Nuclear Reactor Physics

Interaction of radiation with matter

Jan Dufek

2022

KTH Royal Institute of Technology

Info on VR-1 training in Prague

- The VR-1 labs take two days.
- Normally, students arrive to Prague the day before the labs start and leave Prague the day after the labs end, taking 4 days in total for the whole trip.
- Max 10 students can participate in the reactor exercise at the same time, so we need to split students into the groups.
- Choose your group and book it via Canvas Calendar (when set up).
- Soon, I will ask you to fill a spreadsheet with personal info, including your passport number (or ID number for EU citizens), permanent residence address, birthday, nationality, city and country of birth.
- If you wish to stay more days in Prague then you will also need to specify in the spreadsheet the preferred date of flight to Prague and the return flight to Stockholm, and we buy the right flight tickets for you.
- A hotel will be arranged for you in Prague for three nights, covering the standard length of the trip.
- You need to fix your own hotel for the extra nights in Prague - if you stay more than three nights in Prague.
- If you cannot participate then you can do the same exercises on our simulator.

Info on VR-1 training in Prague

Group #1

- Sunday, Nov 27: Travel to Prague
- Nov 28+29: VR-1 Labs
- Nov 30: Travel to Stockholm

Group #2

- Tuesday, Nov 29: Travel to Prague
- Nov 30 + Dec 1: VR-1 Labs
- Dec 2: Travel to Stockholm

Group #3

- Thursday, Dec 1: Travel to Prague
- Dec 2+3: VR-1 Lab, Day #1
- Dec 4: Travel to Stockholm

Group #4

- Saturday, Dec 3: Travel to Prague
- Dec 4+5: VR-1 Labs
- Dec 6: Travel to Stockholm

Neutron interactions

Neutron interactions

Do neutrons lose kinetic energy in elastic scattering?

- In elastic scattering, abbreviated as (n,n), the neutron strikes the nucleus (usually in its ground state) and reappears while the nucleus remains in its ground state.
- The scattering may be thought of as a process in which the neutron is first absorbed by the target nucleus, and the nucleus then decays via neutron emission.
- **The nucleus increases its kinetic energy in the collision (unless the neutron is thermal), and the neutron thus loses a part of its kinetic energy.**

How the inelastic scattering differs from the elastic?

- The neutron strikes the nucleus and reappears while the state of the nucleus changes and becomes excited.
- The excited nucleus decays by the emission of γ photon.
- The symbol (n,n') abbreviates the inelastic scattering of the neutron on a nucleus.

Intensity of the neutron beam

How is the "intensity of the neutron beam" defined and what it represents?

The intensity of the neutron beam I is defined as

$$I = nv$$

and it represents the number of neutrons that fly through the beam per unit area (typically cm^2) per second.

Cross-sections

What is the nuclear cross-section σ? What is it used for?

When a neutron beam with the intensity I is directed perpendicular to a thin target of the area A and thickness X then the number of neutron collisions with atoms in the target material per second is

$$\sigma NIAX$$

where:

- N is the atomic density of the target material, and σ is the cross-section of the nuclei in the target for the **specific neutron kinetic energy**.
- σ represents the **effective** cross-sectional area of the nucleus. (Beware, σ is not the actual geometric cross-sectional area of the nucleus.)

In which unit are cross-sections usually given?

Neutron cross-sections are usually stated in units of **barns**, abbreviated b, that equals 10^{-24}cm^2.

Cross-sections

How is the nuclear cross-section for neutron capture abbreviated?

All types of neutron interactions have corresponding cross-sections, like:

- σ_e - the elastic scattering cross-section
- σ_i - the inelastic scattering cross-section
- σ_γ, also denoted as σ_c - the capture cross-section
- σ_f - the fission cross-section
- etc. . .

Cross-sections

How is the total cross-section σ_t defined?

The total cross-section σ_t is the sum of cross-section of all possible reactions.

How is the "scattering cross-section" σ_s defined?

The scattering cross-section σ_s is the sum of elastic and inelastic scattering cross-sections

$$\sigma_s = \sigma_e + \sigma_i$$

How is absorption cross-section σ_a defined?

The absorption cross-section is the difference between the total cross-section σ_t and the scattering cross-section σ_s,

$$\sigma_a = \sigma_t - \sigma_s$$

Note that the absorption cross-section includes the capture and fission cross-sections!

What is the physical meaning of the macroscopic cross-section?

- The macroscopic cross-section Σ is the product of the cross-section and the atom density N of the target material,

$$\Sigma = N\sigma$$

- We can define macroscopic cross-section for various reactions.
- The unit of the macroscopic cross-section is cm^{-1}.
- **Σ_t has the physical meaning of the probability per unit path that the neutron collides.**
- Exactly,

$$\Sigma_t dx$$

  is the probability that a neutron that survived its travel without a collision till the depth x has a collision in the next dx.
- You can interpret Σ_t as the expected number of neutron collisions per unit path.

What is the probability that a neutron travels the distance x without a collision?

Let's consider a neutron beam perpendicular to a surface of a block of material. The intensity of uncollided neutrons depend on depth x from the surface, and it must obey the equation

$$dI(x) = -I(x)\Sigma_t dx$$

that has a solution

$$I(x) = I_0 e^{-\Sigma_t x}$$

So, the probability must be

$$e^{-\Sigma_t x}$$

How can we calculate the mean free path of the neutron in a material?

The mean free path, *mfp*, can be computed as

$$mfp = \int_0^{\infty} xp(x)dx$$

where $p(x)dx$ is the probability that the neutron travels (without a collision) the distance x (which is $e^{-\Sigma_t x}$) and then it collides within dx, i.e.

$$p(x)dx = e^{-\Sigma_t x}\Sigma_t dx$$

The result is

$$mfp = \Sigma_t \int_0^{\infty} xe^{-\Sigma_t x} dx = \frac{1}{\Sigma_t}$$

Neutron flux

Neutron flux

What is the formal definition of the neutron flux ϕ?

- The neutron flux ϕ is defined as

$$\phi = nv$$

  where n is neutron density and v is neutron speed.
- The unit of the neutron flux is the same as the unit of the neutron beam intensity, i.e. $[1/\mathrm{cm}^2/\mathrm{s}]$.

What is the difference between neutron flux and neutron beam intensity?

- All neutrons in the neutron beam have the same speed and direction, while the neutron flux can be computed in systems where neutrons come from any direction and neutrons can have any kinetic energy.
- Note that, due to the fact that neutrons may not be mono-energetic, the neutron flux has a dependence on neutron energy, and the above equation should be rather written as

$$\phi(E) = n(E)v$$

  where E is the kinetic energy of neutrons.

Neutron flux

What is the one-group neutron flux?

One-group neutron flux is the neutron flux integrated over all neutron energies:

$$\phi(r) = \int_0^\infty \phi(r, E)dE$$

In principle, you can integrate the neutron flux over a specific energy range, and get a group neutron flux for the specific range (e.g. for thermal or fast neutrons).

Neutron flux

In nuclear engineering, the flux is often integrated over small volumes (e.g. small fuel elements) and all energies. What is the physical meaning of the volume and energy integrated neutron flux?

- Let's start with the definition

$$\phi = nv$$

- The velocity v has the physical meaning of the path that the neutron travels in a second.
- So, when integrated over a volume V, the integrated velocity represents the total path that the neutron travels within the volume V in a second.
- When nv is integrated over the volume V then we receive **the total path that all neutrons travel within the volume V in a second**, which is the physical meaning of the volume integrated neutron flux.

Neutron flux

How can you compute the neutron collision density (reaction rate) F, i.e. the number of collisions per unit of volume and per unit of time?

The neutron collision density F can be computed as

$$F = \Sigma_t \phi$$

Neutron cross-section data

What is the cause of the presence of the so-called resonances in the energy dependence of some cross-sections?

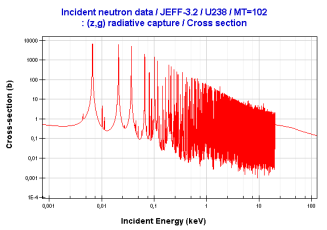


Figure 1: Resonances in capture xs of ^{238}U

- During the collision, the incident neutron and the nucleus first form a compound nucleus.
- The interaction is more likely when the incident neutron energy is such that the possible compound nucleus is created in one of its excited states.
- The excited states energies are thus reflected in the cross-sections (remember that when neutron combines with the nucleus then the nucleus gains the neutron kinetic energy + the binding energy).

Describe general regions in the energy dependence of radiative capture cross-section.

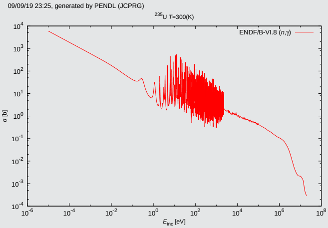
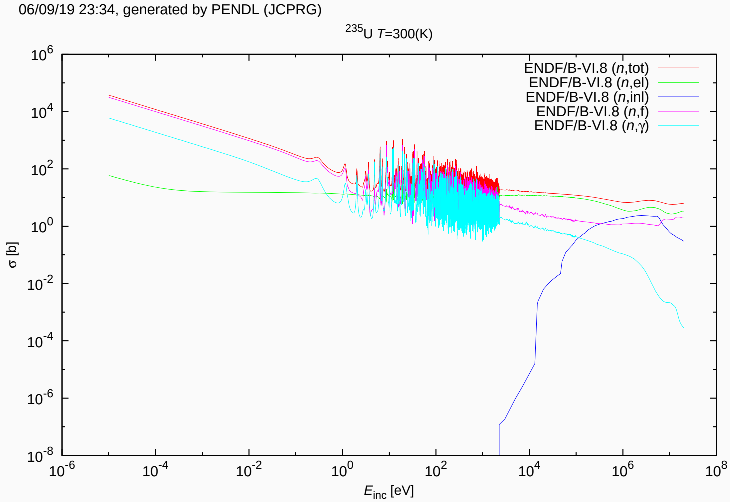


Figure 2: Radiative capture xs of ^{235}U

- In low-energy region of most nuclei, σ_c varies as $1/\sqrt{E}$, where E is the neutron energy. (As the neutron speed v is proportional to $\sqrt{E}$, it follows that σ_c varies as $1/v$. Therefore, this region is called the $1/v$ region.)
- At larger energies, there is a region of resonances (for the same energies as those in scattering cross-sections).

Cross-sections for ^{235}U (present in nuclear fuel)



Energy loss in neutron scattering

Energy loss in neutron scattering collisions

Do neutrons lose more kin. energy at a small or large scattering angle ϑ?

- Neutrons lose more energy in scattering at larger angles ϑ.
- The maximum loss occurs at the scattering angle $\vartheta = \pi$ (or $\pi/2$ in case of scattering on hydrogen). Then the neutron energy after scattering becomes

  $$E' = \alpha E$$

  where

  $$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

  is called the **collision parameter**.

Energy loss in neutron scattering collisions

How much of its kinetic energy the neutron loses, on average, in a collision with the hydrogen nucleus?

- The neutron loses, on average, half of its kinetic energy in a collision with the hydrogen nucleus.
- While the derivation is cumbersome, it can be shown that the average energy loss $\overline{\Delta E}$ for the neutron in the scattering collision on a nucleus with mass number A is approximately

$$\overline{\Delta E} = \frac{1}{2}(1 - \alpha)E$$

  Note that this formula over-estimates the energy loss for high-energy neutrons (above 100 keV) in collisions on heavy nuclei.

How much of its kinetic energy a neutron loses, on average, in a collision with uranium?

The neutron loses, on average, less than 1% of its kinetic energy in a collision with uranium.

Energy loss in neutron scattering collisions

How is the lethargy u of the neutron defined?

The neutron lethargy u is defined as

$$u = \ln\left(E_M/E\right)$$

where E is the kinetic energy of the neutron, and E_M is an arbitrarily chosen reference energy–usually the highest energy of neutrons in the system.

As neutrons slow down in collision, their lethargy grows or decay?

As the neutron loses its kinetic energy, its lethargy increases.

The average change in neutron lethargy, $\xi = \overline{\Delta u}$, does not depend on the incident neutron energy in elastic collisions. It depends only on the nucleus mass number A as

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right)$$

(the derivation is lenthy..)

Polyenergetic neutrons

How can the collision density F of all polyenergetic neutrons be computed?

The collision density can be computed as

$$F = \int_0^\infty \phi(E)\Sigma_t(E)dE$$

What are the thermal neutrons?

Thermal neutrons are neutrons whose energy distribution in the system can be described by the Maxwell–Boltzmann distribution

$$N(E) = \frac{2\pi N}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}$$

where $k = 8.6170 \times 10^{-5}$ eV/K is the Boltzmann's constant and T is the absolute temperature in Kelvin.

Polyenergetic neutrons

We have previously defined the group neutron flux (i.e. the flux that is integrated over all energies). Can we also define group cross-sections? How?

The one-group neutron flux was defines as

$$\overline{\phi}(r) = \int_0^\infty \phi(r, E) dE$$

We can define the group cross-section $\overline{\Sigma_t}(r)$ so that it satisfies the condition

$$F(r) = \int_0^\infty \Sigma_t(r, E)\phi(r, E) dE = \overline{\Sigma_t}(r)\overline{\phi}(r)$$

From there is follows that

$$\overline{\Sigma_t}(r) = \frac{\int_0^\infty \Sigma_t(r, E)\phi(r, E) dE}{\overline{\phi}(r)}$$

Fission

Fission

What is the critical energy of fission?

- Although the heavy nucleus may fission spontaneously, it happens only rarely.
- The nucleus may, however, fission after receiving certain minimal extra energy - called the **critical energy of fission** E_{crit}.
- The critical energy of fission causes the nucleus to become temporarily deformed, setting its repulsive (coulomb) and attractive (strong) forces out of balance, and resulting in its fission.
- A possible way of introducing E_{crit} into the nucleus is by absorbing a neutron. (The nucleus then receives the kinetic energy of the neutron + the binding energy of one nucleon.)

What condition nuclei of a certain nuclide must fulfill to be called "fissile"?

- The nucleus is called fissile when, after absorbing a neutron, the neutron binding energy is bigger than the critical energy of fission of the compound nucleus.
- Fissile nuclei can thus fission after absorbing neutrons of zero kinetic energy.
- Note that when ^{235}U absorbs a neutron and consequently fissions, it is actually the ^{236}U nucleus that fissions. (So it is ^{236}U whose critical energy of fission must be greater than the binding energy.)

Name some fissile nuclides

Fissile nuclides include e.g.: ^{233}U, ^{235}U, ^{239}Pu, ^{241}Pu, and many other of other higher actinides.

Fission

Can fission be triggered in a non-fissile nucleus, such as ^{238}U?

Yes, the fission can be triggered by an energetic neutron when the sum of the neutron kinetic energy energy + the binding energy would be greater than the critical energy of fission of the compound nucleus.

How are called the nuclei that can be fissioned only by energetic neutrons?

Such nuclei are called **fissionable but nonfissile**.

Can fissionable but nonfissile nuclides be used alone to fuel nuclear reactors?

That is not possible in current nuclear power reactors since the fission chain reaction could not be maintained with only these nuclides.

Will fission be always triggered when a neutron strikes fissile nucleus (or when energetic neutron strikes fissionable but nonfissile nucleus)?

Not always. All interactions are possible, including the radiative capture, and elastic and inelastic scattering.

What is the probability that the fissile (or fissionable) nucleus fissions after colliding with a neutron?

The probability p is given by reaction cross-sections of the nucleus, as

$$p = \frac{\sigma_f(E)}{\sigma_t(E)}$$

where E is the incident neutron energy.

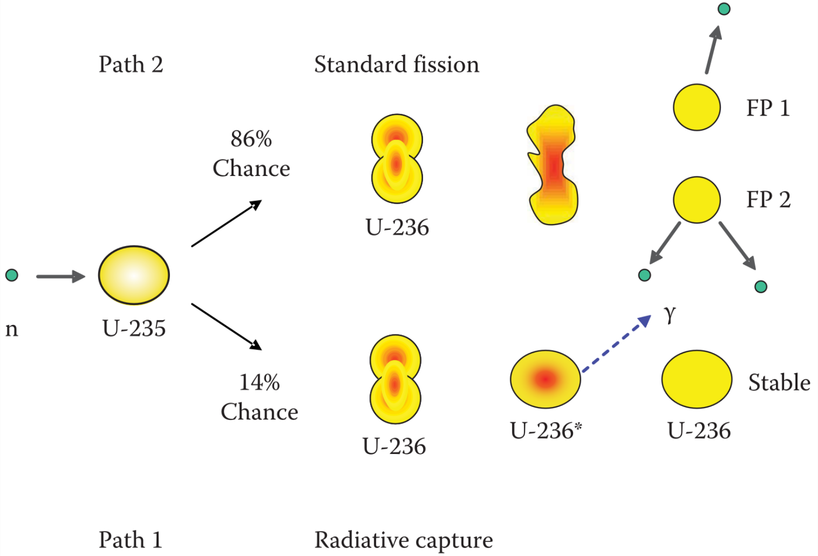


Figure 3: Possible processes after neutron absorption on ^{235}U.

What are the prompt and delayed neutrons?

- Most of fission neutrons (about 99.3%) are emitted at the instant of fission–these are called the **prompt neutrons**.
- There are about 40 fission products that beta-decay into a daughter product and then immediately emit a (delayed) neutron.
- For practical reasons, precursors are represented by about 6 groups (a group contains precursors with a single averaged decay constant).
- The fission products that decay and emit delayed neutrons are called precursors of delayed neutrons.

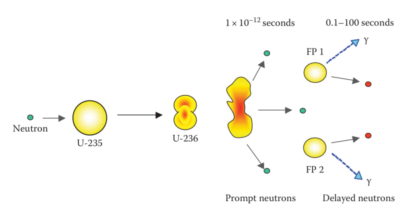


Figure 4: Emission of prompt neutrons from parent nucleus and delayed neutrons from the fission fragments.

How many fission neutrons in total are emitted in nuclear fission, on average?

- The expected total number of fission neutrons (prompt + delayed), ν, depends on the type of nucleus that fissions and kinetic energy of incident neutron. Values of ν for various nuclides (at incident neutron energy of 0.025eV):
  - ^{233}U: 2.492,
  - ^{235}U: 2.418,
  - ^{239}Pu: 2.871,
  - ^{241}Pu: 2.917.
- An additional neutron emitted for every 6-7 MeV increase in incident neutron energy.

What β and β_i represent (with respect to delayed neutrons)?

- The β_i is the fraction of delayed neutrons of group i in all fission neutrons,

$$\beta_i = \frac{\text{expected number of delayed neutrons of group } i \text{ from fission}}{\nu}$$

- β is the fraction of all delayed neutrons in all fission neutrons, $\beta = \sum_i \beta_i$.
- Note that β_i is thus the probability that the fission neutron is born as a delayed neutron in the ith group.
- **β is the probability that the fission neutron is born as delayed neutron.**

Group	Half-life [s]	β_i
1	55.72	0.000215
2	22.72	0.001424
3	6.22	0.001274
4	2.30	0.002568
5	0.61	0.000748
6	0.23	0.000273

Fission

How can we compute the average number of fission neutrons η emitted per neutron absorbed in the fuel composed of a single nuclide (e.g. ^{235}U)?

- The average number of fission neutrons η emitted per neutron absorbed is simply the product of ν and the conditional probability that fission occurs when the incident neutron is absorbed in the nucleus,

$$\eta = \nu \frac{\sigma_f}{\sigma_a}$$

- Values of η for various nuclides (at incident neutron energy of 0.025eV):
    - ^{233}U: 2.287,
    - ^{235}U: 2.068,
    - ^{239}Pu: 2.108,
    - ^{241}Pu: 2.145.

What is the total energy released in fission of ^{235}U?

The total energy released in fission of ^{235}U is about 210 MeV.

Can all of the energy released from fission be utilized completely?

No, the energy of emitting antineutrinos, about 7 MeV, is essentially unrecoverable. The total recoverable energy is about 202.5 MeV.

Fission

What is the expected kinetic energy of fission products from fission of ^{235}U?

About 169 MeV.

What is the expected kinetic energy of fission neutrons coming from fission of ^{235}U?

About 4.8 MeV.

Kinetic energy of fission products and fission neutrons combined does not sum up to 202 MeV. What are we missing?

The processes associated with nuclear fission contribute to heat production too:

- (n,γ) reactions of neutrons that do not cause fission: 8.8 MeV
- prompt γ rays: 7 MeV
- energy from decaying fission products at a later time:
  - energy of β particles: 6.5 MeV
  - energy of delayed γ rays: 6.3 MeV

(A vast majority of γ photons are absorbed in the reactor and contribute to heating the coolant.)

What is the average and the most probable kinetic energy of the prompt neutron coming from thermal fission of ^{235}U?

The average energy is about 1.98 MeV, and the most probable energy is about 0.73 MeV.

The energy spectrum $\chi(E)$ can be approximated by the so-called Watt distribution

$$\chi = 0.453 e^{-1.036E} \sinh \sqrt{2.29E}$$

where $\chi(E)dE$ gives the fraction of prompt neutrons with energies between E and $E + dE$ MeV.