Nuclear Reactor Physics

Reactor Kinetics II

Jan Dufek

2022

KTH Royal Institute of Technology

Contents

The inhour equation

The inhour equation

Reactor period T

- **In the absolute value, the reactor period T is the time that is needed for the system power (number of neutrons) to change e-fold. The period is positive when the power grows, and it is negative when the power decreases.**
- When increasing the power during a standard operation, the reactor periods is maintained larger than about 30 or 40 s.
- If the period drops below about 30 s (while increasing the power) then control systems shut down the reactor automatically.

The inhour equation

The inhour equation gives the relation between the reactivity ρ and reactor period T (or its inverse value ω), assuming that ρ is constant for $t \geq 0$.

The inhour equation

Derivation of the inhour equation

- The inhour equation is derived with the assumption of no external source q.
- We assume now that $\rho = \rho_0$ for $t \geq 0$. To obtain the inhour equation, we need to solve the kinetic equations

$$\frac{dn}{dt} = \frac{\rho_0 - \beta_{\mathrm{eff}}}{\Lambda} n + \sum_i \lambda_i c_i$$

$$\frac{dc_i}{dt} = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n - \lambda_i c_i, \;\; i = 1, \ldots, 6$$

- This can be done easily via Laplace transformation (not done here). The solution is

$$n(t) = \sum_{j=1}^{7} A_j e^{\omega_j t},$$

where ω_j are solutions to the equation

$$\rho_0 = \beta_{\mathrm{eff}} + \Lambda\omega - \sum_i \frac{\beta_{\mathrm{eff}\,i}\lambda_i}{\omega + \lambda_i}$$

The inhour equation

Derivation of the inhour equation

- Since $\beta_{\text{eff}} = \sum_i \beta_{\text{eff}\,i} = \sum_i \frac{\beta_{\text{eff}\,i}\omega + \beta_{\text{eff}\,i}\lambda_i}{\omega+\lambda_i}$ the equation

$$\rho_0 = \beta_{\text{eff}} + \Lambda\omega - \sum_i \frac{\beta_{\text{eff}\,i}\lambda_i}{\omega + \lambda_i}$$

  can also be written as

$$\rho_0 = \Lambda\omega + \sum_i \frac{\beta_{\text{eff}\,i}\omega}{\omega + \lambda_i}$$

  which is known as **the inhour equation**.

- **The inhour equation has 6 real negative roots for ω and one real of the same sign as ρ_0.**

The inhour equation

The solution $n(t)$ has 7 terms $A_j e^{\omega_j t}$. Which term will dominate for large times?

- For large times, the term with the largest ω_j (let's assign it the first index, ω_1) will dominate, and
$$n(t) \rightarrow A_1 e^{\omega_1 t}$$
- For $\rho_0 > 0$, the dominant term is a growing exponential, and the characteristic time
$$T = 1/\omega_1$$
is called the **reactor period** or the **asymptotic period**.

The inhour equation

Based on the kin. equation

$$\frac{dn}{dt} = \frac{\rho - \beta_{\mathrm{eff}}}{\Lambda} n + \sum_i \lambda_i c_i + q$$

what happens when $\rho > \beta_{\mathrm{eff}}$?

The first term in the kinetic equation for $n(t)$ then becomes positive for $\rho > \beta_{\mathrm{eff}}$, and $n(t)$ can then grow even without the need of delayed neutrons or external source.

When a reactor is so-called prompt-critical?

When $\rho = \beta_{\mathrm{eff}}$ then the reactor is called prompt-critical since the chain reaction could be self-sustained just on prompt neutrons.

The inhour equation

It is not possible to calculate analytically the roots of the inhour equation

$$\rho_0 = \beta_{\text{eff}} + \Lambda\omega - \sum_i \frac{\beta_{\text{eff}\,i}\lambda_i}{\omega + \lambda_i}$$

but we can simplify the equation. For large ρ_0 (when $\rho_0 > \beta_{\text{eff}}$) we get:

For large $\rho_0 > \beta_{\text{eff}}$, reactor period T is small (ω is very large) and the term

$$\frac{\beta_{\text{eff}\,i}\lambda_i}{\omega + \lambda_i}$$

can be neglected in the inhour equation, so we can write

$$\rho_0 = \beta_{\text{eff}} + \Lambda\omega$$

from where it follows that

$$T = \frac{\Lambda}{\rho_0 - \beta_{\text{eff}}}$$

Example with $\rho_0 = 2\beta_{\text{eff}}$ (about 1600 pcm)

When we choose e.g. $\rho_0 = 1600pcm$, then the reactor period becomes about $T \approx \frac{10^{-3}}{0.008}s = 0.125s$. The reactor power then increases about $e^{1/0.125} = e^8 \approx 3000$ times during a single second!

The inhour equation

We can also simplify the inhour equation

$$\rho_0 = \Lambda\omega + \sum_i \frac{\beta_{\text{eff}\,i}\omega}{\omega + \lambda_i}$$

for small ρ_0 (close to 0).

When ρ_0 is very small then T is very large (slow increase in power), and so ω is very small, and the term

$$\frac{\beta_{\text{eff}\,i}\omega}{\omega + \lambda_i} \to \frac{\beta_{\text{eff}\,i}\omega}{\lambda_i}$$

Then we can write the inhour equation as

$$\rho_0 = \left(\Lambda + \sum_i \frac{\beta_{\text{eff}\,i}}{\lambda_i}\right)\omega$$

Since ω changes sign with ρ_0 here, it must be the ω_1. The period is then

$$T = \left(\Lambda + \sum_i \frac{\beta_{\text{eff}\,i}}{\lambda_i}\right)/\rho_0$$

The inhour equation

We can write

$$T = \left(\Lambda + \sum_i \frac{\beta_{\mathrm{eff}\,i}}{\lambda_i} \right) / \rho_0$$

also as

$$T = \left(\Lambda + \frac{\beta_{\mathrm{eff}}}{\lambda} \right) / \rho_0$$

where

$$\lambda = \beta_{\mathrm{eff}} / \sum_i \frac{\beta_{\mathrm{eff}\,i}}{\lambda_i}$$

The $\beta_{\mathrm{eff}}/\lambda$ is several orders of magnitude larger than Λ for most reactors ($\beta_{\mathrm{eff}}/\lambda \doteq 0.0847$ for thermal reactors with ^{235}U, or 0.0327 for therm. reactors with ^{239}Pu), so if we can neglect Λ then we get

$$T \approx \frac{\beta_{\mathrm{eff}}}{\lambda} / \rho_0$$

Example for $\rho = 0.1\beta_{\mathrm{eff}}$:

Then $T \approx 0.0847/0.0008 \approx 106s$, and power will increase only $e^{1/106} = 1.0095$ times (by less than 1%) during a single second.

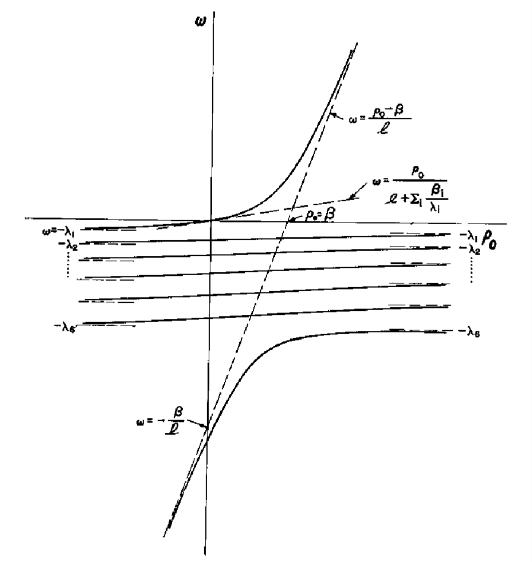


Figure 1: Plot of the inhour equation for six groups of delayed neutrons

Static measurement of reactivity of subcritical system

Apparent neutron multiplication

Let's define the apparent neutron multiplication, M, in a sub-critical system as the total number of neutrons appearing in the system per a neutron from the external source.

The value of M

can be calculated as the sum of neutrons from the source S and from all succeeding fission generations (Sk, Sk^2, etc.) divided by the number of neutrons from the external source,

$$M = \frac{S + Sk + Sk^2 + \cdots}{S} = \frac{1}{1-k}, \quad k < 1$$

Therefore, by measuring M we can obtain k,

$$k = 1 - \frac{1}{M}$$

As $k \to 1$ the value $\frac{1}{M} \to 0$.

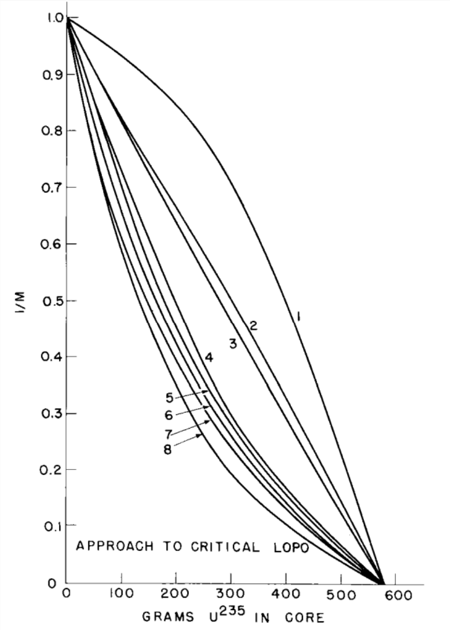
Approaching criticality

The method is commonly used for monitoring reactivity of a subcritical reactor during its approaching to criticality.

Reciprocal multiplication method

The method for safe approach to criticality consists of plotting $1/M$ (reciprocal neutron counting rate) as a function of some parameter that controls reactivity and extrapolating $1/M$ plot to zero after each stepwise increase in the reactivity.

Neutron multiplication measurements



Neutron multiplication measurements

- The actual shape of the $1/M$ curve depends on the system and the position of the neutron detector and source.
- Large separation between the neutron source and detector give positive curvature (e.g. curve 8 on the previous slide).
- Caution must be taken when $1/M$ curve exhibit negative curvature (e.g. curve 1) since the extrapolated critical mass decreases as criticality is approached. (Critical mass can be easily overestimated.)
- The correct multiplication following a reactivity change in a subcritical system is observed only after the power is stabilised, which takes a long time for close-to-critical systems.

Dynamic measurement of reactivity by the rod-drop method

Consider a reactor operating at some equilibrium level, n_0, $\rho_0 = 0$, which is suddenly shut down by the introduction of a negative reactivity ρ_1 (rod drop).

For the equilibrium conditions existing prior to the rod drop, the kinetic equations with no external source become

$$0 = \frac{\rho_0 - \beta_{\mathrm{eff}}}{\Lambda} n_0 + \sum_i \lambda_i c_{i0}$$

$$0 = \frac{{\beta_{\mathrm{eff}}}_i}{\Lambda} n_0 - \lambda_i c_{i0}, \;\; i = 1, \ldots, 6$$

Rod-drop method

From the first equation

$$0 = \frac{\rho_0 - \beta_{\text{eff}}}{\Lambda} n_0 + \sum_i \lambda_i c_{i0}$$

we can write (since $\rho_0 = 0$)

$$n_0 = \frac{\Lambda \sum_i \lambda_i c_{i0}}{\beta_{\text{eff}}}$$

Rod-drop method

Within a few prompt-neutron lifetimes after the drop,

the system adjusts to a lower neutron level determined by the new prompt neutron reproduction and remains nearly constant at this “quasistatic level” until it is ultimately decreased by delayed-neutron decay.

For the “quasistatic level” we can write

$$0 = \frac{\rho_1 - \beta_{\mathrm{eff}}}{\Lambda} n_1 + \sum_i \lambda_i c_{i0}$$

(since the concentration of delayed neutrons remains about the same at this point), from where

$$n_1 = \frac{\Lambda \sum_i \lambda_i c_{i0}}{\beta_{\mathrm{eff}} - \rho_1}$$

Rod-drop method

From equations

$$n_0 = \frac{\Lambda \sum_i \lambda_i c_{i0}}{\beta_{\text{eff}}}$$

and

$$n_1 = \frac{\Lambda \sum_i \lambda_i c_{i0}}{\beta_{\text{eff}} - \rho_1}$$

we can see that

$$\frac{n_1}{n_0} = \frac{\beta_{\text{eff}}}{\beta_{\text{eff}} - \rho_1}$$

from where

$$\boxed{\frac{\rho_1}{\beta_{\text{eff}}} = 1 - \frac{n_0}{n_1}}$$

Hence, the reactivity value of the rod drop in units of dollars can be obtained directly from the observed power ratio.

Dynamic measurement of reactivity by the source-jerk method

Source-jerk method

Consider a subcritical system,

$\rho_0 < 0$, with an external source q at an equilibrium level n_0, from which the source is suddenly removed.

For the equilibrium conditions existing prior to the "source jerk", the kinetic equations are

$$0 = \frac{\rho_0 - \beta_{\mathrm{eff}}}{\Lambda} n_0 + \sum_i \lambda_i c_{i0} + q$$

$$0 = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n_0 - \lambda_i c_{i0}, \;\; i = 1, \ldots, 6$$

From the first equation

$$0 = \frac{\rho_0 - \beta_{\mathrm{eff}}}{\Lambda} n_0 + \sum_i \lambda_i c_{i0} + q$$

we get

$$n_0 = \frac{\Lambda \sum_i \lambda_i c_{i0} + \Lambda q}{\beta_{\mathrm{eff}} - \rho_0}$$

Source-jerk method

Within a few prompt-neutron lifetimes after removal of the source,

the system will adjust to a lower "quasistatic" neutron level n_1 determined by the multiplied delayed-neutron source strength alone:

$$n_1 = \frac{\Lambda \sum_i \lambda_i c_{i0}}{\beta_{\mathrm{eff}} - \rho_0}$$

Source-jerk method

From equations

$$n_0 = \frac{\Lambda \sum_i \lambda_i c_{i0} + \Lambda q}{\beta_{\mathrm{eff}} - \rho_0}$$

and

$$n_1 = \frac{\Lambda \sum_i \lambda_i c_{i0}}{\beta_{\mathrm{eff}} - \rho_0}$$

we get

$$\frac{n_0}{n_1} = 1 + \frac{q}{\sum_i \lambda_i c_{i0}}$$

At steady-state, before the source-jerk,

a specific power level n_0 is established according to the external source q, which can be seen from an equation (derived in one of the previous lectures)

$$n_0 = -\frac{-\Lambda q}{\rho_0}$$

From there we can write

$$q = -\frac{n_0 \rho_0}{\Lambda}$$

which can be substituted into

$$\frac{n_0}{n_1} = 1 + \frac{q}{\sum_i \lambda_i c_{i0}}$$

Also, the term $\sum_i \lambda_i c_{i0}$ in equation

$$\frac{n_0}{n_1} = 1 + \frac{q}{\sum_i \lambda_i c_{i0}}$$

can be expressed from equation

$$0 = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n_0 - \lambda_i c_{i0}, \;\; i = 1, \ldots, 6$$

as

$$\sum_i \lambda_i c_{i0} = \frac{\beta_{\mathrm{eff}}}{\Lambda} n_0$$

Source-jerk method

After substitution of terms q and $\sum_i \lambda_i c_{i0}$ in equation

$$\frac{n_0}{n_1} = 1 + \frac{q}{\sum_i \lambda_i c_{i0}}$$

by the derived expressions

$$q = -\frac{n_0 \rho_0}{\Lambda}$$

$$\sum_i \lambda_i c_{i0} = \frac{\beta_{\mathrm{eff}}}{\Lambda} n_0$$

we get

$$\frac{n_0}{n_1} = 1 - \frac{\rho_0}{\beta_{\mathrm{eff}}}$$

From

$$\frac{n_0}{n_1} = 1 - \frac{\rho_0}{\beta_{\text{eff}}}$$

we can see that reactivity in dollars can be measured as

$$\frac{\rho_0}{\beta_{\text{eff}}} = 1 - \frac{n_0}{n_1}$$

This method requires the rapid removal of only a small mass (the source) compared with the rod-drop method which requires the rapid transfer of one or more control rods.

Another variant of the source-jerk and rod-drop methods

Another variant of the source-jerk and rod-drop methods

After the source-jerk or rod-drop the system is described by kinetic equations without an external source

$$\frac{dn}{dt} = \frac{\rho - \beta_{\mathrm{eff}}}{\Lambda} n + \sum_i \lambda_i c_i$$

$$\frac{dc_i}{dt} = \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n - \lambda_i c_i, \;\; i = 1, \ldots, 6$$

These equations can be integrated from the time of the dynamic change to infinity

$$\int_0^\infty dn = \int_0^\infty \frac{\rho - \beta_{\mathrm{eff}}}{\Lambda} n dt + \sum_i \lambda_i \int_0^\infty c_i dt$$

$$\int_0^\infty dc_i = \int_0^\infty \frac{\beta_{\mathrm{eff}\,i}}{\Lambda} n dt - \lambda_i \int_0^\infty c_i dt, \;\; i = 1, \ldots, 6$$

where

$$\int_0^\infty dn = n_\infty - n_0$$

$$\int_0^\infty dc_i = c_{i,\infty} - c_{i,0}$$

Another variant of the source-jerk and rod-drop methods

Since we assume a subcritical reactor, we can state that $n_\infty = 0$ and $c_{i,0} = 0$, so we can re-write the eqautions as

$$-n_0 = \int_0^\infty \frac{\rho - \beta_{\text{eff}}}{\Lambda} n dt + \sum_i \lambda_i \int_0^\infty c_i dt$$

$$-c_{i,0} = \int_0^\infty \frac{\beta_{\text{eff}\,i}}{\Lambda} n dt - \lambda_i \int_0^\infty c_i dt, \ \ i = 1, \dots, 6$$

We can sum up the equations for precursors over all groups, and we get

$$-\sum_i c_{i,0} = \int_0^\infty \frac{\beta_{\text{eff}}}{\Lambda} n dt - \sum_i \lambda_i \int_0^\infty c_i dt$$

The above equation combines with the equation at the top into

$$-n_0 = \int_0^\infty \frac{\rho}{\Lambda} n dt + \sum_i c_{i,0}$$

Another variant of the source-jerk and rod-drop methods

We know that there is a relation between equilibrium number of neutrons and precursors:

$$c_{i,0} = \frac{\beta_{\mathrm{eff}_i}}{\lambda_i \Lambda} n_0$$

Hence, we can use it for the equation

$$-n_0 = \int_0^\infty \frac{\rho}{\Lambda} n dt + \sum_i c_{i,0}$$

and we get

$$-n_0 = \int_0^\infty \frac{\rho}{\Lambda} n dt + \sum_i \frac{\beta_{\mathrm{eff}_i}}{\lambda_i \Lambda} n_0$$

which can be solved for reactivity as

$$\frac{\rho}{\beta_{\mathrm{eff}}} = -\frac{n_0 \times A}{\int_0^\infty n dt}$$

where

$$A = \frac{\Lambda}{\beta_{\mathrm{eff}}} + \sum_i \frac{\beta_{\mathrm{eff}_i}}{\beta_{\mathrm{eff}} \lambda_i}$$

Another variant of the source-jerk and rod-drop methods

Note

The value of A in the equation

$$\frac{\rho}{\beta_{\text{eff}}} = -\frac{n_0 \times A}{\int_0^\infty n dt}$$

is a characteristic of the reactor. It can be for instance equal to 11s.

The value of $\int_0^\infty n dt$ can be measured during the experimant.