# Course SH2774: Numerical Methods in Nuclear Engineering (NMiNE)

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KTH Reactor Physics

## **Overview**

- 6 ECTS Credits
- Start: August 30, Week 35
- Finish: November 16, Week 46
- 19 Sessions
- Quizzes, HAs, 1 Exam, 1 Numerical Project
- Tick/Tock Times: Tue 10/ Thu 08

W	35	36	37	38	39	40	41	44	45	46	47	48	49	50
٦	2	2	2	2	2	2	2	2	2	1	0	0	0	0

L	Wk	WD	Date	Time	Room	Lecturer	Торіс
1)	35	Tue	30-aug	10-12	СЗ	V. Arzhanov	Course Overview; Numerical Experiment
2)	35	Thu	01-sep	08-10	C3	V. Arzhanov	Computer Arithmetics
3)	36	Tue	06-sep	10-12	C3	V. Arzhanov	Basic Concepts, Convergent Series
4)	36	Thu	08-sep	08-10	СЗ	V. Arzhanov	Polynomial/Spline Interpolation
5)	37	Tue	13-sep	10-12	C3	V. Arzhanov	Nonlinear Equations
6)	37	Thu	15-sep	08-10	C3	V. Arzhanov	Differentiation and Integration
7)	38	Tue	20-sep	10-12	C3	V. Arzhanov	Ordinary Differential Equations (ODE)
8)	38	Thu	22-sep	08-10	C3	V. Arzhanov	Boundary Value Problems (BVP)
9)	39	Tue	27-sep	10-12	C3	V. Arzhanov	Midterm Exam
10)	39	Thu	29-sep	08-10	C3	V. Arzhanov	Linear Algebraic Equations (LAE)
11)	40	Tue	04-okt	08-10	C3	V. Arzhanov	Direct Methods for LAE
12)	40	Thu	06-okt	08-10	C3	V. Arzhanov	FD for NDE
13)	41	Tue	11-okt	08-10	СЗ	V. Arzhanov	Basic Iterative Methods
14)	41	Thu	13-okt	08-10	C3	V. Arzhanov	Successive Over Relaxation
15)	44	Wed	02-nov	13-15	C3	V. Arzhanov	Gradient Methods
16)	44	Thu	03-nov	08-10	C3	V. Arzhanov	Power Method
17)	45	Wed	09-nov	14-16	C3	V. Arzhanov	Partial Differential Equations
18)	45	Thu	10-nov	08-10	C3	V. Arzhanov	Recapitulation
19)	46	Wed	16-nov	13-15	СЗ	V. Arzhanov	Reserved
20)	46	Thu	17-nov		C3		
21)	46	Fri	18-nov		C3		
22)	47	Mon	21-nov		C3		
23)	47	Thu	24-nov		C3		

## **Cancelations**

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# **Beginning of Lectures**

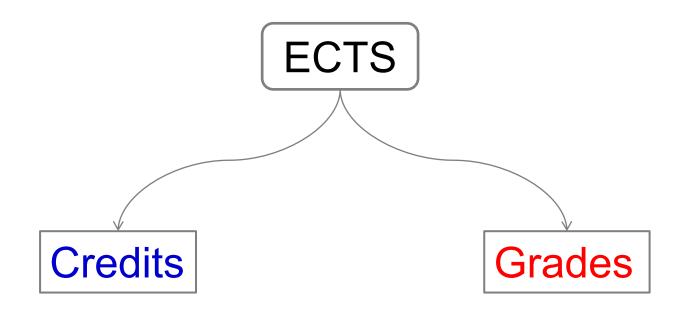
Example: A lecture is announced at 10 – 12.

This actually means:

- The lecture starts at 10:15
- The first (academic) hour runs 45 min (10:15 11:00)
- Break 15 min (11:00 11:15)
- The second hour runs another 45 min (11:15 12:00)

### **ECTS**

European Credit Transfer and Accumulation System (ECTS) is a standard for comparing the study attainment and performance of students of higher education across the European Union and other collaborating European countries.



#### **ECTS Credits**

Sweden joined the Bologna declaration.

1 study year = 1500 ÷ 1800 hours = 60 ECTS credits

1 ECTS = 25 ÷ 30 hours

### **ECTS Grades**

#### Since July 2007, ECTS is compulsory as grading system in KTH

ECTS	N	Verbal	Description
А	7	Excellent	(90 – 100)%
В	6	Very Good	(80 – 90)%
С	5	Good	(70 – 80)%
D	4	Satisfactory	(60 – 70)%
Е	3	Sufficient	(50 – 60)%
Fx	2	Insufficient	(40 – 50)%
F	1	Insufficient	< 40%

# **Necessary Condition**

#### Pass Midterm Written Exam

- Duration: 2 hours.
- Several (8-10) simple problems.
- Problems are theoretical or computational.
- Each problem is of multiple choice.
- Each problem brings 1 4 points.
- Totally 20 points.
- Passing: N ≥ 10.

# Minimal Requirement

- Pass the midterm exam
- All HAs ≥ 50%
- Result = E
- No Project

# **Higher Grading Involves**

• Quizzes (Q) 20	)%
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- Home Assignments (HA) 40%
- Project (P) 40%

## **How It Works**

20%	40%	40%		
Q	HA	Р	Fin	Gr
90.0%	90.0%	90.0%	90.0%	Α
100.0%	0.0%	100.0%	60.0%	D
100.0%	0.0%	99.0%	59.6%	Е
0.0%	100.0%	100.0%	80.0%	В
0.0%	99.0%	100.0%	79.6%	С
90.0%	80.0%	80.0%	82.0%	В
50.0%	75.0%	75.0%	70.0%	С

#### Exam is a must!

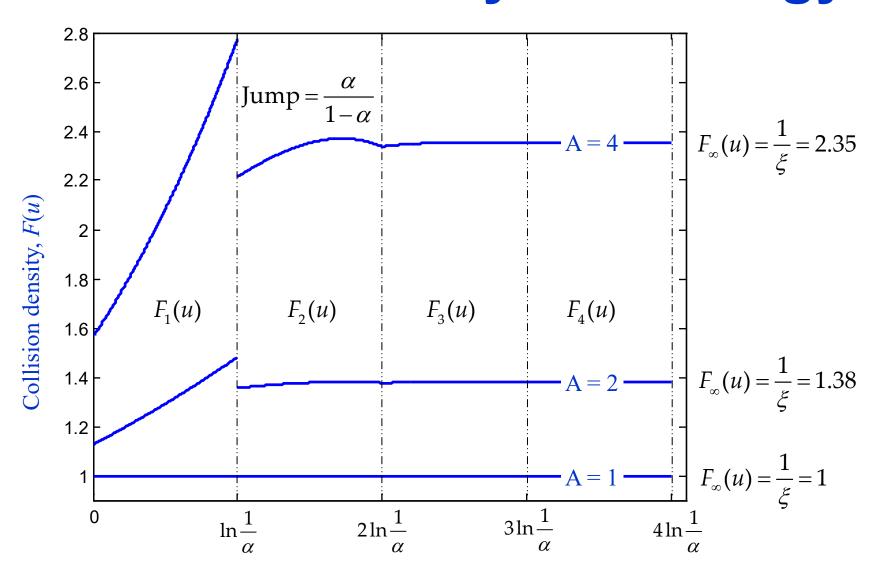
# **Project**

- Larger numerical project
  - Analytic calculations
  - Numerical calculations (programming)
  - Answering questions
- Final report is in a form of scientific paper
- Topics
  - Neutron slowing down spectrum
  - Non-linear least squares fit

# **Project 1**

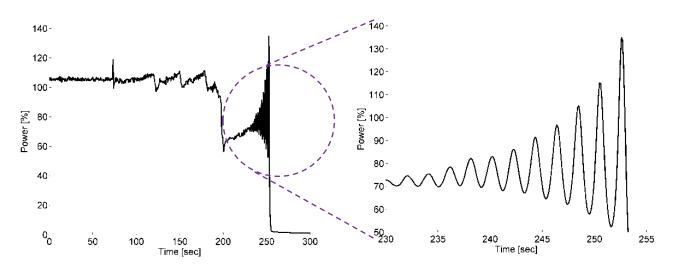
- Neutron slowing down spectrum
- Infinite homogenous medium
- Uniformly distributed neutron sources
- Equation: Σ<sub>t</sub>(E)φ(E) = ∫ Σ<sub>s</sub>(E' → E)φ(E')dE' + S(E)
   Numerical inversion of Laplace transform

## **Collision Density in Lethargy**



# Project 2

- Non-Linear Model Fitting.
- Instability event at BWR in O-2, 1999.
- Model:  $P(t) = ae^{-\gamma t} \cos(\omega t + \phi) + m$



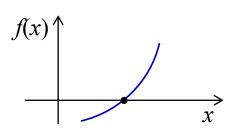
# **Topics**

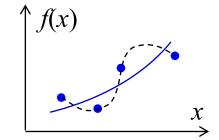
- Role of Numerical Modelling
- Computer Arithmetic
- Polynomial Interpolation, Splines
- Linear/Non-Linear Least Squares
- Linear Algebraic Equations, Eigenvalues
- Numerical Differentiation, Finite Differences
- Numerical Integration, Quadrature
- Solving (numerically) Neutron Diffusion Equation, NDE
- Numerical solution of ODE
- Numerical solution of PDE
- Equivalence Theorem, Spectral Stability

# **Major Numerical Methods**

#### Root finding

$$f(x) = 0$$



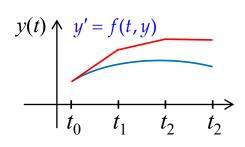




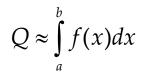
$$f(x_i) \approx y_i$$

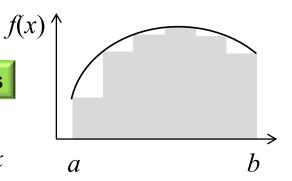
#### Discretization

$$y'(x_i) \approx \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$



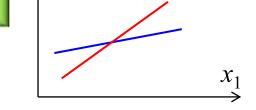
#### Integrals







$$\mathbf{A}\mathbf{x} = \mathbf{f}$$



#### Eigenvalue

$$\mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

$$k = \frac{N_{n+1}}{N_n}$$

## **Prerequisites**

- Basic course in Calculus
- Basic course in Linear Algebra
- Basic knowledge of Matlab/Python

Matlab/Python will be used for all class instructions and home assignments.

The amount of programming will increase after the first few weeks.

### Literature

- Lecture presentations.
- Numerical Methods for Engineers by S.S.Chapra and R.P.Canale, 6<sup>th</sup> edition.
- Numerical Analysis by T.Sauer, 3<sup>d</sup> edition.
- Computer Methods for Mathematical Computations by G.E.Forsythe M.E.Malcolm and C.B.Moler, 1977.
- Numerical Mathematics, A. Quarteroni, 2000.

## Internet Resources

- https://www.python.org
- http://www.maths.dundee.ac.uk/software/ matlab.shtml
- http://www.glue.umd.edu/~nsw/ench250/ primer.htm
- https://www.sololearn.com/
   Learn to code for free
- ?? Open Courseware (MIT, etc.)

### **Numerical Methods**

- Are techniques by which mathematical problems can be solved with elementary operations.
  - Arithmetic: addition, subtraction, multiplication, division;
  - Square, cubic etc. roots;
  - Elementary transcendental functions [ $e^x$ ,  $\log(x)$ ,  $\sin(x)$ , etc.];
  - Special functions:  $J_n(x)$ ,  $\Gamma(x) = (x-1)!$ , hypergeometric, elliptic etc.
- Typically involve a large number of tedious calculations.
- Are characterized by its increasing role.
- Can be classified as: (a) Noncompurer and (b) Computer methods.
- Enhance your (mental, scientific) power
- Give direct (numerical) solution to complicated problems.

# Why to Study NuMeth

- Explosion in use and development of Numerical Methods.
- Extremely effective in solving complicated problems.
- Intelligent use prepackaged computer programs heavily relies on the underlying theory. (Comsol, DIF3D, Serpent, SCALE etc.)
- Many problems cannot be solved by available packages.
- Numerical Methods are efficient in learning computers and thus understanding their strong and weak sides.
- Numerical Methods are efficient to reinforce your understanding of mathematics and physics.

## The END

of the introductory lecture