Nuclear Reactor Physics

Reactor Theory I

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Clarification of home assignments rules

Clarification of home assignments rules

Several years ago:

- The final grade from this course was based on a single big written exam at the end of the course, lasting for 5 hours (10 problems).
- Each problem solution was evaluated personally by the professor. When the result was wrong but the derivation was OK then partial points were given.
- The evaluation took about two weeks for the professor - so students didn't know for few weeks whether they passed. (That's why there were no graded home assignments given to students at that time.)

Today:

The final grade is based on:

- 40 graded home assignment problems with automatic evaluation based on the numerical value of the result,
- short written exam with 5 problems (automatic evaluation),
- 9 quizzes (automatic evaluation),
- two lab reports (human evaluation),
- oral exam (human evaluation).

Clarification of home assignments rules

Question - is the automatic evaluation not fair?

Extreme case #1:

Imagine you have a company, and you need to hire new engineers to solve problems. How much would you pay for:

- an engineer who delivers correct results? The usual salary?
- an engineer who knows exactly how to solve all problems, but always makes some unfortunate mistakes? He/she never delivers correct results, so you can't use his/her results. Would you hire the engineer and pay him/her 75% of the standard salary?

Extreme case #2:

An engineer constructed a bridge. The engineer knew the correct procedure, but an unfortunate mistake in the design resulted in a collapse of the bridge. Will the relatives of the victims glorify the engineer for his good knowledge he had when designing the bridge?

Clarification of home assignments rules

In real life, all errors count the same.

In this course

Even when you do numerical mistakes in about 15 graded assignment problems you can still get the A grade from the course!

If you disagree with these rules then raise your voice now

Let's resolve any disagreements now.

Clarification of home assignments rules

What to do when you wonder why your solution was not accepted:

- Check the derivation of the solution in Möbius Gradebook (the link to it is in Canvas/Assignments).
- Try to figure out why your and Gradebook solutions differ.
- If your solution differs because you made a mistake then learn from your mistake so that you would not repeat it in the future.
- If you think your solution is correct and the solution shown in the Gradebook is wrong (which happened many times in past years) then try to identify the bug in the Gradebook solution and send me an email in which you:
  - identify whether you talk about graded/non-graded assignment,
  - identify the HA number as well as the question number,
  - **attach** a working commented script that returns the exact value that you submitted,
  - suggest the nature of the bug in the official solution.
- If you are correct about the bug then I will fix the points to you and all other affected students, and I will give you an extra bonus point.

Clarification of home assignments rules

Please don't send me:

- emails asking me to give you some points because you made only a small mistake. The cause of the error is irrelevant in the assignments/exam problems in this course. (This may be set up differently in other courses.)
- scripts that don't run and exit with an error message. (I will not be fixing your scripts.)
- scripts that return a value different from the value you submitted to Möbius.
- scripts copied into the text of your email. Always attach the script as a file, so that I can save it and run it.
- questions about problems without specifying what problem you talk about. This is especially common when you ask me about several different problems from various assignments in a single email. Better to send me several emails (one email per problem).

Clarification of points in Canvas

Note that:

- Canvas shows a total grade in Grades. It is not clear how Canvas calculates it, but this value is completely irrelevant and you can ignore it. (Unfortunately, I can't deactivate this feature.) At the end of the course, I will export all points from Canvas and process them myself.
- If you get a bonus point from me for discovering a serious bug in a home assignment then you will not see the bonus point in Canvas. I keep track of these points independently. I'll add these points to your other points at the end of the course.

The group-diffusion method

The group-diffusion method

What is the main idea of the multi-group methods?

In multigroup calculations, the energy dependence of neutron flux and cross sections and other data considers several neutron energies.

- The one group diffusion approximation gives only rough estimates.
- To obtain more accurate results it is necessary to perform multi-group calculations.
- Two-group diffusion approximation is sufficient for thermal reactors (fast and thermal group).
- More than ten groups are needed for fast reactors.

Two-group diffusion approximation is commonly used by solvers used in LWR power plants.

The group-diffusion method

To form multigroup equations for a critical reactor, we need to define new variables.

- The index g of the most energetic group is 1, and the largest index, N, corresponds to the least energetic group (thermal neutrons).
- ϕ_g = neutron flux in Group g (flux integrated over energies corresponding to Group g),

$$\phi_g = \int_g \phi(E)dE$$

- Σ_{ag} = the group-averaged macroscopic absorption cross section,

$$\Sigma_{ag} = \frac{1}{\phi_g} \int_g \Sigma_a(E)\phi(E)dE$$

- ν_g = the average number of fission neutrons emitted from a fission induced by a neutron in the gth group.

The group-diffusion method

As neutrons collide, they lose energy and change groups. This must be accounted for by neutron flux transfer between energy groups.

This can be accounted for by using the **group transfer cross section** $\Sigma_{g\to h}$, via which we can express the transfer rate of neutrons from group g to group h

$$\Sigma_{g\to h}\phi_g$$

The total transfer rate out of group g

$$\sum_{h=g+1}^{N} \Sigma_{g\to h}\phi_g$$

The total transfer rate into group g

$$\sum_{h=1}^{g-1} \Sigma_{h\to g}\phi_h$$

The group-diffusion method

The diffusion can be then expressed for Group g via the above terms. Find a mistake in this equation:

$$D_g \nabla^2 \phi_g - \Sigma_{ag} \phi_g - \sum_{h=g+1}^{N} \Sigma_{g \to h} \phi_g + \sum_{h=1}^{g-1} \Sigma_{h \to g} \phi_h = 0$$

The neutron source (fission or external) was missing. The correct equation is:

$$D_g \nabla^2 \phi_g - \Sigma_{ag} \phi_g - \sum_{h=g+1}^{N} \Sigma_{g \to h} \phi_g + \sum_{h=1}^{g-1} \Sigma_{h \to g} \phi_h + s_g = 0$$

The system is then described by N coupled group equations.

Thermal neutron diffusion

What is the difference between ϕ_0 (2200 meters-per-second neutron flux) and ϕ_T (thermal flux)?

- ϕ_0 is "2200 meters-per-second neutron flux". If a system contains n neutrons then ϕ_0 is defined as

$$\phi_0 = nv_0$$

where $v_0 = 2200$ m/s, so it is the total flux if all neutrons had the same speed v_0. The absorption rate of polyenergetic neutrons on a $1/v$ absorber can be easily computed as

$$F_a = \Sigma_a(E_0)\phi_0$$

where $E_0 = 0.0253$eV corresponds to neutron kinetic energy with v_0.

For this reason, cross sections are often given at energy E_0.

For non-$1/v$ absorbers, the absorption rate can be computed as

$$F_a = g_a(T)\Sigma_a(E_0)\phi_0$$

where $g_a(T)$ is called the "non-$1/v$ factor" that is a tabulated function.

Thermal neutron diffusion

What is the difference between ϕ_0 (2200 meters-per-second neutron flux) and ϕ_T (thermal flux)?

- ϕ_T is the "thermal flux". If a system contains n neutrons then ϕ_T would be the group flux assuming that the neutron kinetic energy spectrum (of all neutrons) is given by the Maxwell–Boltzmann distribution at temperature T.

  It can be shown that

  $$\frac{\phi_0}{\phi_T} = \frac{\sqrt{\pi}}{2}\sqrt{\frac{T_0}{T}}$$

  where $T_0 = 293.61\text{K}$.

  We can use this result when calculating the group absorption cross section as

  $$\bar{\Sigma}_a = \frac{1}{\phi_T}\int_T \Sigma_a(E)\phi(E)dE$$

  For non-$1/v$ absorbers, this becomes

  $$\bar{\Sigma}_a = g_a(T)\Sigma_a(E_0)\phi_0/\phi_T = \frac{\sqrt{\pi}}{2}g_a(T)\Sigma_a(E_0)\sqrt{\frac{T_0}{T}}$$

Two-group calculation of neutron moderation

What is the physical meaning of the thermal diffusion area?

The thermal diffusion area L_T^2 is defined as

$$L_T^2 = \frac{D_T}{\Sigma_{aT}}$$

where T denotes averaging over the thermal group.

The thermal diffusion area equals one-sixth the average of the square of the straight distance from the point where the neuton becomes thermal to the point where it is absorbed.

What is the physical meaning of the neutron age in thermal reactors?

In two-group diffusion approximation, the neutron age τ is defined as

$$\tau = \frac{D_1}{\Sigma_{1\to 2}}$$

The neutron age equals one-sixth the average of the square of the straight distance from the point where a fast neutron is emitted to the point where it becomes thermal.

One-group reactor equation

One-group reactor equation

What is the one-group reactor equation good for?

The solution of one-group reactor equation can provide the (approximate) steady-state spatial distribution of the (group) neutron flux in a fissile system and help to:

- find the size of the critical system (when material properties and shape are given),
- or material properties of the critical system (when the size and shape are given).

One-group reactor equation

Let's assume the only neutron source is fission in the system. How the source in the diffusion equation

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + s$$

can be then specified?

The source term s is proportional to the fission rate,

$$s = \nu\Sigma_f\phi$$

where ν is the average number of neutrons produced per fission. Hence, the diffusion equation can be then written as

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi$$

One-group reactor equation

The solution of

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi$$

is time dependent. Can we get the steady-state solution by setting the time derivative to zero?

We want to have time-independent solution, but setting the time derivative to zero does not help since the equation

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = 0$$

cannot have a physical solution for non-critical systems. (Steady-state solution does not exist for non-critical systems.)

Is it possible to tune the size or materials of the system so that it becomes exactly critical (in order to get a physical solution from the above steady-state equation)?

That would be possible only for the simplest systems, such as homogeneous systems with simplest geometric shapes. Any realistic numerical models of reactors, however, cannot be set exactly criticle, in principle.

One-group reactor equation

How can we find a steady-state flux in our system then? (When we accept that we can never achieve a critical model.)

- **We can change the equation so that it always gives a physical solution (with the understanding that the modified equation is applicable only when the system is close to being critical).**
- We can **force** the steady-state solution by changing the equation

$$D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi = 0$$

into

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

This equation is called the **criticality equation**.

One-group reactor equation

Is the k in

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

the familiar multiplication factor?

Yes, the equation can be rearranged as

$$k = \frac{\nu\Sigma_f\phi}{-D\nabla^2\phi + \Sigma_a\phi}$$

where the right-hand side represents a ratio of the rate at which neutrons are being produced to the rate at which neutrons are being lost from the system due to absorption or leakage, which is the definition of the multiplication factor.

One-group reactor equation

Is it now ensured that equation

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

has always a steady-state solution?

Yes. Let's assume three cases:

- The original source term ($\nu\Sigma_f\phi$) is smaller than the absolute value of the combined neutron leakage and absorption terms. Then, k will have to be < 1 so that the source term is artificially increased and the terms will balance. (This system is sub-critical.)
- The original source term is larger than the sum of other terms. Then, k will have to be > 1 so that the source term is artificially decreased. (This system is super-critical.)
- When the original source term balances the other terms exactly then the fission source does not need to be changed ($k = 1$). Such a system is critical.

One-group reactor equation

Note that k is also called k-eigenvalue. Why?

That is because the equation

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

is, in fact, an eigenvalue equation. This can be easily seen when we define a new operator $\mathscr{L}$,

$$\mathscr{L}\phi \equiv \Sigma_a\phi - D\nabla^2\phi$$

Then the above equation can be written as

$$\nu\Sigma_f\phi = k\mathscr{L}\phi$$

or as

$$\mathscr{F}\phi = k\phi$$

where

$$\mathscr{F}\phi = \mathscr{L}^{-1}\nu\Sigma_f\phi$$

One-group reactor equation

Since

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

represents an eigenvalue equation, with k being the eigenvalue and ϕ being the eigenfunction, it must have infinitely many solutions for ϕ-and-k eigenpairs. Which of the solutions is physical?

- The physical solution is the one that has the largest eigenvalue.
- The physical solution is called the **fundamental mode**.
- The eigenfunction of the fundamental mode is real and non-negative everywhere in the system.
- Other modes are called higher modes.
- All higher modes have eigenfunctions that is negative/complex in some part of the system.
- Any solution for ϕ remains the same solution when multiplied by any real number (even negative). So, depending on the solver, a negative flux distribution may be obtained as the physical solution. Just multiply it by -1 then. This would not help with higher-mode solutions since those are always partly positive and partly negative/complex in (various parts of) the system.

One-group reactor equation

What is the trivial solution of the eigenvalue equation?

Note that there is always a trivial solution $\phi = 0$ to the equation

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

which corresponds to a reactor with no neutrons.

- This solution is theoretically possible for all reactors, including super-critical ones.
- In reality, this mode is, however, not achievable due to spontaneous fission that causes that neutrons are always present in the reactor.

One-group reactor equation

What is the geometric buckling?

Let's define the geometric buckling, B^2, as

$$B^2 = \frac{1}{D}\left(\frac{1}{k}\nu\Sigma_f - \Sigma_a\right)$$

Then, equation

$$D\nabla^2\phi - \Sigma_a\phi + \frac{1}{k}\nu\Sigma_f\phi = 0$$

can be rewritten as

$$\nabla^2\phi + B^2\phi = 0$$

or

$$\nabla^2\phi = -B^2\phi$$

which is known as the **one-group reactor equation**.

Note that

$$\nabla^2\phi + B^2\phi = 0$$

is also an eigenvalue equation, with ϕ and B unknown.

One-group reactor equation

Do the material properties of the system need to be known in order to solve the equation

$$\nabla^2\phi = -B^2\phi$$

for ϕ (assuming a homogeneous reactor)?

- No, the material properties of the system do not need to be known.
- So, the ϕ solution is the same no matter what material is in the homogeneous reactor. The material just must include fissile nuclides.

Note that the equation looks the same for all possible shapes of reactors, and none of the variables in the equation is known. How can we then get different solutions for different systems?

We get different solutions due to different boundary conditions.

One-group reactor equation

What is k_∞?

k_∞ is the multiplication factor of the reactor under the condition that there is no neutron leakage. (This would be achieved in an infinite reactor, or in a numerical model with reflective boundary conditions.)

How is the fuel utilization factor f defined in a homogeneous reactor?

- Fuel utilization f is defined as $f = \frac{\Sigma_{aF}}{\Sigma_a}$, where Σ_{aF} is the one-group absorption cross section of the fuel.
- Fuel utilization represents a conditional probability that a neutron gets absorbed in the fuel when it is absorbed in the system.
- We can express the fission source in terms of f as:

$$s = \nu\Sigma_f\phi = \eta\Sigma_{aF}\phi = \eta\frac{\Sigma_{aF}}{\Sigma_a}\Sigma_a\phi = \eta f\Sigma_a\phi$$

- And then, we can express k_∞ in terms of f as:

$$k_\infty = \frac{s}{\Sigma_a\phi} = \frac{\eta f\Sigma_a\phi}{\Sigma_a\phi} = \eta f$$

One-group reactor equation

If solution of the reactor equation doesn't depend on material properties, how can we find material properties of a critical homogeneous reactor of a specific shape and size?

- First we compute the buckling B^2 from the reactor equation (we call the solution the **geometric buckling** B_g^2).
- Then we set $k = 1$ (critical reactor) in the definition of buckling, and evaluate its value (called **material buckling**) as

$$B_m^2 = \frac{1}{D}\left(\nu\Sigma_f - \Sigma_a\right)$$

- For a critical system, we simply choose Σ_f and Σ_a such that the

$$B_g^2 = B_m^2$$

- Note that above equation can also be written in terms of k_∞ as

$$B^2 = \frac{1}{D/\Sigma_a}\left(\nu\Sigma_f/\Sigma_a - 1\right) = \frac{k_\infty - 1}{L^2}$$

One-group reactor equation

Define the non-leakage probability P_L

The non-leakage probability P_L is the probability that a neutron is absorbed (does not leak out of the system),

$$P_L = \frac{\Sigma_a \phi}{\Sigma_a \phi - D\nabla^2 \phi} = \frac{\Sigma_a}{\Sigma_a + DB^2} = \frac{1}{1 + B^2 L^2}$$

Note that the equation

$$B^2 = \frac{k_\infty - 1}{L^2}$$

can be written in a form

$$\frac{k_\infty}{1 + B^2 L^2} = 1$$

and therefore it holds for critical reactors that

$$k_\infty P_L = 1$$

For non-critical reactors, however, it holds that

$$k_\infty P_L = k$$

To distinguish k from k_∞, k is often called **effective** multiplication factor k_{eff}.