Lecture No 07

Title:
Boiling Channel – Part III: Void Fraction and Pressure Drop

Henryk Anglart
Nuclear Reactor Technology Division
Department of Physics, School of Engineering Sciences
KTH

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Outline of the Lecture

- Void fraction in saturated region
  - Drift Flux Model
  - Void distribution in BWR fuel assembly
- Void fraction in subcooled region
- Pressure drop in saturated region
  - Rod bundle correlations
  - Integral friction multipliers using HEM

Void Fraction Calculation

- Prediction of void fraction is important because it affects the moderator density, thus, it affects power generation in nuclear reactors

- Two models are widely used in saturated region:
  - Homogeneous Equilibrium Model (HEM)
  - Drift-Flux Model (DFM)

- Void fraction in subcooled region
  - Onset of Nucleate Boiling (ONB)
  - Onset of Significant Void (OSV)
  - Actual quality model

Void Fraction - HEM (1)

- In HEM, it is assumed that both phases are in the thermodynamic equilibrium and flow with the same speed

- Void fraction is calculated in two steps:
  - first the value of the equilibrium quality (x_e) is found as $x_e(z) \equiv \frac{i(z) - i_f}{i_{fg}}$
  - next the value of void fraction is calculated from the following equation:

$$\alpha(z) = \begin{cases} 0 & \text{for} \quad x_e \le 0 \\ \dfrac{1}{1 + \dfrac{\rho_g}{\rho_f} \cdot \left(\dfrac{1 - x_e(z)}{x_e(z)} \right)} & \text{for} \quad 0 < x_e < 1 \\ 1 & \text{for} \quad x_e \ge 1 \end{cases}$$

Void Fraction - DFM

- Drift flux model allows for:
  - different velocities for both phases
  - thermodynamic equilibrium/non-equilibrium
- The void fraction is calculated from the following relationship:

$$\boxed{\alpha = \frac{J_v}{C_0 J + U_{vj}}}$$

$$J = J_v + J_l$$

$$J_v = \frac{G_v}{\rho_v} = \frac{xG}{\rho_v}$$

$$J_l = \frac{G_l}{\rho_l} = \frac{(1-x)G}{\rho_l}$$

  - here C_0 and U_{vj} are the distribution parameter and the drift velocity, respectively. They are flow-regime dependent.
  - J_v and J are superficial velocities for vapor and for the mixture, respectively

DFM in Thermodynamic Equilibrium

Flow pattern	Distribution parameter	Drift velocity
Bubbly $0 < \alpha \le 0.25$	$C_0 = \begin{cases} 1 - 0.5 p/p_{cr} & D \ge 0.05m \\ 1.2 \quad p/p_{cr} < 0.5 & D < 0.05m \\ 1.4 - 0.4 p/p_{cr} \quad p/p_{cr} \ge 0.5 & \end{cases}$ [1]	$U_{vj} = 1.41 \left(\frac{\sigma g (\rho_l - \rho_v)}{\rho_l^2} \right)^{0.25}$
Slug/churn $0.25 < \alpha \le 0.75$	$C_0 = 1.15$	$U_{vj} = 0.35 \left(\frac{g D (\rho_l - \rho_v)}{\rho_l} \right)^{0.5}$
Annular $0.75 < \alpha \le 0.95$	$C_0 = 1.05$	$U_{vj} = 23 \left(\frac{\mu_l j_l}{\rho_v D_h} \right)^{0.5} \frac{(\rho_l - \rho_v)}{\rho_l}$
Mist $0.95 < \alpha < 1$	$C_0 = 1.0$	$U_{vj} = 1.53 \left(\frac{\sigma g (\rho_l - \rho_v)}{\rho_v^2} \right)^{0.25}$

[1] p_{cr} – critical pressure σ - surface tension $D = D_h$ – hydraulic diameter

Example: Void in BWR Assembly

- **Example**: Calculate the mean void fraction at the exit of a BWR fuel assembly using Drift Flux Model. Given:

Flow area: (A)	$23.44 * 10^{-4}$ m^2
Hydraulic diameter: (D_h)	11.5 mm
Mass flux: (G)	1770 kg/m^2.s
Pressure: (p)	7 MPa
Inlet subcooling:(ΔT_{sub})	10 K
Total thermal power: (q)	2.3 MW

Exit specific enthalpy is found from the energy balance: $i_{ex} = i_{in} + q/(G*A)$
$i_{in} = i(p, T_{sat} - \Delta T_{sub}) = 1214.5$ kJ/kg

Exit quality is found as:
$x = x_{ex} = (i_{ex} - i_f)/i_{fg} = 0.3332$

Example: Void in BWR Assembly

- **Solution**: We guess that the flow conditions correspond to annular flow, for which we find:

$$C_0 = 1.05 \qquad U_{vj} = 23\left(\frac{\mu_f J_f}{\rho_g D_h}\right)^{0.5} \frac{\left(\rho_f - \rho_g\right)}{\rho_f}$$

From water property tables we get: ρ_f = 739.7 kg/m^3; ρ_g = 36.5 kg/m^3; μ_f = 9.13 10^{-5} Pa.s; J_f = (1-x)G/ρ_f = 1.596 m/s; Thus

$$U_{vj} = 23\left(\frac{9.13\cdot 10^{-5}\cdot 1.596}{36.5\cdot 0.0115}\right)^{0.5} \frac{\left(739.7 - 36.5\right)}{739.7} = 0.407 \text{ m/s}$$

Example: Void in BWR Assembly

- Superficial velocity of vapor is found as: $J_g = x*G/\rho_g$ = 16.15 m/s

- The void fraction is now found as

$$\alpha = \frac{J_g}{C_0 J + U_{vj}} = \frac{16.15}{1.05(16.15+1.60)+0.407} = 0.848$$

As can be seen, this high void fraction corresponds to annular flow.

Example: Void in BWR Assembly

- **Example:** Calculate the mean void fraction at 2 m distance from the inlet in a BWR fuel assembly using Drift Flux model and assuming the cosine axial power distribution. Given: H=3.66 m, d = 10 cm

Flow area:	$23.44 * 10^{-4}$ m^2
Mass flux:	1770 kg/m^2.s
Ref. Pressure:	7 MPa
Inlet subcooling:	10 K
Total thermal power:	2.25 MW
Hydraulic diameter:	11.5 mm

(i_{in} = 1214.5 kJ/kg; i_{fg} = 1505.1 kJ/kg; i_f = 1267.4 kJ/kg)

Example: Void in BWR Assembly

- **Solution:** The heat flux in the bundle is distributed as:

$$q''(z) = q_0'' \cdot \cos\left(\frac{\pi z}{\tilde{H}}\right)$$

Thus the total bundle power can be found as:

$$q = q_0'' \cdot P_H \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{\tilde{H}}\right) dz = q_0'' \cdot \frac{2 P_H \tilde{H}}{\pi} \sin\left(\frac{\pi H}{2\tilde{H}}\right)$$

thus

$$\frac{q\pi}{2 P_H \tilde{H} \sin\left(\frac{\pi H}{2\tilde{H}}\right)} = q_0''$$

and

$$q''(z) = \frac{q\pi}{2 P_H \tilde{H} \sin\left(\frac{\pi H}{2\tilde{H}}\right)} \cdot \cos\left(\frac{\pi z}{\tilde{H}}\right)$$

Example: Void in BWR Assembly

- **Solution:** Energy balance up to 2 m from the inlet (z = 2-H/2 = 0.17 m):

$$q_{2-H/2} = \int_{-H/2}^{2-H/2} P_H q''(z)dz = \frac{q\pi}{2\tilde{H}\sin\left(\frac{\pi H}{2\tilde{H}}\right)} \cdot \int_{-H/2}^{2-H/2} \cos\left(\frac{\pi z}{\tilde{H}}\right)dz =$$

$$\frac{q}{2\sin\left(\frac{\pi H}{2\tilde{H}}\right)}\left[\sin\left(\frac{2-H/2}{\tilde{H}}\pi\right)+\sin\left(\frac{H\pi}{2\tilde{H}}\right)\right]$$

substituting data gives: $q_{2-H/2} = 1.28\, MW$

Example: Void in BWR Assembly

- **Solution:** thermodynamic equilibrium quality at 2 m from inlet:

$$i_{2-H/2} = i_{in} + \frac{q_{2-H/2}}{GA} = 1.523\,MJ/kg$$

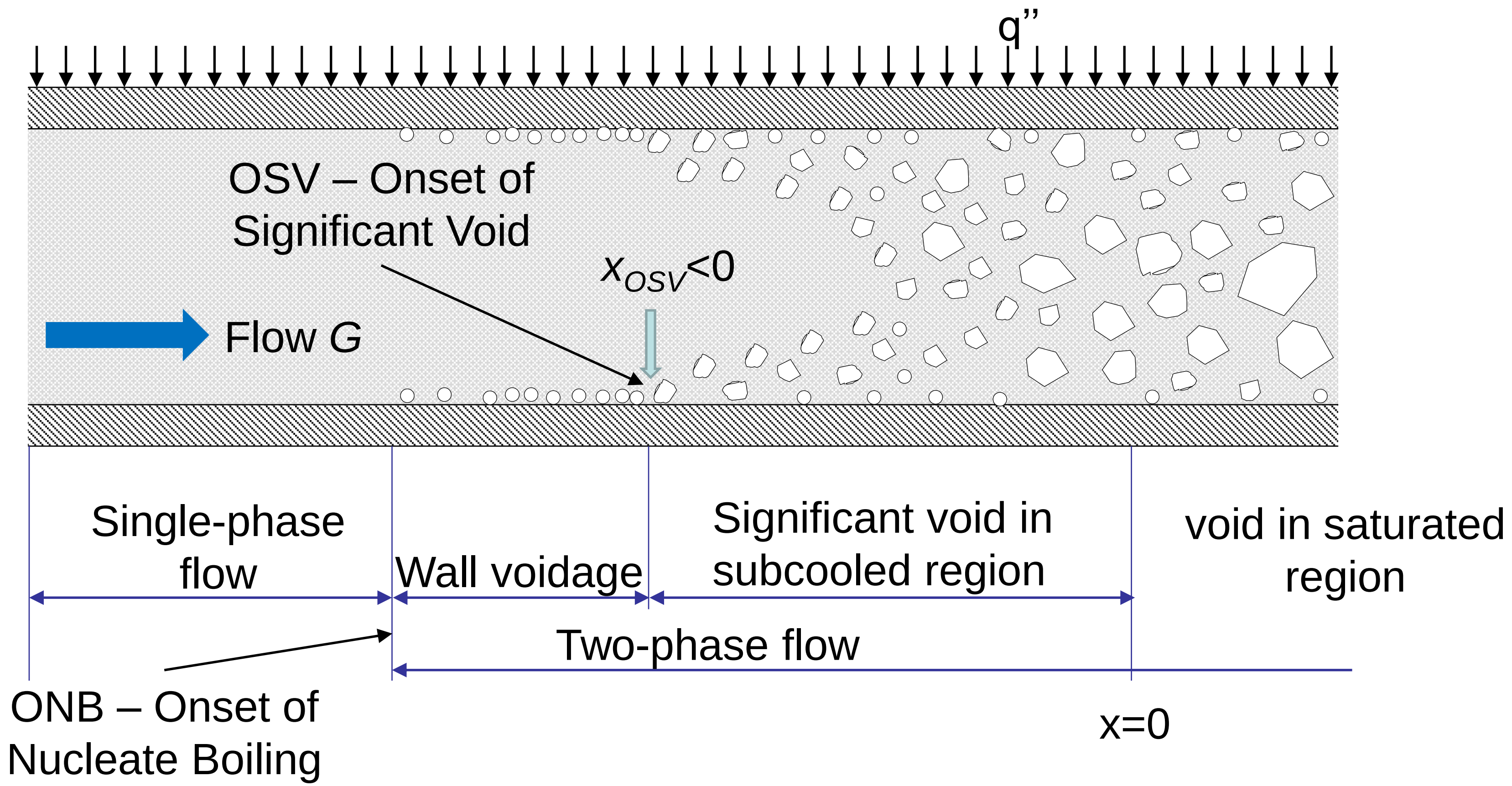
$$x_{2-H/2} = \frac{i_{2-H/2} - i_f}{i_{fg}} = 0.170$$

We guess annular flow and take corresponding values of C_0 and U_{vj} : C_0=1.05; U_{vj}=0.454 m/s, thus the void is found: α = 73.6 %. As can be seen this corresponds to slug/churn flow and iteration is needed

Example: Void in BWR Assembly

- **Solution:** using the slug/churn model we get α = 67.5 %, thus this is taken as the answer.

Void Fraction in Subcooled Region



Void Fraction – Subcooled Boiling (1)

- It can be assumed that void fraction is negligible up to the Onset of Significant Void (OSV) point.
- This point occurs at the location, where equilibrium quality becomes (Saha-Zuber model):

$$x_{e,OSV} = \begin{cases} -0.0022 \dfrac{q'' \cdot D_h \cdot c_{pf}}{i_{fg} \cdot \lambda_f} & \text{for} \quad \mathrm{Pe} < 70000 \\ -154 \dfrac{q''}{G \cdot i_{fg}} & \text{for} \quad \mathrm{Pe} \geq 70000 \end{cases}$$

  – here Pe is the Peclet number, defined as:

$$\mathrm{Pe} = \mathrm{Re} \cdot \mathrm{Pr} = \frac{G \cdot D_h \cdot c_{pf}}{\lambda_f}$$

q'' – heat flux, W/m^2
D_h – hydraulic diameter, m
c_{pf} – fluid spec. heat, J/kgK
λ_f – thermal conduct. W/mK

Void Fraction – Subcooled Boiling (2)

- The actual quality is approximated as (Levy's model):

$$x_a(z) = x_e(z) - x_e(z_{OSV}) \cdot e^{\frac{x_e(z)}{x_e(z_{OSV})} - 1}$$

- The void fraction is then found as:

$$\alpha = \frac{J_v}{C_0 J + U_{vj}}$$

$$J_v = \frac{x_a G}{\rho_g}$$ superficial velocity of vapour

$$J_l = \frac{(1 - x_a) G}{\rho_f}$$ Superficial velocity of liquid

  – where $C_0 = \beta \left[1 + \left(\frac{1}{\beta} \right)^b \right]$

$$\beta = \frac{1}{1 + \frac{\rho_g}{\rho_f} \frac{1 - x_a(z)}{x_a(z)}} \qquad b = \left(\frac{\rho_g}{\rho_f} \right)^{0.1} \qquad U_{vj} = 2.9 \left(\frac{\sigma g (\rho_f - \rho_g)}{\rho_f^2} \right)^{0.25}$$

σ – surface tension, N/m

Henryk Anglart
Nuclear Reactor Technology Division
Department of Physics, KTH

Example 5

- Calculate the void fraction at the exit from a PWR subchannel and find the location of z_{OSV} and z_{SUB}

- Given:
  pressure 15.5 MPa (everywhere the same), inlet temperature T_{in} = 300 °C, uniform heat flux 850 kW/m^2, rod diameter d_r=9.4 mm, fuel rod pitch 12.5 mm, length of fuel assembly H=3.67 m

Example 5

- SOLUTION: we find first z_{SUB} from the energy balance as:

$$z_{SUB} = \frac{GA_{sch}\left(i_f - i_{in}\right)}{q'' P_H} = 3.935\,\mathrm{m} > L$$

- Thus at the exit there is subcooled water. We find the exit quality as:

$$\underbrace{\frac{i_{ex} - i_f}{i_{fg}}}_{x_{ex}} = \underbrace{\frac{i_{in} - i_f}{i_{fg}}}_{x_{in} \approx -0.302} + \underbrace{\frac{q'' P_H H}{GA_{sch} i_{fg}}}_{\Delta x \approx 0.282} = -0.0183$$

- Now we need to find the Peclet number: $\mathrm{Pe} = \dfrac{G \cdot D_h \cdot c_{pf}}{\lambda_f} = 9.1 \cdot 10^5$

Example 5

- Since Pe > 70000, we find x_{OSV} as:

$$x_{e,OSV} = -154 \frac{q''}{G \cdot i_{fg}} = -0.0348$$

- Now from the energy balance we find the location of the OSV point as:

$$x_{e,OSV} = x_{in} + \frac{q'' P_H z_{OSV}}{G A_{sch} i_{fg}} \Rightarrow z_{OSV} = \frac{G A_{sch} i_{fg}}{q'' P_H} \left(x_{e,OSV} - x_{in} \right) \approx 3.482 \text{ m}$$

- Thus significant void starts at about 19 cm upstream of the exit

Example 5

- We will now calculate the actual quality at the exit from the subchannel:

$$x_a(H) = x_e(H) - x_e(z_{OSV}) \cdot e^{\frac{x_e(H)}{x_e(z_{OSV})} - 1}$$

- or:

$$x_{a,ex} = x_{e,ex} - x_{e,OSV} \cdot e^{\frac{x_{e,ex}}{x_{e,OSV}} - 1} =$$

$$-0.0183 + 0.0351 \cdot e^{\frac{-0.0183}{-0.0351} - 1} \approx 0.003366$$

- Thus the actual quality at the exit from the subchannel is about 0.003366
- Using the expressions for subcooled void: $\alpha_{ex} \approx 3.3\%$

Pressure Drop in Two-Phase Flows

- Steady-state momentum equation for a homogeneous two-phase mixture flow in a channel can be written as,

$$\boxed{-\frac{dp}{dz} = \left(\frac{dp}{dz}\right)_w + \rho_m g \sin\varphi + \frac{1}{A}\frac{d}{dz}\left(\frac{G^2 A}{\rho_M}\right)}$$

- Where two definitions of mixture density are introduced:
  - Mixture static density $$\rho_m = \sum_k \rho_k \alpha_k$$
  - Mixture dynamic density $$\rho_M = \left(\sum_k \frac{x_k^2}{\rho_k \alpha_k}\right)^{-1}$$

Local Pressure Loss in Two-Phase Flows

- Local pressure losses in two-phase flows are calculated as:

$$-\Delta p_{loc} = \phi_{lo,d}^2 \xi \frac{G^2}{2\rho_f}$$

Here:

G - total mass flux, kg/m^2.s

ξ - local (single-phase) loss coefficient

$$\phi_{lo,d}^2 = \left[1 + \left(\frac{\rho_f}{\rho_g} - 1\right)x\right]$$ - HEM local two-phase multipl.

Friction pressure loss in two-phase flows

- It can be shown that the ratio of two-phase friction loss to single-phase friction loss is as follows,

$$\left(\frac{dp}{dz}\right)_{w,tp} \Big/ \left(\frac{dp}{dz}\right)_{w,lo} = \frac{C_{f,tp}}{C_{f,lo}} \frac{\rho_l}{\rho_m}$$

- The above ratio is called a two-phase friction multiplier and is as follows

$$\phi_{lo}^2 = \left(\frac{dp}{dz}\right)_{w,tp} \Big/ \left(\frac{dp}{dz}\right)_{w,lo} = \frac{C_{f,tp}}{C_{f,lo}} \frac{\rho_l}{\rho_m}$$

- It should be noted that it is a local variable

Two-Phase Friction Multiplier using HEM

- For Homogeneous Equilibrium Model, it can be shown that the two-phase friction multiplier is the following function of the local equilibrium quality:

$$\phi_{lo}^{2}=\left[1+\left(\frac{\mu_f}{\mu_g}-1\right)x\right]^{-0.25}\left[1+\left(\frac{\rho_f}{\rho_g}-1\right)x\right]$$

- where it is assumed that mixture viscosity is given as:

$$\frac{1}{\mu_m}=\frac{x}{\mu_g}+\frac{1-x}{\mu_f}$$

- it should be noted that other models of mixture viscosity are used as well (see Compendium in Thermal-Hydraulics)

Rod Bundle Correlations for ϕ_{lo}^2

- Local two-phase friction multiplier in general depends on local conditions (pressure, mass flux, heat flux) and geometry (pipe, bundle)
- For a rod bundle geometry the following correlation has been obtained (FRIGG)

$$\phi_{lo}^2 = 1 + (2234 - 0.348G)\left(\frac{x}{p}\right)^{0.96}$$

x –quality
p – pressure (bar)
G – mass flux (kg/m^{2}s)

- To capture the effect of heating:

$$\frac{\left(\phi_{lo}^2\right)_{diabatic}}{\left(\phi_{lo}^2\right)_{adiabatic}} = 1 + C\left(\frac{q''}{G}\right)^{0.7}$$

C – constant coefficient
q´´ - heat flux (W/m^2)
G – mass flux (kg/m^{2}s)

EPRI Correlation for ϕ_{lo}^2

$$\phi_{lo}^2 = \left[1 + x\left(\frac{\rho_f}{\rho_g} - 1\right)C\right]$$

$$C = \begin{cases} 1.02x^{-0.175}G_R^{-0.45} & \text{for} \quad p > 4.137\,\text{MPa} \\ 0.357(1+p_R)x^{-0.175}G_R^{-0.45} & \text{for} \quad 2.068 < p \le 4.137\,\text{MPa} \end{cases}$$

$$p_R = \frac{p}{p_{cr}};\, G_R = \frac{G}{1356.2}$$

x – equilibrium quality
p – pressure (Pa)
G – mass flux (kg/m^{2}s)
p_{cr} – critical pressure (22.1 MPa)

Parameter range: 2.068 < p < 8.963 MPa; 0 < *x* <1; 475 < *G* < 4475 kg/m^{2}s; 5.08 < *d* < 15.24 mm; 127 < *L* < 2540 mm; geometry: round tubes and vertical upflow; based on 1533 experimental points; RMS error: 9.7%

Mean Value of ϕ_{lo}^2 Over Channel Length

- Integration of ϕ_{lo}^2 along a channel length gives

$$r_3 = \frac{1}{L}\int_0^L \phi_{lo}^2 dz \qquad \phi_{lo}^2 = \left[1+\left(\frac{\mu_f}{\mu_g}-1\right)x\right]^{-0.25}\left[1+\left(\frac{\rho_f}{\rho_g}-1\right)x\right]$$

- The integral to calculate r_3 is thus a function of the quality distribution along the channel.

- In particular, if x = const (unheated channel):

$$r_3 = \phi_{lo}^2$$

Enthalpy and Quality in Heated Channel

- For heated channel, we have:

$$di = \frac{q''(z)P_H dz}{W} \Rightarrow d\left(\frac{i - i_f}{i_{fg}}\right) \equiv dx = \frac{q''(z)P_H dz}{W i_{fg}}$$

thus, assuming z = 0 at the inlet:

$$x(z) - x_{in} = \frac{P_H}{W i_{fg}} \int_0^z q''(z')dz'$$

For uniformly heated channel:

$$x(z) = x_{in} + \frac{P_H q''}{W i_{fg}} z$$

Total Pressure Drop in Boiling Channel

- Integration of the momentum eq. gives the total pressure drop for two-phase flows in channel with length L as:

$$-\Delta p = \underbrace{r_3 C_{f,lo} \frac{4L}{D} \frac{G^2}{2\rho_f}}_{friction} + \underbrace{r_4 L \rho_f g \sin\varphi}_{gravity} + \underbrace{r_2 \frac{G^2}{\rho_f}}_{acceleration} + \underbrace{\left(\sum_{i=1}^{N} \phi_{lo,d,i}^2 \xi_i \right) \frac{G^2}{2\rho_f}}_{local}$$

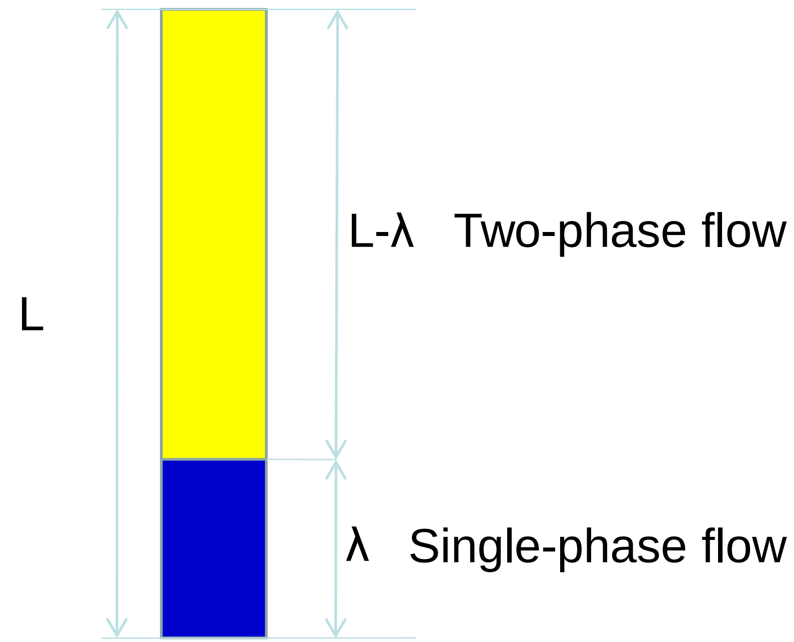
- where:
  - friction multiplier: $r_3 = \frac{1}{L} \int_0^L \phi_{lo}^2 dz$
  - gravity multiplier: $r_4 = \frac{1}{L\rho_f} \int_0^L \left[\alpha\rho_g + (1-\alpha)\rho_f \right] dz$
  - acceleration multiplier: $r_2 \equiv \rho_f \int_0^L \frac{d}{dz} \left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f} \right] dz = \left[\frac{x^2 \rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)} \right]_{ex} - \left[\frac{x^2 \rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)} \right]_{in}$

Friction Loss in BWR Fuel Assembly

- Thus to find friction pressure drop in heated fuel assembly:
  - Find the location of the onset of two-phase flow. If HEM is used, it will be at location where x = 0
  - Let $z = \lambda = z_{SUB}$ where x = 0

$$-\Delta p_{fric} = -\int_0^L \left(\frac{dp}{dz}\right)_{fric} dz =$$

$$-\int_0^\lambda \left(\frac{dp}{dz}\right)_{fric} dz - \int_\lambda^L \left(\frac{dp}{dz}\right)_{fric} dz$$



Friction Loss in BWR Fuel Assembly

- Thus: $$-\Delta p_{fric} = \left(\frac{4C_f \lambda}{D_h} + \frac{4C_{f,lo}}{D_h} \int_\lambda^L \phi_{lo}^2 dz \right) \frac{G^2}{2\rho_f} = \left[\frac{4C_f \lambda}{D_h} + r_3 \frac{4C_{f,lo}(L-\lambda)}{D_h} \right] \frac{G^2}{2\rho_f}$$

- where $$r_3 = \frac{1}{L-\lambda} \int_\lambda^L \phi_{lo}^2 dz$$

Assuming uniform power distributions with q”=const

$$r_3 = \int_0^1 \frac{1 + x_{ex}\left(\frac{\rho_f}{\rho_g} - 1 \right)\zeta}{\left[1 + x_{ex}\left(\frac{\mu_f}{\mu_g} - 1 \right)\zeta \right]^{0.25}} d\zeta$$

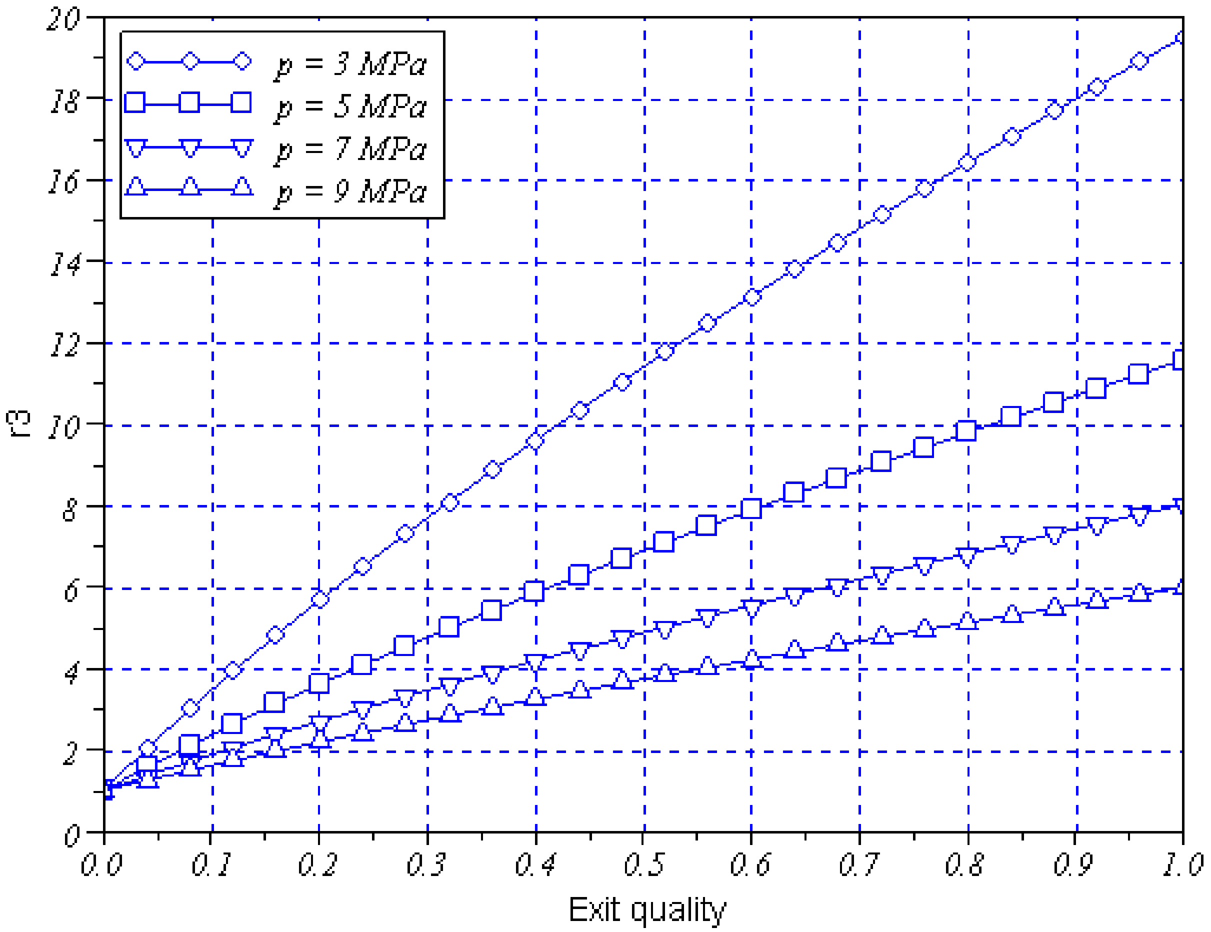
where $$x_{ex} = x_{in} + \frac{q'' P_H}{W i_{fg}} L$$

is the exit quality and x_{in} is the inlet quality

Integral r_3 Multiplier

$$r_3 = \int_0^1 \frac{1 + x_{ex}\left(\frac{\rho_f}{\rho_g} - 1\right)\zeta}{\left[1 + x_{ex}\left(\frac{\mu_f}{\mu_g} - 1\right)\zeta\right]^{0.25}} d\zeta$$

This graph can be used to find the value of the r_3 multiplier for known exit quality and system pressure in uniformly heated channel where inlet quality is zero

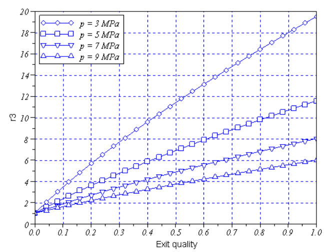


BWR Fuel Assembly Friction Losses

- **Summary:** to find the friction pressure drop in uniformly heated channel, take the following steps:
  - find the single phase flow length λ
  - find C_f and pressure drop in single-phase region as:

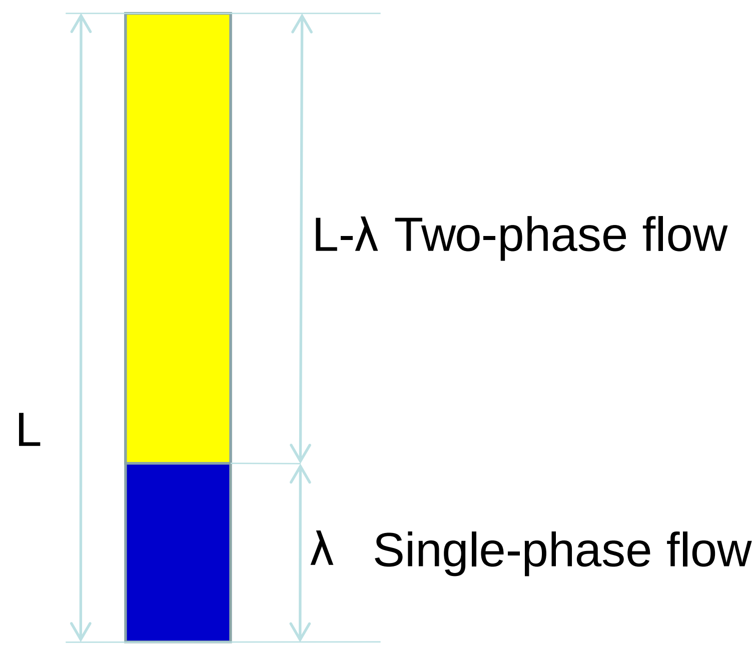
$$-\Delta p_{fric,sp} = \frac{4C_f\lambda}{D_h}\frac{G^2}{2\rho_f}$$

  - find exit quality x_{ex} from energy balance
  - Find r_3 from the plot



  - find two-phase pressure drop:

$$-\Delta p_{fric,tp} = r_3\frac{4C_{f,lo}(L-\lambda)}{D_h}\frac{G^2}{2\rho_f}$$



Gravity Pressure Drop

- The gravity pressure drop multiplier is given as:

$$r_4 = \frac{1}{L\rho_f}\int_0^L \left[\alpha\rho_g + (1-\alpha)\rho_f\right]dz$$

where using HEM

the local void fraction is obtained as:

$$\alpha = \frac{1}{1+\frac{\rho_g}{\rho_f}\cdot\left(\frac{1-x}{x}\right)} \quad for \quad 0<x<1$$

The integral to calculate r_4 is thus a function of the quality distribution along the channel.

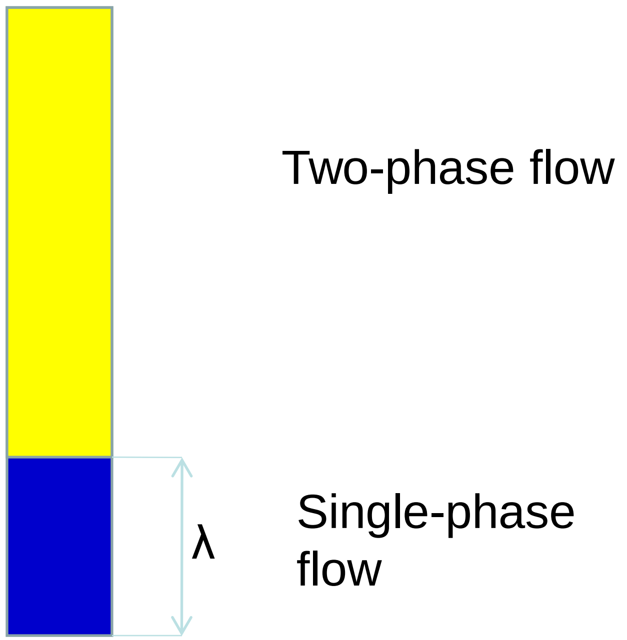
Gravity Pressure Drop in BWR Fuel Assembly

- Thus to find the gravity pressure drop in a heated fuel assembly:

  - Find the location of the onset of two-phase flow. If HEM is used, it will be at location where x = 0
  - Let z = λ where x = 0

$$
-\Delta p_{grav} = -\int_0^L \left(\frac{dp}{dz}\right)_{grav} dz =
$$

$$
-\int_0^\lambda \left(\frac{dp}{dz}\right)_{grav} dz - \int_\lambda^L \left(\frac{dp}{dz}\right)_{grav} dz
$$



Gravity Pressure Drop in BWR Fuel Assembly

- Thus:

$$-\Delta p_{grav} = \int_0^{\lambda} \rho_l g \sin\varphi dz + \int_{\lambda}^{L} \left[\alpha\rho_g + (1-\alpha)\rho_f\right] g \sin\varphi dz =$$

$$\lambda\rho_l g \sin\varphi + r_4 (L-\lambda)\rho_f \sin\varphi$$

where:

$$r_4 = \frac{1}{(L-\lambda)\rho_f} \int_{\lambda}^{L} \left[\alpha\rho_g + (1-\alpha)\rho_f\right] dz$$

assuming uniform power distribution:

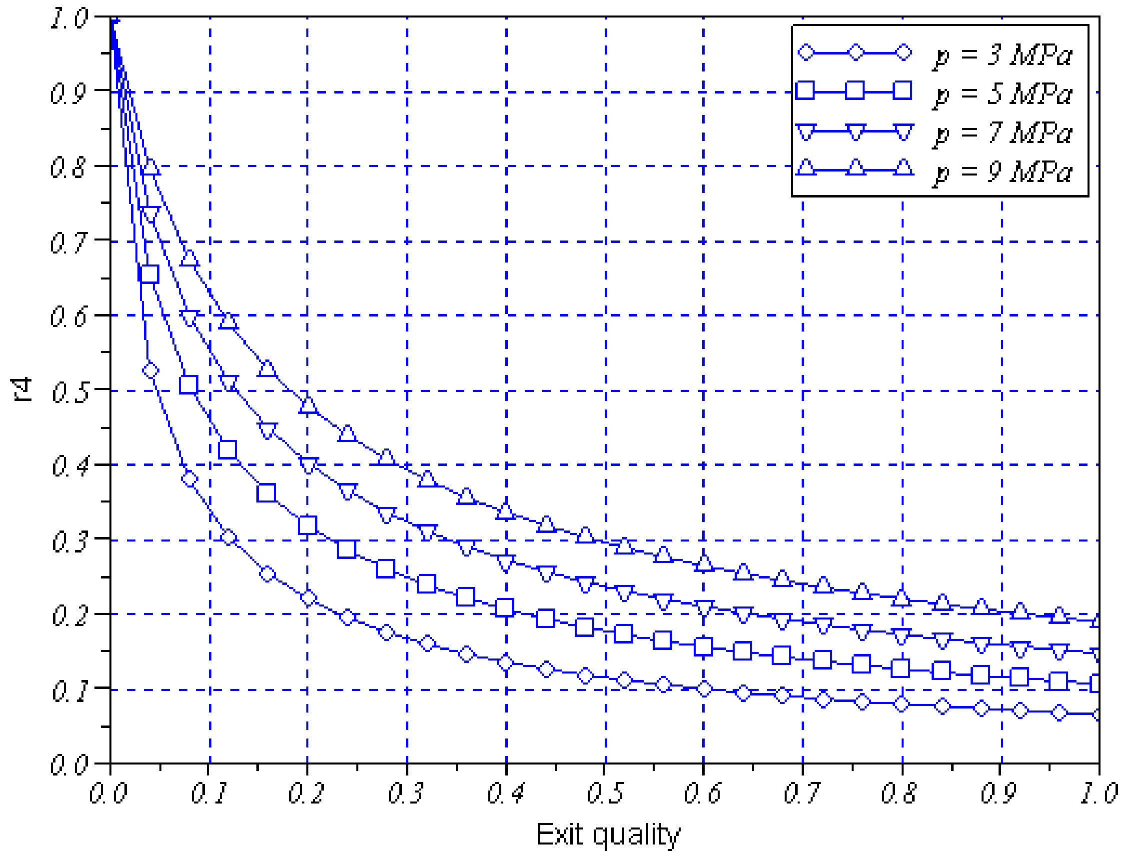
$$r_4 = 1 - x_{ex} \int_0^1 \frac{\zeta}{\rho_g / (\rho_f - \rho_g) + x_{ex}\zeta} d\zeta$$

where x_{ex} is the exit quality

Gravity Pressure Drop Multiplier

$$r_4 = 1 - x_{ex}\int_0^1 \frac{\zeta}{\rho_g/\left(\rho_f - \rho_g\right) + x_{ex}}$$

This graph can be used to find the value of the r_4 multiplier for known exit quality and system pressure in uniformly heated channel and x_{in}=0



The gravity pressure drop is then found as:

$$-\Delta p_{grav} = \lambda \rho_f g \sin\varphi + r_4\left(L - \lambda\right)\rho_f \sin\varphi$$

Acceleration Pressure Drop in Two-Phase Flows

For channel with subcooled water at inlet, the acceleration multiplier can be calculated as:

$$r_2 \equiv \rho_f \int_0^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]dz = \cancelto{0}{\rho_f \int_0^\lambda \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]dz} +$$

$$\rho_f \int_\lambda^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]dz = \left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - \left[\underbrace{\frac{x^2\rho_f}{\alpha\rho_g}}_{0} + \underbrace{\frac{(1-x)^2}{(1-\alpha)}}_{1}\right]_\lambda =$$

$$\left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - 1$$

Thus: $$r_2 = \left[\frac{x^2\rho_f}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - 1$$

Acceleration Pressure Drop Multiplier

$$r_2 = \left[\frac{x^2 \rho_f}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)} \right]_{ex} - 1$$

This graph can be used to find the value of the r_2 multiplier for known exit quality and system pressure in a heated channel

The acceleration pressure drop is then found as:

$$-\Delta p_{acc} = r_2 \frac{G^2}{\rho_f}$$