P and r_P are fixed values of the axial and radial coordinates at which peaking factors are defined

Peaking Factors (2)

- For example for a fuel rod located at $r=r_P$ distance from the centreline, the axial peaking factor is found as:

$$f_A(r_P) = \frac{q_0''' J_0\left(\frac{2.405 r_P}{\tilde{R}}\right)\cos(0)}{\frac{1}{H}\int_{-H/2}^{H/2} q_0''' J_0\left(\frac{2.405 r_P}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)dz} =$$

$$\frac{1}{\frac{1}{H}\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{\tilde{H}}\right)dz} = \frac{\pi H}{2\tilde{H}\sin\left(\frac{\pi}{2}\cdot\frac{H}{\tilde{H}}\right)}$$

- As can be seen the axial peaking factor does not depend on r_P

Peaking Factors (3)

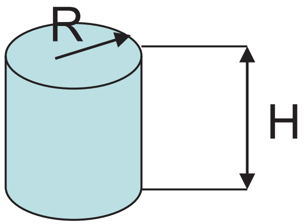
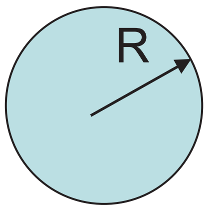
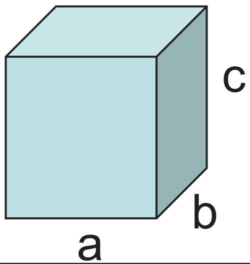
- Similarly for a core cross-section located at $z=z_P$, the radial peaking factor is found as:

$$f_R\left(z_P\right)=\frac{q_0'''J_0\left(0\right)\cos\left(\frac{\pi z_P}{\tilde{H}}\right)}{\frac{1}{\pi R^2}\int_0^R q_0'''J_0\left(\frac{2.405r}{\tilde{R}}\right)2\pi r\cos\left(\frac{\pi z_P}{\tilde{H}}\right)dr}=$$

$$\frac{1}{\frac{1}{\pi R^2}\int_0^R J_0\left(\frac{2.405r}{\tilde{R}}\right)2\pi r dr}=\frac{2.405\cdot R}{2\tilde{R}\cdot J_1\left(\frac{2.405R}{\tilde{R}}\right)}$$

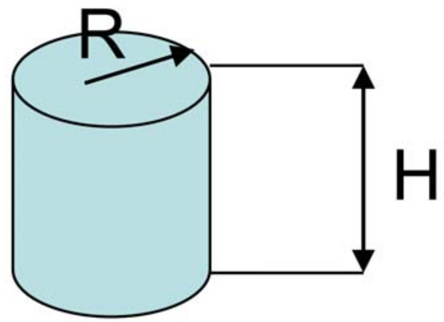
- As can be seen the radial peaking factor does not depend on z_P

Power Distribution – Peaking Factors

	Mean power density	Assuming extrapolation length equal to zero
	$\bar{q}''' = q_0''' \frac{2\tilde{R}}{2.405R} J_1\left(\frac{2.405R}{\tilde{R}}\right) \frac{2\tilde{H}}{H\pi} \sin\left(\frac{\pi H}{2\tilde{H}}\right)$	$\bar{q}''' = 0.274824 q_0'''$ $q_0''' = 3.63869 \bar{q}'''$
	$\bar{q}''' = 3q_0''' \left(\frac{\tilde{R}}{\pi R}\right)^2 \left[\frac{\tilde{R}}{\pi R} \sin\left(\frac{\pi R}{\tilde{R}}\right) - \cos\left(\frac{\pi R}{\tilde{R}}\right)\right]$	$\bar{q}''' = \frac{3q_0'''}{\pi^2} \approx 0.303964 q'''$ $q_0''' = 3.28986 \bar{q}'''$
	$\bar{q}''' = q_0''' \frac{\tilde{a}\tilde{b}\tilde{c}}{abc} \left(\frac{2}{\pi}\right)^3 \sin\left(\frac{\pi a}{2\tilde{a}}\right) \sin\left(\frac{\pi b}{2\tilde{b}}\right) \sin\left(\frac{\pi c}{2\tilde{c}}\right)$	$\bar{q}''' = \frac{8q_0'''}{\pi^3} \approx 0.258012 q'''$ $q_0''' = 3.87579 \bar{q}'''$

E01_P01

- Calculate the peaking factor of a cylindrical reactor core.



- Solution:
  - Peaking factor is defined as $$f_V = \frac{q_0'''}{\overline{q}'''} = \frac{q'''(0,0)}{\frac{1}{V}\int_V q''' dV}$$
  - The power distribution in a cylindrical core gives

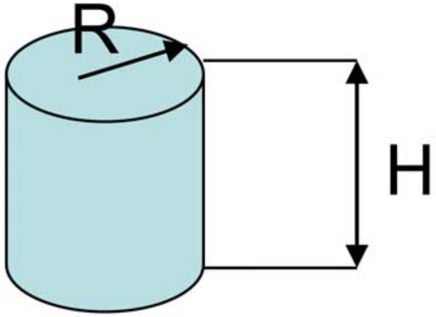
$$\overline{q}''' = q_0''' \frac{2\widetilde{R}}{2.405R} J_1\left(\frac{2.405R}{\widetilde{R}}\right) \frac{2\widetilde{H}}{H\pi} \sin\left(\frac{\pi H}{2\widetilde{H}}\right)$$

  - Use the relation of extrapolation $$\frac{R}{\tilde{R}} \cong \frac{H}{\tilde{H}} \cong \frac{5}{6}$$
  - The peaking factor is 2.356

E01_P02

Calculate the axial peaking factor at a radius of rp = 1.5 m, for a cylindrical reactor core. The core radius is R = 3.3 m, and the height is H = 3.7 m. Assume the core is reflected

with $\frac{R}{\tilde{R}} \cong \frac{H}{\tilde{H}} \cong \frac{5}{6}$



Additional information:

1. The power distribution in a cylindrical core could be found as

$$q'''(r,z) = q_0''' J_0\left(\frac{2.405r}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

(Equation 1-1)

2. In a cylindrical core, the axial peaking factor at radius rp is defined as

$$f_A(r_P) = \frac{q'''(r_P,0)}{\frac{1}{H}\int_{-H/2}^{H/2} q'''(r_P,z)\,dz}$$

(Equation 1-2)

3. Bessel function values:

- J0(0) = 1
- J0(2.0042) = 0.2215
- J0(2.405) = -9.056e-5
- J1(0) = 0
- J1(2.0042) = 0.5765
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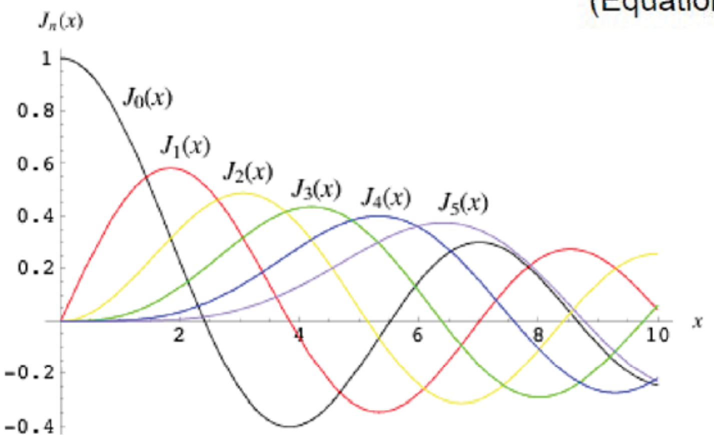
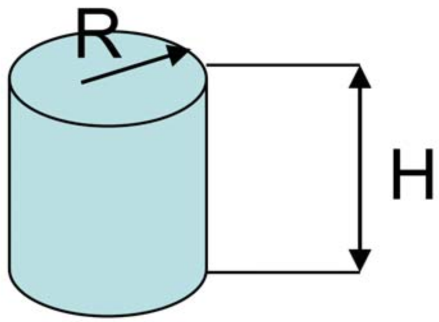


E01_P02

Calculate the axial peaking factor at a radius of rp = 1.5 m, for a cylindrical reactor core. The core radius is R = 3.3 m, and the height is H = 3.7 m. Assume the core is reflected

with $\frac{R}{\tilde{R}} \cong \frac{H}{\tilde{H}} \cong \frac{5}{6}$



- Solution:

From the definition:

$$f_A(r_P) = \frac{q'''(r_P,0)}{\frac{1}{H}\int_{-H/2}^{H/2} q'''(r_P,z)\,dz}$$

$$f_A(r_P) = \frac{q_0'''J_0\left(\frac{2.405r_P}{\tilde{R}}\right)\cos(0)}{\frac{1}{H}\int_{-H/2}^{H/2} q_0'''J_0\left(\frac{2.405r_P}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)dz} =$$

$$\frac{1}{\frac{1}{H}\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{\tilde{H}}\right)dz} = \frac{\pi H}{2\tilde{H}\sin\left(\frac{\pi}{2}\cdot\frac{H}{\tilde{H}}\right)}$$

Use the extrapolation relation, we get

fa = 1.3552

E01_P03

- **Example:** Calculate temperature drops in a fuel pellet, gas gap, clad and thermal boundary layer using the following typical data for PWR:

  Diameters: d_{Fo} = 8.25 mm; d_{Go} = 8.43 mm; d_{Co} = 9.70 mm

  Thermal conductivity: clad – 11 W/mK; gas gap – 0.6 W/mK; fuel (UO_2) – 2.5 W/m.K

  Heat transfer coefficient: h = 45 000 W/m^2.K

  Linear power density: q' = 41 kW/m.

  What is the maximum allowed linear power density if the fuel temperature shouldn't exceed 3073 K and the coolant temperature is equal to 600 K?

Heat conduction in reactor fuel elements (11)

- The total temperature rise in the fuel element is thus

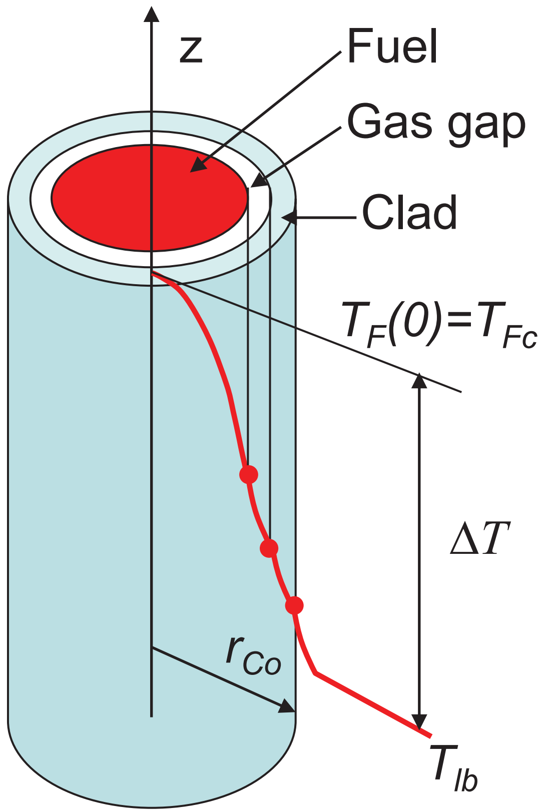
$$\Delta T = \Delta T_F + \Delta T_G + \Delta T_C + \Delta T_{lb} = T_{Fc} - T_{lb}$$

$$\Delta T = \frac{q''' r_{Fo}^2}{4\lambda_F} + \frac{q''' r_{Fo}^2}{2\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{q''' r_{Fo}^2}{2\lambda_C} \ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{q''' r_{Fo}^2}{2 r_{Co} h} =$$

$$\frac{q''' r_{Fo}^2}{4}\left[\frac{1}{\lambda_F} + \frac{2}{\lambda_G}\ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{2}{\lambda_C}\ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{2}{r_{Co} h}\right]$$

Since $q''' \pi r_{Fo}^2 = q'$ (linear power density)

$$\Delta T = \frac{q'}{4\pi}\left[\frac{1}{\lambda_F} + \frac{2}{\lambda_G}\ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{2}{\lambda_C}\ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{2}{r_{Co} h}\right]$$



Fuel element

E01_P03

- Solution:

$$\Delta T_F = \frac{q'}{4\pi \langle \lambda_F \rangle} = \frac{41000}{4\pi \cdot 2.5} = 1305.07 \text{ K}$$

$$\Delta T_G = \frac{q''' r_{Fo}^2}{2\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) = \frac{q'}{2\pi\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) = \frac{41000}{2\pi \cdot 0.6} \ln\left(\frac{8.43}{8.25}\right) = 234.73 \text{ K}$$

$$\Delta T_C = \frac{q'}{2\pi\lambda_C} \ln\left(\frac{r_{Co}}{r_{Go}}\right) = \frac{41000}{2\pi \cdot 11} \ln\left(\frac{9.7}{8.43}\right) = 83.25 \text{ K}$$

$$\Delta T_{lb} = \frac{q''' r_{Fo}^2}{2 r_{Co} h} = \frac{q'}{2\pi r_{Co} h} = \frac{41000}{\pi 0.0097 \cdot 45000} = 29.9 \text{ K}$$

E01_P03

- **The total temperature drop is:**

$$\Delta T = \Delta T_F + \Delta T_G + \Delta T_C + \Delta T_{lb} = 1652.95\ \mathrm{K}$$

- The total temperature drop is given as:

$$\Delta T = \frac{q'}{4\pi}\left[\frac{1}{\lambda_F} + \frac{2}{\lambda_G}\ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{2}{\lambda_C}\ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{2}{r_{Co}h}\right]$$

- Thus:

$$q'_{\max} = \frac{4\pi\left(T_{melt} - T_{cool}\right)}{\left[\frac{1}{\lambda_F} + \frac{2}{\lambda_G}\ln\left(\frac{r_{Go}}{r_{Fo}}\right) + \frac{2}{\lambda_C}\ln\left(\frac{r_{Co}}{r_{Go}}\right) + \frac{2}{r_{Co}h}\right]} = 61.34\ \frac{\mathrm{kW}}{\mathrm{m}}$$

E01_P04

- **Example:** Calculate locations and values of the maximum temperatures of fuel pellets and clad in a PWR fuel assembly :

  Diameters: d_{Fo} = 8.25 mm; d_{Go} = 8.43 mm; d_{Co} = 9.70 mm

  Thermal conductivity: clad – 11 W/mK; gas gap – 0.6 W/mK; fuel (UO_2) – 2.5 W/m.K

  Heat transfer coefficient: h = 45 000 W/m^2.K

  Mean linear power density: q' = 41 kW/m. (assume cosine distribution of the power distribution)

  Fuel element height: H = 3.7 m

  Extrapolation length d = 7.5 cm

  Inlet mass flow rate: W = 10 kg/s

  Heated perimeter: P_H = 0.762 m

  Specific heat: c_p = 5458 J/kg.K

  Inlet coolant temperature: 569 K

Non-uniform heat flux distribution (1)

- For non-uniform (cosine) heat flux distribution

$$q''(z) = q''_0 \cdot \cos\left(\frac{\pi z}{\widetilde{H}}\right) \qquad T_{lb}(z) = \frac{q''_0 \cdot P_H}{W \cdot c_p} \cdot \frac{\widetilde{H}}{\pi}\left[\sin\left(\frac{\pi z}{\widetilde{H}}\right) + \sin\left(\frac{\pi H}{2\widetilde{H}}\right)\right] + T_{lbi}$$

Substituting the above to

$$q'' = h\left(T_{Co} - T_{lb}\right) \Rightarrow T_{Co} = T_{lb} + \frac{q''}{h}$$

yields the following outer clad temperature

$$T_{Co}(z) = \frac{q''_0 \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p} \cdot \left[\sin\left(\frac{\pi z}{\widetilde{H}}\right) + \sin\left(\frac{\pi H}{2\widetilde{H}}\right)\right] + \frac{q''_0}{h} \cdot \cos\left(\frac{\pi z}{\widetilde{H}}\right) + T_{lbi}$$

Non-uniform heat flux distribution (2)

- The temperature distribution can be re-written in short as

$$T_{Co}(z) = A + B\sin\left(\frac{\pi z}{\widetilde{H}}\right) + C_{Co}\cos\left(\frac{\pi z}{\widetilde{H}}\right)$$

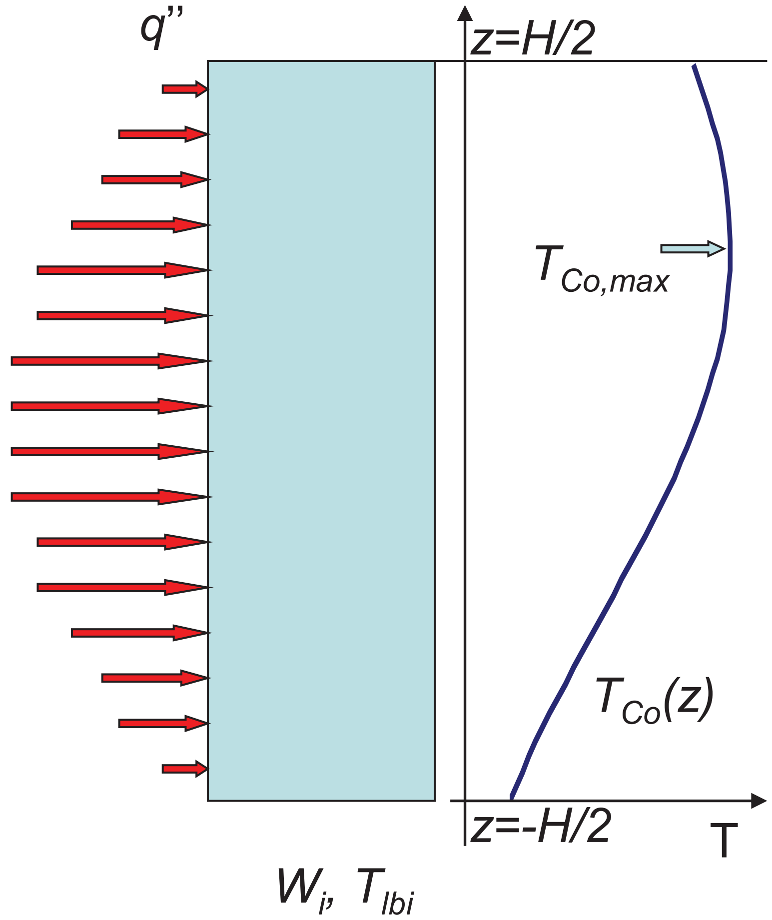
- where

$$A = \frac{q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p}\sin\left(\frac{\pi H}{2\widetilde{H}}\right) + T_{lbi}, \quad B = \frac{q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p}, \quad C_{Co} = \frac{q_0''}{h}$$

Non-uniform heat flux distribution (3)

- Figure to the right shows the clad temperature distribution assuming the cosine axial power distribution

- It should be noted that the temperature of the clad outer surface gets its maximum value $T_{Co,max}$ at a certain location $z_{Co,max}$ different from z=0 and $z=H/2$



Non-uniform heat flux distribution (4)

- The location of the maximum clad temperature can be found as:

$$\frac{dT_{Co}(z)}{dz} = 0 \quad \Rightarrow \quad B\cos\left(\frac{\pi z_{Co,\max}}{\widetilde{H}}\right) - C_{Co}\sin\left(\frac{\pi z_{Co,\max}}{\widetilde{H}}\right) = 0$$

$$\tan\left(\frac{\pi z_{Co,\max}}{\widetilde{H}}\right) = \frac{B}{C_{Co}} \quad \Rightarrow \quad z_{Co,\max} = \frac{\widetilde{H}}{\pi}\arctan\left(\frac{B}{C_{Co}}\right)$$

- Substituting $z = z_{Co,max}$ in the equation for the clad temperature yields the maximum clad temperature

$$T_{Co,\max} = A + B\sin\left(\frac{\pi z_{Co,\max}}{\widetilde{H}}\right) + C_{Co}\cos\left(\frac{\pi z_{Co,\max}}{\widetilde{H}}\right)$$

Non-uniform heat flux distribution (5)

- Noting that:

$$\sin\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) = \pm \frac{\tan\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)}{\sqrt{1+\tan^2\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)}} = \pm \frac{\frac{B}{C_{Co}}}{\sqrt{1+\left(\frac{B}{C_{Co}}\right)^2}}$$

and

$$\cos\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right) = \pm \frac{1}{\sqrt{1+\tan^2\left(\frac{\pi z_{Co,\max}}{\tilde{H}}\right)}} = \pm \frac{1}{\sqrt{1+\left(\frac{B}{C_{Co}}\right)^2}}$$

- The maximum temperature becomes (taking only + sign above, since $z_{Co,max} > 0$):

$$T_{Co,\max} = A + \sqrt{B^2 + C_{Co}^2}$$

Non-uniform heat flux distribution (6)

- Since the clad maximum temperature is located on the inner surface, it is of interest to find it

$$T_{Ci}(z) = \Delta T_C + T_{Co}(z) =$$

$$\frac{q'}{2\pi\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p} \cdot \left[\sin\left(\frac{\pi z}{\widetilde{H}}\right) + \sin\left(\frac{\pi H}{2\widetilde{H}}\right)\right] + \frac{q_0''}{h} \cdot \cos\left(\frac{\pi z}{\widetilde{H}}\right) + T_{lbi} =$$

$$\frac{q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p} \cdot \left[\sin\left(\frac{\pi z}{\widetilde{H}}\right) + \sin\left(\frac{\pi H}{2\widetilde{H}}\right)\right] + q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{1}{h}\right)\cos\left(\frac{\pi z}{\widetilde{H}}\right) + T_{lbi}$$

Non-uniform heat flux distribution (7)

- This temperature can be written again in a short form as

$$T_{Ci}(z) = A + B\sin\left(\frac{\pi z}{\tilde{H}}\right) + C_{Ci}\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

- where

$$A = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}\sin\left(\frac{\pi H}{2\tilde{H}}\right) + T_{lbi}, \quad B = \frac{q_0'' \cdot P_H \cdot \tilde{H}}{\pi \cdot W \cdot c_p}, \quad C_{Ci} = q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{1}{h}\right)$$

location and value of the maximum temperature are found in a similar way as for the outer surface:

$$z_{Ci,\max} = \frac{\tilde{H}}{\pi}\arctan\frac{B}{C_{Ci}} \qquad T_{Ci,\max} = A + \sqrt{B^2 + C_{Ci}^2}$$

Non-uniform heat flux distribution (8)

- The **fuel temperature** can be written in short form as

$$T_{Fc}(z) = A + B\sin\left(\frac{\pi z}{\widetilde{H}}\right) + C_{Fc}\cos\left(\frac{\pi z}{\widetilde{H}}\right)$$

- where

$$C_{Fc} = q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Ci}} + \frac{r_{Co}}{\lambda_G}\ln\frac{r_{Go}}{r_{Gi}} + \frac{r_{Co}}{2\langle\lambda_F\rangle} + \frac{1}{h}\right)$$

and

$$A = \frac{q_0''\cdot P_H\cdot\widetilde{H}}{\pi\cdot W\cdot c_p}\sin\left(\frac{\pi H}{2\widetilde{H}}\right) + T_{lbi}, \quad B = \frac{q_0''\cdot P_H\cdot\widetilde{H}}{\pi\cdot W\cdot c_p}$$

Non-uniform heat flux distribution (9)

- Thus, the location of the maximum fuel temperature and its value are found

$$z_{Fc,\max} = \frac{\widetilde{H}}{\pi} \arctan \frac{B}{C_{Fc}} \qquad T_{Fc,\max} = A + \sqrt{B^2 + C_{Fc}^2}$$

- **Example:** Solution

$$\widetilde{H} = H + 2d = 3.7 + 2 \cdot 0.075 = 3.85 \text{ m} \quad q''_{av} = \frac{q'_{av}}{\pi d_{Co}} = \frac{41000}{\pi \cdot 0.0097} = 1.345 \cdot 10^6 \ \frac{\text{W}}{\text{m}^2}$$

$$q''(z) = q''_0 \cdot \cos\left(\frac{\pi z}{\widetilde{H}}\right) \Rightarrow q''_{av} = q''_0 \cdot \frac{1}{H} \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{\widetilde{H}}\right) dz = q''_0 \cdot \frac{2\widetilde{H}}{\pi H} \sin\left(\frac{\pi H}{2\widetilde{H}}\right)$$

$$q''_0 = \frac{q''_{av} \pi H}{2\widetilde{H} \sin\left(\frac{\pi H}{2\widetilde{H}}\right)} = \frac{\pi \cdot 1.345 \cdot 10^6 \cdot 3.7}{2 \cdot 3.85} \cong 2.035 \cdot 10^6 \ \frac{\text{W}}{\text{m}^2}$$

$$A = \frac{q''_0 \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p} \sin\left(\frac{\pi H}{2\widetilde{H}}\right) + T_{lbi} =$$

$$\frac{2.035 \cdot 10^6 \cdot 0.762 \cdot 3.85}{\pi \cdot 10 \cdot 5458} \sin\left(\frac{\pi \cdot 3.7}{2 \cdot 3.85}\right) + 569 \cong 603.75 \text{ K}$$

- **Example:** Solution

$$B = \frac{q_0'' \cdot P_H \cdot \widetilde{H}}{\pi \cdot W \cdot c_p} = \frac{2.035 \cdot 10^6 \cdot 0.762 \cdot 3.85}{\pi \cdot 10 \cdot 5458} \cong 34.82\ \mathrm{K}$$

$$C_{Co} = \frac{q_0''}{h} = \frac{2.035 \cdot 10^6}{45000} \cong 45.22\ \mathrm{K}$$

$$C_{Ci} = q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Go}} + \frac{1}{h}\right) = 2.035 \cdot 10^6\left(\frac{0.00485}{11}\ln\frac{9.7}{8.43} + \frac{1}{45000}\right) \cong 171.12\ \mathrm{K}$$

$$C_{Fc} = q_0''\left(\frac{r_{Co}}{\lambda_C}\ln\frac{r_{Co}}{r_{Go}} + \frac{r_{Co}}{\lambda_G}\ln\frac{r_{Go}}{r_{Fo}} + \frac{r_{Co}}{2\langle\lambda_F\rangle} + \frac{1}{h}\right) = 2499.97\ \mathrm{K}$$

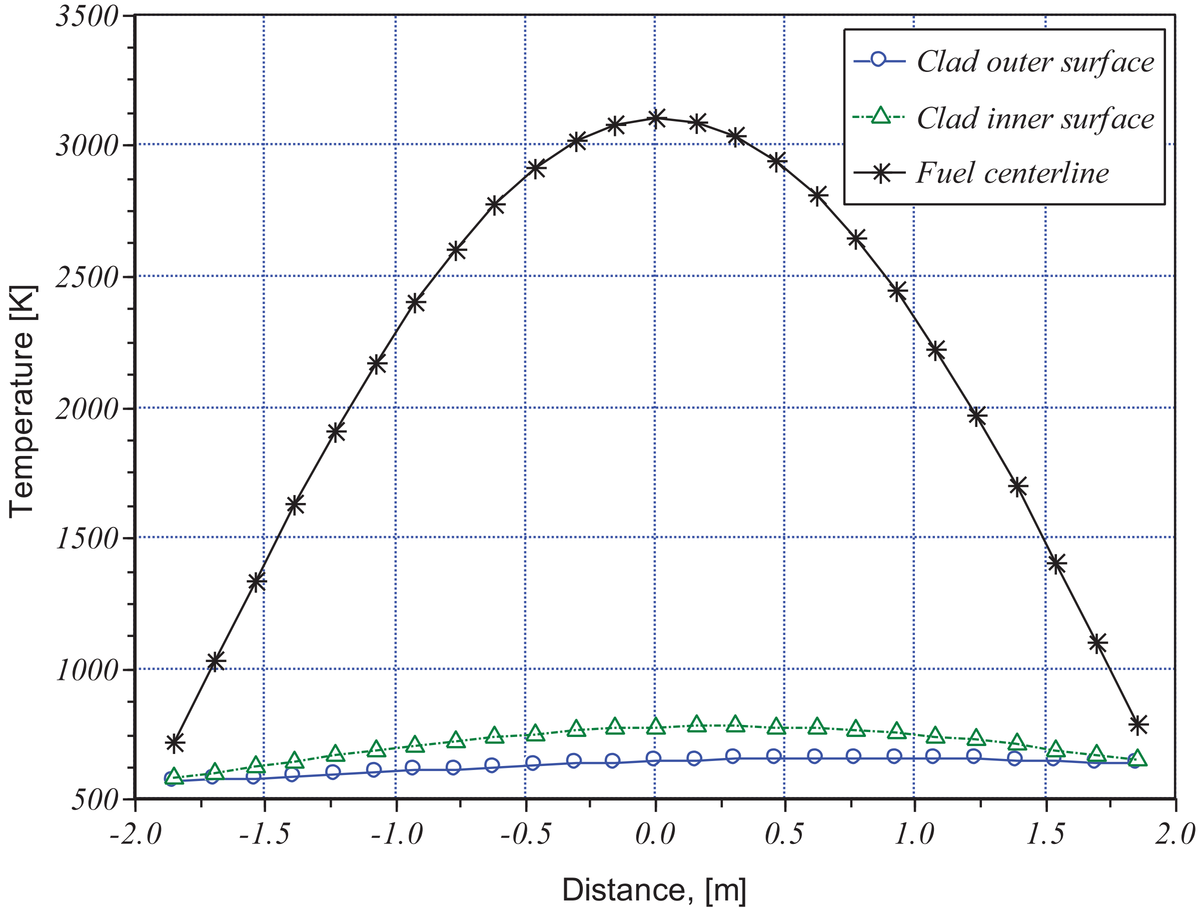
- **Example:** Solution

$$z_{Co,\max} = \frac{\widetilde{H}}{\pi}\arctan\left(\frac{B}{C_{Co}}\right) = 0.804\ \mathrm{m} \qquad T_{Co,\max} = A + \sqrt{B^2 + C_{Co}^2} = 660.82\ \mathrm{K}$$

$$z_{Ci,\max} = \frac{\widetilde{H}}{\pi}\arctan\left(\frac{B}{C_{Ci}}\right) = 0.246\ \mathrm{m} \qquad T_{Ci,\max} = A + \sqrt{B^2 + C_{Ci}^2} = 778.38\ \mathrm{K}$$

$$z_{Fc,\max} = \frac{\widetilde{H}}{\pi}\arctan\left(\frac{B}{C_{Fc}}\right) = 0.017\ \mathrm{m} \qquad T_{Fc,\max} = A + \sqrt{B^2 + C_{Fc}^2} = 3103.96\ \mathrm{K}$$

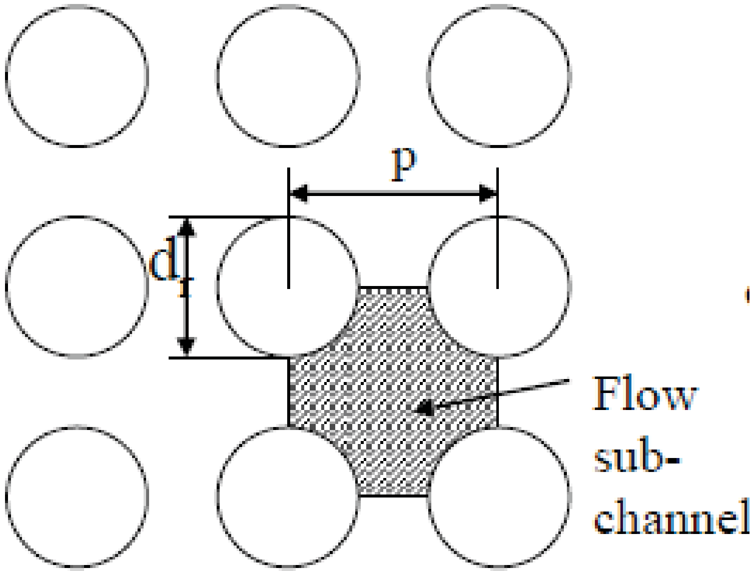
E01_P04



3500
3000
2500
2000
1500
1000
500
Temperature [K]
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
Distance, [m]
Clad outer surface
Clad inner surface
Fuel centerline

E01_P05

- Calculate the maximum temperatures in E01_P04, using heat transfer coefficient from Markoczy (1972) correlation.
  - Square lattice with pitch 12.5 mm
  - Use Isolated Subchannel Model
  - Assume all the rods in the fuel assembly are heated, with the same rod power distribution
  - Assume constant coolant properties
    - Dynamic viscosity $9*10^{-5}$ Pa*s
    - Thermal conductivity 0.56 W/m/K



Heat Transfer in Rod Bundles (5)

Markoczy (1972) performed study of experimental data (over 63 bundles of different geometry)

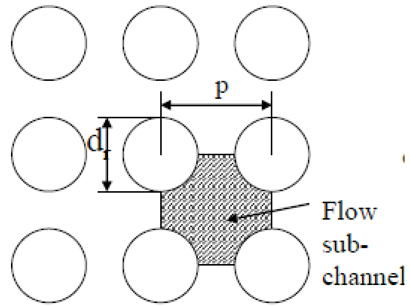
He proposed the following correlation:

$$\frac{\mathrm{Nu}_{bundle}}{\mathrm{Nu}_{DB}} = 1 + 0.91\,\mathrm{Re}^{-0.1}\,\mathrm{Pr}^{0.4}\left(1 - 2e^{-B}\right) \qquad B = \begin{cases} \dfrac{2\sqrt{3}}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1 & \text{triangular} \\ \dfrac{4}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1 & \text{square} \end{cases}$$

Validity region: $3\ 10^3 < \mathrm{Re} < 10^6$; $0.66 < \mathrm{Pr} < 5$; $1.02 < p/d_r < 2.5$

E01_P05

- Calculate the maximum temperatures in E01_P04, using heat transfer coefficient from Markoczy (1972) correlation.
  - Solution
  - Re = rho * U * Dh / mu = G * Dh / mu = W / A * Dh / mu
  - Dh = 4 * A / Pw = 4 * A / PH
  - Re = W * 4 / PH / mu = 5.83 * 10^5
  - Pr = Cp * mu / lambda = 0.877
  - pdr = pitch/dRod = 1.2887
  - Dh = dRod * (4.0 / pi * pdr^2 - 1.0) = 10.81 * 10^{-3}
  - NuDB = 0.023 * Re^0.8 * Pr^0.4 = 894.65
  - hDB = NuDB * lambda / Dh = 4.6 * 104
  - B = 4.0 / pi * pdr^2 – 1.0 = 1.1144
  - NuBundleNuDB = 1.0 + 0.91 * Re^(-0.1) * Pr^0.4 * (1.0 - 2.0 * exp(-B)) = 1.0787
  - NuBundle = NuBundleNuDB * NuDB = 965
  - hBundle = NuBundle * lambda / Dh = 5 * 10^4
  - TcomaxNew = A + sqrt(B^2 + CcoNew^2) = 657.3
  - TcimaxNew = A + sqrt(B^2 + CciNew^2) = 773.9
  - TfcmaxNew = A + sqrt(B^2 + CfcNew^2) = 3099.4



$$D_h = \begin{cases} d_r\left[\dfrac{4}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1\right] & \text{for square lattice} \\ d_r\left[\dfrac{2\sqrt{3}}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1\right] & \text{for triangular lattice} \end{cases}$$

$$\frac{Nu_{bundle}}{Nu_{DB}} = 1 + 0.91\,\mathrm{Re}^{-0.1}\,\mathrm{Pr}^{0.4}\left(1 - 2e^{-B}\right) \qquad B = \begin{cases} \dfrac{2\sqrt{3}}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1 & \text{triangular} \\ \dfrac{4}{\pi}\left(\dfrac{p}{d_r}\right)^2 - 1 & \text{square} \end{cases}$$

Validity region: $3\ 10^3 < Re < 10^6$; $0.66 < Pr < 5$; $1.02 < p/d_r < 2.5$

E01_P06

- A nuclear reactor has been operating at 3000 MWt for one year. Calculate the decay power at 1 min after shutdown, using the one-equation model.

Decay heat in Fission Reactors (3)

- In a simplified analysis, a one-equation model can be used to approximate the decay power after shutdown

- As an example, using this model for a reactor with 3500 MWt during normal operation, the power after shut down drops to 227.5 MWt – still a considerable thermal power that requires efficient reactor cooling

- According to this model, the decay heat is given as follows:

$$\frac{q_D}{q} = \frac{0.065}{t_{op}^{0.2}} \left[\frac{1}{\theta^{0.2}} - \frac{1}{(\theta + 1)^{0.2}} \right]$$

t=0 $\quad$ t=t_{op} $\quad$ Shutdown Reactor t>t_{op}

$$\theta = \left(t - t_{op}\right) / t_{op}$$

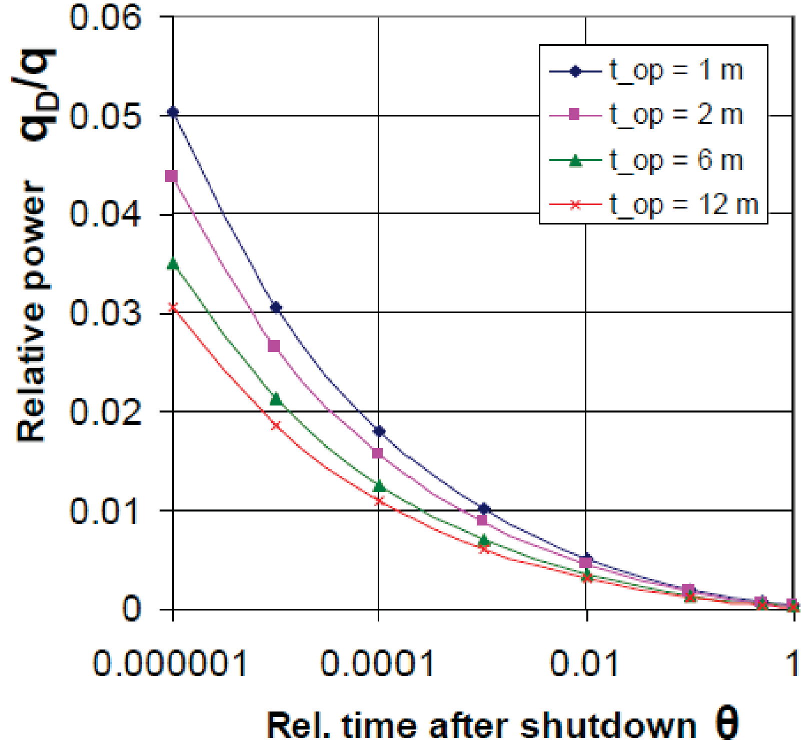
- Here q_D is the decay heat, q is the reactor thermal power before shut-down, t_{op} is the reactor operation time [s] and t is time after reactor start-up [s]

E01_P06

- A nuclear reactor has been operating at 3000 MWt for one year. Calculate the decay power at 1 min after shutdown, using the one-equation model.
  - Solution
  - top = 1.0 * 365.0 * 24.0 * 60.0 * 60.0
  - tsd = 60.0
  - t = top + tsd
  - theta = (t-top)/top = 1.9026e-006
  - qDq

    = 0.065 / top^0.2 * (1.0/theta^0.2 – 1.0/(theta+1.0)^0.2)

    = 0.0266
  - q = 3000
  - qD = qDq * q = 79.8 (MW)



$$\frac{q_D}{q} = \frac{0.065}{t_{op}^{0.2}}\left[\frac{1}{\theta^{0.2}} - \frac{1}{(\theta+1)^{0.2}}\right]$$

t=0 $\quad$ t=t_{op} $\quad$ Shutdown Reactor $t>t_{op}$

$$\theta = \left(t - t_{op}\right)/t_{op}$$

•Here q_D is the decay heat, q is the reactor thermal power before shut-down, t_{op} is the reactor operation time [s] and t is time after reactor start-up [s]