Sustainable Energy Transformation Technologies, SH2706

Lecture No 1

Title:

Part II: Mechanical and Electromagnetic Energy

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Outline of the Lecture

- Mechanical energy
  - Forces and fields
  - Kinetic and potential energy
  - Mechanical energy losses
  - Mechanical energy storage
- Electromagnetic energy
  - Electrostatics and electric current
  - Electromagnetic energy losses
  - Electromagnetic energy storage

Forces and Fields

- A force is an influence on an object from another object or field
- Newton's second law of motion describes the action of force **F** on an object of mass *m* as:

  $\mathbf{F} = d\mathbf{p}/dt = d(m\mathbf{v})/dt$

  "Rate of change of body's momentum equals the force applied to it"

Distant Forces

- Gravitational attraction force

$$\mathbf{F}_G = -\frac{GM_1M_2}{r^2}\hat{\mathbf{r}},$$

where $\mathbf{F}_G$ is a gravitational attraction force exerted by mass M_1 on mass M_2 pointing from mass M_2 to mass M_1, r is the distance between the centers of the masses, $\hat{\mathbf{r}}$ is a unit vector pointing from mass M_1 to mass M_2 and $G = 6.674 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$ is Newton's constant.

- Electromagnetic force

$$\mathbf{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}, \qquad (2.4)$$

where $\mathbf{F}_E$ is an electromagnetic force exerted by body with electric charge q_1 on body with electric charge q_2, $\hat{\mathbf{r}}$ is a unit vector pointing from charge q_1 to charge q_2 and ε_0 is the permittivity of the vacuum with $1/(4\pi\varepsilon_0) = 8.988 \times 10^9\ \mathrm{Nm^2\ C^{-2}}$. In SI units, charge is measured in coulombs (C) and can be either positive or negative. From Eq. 2.4 it follows that same-sign charges repel and opposite-sign charges attract.

A Field

- A field is a local mechanism that mediates a force.
  - The gravitational field is responsible for the gravitational force $\mathbf{F}_G$
  - The electric field is described by a vector $\mathbf{E}(\mathbf{x},t)$ at point $\mathbf{x}$ and time t. This field exerts a force $\mathbf{F} = q\mathbf{E}$ on charge q
- If the electric field results from a presence of charge Q, the expression for the electric field is

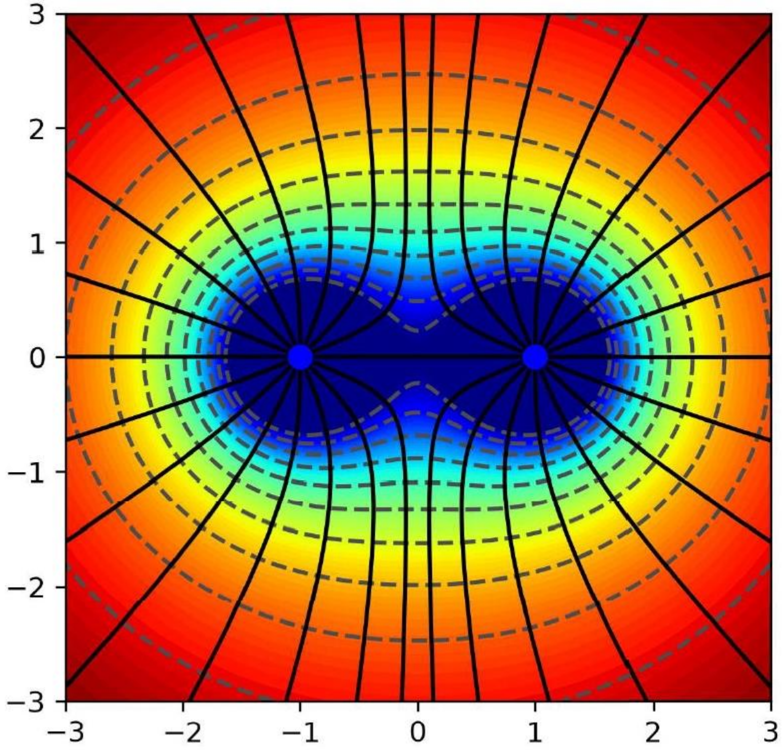
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}},$$

- In general, the electric field around a set of charges Q_i is

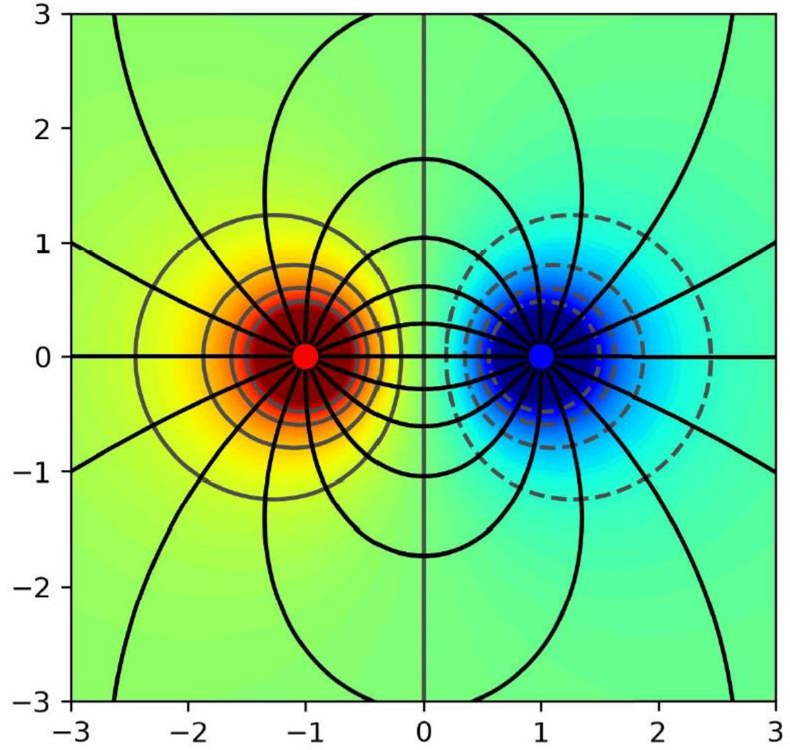
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_i Q_i \frac{\mathbf{x}-\mathbf{x}_i}{|\mathbf{x}-\mathbf{x}_i|^3}$$

A Field

Electric field lines



Two positive charges



Two opposite charges

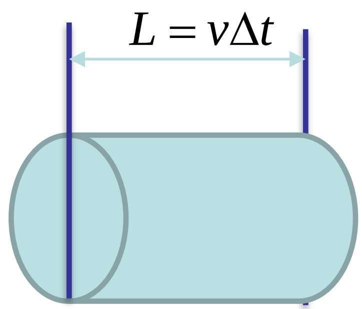
Kinetic and Potential Energy

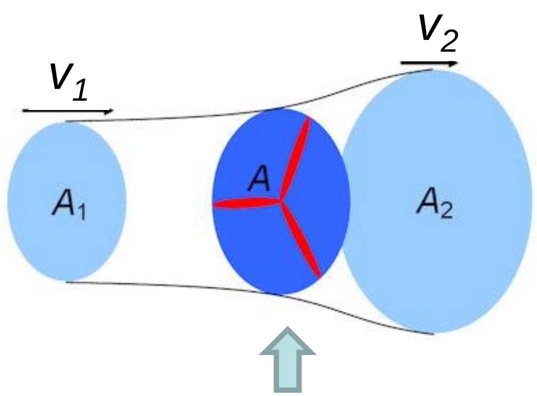
- Kinetic energy of an object with mass *m* and speed *v* is given as $E_k = mv^2/2$
- For example, for air with density *ρ* and speed *v* we have:

v – wind speed

Δt - period of time

V - air volume





$$m = V\rho = A_1 L\rho = v\Delta t A_1 \rho$$

Kinetic energy of flowing wind ~v^3 (!)

$$E = \frac{1}{2} m v^2 = \frac{1}{2} v^3 \Delta t A_1 \rho$$

Harvesting kinetic energy in a wind turbine

Kinetic and Potential Energy

- The potential energy is energy stored in a configuration of objects that interact through forces
  - For a particle moving a distance *dx* in a field with force *F*, the change in potential energy *dV* in time *dt* is given by $dV = -dE_k = -Fdx$
- For body with mass *m* in gravitational field we have

$$V(z) = \int_0^z dV = \int_0^z mgdz' = mgz.$$

z – distance above Earth's surface.
V(0) = 0

Kinetic and Potential Energy

Example 4.3: Escape Velocity from Earth's Surface

Calculate the initial velocity of a body with mass m that is required for the body to reach an infinite distance from Earth with zero kinetic energy.

Solution

The total energy of the body moving with velocity v at distance z from Earth surface is

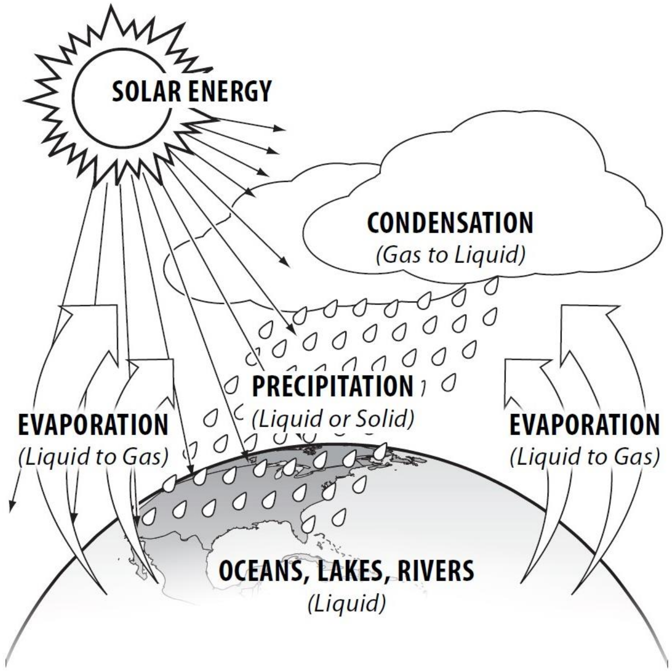
$$E = \frac{1}{2}mv^2 - \frac{G_N m M_\oplus}{R_\oplus + z}, \tag{4.11}$$

where G_N is the Newtonian constant of gravitation, $M_\oplus$ is the Earth's mass and $R_\oplus$ is the Earth's radius. For $z \to \infty$ both the kinetic and the potential energy become zero, thus the total energy is also zero. From the energy conservation principle, the total energy is zero as well when $z = 0$, thus the corresponding body velocity is obtained as,

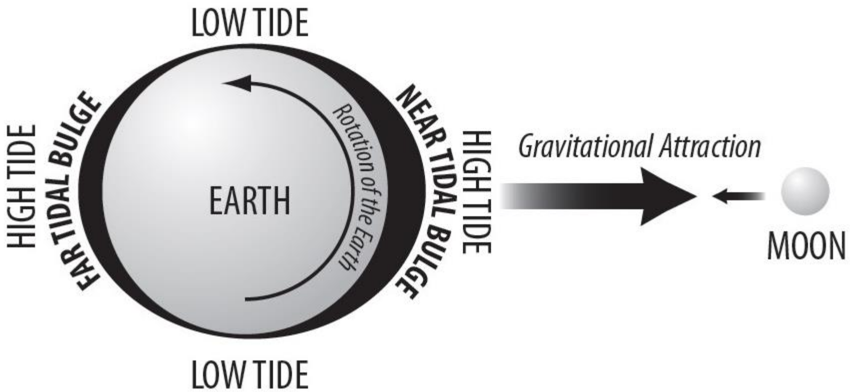
$$v_\oplus = \sqrt{\frac{2G_N M_\oplus}{R_\oplus}}. \tag{4.12}$$

Substituting data yields $v_\oplus$ = 11.2 km/s.

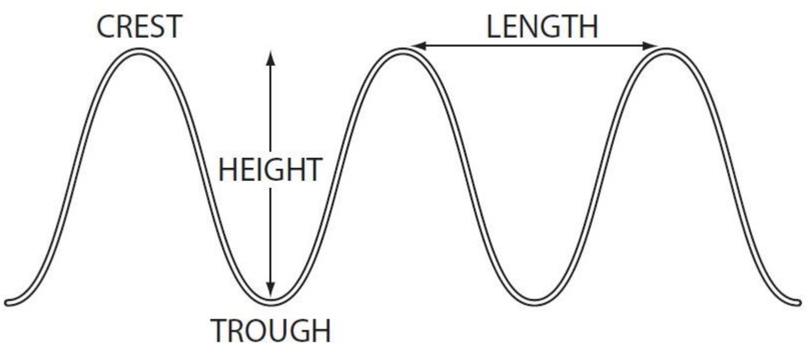
Kinetic and Potential Energy



Potential energy gained by water during water cycle in the hydrosphere



Tidal energy



Wave energy

Mechanical Energy Losses

- Mechanical energy can transform to thermal energy due to dissipative forces, such as:
  - friction between two bodies
  - drag force between a solid and a fluid
  - rolling resistance
- For example, the drag force is as follows

$$\mathbf{F}_D = -\frac{1}{2} C_D A \rho v^2 \hat{\mathbf{v}}$$

where $\hat{\mathbf{v}}$ is the unit relative velocity vector, A is the body's reference area (often defined as the cross-section area of the body in a plane perpendicular to $\hat{\mathbf{v}}$), ρ is the fluid density and C_D is a drag coefficient. The drag coefficient depends on the shape of the body

Mechanical Energy Losses

- Assume a body moving in air. The body drag coefficient is C_D, and the drag force $\mathbf{F}_D$, where

$$\mathbf{F}_D = -\frac{1}{2} C_D A \rho v^2 \hat{\mathbf{v}}$$

- After moving a distance d$\mathbf{x}$ during time dt, the body transfers $\mathbf{F}_D$*d$\mathbf{x}$ energy to the fluid, and the rate of energy loss is $\mathrm{d}E/\mathrm{d}t = \mathbf{F}_D * \mathrm{d}\mathbf{x}/\mathrm{d}t = \mathbf{F}_D * \mathbf{v}$.
- For a body moving a distance D during time T, the total mechanical energy loss is

$$\Delta E = \int_0^T (\mathrm{d}E/\mathrm{d}t)\mathrm{d}t = \int_0^D \mathbf{F}_D \cdot \mathrm{d}\mathbf{x} = \frac{1}{2} C_D A D \rho v^2$$

Example

Calculate the energy loss of a passenger car travelling 330 km with mean speed 28 m/s. The car cross-section area is approximately 2.7 m^2 and its drag coefficient is C_D=0.33. Assume air density $\rho = 1.2$ kg/m^3.

Solution

The total energy loss due to air resistance is found as

$$\Delta E_{air} = \frac{1}{2} C_D A D \rho v^2, \tag{4.26}$$

Substituting the given data yields $\Delta E = 138$ MJ.

Solve the same problem assuming:

(a) the car uniformly accelerates from 25 m/s (start) to 31 m/s (finish)

(b) the car uniformly accelerates from 25 m/s (start) to 31 m/s (half-way) and then uniformly decelerates to 25 m/s (finish)

Mechanical Energy Storage

- Attempts have been made to develop methods for a mechanical energy storage, such as:
  - pumped-storage hydropower (PSH) is storing the potential energy in the upper reservoir (efficiency up to 70-80%)

  - flywheel storage, where the stored kinetic energy is

$$E_{rot} = \frac{1}{2} I \omega^2$$

Mechanical Energy Storage

Example 2.6. Calculate the amount of energy stored in a flywheel made from a uniform steel disc with mass $m = 5$ kg and radius $R = 0.3$ m, rotating at 4×10^4 rpm.

Solution: The moment of inertia of a solid disk of density ρ, radius R and height H is

$$I = \int_0^R 2\pi r H \rho r^2 dr = \frac{\pi}{2}\rho H R^4, \tag{2.28}$$

and the mass is

$$m = \int_0^R 2\pi r H \rho dr = \pi \rho H R^2. \tag{2.29}$$

Thus we have $I = mR^2/2$. At 4×10^4 rpm $\omega = 40\,000 \times 2\pi/60 \approx 4200$ s^{-1}. Substituting the given data yields $E_{rot} = 2$ MJ. $\square$

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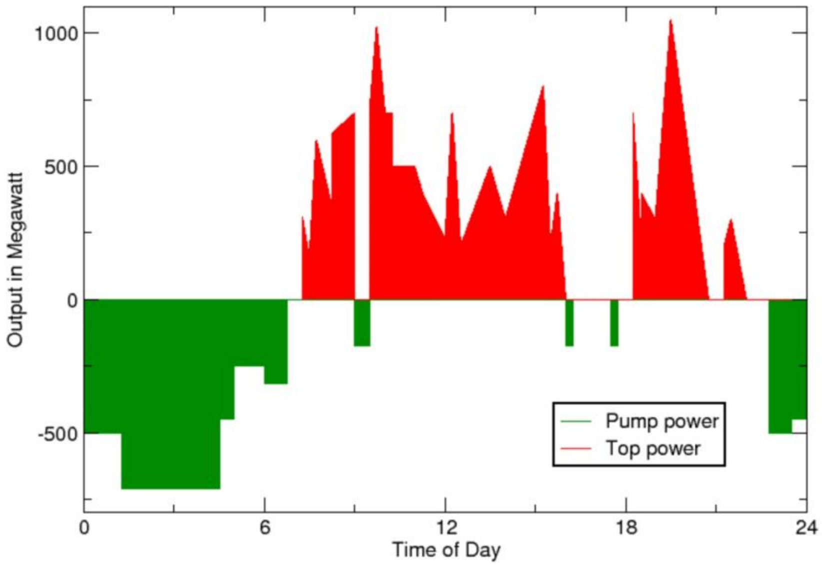
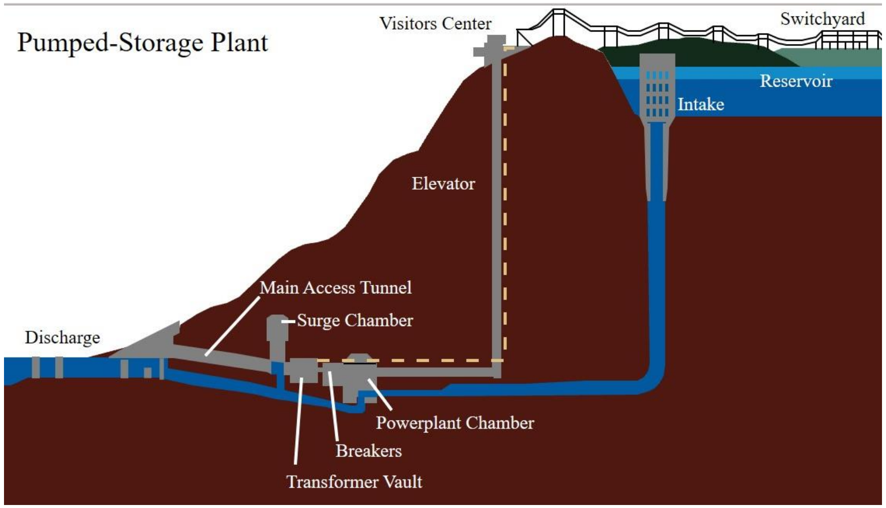
Pumped Hydro Storage





- Used for load balancing
- Allows intermittent energy from solar and wind to be saved for periods of high demands
- The over-all (from producer to user) efficiency is 70-80%
- Will be discussed later in the course (hydropower)

Electromagnetic Energy

- Electromagnetic energy results from forces and fields caused by electrically charged objects
- Human use of the electromagnetic energy involves: (1) generation, (2) transmission, (3) storage, (4) utilization
- Devices using electromagnetic energy are compact, clean and convenient
- Electromagnetic energy can be efficiently transformed into mechanical energy
- Electromagnetic energy can be transmitted over great distances with small losses

Electrostatics

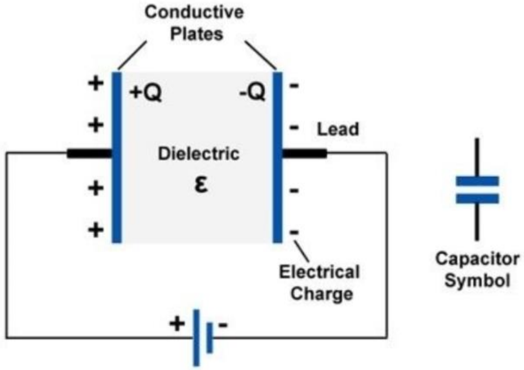
- Electrostatics is concerned with electric fields, forces and potential, resulting from static electric charges
- Let us take a static charge distributed in space as ρ(**x**) : the resulting electric field satisfy the following equations

$$\nabla \cdot \mathbf{E}(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\varepsilon_0} \quad \text{(Gauss law)} \qquad \nabla \times \mathbf{E}(\mathbf{x}) = 0 \quad \text{(Stokes theorem)}$$

- Once **E**(**x**) is known, the electrostatic voltage difference between two points $\mathbf{x}_1$, $\mathbf{x}_2$ is found as $V(\mathbf{x}_1) - V(\mathbf{x}_2) = -\int_{x_1}^{x_2} \mathbf{E}(\mathbf{x}) \cdot d\mathbf{x}$

  and the force exerted by the electric field on a charge q is $\mathbf{F} = q\mathbf{E}$

Electrostatics - Example



$\varepsilon = \kappa\varepsilon_0$, where
ε – permittivity
κ – dielectric constant
ε_0 - vacuum permittivity

Example 2.7. A plate capacitor made of two square metal plates with 1 cm side length, separated by 1 mm gap, suffers a dielectric breakdown and conducts electric current in the presence of electric field greater than $E_{max} \approx 3.3 \times 10^6$ V/m. Calculate the maximum energy that can be stored in the capacitor and the associated energy density.

Solution: The capacitor has capacitance $C = \kappa\varepsilon_0 A/d \approx (1.00059)(8.854 \times 10^{-12}$ F/m$)(0.01\ \text{m})^2/(0.001\ \text{m}) \approx 0.885 \times 10^{-12}$ F. The maximum possible voltage to which the capacitor can be charged is $V_{max} = E_{max}d \approx 3.3 \times 10^3$ V, with the maximum energy $CV^2/2 \approx 4.8 \times 10^{-6}$ J, and energy density $CV^2/(2Ad) \approx 50$ J/m^3. This value is very small compared for example to the energy density of gasoline, which is about 32 GJ/m^3. □

Electric Current

- Electric current measures the net rate at which charges pass a given point in a wire: $I = dQ/dt$ (ampere, A)
- $I = V/R$ – **Ohm's law**; R – resistance (ohm, Ω)

- When electric charge is driven through a resistor, the electromagnetic energy is transformed into thermal energy (**Joule heating**)

- The rate of power dissipation is given by the **Joule's law**
  $P_{res} \equiv N_{Joule} = V*I = I^2*R = V^2/R$ (watts)

Electromagnetic Energy Losses

- Electromagnetic energy is transformed into thermal energy due to Joule heating
- These losses occur in all types of electric devices, such as motors, generators, transformers, etc

Transmission lines

- For example, resistive losses $\overset{N_{Joule}}{\downarrow}$ q_{loss}, when AC power is transmitted to a resistive load R_L through wires with resistance R_T, are the following fraction of the delivered power N_L,

$$\frac{q_{loss}}{N_L} \cong \frac{R_T N_L}{V_{RMS}^2} = \frac{N_L \rho l}{A V_{RMS}^2}$$

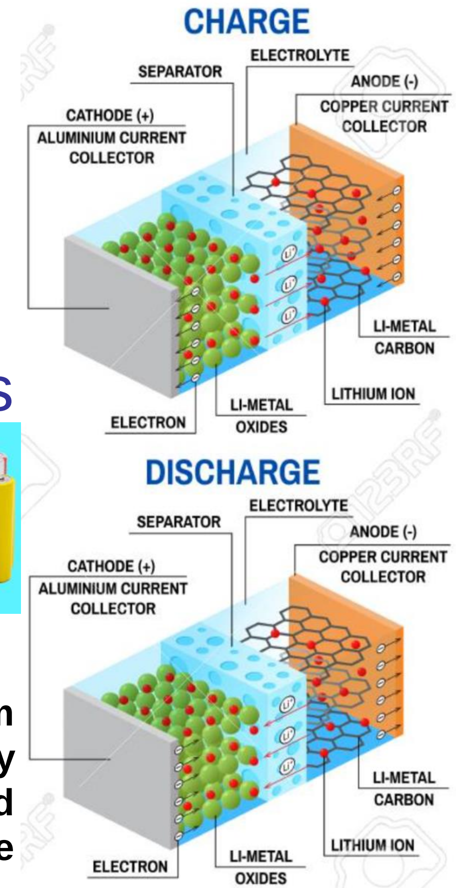
Explains why electric power is transmitted at as high a voltage as possible

where $V_{RMS} = \sqrt{\langle V(t)^2 \rangle}$

l, A, ρ - length, area and resistivity of the transmission wire

Electromagnetic Energy Storage

- Large-scale storage of electromagnetic energy is an important and challenging problem
- Capacitive energy storage is quite limited
- The main technology currently in practical use is the storage in re-chargeable batteries
- Batteries store energy in chemical form and use electrochemical reactions to convert energy to and from electrical form



Lithium battery charge and discharge