SH2706
Sustainable Energy Transformation Technologies

Exercise Session 02

E02_P01

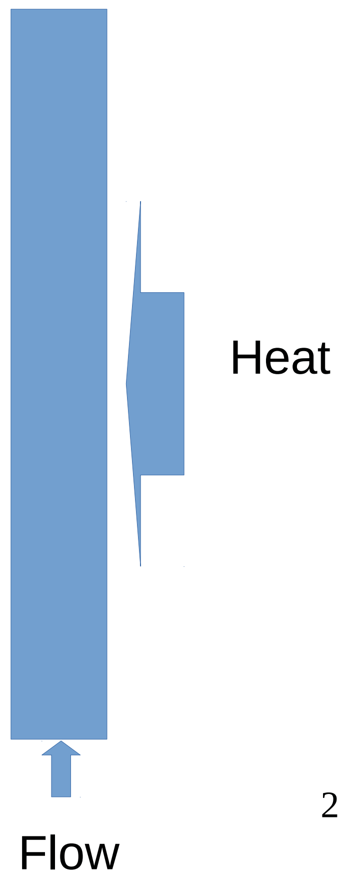
Water flows upward in a vertical round pipe with diameter of 10 mm. The fluid is heated with constant power of 32.31 kW. The mass flow flux is 1500 kg/m^2/s. The system pressure is 7.2 MPa and the inlet temperature is 269°C. Calculate the mean void fraction at the pipe exit using both HEM and DFM.

(Assume the thermodynamic equilibrium quality equal to the actual quality;

Enthalpy of water at 7.2 MPa and 269°C is 1179457 J/kg;

Latent heat of water at 7.2 MPa is 1492273 J/kg;

Enthalpy of saturated water at 7.2 MPa is 1277653 J/kg;
Density of saturated water at 7.2 MPa is 736.2 kg/m^3;
Density of saturated vapor at 7.2 MPa is 37.7 kg/m^3;
Surface tension at 7.2 MPa is 0.0172 N/m;
Viscosity of saturated water at 7.2 MPa is 9x10^{-5} Pa*s;
Viscosity of saturated vapor at 7.2 MPa is 1.9x10^{-5} Pa*s;
Critical pressure of water is 22.1 MPa;
Gravity acceleration is 9.8 m/s^2)



Void-Quality Relationship (1)

- In the same manner, void fraction can be expressed in terms of quality as follows: since $x_a = G_v/G$, we have:

$$x_a\left[\rho_v \alpha U_v + \rho_l (1-\alpha) U_l\right] = \rho_v \alpha U_v$$

$$x_a \rho_v \alpha U_v - x_a \rho_l \alpha U_l - \rho_v \alpha U_v = -x_a \rho_l U_l$$

$$\alpha = \frac{-x_a \rho_l U_l}{x_a \rho_v U_v - x_a \rho_l U_l - \rho_v U_v} = \boxed{\frac{1}{1 + \frac{1-x_a}{x_a} \frac{\rho_v}{\rho_l} \frac{U_v}{U_l}}}$$

Phases usually move with different velocities, and the ratio $S=U_v/U_l$, called “**slip ratio**” is not equal to 1!

Homogeneous Equilibrium Model (4)

- Since the phases are treated as a homogeneous mixture, the slip ratio is equal to 1 (phases are moving with the same velocity)

- Thus the void-quality relationship in HEM reduces to:

$$\boxed{\alpha = \frac{1}{1 + \frac{1 - x_a}{x_a} \frac{\rho_v}{\rho_l}}}$$

- Thus to calculate void fraction, it is enough to know actual quality and density ratio

Drift-Flux Model

- The drift-flux void correlation expresses area-averaged void fraction in terms of superficial velocity of vapor and the total superficial velocity.

$$\langle \alpha \rangle = \frac{J_v}{C_0 J + U_{vj}}$$

- Two additional parameters are needed:
  - C_0 – distribution parameter
  - U_{vj} – drift velocity
- Both these parameters are flow-regime dependent and need to be known to obtain void fraction.

Drift-Flux Model

- C_0 and U_{vj} values (p_c – critical pressure)

Flow pattern	Distribution parameter	Drift velocity
bubbly $0 < \alpha \le 0.25$	$C_0 = \begin{cases} 1-0.5p/p_c & D \ge 0.05m \\ \left.\begin{matrix} 1.2 & p/p_c < 0.5 \\ 1.4-0.4p/p_c & p/p_c \ge 0.5 \end{matrix}\right\} & D < 0.05m \end{cases}$	$U_{vj} = 1.41\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2}\right)^{0.25}$
Slug/churn $0.25 < \alpha \le 0.75$	$C_0 = 1.15$	$U_{vj} = 0.35\left(\frac{gD(\rho_l - \rho_v)}{\rho_l}\right)^{0.5}$
Annular $0.75 < \alpha \le 0.95$	$C_0 = 1.05$	$U_{vj} = 23\left(\frac{\mu_l J_l}{\rho_v D_h}\right)^{0.5}\frac{(\rho_l - \rho_v)}{\rho_l}$
Mist $0.95 < \alpha < 1$	$C_0 = 1.0$	$U_{vj} = 1.53\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2}\right)^{0.25}$

E02_P01

HEM

$$\alpha = \frac{1}{1 + \frac{1 - x_a}{x_a} \frac{\rho_v}{\rho_l}}$$

DFM

$$\langle \alpha \rangle = \frac{J_v}{C_0 J + U_{vj}}$$

Flow pattern	Distribution parameter	Drift velocity
bubbly $0 < \alpha \le 0.25$	$C_0 = \begin{cases} 1 - 0.5p/p_c & D \ge 0.05m \\ \begin{rcases} 1.2 & p/p_c < 0.5 \\ 1.4 - 0.4p/p_c & p/p_c \ge 0.5 \end{rcases} & D < 0.05m \end{cases}$	$U_{vj} = 1.41\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2}\right)^{0.25}$
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E02_P01

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$$\alpha = \frac{1}{1 + \frac{1 - x_a}{x_a} \frac{\rho_v}{\rho_l}}$$

$$\langle \alpha \rangle = \frac{J_v}{C_0 J + U_{vj}}$$

Solution:

Enthalpy at exit (energy balance): $i_{ex} = i_{in} + q/W = 1.4536e6$ where W=G*A

Quality at exit: $x_{ex} = (i_{ex}-i_f)/i_{fg} = 0.1179$

HEM void fraction: $alpha_{HEM} = 1/(1 + (1-x_{ex})/x_{ex}*rho_v/rho_l) = 0.723$

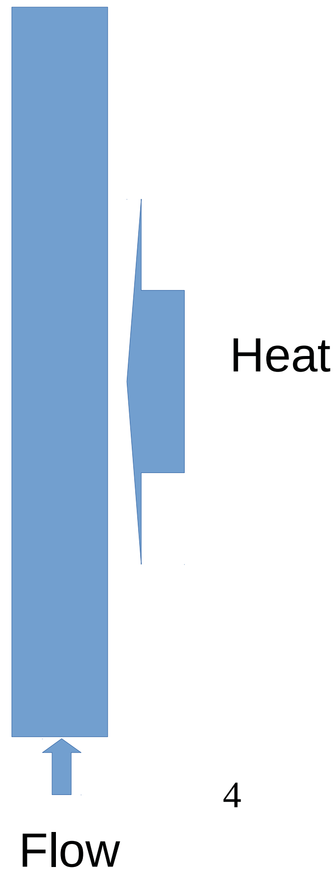
Assume the current flow pattern is slug/churn flow:

$C_0 = 1.15$; $U_{vj} = 0.35*(g*D_h*(rho_l-rho_v)/rho_l)$^0.5

DFM void fraction: $alpha_{DFM} = J_v/(C_0*J+U_{vj}) = 0.6198$

where $J_v = G_v/rho_v = x*G/rho_v$; $J_l = (1-x)*G/rho_l$; $J = J_v+J_l$

Check if it is in the range of the slug/churn flow: yes. Then no further iteration is needed.



A system shown in the following figure is supposed to deliver 150 kg/s of water at atmospheric pressure and temperature 298 K to an open vessel. Calculate the required pressure rise provided by the pump and the required pumping power if the pump efficiency is 88%.

(Assume the water movements in the tank and the vessel can be neglected; Use Equation 1 to calculate the total pressure drop; Use Haaland correlation (Equation 2) to calculate friction factors; Use Equation 5 to calculate the pumping power; Use local loss coefficients from the following figure; Density of water at atmospheric pressure and 298 K is 997 kg/m^3; Viscosity of water at atmospheric pressure and 298 K is 8.9x10^{-4} Pa*s; Gravity acceleration is 9.81 m/s^2)

Total pressure drop between cross sections 1 and 2 in any conduit consisting of segments with lengths Lk and cross-section areas Ak can be found as

$$p_1 - p_2 = \frac{W^2}{2\rho}\left[\sum_k \left(\frac{4L_k}{D_{h,k}} C_{f,k}\right)\frac{1}{A_k^2} + \sum_j \xi_j \frac{1}{A_{j,\min}^2} + \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)\right] + \rho g (H_2 - H_1) \quad \text{(Equation 1)}$$

Darcy friction loss coefficient can be found from Haaland correlation as

$$\frac{1}{\sqrt{\lambda}} = -1.8\log_{10}\left[\left(\frac{k/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] \quad \text{(Equation 2)}$$

where

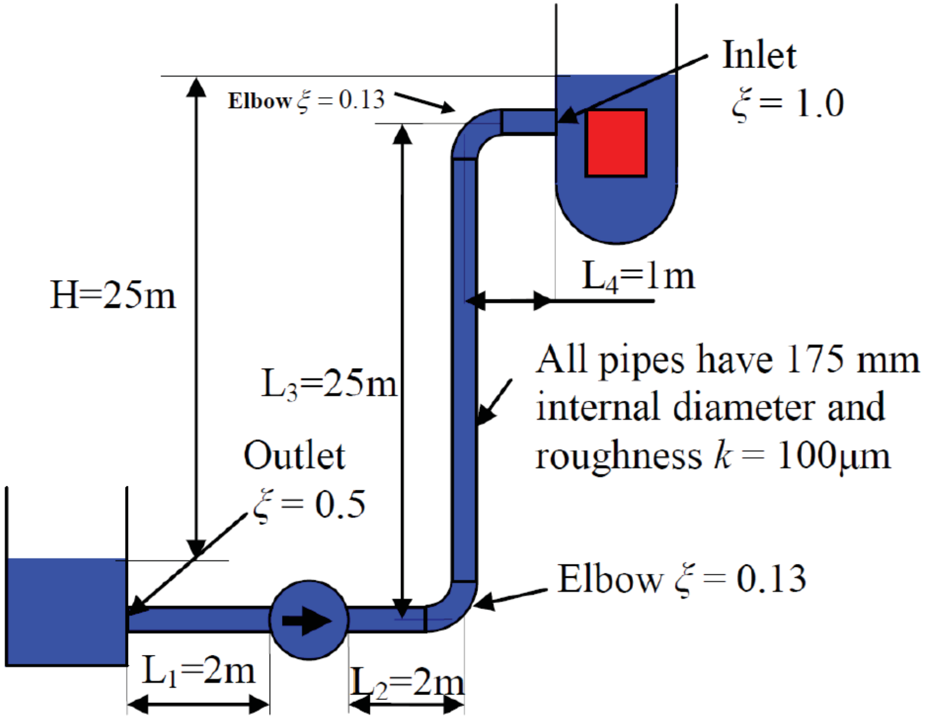
$$\mathrm{Re} = \frac{GD}{\mu_f} \quad \text{(Equation 3)}$$

And the Fanning friction factor is given by

$$C_f = \frac{\lambda}{4} \quad \text{(Equation 4)}$$

The required pumping power can be calculated as

$$P_{pump} = \frac{\Delta p W}{\eta \rho} \quad \text{(Equation 5)}$$



E02_P02

A system shown in the following figure is supposed to deliver 150 kg/s of water at atmospheric pressure and temperature 298 K to an open vessel. Calculate the required pressure rise provided by the pump and the required pumping power if the pump efficiency is 88%.

(Assume the water movements in the tank and the vessel can be neglected; Use Equation 1 to calculate the total pressure drop; Use Haaland correlation (Equation 2) to calculate friction factors; Use Equation 5 to calculate the pumping power; Use local loss coefficients from the following figure; Density of water at atmospheric pressure and 298 K is 997 kg/m3; Viscosity of water at atmospheric pressure and 298 K is 8.9x10-4 Pa*s; Gravity acceleration is 9.81 m/s2)

Solution:

Total pressure drop between cross sections 1 and 2 in any conduit consisting of segments with lengths Lk and cross-section areas Ak can be found as

$$p_1 - p_2 = \frac{W^2}{2\rho}\left[\sum_k \left(\frac{4L_k}{D_{h,k}} C_{f,k}\right)\frac{1}{A_k^2} + \sum_j \xi_j \frac{1}{A_{j,\min}^2} + \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)\right] + \rho g (H_2 - H_1) \; - dp_{pump}$$

$p_1 = p_2$

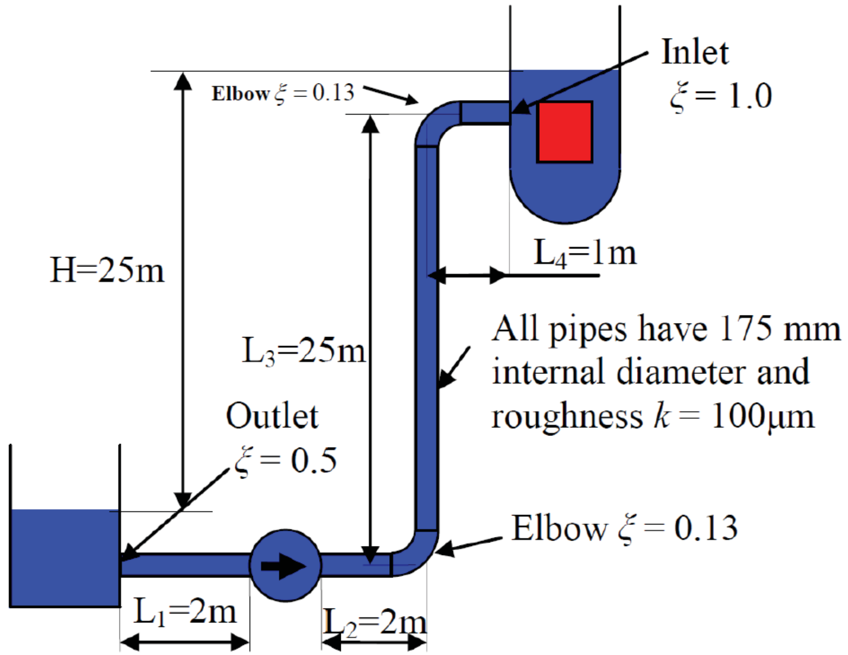
Friction: L1, L2, L3, L4 -dp_f = 5.876e4 Pa

Local: xi1, xi2, xi3, xi4 -dp_l = 3.433e4 Pa

Gravity: H = 25 m -dp_g = 2.445e5 Pa

dp_{pump} = 3.376e5 Pa

Power = dp_{pump} * W / eta / rho = 5.77e4 W

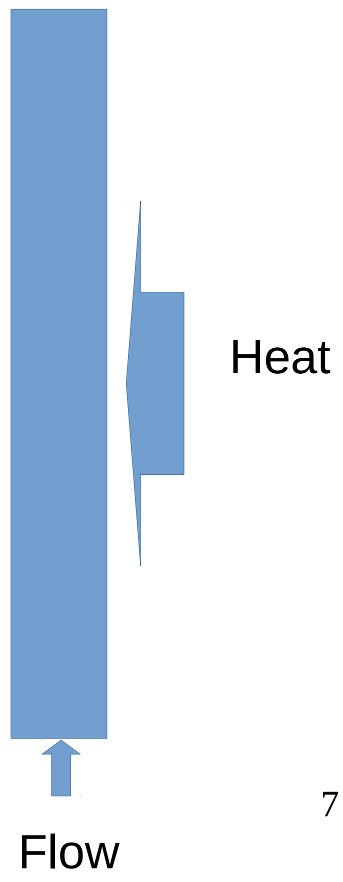


E02_P03

Two phase mixture of saturated steam and water is flowing upward in a uniformly heated vertical pipe with 15 mm internal diameter and 3.5 m in length. The inlet is saturated water ($x=0$) and the outlet is saturated vapor ($x=1$). The total mass flux is 1200 kg/m^2s. Assume constant fluid properties at reference pressure 7 MPa. Calculate the friction, gravity and acceleration pressure drop in the pipe, using HEM.

(Use the figures provided by the lecture slides or compendium for pressure drop multipliers; Use Haaland correlation for friction factor calculation; Assume smooth pipe)

(Enthalpy of water at 7 MPa and 276°C is 1.2154×10^6 J/kg;
Enthalpy of saturated water at 7 MPa is 1.2674×10^6 J/kg;
Latent heat of water at 7 MPa is 1.5051×10^{-6} J/kg;
Density of saturated water at 7 MPa is 739.7 kg/m^3;
Density of saturated vapor at 7 MPa is 36.5 kg/m^3;
Surface tension at 7 MPa is 0.0176 N/m;
Viscosity of saturated water at 7 MPa is 9.1291×10^{-5} Pa*s;
Viscosity of saturated vapor at 7 MPa is 1.8965×10^{-5} Pa*s;
Critical pressure of water is 22.1 MPa;
Gravity acceleration is 9.8 m/s^2)



Friction Pressure Losses (6)

- Using the first expression, the following final form of the two-phase multiplier is obtained:

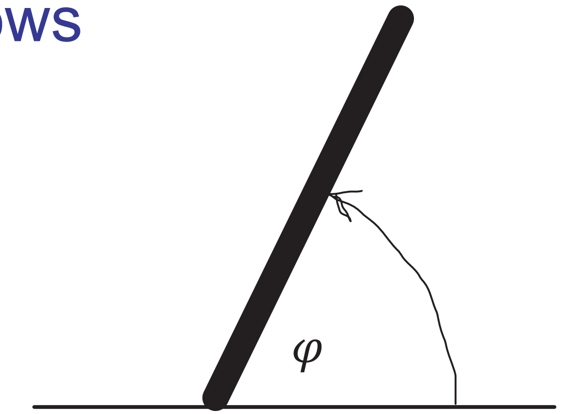
$$\phi_{lo}^2 = \left[1 + \left(\frac{\mu_l}{\mu_v} - 1\right)x\right]^{-0.25}\left[1 + \left(\frac{\rho_l}{\rho_v} - 1\right)x\right]$$

- And the two-phase friction pressure gradient becomes:

$$-\left(\frac{dp}{dz}\right)_{w,tp} = -\phi_{lo}^2\left(\frac{dp}{dz}\right)_{w,lo} = \frac{P_w}{A}\left[1 + \left(\frac{\mu_l}{\mu_v} - 1\right)x\right]^{-0.25}\left[1 + \left(1 - \frac{\rho_l}{\rho_v}\right)x\right]C_{f,lo}\frac{G^2}{2\rho_l}$$

Gravity Pressure Gradient

- The gravity pressure gradient is as follows



$$-\left(\frac{dp}{dz}\right)_{grav} = \rho_m g \sin\varphi$$

- Here sinφ is equal to +1 for upwards flow in vertical channels, -1 for downwards flow, and to 0 for horizontal channels.
- Since, in general, the mixture density ρ_m can change along channel, the pressure gradient will change accordingly.

Acceleration Pressure Gradient (1)

- The acceleration pressure gradient can be evaluated as

$$-\left(\frac{dp}{dz}\right)_{acc} = \frac{1}{A}\frac{d}{dz}\left(\frac{G^2 A}{\rho_M}\right)$$

- For constant *G* and *A*, this equation reduces to:

$$-\left(\frac{dp}{dz}\right)_{acc} = G^2 \frac{d}{dz}\left(\frac{1}{\rho_M}\right) = G^2 \frac{d\upsilon_M}{dz}$$

- As can be seen, the pressure gradient is proportional to the gradient of mixture specific volume, v_M, multiplied with a square of the mass flux.

Acceleration Pressure Gradient (2)

- According to the definition, the dynamic mixture density can be expressed in terms of quality and void fraction as follows

$$-\left(\frac{dp}{dz}\right)_{acc} = G^2 \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]$$

- For HEM, we have

$$-\left(\frac{dp}{dz}\right)_{acc} = G^2 \frac{d}{dz}\left[\frac{x\left(\frac{\rho_f}{\rho_g}-1\right)+1}{\rho_f}\right] = G^2\Bigg(\underbrace{\frac{1}{\rho_g}-\frac{1}{\rho_f}}_{\upsilon_g-\upsilon_f=\upsilon_{fg}}\Bigg)\frac{dx}{dz} = G^2\upsilon_{fg}\frac{dx}{dz}$$

Local Pressure Losses (1)

- Using HEM the irreversible pressure loss at sudden expansion is obtained as,

$$-\Delta p_I = \left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]\left(1 - \frac{A_1}{A_2}\right)^2 \frac{G_1^2}{2\rho_l}$$

- This equation can be compared with its equivalent for the single-phase flow through a sudden expansion.
- As can be seen, a new term appears, which can be identified as a two-phase multiplier for the local pressure loss

$$\phi_{lo,d}^2 = \left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]$$

Local Pressure Losses (2)

- The subscript *lo,d* is used to indicate that the multiplier is valid for local losses, where the viscous effects can be neglected and only the ***d***ensity ratio between the two phases plays any role
- The corresponding irreversible pressure drop for homogeneous two-phase flow through a sudden contraction becomes,

$$-\Delta p_I = \left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]\left(\frac{A_2}{A_c} - 1\right)^2 \frac{G_2^2}{2\rho_l}$$

Local Pressure Losses (3)

- In general, a local irreversible pressure drop for two-phase flows can be expressed as:

$$\Delta p_{I,tp} = \phi_{lo,d}^2 \Delta p_{I,lo}$$

- where *tp* stands for ***t**wo-**p**hase* and *lo* for ***l**iquid **o**nly*.
- As can be seen, the local pressure drop for two-phase flows can be obtained from a multiplication of the corresponding local pressure drop for single-phase flow and a proper local two-phase multiplier.

Total Integral Pressure Drop (1)

- In practical calculation it is usually required to determine the over-all pressure drop in a channel of a given length and shape.
- The total pressure drop can be readily obtained from the integration of the pressure gradient expression along the channel length as follows

$$-\int_0^L \frac{dp}{dz} dz \equiv -\left[p(L) - p(0) \right] \equiv -\Delta p =$$

$$\int_0^L \left(\frac{dp}{dz} \right)_w dz + \int_0^L \rho_m g \sin \phi dz + \int_0^L \frac{1}{A} \frac{d}{dz} \left(\frac{G^2 A}{\rho_M} \right) dz$$

Total Integral Pressure Drop (2)

- Assuming that the channel has a constant cross-section area and using expressions for the friction, gravity and acceleration terms, the following expression is obtained,

$$-\Delta p = C_{f,lo}\frac{4}{D_h}\frac{G^2}{2\rho_l}\int_0^L \phi_{lo}^2 dz + g\sin\varphi\int_0^L\left[\alpha\rho_v + (1-\alpha)\rho_l\right]dz +$$

$$G^2\int_0^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_v} + \frac{(1-x)^2}{(1-\alpha)\rho_l}\right]dz$$

Total Integral Pressure Drop (3)

- It is customary to introduce integral multipliers into the above equations which are defined as follows.

  – The **integral acceleration multiplier**

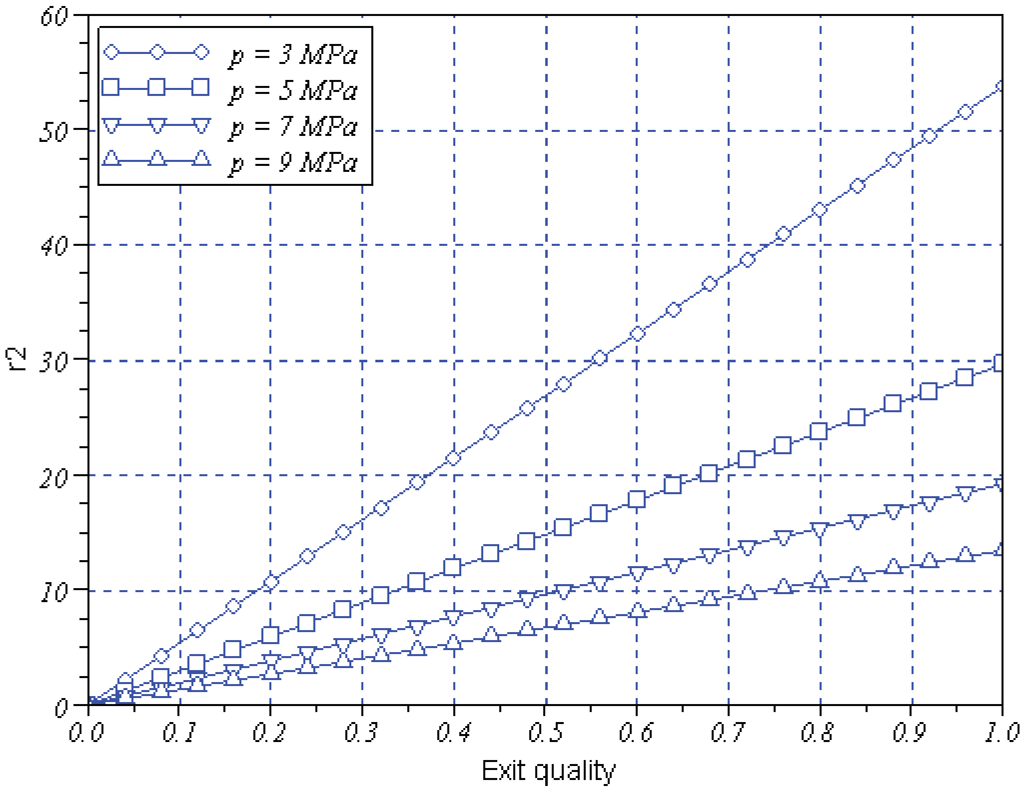
$$r_2 \equiv \rho_l \int_0^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_v} + \frac{(1-x)^2}{(1-\alpha)\rho_l}\right]dz = \left[\frac{x^2\rho_l}{\alpha\rho_v} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - \left[\frac{x^2\rho_l}{\alpha\rho_v} + \frac{(1-x)^2}{(1-\alpha)}\right]_{in}$$

  – Here subscripts *ex* and *in* mean that the expression in the rectangular parentheses is evaluated at the channel *ex*it (z=L) and at the channel *in*let (z=0), respectively.

Total Integral Pressure Drop (4)

- For heated channel with $x = \alpha = 0$ at the inlet and x_{ex} with α_{ex} at the outlet, the multiplier is as follows,
- $r_2 = \left[\frac{x^2 \rho_f}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)} \right]_{ex} - 1$, or for HEM $r_2 = \rho_f \upsilon_{fg} x_{ex}$
- This multiplier describes the pressure change due to flow acceleration caused by mixture expansion.
- It should be noted that it depends only on inlet and outlet values of void and quality.

Total Integral Pressure Drop (4)



r_2 multiplier as a function of the exit quality, for various reference pressures

It is assumed that quality is 0 at the channel inlet.

Total Integral Pressure Drop (5)

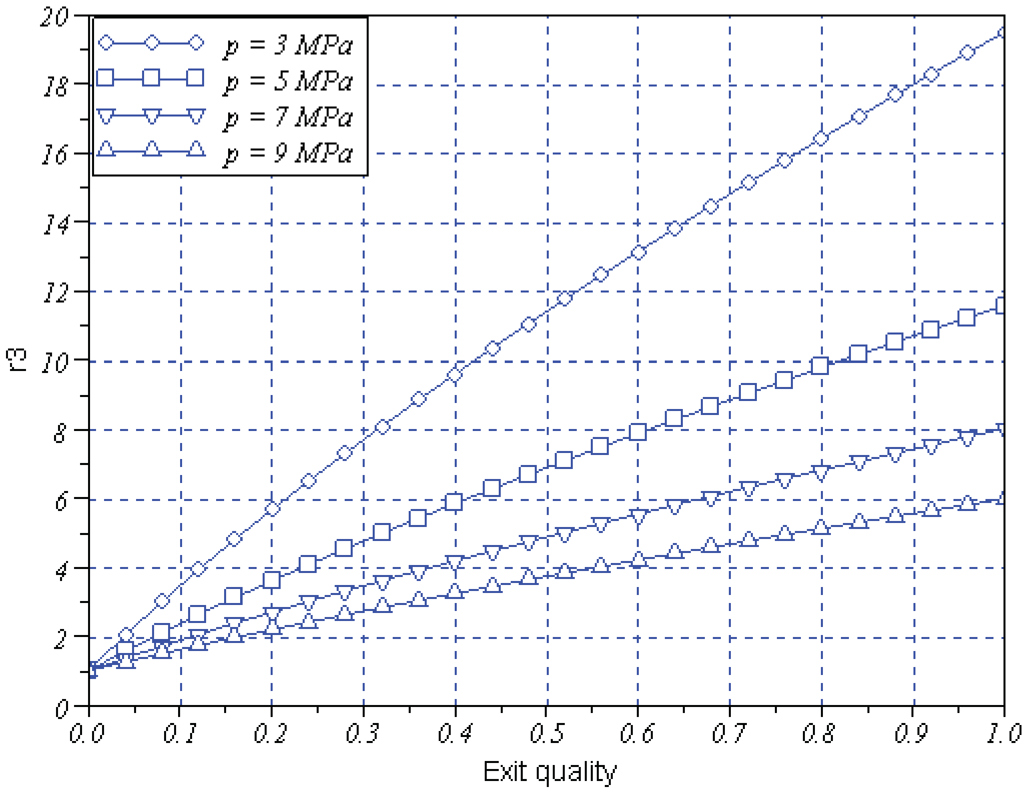
- Integral friction multiplier:

$$r_3 = \frac{1}{L}\int_0^L \varphi_{lo}^2 dz = \frac{1}{L}\int_0^L \left[1 + \left(\frac{\mu_f}{\mu_g} - 1\right)x\right]^{-0.25} \left[1 + \left(\frac{\rho_f}{\rho_g} - 1\right)x\right] dz$$

- This multiplier represents the effect of two-phase flow conditions on the friction pressure loss.

- The value of the integral multiplier depends on the values of local multiplier along the channel

Total Integral Pressure Drop (5)



r_3 multiplier as a function of the exit quality, for various reference pressures

It is assumed that quality is 0 at the channel inlet and that the power is distributed uniformly in the channel.

Total Integral Pressure Drop (6)

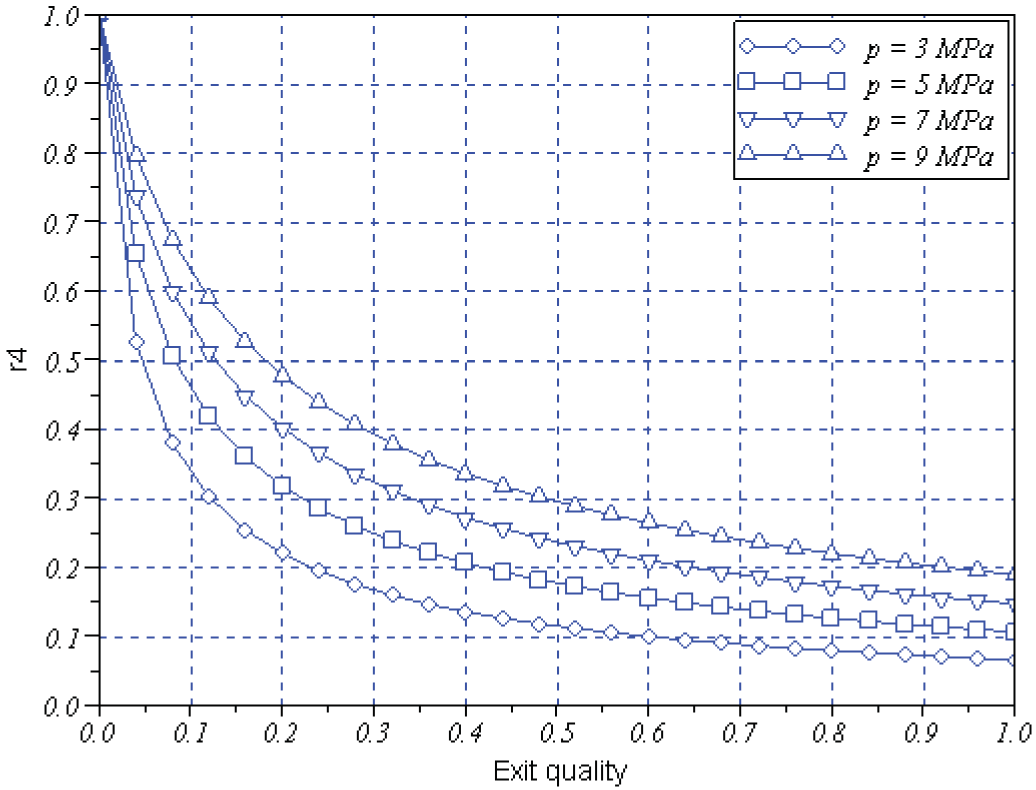
- Integral gravity multiplier:

$$r_4 = \frac{1}{L\rho_l}\int_0^L \left[\alpha\rho_g + \left(1-\alpha\right)\rho_f\right]dz = 1 - \frac{\rho_f - \rho_g}{\rho_f}\frac{1}{L}\int_0^L \alpha dz$$

- This multiplier describes the influence of two-phase flow conditions on the gravity pressure drop.

- The value of the friction multiplier depends on the void fraction distribution along the channel.

Total Integral Pressure Drop (6)



r_4 multiplier as a function of the exit quality, for various reference pressures

It is assumed that quality is 0 at the channel inlet and that the power is distributed uniformly in the channel.

Total Integral Pressure Drop (7)

- **The total channel pressure drop can be then found as,**

$$-\Delta p = r_3 C_{f,lo} \frac{4L}{D} \frac{G^2}{2\rho_f} + r_4 L \rho_f g \sin\varphi + 2r_2 \frac{G^2}{2\rho_f} =$$

$$\left(r_3 \frac{4C_{f,lo}L}{D} + 2r_2 \right) \frac{G^2}{2\rho_f} + r_4 L \rho_f g \sin\varphi$$

- **If the channel contains a number of local losses (*i* = 1, .., *N*), the total pressure drop will be as follows,**

$$-\Delta p = r_3 C_{f,lo} \frac{4L}{D} \frac{G^2}{2\rho_f} + r_4 L \rho_f g \sin\varphi + 2r_2 \frac{G^2}{2\rho_f} + \left(\sum_{i=1}^{N} \phi_{lo,di}^2 \xi_i \right) \frac{G^2}{2\rho_f} =$$

$$\left[r_3 \frac{4C_{f,lo}L}{D} + 2r_2 + \left(\sum_{i=1}^{N} \phi_{lo,di}^2 \xi_i \right) \right] \frac{G^2}{2\rho_f} + r_4 L \rho_f g \sin\varphi$$

Total Integral Pressure Drop (8)

- **NOTE:**

definitions of the integral multipliers used in this course are slightly different from definitions used in literature. This is due to two reasons:

- our definitions give non-dimensional values of multipliers

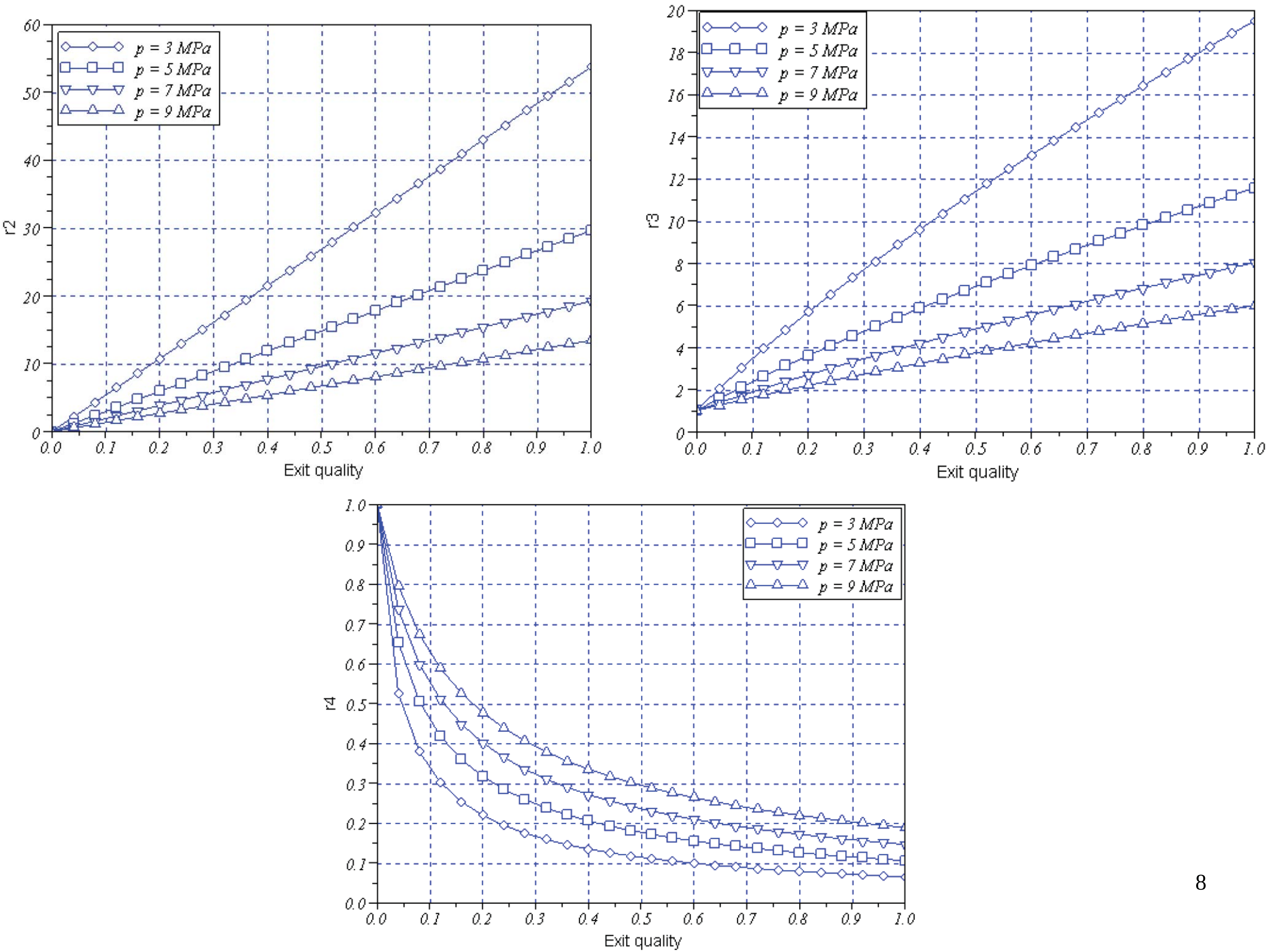
- with definitions used in this course, the two-phase pressure drop equation is a natural extension of the single-phase equation

$$-\Delta p = \left(r_3 \frac{4C_{f,lo}L}{D} + 2r_2 \right) \frac{G^2}{2\rho_f} + r_4 L \rho_f g \sin\varphi$$

two-phase flow pressure drop

$$-\Delta p = \frac{4C_f L}{D} \frac{G^2}{2\rho_f} + L \rho_f g \sin\varphi$$

single-phase flow pressure drop
$r_2 = 0$, $r_3 = r_4 = 1$



p = 3 MPa
p = 5 MPa
p = 7 MPa
p = 9 MPa
r2
Exit quality
r3
r4
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0 10 20 30 40 50 60
0 2 4 6 8 10 12 14 16 18 20

E02_P03

Two phase mixture of saturated steam and water is flowing upward in a uniformly heated vertical pipe with 15 mm internal diameter and 3.5 m in length. The inlet is saturated water (x=0) and the outlet is saturated vapor (x=1). The total mass flux is 1200 kg/m^{2}s. Assume constant fluid properties at reference pressure 7 MPa. Calculate the friction, gravity and acceleration pressure drop in the pipe, using HEM.

Solution:

The total two-phase flow pressure drop is calculated using multipliers as

$$-\Delta p = r_3 C_{f,lo} \frac{4L}{D}\frac{G^2}{2\rho_l} + r_4 L \rho_l g \sin\varphi + r_2 \frac{G^2}{\rho_l} + \left(\sum_{i=1}^{N} \phi^2_{lo,d,i}\xi_i\right)\frac{G^2}{2\rho_l}$$

$$-\Delta p_{tot} = \left(-\Delta p_{fric}\right) + \left(-\Delta p_{grav}\right) + \left(-\Delta p_{acc}\right) + \left(-\Delta p_{local}\right)$$

Since the flow is uniformly heated and the inlet quality is 0, the curves on two-phase flow pressure drop multipliers on the lecture slides could be directly used. The pressure is 7 MPa and the exit quality is 1. The multipliers are therefore obtained as

$r_2 = 20$

$r_3 = 8$

$r_4 = 0.15$

E02_P03

Two phase mixture of saturated steam and water is flowing upward in a uniformly heated vertical pipe with 15 mm internal diameter and 3.5 m in length. The inlet is saturated water (x=0) and the outlet is saturated vapor (x=1). The total mass flux is 1200 kg/m^{2}s. Assume constant fluid properties at reference pressure 7 MPa. Calculate the friction, gravity and acceleration pressure drop in the pipe, using HEM.

For friction pressure drop, the Re number is calculated as

$$\mathrm{Re}_{lo} = \frac{GD}{\mu_f} = 1.97 \times 10^5$$

We use the Haaland correlation to obtain the Fanning friction factor

$$C_{f,lo} = 0.0039$$

Finally we calculate the pressure drops as

$$-\Delta p_{fric} = 2.82 \times 10^4 \quad Pa$$

$$-\Delta p_{grav} = 3.81 \times 10^3 \quad Pa$$

$$-\Delta p_{acc} = 3.89 \times 10^4 \quad Pa$$