Division of Nuclear Engineering

2022 Exercises in Reactor Kinetics and Dynamics using TRIGA Research Reactor Simulator

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Introduction

This document describes exercises in reactor kinetics and dynamics using TRIGA research reactor simulator, created by the Reactor Physics Division of Jožef Stefan Institute in Ljubliana, Slovenia [1]. The exercise set was developed as a complement to the practical exercises at the VR-1 training reactor in Prague [2] as a part of the Reactor Physics course (SH2600) at KTH.

The document further describes the following exercise sets to be performed using the simulator, including

1. Reactor behaviour at supercritical, critical and subcritical conditions
2. Reactor stability during oscillatory reactivity insertion
3. Fuel temperature feedback and Xe poisoning feedback

The description of each exercise set includes an introduction, outlining the main aspects and the objectives of the exercise; the theory part, describing the key physics to be studied during the exercise; the procedure, describing how to perform the exercise using the simulator; and the data processing part, describing how to utilize and process the data collected during the exercise. The introduction part includes several questions that should help students prepare for the exercise. The students may use these questions as guidance for writing the exercise theory part in the report.

The exercise descriptions provided in this document are based on the VR-1 compendium [2], which will be provided to the students separately, as supplementary material. It is expected that the students will study the provided material before performing the exercises, as the theory part in this document gives only a basic summary, without an extended explanation. The basic functionality of the TRIGA simulator will be introduced prior to performing the exercises. Detailed information about the simulator can be found in the research paper by the JSI simulator development group [1].

1 Exercise 1: Reactor behaviour at supercritical, critical and subcritical conditions

In this exercise, we will introduce positive and negative reactivity into an initially critical core and study its kinetic response. The goal of this exercise is to observe and explain the typical neutron population trends when the reactor is critical, super-critical, sub-critical and subject to periodic reactivity changes. We will also investigate the effect of the external neutron source on the initially critical, super-critical, and super-critical core. Before performing this exercise, study Section 4 in the VR-1 compendium [2] and Section 7.2 in ref. [4].

These questions will help you prepare for the exercise:

1. Explain what is supercritical, critical and subcritical states of the reactor and what parameters determine this state
2. Explain the expected behaviour of the reactor power in the presence of an external neutron source
3. Explain the role of an external neutron source during reactor start up

1.1 Theory

Here we will summarize the main conclusions of the reactor kinetic theory when delayed neutrons are and are not considered.

1.1.1 Reactor Without Delayed Neutrons

The kinetic behaviour of a reactor without external neutron sources and without delayed neutrons can be described by

$$n(t) = n_0 e^{\frac{k_{eff}-1}{l}t}\,, \tag{1}$$

which is the solution to the first-order ODE describing change of neutron population in such reactor (see e.g. Eq. (7.11) in ref. [4], or Eq. (4.1) in ref. [2]). It is evident that $n(t) = n_0$ only if k_{eff} is equal to one. When $k_{eff} < 1$, the numerator in the exponent is negative and $n(t)$ decreases exponentially. When $k_{eff} > 1$, the numerator is positive and $n(t)$ increases exponentially. Note how the small value of l significantly affects the time-scale of the exponential changes.

Adding a constant neutron source, S_0, to the original ODE results in a solution

$$n(t) = \frac{lS_0}{1-k_{eff}}\left(1 - e^{\frac{-(1-k_{eff})}{l}t}\right)\,. \tag{2}$$

When $k_{eff} > 1$, the exponent is positive and $n(t)$ increases exponentially. When $k < 1$, $n(t)$ is increasing at first, but as the exponential term decays away, $n(t)$ stabilizes to a time-independent value $n(\infty)$, with

$$n(\infty) = \frac{lS_0}{1-k_{eff}}\,. \tag{3}$$

Because Eq. (2) is undefined for $k_{eff} = 1$, we expand the exponential term in a series and study the limit as $k_{eff} \to 1$, which gives

$$\lim_{k_{eff}\to 1} n(t) = \lim_{k_{eff}\to 1}\left[\frac{lS_0}{1-k_{eff}}\left(1 - \frac{k_{eff}-1}{l}t + \frac{1}{2}\frac{(k_{eff}-1)^2}{l^2}t^2 + \ldots - 1\right)\right] = S_0 t\,. \tag{4}$$

Therefore, $n(t)$ increases linearly in a critical reactor with an external neutron source.

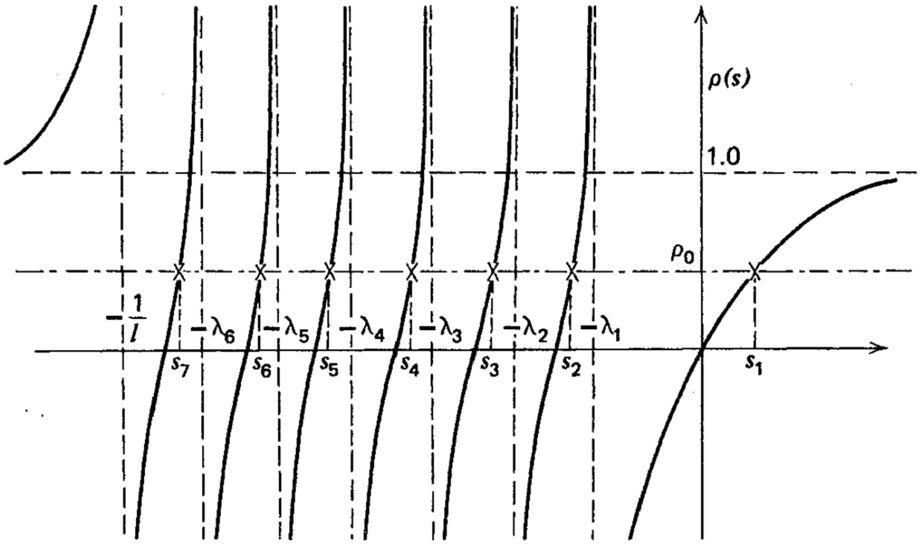

Figure 1: Roots of the inhour equation (from Duderstadt and Hamilton (1976)).

1.1.2 Reactor With Delayed Neutrons

Considering the presence of delayed neutrons results in an ODE for the neutron population

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \sum_i \lambda_i c_i + S_0 \,, \tag{5}$$

where the first term describes the prompt reproduction, the second term describes the delayed neutron emission rate, and S_0 is the external neutron source emitting at some constant rate. Equation (5) is then supplemented by ODEs describing precursor concentrations for each delayed neutron precursor group

$$\frac{\mathrm{d}c_i(t)}{\mathrm{d}t} = \frac{\beta_{eff,i}}{\Lambda} n(t) - \lambda_i c_i(t) \,. \tag{6}$$

The number of precursor equations depends on the number of delayed neutron groups that are considered; commonly, $i = 1, \ldots, 6$ or $i = 1, \ldots, 8$. Describing the kinetic response of a reactor with delayed neutrons, requires solving the ODE system formed by Eqs. (5) and (6). The system may be solved by applying Laplace transformation (see Eq. (4.13) in ref. [2] or p. 334 in ref. [4] for 1 d.n. group). Assuming six delayed neutron groups and $S_0 = 0$, the solution is found as

$$n(t) = \sum_{j=1}^{7} A_j \mathrm{e}^{s_j t} \,, \tag{7}$$

where s_j are the roots of the inhour equation

$$\rho_0 = s \left(\Lambda + \sum_i \frac{\beta_{eff,i}}{s + \lambda_i} \right) \,. \tag{8}$$

Here we assumed $\rho = \rho_0$, for $t \geq 0$. Eq. (8) has six real negative roots $s_{2\ldots 7}$ and one real root s_1, which has the same sign as ρ_0 (see Figure 1). The analysis of the roots shows that a step-change in reactivity will first yield a prompt jump (drop) in the neutron population and, as the terms in Eq. (7) with the higher order roots rapidly decay, the population will change exponentially with a time constant s_1. The reciprocal of s_1 is called the reactor period or asymptotic period, $T = s_1^{-1}$.

For a critical reactor $\rho_0 = 0$ and $s_1 = 0$, so $T = \infty$. In the case of positive reactivity insertion ($\rho_0 > 0$), the neutron population will experience a prompt jump and settle to an exponential increase with period

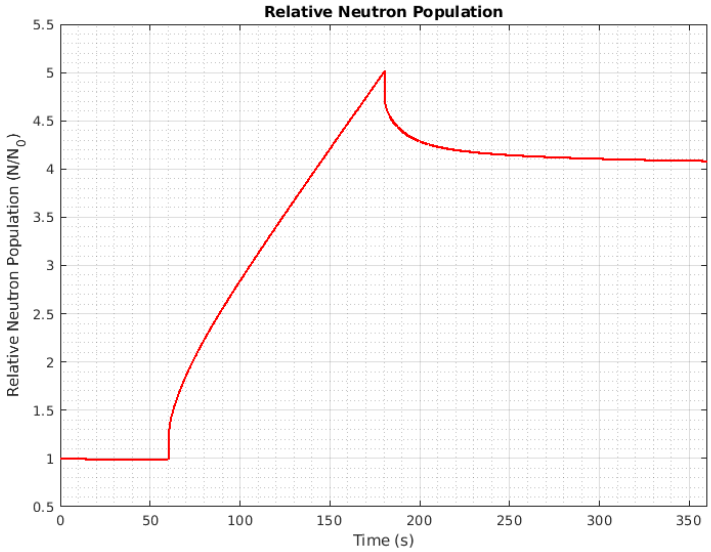

Figure 2: Example simulator output from the kinetics study in a critical reactor. The external neutron source was inserted at $t = 60\,\text{s}$ and removed at $t = 170\,\text{s}$.

T, where T depends on the magnitude of ρ_0. In the case of negative reactivity insertion ($\rho_0 < 0$) the neutron population would decrease in the same matter; however, as $\rho_0 \to -\infty$, the dominating time constant $s_1 \to -\lambda_1$ (see Figure 1). Therefore, no matter how much negative reactivity we introduce, the reactor power cannot be reduced faster than on a period of $T = 1/\lambda_1$.

1.2 Procedure

1.2.1 Critical State

We start by studying the critical reactor response to insertion of an external neutron source:

- Prepare the simulator in the critical state, with the Safety Rod and the Shim Rod fully extracted at 1000 steps. Achieve critical condition by moving the Regulating Rod to 250 steps.
- Wait for approx. 60 s of the steady-state simulation, then insert the neutron source. Wait for additional 60 s.
- After 60 s with the neutron source inserted, remove the neutron source. Wait for 120 s and pause the simulation.

Export the data for further processing, you will need it to explain the observed trends. The expected output is shown in Figure 2. Explain what is happening with the reactor power when the source is inserted and removed.

1.2.2 Super-critical State

First, we will investigate the behaviour of a super-critical reactor without the external neutron source:

- Continue from the reactor state obtained after performing the previous exercise.
- Move the Regulating Rod up to insert a small amount of positive reactivity.
- Wait for approx. 60-120 s, then return the rod to the critical position.
- Pause the simulation and export the data for further processing.

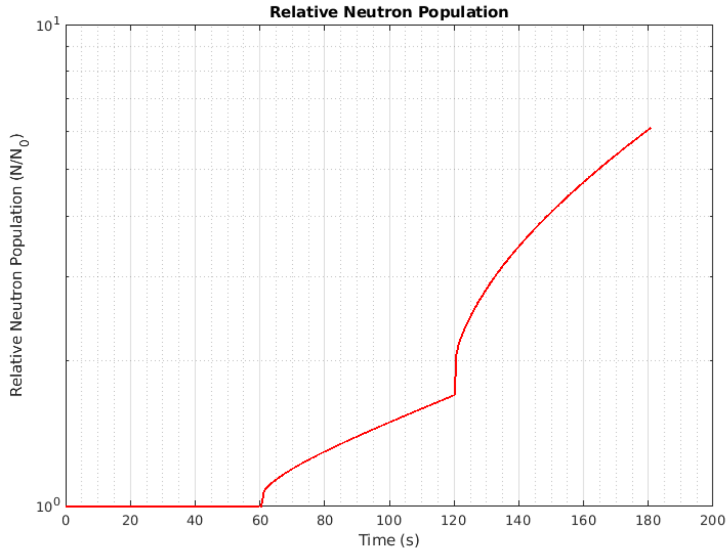

Figure 3: Example simulator output from the kinetics study in a super-critical reactor. The external neutron source was inserted at $t = 120\,\mathrm{s}$.

Explain what is happening with the reactor power when the rod is moved to a super-critical position and then returned to the critical position. How is the observed power trend different from the previous trend?

We will proceed by investigating the effect of the external neutron source on a super-critical core:

- Continue from the reactor state obtained after performing the previous exercise.
- Move the regulating rod again to insert a small amount of positive reactivity.
- Wait for 60 s then insert the neutron source.
- Wait for additional 60 s and pause the simulation.
- Export the data for further processing.

You will need to plot the data in log scale to observe and explain the trends before and after inserting the neutron source. The expected output is shown in Figure 3. How is the power trend after neutron source insertion different compared to inserting the neutron source into a critical system?

1.2.3 Sub-critical State

In this part of the exercise, we will make the initially critical system highly sub-critical, observe the power trend, and then insert the external neutron source.

- Prepare the simulator in the critical state, with the Safety Rod and the Shim Rod fully extracted. Achieve critical condition by moving the Regulating Rod.
- Wait for approx. 60 s and hit the SCRAM button.
- Wait for approx. 60 s after the SCRAM and insert the external neutron source.
- Wait another 30 s, pause the simulation and export the data for further processing.

Explain the observed trends after executing the SCRAM and subsequently and inserting the neutron source. The expected output is shown in Figure 4. What is the power trend after the SCRAM? What happens after the external source is inserted?

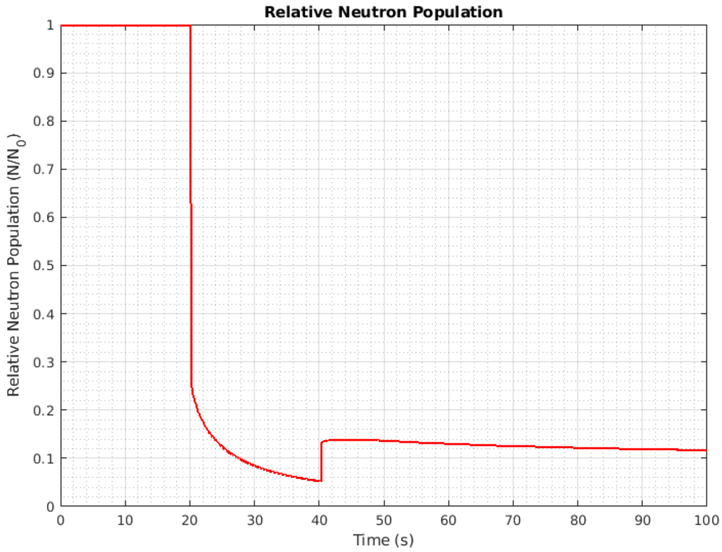

Figure 4: Example simulator output from the kinetics study in a sub-critical reactor. The external neutron source was inserted at $t = 40\,\mathrm{s}$

1.2.4 Delayed neutron importance for reactor operation

The fraction of fission neutrons which come from delayed precursors is $< 1\%$. Even though delayed neutrons comprise of only a small percentage of the total fission neutrons, they are key to understanding how operators can adjust the power levels in the reactor in a controlled manner. To demonstrate this, let us artifically change the physics of U235 fission such that the delayed neutron fission decreases from $< 1\% \rightarrow 0$, i.e. 100% of fission neutrons are prompt, with a neutron generation time, Λ, of 3.9×10^{-5} seconds.

- Put the simulator in a supercritical state with a small positive reactivity with safety and shim rods at 1000 steps and regulating rod at 270 steps. Record the reactor period
- Reduce the simulation speed to "0.05x" and pause the simulation. Turn-off scram triggers and turn off the delayed neutron groups under "Physics settings".
- Resume the simulation and note the new reactor period

Explain what happened in the simulation. How is the power trend different?

2 Exercise 2: Reactor stability during oscillatory reactivity insertion

2.1 Theory

In this part of the exercise, we will investigate a critical reactor response to a periodic reactivity change with angular frequency ω. Given a small inserted reactivity which has a sinusoidal shape, the reactor power will also take on a sinusoidal function as shown in Figure 5.

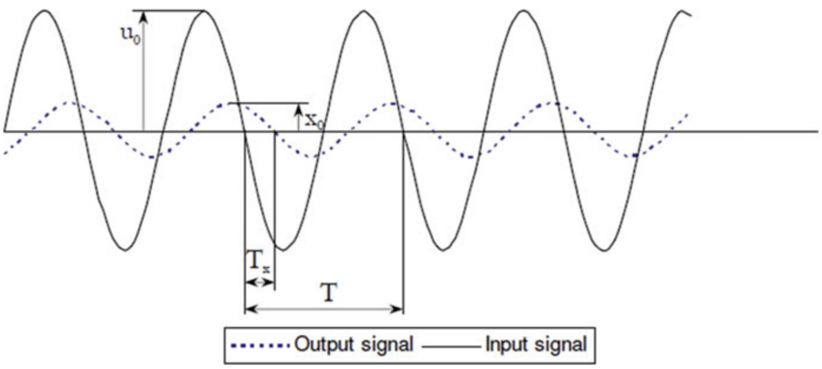

Fig. 13.8 A linear system response to a sinusoidal forcing function at steady state. The system introducing a lag to the signal and reducing its amplitude [4]

Figure 5: Insertion of sinoisoidal positive reactivity taken from [5]

For reactor stability studies, we are primarily interested in two things

- The amplitude of the power sinusoidal curve dependence on the angular frequency of the inserted reactivity
- The phase shift of the power sinusoidal curve from the inserted reactivity

We will experimentally investigate these parameters using kinetic parameters typical of a BWR reactor.

Group number, i	Decay constant $\lambda_i(s^{-1})$	Generation ratio β_i
1	0.0127	0.247×10^{-3}
2	0.0327	1.38×10^{-3}
3	0.115	1.22×10^{-3}
4	0.311	2.64×10^{-3}
5	1.40	0.832×10^{-3}
6	3.87	0.169×10^{-3}

with neutron generation time Λ set to 4.3×10^{-5} s.

2.2 Procedure

- Reset the reactor and load "`BWR_param.rss`" settings. This changes the delayed neutron decay rate, λ_i, the delayed neutron ratio β_i and prompt neutron generation time Λ to parameters which can be found in a typical BWR reactor.

- Prepare the simulator in the critical state, with the Safety Rod and the Shim Rod fully extracted. Achieve critical condition by moving the Regulating Rod to 250 steps. Turn-off the neutron source
- Pause the simulation. Change the Operation mode to Sine Wave, with an amplitude of 20 steps and a period of 0.20s. Record the time of the simulation for easier data processing later
- Unfreeze the simulation and let the simulation run. After at least 3 periods, pause the simulation and change the period to 0.5s.
- Repeat the above two steps for the following periods: 0.20, 0.5, 1, 5, 10, 20, 40, 80, 160. You should have a total of 9 data points
- Export simulation data for further processing.

2.3 Data Processing

Generate a plot showing the reactor amplitude response against angular frequency, ω, with the x-axis and y-axis on a log scale.

Generate a plot showing the reactor phase difference in degrees, or the "lag" between the reactivity peak and the power peak, against angular frequency. The x-axis should be on a log scale.

Compare your results against the curve reported by Suzuki et al. in [6], are your experimental results similar to the analytical results?

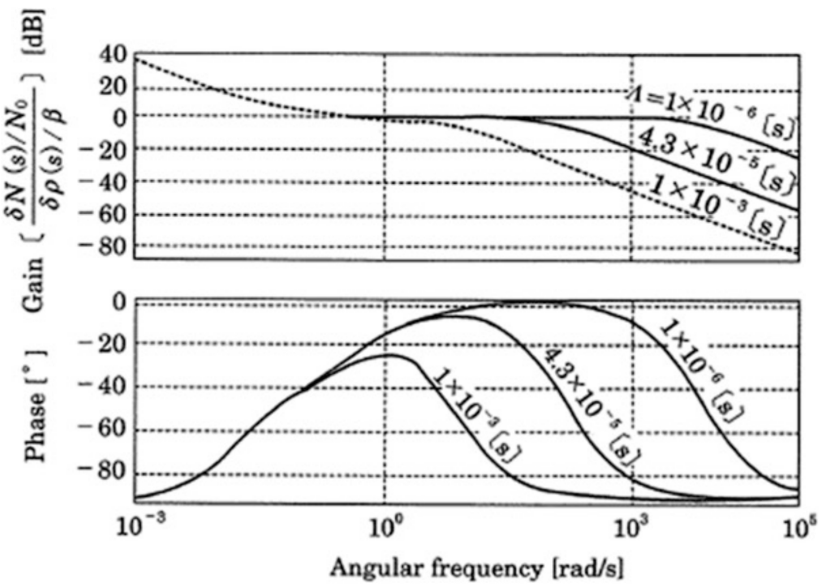

Figure 6: Frequency response curves for critical system from [6]

3 Exercise 3: Temperature and Xe feedback study

In the previous exercises, the reactor has been studied as a system without feedbacks, because we performed the exercises at very low power levels (∼mW to ∼W). In reactors operating at low power levels, the reactor parameters (i.e. temperature, pressure, fuel composition etc.) are either constant or do not change sufficiently enough to induce reactivity feedback effects. In actual reactor plants, the parameters of the core are ever-changing, causing the core properties to change and hence affecting the reactivity of the reactor. These changes are very important when considering short-term transients, such as power excursions, or medium-to-long-term transients, such as fuel composition changes.

In this exercise, we will study two types of reactivity feedbacks: (i) fuel temperature feedback; and (ii) xenon poisoning. The goals of this exercise are (i) to determine the magnitude and the sign of the temperature feedback; and (ii) the reactivity worth of Xenon poisoning with the simulator running at constant power. Before performing the exercise, study Section 5.1 in the VR-1 compendium [2] and Sections 7.4-7.5 in ref. [4].

Find answers to these questions before conducting the exercise:

1. How the reactivity feedbacks work? Specific reactivity feedbacks can be identified due to changes in specific conditions. Name some of those conditions.
2. What are the coefficients of reactivity? How a reactor responds to changes in conditions when its coefficient of reactivity is negative/positive?
3. Describe in general terms the mechanics of the fuel temperature (prompt or Doppler) feedback?
4. What is the mechanics of the moderator temperature (delayed) feedback? Describe the two processes in general terms. What are the under/over-moderated systems?
5. What is the most important fission product in thermal reactors? How is it formed in the reactor? How does its concentration depend on reactor power?

3.1 Theory

In reactor kinetics analysis we assumed that reactivity $\rho(t)$ is a given function of time. However, reactivity depends on various reactor parameters, which in turn depend on the reactor power itself. This dependence arises because the multiplication factor depends on the macroscopic cross-sections of various materials composing the reactor core. Recall that macroscopic cross-section for a material is

$$\Sigma(\mathbf{r}, t) = \sum_i N_i(\mathbf{r}, t)\sigma_i(\mathbf{r}, t), \tag{9}$$

where N_i and σ_i are the atomic number density and the microscopic cross-section for each nuclide i, composing the material. Changes in reactor parameters affect both the atomic number densities and the microscopic cross-sections causing feedback to reactivity. For example, changes in reactor power (the neutron flux) cause changes in temperature that affect the material densities and therefore the atomic number densities. Moreover, atomic number densities of different isotopes are changing due to neutron interactions, e.g. buildup of reactor poisons or burnup of fuel. Microscopic cross-sections are affected by changes in temperature due to the Doppler effect.

Reactivity feedbacks are commonly analysed through various feedback coefficients, defined as

$$\alpha_y \equiv \frac{\partial \rho}{\partial y}, \tag{10}$$

where y denotes some reactor parameter that affects reactivity, e.g. coolant temperature, fuel temperature, coolant void fraction, or reactor power. Feedbacks can be either positive or negative, hence determining the sign of the respective feedback coefficient. In a dynamically stable system, the combination of all reactivity coefficients is necessarily negative.

Changes in various reactor parameters are characterized by different time scales. For example, the fuel temperature changes on a very short time scale following a change in reactor power; hence the reactivity changes due to fuel temperature feedback occur *promptly* and the corresponding feedback coefficient may be called *prompt temperature coefficient*. On the other hand, the time scale of coolant (moderator) temperature change is determined by the heat transfer process from the fuel to the coolant and the heat transport characteristics of the reactor system. This time scale is significantly longer compared to the time scale of the fuel temperature change; hence the reactivity feedbacks occur with a *delay* and the corresponding feedback coefficient may be called *delayed temperature coefficient*.

3.2 Procedure

3.2.1 Fuel Temperature Feedback

In this part of the exercise we will observe the fuel temperature feedback[1]:

- Prepare the simulator in the critical state, with the Safety Rod and the Shim Rod fully extracted. Achieve critical condition by moving the Regulating Rod.
- Record the initial temperature of the fuel.
- Slowly move the Regulating rod up until achieving a 30 s asymptotic period.
- Observe the power trend. Before ending the exercise, record the fuel temperature, pause the simulation and save the data for further processing.

Explain the power trend. How is it different from the one observed during the kinetics study?

3.2.2 Xe Poisoning

In this part of the exercise we will observe reactivity changes due to xenon poisoning[2]:

- Continue from the reactor state obtained during the previous exercise with a power level of 50 kW
- Record the position of the Regulating rod.
- Change the operating mode to Automatic and continue the simulation for at least 90 hours (note that you can increase the simulation speed to 500x).
- Record the position of the Regulating rod after the simulation.

What is happening with the Regulating rod during the simulation?

3.3 Data Processing

The data needed to calculate the fuel temperature feedback coefficient is the temperature change from initial critical state to the state when the power has saturated (ΔT) and the number of steps that the Regulating rod was withdrawn from critical position (Δx). The amount of reactivity inserted by the Regulating rod can be found from the integral calibration curve (ICC), we determined in Exercise 3. The fuel reactivity feedback coefficient can be then calculated as

$$\alpha_f = \frac{ICC(\Delta x)}{\Delta T}. \tag{11}$$

Report and discuss the sign and magnitude of the fuel reactivity coefficient.

[1] Make sure that temperature feedbacks are turned on under the "Physics settings" and cooling is turned on under "Main controls"

[2] Make sure that xenon poisoning is turned on under the "Physics settings"

The xenon reactivity worth can be determined from the difference between the initial and the final positions of the Regulating rod (Δx), and the integral calibration curve

$$\Delta\rho_{Xe} = ICC(\Delta x)\,. \tag{12}$$

Report and discuss the sign and magnitude of the xenon reactivity worth. In the theory part of this exercise, derive the expression for equilibrium xenon concentration.

3.4 Recommended Report Structure

You may submit an earlier submission for comments as well as an initial grade from the instructors. You may revise the report based on the comments to obtain a higher grade for the final submission.

Because of the quantity and the extent of the exercises, it may be difficult to follow the typical IMRAD structure throughout the entirety of the report. Therefore you can consider the following structure:

- Common introduction for the whole report
- Separate chapters for the exercises, where each exercise chapter includes:
  - Exercise-specific introduction;
  - Exercise-specific theory and methods;
  - Exercise-specific results and discussion;
  - Exercise-specific conclusion.
- References.

If you have questions regarding the report structure, contents, or are unsure what should be included under a specific section, contact one of the instructors:

- Yi Meng (`ymchan@kth.se`, or come to the B51 corridor on the 5-th floor).
- Dmitry (`dmitrygr@kth.se`, or come to the C3 corridor on the 3-rd floor).

3.5 Things to Consider Before Submitting

Before submitting the report, you must proofread it. Consider:

- Is it clear and easily understandable?
- Is the linguistic quality of the report appropriate?
- Did you include all required sections?
- Did you check results and units?
- Check Section 4 of the Guidelines document on formatting. Are your tables/figures/equations formatted correctly?

Common reasons that make a report not acceptable:

- The report contains wrong or unphysical statements.
- The report does not contain the required sections or is not complete (missing data, calculations, units, etc.).
- The linguistic quality or the formatting is poor and makes the report difficult to understand.
- Missing references

References

[1] Malec, J., Toškan, D., Snoj, L. (2020). PC-based JSI research reactor simulator. Ann. Nucl. Energy. 146. https://doi.org/10.1016/j.anucene.2020.107630

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