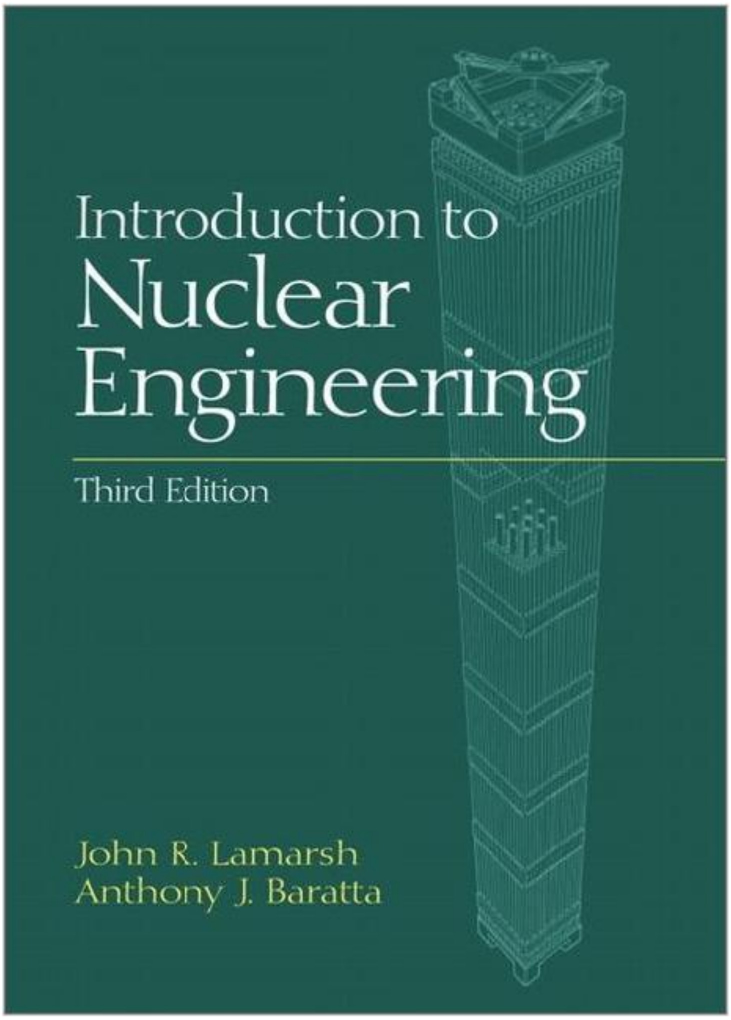
Reference Solution



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Chapter 2

Atomic and Nuclear Physics

1. How many neutrons and protons are there in the nuclei of the following atoms:

(a) ^{7}Li,

[Sol] The nucleus is composed of protons and neutrons and nuclide is expressed by $^{A}_{Z}X$. The total number of nucleon, that is protons and neutrons, is equal to A(mass number) = Z(atomic number) + N (neutron number) and atomic number is represented as the proton number. So proton number of ^{7}Li is 3(atomic number) and the number of neutrons is 4

(b) ^{24}Mg,

[Sol] 24(mass number) = 12(atomic number) + neutron number. So, the neutron number is 12.

(c) ^{135}Xe,

[Sol] 135(mass number) = 54(atomic number) + neutron number. So, the neutron number is 81.

(d) ^{209}Bi,

[Sol] 209(mass number) = 83(atomic number) + neutron number. So, the neutron number is 126.

(e) ^{222}Rn,

[Sol] 222(mass number) = 86(atomic number) + neutron number. So, the neutron number is 139.

2. The atomic weight of ^{59}Co is 58.93319. How many times heavier is ^{59}Co than ^{12}C?

[Sol] $M(^{12}C)$ = 12.00000 amu , $M(^{59}Co)$ = 58.93319 amu
$M(^{59}Co) / M(^{12}C)$ = 58.93319/12.00000 = 4.911 times.

3. How many atoms are there in 10g of ^{12}C?

[Sol] 12g : 0.6022×10^{-24} atoms = 10g : X

X = 10g $\times$ 0.6022×10^{-24} atoms / 12g = 5.0183×10^{23} atoms

4. Using the data given next and in example 2.2, compute the molecular weights of
(a) H_2 gas (b) H_2O (c) H_2O_2

Isotope	Abundance, a/o	Atomic weight
^{1}H	99.985	1.007825
^{2}H	0.015	2.01410

[Sol] (a) H_2 gas

$$M(H_2\ gas) = \{ (99.985 \times 1.007825)/100 + (0.015 \times 2.01410)/100\} \times 2$$
$$= 2.015952 \text{ amu}$$

(b) H_2O

$$M(H_2O) = M(H_2) + M(O)$$
$$= 2.015952 + 0.01(99.759 \times 15.99492 + 0.037 \times 16.99913 + 0.204 \times 17.99916)$$
$$= 18.05532 \text{ amu}$$

(c) H_2O_2

$$M(H_2O_2) = M(H_2) + 2M(O)$$
$$= 2.015952 + 31.99876$$
$$= 34.014712 \text{ amu}$$

5. When H_2 gas is formed from naturally occurring hydrogen, what percentages of the molecules have molecular weights of approximately 2, 3, and 4?

[Sol] (a) In the case of being molecular weights of 2

$^1H + ^1H \rightarrow H_2$

0.99985 x 0.99985 = 0.9991 = 99.91 %

(b) In the case of being molecular weights of 3

$^1H + ^2H \rightarrow H_2$

0.99985 x 0.00015 = 0.000149977 = 0.0149977%

(c) In the case of being molecular weights of4

$^1H + ^3H \rightarrow H_2$

$^2H + ^2H \rightarrow H_2$

0.99985 x 0 + 0.00015 x 0.00015 = 0.00000002%

6. Natural uranium is composed of three isotopes, ^{234}U, ^{235}U, ^{238}U. Their abundances and atomic weights are given in the following table. Compute the atomic weight of natural uranium.

Isotope	Abundance, %	Atomic weight
^{234}U	0.0057	234.0409
^{235}U	0.72	235.0439
^{238}U	99.27	238.0508

[Sol] M(U) = 0.01(0.0057 x 234.0409 + 0.72 x 235.0439 + 99.27 x 238.0508)

= 238.0187 amu.

7. A beaker contains 50g of ordinary (i.e., naturally occurring) water.
(a) How many moles of water are present?
(b) How many hydrogen atoms?
(c) How many deuterium atoms?

[Sol] (a) M(H_2O) = 18.015g/mole

50g/18.0015g·mole^{-1} = 2.775mole

(b) 2.775mole x 0.6022 x 10^{24} atoms/mole x 2

= 3.3422 x 10^{24}

(c) Abundance of ^2H is about 0.0151%

3.3422 x 10^{24} x 0.01 x 0.0151 =5.046 x 10^{20} atoms

8. The glass in Example 2.1 has and inside diameter of 7.5 cm. How high does the water stand in the glass?

[Sol] N(H) = 6.6 x 10^{24}atoms

N(H_2O) = 0.5 N(H)

= 3.3 x 10^{24}atoms

3.3 x 10^{24}atoms / (0.6022 x 10^{24}atoms· mole^{-1}) = 5.4799 mole

ρ(density) = m(mass)/v(volume)

m = 5.4799 mole x 18g/mole = 98.6382g

ρ(H_2O) = 98.6382g/ (3.75^2 x π x H cm^3)

H = 2.2327 cm

9. Compute the mass of a proton in amu.

[Sol] The mass of proton is 1.67261 x 10-^{24}g

Mp = 1.67261 x 10^{-24}g x (amu/1.66053 x 10-^{24}g)
= 1.007277 amu

10. Calculate the mass of a neutral atom of ^{235}U
(a) in amu
(b) in grams

[Sol] M(^{235}U)=235.0439 amu = 235.0439 amu x 1.66053 x 10^{-24}g/amu
= 3.903 x 10^{-22}g

11. Show that 1 amu is numerically equal to the reciprocal of N_A

[Sol] 1 amu = 1/12 x m(12C)
= 1/12 x 1.99264 x 10^{-22} g
= 1.66053 x 10^{-24} g
= 1/(1/1.66053 x 10^{-24}) g
= 1 / 6.02216 x 10^{23} g
= 1/ N_A g

12. Using Eq. (2.3), estimate the radius of the nucleus of ^{238}U. Roughly what fraction of the ^{238}U atom is taken by the nucleus?

[sol] Eq. (2.3) $R_1 = 1.25\text{fm} \times A^{1/3} = 1.25 \times 10^{-13} \times 238^{1/3} = 7.7464 \times 10^{-13}$ **cm**
R_2**(Radius of atom)** $= 2 \times 10^{-8}$ **cm**

$$\text{Fraction} = \frac{\text{Volume of nucleus}}{\text{Volume of atom}} = \frac{\frac{4}{3}\times\pi\times R1^3}{\frac{4}{3}\times\pi\times R2^3} = \left(\frac{R1}{R2}\right)^3 = 5.8104 \times 10^{-14}$$

13. Using Eq. (2.3), estimate the density of nuclear matter in g/cm^3; in Kg/m^3. Take the mass of each nucleon to be approximately 1.5×10^{-24} g.

[sol] Nuclear matter = ^{A}X
R(Radius of nucleus) $= 1.25\text{fm} \times A^{1/3} = 1.25 \times 10^{-13} \times A^{1/3}$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{1.5\times10^{-24}*A}{\frac{4}{3}\times\pi\times R^3} = \frac{1.5\times10^{-24}*A}{\frac{4}{3}\times\pi\times(1.25\times10^{-13}\times A^{1/3})^3} = 1.8335\times10^{14}\text{g/cm}^3 = 1833.5\times10^{14}\text{kg/m}^3$$

14. The planet earth has a mass of approximately 6×10^{24} kg. If the density of the earth were equal to that of nuclei, how big would the earth be?

[sol] Density = 1833.5×10^{14} kg/m^3(2.13 ans)

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{6\times10^{24}}{1833.5\times10^{14}} = 3.2724\times10^7\text{m}^3 = \frac{4}{3}\times\pi\times R^3$$

R = 198.42m (Radius of earth = 6400km)

15. The compete combustion of 1 kg of bituminous coal releases about 3×10^7 J in heat energy. The conversion of 1 g of mass into energy is equivalent to the burning of how much coal?

[sol] E_c **= Energy of 1 kg of bituminous coal = 3×10^7 J**

$$E_m = m\times c^2 = 1g\times(2.9979\times10^{10})^2 = 8.9874\times10^{20}\text{erg} = 8.9874\times10^{13}\text{J}$$
$$E_m = m_{coal}\times E_c$$
$$m_{coal} = E_m/E_c = 8.9874\times10^{13}/3*10^7 = 2.9958\times10^6\text{kg}$$

16. The fission of the nucleus of ^{235}U releases approximately 200 MeV. How much energy (in kilowatt-hours

and megawatt-days) is released when 1 g of ^{235}U undergoes fission?

[sol] $N_A = 0.6022045\times10^{24}$/mol

$$\text{Number of atoms in } {}^{235}\text{U}(1\text{g}) = \frac{1\text{g}}{235\text{g}} \times N_A = 2.5626\times10^{21}\text{atoms}$$

$$2.5626\times10^{21}\times\frac{200\text{MeV}}{1\text{atm}} = 5.1254\times10^{23}\text{MeV}$$

$$1\text{MeV} = 1.60219\times10^{-19}\text{J}\times\frac{1\text{hr}}{3600\text{s}} = 4.4505\times10^{-23}\text{Whr} = 4.4505\times10^{-20}\text{MWhr}$$

$$5.1254\times10^{23}\text{MeV} = 5.1254\times10^{23}\text{MeV}\times\frac{4.4505\times10^{-20}\text{kWh}}{1\text{MeV}} = 2.281\times10^{4}\ \text{kWhr}$$

$$= 2.281\times10^{4}\text{kWhr}\times\frac{1\text{day}}{24\text{hr}} = 0.95042\text{MWD}$$

17. Compute the neutron-proton mass difference in MeV.

[sol] Neutron : $E_1 = 1.67495\times10^{-24}\text{g}\times(2.9979\times10^{10})^2 = 1.5053\times10^{-3}\text{ergs}$
$= 1.5053\times10^{-10}\text{J} = 939.55\text{MeV}$

Proton : $E_2 = 1.67265\times10^{-24}\text{g}\times(2.9979\times10^{10})^2 = 1.5033\times10^{-3}\text{ergs} = 1.5033\times10^{-10}\text{J}$
$= 938.27\text{MeV}$

Neutron-proton mass difference = 1.28MeV

18. An electron starting from rest is accelerated across a potential difference of 5 million volts.
(a) What is its final kinetic energy?

[sol] $K_E = 1(\text{one electron})\times5\times10^{6}\text{V} = 5\text{MeV}$

(b) What is its total energy?

[sol] $E_{total} = E_{rest} + K_E = 0.5110 + 5 = 5.5110\text{MeV}$

$E_{rest} = 9.1095\times10^{-28}\text{g}\times(2.9979\times10^{10})^2 = 8.1871\times10^{-7}\text{ergs} = 8.1871\times10^{-14}\text{J}$
$= 0.5110\text{MeV}$

(c) What is its final mass?

[sol] $E = mc^2$

$$m = \frac{E}{c^2} = \frac{5.5110\text{MeV}\times1.60219\times10^{-6}\frac{\text{erg}}{\text{MeV}}}{(2.9979\times10^{10})^2} = 9.8245\times10^{-27}\text{g}$$

19. Derive Eq. (2.18).

[sol] Eq. (2.5)

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$m^2 = \frac{m_0^2}{1-v^2/c^2}$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

Eq.(2.18)

$$p = mv = \sqrt{m^2c^2 - m_0^2c^2} = \sqrt{\frac{(mc^2)^2}{c^2} - \frac{(m_0c^2)^2}{c^2}} = \frac{1}{c}\sqrt{E_{total}^2 - E_{rest}^2}$$

20. Show that the speed of any particle, relativistic or nonrelativistic, is given by the following formula:

$v = c\sqrt{1-\frac{E_{rest}^2}{E_{total}^2}}$, where Erest and Etotal are its rest-mass energy and total energy, respectively, and c is the speed of light.

[sol] $E_{total} = mc^2$

$E_{rest} = m_oc^2$

$$\left(\frac{E_{rest}}{E_{total}}\right)^2 = \left(\frac{m_0c^2}{mc^2}\right)^2 = \left(\frac{m_0}{m}\right)^2 = 1 - \frac{v^2}{c^2}$$
$$v^2 = c^2\left(1 - \frac{E_{rest}^2}{E_{total}^2}\right)$$
$$v = c\sqrt{\left(1 - \frac{E_{rest}^2}{E_{total}^2}\right)}$$

21. Using the result derived in Problem 2.20, calculate the speed of a 1-MeV electron, one with a kinetic energy of 1 MeV.

[sol] $E_{rest} = 9.1095 \times 10^{-28}g \times (2.9979 \times 10^{10})^2 = 8.1871 \times 10^{-7}ergs = 8.1871 \times 10^{-14}J$
$= 0.5110MeV$
$K_E = 1Mev$
$E_{total} = E_{rest} + K_E = 0.5110 + 1 = 1.5110MeV$
$$v = c\sqrt{\left(1 - \frac{E_{rest}^2}{E_{total}^2}\right)} = 2.9979 \times 10^{10}\sqrt{\left(1 - \frac{0.5110^2}{1.5110^2}\right)} = 2.8213 \times 10^{10}cm/s$$

22. Compute the wavelengths of 1-MeV
(a) photon

[sol] $\lambda = \frac{1.240\times10^{-6}}{E} = \frac{1.240\times10^{-6}}{1\times10^6 eV} = 1.240 \times 10^{-12}m = 1.240 \times 10^{-10}cm$

(b) neutron.

[sol] $\lambda = \frac{2.860\times10^{-9}}{E} = \frac{2.860\times10^{-9}}{1\times10^6 eV} = 2.860 \times 10^{-15}m = 2.860 \times 10^{-13}cm$

23. Show that the wavelength of a relativistic particle is given by
$$\lambda = \lambda_c\frac{m_e c^2}{\sqrt{E_{total}^2 - E_{rest}^2}},$$
where $\lambda_c = h/m_e c = 2.426 \times 10^{-10}$cm is called the Compton wavelength.

[sol] When body is in motion, it's mass increase relative to an observer : $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ ----- **(Eq 2.5),**

So $E_{total} = mc^2 = \frac{m_0c^2}{\sqrt{1-v^2/c^2}}$.

Relativistic momentum : p = mv = $\frac{m_0 v}{\sqrt{1-v^2/c^2}}$ **----(Eq.2.15)**

So $(pc)^2 = \frac{{m_0}^2v^2c^2}{1-v^2/c^2}$, $\quad {E_{total}}^2 = \frac{{m_0}^2c^4}{1-v^2/c^2}$

$${E_{total}}^2 - p^2c^2 = \frac{{m_0}^2c^4}{1-v^2/c^2} - \frac{{m_0}^2v^2c^2}{1-v^2/c^2} = \frac{{m_0}^2c^4(1-v^2/c^2)}{1-v^2/c^2} = {m_0}^2c^4 = {E_{rest}}^2$$

Rearrange this relation.

$$(pc)^2 = {E_{total}}^2 - {E_{rest}}^2 \quad \text{so} \quad pc = \sqrt{{E_{total}}^2 - {E_{rest}}^2} \text{ --- } (1)$$

$p = \frac{h}{\lambda}$ **, replace p in eq -- (1) =>** $\frac{h}{\lambda}c = \sqrt{{E_{total}}^2 - {E_{rest}}^2}$

so $\lambda = \frac{hc}{\sqrt{{E_{total}}^2 - {E_{rest}}^2}} = \frac{h}{m_0c} \times \frac{m_0c^2}{\sqrt{{E_{total}}^2 - {E_{rest}}^2}} = \lambda_c\frac{m_0c^2}{\sqrt{{E_{total}}^2 - {E_{rest}}^2}}$

$$\therefore \lambda = \lambda_c\frac{m_0c^2}{\sqrt{{E_{total}}^2 - {E_{rest}}^2}}$$

24. Using the formula obtained in Problem 2.23, compute the wavelength of a 1-MeV electron.

[sol] in prob 2.23, $\lambda = \lambda_c\frac{m_e c^2}{\sqrt{E_{total}^2 - E_{rest}^2}},$

$E_{total} = E_{kinetic} + E_{rest}$

$E_{rest} = 9.10954 \times 10^{-31}[kg] \times (2.9979\times 10^{8})^2[(m/sec)^2] \cong 81.87\times 10^{-15}[joule]$
1ev = 1.60219× 10^{-19}joule,
$E_{rest} = 81.87\times 10^{-15} \div 1.60219\times 10^{-19} \cong 510.9950\times 10^{3}[eV] \cong 0.511[MeV]$
$\therefore E_{total} = 1[MeV] + 0.511[MeV] = 1.511[MeV]$
$\lambda = 2.426 \times 10[cm]\frac{0.511[MeV]}{\sqrt{1.511[MeV]^2 - 0.511[MeV]^2}} = 8.71\times 10^{-11}[cm]$

25. An electron moves with a kinetic energy equal to its rest-mass energy. Calculate the electron's
(a) total energy in units of $m_e c^2$

[sol] $E_{kinetic} = E_{total} - E_{rest}$, $E_{kinetic} = E_{rest}$ so $E_{total} = 2E_{rest} = 2m_e c^2$
$\therefore E_{total} = 2 \times 0.511[MeV] = 1.022[MeV]$

(b) mass in units of m_e

[sol] $mc^2 = 2m_e c^2$, $m = 2m_e$
$\therefore m = 2 \times 9.10954 \times 10^{-31} \cong 1.822 \times 10^{-30}[Kg]$

(c) speed in units of c

[sol] $E_{kinetic} = mc^2 - m_e c^2 = m_e c^2\left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right]$ **----- Eq 2.8**
$m = 2m_e$, so $2 = \left[\frac{1}{\sqrt{1-v^2/c^2}}\right]$ => $4 = \frac{1}{1-v^2/c^2}$ => $1 - v^2/c^2 = 1/4$
$\therefore v = \frac{\sqrt{3}}{2}c = \frac{\sqrt{3}}{2} \times 2.9979\times 10^{8} \cong 2.60 \times 10^{8} m/s$

(d) wavelength in units of the Compton wavelength

[sol] $\lambda = \lambda_c \frac{m_0 c^2}{\sqrt{E_{total}^2 - E_{rest}^2}} = \lambda_c \frac{E_{rest}}{\sqrt{(2E_{rest})^2 - E_{rest}^2}} = \lambda_c \frac{\sqrt{3}}{3} = 2.426\times 10^{-10} \times \frac{\sqrt{3}}{3} \cong 1.40 \times 10^{12} cm$

26. According to Eq. (2.20), a photon carries momentum, thus a free atom or nucleus recoils when it emits a photon. The energy of the photon is therefore actually less than the available transition energy(energy between states) by an amount equal to the recoil energy of the radiating system.
(a) Given that E is the energy between two states and E_γ.is the energy of the emitted photon, show that

$$E_\gamma \cong E(1 - \frac{E}{2Mc^2})$$

Where M is the mass of the atom or nucleus

[sol] The momentum of zero mass particle is
p = E/c ----(Eq.2.20)
The momentum of the particle having nonrelativistic energy is
p = $\sqrt{2ME}$ ----(Eq.2.15)

$$\therefore E_\gamma = pc - \frac{p^2}{2M} \cong E - \frac{E^2}{2Mc^2} = E(1 - \frac{E}{2Mc^2})$$

(b) Compute E - E_γ for the transitions from the first excited state of atomic hydrogen at 10.19eV to ground and the first excited state of ^{12}C at 4.43 MeV to ground (see Figs. 2.2 and 2.3)

[sol] i) For Hydrogen

$$E - E_\gamma = \frac{E^2}{2Mc^2}$$
$$= \frac{(10.19[eV])^2}{2\times 1.008\times 10^{-3}[kg]\times(2.997\times 10^{8}[m/s])^2} = \frac{(10.19[eV])^2}{5.6274\times 10^{-14}[Joule]} = \frac{103.84[eV]^2}{5.2674\times 10^{-14}[Joule]\times\frac{1[eV]}{1.60219\times 10^{-19}[Joule]}}$$
$$\cong 3.16 \times 10^{-4}[eV]$$

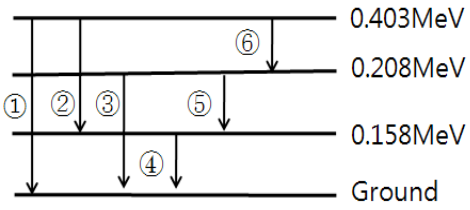
ii) For Carbon

$$\mathbf{E - E_{\gamma}} = \frac{E^2}{2Mc^2}$$

$$= \frac{(4.43\times10^6[\mathrm{eV}])^2}{2\times12.011\times10^{-3[\mathrm{kg}]}\times(2.997\times10^8[{}^{\mathrm{m}}/_{\mathrm{s}}])^2} = \frac{1.962\times10^{13}[\mathrm{eV}]^2}{9.095\times10^{-3}[\mathrm{Joule}]\times\frac{1[\mathrm{eV}]}{1.60219\times10^{-19}[\mathrm{Joule}]}} \cong \mathbf{3.46\times10^{-4}}[\mathbf{eV}]$$

27. The first three excited states of the nucleus of ^{199}Hg are at 0.158 MeV, 0.208MeV, and 0.403 MeV above the ground state. If all transitions between these states and ground occurred, what energy γ-rays would be observed?

[sol]



Possible observed γ-rays are ..

i) $\mathbf{E_1 = 0.403[MeV] - 0 = 0.403[MeV]}$

ii) $\mathbf{E_2 = 0.403[MeV] - 0.158[MeV] = 0.245[MeV]}$

iii) $\mathbf{E_3 = 0.208[MeV] - 0 = 0.208[MeV]}$

iv) $\mathbf{E_4 = 0.158[MeV] - 0 = 0.158[MeV]}$

v) $\mathbf{E_5 = 0.208[MeV] - 0.158[MeV] = 0.050[MeV]}$

vi) $\mathbf{E_6 = 0.403[MeV] - 0.208[MeV] = 0.195[MeV]}$

28. Using the chart of the nuclides, complete the following reactions. If a daughter nucleus is also radioactive, indicate the complete decay chain.

(a) ^{18}N →

[sol] $\mathbf{{}^{18}_{7}N \rightarrow \beta^- \rightarrow {}^{18}_{8}O}$**(stable)**

(b) ^{83}Y →

[sol] $\mathbf{{}^{83}_{39}Y \rightarrow \beta^+ \rightarrow {}^{83}_{38}Sr \rightarrow \beta^+ \rightarrow {}^{83}_{37}Rb \rightarrow \beta^+ \rightarrow {}^{83}_{36}Kr(stable)}$

(c) ^{135}Sb →

[sol] $\mathbf{{}^{135}_{51}Sb \rightarrow \beta^- \rightarrow {}^{135}_{52}Te \rightarrow \beta^- \rightarrow {}^{135}_{53}I \rightarrow \beta^- \rightarrow {}^{135}_{54}Xe \rightarrow \beta^- \rightarrow {}^{135}_{55}Cs \rightarrow \beta^- \rightarrow {}^{135}_{56}Ba(stable)}$

(d) ^{219}Rn →

[sol] $\mathbf{{}^{219}_{86}Rn \rightarrow \alpha \rightarrow {}^{215}_{84}Po \rightarrow \alpha \rightarrow {}^{211}_{82}Pb \rightarrow \beta^- \rightarrow {}^{211}_{83}Bi \rightarrow \alpha \rightarrow {}^{207}_{81}Tl \rightarrow \beta^- \rightarrow {}^{207}_{82}Pb(stable)}$

29. Tritium(^{3}H) decays by negative beta decay with a half-life of 12.26 years. The atomic weight of ^{3}H is 3.016

(a) To what nucleus does ^{3}H decay?

[sol] $\boldsymbol{\beta^-}$ **decay : neutron → proton + $\mathbf{e^-}$+anti-neutrino so it will be cause increase in proton number and same atomic mass So $\boldsymbol{\beta^-}$decay of $\mathbf{{}^3_1H}$, the proton number increase +1 so it will be 2, and same atomic mass 3,**
$\therefore \mathbf{{}^3_1H \rightarrow \beta^- \rightarrow {}^3_2He}$

(b) What is the mass in grams of 1mCi of tritium?

[sol] $\boldsymbol{\alpha(\mathbf{activity}) = \lambda \mathbf{n}(\mathbf{t})}$ **----(Eq.2.23), using Eq.2.26** $\boldsymbol{\lambda} = \frac{\mathbf{ln2}}{\mathbf{T_{1/2}}}$

$$\lambda = \frac{\ln 2}{12.26 \times 365 \times 24 \times 60 \times 60[s]} \cong 1.8 \times 10^{-9}[s^{-}]$$

1mCi = 3.7× 10^7 disintegrations / second = $\lambda n(t)$

Total number of ^3H, n(t)

n(t) = 3.7× $10^7[s^{-1}]/1.8 \times 10^{-9}[s^{-1}]$ ≅2.06× 10^{16}

mass = $\frac{n(t)}{N_{Av}} \times 3.016[g]$=$\frac{2.06\times10^{16}}{6.022045\times10^{23}} \times 3.016 \cong 1.032 \times 10^{-7}[g]$

30. Approximately what mass of $^{90}Sr(T_{1/2} = 28.8yr.)$ has the same activity as 1g of $^{60}Co(T_{\frac{1}{2}} = 5.26yr.)$?

[sol] Activity of 1g of ^{60}Co

$\lambda = \frac{\ln 2}{T_{1/2}}$=$\frac{\ln 2}{5.26\times365\times24\times3600} \cong 4.18 \times 10^{-9}[s^{-1}]$

$n(t)_{1gCo} = \frac{6.022045\times10^{23}}{60[g]} \times 1[g] = 1.003 \times 10^{22}$

$\alpha = \lambda n(t)_{1gCo} = 4.18 \times 10^{-9} \times 1.003 \times 10^{22}$ ≅4.19× $10^{13}[s^{-1}]$

some mass of ^{90}Sr activity same as 1g of Co activity

$\lambda = \frac{\ln 2}{T_{1/2}}$=$\frac{\ln 2}{28.8\times365\times24\times3600} \cong 7.63 \times 10^{-10}[s^{-1}]$

$\alpha = \lambda n(t)_{xgSr} = 4.19 \times 10^{13}[s^{-1}]$, $n(t)_{xgSr}$=$\frac{4.19\times10^{13}}{7.63\times10^{-10}}$ ≅5.50× 10^{22}

$n(t)_{xgSr} = \frac{X}{90} \times N_{Av} = 5.50 \times 10^{22}$ **So, X =** $\frac{5.50\times10^{22}\times90[g]}{6.022045\times10^{23}}$ **=8.2197990088≅ $8.22g$**

$$\therefore 8.22g \text{ Sr activity same as 1g of Co activity}$$

31.Carbon tetrachloride labeled with ^{14}C is sold commercially with an activity of 10 millicuries per millimole(10mCi/mM). What fraction of the carbon atoms is ^{14}C?

[sol] Activity = 10mCi/mM = 3.7× 10^{11}=n(t)× $\frac{\ln 2}{T_{1/2}}$, $T_{1/2}$ of ^{14}C is 5730yr

n(t)= $\frac{5730\times365\times24\times3600}{\ln 2} \times 3.7\times 10^{11}$ **≅9.6458× 10^{22}**

$$\therefore \text{fraction} = \frac{9.6458 \times 10^{22}}{6.020045 \times 10^{23}} \cong 0.1602$$

32. Tritiated water (ordinary water containing some $^1H\ ^3HO$) for biological applications can be purchased in 1-cm^3 ampoules having an activity of 5mCi per cm^3. How many atoms per cm^3. What fraction of the water molecules contains an ^3H atom?

[sol] density of $^1H\ ^3HO = 1.017g/cm^3$, atomic mass : 20.04g/mole

volume of 1mole of $^1H\ ^3HO = \frac{20.04g/mole}{1.017g/cm^3} \cong 19.7cm^3/mole$

$\alpha = \frac{5\times10^{-3}Ci}{1cm^3} \times \frac{19.7cm^3}{mole} = 9.85 \times 10^{-2}Ci/mole$ **= 3.6445× 10^9disintegrations per second**

$\alpha = \lambda n(t) = \frac{\ln 2}{T_{1/2}} n(t),\quad T_{1/2} = 12.26yr \text{ for tritium}$

n(t) =$\frac{12.26\times365\times24\times3600}{\ln 2} \times 3.6445 \times 10^{9} \cong 2.033 \times 10^{18}$

$$\therefore \text{fraction} = \frac{2.033 \times 10^{18}}{6.020045 \times 10^{23}} \cong 3.38 \times 10^{-6}$$

33. After the initial cleanup effort at Three Mile Island, approximately 400,000 gallons of radioactive water remained in the basement of the containment building of the Three Mile Island Unit 2 nuclear plant. The principal sources of this radioactivity were ^{137}Cs at 156μCi/cm^3 and ^{134}Cs at 26μCi/cm^3. How many atoms per cm^3 of these radionuclides were in the water at that time?

[sol] For ^{137}Cs

$\alpha = \lambda n(t) = \frac{\ln 2}{T_{1/2}} n(t)$, $T_{1/2} = 30.07yr$

n(t)= $\frac{30.07\times365\times24\times3600}{\ln 2} \times 156 \times 10^{-6} \times 3.7 \times 10^{10} \cong 7.897 \times 10^{15}/cm^3$

For ^{134}Cs
$T_{1/2} = 2.0648yr$
$n(t)= \frac{2.0648x365\times24\times3600}{ln2} \times 26 \times 10^{-6} \times 3.7 \times 10^{10} \cong 9.037 \times 10^{13}/cm^3$
number of total atoms at that time per cm^3
: 7.897×10^{15} **of** ^{137}Cs **atoms and** 9.037×10^{13} **of** ^{134}Cs **atoms**

34. One gram of ^{226}Ra is placed in a sealed, evacuated capsule $1.2cm^3$ in volume.
(a) At what rate does the helium pressure increase in the capsule, assuming all of the α-particles are neutralized and retained in the free volume of the capsule?

[sol] pressure rate → activity
$\alpha = \lambda N$ **(per volume),** $\lambda = \frac{\ln 2}{T_{\frac{1}{2}}} = \frac{\ln 2}{1600\ year}$
$$= \frac{\ln 2}{1600\times365\times24\times3600s} \times \frac{1g\times(0.602\times10^{24})}{226} \times \frac{1}{1.2cm^3}$$
$$= 3.049 \times 10^{10} disintegrations/cm^3 \cdot s$$

(b) What is the pressure 10 years after the capsule is sealed?

[sol] $\frac{(3.049\times10^{10})disintegrations}{cm^3} \cdot s \times (10 \times 365 \times 24 \times 3600)s$

$=9.6164 \times 10^{18} disintegrations/cm^3$

35. Polonium-210 decays to the ground state of ^{206}Pb by the emission of a 5.305-MeV α-particle with a half-life of 138 days. What mass of 210Po is required to produce 1MW of thermal energy from its radioactive decay?

[sol] $\frac{5.305\times10^6 eV}{disintegrations} \times \frac{3.7\times10^{10} disintegrations/s}{1\ Ci} \times \frac{1.6\times10^{-19}J}{1eV}$
=0.0314056W/Ci
To produce 1MW(thermal),
$1Ci : 0.0314056 = \alpha : 10^6$, $\alpha = 3.184 \times 10^7 Ci$ ← **We need this reactivity** α**.**
$\alpha = 3.184 \times 10^7 Ci \times \frac{3.7\times10^{10} disintegrations/s}{1\ Ci}$
$= 1.1767 \times \frac{10^{18} disintegrations}{s}$ ← $\alpha = \lambda N$
$= \frac{ln2}{138\times24\times3600} \times \frac{m\cdot(0.6022\times10^{24})}{210}$
$\therefore m = 7.058 \times 10^3 g$

36. The radioisotope generator SNAP-9 was fueled with 475 g of ^{238}PuC (plutonium-238 carbide), which has a density of 12.5 g/cm^3. The ^{238}Pu has a half-life of 89 years and emits 5.6MeV per disintegration, all of which may be assumed to be absorbed in the generator. The thermal to electrical efficiency of the system is 5.4%. Calculate
(a) the fuel efficiency in curies per watt (thermal);

[sol] $\frac{disintegrations}{5.6\times10^6 eV} \times \frac{1\ Ci}{3.7\times10^{10} disintegrations/s} \times \frac{1eV}{1.6\times10^{-19}J} = 30.16 Ci/watt$

(b) the specific power in watts (thermal) per gram of fuel;

[sol] $\frac{Cm^3}{12.5g} \times \frac{m\ell}{cm^3} \times \frac{\ell}{10^3 m\ell} \times \frac{mole}{22.4\ell} \times \frac{0.602\times10^{24}}{mole} \times \frac{ln2}{89\times365\times24\times3600s} \times \frac{Ci}{3.7\times10^{10} disintegrations/s} \times \frac{watt}{30.13Ci}$
$= 4.763\times 10^{-4} watts/gram$

(c) the power density in watts (thermal) per cm^3;

[sol] $\frac{12.5g}{cm^3} \times 4.763 \times \frac{10^{-4} watts}{gram} = 5.954 \times 10^{-3}$ **watts/cm^3**

(d) the total electrical power of the generator.

[sol] $\frac{(4.763\times10^{-4})\text{watts}}{\text{gram}}\times 475\text{g}\times 0.054 = 1.22\times10^{-2}\text{watts}$

37. Since the half-life of ^{235}U (7.13 ×10^{8} years) is less than that of 238U (4.51×10^{9} years), the isotopic abundance of ^{235}U has been steadily decreasing since the earth was formed about 4.5 billion years ago. How long ago was the isotopic abundance of ^{235}U equal to 3.0 *a/o*, the enrichment of the uranium used in many nuclear power plants?

[sol] $T_{\frac{1}{2},235} = 7.13\times10^{8}\text{years} \rightarrow \lambda_{235} = \frac{\ln2}{7.13\times10^{8}\text{years}} = 9.722\times10^{-10}\text{years}^{-1}$

$T_{\frac{1}{2},238} = 4.5\times10^{9}\text{years} \rightarrow \lambda_{238} = \frac{\ln2}{4.5\times10^{9}\text{years}} = 1.5369\times10^{-10}\text{years}^{-1}$

(Primary) Abundance of $^{235}\text{U} = 0.72\text{a/o}$

$X_{235} = \frac{0.03\text{mN}_A}{235.0439}$,

$X_{238} = \frac{0.97\text{mN}_A}{238.0289}$

$\frac{X_{235}e^{-\lambda_{235}t}}{X_{235}e^{-\lambda_{235}t}+X_{238}e^{\lambda_{238}t}} = 0.0072$

$1 = 0.0072\left(1+31.9279e^{8.1851\times10^{-10}t}\right)$

$e^{8.1851\times10^{-10}t} = 4.31875848$

$8.1851\times10^{-10}t = \ln(4.31875848) = 1.462967972$

$\therefore t = 1.787\times10^{9}$ **years ago.**

38. The radioactive isotope *Y* is produced at the rate of *R* atoms/sec by neutron bombardment of *X* according to the reaction

X(n,γ)Y.

If the neutron bombardment is carried out for a time equal to the half-life of *Y*, what fraction of the saturation activity of *Y* will be obtained assuming that there is no *Y* present at the start of the bombardment?

[sol] $n(t) = n_0e^{-\lambda t}+\frac{R}{\lambda}\left(1-e^{-\lambda t}\right)$

Assuming that there is no Y present at the start of the bombardment,

$n(t) = \frac{R}{\lambda}\left(1-e^{-\lambda t}\right)$

$\alpha(t) = \lambda n(t) = R\left(1-e^{-\lambda t}\right)$

If the neutron bombardment is carried out for a time equal to the half-life of Y,

$\alpha\left(T_{\frac{1}{2}}\right) = R\left(1-e^{-\lambda T_{\frac{1}{2}}}\right)$

$\leftarrow T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$= R\left(1-e^{-\lambda\frac{\ln 2}{\lambda}}\right)$

$= R\left(1-e^{\ln 2}\right)$ = 0.5R

$\therefore (\text{fraction}) = 0.5$

39. Consider the chain decay

A → B → C →,

with no atoms of B present at t=0.

(a) Show that the activity of B rises to a maximum value at the time t_m given by

$$t_m = \frac{1}{\lambda_B-\lambda_A}\ln\left(\frac{\lambda_B}{\lambda_A}\right),$$

at which time the activities of A and B are equal.

[sol] $\alpha_B = \lambda_B n_{B0}e^{-\lambda_0 t}+\frac{\lambda_A n_0\lambda_B}{(\lambda_A-\lambda_B)}\left[e^{-\lambda_A t}-e^{-\lambda_B t}\right]$

$\leftarrow t = 0, \alpha_B = 0$

$\therefore \alpha_B = \frac{\lambda_A n_0\lambda_B}{(\lambda_A-\lambda_B)}\left[e^{-\lambda_A t}-e^{-\lambda_B t}\right]$

$$\left.\frac{D\alpha_B}{dt}\right|_{t=t_m} = \frac{\lambda_A n_0 \lambda_B}{(\lambda_A-\lambda_B)}\left[-\lambda_A e^{-\lambda_A t_m} + \lambda_B e^{-\lambda_B t_m}\right] = 0$$

$$\therefore\ \lambda_B e^{-\lambda_{B} t_m} = \lambda_A e^{-\lambda_{A} t_m} \quad -①$$

$$\frac{e^{-\lambda_A t_m}}{e^{-\lambda_B t_m}} = \frac{\lambda_B}{\lambda_A}\ , \qquad e^{(\lambda_B-\lambda_A)t_m} = \frac{\lambda_B}{\lambda_A} \quad -②$$

$$(\lambda_B - \lambda_A)t_m = \ln\left(\frac{\lambda_B}{\lambda_A}\right)$$

$$\therefore\ t_m = \frac{1}{(\lambda_B-\lambda_A)}\ln\left(\frac{\lambda_B}{\lambda_A}\right) \quad -③$$

(b) Show that, for $t<t_m$, the activity of B is less than that of A, whereas the reverse is the case for $t> t_m$.

[sol] $\alpha_A(t_m) = \alpha_{A,0} e^{-\lambda_A t_m}$

$$\alpha_B(t_m) = \frac{\alpha_{A,0}\lambda_B}{(\lambda_B-\lambda_A)}\left[e^{-\lambda_A t_m} - e^{-\lambda_B t_m}\right]$$

$$= \frac{\alpha_{A,0}}{(\lambda_B-\lambda_A)}\left[\lambda_B e^{-\lambda_A t_m} - \lambda_B e^{-\lambda_B t_m}\right]$$

$$\leftarrow ①\ :\ \lambda_B e^{-\lambda_B t_m} = \lambda_A e^{-\lambda_A t_m}$$

$$= \frac{\alpha_{A,0}}{(\lambda_B-\lambda_A)}\left[\lambda_B e^{-\lambda_A t_m} - \lambda_A e^{-\lambda_A t_m}\right]$$

$$= \frac{\alpha_{A,0}}{(\lambda_B-\lambda_A)} e^{-\lambda_A t_m}(\lambda_B - \lambda_A)$$

$$= \alpha_{A,0} e^{-\lambda_A t_m} = \alpha_A(t_m) \qquad \therefore\ \alpha_A(t_m) = \alpha_B(t_m)$$

$$\alpha_A - \alpha_B = \alpha_{A,0} e^{-\lambda_A t_m} - \frac{\alpha_{A,0}\lambda_B}{(\lambda_B-\lambda_A)}\left[e^{-\lambda_A t_m} - e^{-\lambda_B t_m}\right]$$

$$= \frac{\alpha_{A,0}}{(\lambda_B-\lambda_A)}\left\{(\lambda_B - \lambda_A)e^{-\lambda_A t} - \lambda_B e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}\right\}$$

$$= \frac{\alpha_{A,0}}{(\lambda_B-\lambda_A)}\left\{\lambda_B e^{-\lambda_B t} - \lambda_A e^{-\lambda_A t}\right\}$$

$$= \frac{\alpha_{A,0}\lambda_A}{(\lambda_B-\lambda_A)}\left\{\frac{\lambda_B}{\lambda_A} e^{-\lambda_B t} - e^{-\lambda_A t}\right\}$$

$$\leftarrow ②\ :\ e^{(\lambda_B-\lambda_A)t_m} = \frac{\lambda_B}{\lambda_A}$$

$$= \frac{\alpha_{A,0}\lambda_A}{(\lambda_B-\lambda_A)} e^{-\lambda_B t}\left\{e^{(\lambda_B-\lambda_A)t_m} - e^{(\lambda_B-\lambda_A)t}\right\}$$

If $t < t_m$, $\lambda_B > \lambda_A$

$(\lambda_B - \lambda_A) > 0$, $\quad e^{(\lambda_B-\lambda_A)t_m} > e^{(\lambda_B-\lambda_A)t} \quad \therefore \alpha_A - \alpha_B > 0$

If $t < t_m$, $\lambda_B < \lambda_A$

$(\lambda_B - \lambda_A) < 0$, $\quad e^{(\lambda_B-\lambda_A)t_m} < e^{(\lambda_B-\lambda_A)t} \quad \therefore \alpha_A - \alpha_B > 0 \quad \therefore\ \alpha_A > \alpha_B\ (t < t_m)$

If $t > t_m$, $\lambda_B > \lambda_A$

$(\lambda_B - \lambda_A) > 0$, $\quad e^{(\lambda_B-\lambda_A)t_m} < e^{(\lambda_B-\lambda_A)t} \quad \therefore \alpha_A - \alpha_B < 0$

If $t < t_m$, $\lambda_B < \lambda_A$

$(\lambda_B - \lambda_A) < 0$, $\quad e^{(\lambda_B-\lambda_A)t_m} > e^{(\lambda_B-\lambda_A)t} \quad \therefore \alpha_A - \alpha_B < 0 \quad \therefore\ \alpha_A < \alpha_B\ (t > t_m)$

40. Show that if the half-life of B is much shorter than the half-life of A, then the activities of A and B in Problem 2.39 eventually approach the same value. In this case, A and B are said to be in *secular equilibrium*.

[sol] $T_{\frac{1}{2},A} = \frac{\ln 2}{\lambda_A} \quad \rightarrow \lambda_A = \frac{\ln 2}{T_{\frac{1}{2},A}}$

$T_{\frac{1}{2},B} = \frac{\ln 2}{\lambda_B} \quad \rightarrow \lambda_B = \frac{\ln 2}{T_{\frac{1}{2},B}} \qquad \Rightarrow \quad T_{\frac{1}{2},A} \gg T_{\frac{1}{2},B} \rightarrow \lambda_A \ll \lambda_B \rightarrow e^{-\lambda_A t} \gg e^{-\lambda_B t}\ ,\ (\lambda_B - \lambda_A) \approx \lambda_B$

$$\alpha_A(t) = \alpha_{A,0} e^{-\lambda_A t}$$

$$\alpha_B(t) = \frac{\alpha_{A,0}\lambda_B}{(\lambda_{\square}-\lambda_A)}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

$$= \frac{\alpha_{A,0}\lambda_B}{\lambda_B}\left[e^{-\lambda_A t}\right] = \alpha_A(t)$$

$\therefore$ If $T_{\frac{1}{2},A} \gg T_{\frac{1}{2},B}$, $\alpha_A(t) = \alpha_B(t)$

41. Show that the abundance of ^{234}U can be explained by assuming that this isotope originates solely from the decay of ^{238}U.

[sol] $\quad \alpha \qquad \beta^-$

$$^{238}U \xrightarrow[4.51\times10^{9}\text{years}]{} {}^{254}Th \xrightarrow[24.1\text{days}]{} {}^{234m}Pa \xrightarrow[6.75\text{years}]{} {}^{234}U$$

→ Half life of ^{238}U is much longer than others (Th, Pa).
So ^{234}U depends on decay of ^{238}U.

42. Radon-222, a highly radioactive gas with a half-life of 3.8 days that originates in the decay of ^{234}U (see chart of nuclides), may be present in uranium mines in dangerous concentrations if the mines are not properly ventilated. Calculate the activity of ^{222}Rn in Bq per metric ton of natural uranium.

[sol] $\lambda_{Radon} = \frac{\ln 2}{3.8\text{days}} = 0.1824\text{days}^{-1}$

$T_{\frac{1}{2},234} = 245500\text{years} = 245500 \times 365\text{ days} = 89607500\text{days}$

$T_{\frac{1}{2},Radon} = 3.8\text{days}$

$T_{\frac{1}{2},234} \gg T_{\frac{1}{2},Radon}$, $\alpha(^{234}U) \approx \alpha(^{222}Ra)$

$\gamma_{^{234}U} = \frac{0.0054\times234.0409}{0.0054\times234.0409+0.7204\times235.0439+99.274\times238.0289} = 0.00531\%$

$\alpha_{^{234}U} = \lambda_{234}N$

$= \frac{\ln 2}{89607500\times24\times3600s} \times \frac{0.602\times10^{24}\times0.00531\times10^{-2}}{234.0409} \times \frac{10^6 g}{1\text{ ton}}$

$= 1.22 \times 10^{10}\text{ disintegrations/}\square\text{on}\cdot s$

$\leftarrow 1Ci = 3.7 \times 10^{10}\text{ disintegrations/s}$

$= 0.33Ci/ton$

43. According to U.S. Nuclear Regulatory Commission regulations, the maximum permissible concentration of radon-222 in air in equilibrium with its short-lived daughters is 3 PCi/liter for nonoccupational exposure. This corresponds to how many atoms of radon-222 per cm^3?

[sol] $\frac{3pCi}{\ell} = 3 \times 10^{-12}Ci/\ell \times \frac{3.7\times10^{10}\text{disintegrations/s}}{1\ \square i} \times \frac{\ell}{1000cm^3}$

$= 0.111 \times 10^{-3}\text{disintegrations/cm}^3\cdot s$

Short-lived daughters

$\sum T_{\frac{1}{2}} = T_{\frac{1}{2},Polonium-218} + T_{\frac{1}{2},Lead-214} + T_{\frac{1}{2},Bismuth-214} + T_{\frac{1}{2},Polonium-214}$

= 3min + 27min + 20min + 180microsec
= 3000.18s

$0.111 \times 10^{-3}\frac{\text{disintegrations}}{cm^3}\cdot s = \frac{\ln 2}{\sum T_{\frac{1}{2}}} \times N$

$\therefore N = 0.48\text{atoms/cm}^3$

44. Consider again the decay chain in Problem 2.39 in which the nuclide A is produced at the constant rate of R atoms/sec. Derive an expression for the activity of B as a function of time.

[sol] 1. $\frac{dn_A}{dt} = -\lambda n_A + R = -(\lambda n_A - R)$

$\int \frac{dn_A}{\lambda n_A - R} = \int -dt$

$\frac{1}{\lambda}\ln(\lambda n_A - R) = -t$

$\ln(\lambda n_A - R) = -\lambda t$

$\lambda n_A - R = Ce^{-\lambda t}$

$\lambda n_A = Ce^{-\lambda t} + R$

$n_A = \frac{C}{\lambda}e^{-\lambda t} + \frac{R}{\lambda}$

If t = 0, $n_A = n_0$

$n_0 = \frac{C}{\lambda} + \frac{R}{\lambda}$

$C + R = \lambda n_0$

$C = \lambda n_0 - R$

$\mathbf{n_A = \frac{\lambda n_0 - R}{\lambda} e^{-\lambda t} + \frac{R}{\lambda}}$

$\mathbf{= n_0 e^{-\lambda t} - \frac{R}{\lambda} e^{-\lambda t} + \frac{R}{\lambda}}$

$\mathbf{= n_0 e^{-\lambda t} - \frac{R}{\lambda}(1 - e^{-\lambda t})}$

2. A → B → C

$\mathbf{\frac{dn_B}{dt} + \lambda_B n_B = \lambda_A n_A}$

$\mathbf{n_A = n_{A0} e^{-\lambda_A t} - \frac{R}{\lambda_A}(1 - e^{-\lambda_A t})}$

$\mathbf{\frac{dn_B}{dt} + \lambda_B n_B = \lambda_A n_{A0} e^{-\lambda_A t} - R(1 - e^{-\lambda_A t})}$ **----------- a**

2-1. homogeneous

$\mathbf{\frac{dn_B}{dt} + \lambda_B n_B = 0}$

$\mathbf{dn_B = -\lambda_B n_B d_t}$

$\mathbf{\int \frac{1}{n_B} dn_B = \int -\lambda_B d_t}$

$\mathbf{\ln n_B = -\lambda_B t + C}$

$\mathbf{n_B = Ce^{-\lambda_B t}}$

if $\mathbf{t = 0,\ n_B = n_{B0}}$

$\mathbf{n_{B0} = C}$

$\mathbf{n_B = n_{B0} e^{-\lambda_B t}}$

2-2. non-homogeneous

Let, $\mathbf{\lambda_B t = I}$

$\mathbf{\lambda_B dt = dI}$

$\mathbf{\frac{dI}{dt} = \lambda_B}$

$\mathbf{n_B = n_{B0} e^{-I}}$

$\mathbf{n_B e^{I} = n_{B0}}$

$\mathbf{\frac{d}{dt}(n_B e^{I}) = n'_B e^{I} + n_B e^{I} \frac{dI}{dt}}$

$\mathbf{= n'_B e^{I} + n_B e^{I} \lambda_B}$

Eq. a × $\mathbf{e^{I}}$

$\mathbf{\frac{dn_B}{dt} e^{I} + \lambda_B n_B e^{I} = \lambda_A n_{A0} e^{-\lambda_A t} e^{I} - R(1 - e^{-\lambda_A t})\, e^{I} = \frac{d}{dt}(n_B e^{I})}$

$\mathbf{n_B e^{I} = \int \lambda_A n_{A0} e^{-\lambda_A t} e^{I} dt - \int R(1 - e^{-\lambda_A t})\, e^{I} dt + C}$

$\mathbf{n_B e^{\lambda_B t} = \int \lambda_A n_{A0} e^{-\lambda_A t} e^{\lambda_B t} dt - \int R(1 - e^{-\lambda_A t})\, e^{\lambda_B t} dt + C}$

$\mathbf{= \int \lambda_A n_{A0} e^{(\lambda_B - \lambda_A)t} dt - \int R(e^{\lambda_B t} - e^{(\lambda_B - \lambda_A)t}) dt + C}$

$\mathbf{= \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A} e^{(\lambda_B - \lambda_A)t} - R(\frac{1}{\lambda_B} e^{\lambda_B t} - \frac{1}{\lambda_B - \lambda_A} e^{(\lambda_B - \lambda_A)t}) + C}$

$\mathbf{n_B = \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A} e^{-\lambda_A t} - R(\frac{1}{\lambda_B} - \frac{1}{\lambda_B - \lambda_A} e^{-\lambda_A t}) + Ce^{-\lambda_B t}}$

if $\mathbf{t = 0,\ n_B = 0}$

$\mathbf{\frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A} - R(\frac{1}{\lambda_B} - \frac{1}{\lambda_B - \lambda_A}) + C = 0}$

$\mathbf{C = R(\frac{1}{\lambda_B} - \frac{1}{\lambda_B - \lambda_A}) - \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A}}$

$\mathbf{n_B = \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A} e^{-\lambda_A t} - R(\frac{1}{\lambda_B} - \frac{1}{\lambda_B - \lambda_A} e^{-\lambda_A t}) + \{R(\frac{1}{\lambda_B} - \frac{1}{\lambda_B - \lambda_A}) - \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A}\} e^{-\lambda_B t}}$

$\mathbf{= \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) + \frac{R}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) + \frac{R}{\lambda_B}(e^{-\lambda_B t} - 1)}$

3. homogeneous + non-homogeneous

$\mathbf{n_B = n_{B0} e^{-\lambda_B t} + \frac{\lambda_A n_{A0}}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) + \frac{R}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) + \frac{R}{\lambda_B}(e^{-\lambda_B t} - 1)}$

activity $\mathbf{\alpha_B = \alpha_{B0} e^{-\lambda_B t} + \frac{\lambda_A \alpha_{A0}}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) + \frac{R}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) + \frac{R}{\lambda_B}(e^{-\lambda_B t} - 1)}$

45. Complete the following reactions and calculate their Q values. [*Note:* The atomic weight of ^{14}C is 14.003242.]

[a] $^{4}He(p, d)$

[sol] $\mathbf{^{4}He(p, d) = {}^{4}He(p, d){}^{3}He : {}^{4}He + {}^{1}H \rightarrow {}^{3}He + {}^{2}H}$

$\mathbf{Q = \{[M(^{4}He) + M(^{1}H)] - [M(^{3}He) + M(^{2}H)]\} \times 931.502\ MeV}$ **<Eq. (2.38)>**

$\mathbf{= [(4.002603 + 1.007825) - (3.016029 + 2.014102)] \times 931.502\ MeV}$

$\mathbf{= -18.3534\ MeV}$

[b] ^{9}Be(α, n)

[sol] ^{9}Be(α, n) = ^{9}Be(α, n)^{12}C : ^{9}Be + ^{4}He → ^{12}C + n
Q = {[M(^{9}Be) + M(^{4}He)] – [M(^{12}C) + M(n)]} × 931.502 MeV
= [(9.012182 + 4.002603) – (12 + 1.008665)] × 931.502 MeV
= 5.70079 MeV

[c] ^{14}N(n, p)

[sol] ^{14}N(n, p) = ^{14}N(n, p)^{14}C : ^{14}N + n → ^{14}C + ^{1}H
Q = {[M(^{14}N) + M(n)] – [M(^{14}C) + M(^{1}H)]} × 931.502 MeV
= [(14.003074 + 1.008665) – (14.003242 + 1.007825)] × 931.502 MeV
= 0.625969 MeV

[d] ^{115}In(d, p)

[sol] ^{115}In(d, p) = ^{115}In(d, p)^{116}In : ^{115}In + ^{2}H → ^{116}In + ^{1}H
Q = {[M(^{115}In) + M(^{2}H)] – [M(^{116}In) + M(^{1}H)] × 931.502 MeV
= [(114.903879 + 2.014102) – (115.905262 + 1.007825)] × 931.502 MeV
= 4.55877 MeV

[e] ^{207}Pb(γ, n)

[sol] ^{207}Pb(γ, n) = ^{207}Pb(γ, n)^{206}Pb : ^{207}Pb + γ → ^{206}Pb + n
Q = {[M(^{207}Pb) + M(γ)] – [M(^{206}Pb) + M(n)]} × 931.502 MeV
= [(206.975880 + 0) – (205.974449 + 1.008665)] × 931.502 MeV
= -6.73849 MeV

46. (a) Compute the recoil energy of the residual, daughter nucleus following the emission of a 4.782-MeV α-particle by ^{226}Ra.

[sol] ^{226}Ra → ^{222}Rn + α : ^{226}Ra → ^{222}Rn + ^{4}He
Recoiling Energy = Q(E) – E_α
Q = {[M(^{226}Ra)] – [M(^{222}Rn) + M(^{4}He)]} × 931.502 MeV
= [226.025402 – (222.017570 + 4.002603)] × 931.502 MeV
= 4.87082 MeV ≒ 4.871 MeV
Recoiling Energy = E – E_α = (4.871 – 4.782) MeV = 0.089 MeV

(b) What is the total disintegration energy for this decay process?

[sol] Total disintegration energy = Q value
= 0.089 MeV

47.In some tabulations, atomic masses are given in terms of the mass excess rather than as atomic masses. The mass excess, Δ, is the difference

$$\Delta = M - A,$$

Where M is the atomic mass and A is the atomic mass number. For convenience, Δ, which may be positive or negative, is usually given in units of MeV. Show that the Q

$$Q = (\Delta_a + \Delta_b) - (\Delta_c + \Delta_d),$$

[sol] Q = [(M_a + M_b) – (M_c + M_d)]931 MeV <Eq. (2.38)>
= [(Δ_a + A_a + Δ_b + A_b) – (Δ_c + A_c + Δ_d + A_d)] <Δ = M – A and usually given in units of MeV>
= (Δ_a + Δ_b) – (Δ_c + Δ_d) <A_a +A_b = A_c + A_d>

48. According to the tables of Lederer et al. (See References), the mass excesses for the (neutral) atoms in the reaction in Example 2.8 are as follows: Δ(^{3}H) = 14.95 MeV, Δ(^{2}H) = 13.14 MeV, Δ(n) = 8.07 MeV, and Δ(^{4}He) = 2.42 MeV. Calculate the Q value of this reaction using the results of Problem 2.47.

[sol] a + b → c + d
^{3}H + ^{2}H → ^{4}He + n
Q = (Δa + Δb) – (Δc + Δd) <Problem 47>
= [(Δ(^{3}H) + Δ(^{2}H)) – (Δ(^{4}He) + Δ(n))]
= [(14.95 + 13.13) – (2.42 + 8.07)] MeV

= 17.59 MeV

49. The atomic weight of ^{206}Pb is 205.9745. Using the data in Problem 2.35, calculate the atomic weight of ^{210}Po. [*Caution:* See Problem 2.46]

[sol] E_α = 5.305 MeV <Problem 2.35>

$^{210}\mathrm{Pb} \rightarrow {}^{206}\mathrm{Pb} + \alpha$: $^{210}\mathrm{Pb} \rightarrow {}^{206}\mathrm{Pb} + {}^{4}\mathrm{He}$

$Q = \{[M(^{210}\mathrm{Pb})] - [M(^{206}\mathrm{Pb}) + M(^{4}\mathrm{He})]\} \times 931.502$ MeV

$= [M(^{210}\mathrm{Pb}) - (205.9745 + 4.002603)] \times 931.502$ MeV = 5.305 MeV <Recoiling Energy = 0>

$M(^{210}\mathrm{Pb}) = 209.9828$

50. Tritium (^{3}H) can be produced through the absorption of low-energy neutrons by deutrerium. The reaction is

$$^{2}\mathrm{H} + n \rightarrow {}^{3}\mathrm{H} + \gamma,$$

Where the γ-ray has an energy of 6.256 MeV.

(a) Show that the recoil energy of the ^{3}H nucleus is approximately 7 KeV.

[sol] E_γ = 6.256 MeV

Recoiling Energy = $\frac{1}{2}MV^2 = \frac{p^2}{2M}$ <p=mv> = $\frac{E_\gamma{}^2}{2MC^2}$ <p=$\frac{E}{c}$>

= (6.256 MeV)^2 / (2 × M(^{3}H) × 931.502 MeV)

= (6.256 MeV)^2 / (2 × 3.016049 × 931.502 MeV)

= 0.006965 MeV ≒ 7 KeV

(b) What is the Q value of the reaction?

[sol] $Q = \{[M(^{2}\mathrm{H}) + M(n)] - M(^{3}\mathrm{H})\} \times 931.502$ MeV <Eq. (2.38)>

= [(2.014102 + 1.008665) – 3.016049] × 931.502 MeV

= 6.25783 MeV

(c) Calculate the separation energy of the last neutron in ^{3}H.

[sol] Separation Energy = Binding Energy of the last neutron

$E_s = [M_n + M(^{A-1}Z) - M(^{A}Z)] \times 931.502$ MeV <Eq. 2.45>

$= [M_n + M(^{2}\mathrm{H}) - M(^{3}\mathrm{H})] \times 931.502$ MeV

= 6.25783 MeV

(d) Using the binding energy for ^{2}H of 2.23 MeV and the result from part (c), compute the total binding energy of ^{3}H.

[sol] $BE_{(A,Z)} = [ZM(H) + NM_n - M(^{A}Z)] \times 931.502$ MeV

$BE_{(A-1,Z)} = [ZM(H) + (N-1)M_n - M(^{A-1}Z)] \times 931.502$ MeV

$BE_{(A,Z)} - BE_{(A-1,Z)} = [ZM(H) + NM_n - M(^{A}Z) - ZM(H) - (N-1)M_n + M(^{A-1}Z)] \times 931.502$ MeV

$= [M_n + M(^{A-1}Z) - M(^{A}Z)] \times 931.502$ MeV $= E_s$

$BE_{(A,Z)} = E_s + BE_{(A-1,Z)} = BE(^{3}\mathrm{H}) = E_s + BE(^{2}\mathrm{H})$ = (6.25783 + 2.23) MeV = 8.48783 MeV

51. Consider the reaction

$$^{6}\mathrm{Li}(\alpha, p)^{9}\mathrm{Be}$$

Using atomic mass data, compute:

(a) the total binding energy of ^{6}Li, ^{9}Be, and ^{4}He;

[sol] $BE(a) = [Z_aM(^{1}\mathrm{H}) + N_aM_n - M_a] \times 931.502$ MeV

$M(^{1}\mathrm{H})$ = 1.007825

M_n = 1.008665

BE(^{6}Li) = (3 × 1.007825 + 3 × 1.008665 – 6.015122) × 931.502 MeV

= 31.9952 MeV

BE(^{9}Be) = (4 × 1.007825 + 5 × 1.008665 – 9.012182) × 931.502 MeV

= 58.1658 MeV

BE(^{4}He) = (2 × 1.007825 + 2 × 1.008665 – 4.002603) × 931.502 MeV

= 28.2962 MeV

(b) the Q value of the reaction using the results of part (a).

[sol] $Q = [(M_a + M_b) - (M_c + M_d)]c^2$ <Eq. 2.37>

$= [M(^6Li) + M(^4He) – M(^9Be) – Mp]c^2$
$= [\underline{3M(^1H)} \underline{+ 3Mn} – BE(^6Li)/c^2 \underline{+ 2M(^1H)} \underline{+ 2Mn} – BE(^4He)/c^2$
$\underline{- 4M(^1H)} \underline{– 5Mn} + BE(^9Be)/c^2 \underline{– M(1H)}]c^2$
$= BE(^9Be) - BE(^6Li) - BE(^4He)$
= (58.1658 – 31.9952 – 28.2962) MeV
= -2.1256 MeV

52. Using atomic mass data, compute the average binding energy per nucleon of the following nuclei:
(a) ^2H

[sol] $BE(^2H) = (1 \times 1.007825 + 1 \times 1.008665 – 2.014102) \times 931.502$ **MeV**
= 2.22443 MeV
$BE(^2H)/2 = 1.11221$ **MeV**
(b) ^4He

[sol] $BE(^4He) = (2 \times 1.007825 + 2 \times 1.008665 – 4.002603) \times 931.502$ **MeV**
= 28.2962 MeV
$BE(^4He)/4 = 7.07406$ **MeV**
(c) ^{12}C

[sol] $BE(^{12}C) = (6 \times 1.007825 + 6 \times 1.008665 – 12) \times 931.502$ **MeV**
= 92.1628 MeV
$BE(^{12}C)/12 = 7.68023$ **MeV**
(d) ^{51}V

[sol] $BE(^{51}V) = (23 \times 1.007825 + 28 \times 1.008665 – 50.943964) \times 931.502$ **MeV**
= 445.846 MeV
$BE(^{51}V)/51 = 8.74207$ **MeV**
(e) ^{138}Ba

[sol] $BE(^{138}Ba) = (56 \times 1.007825 + 82 \times 1.008665 – 137.905242) \times 931.502$ **MeV**
= 1158.31 MeV
$BE(^{138}Ba)/138 = 8.39356$ **MeV**
(f) ^{235}U

[sol] $BE(^{235}U) = (92 \times 1.007825 + 143 \times 1.008665 – 235.043922) \times 931.502$ **MeV**
= 1783.89 MeV
$BE(^{235}U)/235 = 7.59104$ **MeV**

53. Using the mass formula, compute the binding energy per nucleon for the nuclei in Problem 2.52. Compare the results with those obtained in that problem.
(a) ^2H

[sol] $M = NM_n + ZM_p – \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \zeta(A – 2Z)^2/A$ **<Eq. 2.50>**
$M = NM_n + ZM_p – \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \zeta(A – 2Z)^2/A + \delta$ **<Eq. 2.51> (Nuclei with odd numbers of neutrons and odd numbers of protons would thus be less strongly bound together.)**
$BE = \alpha A - \beta A^{2/3} - \gamma Z^2/A^{1/3} - \zeta(A – 2Z)^2/A – \delta$
$BE(^2H) = (15.56 \times 2 – 17.23 \times 2^{2/3} – 0.697 \times 1^2/2^{1/3} – 23.285 \times (2 – 2 \times 1)^2/2)$ **MeV <N=1, Z=1>**
= 3.21587 MeV
$BE(^2H)/2 = 1.60794$ **MeV (> 1.11221 MeV)**
(b) ^4He

[sol] $BE(^4He) = (15.56 \times 4 – 17.23 \times 4^{2/3} – 0.697 \times 2^2/4^{1/3} – 23.285 \times (4 – 2 \times 2)^2/4 – 12.0 \times -1)$ **MeV <N=2, Z=2>**
= 29.0668 MeV
$BE(^4He)/4 = 7.2667$ **MeV (> 7.07406 MeV)**
(c) ^{12}C

[sol] $BE(^{12}C) = (15.56 \times 12 – 17.23 \times 12^{2/3} – 0.697 \times 6^2/12^{1/3} – 23.285 \times (12 – 2 \times 6)^2/12 – 12.0 \times -1)$ **MeV**

<N=6, Z=6>
= 97.4493 MeV
BE(^{12}C)/12 = 8.12078 MeV (> 7.68023 MeV)

(d) ^{51}V

[sol] BE(^{51}V) = (15.56 × 51 – 17.23 × 51$^{2/3}$ – 0.697 × 23^{2}/51$^{1/3}$ – 23.285 × (51 – 2 × 23)^{2}/51 – 12.0 × 0) MeV <N=23, Z=28>
= 445.765 MeV
BE(^{51}V)/51 = 8.7405 MeV (< 8.74207 MeV)

(e) ^{138}Ba

[sol] BE(^{138}Ba) = (15.56 × 138 – 17.23 × 138$^{2/3}$ – 0.697 × 56^{2}/138$^{1/3}$ – 23.285 × (138 – 2 × 56)^{2}/138 – 12.0 × -1) MeV <N=56, Z=82>
= 1162.12 MeV
BE(^{138}Ba)/138 = 8.42117 MeV (> 8.39356 MeV)

(f) ^{235}U

[sol] BE(^{235}U) = (15.56 × 235 – 17.23 × 235$^{2/3}$ – 0.697 × 92^{2}/235$^{1/3}$ – 23.285 × (235 – 2 × 92)^{2}/235 – 12.0 × 0) MeV <N=92, Z=143>
= 1786.75 MeV
BE(^{235}U)/235 = 7.60319 MeV (>7.59104 MeV)

54. Compute the separation energies of the last neutron in the following nuclei:

$$E_s=[M_n + M(^{A-1}Z) - M(^{A}Z)]931 \text{ Mev/amu}$$

(a) ^{4}He
[sol] E_s=[1.008665 + 3.016029 – 4.002603]931=20.567 MeV

(b) ^{7}Li
[sol] E_s=[1.008665 + 6.015122 – 7.016004]931=7.246 MeV

(c) ^{17}O
[sol] E_s=[1.008665 + 15.99491 – 16.99913]931=4.141 MeV

(d) ^{51}V
[sol] E_s=[1.008665 + 49.947163 – 50.943964]931=11.045 MeV

(e) ^{208}Pb
[sol] E_s=[1.008665 + 206.97588 – 207.97664]931=7.3633 MeV

(f) ^{235}U
[sol] E_s=[1.008665 + 234.040945 – 235.043922]931=5.296 MeV

55. Derive Eq.(2.53).[Hint: Try taking the logarithm of Eq.(2.52) before differentiating.]

[sol] In Eq. (2.52), let $\frac{2\pi N}{(\pi kT)^{\frac{3}{2}}}$ be C, then take the logarithm

$$\ln(N(E)) = \ln C + \frac{1}{2}\ln E - \frac{E}{kT}$$

At the Fig 2.9 the most probable energy in distribution is at the top.

$$\text{So,} \frac{d\ln(N(E))}{dN(E)} = \frac{1}{2E} - \frac{1}{kT} = 0$$

$$\therefore E = \frac{1}{2}kT$$

56. What is 1 atmosphere pressure in units of eV/cm^{3}? [Hint: At standard temperature and pressure (0℃ and 1 atm), 1 mole of gas occupies 22.4 liters.]

[sol] The Eq.(2.58), P=NkT

$$\mathbf{P = \left(\frac{0.6022 \times 10^{24}}{22400} \times 8.6170 \times 10^{-5} \times 273.15\right) = 6.3278 \times 10^{17}\,\frac{eV}{cm^3}}$$

57. Calculate the atom density of graphite having density of 1.60 g/cm^3.

[sol] In Eq.(2.59) $\mathbf{N = \frac{\rho N_A}{M}}$**, so**

$$\mathbf{\frac{(1.6) \times 0.6022 \times 10^{24}}{12} = 0.08029 \times 10^{24}}$$

$$\mathbf{\therefore N = 0.08029 \times 10^{24} atoms/cm^3}$$

58. Calculate the activity of 1 gram of natural uranium.

[sol]

	Abundance, a/o	Atomic weight	Half life (T 1/2)
U-234	0.0057	234.0409	2.46×10E5 y
U-235	0.72	235.0439	7.038×10E8 y
U-238	99.27	238.0508	4.68×10E9 y

Atomic weight of natural uranium is;

$$\mathbf{M(U) = 0.01[0.0057 \times 234.0409 + 0.72 \times 235.0439 + 99.27 \times 238.0508] = 238.0187}$$

Total number of natural uranium nuclei in 1 gram of uranium is;

$$\mathbf{N = \frac{0.6022 \times 10^{24}}{238.0187} = 25.301 \times 10^{20} atoms/gram}$$

In Eq.(2.26), $\lambda = \ln 2 / T_{1/2}$

$$\mathbf{\alpha(U) = \alpha(^{234}U) + \alpha(^{235}U) + \alpha(^{238}U)}$$

$$\mathbf{\alpha(U) = \lambda_{234} n_{234} + \lambda_{235} n_{235+} \lambda_{238} n_{238}}$$

$$\mathbf{\alpha(U) = \frac{\ln 2}{2.46 \times 10^5} \times \left(\frac{0.0057}{100} \times 25.301 \times 10^{20}\right] + \frac{\ln 2}{7.038 \times 10^8} \times \left(\frac{0.72}{100} \times 25.301 \times 10^{20}\right)}$$

$$\mathbf{+ \frac{\ln 2}{4.68 \times 10^9} \times \left(\frac{99.27}{100} \times 25.301 \times 10^{20}\right) = 7.96287 \times 10^{11}\,\frac{dis}{yr}}$$

$$\mathbf{= \frac{7.96287 \times 10^{11}}{365 \times 24 \times 3600} \times \frac{1}{3.7 \times 10^{10}} = 6.82435 \times 10^{-7}\ Ci}$$

$$\mathbf{\therefore \alpha(U) = 68.2435\ \mu Ci}$$

59. What is the atom density of ^{235}U in uranium enriched to 2.5 a/o in this isotope if the physical density of the uranium is 19.0 g/cm^3?

[sol] $\mathbf{M(U) = 0.01[0.0057 \times 234.0409 + 2.5 \times 235.0439 + 97.4943 \times 238.0508] = 237.9754}$

$$\mathbf{N(U) = \frac{19 \times 0.6022 \times 10^{24}}{237.9754} = 4.80798 \times 10^{22}\ \frac{atoms}{cm^3}}$$

$$\mathbf{N(^{235}U) = \frac{2.5}{100} \times N(U) = 1.202 \times 10^{21}\ atoms/cm^3}$$

$$\mathbf{\therefore N(^{235}U) = 12.02 \times 10^{20}\ atoms/cm^3}$$

60.Plutonium-239 undergoes α-decay with a half-life of 24,000 years. Compute the activity of 1 gram of plutonium dioxide, $^{239}PuO_2$. Express the activity in terms of Ci and Bq.

[sol] $\mathbf{M(^{239}PuO_2)\ = 239.0522 + 2 \times 15.9994 = 271.051\ amu}$

The weight percent of plutonium is $\mathbf{\frac{239.0522}{271.051} = 0.88195 \cong 88.2\%}$

The weight percent is almost the same with a/o.

$$\mathbf{so, \alpha(^{239}PuO_2) = \lambda T_{\frac{1}{2}} = \frac{\ln 2}{24000 \times 365 \times 24 \times 3600} \times \left(\frac{0.88198 \times 0.6022 \times 10^{24}}{239.0522}\right)}$$

$$\mathbf{= 2.0347 \times 10^9\ Bq}$$

$$\mathbf{\therefore 0.054992\ Ci = 2.0347 \times 10^9\ Bq}$$

61. it has been proposed to use uranium carbide (UC) for the initial fuel in certain types of breeder reactors, with the uranium enriched to 25 w/o. The density of UC is 13.6 g/cm^3.

(a) What is the atomic weight of the uranium?

[sol] $\frac{1}{M(U)} = \frac{1}{100}\sum\frac{w_i}{M_i}$

$$= \frac{1}{100}\left(\frac{25}{235.0439} + \frac{75}{238.0508}\right) = 4.2142 \times 10^{-3}$$

$$\therefore M(U) = 237.292$$

(b) What is the atom density of the ^{235}U?

[sol] Molecular weight of UC is;

$$237.292 + 12 = 249.292$$

The physical density of U is;

$13.6 \times 0.95186 = 12.945 \quad (\because \frac{237.292}{249.292} = 0.95186),$

$\text{so}, N(^{235}U) = \frac{25}{100} \times \left(\frac{12.945 \times 0.6022 \times 10^{24}}{235.0439}\right) = 8.2915 \times 10^{21} \text{ atoms/cm}^3$

62. Compute the atom densities of ^{235}U and ^{238}U in UO_2 of physical density 10.8 g/cm^3 if the uranium is enriched to 3.5 w/o in ^{235}U.

[sol] Eq.(2.65) $\frac{1}{M} = \frac{1}{100}\sum\frac{w_i}{M_i}$

so, $\frac{1}{M(U)} = \left(\frac{3.5}{235.0439} + \frac{96.5}{238.0508}\right) \times \frac{1}{100} = 4.20266 \times 10^{-3}$

$$M(U) = 237.9443$$

$$M(UO_2) = 237.9443 + 2 \times 15.9994 = 269.9431$$

The weight percent of U in UO_2 is;

$$\frac{237.9443}{269.9431} = 0.88146$$

The physical density of U is;

$$10.8 \times 0.88146 = 9.52 \text{ g/cm}^3$$

$$\therefore N(^{235}U) = \frac{3.5}{100} \times \frac{9.52 \times 0.6022 \times 10^{24}}{235.0439} = 8.5368 \times 10^{20} \text{ atoms/cm}^3$$

$$N(^{238}U) = \frac{96.5}{100} \times \frac{9.52 \times 0.6022 \times 10^{24}}{238.0508} = 2.324 \times 10^{22} \text{ atoms/cm}^3$$

63. The fuel for a certain breeder reactor consists of pellets composed of mixed oxides, UO_2 and PuO_2, with the PuO_2 comprising approximately 30 w/o of the mixture. The uranium is essentially all ^{238}U, whereas the plutonium contains the following isotopes: ^{239}Pu (70.5 w/o), ^{240}Pu(21.3 w/o), ^{241}Pu(5.5 w/o), and ^{242}Pu(2.7 w/o). Calculate the number of atoms of each isotope per gram of the fuel.

[sol] **Mass of UO_2, 0.7g and mass of PuO_2, 0.3g**

$$\text{Mass of } ^{238}U = 0.7 \times \frac{238.0508}{238.0508 + 2 \times 15.9994} = 0.6171 \text{ g}$$

$$N(^{238}U) = \frac{0.6022 \times 10^{24}}{238.0508} = 1.561 \times 10^{21} \text{ atoms/g}$$

Atomic weight of Pu; M(Pu)

$$= \frac{70.5}{100} \times 239.0522 + \frac{21.3}{100} \times 240.0538 + \frac{5.5}{100} \times 241.0568 + \frac{2.7}{100} \times 242.0587$$
$$= 239.457$$

$\text{Mass of Pu} = 0.3 \times \frac{239.404}{239.404 + 2 \times 15.9994} = 0.26464 \text{ g}$

$N(^{239}Pu) = \frac{0.6022 \times 10^{24}}{239.0522} \times 0.26464 \times 0.705 = 4.7074 \times 10^{20} \text{ atoms/g}$

$N(^{240}Pu) = \frac{0.6022 \times 10^{24}}{240.\square 538} \times 0.26464 \times 0.213 = 1.4141 \times 10^{20} \text{ atoms/g}$

$N(^{241}Pu) = \frac{0.6022 \times 10^{24}}{241.0568} \times 0.26464 \times 0.055 = 3.636 \times 10^{19} \text{ atoms/g}$

$$\mathbf{N(^{242}Pu) = \frac{0.6022\times10^{24}}{242.0587} \times 0.26464 \times 0.027 = 1.7\square76 \times 10^{19}\ atoms/g}$$

Chapter 3

Interaction of Radiation with Matter

1. Two beams of 1-eV neutrons intersect at an angle of 90°. The density of neutrons in both beams is 2×10^8 neutrons/cm^3

(a) Calculate the intensity of each beam.

[sol] $\mathbf{v = 1.383 \times 10^6\sqrt{E}}$ **cm/s <Eq. 2.12>**

$\mathbf{= 1.383 \times 10^6\sqrt{1}}$ **cm/s**

$\mathbf{= 1.383 \times 10^6}$ **cm/s**

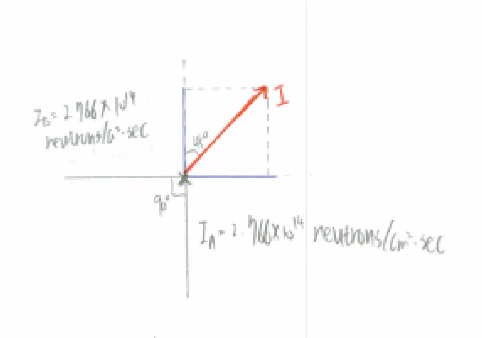
$\mathbf{\emptyset = nv}$ **<Eq. 3.20>**

$\mathbf{= (2 \times 10^8\ neutrons/cm^3) \times (1.383 \times 10^6\ cm/s)}$

$\mathbf{= 2.766 \times 10^{14}\ neutrons/cm^2\text{-}sec}$

(b) What is the neutron flux where the two beams intersect?

[sol]



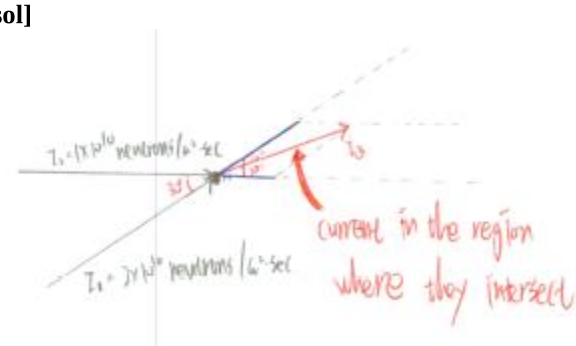
the neutron flux in the region where they intersect <law of cosines>

$$= \sqrt{(2.766 \times 10^{14})^2 - (2.766 \times 10^{14})^2 + 2 \times (2.766 \times 10^{14}) \times (2.766 \times 10^{14}) \times \cos 45°}$$

$\mathbf{= 3.289 \times 10^{14}\ neutrons/cm^2\text{-}sec}$

2. Two monoenergetic neutron beams of intensities $I_1 = 2 \times 10^{10}$ neutrons/cm^2-sec and $I_2 = 1 \times 10^{10}$ neutrons/cm^2-sec intersect at an angle of 30°. Calculate the neutron flux and current in the region where they intersect.

[sol]



the neutron flux in the region where they intersect <law of cosines>

$$= \sqrt{(1 \times 10^{10})^2 - (2 \times 10^{10})^2 + 2 \times (2 \times 10^{10}) \times (1 \times 10^{10}) \times \cos 10°}$$

$\mathbf{= 0.969 \times 10^{10}\ neutrons/cm^2\text{-}sec}$

3. A monoenergetic beam of neutrons, $\emptyset = 4 \times 10^{10}$ neutrons/cm^2-sec, impinges on a target 1 cm^2 in area and 0.1 cm thick. There are 0.048×10^{24} atoms per cm^3 in the target, and the total cross-section at the energy of the beam is 4.5 b

(a) What is the macroscopic total cross-section?

[sol] $\mathbf{\Sigma_t = N \times \sigma_t}$

$\mathbf{= (0.048 \times 10^{24}) \times (4.5 \times 10^{-24})}$

$\mathbf{= 0.216\ cm^{-1}}$

(b) How many neutron interactions per second occur in the target?

[sol] interactions rate = σ_tINAX (I : intensity of the beam, A : area, X : thickness) <Eq. 3.2>

= Σ_tIAX

= 0.216 cm^{-1} × (4 × 10^{10} neutrons/cm^2-sec) × 1 cm^2 × 0.1 cm

= 8.64 × 10^8 interactions(interaction neutrons)/sec

(c) What is the collision density?

[sol] collision density F = INσ_t (Eq. 3.7>

= IΣ_t

= (4 × 10^{10} neutrons/cm^2-sec) × 0.216 cm^{-1}

= 8.64 × 10^9 collisions(collision neutrons)/cm^3-sec

4. The β^--emitter ^{28}Al (half-life 2.30 min) can be produced by the radiative capture of neutrons by ^{27}Al. The 0.0253-eV cross-section for this reaction is 0.23 b. Suppose that a small, 0.01-g aluminum target is placed in a beam of 0.0253-eV neutrons, Ø = 3 × 10^8 neutrons/cm^2-sec, which strikes the entire target. Calculate

(a) the neutron density in the beam;

[sol] I = nv <Eq. 3.1>

n = I / v

= (3 × 10^8 neutrons/cm^2-sec) / (2.2 × 10^5 cm/sec)

<the speed of thermal neutron = 2.2 km/sec)

= 1363.64 neutrons/cm^3

(b) the rate at which ^{28}Al is produced;

[sol] Production rate = INσ_t

N = ρN_A / M <Eq. 2.59>

INσ_t = (3 × 10^8 neutrons/cm^2-sec) × {(0.01 g/cm3 × 0.6022 × 10^{24}) / 26.981538g}

× 0.23 × 10^{-24} cm^2

<M_{27Al} = 26.981538>

= 1.54 × 10^4 atoms/cm^3-sec

(c) the maximum activity (in curies) that can be produced in this experiment.

[sol] $\frac{dN(28_{Al})}{dt} = IN(^{27}Al)\sigma_t - \lambda N(^{28}Al)$

Where $IN(^{27}Al)\sigma_t = C$ **and** $N(^{28}Al) = N$, $\frac{dN}{dt} = C - \lambda N \leftarrow \times\ e^{\lambda t}$

$e^{\lambda t}\frac{dN}{dt} = Ce^{\lambda t} - \lambda Ne^{\lambda t}$

$e^{\lambda t}\frac{dN}{dt} + \lambda Ne^{\lambda t} = Ce^{\lambda t}$

$\frac{d}{dt}(e^{\lambda t}N) = Ce^{\lambda t}$

$[e^{\lambda t}N(t)]_0^t = \int_0^t Ce^{\lambda t}\,dt$

$e^{\lambda t}N(t) - N(0) = \frac{1}{\lambda}Ce^{\lambda t} - \frac{1}{\lambda}C$

$e^{\lambda t}N(t) = \frac{C}{\lambda}(e^{\lambda t} - 1)$

$N(t) = \frac{C}{\lambda}(1 - e^{-\lambda t})$

$\alpha(t) = C(1 - e^{-\lambda t})$

$\alpha_{\text{Maximum activity}} = \lim_{t\to\infty} C(1 - e^{-\lambda t})$

$= C = IN(^{27}Al)\sigma_t$

= 1.54 × 10^4 atoms/cm^3-sec

= 4.16 × 10^{-7} Ci <1Ci = 3.7 × 10^{10} disintegrations/sec>

5. Calculate the mean free path of 1-eV neutrons in graphite. The total cross-section of carbon at this energy is 4.8b

[sol] $\Sigma_t = N \times \sigma_t$
$= (0.08023 \times 10^{24}) \times (4.8 \times 10^{-24})$ <the density of carbon ➔ Table II.3>
$= 0.3851\ cm^{-1}$
mean free path $\lambda = 1 / \Sigma_t = 2.5967$ cm

6. A beam of 2-MeV neutrons is incident on a slab of heavy water (D_2O). The total cross-sections of deuterium and oxygen at this energy are 2.6 b and 1.6 b, respectively.

(a) What is the macroscopic total cross-section of D_2O at 2MeV?

[sol] $\Sigma_t = N \times \sigma_t$
$N(D_2O) = 0.03323 \times 10^{24}$ molecules/cm^3 <Table II.3>
σ of D_2O = $(2 \times \sigma$ of deuterium$) + \sigma$ of oxygen
$= (2 \times 2.6) + 1.6 = 6.8$ b
$\Sigma_t = (0.03323 \times 10^{24}) \times (6.8 \times 10^{-24}) = 0.225964\ cm^{-1}$

(b) How thick must the slab be to reduce the intensity of the uncollided beam by a factor of 10?

[sol] $I = I^0 e^{-\Sigma_t x}$ **<Eq. 3.11>**
$\frac{I}{I_0} = e^{-\Sigma_t x} = \frac{1}{10}$
$-\Sigma_t x = -\ln 10$
$x = \ln 10 / \Sigma_t = \ln 10 / 0.225964\ cm^{-1} = 10.19$ cm

(c) If an incident neutron has a collision in the slab, what is the relative probability that it collides with deuterium?

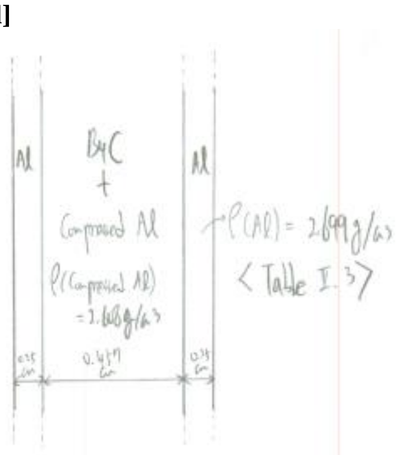
[sol] relative probability = Σ_t of deuterium / Σ_t of heavy water
$= \{N(D_2O) \times (2 \times \sigma$ of deuterium$)\} / 0.225964$
$= \{(0.03323 \times 10^{24}) \times (2 \times 2.6 \times 10^{-24})\} / 0.225964$
$= 0.7647$

7. A beam of neutrons is incident from the left on a target that extends from x = 0 to x = a. Derive an expression for the probability that a neutron in the beam will have its first collision in the second half of the target that is in the region a/2 < x < a.

[sol] $p(x)\,dx = e^{-\Sigma_t x} \times \Sigma_t\,dx$ **<page 59>**
$P = \int_{a/2}^{a} \Sigma t\, e^{-\Sigma_t x}\, dx = [-e^{-\Sigma_t x}]_{a/2}^{a} = -e^{-a\Sigma_t} + e^{-\frac{a}{2}\Sigma_t}$

8. *Boral* is commercial shielding material consisting of approximately equal parts by weight of boron carbide (B_4C) and aluminum compressed to about 95% theoretical density (2.608 g/cm^3) and clad with thin sheets of aluminum 0.25 cm thick. The manufacturer specifies that there are 0.333g of boron per cm^2 of a boral sheet 0.457 cm in overall thickness. What is the probability that a 0.0253-eV neutron incident normally on such a sheet will succeed in penetrating it?

[sol]



Penetrating probability
= the probability that a neutron can move through this distance without having a collision
= $e^{-\Sigma_t x}$ <page 58>
$M(B_4C) = 4 \times 10.811 + 12 = 55.244$
mass of C per cm^2 = $\frac{12}{55.244} \times 0.333g = 0.0723g$
mass of B_4C per cm^2 = 0.333g + 0.0723g = 0.4053g
$\rho(B_4C) = \frac{0.4053g}{0.457cm \times 1cm^2} = 0.8869g/cm^3$
$N = \rho N_A / M$ <Eq. 2.59>
$N(B_4C) = \frac{0.8869 \times 0.6022 \times 10^{24}}{55.244} = 9.6679 \times 10^{-3} \times 10^{24}$ atoms/cm^3-sec
$\sigma_t(B_4C) = 4\sigma_t(B) + \sigma_t(C)$
$= 4(\sigma_a(B) + \sigma_s(B)) + (\sigma_a(C) + \sigma_s(C))$
= 4(759 + 3.6) + (0.0034 + 4.75) <Table II.3>
= 3055.15b
$\Sigma_t(B_4C) = N(B_4C) \times \sigma_t(B_4C) = 9.6679 \times 10^{-3} \times 10^{24} \times 3055.15 \times 10^{-24} = 29.5369\ cm^{-1}$
M(Al) = 26.9815
$N(Al) = \frac{2.699 \times 0.6022 \times 10^{24}}{26.9815} = 6.0239 \times 10^{-2} \times 10^{24}$ atoms/cm^3-sec
$\sigma_t(Al) = \sigma_a(Al) + \sigma_s(Al) = 0.23 + 1.49 = 1.72b$
$\Sigma_t(Al) = N(Al) \times \sigma_t(Al) = 6.0239 \times 10^{-2} \times 10^{24} \times 1.72 \times 10^{-24} = 0.1036\ cm^{-1}$
$N(\text{Compressed Al}) = \frac{2.608 \times 0.6022 \times 10^{24}}{26.9815} = 5.8208 \times 10^{-2} \times 10^{24}$ atoms/cm^3-sec
$\Sigma_t(\text{Compressed Al}) = N(\text{Compressed Al}) \times \sigma_t(Al) = 5.8208 \times 10^{-2} \times 10^{24} \times 1.72 \times 10^{-24}$
$= 0.1001\ cm^{-1}$
Penetrating probability
$= e^{-\Sigma_t x} = e^{-\{(29.5369+0.1001)\times 0.457+0.1036\times 0.25\times 2\}}$
$= 1 \times 10^{-6}$

9. What is the probability that a neutron can move one mean free path without interacting in a medium?

[sol] the probability that a neutron can move through this distance without having a collision
= $e^{-\Sigma_t x}$ <page 58>
= $e^{-\frac{1}{\lambda}\lambda}$ <one mean free path, $\Sigma_t = 1 / \lambda$>
= e^{-1} = 0.3679

10. A wide beam of neutrons of intensity $\emptyset_0$ is incident on a thick target consisting of material for which $\sigma_a \gg \sigma_s$. The target area is A and its thickness is X. Derive an expression for the rate at which neutrons are absorbed in this target.

[sol] neutrons absorption rate = $\sigma_t INAX$ (I : intensity of the beam, A : area, X : thickness) <Eq. 3.2>
a wide beam of neutrons of intensity : $\emptyset_0$
target area : A
target thickness : X
$\sigma_t = \sigma_a$ <$\sigma_a \gg \sigma_s$>
neutrons absorption rate = $\sigma_a \emptyset_0 NAX = \emptyset_0 \Sigma_a AX$

11. Stainless steel, type 304 having a density of 7.86 g/cm^3, has been used in some reactors. The nominal composition by weight of this material is as follows: carbon, 0.08%; chromium, 19%; nickel, 10%; iron, the remainder. Calculate the macroscopic absorption cross-section of SS-304 at 0.0253 eV.

[sol] $\Sigma_a = N\sigma_a$
$= \Sigma(N_i \sigma_{a(i)})$
$N_i = (w_i \rho N_A) / (100M_i)$ <Eq. 2.63>
M_{carbon} = 12.01115 <Table II.3>
$M_{chromium}$ = 51.996
M_{nickel} = 58.71
M_{iron} = 55.847

N_{carbon} = (0.08 × 7.86 × 0.6022 × 10^{24}) / (100 × 12.01115) = 3.1526 × 10^{-4} × 10^{24} atoms/cm^3
$N_{chromium}$ = (19 × 7.86 × 0.6022 × 10^{24}) / (100 × 51.996) = 1.7296 × 10^{-2} × 10^{24} atoms/cm^3
N_{nickel} = (10 × 7.86 × 0.6022 × 10^{24}) / (100 × 58.71) = 8.0622 × 10^{-3} × 10^{24} atoms/cm^3
N_{iron} = (70.92 × 7.86 × 0.6022 × 10^{24}) / (100 × 55.847) = 6.0108 × 10^{-2} × 10^{24} atoms/cm^3
$\sigma_{a(carbon)}$ = 0.0034 b <Table II.3>
$\sigma_{a(chromium)}$ = 3.1 b
$\sigma_{a(nickel)}$ = 4.43 b
$\sigma_{a(iron)}$ = 2.55 b
Σ_a = (3.1526 × 10^{-4} × 0.0034) + (1.7296 × 10^{-2} × 3.1) + (8.0622 × 10^{-3} × 4.43)
+ (6.0108 × 10^{-2} × 2.55) = 0.2426 cm^{-1}

12. Calculate at 0.0253 eV the macroscopic absorption cross-section of uranium dioxide (U_2O), in which the uranium has been enriched to 3 w/o in ^{235}U. The density of UO_2 is approximately 10.5 g/cm^3.

[sol] Σ_a = N × σ_a
Atomic weight of U : 1/M_u = 1/100(3/235.0439 + 97/238.0508) <Eq. 2.65>
→ M_u = 237.9595
Molecular weight of UO_2 = 237.9595 + 2 × 15.9994 = 269.9583
N = ρN_A / M <Eq. 2.59>
N(UO_2) = (10.5 × 0.6022 × 10^{24}) / 269.9583 = 2.3423 × 10^{22}
3% is not a volume percent → volume percent of ^{235}U = 237.9595/235.0439 × 0.03 = 0.030372
volume percent of ^{238}U = 1 - 0.030372 = 0.969628
Σ_a(UO_2) = N(UO_2) × σ_a(UO_2)
= (2.3423 × 10^{22}) × {0.030372 × 680.8 + 0.969628 × 2.7 + 2 × 0.00027) × 10^{-24}}
= 0.5457 cm^{-1}

13. The compositions of nuclear reactors are often stated in volume fractions, that is, the fractions of the volume of some region that are composed of particular materials. Show that the macroscopic absorption cross-section for the equivalent homogeneous mixture of materials is given by

$$\sum_a = f_1 \sum_{a1} + f_2 \sum_{a2} + \cdots\cdots$$

Where f_i and $\sum_{ai}$ are, respectively, the volume fraction and macroscopic absorption cross-section of the ith constituent at its normal density

[Sol]

$$\sum_a = \mathbf{N_1\sigma_{a1} + N_2\sigma_{a2} + N_3\sigma_{a3}} \cdots\cdots$$

,where $N_1=Nf_1$, $N_2=Nf_2$, $N_i=Nf_i$ ...

$$\sum_a = \mathbf{N_1\sigma_{a1} + N_2\sigma_{a2} + N_3\sigma_{a3} \cdots = Nf_1\sigma_{a1} + Nf_2\sigma_{a2} + Nf_3\sigma_{a3}} \text{....}$$

N σ = $\sum_a$ So.

$$\therefore \sum_a = \mathbf{f_1} \sum_{a1} + \mathbf{f_2} \sum_{a2} + \cdots\cdots$$

14. Calculate $\sum_a$ and $\sum_f$ (at 0.0253eV) for the fuel pellets described in Problem 2.63. The pellet density is about 10.6 g/cm^3

[sol]
At Prob. 2.63 pellet composed of mixed oxides, UO_2 and PuO_2 with the PuO_2 comprising approximately 30w/o of the mixture. 100% ^{238}U and plutonium contains the following isotopes ^{239}Pu(70.5w/o), ^{240}Pu(21.3w/o), ^{241}Pu(5.5w/o) and ^{242}Pu(2.75w/o)
$\boldsymbol{\rho = 10.6 g/cm^3}$

$\frac{M(PuO_2)}{M(UO_2 + PuO_2)} = \mathbf{30\%}$ **,** $\rho(UO_2)$**=0.7×** $\mathbf{10.6 = 7.42 g/cm^3}$**,** $\rho(PuO_2)$**=0.3×** $\mathbf{10.6 = 3.18 g/cm^3}$

$$N(UO_2) = \frac{7.42\times0.6022\times10^{24}}{(238.0508+2\times15.999)} = 0.0165\times10^{24}\ \text{cm}^{-3}$$

$$N(PuO_2) = \frac{3.18\times0.6022\times10^{24}}{(239.404+2\times15.999)} = 7.056\times10^{21}\ \text{cm}^{-3}$$

$$\sigma_a(UO_2) = \sigma_a(U) + 2\sigma_a(O) = 2.7b + 2\times0.00027b = 2.70054b$$
$$\sigma_a(PuO_2) = \sigma_a(Pu) + 2\sigma_a(O)$$
$$\sigma_a(Pu) = a(^{239}Pu)\,\sigma_a(^{239}Pu) + a(^{240}Pu)\,\sigma_a(^{240}Pu) + a(^{241}Pu)\,\sigma_a(^{241}Pu) + a(^{242}Pu)\,\sigma_a(^{242}Pu)$$

$$a(^{239}Pu) = \frac{M(^{239}Pu)}{M(Pu)}\times0.705 = \frac{239}{239.404}\times0.705 = 0.7038$$

$$a(^{240}Pu) = \frac{M(^{240}Pu)}{M(Pu)}\times0.213 = \frac{240}{239.404}\times0.213 = 0.2135$$

$$a(^{241}Pu) = \frac{M(^{241}Pu)}{M(Pu)}\times0.055 = \frac{241}{239.404}\times0.055 = 0.0554$$

$$a(^{242}Pu) = \frac{M(^{242}Pu)}{M(Pu)}\times0.0275 = \frac{242}{239.404}\times0.0275 = 0.0277$$

$$\therefore\ \sigma_a(PuO_2) = \sigma_a(Pu) + 2\sigma_a(O) = 0.7038\times1011.3 + 0.2135\times289.5 + 0.0554\times1377 + 0.0277\times18.5 + 2\times0.00027 = 850.864840 \cong 850.86b$$

$$\Sigma_a(UO_2) = N(UO_2)\sigma_a(UO_2) = 0.0165\times2.70054 = 0.044559\text{cm}^{-1}$$
$$\Sigma_a(PuO_2) = N(PuO_2)\sigma_a(PuO_2) = 7.056\times10^{-3}\times850.86 = 6.0037\text{cm}^{-1}$$

$$a(UO_2) = \frac{M(UO_2+PuO_2)}{M(UO_2)}\times\gamma(UO_2) = \left(\frac{(238.0508+2\times15.999)\times0.7+(239.404+2\times15.999)\times0.3}{238.0508+2\times15.999}\right)\times0.7 = 0.701$$
$$a(PuO_2) = 0.299$$

$$\therefore\ \Sigma_a = 0.701\times0.044559 + 0.299\times6.0037 = 1.82634 \cong 1.826\text{cm}^{-1}$$

$$\sigma_f(UO_2) = 0\ (\text{at } 0.0253\text{eV}),$$
$$\sigma_f(PuO_2) = 0.7038\times742.5 + 0.2135\times0.03 + 0.0554\times1009 + 0.0273\times0.2 = 578.48b$$

$$\Sigma_f(UO_2) = 0$$
$$\Sigma_f(PuO_2) = 7.056\times10^{-3}\times578.48 = 4.0818\text{cm}^{-1}$$

$$\therefore\ \Sigma_f = 0.299\times4.0818 = 1.22\text{cm}^{-1}$$

15. Using the fact that the scattering cross-section of ^{209}Bi is approximately 9 b from 0.01 eV to 200 eV, estimate the radius of the ^{209}Bi nucleus and compare with Eq.(2.3)

[Sol]

1) At low energy .. assume that the cross-section is $4\pi R^2$
So, $4\pi R^2 = 9b = 9\times10^{-24}\text{cm}^2$

$$\therefore R = \sqrt{\frac{9\times10^{-24}\text{cm}^2}{4\pi}} = 8.462843753\times10^{-13}\text{cm} \cong 8.46\times10^{-13}\text{cm}$$

2) using Eq 2.3
$R = 1.25\text{fm}\times A^{1/3}$ - Eq(2.3)
$R = 1.25\times10^{-13}\times\sqrt[3]{209} = 7.418090175\times10^{-13}\text{cm} \cong 7.42\times10^{-13}\text{cm}$

16. Using the Breit-Wigner formula. Compute and plot σ_γ in the vicinity of the first resonance in ^{238}U, which occurs at an energy of 6.67eV. The parameters of this resonance are: Γ_n=1.52meV(meV = millielectron volts), $\Gamma_\gamma = 26$meV, and g = 1. [Note: For reasons given in Section 7.3(see in particular Fig.7.12), the value of σ_γ computed from the Breit-Wigner formula do not concide with those measured with targets at room temperature.]

[Sol]
Breit-Wigner Formula

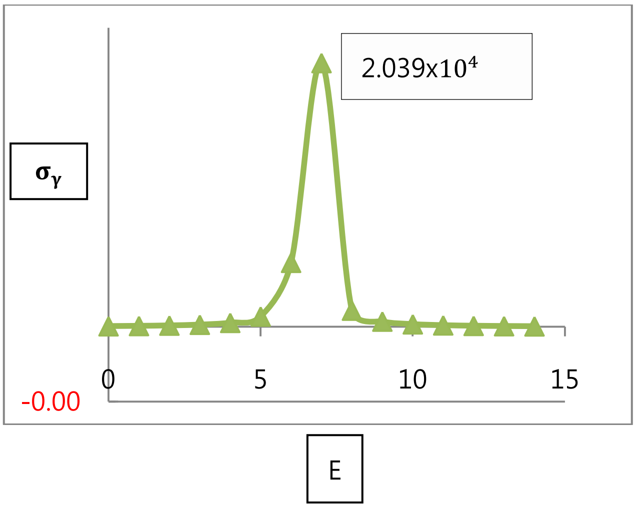
$$\sigma_\gamma = \frac{\gamma_r^2 g}{4\pi}\frac{\Gamma_n \Gamma_g}{(E-E_r)^2+\Gamma^2/4} \quad \cdots\cdots (Eq3.23)$$

g = 1, Γ_n=1.52meV, $\Gamma_\gamma = 26$meV, $\Gamma = \Gamma_n + \Gamma_\gamma$= 1.52 + 26 = 27.52meV

$$\gamma_r = \frac{2.86\times10^{-9}}{\sqrt{E_r}} = \frac{2.86\times10^{-9}}{\sqrt{6.67eV}} = 1.107396423\times10^{-9} \cong 1.107\times10^{-9}cm$$

$$\sigma_\gamma = \frac{\gamma_r^2 g}{4\pi}\frac{\Gamma_n \Gamma_g}{(E-E_r)^2+\Gamma^2/4} = \frac{(1.107\times10^{-9})^2}{4\pi}\frac{1.52\times10^{-3}\times26\times10^{-3}}{(E-6.67)^2+(27.52\times10^{-3})^2/4}$$

$$= \frac{3.853916613\times10^{-24}}{(E-6.67)^2+1.89\times10^{-4}} \cong \frac{3.85\times10^{-24}}{(E-6.67)^2+1.89\times10^{-4}}$$



17. Demonstrate using the Breit-Wigner formula that the width of a resonance at half its height is equal to Γ.

[Sol]

$$\sigma(E) = \frac{\gamma_r^2 g}{4\pi}\frac{\Gamma_n \Gamma_g}{(E-E_0)^2+\Gamma^2/4} \quad \cdots\cdots (Eq3.23)$$

$\sigma(E_0) \rightarrow (E - E_0) = 0$ So,

$\sigma(E_0) = \frac{\gamma_r^2 g}{4\pi}\frac{\Gamma_n\Gamma_g}{\Gamma^2/4}$ **so** $\sigma(E) = \frac{\sigma(E_0)}{1+(\frac{E-E_0}{\Gamma/2})^2}$

$$\frac{\sigma(E)}{\sigma(E_0)} = \frac{1}{2} = \frac{1}{1+(\frac{E-E_0}{\Gamma/2})^2} \rightarrow \frac{\Gamma^2}{4} = (E-E_0)^2$$

$$\therefore E = E_0 \pm \sqrt{\frac{\Gamma^2}{4}} = E = E_0 \pm \frac{\Gamma}{2}$$

18. There are no resonances in the total cross-section of ^{12}C from 0.01 eV to cover 1 Me V. If the radiative capture cross-section of this nuclide at 0.0253 eV is 3.4 mb, what is the value of σ_γ at 1 eV?

[sol] $\boldsymbol{\sigma_\gamma}$ **is $\mathbf{1/v}$, it can be written as**

$$\boldsymbol{\sigma_\gamma(E) = \sigma_\gamma(E_0)\sqrt{\frac{E_0}{E}} - - - - (Ex\ 3.9)}$$

so

$$\boldsymbol{\sigma_\gamma(1eV) = \sigma_\gamma(0.0253eV)\sqrt{\frac{0.0253}{1}} = 3.4 \times 10^{-3}b \times \sqrt{0.0253} = 540.803 \times 10^{-6}b \cong 0.541mb}$$

19. The first resonances in the cross-section of aluminum, which is due entirely to scattering, occurs at 5.8keV. The absorption cross-section at 0.0253 eV is 0.23b. Calculate for 100 eV:

(a) σ_a

[Sol]

$\boldsymbol{\sigma_a(E) = \sigma_a(E_0)\sqrt{\frac{E_0}{E}}}$**, so**

$\boldsymbol{\sigma_a(100eV) = \sigma_a(0.0253)\sqrt{\frac{0.0253}{100}} = 0.23b \times \sqrt{\frac{0.0253}{100}} = 3.658373956 \times 10^{-3}b \cong 3.658mb}$

(b) σ_s

[Sol]

Low energy so it can assume $\boldsymbol{\sigma_s = 4\pi R^2 = 4\pi(1.25fm \times A^{1/3})^2}$

So $\boldsymbol{\sigma_s = 4\pi \times (1.25 \times 10^{-13} \times (27)^{1/3})^2 = 1.767145868 \times 10^{-24} \cong 1.767b}$

(c) σ_t

[Sol]

$\boldsymbol{\sigma_t = \sigma_a + \sigma_s = 3.658mb + 1.767b = 1.770658 \cong 1.771b}$

20. Calculate : $\sum_a$ for

(a) water of unit density at 0.0253 eV;

[Sol]

$\boldsymbol{\rho = 1g/cm^3}$, $\boldsymbol{\sigma_a}$ **of hydrogen : 333mb**, $\boldsymbol{\sigma_a}$ **of oxygen : 0.190mb at 0.0253eV**

$\boldsymbol{\sigma_a}$ **of water** $\boldsymbol{= 2 \times 333mb + 0.190mb = 666.19mb = 0.666b}$

$N = \frac{\rho N^{Av}}{M} = \frac{1 \times 0.6022 \times 10^{24}}{18.0153} = \boldsymbol{3.34271424 \times 10^{22} \cong 3.343 \times 10^{22}cm^{-3}}$

$$\sum\nolimits_a = N\,\boldsymbol{\sigma_a = 3.343 \times 10^{22} \times 0.666 \times 10^{-24} = 0.0226438 \cong 0.0226cm^{-1}}$$

(b) water of density $0.7g/cm^3$ at 0.0253eV;

[Sol]

$\boldsymbol{\rho = 0.7g/cm^3}$

$N = \frac{\rho N^{Av}}{M} = \frac{0.7 \times 0.6022 \times 10^{24}}{18.0153} \cong \boldsymbol{2.34 \times 10^{22}cm^{-3}}$

$$\sum\nolimits_a = N\,\boldsymbol{\sigma_a = 2.34 \times 10^{22} \times 0.666 \times 10^{-24} = 0.0155844 \cong 0.0156cm^{-1}}$$

(c) water of density $0.7g/cm^3$ at 1 eV.

[Sol]

$$\boldsymbol{\sum\nolimits_a(1eV) = \sum\nolimits_a(0.0253)\sqrt{\frac{0.0253}{1}} = 0.0156\sqrt{\frac{0.0253}{1}} = 2.48133319 \times 10^{-3} = 2.481 \times 10^{-3}cm^{-1}}$$

21. The first resonance in the scattering cross-section of the nuclide ^{A}Z occurs at 1.24 MeV. The separation energies of nuclides ^{A-1}Z, ^{A}Z, and ^{A+1}Z are 7.00, 7.50, and 8.00 Me V, respectively. Which nucleus and at what energy above the ground state is the level that gives rise to this resonance?

[Sol]

$\mathbf{E_s\left(^{A-1}Z\right) = 7.00MeV,\ E_s\left(^{A}Z\right) = 7.50MeV,\ E_s\left(^{A+1}Z\right) = 8.00MeV}$

$\mathbf{E_s}$ **is just sufficient to remove a neutron from the nucleus without providing it with any kinetic energy.**

. The first resonance in the scattering cross-section of the nuclide ^{A}Z occurs at 1.24 MeV

So it should occur the ^{A+1}Z and the energy is

8.00MeV + 1.24MeV = 9.24MeV

22. There is a prominent resonance in the total cross-section of ^{56}Fe at 646.4keV. At what energy, measured from the ground state, is the energy level in ^{57}Fe that corresponds to this resonance?[Hint: Use the masses of neutral ^{56}Fe and ^{57}Fe to compute the binding energy of the last neutron in ^{57}Fe]

[Sol]

$$\mathbf{E_s = \left[M_n + M\left(^{A-1}Z\right) - M\left(^{A}Z\right)\right] \times 931.481MeV - - - -Eq(2.45)}$$

$$\mathbf{E_s\left(^{57}Fe\right) = [1.008665 + 55.9349393 - 56.9353958] \times 931.481 = 7.646061789 \cong 7.646MeV}$$

∴ The energy level is 7.646MeV + 646.4$\times 10^{-3}$=8.2964MeV

23. The excited states of ^{17}O occur at the following energies (in Me V) measured from the ground state : 0.871, 3.06, 4.55,5.08,5.38,5.70,5.94, etc. At roughly what energies would resonances be expected to appear in the neutron cross-section of ^{16}O

[Sol]

$$\mathbf{E_s = \left[M_n + M\left(^{A-1}Z\right) - M\left(^{A}Z\right)\right] \times 931.481MeV - - - -Eq(2.45)}$$

$$\mathbf{E_s\left(^{17}O\right)=[1.008665 + 15.994915 - 16.991312] \times 931.481 = 4.141589694 \cong 4.144MeV}$$

The first energy above the 4.144MeV is 4.55 MeV s

∴ 4.55 – 4.144 =0.406MeV this energy will be appear when the resonances

24. Using Eq. (3.28), compute and plot E'/E as a function of angle from 0 to π for A = 1, 12 and 238.

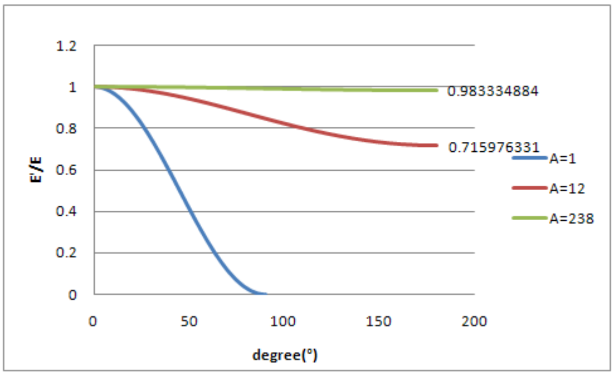
[Sol] Eq. (3.28) $\mathbf{E' = \frac{E}{(A+1)^2}[\cos\vartheta + \sqrt{(A^2 - \sin^2\vartheta)}]^2}$

$$\mathbf{\frac{E'}{E} = \frac{[\cos\vartheta + \sqrt{(A^2 - \sin^2\vartheta)}]^2}{(A+1)^2}}$$

$$\mathbf{A= 1,\ \frac{E'}{E} = \cos^2\vartheta\ (0 \le \vartheta \le \frac{\pi}{2}),\ \frac{E'}{E} = 0\ (\frac{\pi}{2} \le \vartheta \le \pi)}$$

$$\mathbf{A=12,\ \frac{E'}{E} = \frac{[\cos\vartheta + \sqrt{(144 - \sin^2\vartheta)}]^2}{13^2}(0 \le \vartheta \le \pi)}$$

$$\mathbf{A=238,\ \frac{E'}{E} = \frac{[\cos\vartheta + \sqrt{(238^2 - \sin^2\vartheta)}]^2}{239^2}(0 \le \vartheta \le \pi)}$$



25. A 2-MeV neutron traveling in water has a head-on collision with an ^{16}O nucleus.

(a) What are the energies of the neutron and nucleus after the collision?

[Sol] $E' = \frac{E}{(A+1)^2}[\cos\vartheta + \sqrt{(A^2 - \sin^2\vartheta)}]^2$; **E=2 MeV, A=16, θ=π**

$$E' = \frac{2}{17^2}[\cos\pi + \sqrt{(16^2 - \sin^2\pi)}]^2 = 1.557\ \text{MeV}$$

$$E_A = E - E' = 2 - 1.557 = 0.443\ \text{MeV}$$

(b) Would you expect the water molecule involved in the collision to remain intact after the event?

[Sol] By the laws of conservation of momentum, the water molecule was moved towards direction of initial neutron.

26. A 1-MeV neutron strikes a ^{12}C nucleus initially at rest. If the neutron is elastic scattered through an angle of 90°:

(a) What is the energy of the scattered neutron?

[Sol] $E' = \frac{E}{(A+1)^2}[\cos\vartheta + \sqrt{(A^2 - \sin^2\vartheta)}]^2$; **E=1 MeV, A=12, θ=π/2**

$$E' = \frac{1}{13^2}[\cos\frac{\pi}{2} + \sqrt{(12^2 - \sin^2\frac{\pi}{2})}]^2 = 0.846\ \text{MeV}$$

(b) What is the energy of the recoiling nucleus?

[Sol] $E_A = E - E' = 1 - 0.846 = 0.154\text{MeV}$

(c) At what angle does the recoiling nucleus appear?

[Sol] $p' = \sqrt{2mE'}$

$P = \sqrt{2ME_a}$

Y-axle) $p' = P\sin\varphi$

$\sin\varphi = \sqrt{(mE'/ME_A)} = \sqrt{(1.00866 \times 0.846)/(12 \times 0.154)}$

$\varphi = \sin^{-1}(0.6795) = 42.8\text{degree}$

Other solution

$p = \sqrt{2mE}$

$P = \sqrt{2ME_a}$

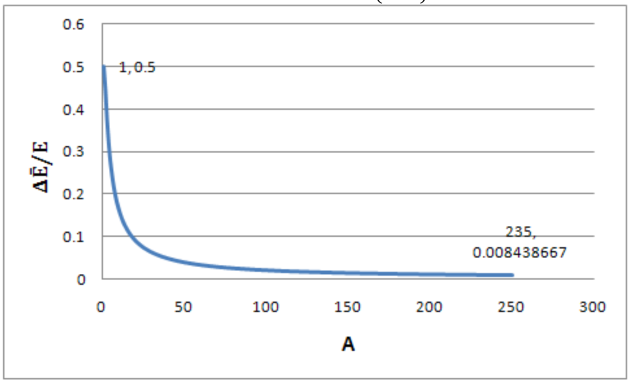
X-axle) p= $P\cos\varphi$

$\cos\varphi = \sqrt{(mE/ME_A)} = \sqrt{(1.00866 \times 1)/(12 \times 0.154)}$

$\varphi = \cos^{-1}(0.7388) = 42.4\text{degree}$

27. Compute and plot the average fractional energy loss in elastic scattering as a function of the mass number of the target nucleus.

[Sol] Eq. (3.32) $\frac{\overline{\Delta E}}{E}=\frac{1}{2}(1-\alpha)=\frac{2A}{(A+1)^2}$



28. Show what the average fractional energy loss in% in elastic scattering for large A is given approximately by

$$\frac{\overline{\Delta E}}{E}\cong\frac{200}{A}$$

[Sol] $\frac{\overline{\Delta E}}{E}=\frac{1}{2}(1-\alpha)=\frac{2A}{(A+1)^2}=\frac{2A}{A^2+2A+1}=\frac{2}{A+2+\frac{1}{A}}\cong\frac{2}{A}=\frac{200}{A}(\%)$

29. A 1.5-Me V neutron in a heavy water reactor collides with an ^2H nucleus. Calculate the maximum and average changes in lethargy in such a collision.

[Sol] Eq. (3.28) $E'=\frac{E}{(A+1)^2}[\cos\vartheta+\sqrt{(A^2-\sin^2\vartheta)}]^2$

At $\theta=\pi$, Lethargy is maximum.

A=2, $E'=(\frac{A-1}{A+1})^2E=\frac{E}{9}$

$u=\ln(E_M/E)=\ln(E/E')=\ln 9=2.197$

Average change, $\xi=\overline{\Delta u}=1-\frac{(A-1)^2}{2A}\ln\left(\frac{A+1}{A-1}\right)=1-\frac{1}{4}\ln 3=0.725$

30. Suppose that a fission neutron, emitted with energy of 2 MeV, shows down to an energy of 1 eV as the result of successive collisions in a moderator. If, on the average, the neutron gains in lethargy the amount ξ in each collision, how many collisions are required if the moderator is

(a) Hydrogen,

[Sol] $\overline{E'}=\frac{1}{2}(1+\alpha)E$

After the number of n time collisions was occurred, $(\overline{E'})_n=[\frac{1}{2}(1+\alpha)]^nE$

$\alpha=0,(\overline{E'})_n=[\frac{1}{2}(1+\alpha)]^nE=(\frac{1}{2})^nE$

$1_n=(\frac{1}{2})^n\times 2\times 10^6$

$n=20.93$

(b) graphite?

[Sol] $\overline{E'}=\frac{1}{2}(1+\alpha)E$

After the number of n time collisions was occurred, $(\overline{E'})_n=[\frac{1}{2}(1+\alpha)]^nE$

$\alpha=(\frac{A-1}{A+1})^2=0.716,\ (\overline{E'})_n=[\frac{1}{2}(1+\alpha)]^nE=(\frac{1.716}{2})^nE$

$1_n=(\frac{1.716}{2})^n\times 2\times 10^6$

$n=94.73$

31. The 2,200 meters-per second flux in an ordinary water reactor is 1.5 × 10^{13} neutrons/cm^2-sec. Ate what rate are the thermal neutrons absorbed by the water?

[Sol] $\mathbf{F_a = \Sigma_a(E_0)\phi_0 = N\sigma_a\phi_0 = 0.03343 \times 0.664 \times 1.5 \times 10^{13} = 3.330 \times 10^{11} netrons/cm^3 sec}$

32. At one point in a reactor, the density of thermal neutrons is 1.5 × 10^8 neutrons/cm^2. If the temperature is 450℃, what is the 2,200 meters-per-second flux?

[Sol] $\mathbf{at\ T = 450℃, n = 1.5 \times 10^8 neutrons/cm^3}$

$\mathbf{\phi_0 = nv_0 = 1.5 \times 10^8 \times 2200 \times 10^2 = 3.3 \times 10^{13} nutrons/cm^2 sec}$

33. A tiny beryllium target located at the center of a three-dimensional Cartesian coordinate system is bombarded by six beams of 0.0253-eV neutrons of intensity 3 × 10^8 neutrons/cm^2-sec, each incident along a different axis.

(a) What is the 2,200 meters0per-second flux at the target?

[Sol] $\mathbf{\phi = 6 \times 3 \times 10^8 = 1.8 \times 10^9 neutrons/cm^2 sec}$

(b) How many neutrons are absorbed in the target per cm^2-sec?

[Sol] $\mathbf{F_a = \Sigma_a(E_0)\phi_0 = N\sigma_a\phi_0 = 0.1236 \times 0.0092 \times 1.8 \times 10^9 = 2.047 \times 10^6 netrons/cm^3 sec}$

34. When thermal neutrons interact with ^{14}N, what is the probability that absorption leads to radioactive capture?

[Sol] $\mathbf{Probability\ of\ the\ catpture\ to\ absorption = \frac{\sigma_r}{\sigma_a} = \frac{\sigma_r}{\sigma_r+\sigma_f} = \frac{0.075}{0.075+1.366} = 0.052 = 5.2}\%$

35. The control rods for a certain reactor are made of an alloy of cadmium (5 w/o), indium (15 w/o), and silver (80 w/o). Calculate the rate at which thermal neutrons are absorbed per gram of this material at a temperature of 400℃ in a 2,200 meters-per-second flux of 5*10^{13}neutrons/cm^2-sec. [Note: Silver is a 1/v absorber.]

[sol] Eq.(3.42)~(3.44)

$\mathbf{The\ absorption\ rate, F_a = g_a(T)\Sigma_a(E_0)\phi_0 = g_a(T)\sigma_a N\phi_0}$

To calculate the atomic abundance of each elements,

$\mathbf{\frac{1}{M} = \frac{1}{100}\left(\frac{5}{112.4} + \frac{15}{114.82} + \frac{80}{107.87}\right) = 9.1724 \times 10^{-13}, M = 109.02\ amu}$

$\mathbf{a(Cd) = \frac{109.02}{112.4} \times \frac{5}{100} = 0.049}$

$\mathbf{a(In) = \frac{109.02}{114.82} \times \frac{15}{100} = 0.1424}$

$\mathbf{a(Ag) = \frac{109.02}{107.87} \times \frac{80}{100} = 0.8085}$

$\mathbf{F_a = (g_a(T)\sigma_a N\phi_0)_{Cd} + (g_a(T)\sigma_a N\phi_0)_{In} + (N\phi_0)_{Ag}}$

$\mathbf{= (2.559 \times 5.36 \times 10^{21} \times 2450 \times 10^{-24} \times 5 \times 10^{13} \times 0.049) + (1.0101 \times 5.24 \times 10^{21} \times 193.5 \times 10^{-24} \times 5 \times 10^{13} \times 0.143) + (5.58 \times 10^{21} \times 63.6 \times 10^{-24} \times 5 \times 10^{13} \times 0.808 = 1.0465 \times 10^{14} neutrons/(g * sec)}$

36. From the data in Table 3.3, would you expect ^{232}Th to be fissile? If not, at what neutron energy would you expect fission to be possible?

[sol] ^{232}Th + ^{1}n = ^{233}Th, the critical energy of ^{233}Th is 5.1Me V, and B.E is 6.5Me V.

K.E = critical energy - B.E = 1.4Me V

∴ Higher than 1.4Me V

37. Two hypothetical nuclei, ^AZ and ^{A+1}Z, of atomic weights M(^AZ)=241.0600 and M(^{A+1}Z)=242.0621 have critical fission energies of 5.5 MeV and 6.5 MeV, respectively. Is the nuclei ^AZ fissile?

[sol] $\mathbf{E_s = [1.008665 + 241.0600 - 242.0621] \times 931 = 6.11 Me\ V}$

$\mathbf{B.E} < \textit{critical energy , so this hypothetical nuclei} \ (^{A}\mathbf{Z})$ **is not a fissile.**

38. Fission can be induced when γ-rays are absorbed by a heavy nucleus. What energy γ-rays are necessary to induce fission in
(a) ^{235}U

[sol] 5.75 Me V

(b)^{238}U

[sol] 5.85Me V

(c)^{239}Pu

[sol] 5.5Me V

39. Cross-sections of ^{235}U at 1 MeV are as follows: σ_s=4.0b, σ_i=1.4b, σ_f=1.2b, σ_a=1.3b. The cross-sections for neutron-producing and charged particle reactions are all negligible. Compute at this energy
(a) the total cross-section

[sol] $\boldsymbol{\sigma_s = \sigma_e + \sigma_i,\ \sigma_a = \sigma_f + \sigma_{\gamma+\cdots}}$ **so,**

$$\boldsymbol{\sigma_t = \sigma_s + \sigma_a = 4.0 + 1.3 = 5.3\ b}$$

(b) the capture-to-fission ratio α

[sol] $\boldsymbol{\alpha = \frac{\sigma_\gamma}{\sigma_f} = \frac{\sigma_a - \sigma_f}{\sigma_f} = \frac{1.3-1.2}{1.2} = 0.08333}$

40. The fission cross-product ^{131}I has a half-life of 8.05 days and is produced in fission with a yield of 2.9%-that is, 0.029 atoms of ^{131}I are produced per fission. Calculate the equilibrium activity of this radionuclide in a reactor operating at 3,300 MW.

[sol] $\boldsymbol{\frac{dn}{dt} = -\lambda t + R}$

$$\boldsymbol{n = n_0 e^{-\lambda t} + \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right),\ \text{and}\ n_0 = 0}$$

$$\boldsymbol{\alpha = R\left(1 - e^{-\lambda t}\right)}$$

$$\boldsymbol{\alpha_E = \lim_{t\to\infty} R\left(1 - e^{-\lambda t}\right) = R = 2.987 \times 10^{18}\ dps = 8.073 \times 10^{7} Ci}$$

41. Fission-product activity measured by at the time t_0 following the burst of a nucleus weapon is found to be α_0. Show that the activity at the time $t = 7^n t_0$ is given approximately by $\alpha = \alpha_0/10^n$. This is known as the 7-10 rule in civil defense.

[sol] At 83p, fission product activity

$$\boldsymbol{\cong 1.03 \times 10^{-16} t^{-1.2} Ci = 1.03 \times 10^{-16} (7^n t_0)^{-1.2} = 1.03 \times 10^{-16} \times t_0^{-1.2} \times (7^n)^{-1.2}}$$

$$\boldsymbol{= \alpha_0 (7^{1.2})^{-n}}$$

$$\boldsymbol{\alpha = \frac{\alpha_0}{10.33^n}}$$

42. Suppose that radioactive fallout from a nuclear burst arrives in a locality a hour after detonation. Use the result of Problem 3.41 to estimate the activity 2 week later.

[sol] $\boldsymbol{\alpha = 1.03 \times 10^{-16} t^{-1.2},\quad t_0 = 1hr = 0.04167\ d}$

$$\boldsymbol{\alpha_0 = 1.03 \times 10^{-16} t_0^{-1.2} = 4.663 \times 10^{-15} Ci}$$

$$\boldsymbol{t = 7^n t_0,}$$

$$\boldsymbol{7^n = \frac{t}{t_0},\quad n\ln 7 = \ln\left(\frac{t}{t_0}\right)}$$

$$\boldsymbol{n = \frac{1}{\ln 7} \ln\left(\frac{14}{0.04167}\right) = 2.989}$$

$$\boldsymbol{\alpha = \frac{\alpha_0}{10^n} = \frac{4.663 \times 10^{-15}}{10^{2.989}} = 4.7826 \times 10^{-18} Ci}$$

43. The yields of nuclear weapons are measured in kilotons (KT), where 1 KT =2.6*10^{25} MeV. With this in mind, (a) How much ^{235}U is fissioned when a 100-KT bomb is expoded?

[sol] $\mathbf{100KT = 2.6 \times 10^{27} Me\ V,}$

$$\mathbf{2.6 \times 10^{27} Me\ V \times \frac{1\ fission}{200Me\ V} \times \frac{1\ mole}{0.6022 \times 10^{24} fissions} \times \frac{235.0439\ g}{1\ mole} = 5074\ g}$$

$\mathbf{\therefore 5.074\ kg}$

(b) What is the total fission-product activity due to this bomb 1 min, 1 hr, and 1 day after detonation? [Note: Assume a thermal energy release of 200 MeV per fission.]

[sol] $\mathbf{\alpha = 1.03 \times 10^{-16} t^{-1.2}}$

$$\mathbf{for\ 1\ min,\ \ \alpha = 1.03 \times 10^{-16} \left(1min \times \frac{1hr}{60min} \times \frac{1\ d}{24hr}\right)^{-1.2} = 6.351 \times 10^{-13} Ci}$$

$$\mathbf{for\ 1\ hr,\ \ \alpha = 1.03 \times 10^{-16} \left(1hr \times \frac{1d}{24hr}\right)^{-1.2} = 4.667 \times 10^{-15} Ci}$$

$$\mathbf{for\ 1\ day,\ \ \alpha = 1.03 \times 10^{-16} (1)^{-1.2} = 1.03 \times 10^{-16} Ci}$$

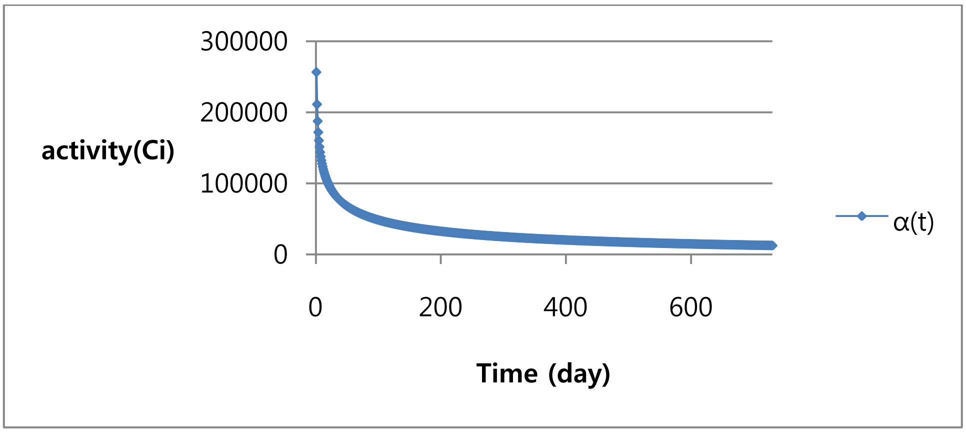
44. A research reactor is operated at a power of 250 kilowatts 8 hours a day, 5 days a week, for 2 years. A fuel element, 1 of 24 in the reactor, is then removed for examination. Compute and plot the activity of the fuel element as a function of time up to 2 years after removal.

[sol] Fission product activity = $\mathbf{1.4 \times 10^{16} P[t^{-0.2} - (t+T)^{-0.2}] Ci}$

$$\mathbf{T = \frac{8\ hr}{1\ d} \times \frac{5\ d}{1\ wk} \times \frac{\frac{365}{7} wk}{1\ yr} \times 2\ yr \times \frac{1\ d}{24\ hr} = 173.8\ d}$$

$$\mathbf{P = 250\ kW = 0.25MW}$$

$$\mathbf{\alpha = 1.4 \times 10^{16} \times 0.25[(730)^{-0.2} - (730 + 173.8)^{-0.2}] = 3.915 \times 10^{3} Ci}$$



45. The spontaneous fission rate of ^{238}U is 1 fission per gram per 100 sec. Show that this is equivalent to a half-life for fission of 5.5*10^{15} years.

[sol] $\mathbf{Fission\ rate = \frac{1\ fission}{g * 100sec} = 0.01\ fission/(g * sec)}$

$$\mathbf{\lambda N = \frac{\ln 2}{T_{\frac{1}{2}}} \times \frac{1 \times 0.6022 \times 10^{24}}{238.0508} = 0.01}$$

$$\mathbf{So,\ T_{\frac{1}{2}} = \frac{0.01 \times 238.0508 \times 5.5 \times 10^{15}}{\ln 2 \times 0.6022 \times 10^{24}} = 1.753 \times 10^{26} sec}$$

$$\mathbf{1.753 \times 10^{26} \times \frac{1\ hr}{3600\ sec} \times \frac{1\ d}{24\ hr} \times \frac{1\ yr}{365\ d} = 5.56 \times 10^{15}\ yr}$$

46. Compute and plot the parameter ŋ at 0.0253 eV for uranium enriched in ^{235}U as a function of its enrichment in weight percent ^{235}U.

[Sol]

$N_i = (x/100) \cdot (\rho_{Ni} \ / M_i)$ where x = w/o, ρ =19.1 g/cm^3

$\eta = \nu(235) \bullet \sum_f(235) \ / [\ \sum_a(235) + \sum_a(238)\ ]$

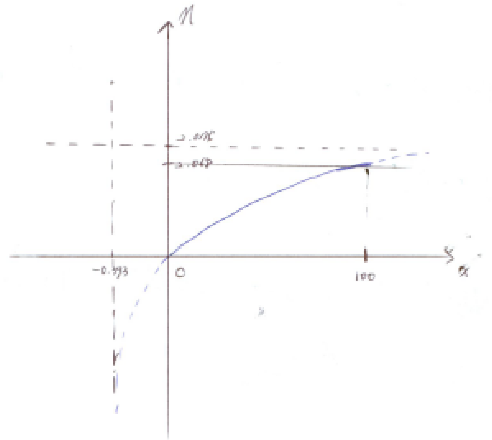
$= \nu(235) \bullet N(235) \bullet \sigma_f(235) \ / \ N(235) \bullet \sigma_a(235) + N(238) \bullet \sigma_a(238)$

$$= \frac{2.418 \ \text{x} \left(\frac{x}{100}\right) \text{x} \left(19.1 \ \text{x} \ 0.6022 \times 582.2 \div 235.0439\right)}{\left(\frac{x}{100}\right) \times \left(19.1 \ \text{x} \ 0.6022 \times 680.8 \div 235.0439\right) + \left(\frac{100-x}{100}\right) \times \left(19.1 \ \text{x} \ 0.6022 \times 2.70 \div 238.0508\right)}$$

$= 0.6889X / [0.333X + (100-X)1.305 \times 10^{-3}]$

$= 0.6889X / (0.1305 + 0.332X) = 2.075(X + 0.332 - 0.332)/(X + 0.393)$

$= 2.075 - 0.6889/(X + 0.393)$



47. Suppose that 1 kg of ^{235}U undergoes fission by thermal neutron. Compute the masses (or mass equivalents) in grams for the following, which are produced:
(a) neutrons
(b) β-rays
(c) γ-rays
(d) neutrinos,
(e) kinetic energy
(f) fission products.

[Sol]

A number of fission occurred by 1kg of ^{235}U

$1000 \times 0.6022 \times 10^{24}/(235.0439) = 2.56 \times 10^{24}$ **fissions**

(a) neutron

$\nu(235) = 2.418$

$2.418 \times 2.56 \times 10^{24}$ **neutrons**

(b) β-rays

2.56×10^{24} **β-rays x (8 Me V/β-ray) =** 2.408×10^{25} **Me V**

(c) γ-rays

$2.56 \times 10^{24} \times 7 = 1.792 \times 10^{25}$ **Me V**

d) neutrinos

$2.56 \times 10^{24} \times 12 = 3.072 \times 10^{25}$ **Me V**

(e) kinetic energy

$2.56 \times 10^{24} \times 5 = 1.28 \times 10^{24}$ **Me V**

(f) fission products.

$2.56 \times 10^{24} \times (8+7+12) = 6.912 \times 10^{25}$ Me V

48. The reactor on the nuclear ship Savannah operated at a power of 69 MW
(a) How much ^{235}U was consumed on a 10.000-nautical mile voyage at an average speed of 20 knots?
(b) This is equivalent to how many barrels of 6.5-millon-Btu/barrel bunker-C oil?

[Sol]
a) Consumption rate = $1.05(1+\alpha)\cdot P$ (g/day)
$=1.05(1+0.169)\cdot 69$ α value can be seen from Table 3.4
$=84.694$ (g/day)

Hours under voyage
t = 10000 nautical mile/20knots = 500Hr = 20.83days
Masses of consumed ^{235}U
$= 84.694 \times 20.83 = 1764.46$ g

b) Total work = 69MW $\times$ 500Hr
$= 34500$ MW·Hr $\times$ (1Btu/2.931×10^{-7} MW·Hr)
$= 1.777 \times 10^{11}$ Btu
The amounts of consumed bunker-C oil
$=1.777 \times 10^{11}$ Btu / (6.5×10^{6} Btu/barrel) $=1.811 \times 10^{4}$ barrels

49. Consolidated Edison's Indian Point No 2 reactor is designed to operate at a power of 2.758 MW. Assuming that all fission occur in ^{235}U, calculate in grams per day the rate at which ^{235}U is
(a) fissioned,
(b) consumed.

[Sol]
a) Fission rate
$= 2758$ MW $\times$ (10^{6} J/ MW) $\times$ (fission/200 Me V) $\times$ (Me V/1.6×10^{-13} J) $\times$ (86400 sec/day)
$=7.447 \times 10^{24}$ fissions/day
b) Consumption rate
$= 7.447 \times 10^{24}$ fissions/day $\times$ 235 g/mole $\div$ 0.6022×10^{24} fissions/mole $\times$ $(1+0.169)$
$= 3397.2$ g/day

50. Referring to the preceeding problem, what is the total accumulated activity of the fission products in the Indian Point No. 2 reactor 1 day after shutdown following 1 year of operation?

[Sol]

$$\alpha = 1.4 \times 10^{6} \times P\,[t^{-0.2} - (t+T)^{-0.2}]$$
$$= 1.4 \times 10^{6} \times 2758 \times [1^{-0.2} - (1+365)^{-0.2}]$$
$$= 2.675 \times 10^{9}\,\text{Ci}$$

51. The photoelectric cross-section of lead at 0.6 Me V is approximately 18b. Estimate σ_{pe} at this energy for Uranium.

[Sol]
$\sigma_{pe} \sim Z^{n}$
$\sigma_{pe} = \text{const.}\ Z^{n}$
From fig. 3.14, the value of n for 0.6 Me V-γ-ray $\cong$ 4.35
$\therefore$ 18b=const.$(82)^{4.35}$

Const. = $18 \times (82)^{-4.35} = 8.515 \times 10^{-8}$C

For Uranium

$$\sigma_{pe} = 8.515 \times 10^{-8} Z^{n}$$
$$= 8.515 \times 10^{-8}(92)^{4.35}$$
$$= 29.69\text{b}$$

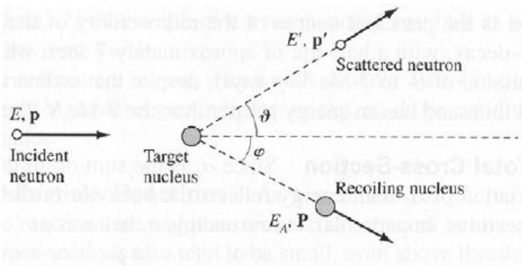
52. A 2-Me V photon is Compton scattered through an angle of 30°.
(a) What is its energy after scattering?
(b) What is the recoil energy of struck electron?
(c) At what angle does the electron appear?

[Sol]

a) $E' = E E_e /2(1-\text{COS}\theta) + E_e = (2 \times 0.511) \div [2 \times (1-\text{COS}30°)+0.511] = 1.312$ Me V

b) Recoil energy = $E - E' = 2-1.312 = 0.688$ Me V

c) $\theta = 30°$



$P\sin\varphi = p'\sin 30°$

$\sin\varphi = p'/P \times 1/2$

$p' = h/\lambda' = h/[(1.240 \times 10^{-4}) \div E'] = 4.1357 \times 10^{-15}\text{eV}\cdot\text{sec}/[(1.240 \times 10^{-4}) \div 1.312 \times 10^{6}]\cdot\text{cm}$

$= 4.376 \times 10^{-7}$ Me V·sec/cm

$= (4.376 \times 10^{-7}$ Me V·sec/cm$) \times [(1.6 \times 10^{-6}$ erg)/Me V$] \times [(\text{g}\cdot\text{cm}^2/\text{sec}^2)/\text{erg}]$

$= 7.01 \times 10^{-17}$ g·cm/sec

$P = \sqrt{m_e \cdot E_e'} = \sqrt{2} \times 9.10956 \times 10^{-28} \times 0.688 \times 1.602 \times 10^{-6} = 4.48 \times 10^{-17}$ g·cm/sec

$\sin\varphi = p'/2P = 7.01 \times 10^{-17}/4.48 \times 10^{-17} = 0.780$

$\therefore \varphi = 51.44°$

53. Show that eq. (3.62) follows from eq. (3.61).

[Sol]

$E' = E E_e /2(1-\text{COS}\theta) + E_e$

$\lambda = h/p = hc/E$, $E = hc/\lambda$

$hc/\lambda' = (hc/\lambda \cdot hc/\lambda_e) /[hc/\lambda \cdot (1-\text{COS}\theta) + hc/\lambda_e)]$

$hc/\lambda'[hc/\lambda \cdot (1-\text{COS}\theta) + hc/\lambda_e)] = (hc)^2/\lambda \cdot \lambda_e$

$(1-\text{COS}\theta)/\lambda' \cdot \lambda + (1/\lambda' \cdot \lambda_e) = 1/\lambda \cdot \lambda_e)$

$1/\lambda_e \cdot (1/\lambda - 1/\lambda') = (1-\text{COS}\theta)/\lambda' \cdot \lambda$

$1/\lambda_e = 1/(\lambda' - \lambda)(1-\text{COS}\theta)$

$\therefore \lambda' - \lambda = \lambda_e(1-\text{COS}\theta)$

54. Show that the minimum energy of the scattered photon in Compton scattering is given by

$$(E')_{min} = E E_e / 2E + E_e$$

And that for $E >> E_e$,

$$(E')_{min} = E_e / 2 = 0.255 \text{ Me V}.$$

[Sol]

a) $E' = E E_e / 2(1-\cos\theta) + E_e$

At $\theta=\pi$ $E' = (E')_{min}$

$E' = EE'/2(1-\cos\pi) + E_e = EE'/2E + E_e$

b) For $E \gg E_e$

$2E + E_e \cong 2E$

$(E')_{min} = E E_e / 2E = E_e / 2 = 0.255$ Me V

55. What is the minimum energy of a Compton-scattered photon if its original energy is
(a) 0.1Me V,
(b) 1 Me V,
(c) 10 Me V ?

[Sol]

a) 0.1 Me V

$(E')_{min} = \frac{EEe}{2E+Ee}$ = (0.1 x 0.511) x [(2 x 0.1)+0.511] = 0.072 Me V

b) 1 Me V

$(E')_{min}$=(1 x 0.511) x [(2 x 0.1)+0.511]= 0.204 Me V

c) 10 Me V

$(E')_{min}$=(10 x 0.511) x [(2 x 0.1)+0.511]= 0.249 Me V

56. Calculate the mass attenuation coefficient of silica glass (SiO_2, ρ = 2.21 g/cm^3) for 3- Me V γ-rays.

[Sol]

$M(SiO_2)$ = 28.089+(2 x 15.9994) = 60.0848

$(u/\rho)_{Si}$ = 0.0367 cm^2/g

$(u/\rho)_O$= 0.0359 cm^2/g , for 3 Me V- photon

$u/\rho = \sum_i \omega_i (u/\rho)$ = (28.086/60.0848)·0.0367 +[(2 x 15.9994)/60.0848]·0.0359

= 0.0363cm^2/g

57. derive Eq. (3.69)

[sol] $\mu = \mu_1 + \mu_2 + \cdots$

$$\frac{\mu}{\rho} = \frac{\mu_1}{\rho} + \frac{\mu_2}{\rho} + \cdots$$

$$= \frac{\mu_1}{\rho_1/\omega_1} + \frac{\mu_2}{\rho_2/\omega_2} + \cdots$$

58. The mass attenuation coefficient of lead at 0.15MeV is 1.84cm^2/g. At this energy, the principal mode of interaction is by the photoelectric effect. What thickness of lead is required to reduce the intensity of a 0.15-MeV γ-ray beam by a factor of 1000?

[sol] Table II.3 $\rho_{lead} = 11.34 g/cm^2$

$$\mu = \frac{\mu}{\rho} \times \rho = (1.84 cm^2/g) \times (11.34 g/cm^2) = 20.8656 cm^{-1}$$

$$I = I_0 e^{-\mu X}$$

$$\frac{I}{I_0} = e^{-\mu X}$$

$$-\mu X = \ln\frac{I}{I_0} \quad ,$$
$$\therefore X = -\frac{1}{\mu}\ln\frac{I}{I_0} = -\frac{1}{20.8656}\ln\frac{1}{1000} \cong 0.331\text{cm}$$

59. The density of air at standard temperature and pressure (0℃ and 1atm) is 1.293×10^{-3}g/cm^3. Compute the man free paths of photons in air under these conditions and compare with the corresponding mean free paths in unit-density water at the following energies:
(a) 0.1 MeV

[sol] (1) Air
Table II.4 $\frac{\mu}{\rho} = 0.151\text{cm}^2/\text{g}$
$$\mu = (0.151\text{cm}^2/\text{g}) \times (1.293 \times 10^{-3}\text{g/cm}^3) \cong 0.195243 \times 10^{-3}\text{cm}^{-1}$$
$$\therefore \lambda = \frac{1}{\mu} = \frac{1}{0.195243\times10^{-3}} = 5121.82\text{cm}$$

(2) Water
Table II.4 $\frac{\mu}{\rho} = 0.167\text{cm}^2/\text{g}$
$$\mu = (0.167\text{cm}^2/\text{g}) \times (1.0\text{g/cm}^3) \cong 0.167\text{cm}^{-1}$$
$$\therefore \lambda = \frac{1}{\mu} = \frac{1}{0.167} = 5.988\text{cm}$$

(b) 1 MeV

[sol] (1) Air
Table II.4 $\frac{\mu}{\rho} = 0.0636\text{cm}^2/\text{g}$
$$\mu = (0.0636\text{cm}^2/\text{g}) \times (1.293 \times 10^{-3}\text{g/cm}^3) \cong 0.0822348 \times 10^{-3}\text{cm}^{-1}$$
$$\therefore \lambda = \frac{1}{\mu} = \frac{1}{0.0822348\times10^{-3}} = 12160.30\text{cm}$$

(2) Water
Table II.4 $\frac{\mu}{\rho} = 0.0706\text{cm}^2/\text{g}$
$$\mu = (0.0706\text{cm}^2/\text{g}) \times (1.0\text{g/cm}^3) \cong 0.0706\text{cm}^{-1}$$
$$\therefore \lambda = \frac{1}{\mu} = \frac{1}{0.0706} = 14.16\text{cm}$$

(c) 10 MeV
[sol] (1) Air
Table II.4 $\frac{\mu}{\rho} = 0.0202\text{cm}^2/\text{g}$
$$\mu = (0.0202\text{cm}^2/\text{g}) \times (1.293 \times 10^{-3}\text{g/cm}^3) \cong 0.0261 \times 10^{-3}\text{cm}^{-1}$$
$$\therefore \lambda = \frac{1}{\mu} = \frac{1}{0.0261\times10^{-3}} = 38286.89\text{cm}$$

(2) Water
Table II.4 $\frac{\mu}{\rho} = 0.0219\text{cm}^2/\text{g}$
$$\mu = (0.0219\text{cm}^2/\text{g}) \times (1.0\text{g/cm}^3) \cong 0.0219\text{cm}^{-1}$$
$$\therefore \lambda = \frac{1}{\mu} = \frac{1}{0.0219} = 45.66\text{cm}$$

60. At 1 MeV, the Compton cross-section per electron is 0.2112 b, and the Compton energy absorption cross-section per electron is 0.0929 b.
(a) What is the average energy of the recoiling electron in a Compton interaction at this energy?

[sol] $E\sigma_{Ca} = \overline{T}\sigma_C$
$$\overline{T} = \frac{E\sigma_{Ca}}{\sigma_C} = (1\text{MeV}) \times \frac{0.0929\text{b}}{0.2112\text{b}} \cong 0.43987\ \text{MeV}$$

(b) Compute the Compton mass attenuation and mass absorption coefficients at 1 MeV for (i) aluminum, (ii) water.

[sol] (1) Aluminum

Table II.3 M=26.9815

$$\frac{\mu}{\rho}=\frac{N_A\sigma_C}{M}=\frac{(0.6022\times10^{24})\times(0.2112\times10^{-24})\times13}{26.9815}=\mathbf{0.06123cm^2/g}$$

$$\frac{\mu_a}{\rho}=\frac{N_A\sigma_{Ca}}{M}=\frac{(0.6022\times10^{24})\times(0.0929\times10^{-24})\times13}{26.9815}=\mathbf{0.02691cm^2/g}$$

(2) Water

Table II.3 M=18.0153

$$\frac{\mu}{\rho}=\frac{N_A\sigma_C}{M}=\frac{(0.6022\times10^{24})\times(0.2112\times10^{-24})\times10}{18.0153}=\mathbf{0.071cm^2/g}$$

$$\frac{\mu_a}{\rho}=\frac{N_A\sigma_{Ca}}{M}=\frac{(0.6022\times10^{24})\times(0.0929\times10^{-24})\times10}{18.0153}=\mathbf{0.031cm^2/g}$$

61. A beam of 0.1-MeV γ-rays with an intensity of 5×10^{-6} γ-rays/cm^2-sec is incident on thin foils of (i) aluminum, (ii) water. At this energy, the Compton cross-section per electron is 0.4929 b, and the Compton energy absorption cross-section per electron is 0.0685 b. Calculate the energy extracted from the beam per unit volume of the foils due to

(a) Compton scattering

[sol] (1) Aluminum

Table II.3 M=26.9815, $\rho=2.699\mathrm{g/cm^3}$

$$\sigma_c=Z({}_e\sigma_C-\sigma_{Ca})=13(0.4926-0.0685)\mathrm{b}=5.5133\mathrm{b}$$

$$\mu_c=N\sigma_c=\frac{\rho N_A}{M}\sigma_c=\frac{(2.699\mathrm{g/cm^3})\times(0.6022\times10^{24})}{26.9815}\times(5.5133\times10^{-24})=0.332\mathrm{cm^{-1}}$$

$$\begin{aligned}\therefore W=EI\mu_C&=(0.1\mathrm{MeV})\times(5\times10^6\gamma-\mathrm{rays/cm^2\cdot s})\times(0.332\mathrm{cm^{-1}})\\&=0.166\times10^6\mathrm{MeV/cm^3\cdot s}\\&=0.2656\times10^{-7}\mathrm{W/cm^3}\end{aligned}$$

(2) Water

Table II.3 M=18.0153, $\rho=1.0\mathrm{g/cm^3}$

$$\mu_c=N\sigma_c=\frac{\rho N_A}{M}\sigma_c=\frac{(1.0\mathrm{g/cm^3})\times(0.6022\times10^{24})}{18.0153}\times(4.241\times10^{-24})\cong0.1418\mathrm{cm^{-1}}$$

$$\begin{aligned}\therefore W=EI\mu_C&=(0.1\mathrm{MeV})\times(5\times10^6\gamma-\mathrm{rays/cm^2\cdot s})\times(0.1418\mathrm{cm^{-1}})\\&=0.0709\times10^6\mathrm{MeV/cm^3\cdot s}\\&=0.11344\times10^{-7}\mathrm{W/cm^3}\end{aligned}$$

(b) the photoelectric effect

[sol] (1) Aluminum

Assume $\mu_a=\mu_{pe}+\mu_{Ca}$, $\therefore\ \mu_{pe}=\mu_a-\mu_{Ca}$

$$\frac{\mu_a}{\rho}=0.0373\mathrm{cm^2/g}\quad,\ \mu_a=0.0373\rho$$

$$\mu_{pe}=\mu_a-\mu_{Ca}=0.0373\times2.699-\frac{2.699\times0.6022\times10^{24}}{26.9815}\times(0.8905\times10^{-24})=0.04703\mathrm{cm^{-1}}$$

$$\begin{aligned}\therefore W=EI\mu_{pe}&=(1\mathrm{MeV})\times\left(5\times10^6\gamma-\frac{\mathrm{rays}}{\mathrm{cm^2}}\cdot\mathrm{s}\right)\times(0.04703\mathrm{cm^{-1}})\\&=0.2352\times10^6\mathrm{MeV/cm^3\cdot s}\\&=0.3763\times10^{-7}\mathrm{W/cm^3}\end{aligned}$$

(2) Water

$$\frac{\mu_a}{\rho}=0.0253\mathrm{cm^2/g}$$

$$\mu_{pe}=\mu_a-\mu_{Ca}=0.0253\times1.0-\frac{1.0\times0.6022\times10^{24}}{18.0153}\times(0.685\times10^{-24})=0.0024\mathrm{cm^{-1}}$$

$$\begin{aligned}\therefore W=EI\mu_{pe}&=(1\mathrm{MeV})\times\left(5\times10^6\gamma-\frac{\mathrm{rays}}{\mathrm{cm^2}}\cdot\mathrm{s}\right)\times(0.0024\mathrm{cm^{-1}})\\&=0.012\times10^6\mathrm{MeV/cm^3\cdot s}\\&=0.0192\times10^{-7}\mathrm{W/cm^3}\end{aligned}$$

62. The absorption of radiation is often measured in units called *rads*, where 1 rad is equal to the absorption of 100 ergs per gram. What intensity of 1MeV γ-rays incident on a thin slab of water is required to give an absorption rate of 1 rad per second?

[sol] 1rad=100erg/g=10^{-5}J/g

(Absorption rate)= 10^{-5}J/g · s=$EI\frac{\mu_a}{\rho}$

$$\therefore I = \frac{10^{-5}J/g\cdot s}{E\frac{\mu_a}{\rho}} = \frac{10^{-5}J/g\cdot s}{(1\times10^{6}\times1.6\times10^{-19}J)\times0.031cm^2/g} \cong 2.02\times10^{9}\gamma-rays/cm^2\cdot s$$

63. Determine the range of 5-MeV α-particles in the following media:
(a) air at 15℃, 1atm;

[sol] Figure 3.23 5MeV $\alpha-$particles : (about)3.5cm

(b) aluminum;

[sol] Table II 3 M=26.9815 $\rho = 2.699g/cm^3$

$$R = 3.2\times10^{-4}\frac{\sqrt{M}}{\rho}R_a = 3.2\times10^{-4}\frac{\sqrt{26.9815}}{2.699}\times3.5cm \cong 0.0022cm$$

(c) lead;

[sol] Table II.3 M=207.19 $\rho = 11.34g/cm^3$

$$R = 3.2\times10^{-4}\frac{\sqrt{M}}{\rho}R_a = 3.2\times10^{-4}\frac{\sqrt{207.19}}{11.34}\times3.5cm \cong 0.0014cm$$

(d) unit-density water;

[sol] Table II.3 M=18.0153 $\rho = 1.0g/cm^3$

$$R = 3.2\times10^{-4}\frac{\sqrt{M}}{\rho}R_a = 3.2\times10^{-4}\frac{\sqrt{18.0153}}{1}\times3.5cm \cong 0.0048cm$$

(e) air at 300℃, 10atm;

[sol] ① : Air at 15℃, 1atm, ② :Air at 300℃, 10atm

$$P_1V_1 = nRT_1 \quad , \ P_2V_2 = nRT_2$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad , \quad \frac{(1)V_1}{15+273} = \frac{(100)V_2}{300+273} \quad , \ V_1 = 50.2617V_2$$

$$R = \frac{C}{\rho} \ (c = const),$$

$$\frac{R_1}{R_2} = \frac{(1/\rho_1)}{(1/\rho_2)} = \frac{(V_1/M_1)}{(V_2/M_2)} = \frac{V_1}{V_2} = \frac{50.2617V_2}{V_2}$$

$$R_1 = R_a = 3.5cm$$

$$\therefore R_2 = 0.0696cm$$

64. Determine the relative stopping powers of the media in the preceding problem.

[sol] (a) Air

$$\frac{R_a}{R} = 1$$

(b) Aluminum

$$\frac{R_a}{R} = \frac{3.5cm}{0.0022cm} = 1590.91$$

(c) Lead

$$\frac{R_a}{R} = \frac{3.5cm}{0.0014cm} = 2500$$

(d) Unit-density water

$$\frac{R_a}{R} = \frac{3.5cm}{0.0048cm} = 729.17$$

(e) Air at 300°C, 10atm

$$\frac{R_a}{R} = \frac{3.5\text{cm}}{0.0696\text{cm}} = 50.29$$

65. Compare the apparent mass attenuation coefficient of 2-MeV (maximum energy) β-rays with the mass attenuation coefficient of 2-MeV γ-rays in aluminum.

[sol] $\beta-\text{rays}: \frac{\mu_a}{\rho} = \frac{17}{E_{max}^{1.14}} = \frac{17}{(2\text{MeV})^{1.14}} \cong 7.714$

$$\gamma-\text{rays}: \text{Table II.5} - \text{ 2MeV aluminum } \rightarrow \frac{\mu_a}{\rho} = 0.0232$$

66. Compare the maximum ranges of 3-MeV α-rays and β-rays in air at standard thmperature and pressure.

[sol] $\alpha-\text{rays}: \text{Fig. 3.23} - 3\text{MeV } \alpha - \text{particles} \rightarrow 1.8\text{cm}$

$\beta-\text{rays}$

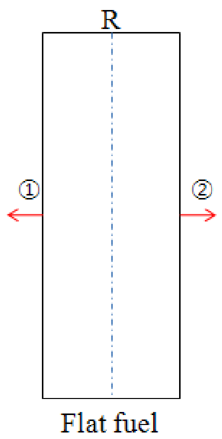
$E_{max} > 2.5MeV, \quad R_{max}\rho = 0.530E_{max} - 0.106$

$\rho_{air} = \frac{1.2041\text{kg}}{\text{cm}^3} = 1.2041 \times 10^{-3}\text{g/cm}^3$ **(At 20°C, 101.325kPa)**

$$R_{max} = \frac{0.530E_{max} - 0.106}{\rho} = \frac{0.530 \times (3\text{MeV}) - 0.106}{1.2041 \times 10^{-3}\text{g/cm}^3} \cong 1232.56\text{cm}$$

67. Near the surface of a flat fuel element in an operating reactor, fissions are occurring at the constant rate of S fissions/cm^3-sec. Given that the average range of the fission fragments is R, show that the rate at which such fragments would escape per cm^2/sec from the surface of the fuel if it were not clad is equal to SR/2.

[sol]



(The rate at which such fragments would escape per cm^2/sec**)**

$= (S\text{ fissions/cm}^3 \cdot \text{sec}) \times (R\text{ cm})$

= **SR fissions/cm^2 · sec** =① + ②

$$\therefore ① = ② = \frac{SR}{2}$$

Chapter 4

Nuclear Reactors and Nuclear Power

1. If 57.5% of the fission neutrons escape from a bare sphere of ^{235}U, what is the multiplication factor of the sphere? In this system, the average value of η is 2.31.

[sol] $\mathbf{k = \frac{x-0.575x}{x} \times 2.31 = 0.98175}$

2. Measurements on an experimental thermal reactor show that, for every 100 neutrons emitted in fission, 10 escape while slowing down and 15 escape after having slowed down to thermal energies. No neutrons are absorbed within the reactor while slowing down. Of those neutrons absorbed at thermal energies, 60% are absorbed in fission material. (a) What is the multiplication factor of the reactor at the time these observations are made? (b) Suppose the thermal leakage is reduced by one third. How would this change the value of *k*? [*Note:* The values of η and ν for the reactor fuel are 2.07 and 2.42, respectively.]

[sol] (a) $\mathbf{k = \frac{(100-10-15)}{100} \times 2.07 \times 0.6 = 0.9315}$

(b) $\mathbf{k = \frac{(100-10-5)}{100} \times 2.07 \times 0.6 = 1.0557}$

3. (a) Show that the energy released in the nth generation of a fission chain reaction, initiated by one fission, is given by

$$E_n = k^n E_R \ ,$$

where k is the multiplication factor and E_R is the recoverable energy per fission. (b) Show that the total energy released up to and including the nth generation is given by

$$E_n = \frac{k^{n+1}-1}{k-1} E_R$$

[sol] (a) One generation : $\mathbf{k^1 E_R}$

Two generation : $\mathbf{k^2 E_R} \ \cdots$

$\therefore$ **nth generation :** $\mathbf{k^n E_R}$

(b) Total energy : $\mathbf{\sum E_n = k^0 E_R + k^1 E_R + k^2 E_R + \cdots k^n E_R = \frac{E_R(k^{n+1}-1)}{k-1}}$

4. Show that the fraction, F , of the energy released from a supercritical chain reaction that originates in the final m generations of the chain is given approximately by

$$F = 1 - k^{-m} \ ,$$

provided the total number of generations is large.

[sol]

$$\mathbf{\frac{\sum E_{m-1}}{\sum E_m} + \frac{E_m}{\sum E_m} = 1}$$

$$\mathbf{\leftarrow \sum E_{m-1} = \frac{k^m-1}{k-1}, \qquad \sum E_m = \frac{k^{m+1}-1}{k-1}}$$

$$\mathbf{\frac{E_m}{\sum E_m} = 1 - \frac{\frac{k^m-1}{k-1}}{\frac{k^{m+1}-1}{k-1}} = 1 - \frac{k^m-1}{k^{m+1}-1} = 1 - \frac{k^m\left(1-\frac{1}{k^m}\right)}{k^m\left(k-\frac{1}{k^m}\right)}}$$

$$\mathbf{\leftarrow k - \frac{1}{k^m} \approx k}$$

($\because$ **the chain is given approximately by provided the total number of generations is large**)

$$\mathbf{= 1 - \frac{1}{k}\left(1 - \frac{1}{k^m}\right)}$$

$$\mathbf{\rightarrow \therefore F = 1 - \frac{1}{k^m}}$$

5. (a) Most of the energy from a nuclear explosion is released during the final moments of the detonation. Using the result of the previous problem, compute the number of fission generations required to release 99% of the total explosive yield. Use the nominal value $k = 2$. (b) If the mean time between generations is the order of 10^{-8} sec over what period of time is energy released during a nuclear explosion?

[sol] (a) $\frac{E_m}{\sum E_m} = 1 - \frac{1}{k}\left(1 - \frac{1}{k^m}\right)$

0.99=$1 - \frac{1}{k}\left(1 - \frac{1}{k^m}\right)$ **←k=2**

$2^{-m} = 0.08$, $2^m = 12.5$

$\therefore m \cong 3.64$

(b) (Period of time)=$m \times 10^{-8}sec = 3.64 \times 10^{-8}sec$

6. A burst of 1×10^9 neutrons from a pulsed accelerator is introduced into a subcritical assembly consisting of an array of natural uranium rods in water. The system has a multiplication factor of 0.968. Approximately 80% of the incident neutrons are absorbed in uranium. (a) How many first-generation fissions do the neutrons produce in the assembly? (b) What is the total fission energy in joules released in the assembly by the neutron burst?

[sol] (a) $0.968 = \frac{x}{(1.6\times10^9)\times0.8} \times 2.068$

$x = 5.9915 \times 10^8$

(b) $E = x \times (200MeV)$

$= (5.9915 \times 10^8) \times (200MeV) \times \frac{1.6\times10^{-13}J}{1MeV}$ **= 0.01917J**

7. It is found that, in a certain thermal reactor, fueled with partially enriched uranium, 13% of the fission neutrons are absorbed in resonances of ^{238}U and 3% leak out of the reactor, both while these neutrons are slowing down; 5% of the neutrons tat slow down in the reactor subsequently leak out; of those slow neutrons that do not leak out, 82% are absorbed in fuel, 74% of these in ^{235}U. (a) What is the multiplication factor of this reactor? (b) What is its conversion ratio?

[sol] $\text{Conversion ratio} = \frac{\text{\# of fissile atoms produced}}{\text{\# of fissile fuel atom consumed}}$

(a) $(1 - 0.13 - 0.03) \times (1 - 0.05) \times 2.068 \times 0.82 \times 0.74 = 1.001$

(b) $C = \frac{0.13+(1-0.03)\times(1-0.05)\times(1-0.74)\times0.82}{1} = 0.303$

8. A natural uranium-fueled converter operates at a power of 250MWt with a conversion ratio of 0.88. At what rate is ^{239}Pu being produced in this reactor in kg/year?

[sol] $C = \frac{\text{239Pu produced}}{^{235}\text{U consumed}}$, $(\text{239Pu produced}) = C \times (^{235}\text{U consumed})$

$(\text{Fission rate}) = 250MW \times \frac{10^6 J}{MW\cdot s} \times \frac{1\ \text{fission}}{200MeV} \times \frac{1\ MeV}{1.60\times10^{-13}J} \times \frac{365\times24\times3600s}{1\ \text{year}} = 2.464 \times 10^{26}\ \text{fissions/year}$

$(\text{239Pu produced}) = C \times (^{235}\text{U consumed})$

$= 0.88 \times 2.464 \times 10^{26} \times \frac{\sigma_f}{\sigma_a} \times \frac{M}{N_A}$

←Table 3.4

$= 0.88 \times 2.464 \times 10^{26} \times \frac{582.2}{680.8} \times \frac{239}{0.6022\times10^{24}}$

= 73.59kg/year

9. Assuming a recoverable energy per fission of 300 Me V, calculate the fuel burnup and consumption rates in g/MWd for

(a) thermal reactors fueled with 233U or 239Pu

[sol] (1) case of U-233

Fission rate = 2.88 * 10^{21}P fissions/day (replace 200 Me V to 300 Me V)

$\mathbf{Burn\ up\ rate: fission\ rate} \times \frac{\mathbf{amu}}{\mathbf{Na}} = \mathbf{2.88 \times 10^{21} P} \times \frac{\mathbf{233 g/mol}}{\mathbf{6.022 \times 10^{23} \#/mol}} = \mathbf{1.1382P} \left[\frac{\mathbf{g}}{\mathbf{day}}\right]$

$\mathbf{Consumption\ rate: 1.1382(1+\alpha)P} \left[\frac{\mathbf{g}}{\mathbf{day}}\right]$, $1+\alpha = \frac{\sigma_a}{\sigma_f} = \frac{575}{529} = \mathbf{1.087}$

Consider devided by unit power for making unit $[\frac{g}{MWd}]$

$\therefore$ **Burn up rate** : $\mathbf{1.1382} \frac{g}{MWd}$

Consumption rat : 1.1382(1.087)=1.1355 g/MWd

(2) case of Pu-239

Burn-up rate : $\mathbf{2.88 \times 10^{21} P} \times \frac{239 \frac{g}{mol}}{6.022 \times 10^{23} \# \frac{}{mol}} = \mathbf{1.143P} \frac{g}{day}$

Consumption rate : 1.142(1+α)P g/day

1+α=$\frac{\sigma_a}{\sigma_f} = \frac{1020}{749}$**, so consumption rate is 1.4593 g/day**

$\therefore$ **burn up rate** : $\mathbf{1.143} \frac{g}{MWd}$

$\therefore$ **consumption rate** : $\mathbf{1.4593} \frac{g}{MWd}$

(b) fast reactors fueled with 239Pu. [Note: In part (b), take the capture to fission ratio to be 0.065.]

[sol] Burn up rate : $\mathbf{2.88 \times 10^{21} P} \times \frac{239 \frac{g}{mol}}{6.022 \times 10^{23} \# \frac{}{mol}} = \mathbf{1.143P} \frac{g}{day}$

Consumption rate : $\mathbf{1.143(1+0.065)P = 1.2173P} \frac{g}{day}$

Devide by unit power[MW]

$\therefore$ **Burn up rate**: $\mathbf{1.143} \frac{g}{MWD}$, **consumption rate** : $\mathbf{1.2173} \frac{g}{MWd}$

10. Because of an error in design, a thermal reactor that was supposed to breed on the 232Th-233U cycle unfortunately has a breeding ratio of only 0.96. If the reactor operates at a thermal power level of 500 megawatts, how much 232Th does it convert in 1 year?

[sol] in 120page

$^{232}\mathbf{Th(n,\gamma)}^{233}\mathbf{Th} \rightarrow^{\beta-} {}^{233}\mathbf{Pa} \rightarrow^{\beta-} {}^{233}\mathbf{U}$

Consumption rate : $\mathbf{500MW} \times \frac{10^6 J}{MW-sec} \times \frac{fission}{200\ Me\ V} \times \frac{Me\ V}{1.6 \times 10^{-13} J} \times \frac{86400\ sec}{1\ day} \times \frac{233\ g/mol}{6.022 \times 10^{23} \#/mol} \times \frac{365\ day}{1\ yr} \times (97.67) = \mathbf{1.906 \times 10^5} \frac{g}{yr}$ **of Th − 233**

C(breeding ratio)=0.96, in 1 yr

$\mathbf{NC + NC^2 + NC^3 + \cdots = \frac{NC}{1-C}}$ **fertile atoms,**

$$\left[\frac{(1.8616 \times 6.022 \times 10^{23}}{233}) \times 0.96/(1-0.96)\right] \mathbf{atoms} \times \frac{233\ amu}{6.022 \times 10^{23} \# \frac{}{mol}} = \mathbf{4.468 \times 10^8\ g}$$

$\therefore \mathbf{4.468 \times 10^8\ g = 446.8\ ton}$

11. What value of the breeding gain is necessary for a fast breeder operating on the 238U-239Pu cycle to have an exponential doubling time of 10 years if the specific power for this type of reactor is 0.6 megawatts per kilogram of 239Pu?

[sol] Consumption rate per unit power [per MW]

$$\mathbf{1MW} \times \frac{10^6 J}{MW-sec} \times \frac{fission}{200Me\ V} \times \frac{Me\ V}{1.6 \times 10^{-13} J} \times \frac{86400\ sec}{1\ day} \times \frac{238 \frac{g}{mol}}{6.022 \times 10^{23} \# \frac{}{mol}} \times (1+\alpha) = \mathbf{1.931\ g/MWd}$$

$$[\mathbf{1+\alpha=1.81}\ (\textbf{cross sections of natural uranium}\ \approx \textbf{that of 238} - \mathbf{U})]$$
$$\mathbf{w=1.931*10^{-3}\ kg/MWd}$$
$$\mathbf{G=\frac{\ln 2}{t_{De}w\beta}=\frac{\ln 2}{3560\ day\times 1.931\times 10^{-13}\frac{kg}{MWd}\times 0.6\frac{MW}{Kg}}=0.1639}$$

12. Per GWe-year, a typical LMFBR produces 558 kg and consumes 789 kg of fissile plutonium in the core while it produces 455 kg and consumes 34 kg of fissile plutonium in the blanket. What is the breeding ratio for this reactor? [Note: As would be expected, there is a net consumption of plutonium in the core and a net production in the blanket, with a positive output from the reactor as as whole. By adjusting the properties of the blanket, it is easy to make breeders into net consumers of plutonium if dangerously large stockpiles of this materials should ever accumulate in the world.]

[sol] $\mathbf{C=\frac{\text{the average number of fissile atoms produced in a reactor}}{\text{\# of fissile fuel atoms consumed}}=\frac{558+455}{789+34}=1.2309}$

13. A certain fossile fueled generating station operates at a power of 1,000 MWe-at an overall efficiency of 38% and an average capacity factor of 0.70.

$$\text{eff}=\frac{W}{Q_R},\quad Q_R=\frac{1000}{0.38}=2631.6\ \text{MW th}$$

(a) How many tons of 13,000 Btu per pound of coal does the plant consume in 1 year?

[sol] $\mathbf{13000\frac{Btu}{lb}\times\frac{1055\ J}{1\ Btu}\times\frac{1\ lb}{0.4536\ kg}=30235890\frac{J}{kg}}$

$$\mathbf{2631.6\times 10^6\frac{J}{s}\times 0.7\times 1yr\times\frac{365\ d}{1\ yr}\times\frac{86400\ sec}{1\ day}=5.213\times 10^{16}\ J}$$
$$\mathbf{\frac{5.213\times 10^{16}}{30235890}=1.724\times 10^6\ ton}$$

(b) It an average coal-carrying railroad car carries 100 tons of coal, how many car loads much be delivered to the plant on an average day?

[sol] $\mathbf{\frac{1.724\times 10^6}{365}=4723\ tons}$

(c) If the coal contains 1.5% by weight sulfur and in the combustion process this all goes up the stack as SO2. how much SO2 does the plant produce in 1 year?

[sol] $\mathbf{1.724\times 10^6\times 0.015\times\frac{32.064+16*2}{32.064}=51668.4\ ton}$

14. Had the plant described in Problem 4.13 been fueled with 6.5 million Btu per barrel bunker-C fuel oil containing .37% sulfur,

(a) how many barrels and tons of oil would the plant consume in 1 year ?

[sol] 1 barrel=117.35 liter

Density of Bunker C oil = 0.96 g/ml

112.7 kg/barrel

$$\mathbf{6.5\times 10^6\times\frac{Btu}{112.7\ kg}\times\frac{1055\ J}{1\ Btu}=60847382\frac{J}{Kg}}$$
$$\mathbf{From\ pb. 4.13,\frac{5.213\times 10^{16}}{60847382}=8.567\times 10^8\ kg=8.567\times 10^5\ ton}$$

(b) how much SO2 would the plant release in 1 year? [Note: 1 U.S. petroleum barrel=5.61 cubic feet=42 U.S gallons; the density of bunker-C oil is approximately the same as water.]

[sol] $\mathbf{8.5673\times 10^5\times 0.0037\times\frac{32.064+2*16}{32.064}=6333\ kg}$

15. Rochester Gas and Electric's Robert Emmett Ginna nuclear power plant operates at a net electric output of 470 megawatts. The overall efficiency of the plant is 32.3%. Appoximately 60% of the plant's power comes from fissions in 235U, the remainder from fissions in converted plutonium, mostly 239Pu. If the plant were

operated at full power for 1 year, how many kiomgrams of 235U and 239Pu would be

(a) fissioned?

[sol] $\mathbf{Q_R = 1455\ MWth}$

Burn up rate of each

$\mathbf{1)\ U-235}$

$$\mathbf{1455 \times 0.6 \times 2.7 \times 10^{21} \times \frac{235}{6.022\times10^{23}} \times 365 = 335.7\frac{kg}{yr}}$$

2) Ph-239

$$\mathbf{1455 \times 0.4 \times 2.7 \times 10^{21} \times \frac{239}{6.022\times10^{23}} \times 365 = 227.6\frac{kg}{yr}}$$

(b) consumed?

[sol] 1) U-235

$$\mathbf{1+\alpha = 1.17, 1.17 * 335.7 = 392.9\frac{kg}{yr}}$$

2)Ph-239

$$\mathbf{1+\alpha = 1.3618,\ 1.3618 * 227.6 = 309.9\frac{kg}{yr}}$$

16. Consider two electrical generating stations-one a fossil fuel plant and the other nuclear-both producing the same electrical power. The efficiency of the fossil fuel plant is (eff), whereas that of the nuclear plant is $(\mathrm{eff})_n$.
(a) Show that the ratio of the heat rejected to the environment by the two plants is given by

$$\frac{Q_{cn}}{Q_{cf}} = \frac{1-(\mathrm{eff})_n}{1-(\mathrm{eff})_f} \times \frac{(\mathrm{eff})_f}{(\mathrm{eff})_n}$$

[sol] $\mathbf{Q_R}$ **: thermal energy output from the heat source.**

$\mathbf{Q_C}$ **: heat energy rejected to the environment**

$$\mathbf{work = W = Q_R - Q_C = eff \times Q_R}$$

$$\mathbf{eff = \frac{W}{Q_R} = \frac{Q_R - Q_C}{Q_R} = 1 - \frac{Q_C}{Q_R}}$$

$$\mathbf{Q_R = Q_c/(1 - eff)}$$

In the exercise, W = $\mathbf{(eff)_f \times Q_{Rf} = (eff)_n \times Q_{Rn}}$

$$\mathbf{\frac{Q_{Rn}}{QRf} = \frac{(eff)_f}{(eff)_n} = \frac{[Q_{Cn}/(1-(eff)_n]}{[Q_{Cf}/(1-(eff)_f]}}$$

$$\mathbf{\frac{Q_{Cn}}{Q_{Cf}} = \frac{1-(eff)_n}{1-(eff)_f} \times \frac{(eff)_f}{(eff)_n}}$$

(b) Evaluate Q_{cn}/Q_{cf} for the case where $(\mathrm{eff})_f = 38\%$ and $(\mathrm{eff})_n = 33\%$

[sol] $\mathbf{\frac{Q_{Cn}}{Q_{Cf}} = \frac{1-(eff)_n}{1-(eff)_f} \times \frac{(eff)_f}{(eff)_n} = \frac{1-0.33}{1-0.38} \times \frac{0.38}{0.33} = 1.244}$

17.An MSBR power plant produces 1,000 MWe at an overall efficiency of 40%. The breeding ratio for the reactor is 1.06, and the specific power is 2.5 MWt per kilogram of ^{233}U.
(a) Calculate the linear and exponential doubling times for this reactor.

[sol] $\mathbf{w = 1.05(1+\alpha) = 1.05(1+0.089) = 1.143 g/dayMW = 1.143 \times 10^{-3} kg/dayMW}$

$$\mathbf{t_{De} = \frac{\ln 2}{Gw\beta} = \frac{\ln 2}{(C-1)w\beta} = \frac{\ln 2}{(1.06-1)\times1.143\times10^{-3}\times2.5} = 4042.9 days}$$

$$\mathbf{t_{Dl} = \frac{t_{De}}{\ln 2} = \frac{4042.9}{\ln 2} = 5832.7 days}$$

(b) What is the net production rate of ^{233}U in kg/year?

[sol] $\mathbf{m_0 = GwP_0 = (C-1) \times w \times \frac{W}{eff} = (1.06-1) \times 1.143 \times 10^{-3} \times \frac{1000}{0.4}}$

$$\mathbf{m_0 = 0.172 kg/day = 67.579 kg/yr}$$

18. A 3,000MW reactor perates for 1 year. How much does the mass of the fuel change during this time as the result of the energy release?

[sol] The fissioning of 1.05g of ^{235}U yields 1MWd.

$$\mathbf{3000MW\ for\ 1year} = \frac{\mathbf{1MWd}}{\mathbf{1.05g}} \times \frac{1\text{year}}{365\text{day}} \times \mathrm{M(g)}$$

$$\mathbf{M(g) = 3000 \times 1.05 \times 354 = 1.15 \times 10^6 g = 1.15ton}$$

19. Referring to the nominal LWR cycle described in Fig. 4.37, The plant operates at an overall efficiency of 33.4%,compute

(a)The specific burnup of the fuel in MWd/t

[sol] $\mathbf{Q_R = \frac{W}{eff} = \frac{0.75GW-yr}{0.334} = 2.246GW-yr}$

$$\mathbf{specific\ burnup = \frac{2.246GW-yr}{26.977} = \frac{2.246 \times 10^3 \times 365}{26.977} = 30388.5MWd/t}$$

(b) The fractional burnup of the fuel

[sol] $\mathbf{\beta = \frac{30388.5}{950000} = 0.032 = 3.2\%}$

(c) The enrichment of the fresh fuel

[sol] $\mathbf{\frac{813}{26977} = 0.03 = 3w/o}$

(d) The enrichment (235U) of the spent fuel

[sol] $\mathbf{\frac{220}{25858} = 0.0085 = 0.85w/o}$

(e) The fraction of the power originating in fissions in ^{235}U and plutonium, respectively

[sol] fissioning of 1.05g of ^{235}U yields 1MWd.

$$\mathbf{\frac{813 \times 10^3 \times \frac{1}{1.05}}{2246 \times 365} = 0.944 = 94.4\%}$$

fissioning of 1.05g of ^{239}Pu yields 1MWd.

$$\mathbf{\frac{813 \times 10^3 \times \frac{1}{1.05}}{2246 \times 365} = 0.944 = 94.4\%}$$

(f) The 30-year requirement for uranium for this system.

[sol] 150,047kg of u3O8 is converted to UF6 in year.

$$\mathbf{150047 \times 30 = 4501410kg}$$

22. Using the data in Fig. 4.41[Note : The reactor operates at an efficiency of 30%.], compute

(a) The enrichment of the fresh fuel

[sol] $\mathbf{M_F = \frac{238 \times 3}{238 \times 3 + 16 \times 8} \times 145051 = 123000kg}$

$$\mathbf{M_T = 1451kg}$$

$$\mathbf{M_P = M_F - M_T = 123000 - 1451 = 121549kg}$$

$$\mathbf{x_T = 0.002}$$

$$\mathbf{x_F = 0.00711}$$

$$\mathbf{x_F M_F = x_P M_P + x_T M_T}$$

$$\mathbf{x_P = 0.00717 = 0.717\%}$$

(b) The residual enrichment of the spent fuel

[sol] mass of ^{235}U = $\mathbf{0.00717 \times 121349 = 871.5kg}$

A mount of 60% of neutrons are absorbed in fission material.

$$\mathbf{M_R = 871.5 \times 0.6 = 522.9kg}$$

$$\mathbf{x_R = \frac{871.5-522.9}{121549-522.9} = 0.00288 = 0.288\%}$$

(c) The fraction of power originating in fissions in ^{235}U

[sol] $\mathbf{P = \frac{Q_R}{eff} = \frac{0.75GWe-yr}{0.3} = 2500MWe-yr}$

To yield 2500MWe-yr, $\mathbf{M_{235U} = \frac{1.05g}{1MWd} \times 2500MWe-yr \times \frac{365d}{1yr} \times \frac{1kg}{1000g} = 958.125kg}$

Fraction=$\mathbf{\frac{522.9}{958.125} = 0.546}$

(d) The burnup of the fuel.

[sol] $\mathbf{burnup = \frac{2500MW-yr}{121549kg} \times \frac{365d}{1yr} \times \frac{1000kg}{1t} = 7507.3MWd/t}$

23. Reliable nuclear weapons cannot be made using plutonium containing much more than 7 w/o ^{240}Pu because this isotope has a high spontaneous fission rate that tends to pre-initiate - that is, fizzle – the device. Using Fig. 4.37 and Fig. 4.39 through4.40, determine whether any of these commercial power systems produce weapons-grade plutonium (<7 w/o ^{240}Pu). Assume that all non-fissioning plutonium is ^{240}Pu (Some of it is actually ^{242}Pu).

[sol]

24. Compute and plot annual and cumulative uranium requirements through the year 2015 for the projected nuclear electric capacities given in Table 4.8 for each of following assumptions: (a) all reactors are PWRs with no recycling; (b) all reactors are PWRs, and all operate with recycling after 2000; (c) all reactors are PWRs until 2000 after which 20% of new reactors are LMFBRs. [note: for simplicity, ignore the extra fuel required for startup; that is, take the annual uranium needs to be the values in Table 4.7 divided by 30.]

.

[sol] According to Table 4.7 Natural U requirements for 1 yr in LWR

Once through: 4260/30= 142t

Recycling: 2665/30=90t

For once through cycle,

a)

Year	Breeding ratio	Breeding ratio(cumulation)
1985	1.42	1.42
1990	2.23	3.65
1995	2.84	6.49
2000	3.62	10.11
2005	3.91	14.02
2010	4.19	18.21
2015	4.33	22.54

b)

Year	Breeding ratio	Breeding ratio(cumulation)
1985	1.42	1.42
1990	2.23	3.65
1995	2.84	6.49
2000	3.62	10.11
2005	2.44	12.55
2010	2.62	15.17
2015	2.71	17.88

c) In LMFBR, 36/30=1.2t/yr for U-Pu recycling

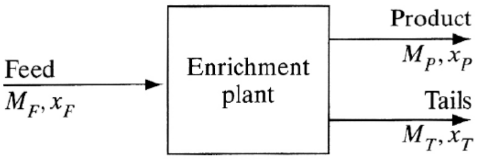
Year	Breeding ratio	Breeding ratio(cumulation)
1985	0.89	0.89
1990	1.39	2.28
1995	1.78	4.06
2000	2.27	6.33

2005	**1.96**	**8.29**
2010	**2.10**	**10.39**
2015	**2.17**	**12.56**

25. A nuclear fuel-fabricating company needs 10,000 kg of 3 w/o uranium and furnished natural uranium (as UF_6) as feed. Assuming a tails assay of 0.2 w/o, (a) how much feed is required? (b) How much more will the enrichment cost?

[sol]

a)



X_F= 0.00711, X_T= 0.002, X_P= 0.03

When the 10,000 kg of U is M_p, mass of UO_2 is

10,000 x [(238+2 x 16)/238]=11345 kg

$M_F = M_T + M_P$

$X_F M_F = X_T M_T + X_P M_P$

→ $M_F = (X_P - X_T)/(X_F - X_T) \times M_P$ =54795 kg

=(0.03-0.002)/(0.0071-0.002) x 10000 = 54795 kg

In UF_6, 54795 x [(238+19 x 6)/238]=81041 kg

b)

$\gamma(0.03)=(1-2\cdot0.03)\ln[(1-0.03)/0.03]=3.2675$

$\gamma(0.00711)= (1-2\cdot0.00711)\ln[(1-0.00711)/0.00711]=4.869$

$\gamma(0.002)= (1-2\cdot0.002)\ln[(1-0.002)/0.002]=6.188$

SWU= $M_P \cdot\gamma(0.03)+ M_T \cdot\gamma(0.002)- M_F \cdot\gamma(0.00711)$

= 10000·3.2675+(54795-10000)·6.188 - 54795·4.869

= 43070 kg

$130.75/SWU

Enrichment cost= 130.75·43070=$5,631,400

26. Suppose that the tails enrichment in the preceding problem were decreased to 0.15 w/o. (a) How much uranium feed would this save? (b) How much more would the enriched fuel cost? (c) What is the cost per kg of feed saved?

[sol]

a)

$M'_F = (X_P - X'_T)/(X_F - X'_T) \times M_P$

=(0.03-0.015)/(0.0071-0.015) x 10000 = 50802 kg

M_F - M'_F=54795-50802= 3993kg

In UF_6, 3993 · [(238+19 x 6)/238]= 5906kg of UF_6 ore saved

b)

$\gamma(0.03)=(1-2\cdot0.03)\ln[(1-0.03)/0.03]=3.2675$

$\gamma(0.00711) = (1-2\cdot 0.00711)\ln[(1-0.00711)/0.00711]=4.869$

$\gamma(0.015) = (1-2\cdot 0.015)\ln[(1-0.015)/0.015]=6.4813$

$$SWU = M_P\cdot\gamma(0.03) + M_T\cdot\gamma(0.015) + M_F\cdot\gamma(0.00711)$$
$$= 10000\cdot 3.2675+(54795-10000)\cdot 6.4813 - 54795\cdot 4.869$$
$$= 49770kg$$

Enrichment cost = 130.75·49770=$6,507,428

$6,507,428-$5,631,400=$876,028

c) 876,028/3,993=$219.39

27. If a customer supplies DOE as part of his feed-uranium that is partially enriched above (or depleted below) the natural uranium level-the total amount of natural uranium feed that he must furnish to obtain a given amount of product is reduced (or increased) by an amount equal to the natural uranium feed needed to provide the partially enriched (or depleted) feed. The customer receives a credit (or debit) for the separative work represented by his enriched (or depleted) feed.

Suppose that the fuel-fabricating company in problem 4.25 offers to supply as part of its feed 10,000 kg of 1% assay uranium. How much natural uranium feed is required in addition, and how much will the total job cost?

[sol]

a)

The amount of M_P generated from the 10000of 1 w/o U

$$M_{P1} = (X_F - X_T)/(XP_F - X_T) \times M_F$$

$$=(0.01-0.002)/(0.0071-0.002) \times 10000 = 2857 \text{ kg}$$

The amount of 3 w/o U produced by the Natural U

10000-2857=7143kg

Natural U needed to make the 7143kg of 3 w/o U

$$M=(X_P - X_T)/(X_F - X_T) \times M_{P2}$$

$$=(0.03-0.002)/(0.00711-0.002) \times 7143 = 39139.7kg$$

b)

$\gamma(0.01)=4.503$

$\gamma(0.00711)= 4.869$

$\gamma(0.002)=6.188$

$\gamma(0.03)= 3.267$

i) For 1 w/o U

$$SWU = M_P\cdot\gamma(X_P) + M_T\cdot\gamma(X_T) - M_F\cdot\gamma(X_F)$$
$$= M_P[\gamma(X_P)-\gamma(X_T)] - M_F[\gamma(X_F)-\gamma(X_T)]$$
$$=2857(3.267-6.188)-10000(4.503-6.188)$$
$$=85047 \text{ kg}$$

ii) For natural U

$$SWU = 7143(3.267-6.188)-39139.7(4.869-6.188)$$
$$= 30760.6kg$$

8504.7+30760.6=39265.3kg

Enrichment cost=$130.75 x 39265.3=$1,876,881

28. Repeat problem 4.27 for the case in which the customer furnishes 10,000 kg of 6% feed.

[sol]

a)

The amount of M_P generated from the 10000 of 0.6 w/o U

$M_{P1} = (X_F\text{-}X_T)/(XP_F\text{-}X_T) \times M_F$

$=(0.006\text{-}0.002)/(0.03\text{-}0.002) \times 10000 = 1429$ kg

The amount of 3 w/o U Produced by the Natural U

10000-1249=8571kg

Natural U needed to make the 7143kg of 3 w/o U

$M=(X_P\text{-}X_T)/(X_F\text{-}X_T) \times M_{P2}$

$=(0.03\text{-}0.002)/(0.00711\text{-}0.002) \times 8571 = 46967$kg

b)

$\gamma(0.006)=5.049$

$\gamma(0.00711)= 4.869$

$\gamma(0.002)=6.188$

$\gamma(0.03)= 3.267$

i) For 1 w/o U

$SWU= M_P \cdot\gamma(X_P)+ M_T \cdot \gamma(X_T) - M_F \cdot\gamma(X_F)$

$= M_P[\gamma(X_P)\text{-}\gamma(X_T)] - M_F[\gamma(X_F)\text{-} \gamma(X_T)]$

$=1429(3.267\text{-}6.188)\text{-}10000(5.049\text{-}6.188)$

$=7217$ kg

ii) For natural U

$SWU= 8571(3.267\text{-}6.188)\text{-}46967(4.869\text{-}6.188)$

$= 36912$

7217+369124=44129kg

Enrichment cost=$130.75 x 44129=$5,769,866

29. Referring to problem 4.28, suppose exactly 33,000 kg of 3.2 w/o UO2 is required per reload. (a) How much UF6 must be given to the enrichment plant, assuming 1% loss in fabrication? (b) How much yellow cake must be used in the conversion to UF6 assuming 0.5% loss in conversion?

[sol]

a)

Mass of U in UO_2

$= 238/(238+16\cdot2) \cdot33000=29088.9$kg

Mass of U in UF_6 after enrichment, M_p

$M_p=1.00/0.99\cdot29088.9=29382.7$kg

$M_F = (X_P\text{-}X_T)/(X_F\text{-}X_T) \times M_P =54795$ kg

$=(0.032\text{-}0.002)/(0.0071\text{-}0.002) \times 29382.7\text{kg} = 172500$kg

Mass of UF_6 given to enrichment plant

$=(238+19\cdot6)/238 \times 172500=255126$kg

b)

U_3O_8 → Conversion to UF_6 (0.5% loss of total U) → UF_6

Mass of U in U_3O_8

$=(1.00/0.995) \times 172500=173367$kg

Mass of U_3O_8M

$=[(3 \cdot 238+16 \cdot 8)/(3 \cdot 238)] \cdot 173367=204447$kg

30. It is proposed to produce 25 kg of 90 *w/o* for a nuclear weapon by enriching 20 *w/o* fuel from a research reactor. (a) How much fresh reactor fuel would be required? (b) Compute the total SWU required. (c) Compare the SWU/kg required to produce 20 *w/o* fuel starting from natural uranium with the SWU/kg for 90 *w/o* material beginning at 20 *w/o*. [*Note*: Assume the tails enrichment to be 0.2 w/o.]

[sol] (a) $M_F = \left(\frac{x_P - x_T}{x_F - x_T}\right)M_P$ **<Eq. 4.33>**

$x_P = 0.9$, $x_T = 0.002$, $x_F = 0.2$, $M_P = 25$ kg

$M_F = \left(\frac{0.9-0.002}{0.2-0.002}\right) \times 25\text{ kg} = 113.384\text{ kg}$

(b) $SWU = M_P[V(x_P) - V(x_T)] - M_F[V(x_F) - V(x_T)]$ **<Eq. 4.37>**

$V(x) = (1 - 2x)\ln\left(\frac{1-x}{x}\right)$ **<Eq. 4.35>**

$V(x_P) = V(0.9) = (1 - 1.8)\ln\left(\frac{0.1}{0.9}\right) = 1.758$

$V(x_T) = V(0.002) = (1 - 0.004)\ln\left(\frac{0.998}{0.002}\right) = 6.188$

$V(x_F) = V(0.2) = (1 - 0.4)\ln\left(\frac{0.8}{0.2}\right) = 0.832$

$SWU = 25\text{ kg} \times (1.758 - 6.188) - 113.384\text{ kg} \times (0.832 - 6.188)$

$= 496.535\text{ kg}$

(c) $x_P = 0.2$, $x_T = 0.002$, $x_F = 0.00711$, $M_P = 113.384$ kg

$M_F = \left(\frac{0.2-0.002}{0.00711-0.002}\right) \times 113.384\text{ kg} = 4393.35\text{ kg}$

$V(x_F) = V(0.00711) = (1 - 0.01422)\ln\left(\frac{0.99289}{0.00711}\right) = 4.869$

$SWU = 113.384\text{ kg} \times (0.832 - 6.188) - 4393.35\text{ kg} \times (4.869 - 6.188)$

$= 5187.54\text{ kg}$

31. (a) Derive an explicit expression for the mass of tails produced in a specified enrichment requiring a given amount of separative work. (b) Show that in enriching natural uranium to 3 *w/o*, approximately 1 kg of 0.2 *w/o* tails are produced per SWU expended. (c) In 1981, the total enrichment capacity in operation in the world was 28.4×10^6 SWU/yr. At what rate was depleted uranium being produced, assuming all the capacity was used to produce 3 *w/o* fuel?

[sol] (a) $M_F = \left(\frac{x_T - x_P}{x_F - x_P}\right)M_T$

$M_P = M_F - M_T$

$SWU = M_P V(x_P) + M_T V(x_T) - M_F V(x_F)$ **<Eq. 4.36>**

$= (M_F - M_T)V(x_P) + M_T V(x_T) - M_F V(x_F)$

$= M_T[V(x_T) - V(x_P)] - M_F[V(x_F) - V(x_P)]$

$= M_T[V(x_T) - V(x_P)] - \left(\frac{x_T - x_P}{x_F - x_P}\right)M_T[V(x_F) - V(x_P)]$

$M_T = \frac{SWU(x_F - x_P)}{(x_F - x_P)[V(xT) - V(xP)] - (x_T - x_P)[V(xF) - V(xP)]}$

(b) $x_F = 0.00711$, $x_P = 0.03$, $x_T = 0.002$

$V(0.03) = 3.268$

$V(0.002) = 6.188$

$V(0.00711) = 4.869$

$\frac{M_T}{SWU} = \frac{(x_F - x_P)}{(x_F - x_P)[V(xT) - V(xP)] - (x_T - x_P)[V(xF) - V(xP)]}$

$= \frac{0.00711-0.03}{(0.00711-0.03)(6.188-3.268) - (0.002-0.03)(4.869-3.268)}$

$= 1.04 \fallingdotseq 1$

Approximately 1 kg of 0.2% tails are produced per SWU

(c) Depleted uranium production rate is 28.4×10^6 kg/yr, because approximately 1 kg of 0.2% tails are produced per SWU.

32. In 1980, the United States had approximately 300,000 tons of depleted uranium in storage at its gaseous diffusion plants. If the entire 1980 electrical capacity of 600 GWe were furnished by LMFBRs fueled with this uranium, how long would the 1980 depleted uranium stock (which is continuing to grow; see preceding problem) last? Use LMFBR data from Fig. 4.40.

[sol] $M_F = 2{,}513 \times \frac{600\ \text{GWe}}{0.75\ \text{GWe}}$ **<Mass flows in kg's per 0.75 GWe-yr>**

$= 2{,}010{,}400$ kg

$x_F = 0.00711$, $x_P = 0.03$, $x_T = 0.002$

$M_P = (\frac{x_F - x_T}{x_P - x_T})M_F = (\frac{0.00711-0.002}{0.03-0.002}) \times 2{,}010{,}400\ \text{kg} = 366{,}898\ \text{kg}$

$M_T = M_F - M_P = 2{,}010{,}400\ \text{kg} - 366{,}898\ \text{kg} = 1{,}643{,}502\ \text{kg}$

Lasting time $= \frac{300{,}000{,}000\ \text{kg}}{1{,}643{,}502\ \text{kg/yr}} = 182.537$ years

33. Show that approximately 116,000 SWU of separative work is required annually to maintain the nominal 1,000 MWe LWR described in Fig. 4.37.

[sol] $SWU = M_P[V(x_P) - V(x_T)] - M_F[V(x_F) - V(x_T)]$

$M_P = 27{,}249$ kg, $M_F = 149{,}297$ kg

$x_P = 0.03$, $x_T = 0.002$, $x_F = 0.00711$

$V(0.03) = (1 - 0.06)\ln(\frac{0.97}{0.03}) = 3.268$

$V(0.002) = 6.188$

$V(0.00711) = 4.869$

$SWU = 27{,}249\ \text{kg} \times (3.268 - 6.188) - 149{,}297\ \text{kg} \times (4.869 - 6.188)$

$= 117356\ \text{kg} \fallingdotseq 116{,}000\ \text{kg}$

34. The enrichment consortium EURODIF operates gaseous diffusion plants in France with a capacity of 10.8 million SWU/year. Using the result of the preceding problem, how much LWR capacity can this company service?

[sol] 1,000 MWe : 116,000 SWU/yr = x : 10,800,000 SWU/yr

$x = \frac{1{,}000\ \text{MWe} \times 10{,}800{,}000\ \text{SWU/yr}}{116{,}000\ \text{SWU/yr}} = 93103.4\ \text{MWe}$

35. Compare and plot the relative cost in SWU of enriched uranium, per unit of contained ^{235}U, as a function of enrichment to 93 *w/o*. Compare with Fig. 4.45.

[sol] $x_T = 0.002$, $x_F = 0.00711$

$SWU = M_P[V(x_P) - V(x_T)] - M_F[V(x_F) - V(x_T)]$

$\frac{SWU}{M_P} = [V(x_P) - V(x_T)] - \frac{M_F}{M_P}[V(x_F) - V(x_T)]$

$= [V(x_P) - V(0.002)] - (\frac{x_P - x_T}{x_F - x_T})[V(0.00711) - V(0.002)]$

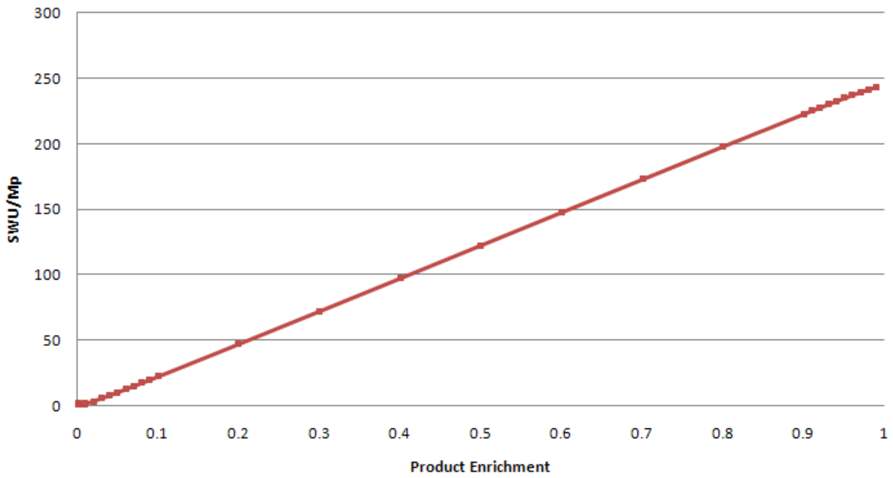
$= (1 - 2x_P)\ln(\frac{1-x_P}{x_P}) - 6.188 - \frac{x_P - 0.002}{0.00711-0.002}(4.869 - 6.188)$

$= (1 - 2x_P)\ln(\frac{1-x_P}{x_P}) + \frac{x_P - 0.002}{0.00387} - 6.188$

If $x_P = 0.93$

$\frac{SWU}{M_P}(x_P = 0.93) = (1 - 2 \times 0.93)\ln(\frac{1-0.93}{0.93}) + \frac{0.93 - 0.002}{0.00387} - 6.188$

$= 235.83$



36. Based on the projections of future installed nuclear electric generating capacity given in Table 4.8, how large an industry (dollars per year) can isotope enrichment be expected to be in 2010 based on current U.S. SWU prices if most of the capacity is in LWRs?

[sol]

37. The principle of enrichment by gaseous diffusion can be seen in the following way. Consider a chamber (diffuser) split into two volumes, A and B, by a porous barrier with openings having a total area S as shown in Fig. 4.48. Volume A contains two isotopic species of mass M_1 and M_2 at atom densities N_{10} and N_{20}, respectively. Volume B is at low pressure and is pumped out at the rate of F m3/sec. According to kinetic theory, the number of atoms or molecules in a gas striking the wall of a container per m^2/sec is

$$J = \frac{1}{4}N\bar{v}$$

where N is the atom density and $\bar{v}$ is the average speed given by

$$\bar{v} = 2\sqrt{\frac{2kT}{\pi M}},$$

where M is the mass of the atom and T is the absolute temperature. (a) Show that in equilibrium the atom concentration in B is

$$\frac{N}{V_B} = (J_1 + J_2)\frac{S}{F},$$

where J_1 and J_2 refer to the two isotopic species. (b) Show that the ratio of equilibrium concentrations in B is given by

$$\frac{N_1}{N_2} = \frac{N_{10}}{N_{20}}\sqrt{\frac{M_2}{M_1}},$$

[sol]

a)

J= $\mathbf{J_1 + J_2}$

Number pumped out = $\mathbf{\frac{F}{V_B} \times N}$

Number of atoms stibing the wall per second **JS =($\mathbf{J_1}$+$\mathbf{J_2}$)S**

$$\mathbf{\frac{F}{V_B} \times N = JS \ = (J_1 + J_2)S}$$

=> $\mathbf{\frac{N}{V_B} = ((J_1 + J_2)S/F}$

b)

$\frac{J_1}{J_2} = \frac{N_{10}\bar{v}}{N_{20}\bar{v}} = \frac{N_{10}}{N_{20}}\sqrt{\frac{M_2}{M_1}}$, $\quad \frac{N_1}{V_B} = J_1\frac{S}{F}$, $\quad \frac{N_2}{V_B} = J_2\frac{S}{F}$,

$$\therefore \frac{N_1}{N_2} = \frac{J_1}{J_2} = \frac{N_{10}}{N_{20}}\sqrt{\frac{M_2}{M_1}}$$

38. The quantity

$$\frac{N_1/N_{10}}{N_2/N_{20}} = \sqrt{\frac{M_2}{M_1}}$$

in the preceding problem is called the ideal separation factor α_0.Compute α_0 for a gas consisting of molecules of $^{235}UF_6$ and $^{238}UF_6$. (The small value of α_0 in this case means that a great many stages must be used to enrich uranium to useful levels.)

[sol]

$$M_1 = M(^{235}UF_6) = 235 + 19 \times 6 = 349$$
$$M_2 = M(^{235}UF_6) = 238 + 19 \times 6 = 352$$
$$\alpha_0 = \frac{N_1/N_{10}}{N_2/N_{20}} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{352}{349}} = 1.00429$$

39. If, using LIS, it is possible to enrich diffusion plant tails from 0.2 w/o to 3 w/o with residual tails of 0.08w/o, verify that the utilization of natural uranium for 3w/o fuel would be increased by 18%.

[sol]

i) $M_p = \left(\frac{x_F - x_T}{x_P - x_T}\right) M_F = \frac{0.00711-0.002}{0.03-0.002} M_F = 0.1825 M_F$

Mass of ^{235}U **in** $M_F = 0.00711 M_F$
Mass of ^{235}U **in** $M_P = 0.03 \times 0.1325 M_F = 5.475 \times 10^{-3} M_F$

$$\therefore \text{utilization} = \frac{5.475 \times 10^{-3} M_F}{0.00711 M_F} = 77\%$$

ii) $M_{P1} = 0.1825 M_F$
$M_{p2} = \frac{0.002-0.0008}{0.03-0.0008} \times (1-0.1825) M_F = 0.033596 M_F$
$M_P = 0.033596 + 0.1825) M_F = 0.2161 M_F$

Mass of ^{235}U **in** $M_P = 0.03 \times 0.2161 M_F = 6.4829 \times 10^{-3} M_F$
Utilization = $\frac{6.4829 \times 10^3}{0.00711} = 91\%$

$$\therefore 91 - 77 = 14\% \text{ increase}$$

40. To how much energy in e V does the isotopic shift of 0.1 Å shown in Fig. 4.52 correspond?

[sol]

$$\lambda = \frac{1.240 \times 10^{-6}}{E} \quad \ldots(2.22)$$

$E = \frac{1.240 \times 10^{-6}}{\lambda}$, $\quad E_1 = \frac{1.240 \times 10^{-6}}{5027.3 \times 10^{-10}(m)} = 2.466532731[eV]$

$$E_2 = \frac{1.240 \times 10^{-6}}{5027.4 \times 10^{-10}(m)} = 2.466483669[eV]$$

$E_1 - E_2 = 2.46632731 - 2.466483669 = 0.000049062 \cong 0.049 \times 10^{-3}[eV]$

41. (a) Show that the specific activity of fuel irradiated for T days up to a burnup of B MWdlkg, t days after removal from a reactor, is given by

$$\alpha = 1.4 \times 10^{6}\frac{B}{T}[t^{-0.2} - (t+T)^{-0.2}]\mathrm{Ci/kg}$$

[sol]
Fission product activity = $1.4\times 10^{6}[t^{-0.2} - (t+T)^{-0.2}]$Ci…..(3.49)
Burnup rate = 1.05P g/day…(3.57) $\cong$ P g/day
The fission of 1g yields 1Mwd
B = $\frac{\mathrm{PMw\times Tday}}{\mathrm{BTg}} = \frac{\mathrm{P\times TMwd\times 10^{3}}}{\mathrm{P\times Tkg}}$

$\rightarrow$ PMw= B$\times$ $(\mathrm{P} \times \mathrm{T} \times 10^{-3})$kg MWd/kg = $\frac{\mathrm{B\times(P\times T\times 10^{-3})}}{\mathrm{T}}$MW

Mass of ^{235}U t operate on PMW for T days

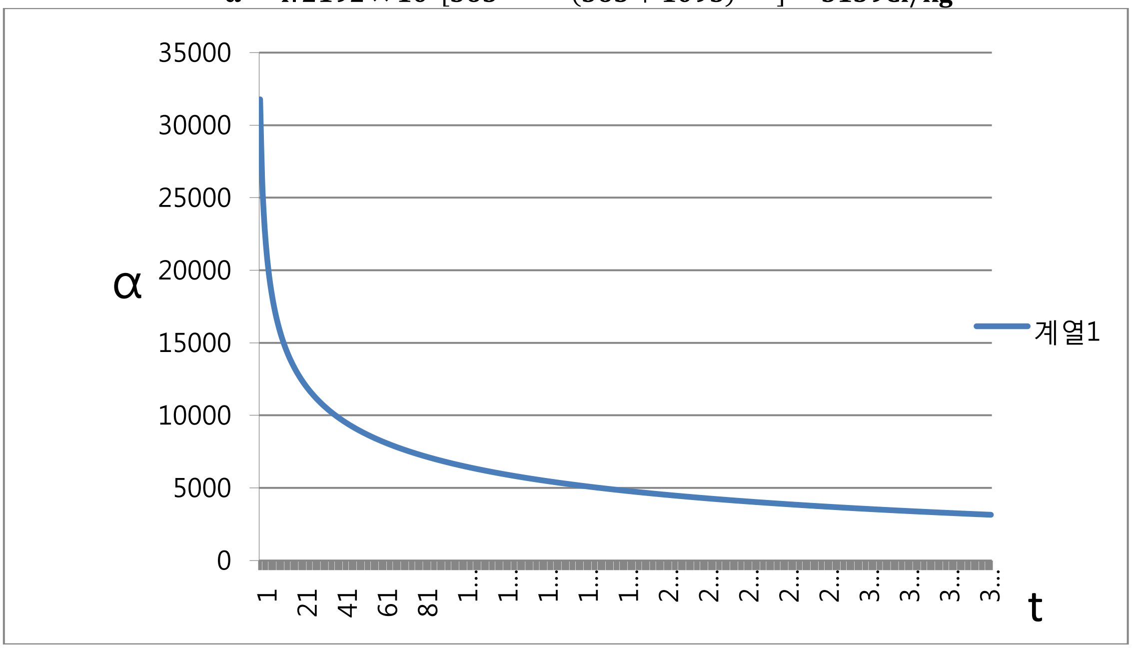
=(P$\times$ T)g = $(\mathrm{P}\times \mathrm{T} \times 10^{-3})$kg

∴ specific activity
$$\alpha = 1.4 \times 10^{6}\mathrm{P}[t^{-0.2} - (t+T)^{-0.2}]\mathrm{Ci} \div (\mathrm{P}\times\mathrm{T}\times 10^{-3})\mathrm{kg}$$
$$= 1.4 \times 10^{6} \times \frac{\mathrm{B\times(P\times T\times 10^{-3})}}{\mathrm{T(P\times T\times 10^{-3})}}[t^{-0.2} - (t+T)^{-0.2}]\mathrm{Ci/kg} = 4 \times 10^{6} \times \frac{B}{T}[t^{-0.2} - (t+T)^{-0.2}]\mathrm{Ci/Kg}$$

(b) Compute and plot the specific activity for fuel irradiated for 3 years to a burnup of 33MWd/kg from 1 day to 1 year after removal from the reactor

[sol]
B= 33MWd/kg, T =.$\times$ 365 = 1095d
$$\alpha = 1.4 \times 10^{6} \times \frac{33}{1095}[t^{-0.2} - (t+1095)^{-0.2}] = 4.2192 \times 10^{4}[t^{-0.2} - (t+1095)^{-0.2}]\mathrm{Ci/kg}$$
i) t =1day
$$\alpha = 4.2192 \times 10^{4}[1^{-0.2} - (1+1095)^{-0.2}] = 3.1786 \times 10^{4}\mathrm{Ci/kg}$$
ii) t=365day

$$\alpha = 4.2192 \times 10^{4}[365^{-0.2} - (365+1095)^{-0.2}] = 3139\mathrm{Ci/kg}$$



42. The fission yields of ^{90}Sr and ^{137}Cs are 0.0593 atoms per fission, respectively. Calculate the specific

activity due to these nuclides and to ^{90}Y in spent fuel irradiated to 33MWd/kg from the time the fuel is removed from the reactor up to 1,000 years.

[sol]

Fission rate $=2.7\times 10^{21}\mathbf{P\ fission\ /day}$

For $^{90}\mathbf{Sr}$
N$=2.7\times 10^{21}\times 33\times 0.0593=5.2836\times 10^{21}$ **/ kg**
$T_{1/2}=28.8yr$
$\lambda=7.632\times 10^{-10}$
$\therefore\ \alpha_0^{90}=4.2365\times 10^{12}dps=114.5Ci$
$d\alpha(t)=\alpha_0 e^{-\lambda t}dt$

$$\therefore\ \alpha=\int_0^t \alpha_0 e^{-\lambda t}dt=\int_0^{1000\times 365\times 24\times 3600} 114.5\times e^{-\lambda t}dt=\left[-\frac{114.5}{7.632\times 10^{-10}}e^{-7.632\times 10^{-10}t}\right]_0^{3.1536\times 10^{10}}$$

$=1.5\times 10^{11}$**Ci/kg**

For $^{137}\mathbf{Cs}$
N$=2.7\times 10^{21}\times 33\times 0.0623=5.551\times 10^{21}$ **/ kg**
$T_{1/2}=30.17yr$
$\lambda=7.285\times 10^{-10}$
$\therefore\ \alpha_0^{137}=4.044\times 10^{12}dps=109.3Ci$

$$\alpha=\int_0^{1000\times 365\times 24\times 3600} 109.3e^{-7.285\times 10^{-10}t}dt=\left[-\frac{109.3}{73285\times 10^{-10}}e^{-7.285\times 10^{-10}t}\right]_0^{3.1536\times 10^{10}}$$

$=1.5\times 10^{11}$**Ci/kg**

For $^{90}\mathbf{Y}$

$^{90}Sr\xrightarrow{28.8y,\beta^-}{}^{90}Y\xrightarrow{64.1h,\beta^-}{}^{90}Zr$
$\lambda_1\ll\lambda_2\rightarrow$ **secular Equilibium**

$$\therefore\ \alpha(^{90}Sr)\cong\alpha(^{90}Y)\cong 1.5\times 10^{11}Ci/kg$$

43. Carbon-14, with a half-life of 5,730 years, is produced in LWRs by way of an (n,p) reaction with nitrogen impurities (in both the fuel and the coolant water) and via an (n,α) reaction with ^{17}O. About 60 to 70 Ci of ^{14}C are generated this way per GWeyear. (a) What is the total ^{14}C activity in the fuel unloaded each year from a 1 ,000MWe reactor that has operated with a 0.70 capacity factor? Assume that one third of the core is unloaded each year. (b) How much ^{14}C will be produced per year in the year 2010 by all the reactors in the world if high or low projections of Table 4.8 come to pass and all reactors are LWRs?
(a)

[sol] 0.7 capacity factor and 1/3 of the core is unloaded, the total activity is

$\alpha=0.7\times\frac{1}{3}\times(60\sim 70)=14\sim 16Ci$

b)

[sol]
assuming that about 65Ci of $^{14}\mathbf{C}$ **one generated per GWe**
i) for low projections
activity $=65\times 850=55250Ci$
N $=\frac{55252Ci}{\lambda}=\frac{5730\times 3.1536\times 10^7}{\ln 2}\times 55250\times 3.7\times 10^{10}=5.329\times 10^{26}atoms$
M$=\frac{5.329\times 10^{26}}{6.022\times 10^{23}}$**=12390g=12.39kg**

ii) for high projections

activity = $65\times 1200 = 78000$Ci

N=$\frac{5730\times 3.1536\times 10^{7}}{\ln 2}\times 78000\times 3.7\times 10^{10} = 7.523\times 10^{26}$

M=$\frac{7.523\times 10^{26}}{6.022\times 10^{23}}$**=17.49kg**

Chapter 5
Neutron Diffusion and Moderation

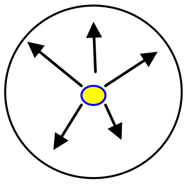
1. A point source emits S neutron/sec isotropically in an finite vacuum.
(a) Show that the neutron flux at the distance r from the source is given by

$$\emptyset = \frac{S}{4\pi r^2}$$

[sol] It is important to note that this solution is quite different than would be obtained for a point source in a vacuum. This difference occurs since the diffusing media both absorbs and scatters neutrons, whereas a vacuum does not.

$\mathbf{4\pi r^2 \emptyset(r) = S}$

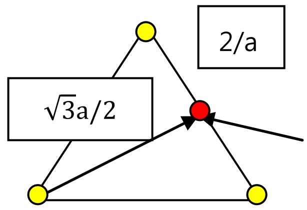
So, $\mathbf{\emptyset(r) = \frac{S}{4\pi r^2}}$



(b) What is the neutron current density vector at the same point? [Note: Neutrons do not diffuse in a vacuum.]

[sol] $\mathbf{J = -D\frac{d}{dr}\left(\emptyset(r)\right) = -D\frac{d}{dr}\left(\frac{S}{4\pi r^2}\right) = \frac{DS}{2\pi r^3}}$

2. Three isotopic neutron source, each emitting S neutons/sec, are located in an infinite vacuum at the three corners of an equilateral triangle of side a. Find the flux and current at the midpoint of one side.



[sol] reference problem (Example 5.3)
The flux

$\mathbf{\emptyset(p) = \frac{S}{4\pi(\frac{a}{2})^2} \times 2 + \frac{S}{4\pi(\frac{\sqrt{3}a}{2})^2} = \frac{7S}{3\pi a^2}}$ **neutrons/cm²sec**

The current,

$\mathbf{J(p) = -D\frac{d}{dx}\left(\emptyset(r)\right) = -D\frac{d}{dx}\left(\frac{S}{4\pi r^2}\right)}$

$\mathbf{= \frac{DS}{2\pi r^3}}$

$\mathbf{J\left(\frac{\sqrt{3}}{2}a\right) = \frac{4\sqrt{3}DS}{9\pi a^3}}$

(Due to characteristics of vector, other two currents are canceling each other.)

3. Using Eqs. (5.10) and (5.11), estimate the diffusion coefficients of (a) beryllium, (b) graphite, for monoenergetic 0.0253 eV neutrons.

[sol] Equ 5. 10 and 11 :

$\mathbf{D = \frac{\lambda_{tr}}{3}}$ **and** $\mathbf{\lambda_{tr} = \frac{1}{\Sigma_{tr}} = \frac{1}{\Sigma_s(1-\bar{\mu})}}$

(a) Beryllium

$\mathbf{\Sigma_s = 0.7589,}$

$\mathbf{\lambda_{tr} = \frac{1}{0.7599\left(1-\frac{1}{6}\right)} = 1.589, D = 0.527cm}$

(b) Graphite

$\mathbf{\Sigma_s = 0.3811}$

$\mathbf{\lambda_{tr} = \frac{1}{0.3811\left(1-\frac{1}{9}\right)} = 2.952, D = 0.984\ cm}$

4. The neutron flux in a bare spherical reactor of spherical reactor of radius 50 cm is given by

$$\emptyset = 5 \times 10^{13}\frac{\sin 0.0628r}{r}\frac{\text{neutrons}}{\text{cm}^2\ \text{sec}}$$

where r is measured from the center of the reactor. The diffusion coefficient for the system is 0.80 cm. (a) What is the maximum value of the flux in the reactor? (b) Calculate the neutron current density as a function of position in the reactor. (c) How many neutrons escape from the reactor per second?

[sol] (a) $\mathbf{\phi_{max}(r) = \lim_{r\to 0}\phi(r) = \lim_{r\to 0} 5 \times 10^{13}\frac{\sin 0.0628r}{r} = 5 \times 10^{13} \times 0.0628 = 3.14}$ **neutrons/**

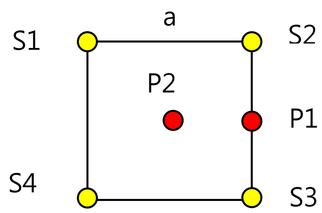
cm^2sec
(b)

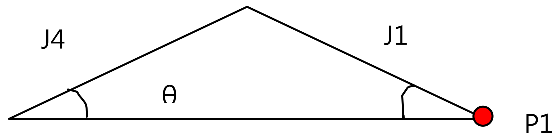
$$\mathbf{J(r) = -D\frac{d}{dr}\left(\emptyset(r)\right) = -D\frac{d}{dr}\left(5\times10^{13}\frac{\sin 0.0628r}{r}\right)}$$
$$\mathbf{= -4\times10^{13}\left(\frac{0.0628\cos 0.0628r}{r} - \frac{\sin 0.0628r}{r^2}\right)}$$

(c) Leakage rate

$$\mathbf{\int_A \vec{J}\cdot\vec{r}da = \int_A J_r da = 4\pi r^2 J(r) = 4\pi r^2 \times (-4\times10^{13}\left(\frac{0.0628\cos0.0628*50}{50} - \frac{\sin0.0628*50}{50^2}\right) = 1.578\times 10^{15}\#/sec}$$

5. Isotope point sources each emitting S neutrons/sec are placed in an infinite moderator at the four corners of a square of side a. Compute the flux and current at the midpoint of any side of the square and at its center.





[sol] $\mathbf{\emptyset(r) = \frac{Se^{-r/L}}{4\pi Dr}}$

$$\mathbf{(a)\emptyset(p1) = 2\times\left[\frac{Se^{-\frac{\frac{\sqrt{5}a}{2}}{L}}}{4\pi D\left(\frac{\sqrt{5}a}{2}\right)} + \frac{Se^{-\frac{a}{2L}}}{4\pi D\left(\frac{a}{2}\right)}\right] = \frac{S}{\pi Da}\left[\frac{1}{\sqrt{5}}e^{-\frac{\frac{\sqrt{5}}{2}a}{L}} + e^{-\frac{a}{2L}}\right]}$$

$$\mathbf{\emptyset(p2) = 4\times\frac{Se^{-\frac{\frac{\sqrt{2}}{2}a}{L}}}{4\pi D\left(\frac{\sqrt{2}}{2}a\right)} = \frac{\sqrt{2}e^{-\frac{\frac{\sqrt{2}}{2}a}{L}}}{\pi Da}}$$

(b) at P1

$$\mathbf{J1 = J4 = -D\frac{d}{dr}\left(\emptyset(r)\right)_{r=\frac{\sqrt{5}}{2}a}}$$
$$\mathbf{= -\frac{S}{4\pi}\left(\frac{-\frac{1}{L}e^{-\frac{r}{L}}}{r} - \frac{e^{-\frac{r}{L}}}{r^2}\right)_{r=\frac{\sqrt{5}}{2}a} = \frac{S}{\pi a^2}e^{-\frac{\sqrt{5}a}{2L}}\left[\frac{a}{2\sqrt{5}L} + \frac{1}{5}\right]}$$
$$\mathbf{J(P1) = \sqrt{J1^2 + J4^2 - 2J1J4\cos(\pi - 2\theta)} = \frac{4}{\sqrt{5}}J1 \quad \left(\cos2\theta = \frac{3}{5}\right)}$$

$$\mathbf{J(P1) = \frac{4}{\sqrt{5}}J1 = \frac{4}{\sqrt{5}}\left[\frac{S}{\pi a^2}e^{-\frac{\sqrt{5}a}{2L}}\left[\frac{a}{2\sqrt{5}L} + \frac{1}{5}\right]\right] = \frac{4S}{5\pi a^2}e^{-\frac{\sqrt{5}a}{2L}}\left[\frac{a}{5L} + \frac{1}{\sqrt{5}}\right]}$$

at P2
$\mathbf{J(P2) = 0}$

6. Find expressions for the flux and current at the point P2 in Fig. 5.5

[sol]

$$\mathbf{(a)\emptyset(P) = 2\times\frac{Se^{-\sqrt{2}a/L}}{4\pi D(\sqrt{2}a)} = \frac{Se^{-\sqrt{2}a/L}}{2\sqrt{2}\pi Da}}$$
$$\mathbf{(b)J = \sqrt{J1^2 + J2^2 - 2J1J2\cos90} = \sqrt{2}J1}$$
$$\mathbf{= -\sqrt{2}D\frac{d}{dr}\left(\emptyset(P)\right)_{r=\sqrt{2}a} = -\sqrt{2}\frac{S}{4\pi}\frac{d}{dr}\left(\frac{e^{-\frac{r}{L}}}{r}\right)_{r=\sqrt{2}a} = \frac{\sqrt{2}S}{4\pi a^2}\left(\frac{1}{2} + \frac{a}{\sqrt{2}L}\right)e^{-\sqrt{2}a/L}}$$

7. An isotropic point source emits S neutrons/sec in a n infinite moderator.
   (a) Compute the net number of neutrons passing per second through a spherical surface of radius r centered on the source.

[sol]

$\mathbf{\emptyset(r) = \frac{Se^{-r/L}}{4\pi Dr}}$ **… (Eq 5.33)**

$$\mathbf{J(r) = -D\frac{d\emptyset}{dr} = \frac{S}{4\pi}\left(\frac{1}{r^2} + \frac{1}{rL}\right)e^{-r/L}}$$

net number of neutrons passing per second through a spherical surface of radius r centered

$$\mathbf{= \ 4\pi r^2 J(r) = S(1 + \frac{r}{L})e^{-r/L}}$$

(b) Compute the number of neutrons absorbed per second within the sphere.

[sol]

$$\int_0^r \Sigma_a \emptyset(r)dv = \int_0^r \Sigma_a \frac{Se^{-r/L}}{4\pi Dr}4\pi r^2 dr = \int_0^r \frac{\Sigma_a S}{D} re^{-r/L}\,dr = \frac{\Sigma_a S}{D}\left[-Lre^{-r/L}\right]_0^r + L\int_0^r e^{-r/L}dr$$

$$= \frac{\Sigma_a S}{D}\left(-Lre^{-\frac{r}{L}} - \ L^2e^{-\frac{r}{L}} + L^2\right)$$

$$= \frac{\Sigma_a SL^2}{D}\left[1 - \left(1 + \frac{r}{L}\right)e^{-r/L}\right]$$

(c) Verify the equation of continuity for the volume within the sphere.

[sol]

$$\frac{\partial n}{\partial t} = \left[S - S\left(1 + \frac{r}{L}\right)e^{-\frac{r}{L}} - \frac{\Sigma_a SL^2}{D}\left[1 - \left(1 + \frac{r}{L}\right)e^{-\frac{r}{L}}\right]\right] \div \left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{1}{\frac{4}{3}\pi r^3}\left[S - S\left(1 + \frac{r}{L}\right)e^{-\frac{r}{L}} - \frac{\Sigma_a SL^2}{D}\left[1 - \left(1 + \frac{r}{L}\right)e^{-\frac{r}{L}}\right]\right]$$

8. Two infinite planar sources each emitting S neutrons/cm2 are placed parallel to one another in an infinite moderator at the distance a apart. Calculate the flux and current as a function of distance from a plane midway between the two.

[sol]

i) flux is scalar factor. So the flux at x =0

$$\emptyset(0) = \ 2 \times \frac{SL}{2D}e^{-\frac{a}{2}/L} = \frac{SL}{D}e^{-\left(\frac{a}{2L}\right)}$$

ii) from the planar source, current vetors have same magnitude and opposite direction. So current at x = 0 is zero.

$$\mathbf{J(o) = 0}$$

9. Suppose the two planar sources in the preceding problem are placed at right angles to one another. Derive expressions for the flux and current as a function of distance from the line of intersection of the sources in a plane bisecting the angle between the sources.

[sol]

Flux from the distance r

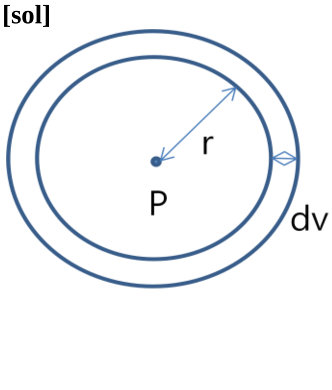
$$\emptyset(r) = 2 \times \frac{SL}{2D}e^{-r/L},$$

$$J_1 = J_2 = -D\frac{d}{dx}\left(2 \times \frac{SL}{2D}e^{-\frac{r}{L}}\right) = -D\left(-\frac{S}{2D}e^{-\frac{r}{L}}\right) = \frac{1}{2}Se^{-\frac{r}{L}}, \qquad x = r$$

$$\therefore J = \sqrt{J_1^2 + J_2^2 - 2J_1J_2\cos\alpha} = \sqrt{2J_1^2 + 2J_1^2\cos\theta} = \sqrt{2}J_1\sqrt{1 + \cos\theta} = \frac{\sqrt{2}}{2}Se^{-\frac{r}{L}}\sqrt{1 + \cos\theta}$$

10 An infinite moderator contains unifonnly distributed isotropic sources emitting S neutrons/cm3 -sec. Detennine the steady-state flux and current at any point in the medium.

[sol]



Netron source in dv = Sdv

$$d\emptyset(P) = \frac{Sdv}{4\pi Dr}e^{-r/L}$$

$$\therefore \emptyset(P) = \int_0^\infty \frac{Sdv}{4\pi Dr}e^{-\frac{r}{L}} = \int_0^\infty \frac{S}{4\pi Dr}e^{-\frac{r}{L}}4\pi r^2dr = \frac{S}{D}\int_0^\infty re^{-\frac{r}{L}}dr$$

$$= \frac{S}{D}\left[\left(-Lre^{\frac{-r}{L}}\right)_0^{\infty} + L\int_0^{\infty} e^{-\frac{r}{L}}dr\right] = -\frac{SL^2}{D}e^{\frac{-r}{L}}\Big|_0^{\infty} = \frac{SL^2}{D}$$

$$= \frac{S}{\Sigma_a} \qquad , J(P)=0$$

11. An infinite bare slab of moderator of thickness 2a contains unifonnly distributed sources emitting S neutrons/cm3 -sec. (a) Show that the flux in the slab is given by

$$\phi = \frac{S}{\Sigma_a}\left(1 - \frac{\cosh x/L}{\cosh\left(\frac{a+d}{L}\right)}\right),$$

where x is measured from the center of the slab. (b) Verify the equation of continuity by computing per unit area of the slab the total number of neutrons (i) produced per sec within the slab; (ii) absorbed per second within the slab; and (iii) escaping per second from the slab. [Hint: The solution to an inhomogeneous differential equation is the sum of solutions to the homogeneous equation plus a particular solution. Try a constant for the particular solution.]

[sol]

(a)

$$D\nabla^2\emptyset - \Sigma_a\phi = -S \text{ (steady state)}$$

$$\nabla^2\emptyset - \frac{1}{L^2}\phi = -\frac{S}{D}$$

At slab geometry

$$\frac{d^2\phi}{dx^2} - \frac{1}{L^2}\phi = -\frac{S}{D}$$

i) homogeneous solution

$$\phi_H = A\sinh\frac{x}{L} + B\cosh\frac{x}{L}$$

ii) particular solution

$$\phi_p = C$$

$$0 - \frac{C}{L^2} = -\frac{S}{D} \rightarrow \phi_p = C = \frac{SL^2}{D}$$

$$\therefore \emptyset = \emptyset_H + \emptyset_p = A\sinh\frac{x}{L} + B\cosh\frac{x}{L} + \frac{SL^2}{D}$$

boundary condition

i) $\emptyset(a+d) = \emptyset(-a-d)$

$$A\sinh\frac{a+d}{L} + B\cosh\frac{a+d}{L} + \frac{SL^2}{D} = A\sinh[-\frac{(a+d)}{L}] + B\cosh[-\frac{(a+d)}{L}] + \frac{SL^2}{D}$$

Sinhx = -sinh(-x), conshx = cosh(-x), A = 0

$$\therefore \emptyset = B\cosh\frac{x}{L} + \frac{SL^2}{D}$$

ii) $\emptyset(a+d) = 0$

$$\therefore \emptyset(a+d) = B\cosh\frac{(a+d)}{L} + \frac{SL^2}{D} = 0, \quad B = -\frac{SL^2}{D}\frac{1}{\cosh\frac{(a+d)}{L}}$$

$$\therefore \emptyset = -\frac{SL^2}{D}\left[\frac{\cosh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)}\right] + \frac{SL^2}{D} = \frac{SL^2}{D}\left[1 - \frac{\cosh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)}\right] = \frac{S}{\Sigma_a}\left[1 - \frac{\cosh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)}\right]$$

(b)

i) S neutron / cm^3**-sec**

ii) $\Sigma_a\phi(x)$

iii) $J(x) = -D\frac{d}{dx}\phi = SL^2 * \frac{\sinh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)} * \frac{1}{L} = SL\frac{\sinh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)}$

$$\mathbf{DivJ - D\frac{d^2\phi}{dx^2} = S\frac{\cosh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)}}$$

Continuity Eq

$$\mathbf{\frac{dn}{dt} = S - \Sigma_a\phi - S\frac{\cosh\left(\frac{x}{L}\right)}{\cosh\left(\frac{a+d}{L}\right)}}$$

12. A point source emitting S neutrons/sec is placed at the center of a sphere of moderator of radius R. (a) Show that the flux in the sphere is given by

$$\phi = \frac{S}{4\pi D\sinh(\frac{R+d}{L})}\frac{\sinh\frac{1}{L}(R+d-r)}{r},$$

where r is the distance from the source.

[sol]

No source with r≠ 0

$\mathbf{D\nabla^2\emptyset - \Sigma_a\phi = 0, \quad \frac{1}{r^2}\frac{d}{dr}r^2\frac{d\emptyset}{dr} - \frac{\emptyset}{L^2} = 0}$ **-------- A**

Let w=rϕ

$\mathbf{\emptyset(r) = C_1\frac{\sinh\frac{r}{L}}{r} + C_2\frac{\cosh\frac{r}{L}}{r}}$ **----------- B**

Boundary condition

$$\mathbf{\emptyset(R) = \frac{1}{R}\left[C_1\sinh\frac{R}{L} + C_2\cosh\frac{R}{L}\right] = 0\,, where\ R = r + d}$$

$\mathbf{C_2 = -C_1\tanh\frac{R}{L}}$ **------- C**

C → B

$\mathbf{\emptyset(r) = \frac{C_1}{r}[\sinh\frac{r}{L} - \cosh\frac{r}{L}\tanh\frac{R}{L}]}$ **------------ D**

B.C

$\mathbf{\lim_{r=0} 4\pi r^2 J(r) = S}$ **--------------------------- E**

$$\mathbf{\frac{d\emptyset(r)}{dr} = \frac{C_1}{r^2}\left[\left(\frac{1}{L}\cosh\frac{r}{L} - \frac{1}{L}\sinh\frac{r}{L}\tanh\frac{R}{L}\right)r - (\sinh\frac{r}{L} - \cos\frac{r}{L}\tanh\frac{R}{L}\right]}$$

$\mathbf{\lim_{r=0} 4\pi r^2 J(r) = 4\pi C_1\left[\left(\frac{1}{L} - 0\right) * 0 - (0 - \tanh\frac{R}{L})\right](-D) = -4\pi D\tanh\frac{R}{L} = S}$

$\mathbf{C_1 = \frac{-S}{4\pi D}\coth\frac{R}{L}}$ **--------- F**

F → D

$$\mathbf{\emptyset(r) = \frac{S}{4\pi rD}\left[\cosh\frac{r}{L} - \coth\frac{R}{L}\sinh\frac{r}{L}\right] = \frac{S}{4\pi D\sinh\frac{R}{L}}\left(\frac{\sinh\frac{R}{L}\cosh\frac{r}{L} - \cosh\frac{R}{L}\sinh\frac{r}{L}}{r}\right)}$$

$$\mathbf{= \frac{S}{4\pi D\sinh\frac{R+d}{L}} * \frac{\sinh\frac{1}{L}(R+d-r)}{r}}$$

(b) Show that the number of neutrons leaking per second from the surface of the sphere is given by

$$\text{No.Leaking/sec} = \frac{(R+d)S}{L\sinh(\frac{R+d}{L})}$$

[sol]

$\mathbf{J(r) = -D\frac{d\emptyset}{dr} = \frac{-S}{4\pi\sinh((R+d)/L)} * \frac{-\frac{r}{L}\cosh\frac{1}{L}(R+d-r) - \sinh\frac{1}{L}(R+d-r)}{r^2} = 4\pi R^2 J(R) = \frac{S}{\sinh\left(\frac{R+d}{L}\right)}(\frac{R}{L}\cosh\frac{R}{L} + \sinh\frac{d}{L})}$

$\mathbf{d/L \approx 0}$
$\mathbf{\cosh(d/L) \approx 1 \; , \; \sinh(d/L) \approx (d/L)}$

$$\therefore 4\pi R^2 J(R) = \frac{S}{\sinh\left(R+\frac{d}{L}\right)}\left[\frac{R}{L}+\frac{d}{L}\right] = \frac{R+d}{L\sinh\left(\frac{R+d}{L}\right)}$$

(c) What is the probability that a neutron emitted by the source escapes from the surface?

[sol]

$$P = \frac{4\pi R^2 J(R)}{S} = \frac{R+d}{L\sinh\left(\frac{R+d}{L}\right)}$$

13. An infinite planar source emitting S neutrons/cm^2-sec is placed between infinite slabs of beryllium and graphite of thickness a and b, respectively, as shown in Fig. 5.9. Derive an expression for the neutron flux in the system. [*Note*: Since the media are different on opposite sides of the source, this problem is not symmetric and the source condition Eq. (5.28) is not valid. The appropriate boundary conditions for this problem are

(i) $\lim_{x\to 0}[\text{Ø}\,(x > 0) - \text{Ø}\,(x < 0)] = 0$

(ii) $\lim_{x\to 0}[J(x > 0) - J(x < 0)] = S$

Condition (i) states, in effect, that Ø is continuous at the source, whereas (ii) accounts for the neutrons emitted from the source.]

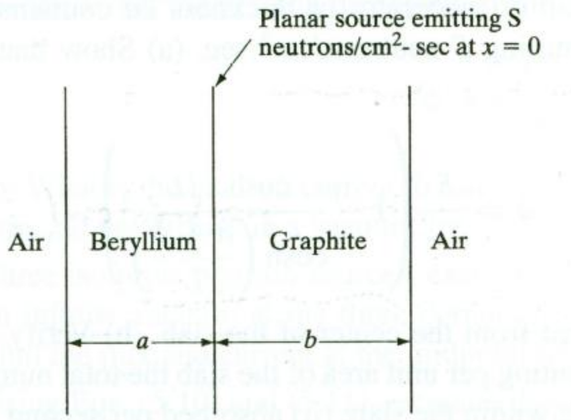


Figure 5.9 Infinite planar source between two finite slabs of material.

[sol] $\frac{d^2\text{Ø}}{dx^2} - \frac{1}{L^2}\text{Ø} = 0, x \neq 0$ **<Eq. 5.25>**

general solutions : $\text{Ø}_{(x<0)} = C_1\cosh\frac{x}{L_1} + C_2\sinh\frac{x}{L_1}$

$\text{Ø}_{(x>0)} = C_3\cosh\frac{x}{L_2} + C_4\sinh\frac{x}{L_2}$

Let, $\tilde{a} = a + d_1$, $\tilde{b} = b + d_2$

Boundary Condition 1 : $\text{Ø}_1(-\tilde{a}) = 0$, $\text{Ø}_2(\tilde{b}) = 0$

$\text{Ø}_1(-\tilde{a}) = C_1\cosh\frac{\tilde{a}}{L_1} - C_2\sinh\frac{\tilde{a}}{L_1} = 0$

$C_2 = C_1\frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}$

$\text{Ø}_2(\tilde{b}) = C_3\cosh\frac{\tilde{b}}{L_2} + C_4\sinh\frac{\tilde{b}}{L_2} = 0$

$C_4 = -C_3\frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}}$

$\text{Ø}_{(x<0)} = C_1\left(\cosh\frac{x}{L_1} + \frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}\sinh\frac{x}{L_1}\right)$

$$\emptyset_{(x>0)} = C_3\left(\cosh\frac{x}{L_2} - \frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}}\sinh\frac{x}{L_2}\right)$$

Boundary Condition 2 : $\lim_{x\to 0}[\emptyset\,(x>0) - \emptyset\,(x<0)] = 0$

$$\lim_{x\to 0}\left[C_3\left(\cosh\frac{x}{L_2} - \frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}}\sinh\frac{x}{L_2}\right) - C_1\left(\cosh\frac{x}{L_1} + \frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}\sinh\frac{x}{L_1}\right)\right] = 0$$

$$C_3(1 - 0) - C_1(1 - 0) = 0,\ C_3 = C_1$$

$$\emptyset_{(x<0)} = C_1\left(\cosh\frac{x}{L_1} + \frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}\sinh\frac{x}{L_1}\right)$$

$$\emptyset_{(x>0)} = C_1\left(\cosh\frac{x}{L_2} - \frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}}\sinh\frac{x}{L_2}\right)$$

Boundary Condition 3 : $\lim_{x\to 0}[J(x>0) - J(x<0)] = S$

$$\lim_{x\to 0}\left[-D_2C_1\frac{1}{L_2}\left(\sinh\frac{x}{L_2} - \frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}}\cosh\frac{x}{L_2}\right) + D_1C_1\frac{1}{L_1}\left(\sinh\frac{x}{L_1} + \frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}\cosh\frac{x}{L_1}\right)\right] = S$$

$$D_2C_1\frac{1}{L_2}\frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}} + D_1C_1\frac{1}{L_1}\frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}} = S$$

$$C_1 = \frac{S}{D_2\frac{1}{L_2}\frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}} + D_1\frac{1}{L_1}\frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}}$$

Solution :

$$\emptyset_{(x<0)} = \frac{S}{D_2\frac{1}{L_2}\frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}} + D_1\frac{1}{L_1}\frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}}\left(\cosh\frac{x}{L_1} + \frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}\sinh\frac{x}{L_1}\right)$$

$$= \frac{S}{D_2\frac{1}{L_2}\frac{\cosh\frac{b+d_2}{L_2}}{\sinh\frac{b+d_2}{L_2}} + D_1\frac{1}{L_1}\frac{\cosh\frac{a+d_1}{L_1}}{\sinh\frac{a+d_1}{L_1}}}\left(\cosh\frac{x}{L_1} + \frac{\cosh\frac{a+d_1}{L_1}}{\sinh\frac{a+d_1}{L_1}}\sinh\frac{x}{L_1}\right)$$

$$\emptyset_{(x>0)} = \frac{S}{D_2\frac{1}{L_2}\frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}} + D_1\frac{1}{L_1}\frac{\cosh\frac{\tilde{a}}{L_1}}{\sinh\frac{\tilde{a}}{L_1}}}\left(\cosh\frac{x}{L_2} - \frac{\cosh\frac{\tilde{b}}{L_2}}{\sinh\frac{\tilde{b}}{L_2}}\sinh\frac{x}{L_2}\right)$$

$$= \frac{S}{D_2\frac{1}{L_2}\frac{\cosh\frac{b+d_2}{L_2}}{\sinh\frac{b+d_2}{L_2}} + D_1\frac{1}{L_1}\frac{\cosh\frac{a+d_1}{L_1}}{\sinh\frac{a+d_1}{L_1}}}\left(\cosh\frac{x}{L_2} - \frac{\cosh\frac{b+d_2}{L_2}}{\sinh\frac{b+d_2}{L_2}}\sinh\frac{x}{L_2}\right)$$

14. A sphere of moderator of radius R contains uniformly distributed sources emitting S neutrons/cm^3-sec. (a) show that the flux in the sphere is given by

$$\emptyset = \frac{S}{\Sigma_a}\left[1 - \frac{R+d}{r}\,\frac{\sinh r/L}{\sinh(\frac{R+d}{L})}\right]$$

(b) Derive an expression for the current density at any point in the sphere. (c) How many neutrons leak from the sphere per second? (d) What is the average probability that a source neutron will escape from the sphere?

[sol] (a) $\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\emptyset}{dr} - \frac{1}{L^2}\emptyset = -\frac{S}{D}$ **<Eq. 5.19, 5.30>**

Let, $w = r\emptyset$

$$\frac{d^2w}{dr^2} - \frac{1}{L^2}w = -\frac{Sr}{D}$$

(i) if homogeneous,

$$W_h = A\sinh\frac{r}{L} + B\cosh\frac{r}{L}$$

$$\emptyset_h = \frac{A}{r}\sinh\frac{r}{L} + \frac{B}{r}\cosh\frac{r}{L}$$

(ii) if particular

$\mathbf{W_p = Cr}$

$$\frac{d^2w}{dr^2} - \frac{1}{L^2}w = -\frac{Sr}{D}$$

$$0 - \frac{Cr}{L^2} = -\frac{Sr}{D}$$

$$W_p = \frac{SrL^2}{D}$$

$$\emptyset_p = \frac{SL^2}{D}$$

$$\emptyset = \emptyset_h + \emptyset_p$$

$$= \frac{A}{r}\sinh\frac{r}{L} + \frac{B}{r}\cosh\frac{r}{L} + \frac{SL^2}{D}$$

Boundary Condition 1 : $\lim_{r\to 0}\emptyset$ **= finite**

$$\emptyset = 0 + \frac{B}{0} + \frac{SL^2}{D} = \text{finite}$$

$$B = 0$$

$$\emptyset = \frac{A}{r}\sinh\frac{r}{L} + \frac{SL^2}{D}$$

Boundary Condition 2 : $\emptyset_{(R+d)} = 0$

$$\frac{A}{R+d}\sinh\frac{R+d}{L} + \frac{SL^2}{D} = 0$$

$$A = -\frac{SL^2}{D}\frac{R+d}{\sinh\frac{R+d}{L}}$$

$$\emptyset = \frac{SL^2}{D}\left(1 - \frac{R+d}{r}\frac{\sinh r/L}{\sinh(\frac{R+d}{L})}\right) \quad \langle L^2 = \frac{D}{\Sigma a}\rangle$$

$$= \frac{S}{\Sigma a}\left(1 - \frac{R+d}{r}\frac{\sinh r/L}{\sinh(\frac{R+d}{L})}\right)$$

(b) current density J <Eq. 5.8>

$$= -D\nabla\emptyset = -D\frac{d}{dr}\left\{\frac{S}{\Sigma a}\left(1 - \frac{R+d}{r}\frac{\sinh r/L}{\sinh(\frac{R+d}{L})}\right)\right\}$$

$$= -D\left\{-\frac{S(R+d)}{\Sigma a}\left(-r\text{-}2\frac{\sinh r/L}{\sinh(\frac{R+d}{L})} + r\text{-}1\frac{1}{L\sinh(\frac{R+d}{L})}\cosh\frac{r}{L}\right)\right\}$$

$$= \frac{DS(R+d)}{\Sigma a}\frac{1}{\sinh(\frac{R+d}{L})}\left(\frac{\cosh\frac{r}{L}}{rL} - \frac{\sinh\frac{r}{L}}{r^2}\right)$$

(c) leakage rate

$$= 4\pi R^2 \times J$$

$$= 4\pi R^2\frac{DS(R+d)}{\Sigma a}\frac{1}{\sinh(\frac{R+d}{L})}\left(\frac{\cosh\frac{r}{L}}{rL} - \frac{\sinh\frac{r}{L}}{r^2}\right) \text{ neutrons/s}$$

(d) The probability that a neutron escapes is equal to the number that escape per cm^2/sec divided by the number emitted per cm^2/sec by the source

The number that escape per cm^2/sec = $4\pi R^2\frac{DS(R+d)}{\Sigma a}\frac{1}{\sinh(\frac{R+d}{L})}\left(\frac{\cosh\frac{r}{L}}{rL} - \frac{\sinh\frac{r}{L}}{r^2}\right)$

The number emitted per cm^2/sec by the source = S

The probability that a neutron escapes = $\frac{\text{The number that escape per cm2/sec}}{\text{The number emitted per cm2/sec by the source}}$

$$= 4\pi R^2\frac{D(R+d)}{\Sigma a}\frac{1}{\sinh(\frac{R+d}{L})}\left(\frac{\cosh\frac{r}{L}}{rL} - \frac{\sinh\frac{r}{L}}{r^2}\right)$$

15. The three-group fluxes for a base spherical fast reactor of radius R = 50 cm are given by the following expressions:

$$\emptyset_1(r) = \frac{3\times 10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$

$$\emptyset_2(r) = \frac{2\times 10^{16}}{r}\sin\left(\frac{\pi r}{R}\right)$$

$$\emptyset_3(r) = \frac{1\times 10^{16}}{r}\sin\left(\frac{\pi r}{R}\right).$$

The group-diffusion coefficients are D_1 = 2.2 cm, D_2 = 1.7 cm, and D_3 = 1.05 cm. Calculate the total leakage of neutrons from the reactor in all three groups. [*Note*: Ignore the extrapolation distance.]

[sol] leakage in group g : div·J

Total leakage of neutrons : $\sum_{g=1}^{3} -D_g\nabla^2 \emptyset_g$ = $- D_1\nabla^2\emptyset_1 - D_2\nabla^2\emptyset_2 - D_3\nabla^2\emptyset_3$

$\nabla^2\emptyset_1 = \frac{d}{dr}\frac{d}{dr}\{\frac{3\times10^{15}}{r}\sin(\frac{\pi r}{R})\}$

$= (3 \times 10^{15})\frac{d}{dr}\{\frac{d}{dr}(r^{-1}\sin\frac{\pi r}{R})\}$

$= (3 \times 10^{15})\frac{d}{dr}(-r^{-2}\sin\frac{\pi r}{R} + r^{-1}\frac{\pi}{R}\cos\frac{\pi r}{R})$

$= 3 \times 10^{15} \times (2r^{-3}\sin\frac{\pi r}{R} - r^{-2}\frac{\pi}{R}\cos\frac{\pi r}{R} - r^{-2}\frac{\pi}{R}\cos\frac{\pi r}{R} + r^{-1}\frac{\pi}{R}\frac{\pi}{R}(-\sin\frac{\pi r}{R}))$

<Ignore the extrapolation distance>

$= 3 \times 10^{15} \times (\frac{\pi}{R^3} + \frac{\pi}{R^3}) = \frac{6\pi\times10^{15}}{R^3}$

$\nabla^2\emptyset_2 = \frac{4\pi\times10^{15}}{R^3}$, $\nabla^2\emptyset_3 = \frac{2\pi\times10^{15}}{R^3}$

Total leakage of neutrons $= -2.2 \times \frac{6\pi\times10^{15}}{50^3} - 1.7 \times \frac{4\pi\times10^{15}}{50^3} - 1.05 \times \frac{2\pi\times10^{15}}{50^3}$

$= -2.57 \times 10^{12}$ **neutrons**

16. The thermal flux in the center of a beam tube of a certain reactor is 2×10^{13} neutrons/cm^2-sec. The temperature in this region is 150℃. Calculate (a) the thermal neutrons density; (b) the energy E_T; (c) the 2,200 meters-per-second flux.

[sol] (a) $\emptyset_T = \frac{2}{\sqrt{\pi}}nv_T$ **<Eq. 5.56>**

$n = \frac{\emptyset_T\sqrt{\pi}}{2v_T}$

$v_T = 1.284T^{1/2} \times 10^4$ **cm/sec <Eq. 5.55>**

$= 1.284(273 + 150)^{1/2} \times 10^4$ **cm/sec**

$= 2.64 \times 10^5$ **cm/sec**

$n = (2 \times 10^{13}$ **neutrons/cm^2-sec** $\times \sqrt{\pi}) / (2 \times 2.64 \times 10^5$ **cm/sec)**

$= 6.71 \times 10^7$ **neutrons/cm^3**

(b) $E_T = 8.617T \times 10^{-5}$ **eV <Eq. 5.54>**

$= 8.617(273 + 150) \times 10^{-5}$ **eV**

$= 0.0345$ **eV**

(c) $\emptyset_0 = nv_0$ **<Page 253>**

$= 6.71 \times 10^7$ **neutrons/cm^3** $\times 2.2 \times 10^5$ **cm/sec**

$= 1.48 \times 10^{13}$ **neutrons/cm^2-sec**

17. The thermal flux at the center of a graphite research reactor is 5×10^{12} neutrons/cm^2-sec. The temperature of the system at this point is 120℃. Compare the neutron density at this point with the atom density of the graphite.

[sol] $\emptyset_T = \frac{2}{\sqrt{\pi}}nv_T$ **<Eq. 5.56>**

$n = \frac{\emptyset_T\sqrt{\pi}}{2v_T}$

$v_T = 1.284T^{1/2} \times 10^4$ **cm/sec <Eq. 5.55>**

$= 1.284(273 + 120)^{1/2} \times 10^4$ **cm/sec**

$= 2.55 \times 10^5$ **cm/sec**

$n = (5 \times 10^{12}$ **neutrons/cm^2-sec** $\times \sqrt{\pi}) / (2 \times 2.55 \times 10^5$ **cm/sec)**

$= 1.74 \times 10^7$ **neutrons/cm^3**

atom density of the graphite = (1.6 g/cm3 $\times 0.6022 \times 10^{24}$ **atoms) / 12g <1.6 g/cm^3 – Page 254>**

$= 8.03 \times 10^{22}$ **atoms/cm^3**

18. The thermal flux in a bare cubical reactor is given approximately by the function

$$\emptyset_T(x, y, z) = A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right),$$

where A is a constant, a is the length of a side of the cube, ã is a plus 2d, d is the extrapolation distance, and x, y, and z are measured from the center of the reactor. Derive expressions for the (a) thermal neutron current as a function of position in the reactor; (b) number of thermal neutrons leaking per second from each side of the reactor; and (c) total number of thermal neutrons leaking per second from the reactor.

[sol] (a) thermal neutron current J

$$= -D\nabla\emptyset$$

$$J(x,y,z) = -D\left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\emptyset\,(x,y,z)$$

$$= -D\frac{\partial}{\partial x}\emptyset\,(x,y,z)i - D\frac{\partial}{\partial y}\emptyset\,(x,y,z)j - D\frac{\partial}{\partial z}\emptyset\,(x,y,z)k$$

$$= -D\frac{\partial}{\partial x}\left\{A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right\}i$$

$$- D\frac{\partial}{\partial y}\left\{A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right\}j$$

$$- D\frac{\partial}{\partial z}\left\{A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right\}k$$

$$= DA\frac{\pi}{\tilde{a}}\left\{\sin\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)i + \cos\left(\frac{\pi x}{\tilde{a}}\right)\sin\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)j + \cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\sin\left(\frac{\pi z}{\tilde{a}}\right)k\right\}$$

(b) each six surface is same.

number of thermal neutrons leaking per second from each side of the reactor

$$= \int_{surface} J(x, y, z)dA$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} J(x, y, z)i\,dydz$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} DA\frac{\pi}{\tilde{a}}\left\{\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right\}dydz$$

$$= DA\frac{\pi}{\tilde{a}}\frac{\tilde{a}}{\pi}\left[\sin\left(\frac{\pi y}{\tilde{a}}\right)\right]_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos\left(\frac{\pi z}{\tilde{a}}\right)dz$$

$$= DA\frac{\pi}{\tilde{a}}\frac{\tilde{a}}{\pi}\left\{\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right\}\frac{\tilde{a}}{\pi}\left\{\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right\}$$

$$= DA \times 2 \times \frac{\tilde{a}}{\pi} \times 2 = \frac{4\tilde{a}DA}{\pi}\ \text{neutrons/sec}$$

(c) total number of thermal neutrons leaking per second from the reactor

$$= 6 \times \frac{4\tilde{a}DA}{\pi} = \frac{24\tilde{a}DA}{\pi}\ \text{neutrons/sec}$$

19. A planar source at the center of an infinite slab of graphite 2 meters thick emits 10^8 thermal neutrons per cm^2/sec. Given that the system is at room temperature, calculate the: (a) total number of thermal neutrons in the slab per cm^2 at any time; (b) number of thermal neutrons absorbed per cm^2/sec of the slab; (c) neutron current as a function of position in the slab; (d) total number of neutrons leaking per cm^2/sec from the two surfaces of the slab; (e) probability that a source neutron does not leak from the slab.

[sol] (a) $\phi_T = A\cdot\sinh[(a + d - |x|/L)]$

$$\phi_T = \int_{-1}^{1} A\cdot\sinh[(a + d - |x|/L)]\ dx = -2AL\left[\cosh\left(\frac{a+d-x}{L}\right)\right]_0^a$$

$$= -2AL\left[\cosh\left(\frac{d}{L}\right) - \cosh\left(\frac{a+d}{L}\right)\right]$$

$$= 2AL\left[\cosh\left(\frac{a+d}{L}\right) - \cosh\left(\frac{d}{L}\right)\right]$$

$$= 2(2.275\times10^9\#/cm^2\cdot s)(59.1608cm)\left[\cosh\left(\frac{100+1.7892}{59.1608}\right) - \cosh\left(\frac{1.7892}{59.1608}\right)\right]$$

$$= 5.068\times10^{11}\#/cm\cdot s$$

$$n = \frac{\phi}{v} = \frac{5.068\times10^{11}\#/cm\cdot s}{2200\times10^2 cm/s} = 2303636.364\#/cm^2$$

(b) $\overline{\Sigma_a}\phi_T = (2.4\times10^{-4}cm^{-1})(5.068\times10^{11}\#/cm^2\cdot s) = 1.216\times10^8\#/cm^2\cdot s$

(c) $J = -D\cdot\frac{d\phi}{dx}$

$$J(x) = -D\cdot\frac{d\phi}{dx}\frac{SL}{2D}\frac{\sinh[(a+d-x)/L]}{\cosh[(a+d)/L]} = \frac{S}{2}\frac{\cosh[d/L]}{\cosh[(a+d)/L]}$$

(d) $\mathbf{2J(a) = \frac{2S}{2}\frac{\cosh[d/L]}{\cosh[(a+d)/L]} = (10^8\#/cm^2\cdot s)\frac{\cosh[1.7892/59.1608]}{\cosh[(100+1.7892)/59.1608]} = 3.4698\times10^7\#/cm^2\cdot s}$

(e) Probability $\mathbf{1-\frac{2J(a)}{S} = 1-\frac{3.4698\times10^7\#/cm^2\cdot s}{10^8\#/cm^2\cdot s} = 0.653}$

20. The thermal flux in a bare cubical reactor of side $a = 800\text{cm}$ is given by the expression

$$\phi_T(x,y,z) = 2\times10^{12}\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right),$$

where x,y, and z are measured from the center of the reactor and $\tilde{a} = a + 2d$. The temperature is 400℃, and the measured values of the thermal diffusion coefficient and diffusion length are 0.84cm and 17.5cm, respectively. (a) How many moles of thermal neutrons are there in the entire reactor? (b) Calculate the neutron current density vector as a function of position in the reactor. (c) How many thermal neutrons leak from the reactor per second? (d) How many thermal neutrons are absorbed in the reactor per second? (e) What is the relative probability that a thermal neutron will leak from the reactor?

[sol] (a) $\mathbf{v_T = 1.284T^{1/2}\times10^4 cm/s}$
$\mathbf{= 1.284\times(673)^{1/2}\times10^4 cm/s = 3.331\times10^5 cm/s}$

$$\frac{\phi_0}{\phi_T} = \frac{\sqrt{\pi}}{2}\frac{v_0}{v_T} \qquad \Leftarrow \ \phi_0 = nv_0$$

$$\therefore n = \frac{\sqrt{\pi}}{2}\frac{\phi_T}{v_T} = \frac{\sqrt{\pi}}{2}\frac{(2\times10^{12})\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)}{3.331\times10^{15}} = (5.321\times10^6)\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)$$

(Total #)= $\iiint n\,dV = \int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}(5.321\times10^6)\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)dxdydz$

$$= (5.321\times10^6)\left(\left[\frac{\tilde{a}}{\pi}\sin\left(\frac{\pi x}{\tilde{a}}\right)\right]_{-400}^{400}\right)^3$$
$$= (5.321\times10^6)\left(\frac{803.5784}{\pi}\right)^3\left[\sin\left(\frac{\pi\times400}{803.5784}\right)-\sin\left(\frac{\pi\times-400}{803.5784}\right)\right]^3$$
$$= 4.8603\times10^{12}\text{neutrons}$$

$$4.8603\times10^{12}\times\frac{1\text{mole}}{0.6022\times10^{24}} = 8.0709\times10^{-12}\text{moles}$$

(b) $\bar{J}(x,y,z) = -D\frac{\partial\phi}{\partial x}\hat{i} - D\frac{\partial\phi}{\partial y}\hat{j} - D\frac{\partial\phi}{\partial z}\hat{k} = -D\left[\frac{\partial\phi}{\partial x}\hat{i}+\frac{\partial\phi}{\partial y}\hat{j}+\frac{\partial\phi}{\partial z}\hat{k}\right]$

$$= -(2\times10^{12})D\left\{\left[-\frac{\pi}{\tilde{a}}\sin\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right]\hat{i}+\left[-\frac{\pi}{\tilde{a}}\cos\left(\frac{\pi x}{\tilde{a}}\right)\sin\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right]\hat{j}+\left[-\frac{\pi}{\tilde{a}}\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\sin\left(\frac{\pi z}{\tilde{a}}\right)\right]\hat{k}\right\}$$

$$= \frac{(2\times10^{12})\pi D}{\tilde{a}}\left\{\left[\sin\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right]\hat{i}+\left[\cos\left(\frac{\pi x}{\tilde{a}}\right)\sin\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)\right]\hat{j}+\left[\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\sin\left(\frac{\pi z}{\tilde{a}}\right)\right]\hat{k}\right\}$$

(c) Leakage rate= $\int_A \vec{J}\cdot\vec{n}dA = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2}\vec{J}\cdot\hat{i}dydz$

$$= \frac{(2\times10^{12})\pi D}{\tilde{a}}\left[\sin\left(\frac{\pi x}{\tilde{a}}\right)\right]_{x=2/a}\cdot\left(\left[\frac{\tilde{a}}{\pi}\sin\frac{\pi y}{\tilde{a}}\right]_{-a/2}^{a/2}\right)^2$$

$$= 3.4935\times10^{10}\text{neutrons/s}$$
$$\therefore 6J = 2.0961\times10^{11}\text{ neutrons/s}$$

(d) Absorption rate= $\int_V \Sigma_a\phi dV = \Sigma_a\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}(2\times10^{12})\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)dxdydz$

$$= (2 \times 10^{12})\Sigma_a \left\{ \left[\frac{\tilde{a}}{\pi} \sin\frac{\pi x}{\tilde{a}} \right]_{-400}^{400} \right\}^3 = 2.2816 \times 10^8 \text{ neutrons/s}$$

(e) $\frac{\text{Absorption}}{\text{Leakage}} = \frac{2.2816\times10^8}{2.0961\times10^{11}} = 0.001$

21. The thermal (0.0253 eV) cross-section for the production of a 54-minute isomer of ^{116}In via the (n,γ) reaction with ^{115}In is 157b. A thin indium foil weighing 0.15g is placed in the beam tube described in Problem 5.16. (a) At what rate are thermal neutrons absorbed by the foil? (b)
After 1 hour in the beam tube, what is the 54-minute activity of ^{116}In? [*Note*: 115In has an isotopic abundance of 95.7% and is a non-1/v absorber.]

[sol] (Table II.3) $\Sigma_a = 7.419\text{cm}^{-1}$
(Table 3.2) $g_a(150°C) = 1.0454$

(a) $\overline{\Sigma_a} = \frac{\sqrt{\pi}}{2} g_a(T)\Sigma_a(E_0)\left(\frac{T_0}{T}\right)^{1/2}$

$= 5.72648\text{cm}^{-1}$
$\therefore \overline{\Sigma_a}\phi_T = (5.72648\text{cm}^{-1})(2 \times 10^{13}\#/\text{cm}^2 \cdot s) = 1.1453 \times 10^{14}\#/\text{cm}^3 \cdot s$

(b) $^{115}\text{In} + n \to {}^{116}\text{In} + \gamma$

$T_{\frac{1}{2}, {}^{115}\text{In}} = 4.4 \times 10^{14}\text{years},$ $\quad T_{\frac{1}{2}, {}^{116}\text{In}} = 54\text{minutes}$

$$A_B = \alpha_{B0}e^{-\lambda_B t} + \frac{\alpha_{A0}\lambda_B}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) = \alpha_{A0}e^{-\lambda_A t} = \lambda_A n_A e^{-\lambda_A t} = 0.037611\text{dis/s}$$
$$= 1.0165 \times 10^{-12}\text{Ci}$$

22. The thermal flux in a bare spherical reactor 1m in diameter is given approximately by

$$\phi_T(r) = 2.29 \times 10^{14} \frac{\sin 0.0628r}{r} \text{neutrons/cm}^2\text{-sec.}$$

If the reactor is moderated and cooled by unit density water that takes up to one-third of the reactor volume, how many grams of ^{2}H are produced per year in the reactor? Assume that the water is only slightly warmed by the heat of the reactor.

[sol] Table II.3 $\sigma_{a,H} = 0.332b,\ M_{H_2O} = 18.0153$
${}_1^1H + {}_0^1n \to {}_1^2H + \gamma$

(Absorption rate) $= \int_V \Sigma_a \phi_T(r) dV \quad \leftarrow \frac{4}{3}\pi r^3 dV = 4\pi r^2 dr$
$= \int_0^{50} \Sigma_a \phi_T(r) \cdot 4\pi r^2 dr$

$N'_{H_2O} = \frac{(1g/cm^3)(0.6022\times10^{24})}{18.0153} = 3.3427 \times 10^{22}\#/\text{cm}^3, \quad \therefore N_{H_2O} = \frac{1}{3}N'_{H_2O} = 1.114 \times 10^{22}\#/\text{cm}^3$

$N_H = 2N_{H_2O} = 2.228 \times 10^{22}\#/\text{cm}^3$
$\therefore \Sigma_a = N_H\sigma_{a,H} = (2.228 \times 10^{22}\#/\text{cm}^3) \cdot (0.332 \times 10^{-24}\text{cm}^2) = 0.0074\text{cm}^{-1}$

$\therefore$**(Absorption rate)** $= \int_0^{50} \Sigma_a \phi_T(r) \cdot 4\pi r^2 dr$
$= \int_0^{50} (0.0074)(4\pi)(2.29 \times 10^{14}) \frac{\sin 0.0628r}{r} \times r^2 dr$
$= 1.6993 \times 10^{16}\#/s \times 365\text{day/yr} \times 24\text{hr/day} \times \frac{3600s}{hr} = 5.3589 \times 10^{23}\#/\text{yr}$

$\therefore ({}^2\text{H production rate}) = (5.3589 \times 10^{23}\#/\text{yr}) \times \frac{2g/\text{mole}}{0.6022\times10^{24}\#/\text{mole}} = 1.7798\text{g/yr}$

23. Tritium (^3H) is produced in nuclear reactors by the absorption of thermal neutrons by ^6Li via the reaction $^6Li(n,\alpha)^3H$. The cross-section for this reaction at 0.0253eV is 940b and it is 1/v. (a) Show that the annual production of 3H in a thermal flux ϕ_T, per gram of ^6Li, is given by

Production rate/g = $2.28 \times 10^9 \phi_T$ atoms/g-yr.

(b) Compute the annual production in curies of ^3H per gram of 6Li in a flux of $\phi_T = 1 \times 10^{14}$ neutrons/cm^2-sec.

[sol] $\mathbf{^6Li + n \rightarrow\ ^3H + \alpha}$ $\qquad\qquad$ $\boldsymbol{\sigma = 940b}$ $(\mathbf{E = 0.0253eV})$

(a) **Production rate=absorption rate of thermal neutrons by** $\mathbf{^6Li}$

$$\overline{\Sigma_a}\phi_T = \frac{\sqrt{\pi}}{2} g_a(T)\Sigma_a(E_0)\left(\frac{T_0}{T}\right)^{1/2} \phi_T = 1.22\times 10^9 \phi_T \text{ atoms/g} \cdot \text{yr}$$

(b) $\boldsymbol{\phi_T = 1 \times 10^{14} \#/cm^2 \cdot s}$

Production=$\mathbf{38.677 \times (1 \times 10^{14}\#/cm^2 \cdot s) \times \frac{1Ci}{3.7\times10^{10}\#/s} = 1.045 \times 10^{-5} Ci/g}$

24. A radioactive sample with half-life $T_{1/2}$ is placed in a thermal reactor at a point where the thermal flux is ϕ_T. Show that the sample disappears as the result of its own decay and by neutron absorption with an effective half-life given by

$$\left(\frac{1}{T_{1/2}}\right)_{eff} = \frac{1}{T_{1/2}} + \frac{\overline{\sigma_a}\phi_T}{\ln 2},$$

where $\overline{\sigma_a}$ is the average thermal absorption cross-section of the sample.

[sol] $\boldsymbol{\alpha_{eff} = \lambda_{eff} N = \lambda N + \overline{\Sigma_a}\phi_T = \lambda N + N\overline{\sigma_a}\phi_T = N(\lambda + \overline{\sigma_a}\phi_T)}$

$$\therefore \lambda_{eff} = \lambda + \overline{\sigma_a}\phi_T, \qquad \leftarrow \quad \lambda_{eff} = \frac{\ln 2}{T_{eff,\frac{1}{2}}}\ ,\ \lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$$

$$\therefore \left(\frac{1}{T_{\frac{1}{2}}}\right)_{eff} = \frac{1}{T_{\frac{1}{2}}} + \frac{\overline{\sigma_a}\phi_T}{\ln 2}$$

25. Calculate the thermal diffusion coefficient and diffusion length of water near the outlet of a pressurized-water reactor, that is at about 300°C and density of 0.68g/cm^3.

[sol]

$\boldsymbol{\bar{D}(\rho_o, T_o)=0.16cm}$, $\qquad \boldsymbol{\rho_o = 1g/cm^3}$, $\qquad \boldsymbol{\rho=0.68g/cm^3}$

T= 300+273=573°K, $\quad \boldsymbol{T_o = 293°K}$, $\qquad$ **m=0.470**

$\boldsymbol{L_T^2 (\rho_o, T_o)=8.1cm^2}$

i) $\boldsymbol{\bar{D}(\rho, T) = \bar{D}(\rho_o, T_o)(\rho_o/\rho)(T/T_o)^m}$

$\boldsymbol{= 0.16 \times (1/0.68)(533/293)^{0.470}=0.312cm}$

ii) $\boldsymbol{L_T^2 (\rho, T)= L_T^2 (\rho_o, T_o) (\rho_o/\rho)^2(T/T_o)^{m+1/2}}$

$\boldsymbol{=8.1 \times (1/0.68)^2 (533/293)^{0.970}}$

$\boldsymbol{=23.2cm^2}$

$\boldsymbol{L_T (\rho, T)=4.8cm}$

26. Calculate the for natural uranium at room temperature and 350°C the value of $\sum_a$, D, and L_T. The measured value of L_T at room temperature is 1.55 cm. [Note: The value of σs is constant at low neutron energies and is

approximately the same for 235U and 238U. The densities of U at 350°C and room temperature are essentially the same]

[sol]

$\rho = \rho_o$, T_o=293°K , T=623°K, m=0 , σ_a(235U)=680.8b, σ_a(238U)=2.70b

$\sum_s$=0.4301cm^{-1}

$g_a(T) \rightarrow$ i) for 235U, 0.9780 at T_o , 0.9335 at T

ii) for 239U, 1.0017 at T_o , 1.0067 at T

σ_a(235U)=680.8b , σ_a(238U)=2.70b

i) $\bar{D}(\rho_o, T_o)= \lambda_{tr}/3 = 1/3\sum_s(1-\bar{\mu})$

$=1/3 \times 0.4301 \times (1-2.801 \times 10^{-3})$, where $-\bar{\mu}$ =2/3 x 238=1-2.801 x 10^{-3}

=0.777cm^{-1}

ii) $L^2_T(\rho_o, T_o)=1.55^2=2.4025\text{cm}^2$

$L^2_T(\rho, T)=)= L^2_T(\rho_o, T_o)(\rho_o/\rho)^2(T/T_o)^{m+1/2}$

$= 2.4025 \times (693/293)^{1/2}=3.5\text{cm}^2$, L_T =1.87cm

iii) $\overline{\sum a}(T_o)=\sqrt{\pi}/2$ x $[g_a(T_o)\ \sum_a(E_o)\ (T_o / T_o)^{1/2}]$ N(U)=0.04833 x 10^{24}#/cm3

$= \sqrt{\pi}/2(g^{235}{}_a N(235U)\ \sigma_a(235U)+ g^{238}{}_a N(238U)\ \sigma_a(238U)$

$= \sqrt{\pi}/2(0.9780 \times 0.0072 \times 0.04833 \times 680.8+1.0017 \times 0.9927 \times 0.04833 \times 2.70)$

= 0.32 cm^{-1}

$\overline{\sum a}(T)= \sqrt{\pi}/2 g_a(T_o)\ \sum_a(E_o)\ (T_o/T)^{1/2}$

$=\sqrt{\pi}/2[(g^{235}{}_a N(235U)\ \sigma_a(235U)+ g^{238}{}_a N(238U)\ \sigma_a(238U)]$ x $(293/623)^{1/2}$

$=\sqrt{\pi}/2[(0.9780 \times 0.0072 \times 0.04833 \times 680.8+1.0017 \times 0.9927 \times 0.04833 \times 2.70)] \times (293/623)^{1/2}$

=0.215 cm^{-1}

$\overline{\sum a}(T_o)= \bar{D}(\rho_o, T_o)/ L^2_T(\rho_o, T_o)=0.777/1.55^2=0.32\text{cm}^{-1}$

$\overline{\sum a}(T)= \bar{D}(\rho, T)/ L^2_T(\rho, T)=0.777/3.5=0.22\text{cm}^{-1}$

27. Repeat the calculation of problem 5.26 for U enriched to 2 weight/percent in 235U.

[sol]

Volume percent of 235U

= [(0.02 x 235.0439+0.98 x 238.0508)/235.0439] x 0.02=0.0203

=2.03percent

σ_s(235U)= σ_s(238U)= σ_s at low energy

$\sum_s$= N(235U) σ_s(235U)+ N(238U) σ_s(238U)

= [N(235U) +N(238U)] x σ_s

=(19.1 x 0.602217 x 8.9)/0.01(2.03 x 235.0439+97.97 x 238.0508)=0.43015 cm^{-1}

i) $\bar{D}(\rho_o, T_o)= \lambda_{tr}/3 = 1/3\sum_s(1-\bar{\mu})$

$=1/3 \times 0.43015 \times (1-2.801 \times 10^{-3})$, where $-\bar{\mu}$ =2/3 x 238=1-2.801 x 10^{-3}

=0.777 cm^{-1}

ii) $L^2_T(\rho_o, T_o)=1.55^2=2.4025\text{cm}^2$

$L^2_T(\rho, T) = L^2_T(\rho_o, T_o)(\rho_o/\rho)^2(T/T_o)^{m+1/2}$

$= 2.4025 \times (693/293)^{1/2}=3.5\text{cm}^2$, L_T =1.87cm

iii) $\overline{\sum a}(T_o)=\sqrt{\pi}/2 g_a(T_o)\ \sum_a(E_o)\ (T_o / T_o)^{1/2}$ N(U)=0.04833 x 10^{24}#/cm3

$= \sqrt{\pi}/2(g^{235}_a N(235U)\ \sigma_a(235U) + g^{238}_a N(238U)\ \sigma_a(238U)$

$= \sqrt{\pi}/2(0.9780 \times 0.0072 \times 0.04833 \times 680.8 + 1.0017 \times 0.9927 \times 0.04833 \times 2.70)$

$= 0.32\ cm^{-1}$

$\overline{\sum a}(T) = \sqrt{\pi}/2 g_a(T_o)\ \sum_a(E_o)\ (T_o/T)^{1/2}$

$= \sqrt{\pi}/2[(g^{235}_a N(235U)\ \sigma_a(235U) + g^{238}_a N(238U)\ \sigma_a(238U)] \times (293/623)^{1/2}$

$= \sqrt{\pi}/2[(0.9780 \times 0.0072 \times 0.04833 \times 680.8 + 1.0017 \times 0.9927 \times 0.04833 \times 2.70)] \times (293/623)^{1/2}$

$= 0.215\ cm^{-1}$

$\overline{\sum a}(T_o) = \bar{D}(\rho_o, T_o)/ L^2_T(\rho_o, T_o) = 0.777/1.55^2 = 0.32 cm^{-1}$

$\overline{\sum a}(T) = \bar{D}(\rho, T)/ L^2_T(\rho, T) = 0.777/3.5 = 0.22 cm^{-1}$

28. Compute and plot for mixtures of D_2O and H_2O, at H_2O concentrations up to 5 weight/percent: (a) $\sum_a$ at 0.0523 eV; (b) L_T at room temoerature.

[sol]

Let weight percent of H_2O x, volume percent of H_2O a

$M = 1/100[18.0153a + 20.0276(100-a)] = \{[(x/18.0153) + (100-x)/(20.0276)] \times 1/100\}^{-1}$

$N = \rho N_A/M = (20.0276 \times 0.6022 \times 10^{24})/[10^{-2}(2002.76 - 2.0123a)] = (12.06 \times 10^{24})/(20.0276 - 0.020123a)$

$a = \{[(x/18.0153) + (100-x)/(20.0276)] \times 1/100\}^{-1} \times (1/18.0153)x$

$= 995.259 - (8.91 \times 10^5/x + 895.26)$

i) $\sum_a = N(D_2O)\ \sigma_a(D_2O) + N(H_2O)\ \sigma_a(H_2O)$

$= 1/100[(100-a)N\ \sigma_a(D_2O) + a) + aN\ \sigma_a(H_2O)]$

$= 1/(2002.76 - 2.0123a) \times [(100-a)0.00133 + 0.644a]$

$= (0.329a + 0.066)/(a - 995.259)$ $\quad a = 995.259 - (8.91 \times 10^5/x + 895.26)$

$= (327.442 + 5.189)/8.91 \times 10^5$

$= 3.675 \times 10^{-4}x + 5.824 \times 10^{-6}$

ii) $\bar{D} \approx \bar{D}(D_2O) = 0.87cm$

$L^2_T = \bar{D}/\sum_a = \bar{D}/[\sqrt{\pi}/2 \times g_a(T)\ \sum_a(E_o)\ (T_o/T)^{1/2}]$

$= 0.87/[\sqrt{\pi}/2 \times 1 \times (3.675 \times 10^{-4}x + 5.824 \times 10^{-6}) \times 1$

$= 0.87/(3.257 \times 10^{-4}x + 4.683 \times 10^{-6})$

$= 2.671 \times 10^3/(x + 0.0144)$

$L_T = [2.671 \times 10^3/(x + 0.0144)]^{1/2}$

29. Calculate the thermal neutron diffusion length at room temperature in water solutions of boric acid(H3BO3)at the following concentrations: (a) 10g/liter, (b) 1g/liter and (c) 0.1 g/liter. [Hint: Because of the small concentration of the boric acid, the diffusion coefficient for the mixture is essentially the same as that of pure water.]

[sol]

$\rho \approx \rho(H_2O) = 1 g/cm^3$, $\bar{D} = 0.16cm$

Let weight percent of H_3BO_3 x, volume percent of H_3BO_3 a

$M = 1/100[18.0153(100-a) + (3\cdot1.00797 + 10.811 + 3\cdot15.9994)a]$

$= 0.438a + 18.0153$

$= [(100-x)/(18.0153) + (x/(3\cdot1.00797 + 10.811 + 3\cdot15.9994)]^{-1}$

$N = \rho N_A/M = (0.602210^{24})/(0.438a + 18.0153)$ molecules/cm^3

$a=[(100-x)/(18.0153)+(x/(61.833)]^{-1}\cdot(x/61.833)$
$=100x/(343.2-2.433x)$

i) $10g/cm^3 = 0.01g/cm^3 \rightarrow 1$ w/o
$a=100/(343.2-2.433)=0.293$
$\sigma_a(B) \approx \sigma_a(H_3BO)=759b$

$\sum_a = N(H_2O)\,\sigma_a(H_2O)+N(H_3BO_3)\,\sigma_a(H_3BO_3)$
$=[(100-0.293)N\,\sigma_a(H_2O)+0.293\,N\,\sigma_a(H_3BO_3)]\cdot 1/100$
$N=(1 \times 0.6022 \times 10^{24})/(0.438 \times 0.293+18.0153)=0.0332 \times 10^{24}$
$\sum_a=[(100-0.293)\cdot 0.0332\cdot 0.664+0.293\cdot 0.0332\cdot 759]\cdot 1/100$
$=0.0958cm^{-1}$

$L^2_T=\bar{D}/\sum_a=0.16/[\sqrt{\pi}/2 \times g_a(T)\ \sum_a(E_o)\ (T_o/T)^{1/2}]$
$=0.166/(\sqrt{\pi}/2\cdot 0.0958)$
$=1.884cm^2$
$L_T=1.372cm$

ii) $1g/cm^3=0.001g/cm^3 \rightarrow 0.1$ w/o
$a=100\cdot 0.1/(343.2-2.433\cdot 0.1)=0.029$ w/o
$N=(0.6022 \times 10^{24})/(0.438 \times 0.029+18.0153)=0.0334 \times 10^{24}$
$\sum_a=[(100-0.029)\cdot 0.0334\cdot 0.664+0.029\cdot 0.0334\cdot 759]\cdot 1/100$
$=0.0295cm^{-1}$

$L^2_T=0.166/(\sqrt{\pi}/2\cdot 0.0295)$
$=6.12cm^2$
$L_T=2.47cm$

iii) $0.1g/cm^3=0.0001g/cm^3 \rightarrow 0.01$ w/o
$a=100\cdot 0.01/(343.2-2.433\cdot 0.01)=0.0029$ w/o
$N=(0.6022 \times 10^{24})/(0.438 \times 0.0029+18.0153)=0.0334 \times 10^{24}$
$\sum_a=[(100-0.0029)\cdot 0.0334\cdot 0.664+0.0029\cdot 0.0334\cdot 759]\cdot 1/100$
$=0.0229cm^{-1}$

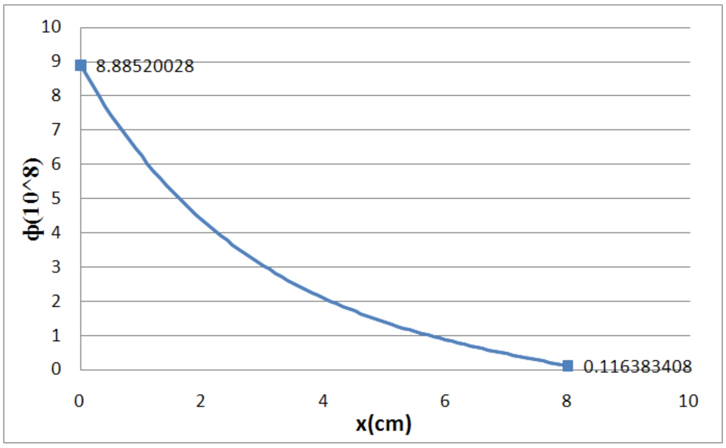
$L^2_T=0.166/(\sqrt{\pi}/2\cdot 0.0229)$
$=7.884cm^2$
$L_T=2.8cm$

30. An infinite slab of ordinary water 16 cm thick contains a planar source at its center emitting 10^8 thermal neutrons per cm^2/sec. Compute and plot the thermal flux within the slab.

[sol] $\varnothing_T=\frac{SL_T}{2\bar{D}}\frac{e^{(a+d-|x|)/L_T}-e^{-(a+d-|x|)/L_T}}{e^{(a+d)/L_T}+e^{-(a+d)/L_T}}=\frac{SL_T}{2\bar{D}}\frac{\sinh[(a+d-|x|)/L_T}{\cosh[(a+d)/L_T}$

Where) $S=10^8cm^2$/sec, $L_T=2.85cm$, $\bar{D}=0.16cm$, $a=8cm$, $d=2.13\bar{D}=0.341cm$

$\varnothing_T=\frac{SL_T}{2\bar{D}}\frac{\sinh[(a+d-|x|)/L_T}{\cosh[(a+d)/L_T}=\frac{10^8\times 2.85}{2\times 0.16}\times\frac{\sinh[(8+0.341-|x|)/2.85]}{\cosh[(8+0.341)/2.85]}=\mathbf{9.516\times 10^7\sinh(2.927-0.351|x|)}$



31. Repeat the calculations of Problem 5.30 and plot the thermal flux for boric acid solutions having the concentrations given in Problem 5.29.

[sol] at L_T=1.884cm

$$\emptyset_T = \frac{SL_T}{2\bar{D}}\frac{e^{(a+d-|x|)/L_T}-e^{-(a+d-|x|)/L_T}}{e^{(a+d)/L_T}+e^{-(a+d)/L_T}} = \frac{SL_T}{2\bar{D}}\frac{\sinh[(a+d-|x|)/L_T}{\cosh[(a+d)/L_T}$$

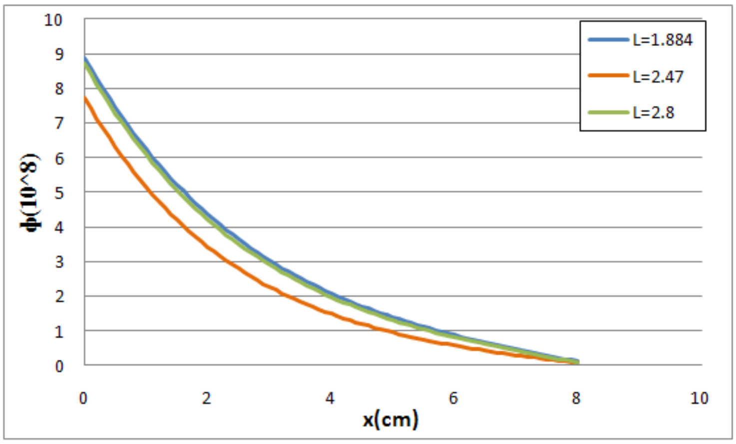
$$=\frac{10^8\times1.884}{2\times0.16}\times\frac{\sinh[(8+0.341-|x|)/1.884]}{\cosh[(8+0.341)/1.884]} = \mathbf{1.41\times10^7\sinh(4.43-0.53|x|)}$$

at L_T=2.47cm

$$\emptyset_T=\frac{10^8\times2.47}{2\times0.16}\times\frac{\sinh[(8+0.341-|x|)/2.47]}{\cosh[(8+0.341)/2.47]} = \mathbf{5.27\times10^7\sinh(3.38-0.405|x|)}$$

at L_T=2.8cm

$$\emptyset_T=\frac{10^8\times2.8}{2\times0.16}\times\frac{\sinh[(8+0.341-|x|)/2.8]}{\cosh[(8+0.341)/2.8]} = \mathbf{8.89\times10^7\sinh(2.98-0.36|x|)}$$



32. Verify that Eq. (5.72) is in fact the solution to Eq. (5.68). Discuss this solution in the cases where $\tau_T \rightarrow 0$ and $\tau_T \rightarrow L^2{}_T$

[sol] Eq.(5.72), $\emptyset_T = \frac{SL_T}{4\pi r\bar{D}(L_T^2-\tau_T)}(e^{-r/L_T}-e^{-r/\sqrt{\tau_T}})$

Eq.(5.68), $\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\emptyset_1}{r} - \frac{1}{\tau_T}\emptyset_1 = 0$

at $\tau_T \rightarrow 0,\ \emptyset_T = \frac{Se^{-r/L_T}}{4\pi r\bar{D}}$

33. Calculate the neutron age of fission neutrons in water for the conditions given in Problem 5.25 .

[sol] $\tau_T(\rho) = \tau_T(\rho_0)\left(\frac{\rho_0}{\rho}\right)^2 = 27x\left(\frac{1}{0.68}\right)^2 = 58.3㎡$

34. An infinite slab of moderator of extrapolated thickness ã contains sources of fast neutrons distributed according to the function

$$s(x) = S\cos(\frac{\pi x}{\tilde{a}})$$

Using the two-group method, derive an expression for the thermal flux in the slab.

[sol] $D_1\nabla^2\phi_1 - \sum_{a1}\phi_1 - \sum_{1\to 2}\phi_1 = -S\cos(\frac{\pi x}{a})$

$D_2\nabla^2\phi_2 - \sum_{a2}\phi_2 - \sum_{1\to 2}\phi_1 = 0$

$J^2\phi_1 - \frac{\Sigma}{D_1}\phi_1 = -\frac{S}{D_1}\cos\left(\frac{\pi x}{a}\right)$ ----(1)

$\nabla^2\phi_1 - \frac{1}{\tau_T}\phi_1 = -\frac{S}{D_1}\cos\left(\frac{\pi x}{a}\right)$

Particular Solution

$\phi = A\cos\left(\frac{\pi x}{a}\right) + B\sin\left(\frac{\pi x}{a}\right)$

$\phi' = -\frac{\pi}{a}A\sin\left(\frac{\pi x}{a}\right) + \frac{\pi}{a}B\cos\left(\frac{\pi x}{a}\right)$

$\phi'' = -(\frac{\pi}{a})^2A\sin\left(\frac{\pi x}{a}\right) - (\frac{\pi}{a})^2B\cos\left(\frac{\pi x}{a}\right) = -(\frac{\pi}{a})^2\phi$ **--->insert the above eq(1)**

$-(\frac{\pi}{a})\left\{A\cos\left(\frac{\pi x}{a}\right) + B\sin\left(\frac{\pi x}{a}\right)\right\} - \frac{1}{\tau_T}\left\{A\cos\left(\frac{\pi x}{a}\right) + B\sin\left(\frac{\pi x}{a}\right)\right\} = -\frac{S}{D_1}\cos\left(\frac{\pi x}{a}\right)$

$\{(\frac{\pi}{a})^2 + \frac{1}{\tau_T}\}A = -\frac{S}{D_1}, A = (S/D_1)/\{(\pi/a)^2 + (1/\tau_T)\}$

$\{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}\}B = 0, B = 0$

$\phi = \frac{S/D_1}{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}}\cos(\frac{\pi x}{a})$

Homogeneous solution

$\phi = C\cos\left(\frac{x}{\sqrt{\tau_T}}\right) + D\sin\left(\frac{x}{\sqrt{\tau_T}}\right)$

$\Rightarrow \phi = C\cos\left(\frac{x}{\sqrt{\tau_T}}\right) + D\sin\left(\frac{x}{\sqrt{\tau_T}}\right) + \frac{S/D_1}{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}}\cos(\frac{\pi x}{a}) = A^*e^{x/\sqrt{\tau_T}} - B^*e^{-x/\sqrt{\tau_T}} + \frac{S/D_1}{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}}\cos(\frac{\pi x}{a})$

Boundary condition

1) $\phi\left(\frac{a}{2}\right) = \phi\left(-\frac{a}{2}\right) = 0$

$A^*e^{\frac{x}{\sqrt{\tau_T}}} - B^*e^{-\frac{x}{\sqrt{\tau_T}}} + 0 = 0$

$B^* = A^*e^{-a/\sqrt{\tau_T}}$

$\phi = A^*[e^{x/\sqrt{\tau_T}} - e^{-a/\sqrt{\tau_T}}e^{-x/\sqrt{\tau_T}}] + \frac{S/D_1}{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}}\cos(\frac{\pi x}{a})$

2) $\lim_{x\to 0} S(x) = S$

$\lim_{x\to 0} J_1(x) = S/2$

$J_1(x) = -D_1\frac{d\phi}{dx} = -D_1[\frac{A}{\sqrt{\tau_T}}(e^{x/\sqrt{\tau_T}} - e^{-a/\sqrt{\tau_T}}e^{-x/\sqrt{\tau_T}}) - (\frac{\pi}{a})\frac{S/D_1}{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}}\cos(\frac{\pi x}{a})]$

$\lim_{x\to 0} J_1(x) = -\frac{D_1A^*}{\sqrt{\tau_T}}(1 - e^{-a/\sqrt{\tau_T}}) = \frac{S}{2}$

$A^* = \frac{S\sqrt{\tau_T}}{2D_1}\frac{1}{(1 - e^{-a/\sqrt{\tau_T}})}$

$\Rightarrow \phi = \frac{S\sqrt{\tau_T}}{2D_1}\frac{1}{(1 - e^{-a/\sqrt{\tau_T}})}[e^{x/\sqrt{\tau_T}} - e^{-a/\sqrt{\tau_T}}e^{-x/\sqrt{\tau_T}}] + \frac{S/D_1}{\left(\frac{\pi}{a}\right)^2 + \frac{1}{\tau_T}}\cos(\frac{\pi x}{a})$

Chapter 6
Neutron Reactor Theory

1. Calculate the fuel utilization and infinite multiplication factor for a fast reactor consisting of a mixture of liquid sodium and plutonium, in which the plutonium is present to 3.0 w/o. The density of the mixture is approximately 1.0 g/cm3

[sol] $\mathbf{f = \frac{\Sigma_{aF}}{\Sigma_a} = \frac{\Sigma_{aF}}{\Sigma_{aF}+\Sigma_{aS}} = \frac{1}{1+\Sigma_{aS}/\Sigma_{aF}}}$

$$\frac{\Sigma_{aS}}{\Sigma_{aF}} = \frac{N_S\sigma_{aS}}{N_F\sigma_{aF}} = \frac{\rho_S M_F}{\rho_F M_S}\frac{\sigma_{aS}}{\sigma_{aF}}$$

$$\frac{\rho_F}{\rho_S+\rho_F} = 0.03$$

$$(1-0.03)\rho_F = 0.03\rho_S$$

$$\frac{\rho_S}{\rho_F} = \frac{0.97}{0.03}$$

$$\frac{\Sigma_{aS}}{\Sigma_{aF}} = \frac{0.97}{0.03}\times\frac{239}{23}\times\frac{0.0008}{2.11} = 1.274\times10^{-1}$$

$$f = \frac{1}{1+1.274\times10^{-1}} = 8.870\times10^{-1}$$

$$k_\infty = \eta f = 2.61\times8.870\times10^{-1} = 2.315$$

2. The core of a certain fast reactor consists of an array of uranium fuel elements immersed in liquid sodium. The uranium is enriched to 25.6 w/o in 235 U and comprises 37% of the core volume. Calculate for this core (a) the average atom densities of sodium, ^{235}U, and ^{238}U; (b) the fuel utilization; (c) the value of η; (d) the infinite multiplication factor.

[sol] (a) $\mathbf{N(Na) = \frac{0.97\times0.6022\times10^{24}}{22.9898}\times0.63 = 1.6\times10^{22}atoms/cm^3}$

$$N(^{235}U) = \frac{0.256\times19.1\times0.6022\times10^{24}}{235.0439}\times0.37 = 4.64\times10^{21}atoms/cm^3$$

$$N(^{238}U) = \frac{(1-0.256)\times19.1\times0.6022\times10^{24}}{238.0508}\times0.37 = 1.33\times10^{22}atoms/cm^3$$

(b) $\mathbf{f = \frac{\Sigma_{aF}}{\Sigma_a} = \frac{\Sigma_{a235}+\Sigma_{a238}}{\Sigma_{a235}+\Sigma_{a238}+\Sigma_{aNa}} = \frac{N_{235}\sigma_{a235}+N_{238}\sigma_{a238}}{N_{235}\sigma_{a235}+N_{238}\sigma_{a238}+N_{Na}\sigma_{aNa}}}$

$$=\frac{4.64\times10^{-3}\times1.65+1.33\times0.255\times10^{-2}}{4.64\times10^{-3}\times1.65+1.33\times10^{-2}\times0.255+1.6\times10^{-2}\times0.0008} = 0.9999$$

(c) $\mathbf{\eta = \upsilon\frac{\Sigma_F}{\Sigma_{aF}} = \frac{\nu_{235}\Sigma_{f235}+\nu_{238}\Sigma_{f238}}{\Sigma_{aF235}+\Sigma_{aF238}} = \frac{2.6\times(4.64\times10^{-3}\times1.4+1.33\times10^{-2}\times0.095)}{4.64\times10^{-3}\times1.65+1.33\times10^{-2}\times0.255} = 1.83}$

(d) $\mathbf{k_\infty = \nu\frac{\Sigma_F}{\Sigma_a} = \eta f = 1.93\times0.9999 = 1.83}$

3. A bare-cylinder reactor of height 100 cm and diameter 100 cm is operating at a steady-state power of 20 MW. If the origin is taken at the center of the reactor, what is the power density at the point r = 7 cm, Z = -22.7 cm?

[sol] $\mathbf{\phi(r.z) = \frac{3.63P}{VE_R\Sigma_F}J_o\left(\frac{2.405r}{R}\right)\cos(\frac{\pi z}{H})}$

$$E_R = 200MeV = 3.2\times10^{-11}J$$

$$P = 20MW = 2\times10^7 J\cdot sec = 2\times10^7 watt$$

$$P(r,z) = E_R\Sigma_F\phi(r,z) = \frac{.3.63P}{V}J_o\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

$$\therefore P(7,-22.7) = \frac{3.63\times2\times10^7}{\pi\times50\times100}J_o\left(\frac{2.425\times7}{50}\right)\cos\left(\frac{\pi\times(-22.7)}{100}\right) = 67.8watt/cm^3$$

4. In a spherical reactor of radius 45 cm, the fission rate density is measured as 2.5 × 10^{11} fissions/cm^3-sec at a point 35 cm from the center of the reactor. (a) At what steady-state power is the reactor operating? (b) What is the fission rate density at the center of the reactor?

[sol] (a) $\mathbf{R = \Sigma_F\phi(r) = 2.5\times10^{11}\#/cm^3\cdot sec}$

$$\phi(r) = A\frac{\sin(\pi r/R)}{r}$$

$$\Sigma_F\phi(r)_{r=35} = \Sigma_F A\left.\frac{\sin(\pi r/R)}{r}\right|_{r=35} = \Sigma_F A\frac{\sin(35\pi/45)}{35} = 2.5\times10^{11}\#/cm^3\cdot sec$$

$$\Sigma_F A = 1.36\times10^{13}$$

$$P = E_R\Sigma_F\int_V \phi(r)dV = 4\pi E_R\Sigma_F A\int_0^R \sin\left(\frac{\pi r}{R}\right)rdr$$

$$= 4\pi E_R\Sigma_F A\frac{R^2}{\pi} = 4\pi E_R\Sigma_F AR^2 = 4\times3.2\times10^{-11}\times1.36\times10^{13}\times(45)^2 = 3.53MW$$

(b) $\lim_{r \to 0} \Sigma_F \phi(r) = \lim_{r \to 0} \Sigma_F A \frac{\sin\left(\frac{\pi r}{R}\right)}{r} = \Sigma_F A \frac{\pi}{R} = 1.36 \times 10^{13} \times \frac{\pi}{45}$
$= 9.5 \times 10^{11} \text{fissions/cm}^3 \cdot \text{sec}$

5. Using the flux function given in Table 6.2 for a critical finite cylindrical reactor, derive the value of the constant A.

[sol] $P = E_R \Sigma_F \int_V \phi(r,z) dV = 2\pi A E_R \Sigma_F \int_0^R \int_{-\frac{H}{2}}^{\frac{H}{2}} J_o\left(\frac{2.405r}{R}\right) \cos\left(\frac{\pi z}{H}\right) dz r dr$

$\int_0^R J_o\left(\frac{2.405r}{R}\right) r dr = \left(\frac{R}{2.405}\right)^2 \int_0^R J_o\left(\frac{2.405r}{R}\right)\left(\frac{2.405r}{R}\right) d\left(\frac{2.405r}{R}\right)$

$= \left(\frac{R}{2.405}\right)^2 \int_0^{2.405} J_o(u) u du = \left(\frac{R}{2.405}\right)^2 u J_1(u)\Big|_0^{2.405} = \frac{R^2}{2.405} J_1(2.405)$

$P = E_R \Sigma_F A 2\pi \left[\frac{R^2}{2.405} J_1(2.405)\right]\left[\frac{\sin(\pi z/H)}{\pi/H}\right]_{-\frac{H}{2}}^{\frac{H}{2}} = 2\pi A E_R \Sigma_F \frac{R^2}{2.405} J_1(2.405) \frac{2H}{\pi} = 4 A E_R \Sigma_F \frac{V}{\pi} \frac{J_1(2.405)}{2.405}$

$A = \frac{2.405\pi P}{4 V E_R \Sigma_F J_1(2.405)} = \frac{3.63P}{V E_R \Sigma_F}$

6. The core of a certain reflected reactor consists of a cylinder 10-ft high and 10 ft in diameter. The measured maximum-to-average flux is 1.5. When the reactor is operated at a power level of 825 MW, what is the maximum power density in the reactor in kW/liter?

[sol] $P = E_R \Sigma_F \int_V \phi dv \rightarrow \int_V \phi_c dV = \frac{P}{E_R \Sigma_F}$

$\phi_{av} = \frac{1}{V} \int_V \phi_c dV, \ \Omega = \frac{\phi_{max}}{\phi_{av}} = 1.5$

$P_{max} = E_R \Sigma_F \phi_{max} = E_R \Sigma_F \times 1.5\phi_{av} = 1.5 E_R \Sigma_F \int_V \phi dv = \frac{1.5}{V} P = \frac{1.5 \times 825}{\pi \times 10 \times 10} = 3.939 \text{MW/ft}^2$

7. Suppose the reactor described in Example 6.3 was operated at a thermal power level of 1 kilowatt. How many neutrons would escape from the reactor per second? [Hint: See Example 6.4.]

[sol] $B^2 = \left(\frac{\pi}{R}\right)^2 = 4.196 \times 10^{-3}, L^2 = 384\text{cm}^2$

$k_\infty = 2.61, \Sigma_a = 0.00835\text{cm}^{-1}$

$\Sigma_f = N_{fF}\sigma_{fF} + N_{fS}\sigma_{fS} = 0.00395 \times 10^{24} \times 1.85 \times 10^{-24} + 0 = 7.31 \times 10^{-3}\text{cm}^{-1}$

Non-leakage probability

$P_{NL} = \frac{1}{1+B^2L^2} = \frac{1}{1+4.196\times10^{-3}\times384} = 0.383$

Leakage probability

$P_L = 1 - P_{NL} = 0.617$

Number of produced neutrons by fission reaction

$k_\infty \Sigma_a \int_V \phi dv = k_\infty \Sigma_a A \int_0^R \frac{\sin(\pi r/R)}{r} 4\pi r^2 dr = 4\pi k_\infty \Sigma_a A \int_0^R \sin\left(\frac{\pi r}{R}\right) r dr = 4 k_\infty \Sigma_a R^2 A$

$= \frac{P}{4R^2 E_R \Sigma_f} \times 4 k_\infty \Sigma_a R^2 = \frac{P \Sigma_a R^2}{E_R \Sigma_f} = \frac{10^3 \times 2.61 \times 0.00835}{3.2\times10^{-11}\times7.31\times10^{-3}} = 9.30 \times 10^{13} \text{neutrons/sec}$

$\therefore P_L k_\infty \Sigma_a \int_V \phi(r) dr = 0.617 \times 9.30 \times 10^{13} = 5.74 \times 10^{13} \text{neutrons/sec}$

8. Show that in a one-group model, the power produced by a reactor per unit mass of fissile material is given by

$$\frac{\text{watts}}{\text{g}} = \frac{\text{kW}}{\text{kg}} = \frac{3.2 \times 10^{11} \sigma_f \bar{\phi} N_A}{M_F}$$

Where σ_f is the one-group fission cross-section $\bar{\phi}$ is the average one-group flux, N_A is Avogadro's number, and M_F is the gram atomic weight of the fuel.

[sol] A one-group model

$P = E_R \Sigma_f \int \phi dV = E_R N_F \sigma_f \bar{\phi} V = \frac{E_R \rho N_A \sigma_f \bar{\phi} V}{M_F}$

$\therefore (\text{Power per unit mass}) = \frac{P}{m} = \frac{p}{\rho V} = \frac{E_R N_A \sigma_f \bar{\phi}}{M_F}$

$$\leftarrow E_R = 200\text{MeV} = 3.1\times 10^{11}\text{J}$$

$$= \frac{3.1\times 10^{11} N_A \sigma_f \bar{\Phi}}{M_F}$$

9. (a) Estimate the critical radius of a hypothetical bare spherical reactor having the same composition as the reactor in Problem 6.1. (b) If the reactor operates at a thermal power level of 500 MW, what is the maximum value of the flux? (c) What is the probability that a fission neutron will escape from the reactor?

[sol] Table II.3 $M_{sodium} = 22.9898, M_{plutonium} = 239.0522$

Table 6.1

$\sigma_{aF} = 2.11\times 10^{24}\text{cm}^2, \sigma_{aM} = 0.0008\times 10^{24}\text{cm}^2, \sigma_{tr,Na} = 3.3\times 10^{24}\text{cm}^2, \sigma_{tr,F} = 6.8\times 10^{24}\text{cm}^2$

$\sigma_{f,Na} = 0, \sigma_{f,Pu} = 1.85\times 10^{24}\text{cm}^2, \eta(^{239}\text{Pu}) = 2.61$

(a) $f = \frac{\Sigma_{aF}}{\Sigma_{aF}+\Sigma_{aM}} = \frac{N_F\sigma_{a,F}}{N_F\sigma_{a,F}+N_M\sigma_{a,M}} =$

$$\frac{(0.0000755\times 10^{24}\#/\text{cm}^3)(2.11\times 10^{24}\text{cm}^2)}{(0.0000755\times 10^{24}\#/\text{cm}^3)(2.11\times 10^{24}\text{cm}^2)+(0.02541\times 10^{24}\#/\text{cm}^3)(0.0008\times 10^{24}\text{cm}^2)} = 0.887$$

$k_\infty = \eta f = 2.61\times 0.887 = 2.315$

$D = \frac{\lambda_{tr}}{3} = \frac{1}{3\Sigma_{tr}} = 3.973$

$L^2 = \frac{D}{\Sigma_a} = \frac{D}{\Sigma_{a,F}+\Sigma_{a,M}} = \frac{3.973}{0.00018} = 22072.22\text{cm}^2$

D=2.13D=8.462cm

$\widetilde{R_c} = \pi\sqrt{\frac{L^2}{k_\infty - 1}} = \pi\sqrt{\frac{22072.22}{2.315-1}} = 407.01\text{cm}$

$R_c = \widetilde{R_c} - d = 398.55\text{cm}$

(b) $\phi = \frac{P}{4E_R\Sigma_f R^2}\frac{\sin(\pi r/\tilde{R})}{r}$

$$\phi_{max} = \frac{P}{4E_R\Sigma_f R^2}\frac{\pi}{\tilde{R}}\lim_{r\to\infty}\frac{\sin(\pi r/\tilde{R})}{\pi r/\tilde{R}} = \frac{\pi P}{4E_R\Sigma_f R^2\tilde{R}} = 1.446\times 10^{15}\#/\text{cm}^2\cdot\text{s}$$

(c) $P = \frac{1}{1+B^2L_T^2} = \frac{1}{1+(5.998\times 10^{-5}\text{cm}^{-2})(22072.22\text{cm}^2)} = 0.432$

(Probability)=1-0.432=0.568

10. An infinite slab of moderator of thickness 2a contains at its center a thin sheet of ^{235}U of thickness t. Show that in one-group theory the condition for criticality of this system can be written approximately as

$$\frac{2D}{(\eta-1)L\Sigma_{aF}t}\cosh\frac{a}{L} = 1$$

Where D and L are the diffusion parameters of the moderator and Σ_{aF} is te macroscopic absorption cross-secton of the ^{235}U.

[sol] <Fuel>

$$\frac{d^2\phi_F}{dx^2} + B^2\phi_F = 0$$

$\phi_F(x) = A\cos Bx + C\sin Bx \rightarrow \phi_F(x) = A\cos Bx$

<Moderator>

$D_M\nabla^2\phi - \Sigma_{aM}\phi_M = 0 \rightarrow \phi_M(x) = A_1\sinh\frac{x}{L} + B\cosh\frac{x}{L}$

$\rightarrow \phi_M(a) = A_1\sinh\frac{a}{L} + B\cosh\frac{a}{L} = 0 \rightarrow B = -A_1\tanh\frac{a}{L}$

$\therefore \phi_M(x) = A_1\sinh\frac{x}{L} - A_1\tanh\frac{a}{L}\cosh\frac{x}{L}$

(i) $\phi_F\left(\frac{t}{2}\right) = \phi_M\left(\frac{t}{2}\right)$

$\phi_F\left(\frac{t}{2}\right) = A\cos\frac{Bt}{2}, \quad \phi_M\left(\frac{t}{2}\right) = A_1\sinh\left(\frac{t}{2L}\right) - A_1\tanh\frac{a}{L}\cosh\left(\frac{t}{2L}\right)$

$\therefore A\cos\frac{Bt}{2} = A_1\sinh\left(\frac{t}{2L}\right) - A_1\tanh\frac{a}{L}\cosh\left(\frac{t}{2L}\right)$

$= \frac{A_1}{\cosh\left(\frac{a}{L}\right)}\left[\cosh\frac{a}{L}\sinh\frac{t}{2L} - \sinh\frac{a}{L}\cosh\frac{t}{2L}\right]$

$$= \mathbf{A_1\, sech}\frac{a}{L}\sinh\left(\frac{t-2a}{2L}\right)$$

(ii) $J_F\left(\frac{t}{2}\right) = J_M\left(\frac{t}{2}\right)$

$$-D_F\frac{d\phi_F}{dx}\left(\frac{t}{2}\right) = -D_M\frac{d\phi_M}{dx}\left(\frac{t}{2}\right)$$

$$-D_F\left[-AB\sin\left(\frac{Bt}{2}\right)\right] = -D_M\left[\frac{A_1}{L}\cosh\left(\frac{t}{2L}\right) - \frac{A_1}{L}\tanh\frac{a}{L}\sinh\left(\frac{t}{2L}\right)\right]$$

$$\therefore ABD_F\sin\left(\frac{Bt}{2}\right) = -\frac{D_M A_1}{L}\mathrm{sech}\frac{a}{L}\cosh\left(\frac{2a-t}{2L}\right)$$

The condition for criticality of this system

$$\begin{vmatrix} \cos\frac{Bt}{2} & \mathrm{sech}\frac{a}{L}\sinh\left(\frac{t-2a}{2L}\right) \\ BD_F\sin\left(\frac{Bt}{2}\right) & -\frac{D_M}{L}\mathrm{sech}\frac{a}{L}\cosh\left(\frac{2a-t}{2L}\right) \end{vmatrix} = 0$$

$$\frac{D_M}{L}\coth\left(\frac{2a-t}{2L}\right) = BD_F\tan\left(\frac{Bt}{2}\right), \qquad t \approx 0,\quad \coth\left(\frac{2a-t}{2L}\right) \approx \coth\frac{a}{L}$$

$$\tan\left(\frac{Bt}{2}\right) \approx \frac{Bt}{2}$$

$$\therefore \frac{D_M}{L}\coth\frac{a}{L} = BD_F\frac{Bt}{2} = D_F\frac{B^2 t}{2}$$

$$\frac{2D_M}{LD_F B^2 t}\coth\frac{a}{L} = 1$$

$$\leftarrow B^2 = (\eta - 1)\frac{\Sigma_{aF}}{D_F}, \quad D_M \approx D$$

$$\therefore \frac{2D}{L(\eta-1)t}\coth\frac{a}{L} = 1$$

11. A large research reactor consists of a cubical array of natural uranium rods in a graphite moderator. The reactor is 25ft on a side and operates at a power of 20MW. The average value of $\bar{\Sigma}_f$ of $2.5 \times 10^{-3}\mathrm{cm}^{-1}$. (a) Calculate the buckling. (b) What is the maximum value of the thermal flux? (c) What is the average value of the thermal flux? (d) At what rate is ^{235}U being consumed in the reactor?

[sol] (a) $B = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = \left[\left(\frac{\pi}{25}\right)^2 + \left(\frac{\pi}{25}\right)^2 + \left(\frac{\pi}{25}\right)^2\right]\mathrm{ft}^2 = 0.0474\mathrm{ft}^2$

(b) $\emptyset_{max} = \phi(0,0,0) = A$

$$A = \frac{3.87P}{VE_R\Sigma_f} = \frac{3.87\times(20\times10^6\mathrm{J/s})}{(25\times30.48)^3\mathrm{cm}^3(3.2\times10^{-11}\mathrm{J})(2.5\times10^{-3}\mathrm{cm}^{-1})} = 2.187\times10^{12}\#/\mathrm{cm}^2\cdot\mathrm{s}$$

(c) $\phi_{av} = \frac{1}{V}\int\phi dV = \frac{A}{V}\int_{-a/2}^{a/2}\cos\left(\frac{\pi x}{a}\right)dx\int_{-a/2}^{a/2}\cos\left(\frac{\pi y}{a}\right)dy\int_{-a/2}^{a/2}\cos\left(\frac{\pi z}{a}\right)dz = 5.643\times10^{11}\#/\mathrm{cm}^2\cdot\mathrm{s}$

(d) $\text{Consumption rate} = 1.05(1+\alpha)P\ \mathrm{g/day} = 24.75\mathrm{g/day}$

12. Show with a recoverable energy per fission of 200MeV the power of a 235U-fueled reactor operating at the temperature T given by either of the following expressions :

$$P = 4.73m_F g_F(T)\overline{\phi_0}\times10^{-14}\mathrm{MW}$$

or

$$P = 7.19m_F g_F(T)^{-1/2}\overline{\phi_T}\times10^{-13}\mathrm{MW}$$

where m_F is the total amount in kg of 235U in the reactor, $g_F(T)$ is the non-1/ν factor for fission, $\overline{\phi_0}$ is the average 2,200 meters-per-second flux, and $\overline{\phi_T}$ is the average thermal flux.

[sol] Table 3.4 $\sigma_{f,U^{235}} = 582.2\mathrm{barns}$

$$P = E_R\Sigma_f\int\phi dV = E_R\Sigma_f\overline{\phi_0}V$$

$$= E_R g_F(T)\Sigma_f(E_0)\overline{\phi_0}\frac{m_F N_A}{N_F M_F} = m_F g_F(T)\overline{\phi_0}\left[E_R\frac{N_A\sigma_f}{M_F}\right]$$

$$= m_F\times10^{-3}\times g_F(T)\overline{\phi_0}\times\left[(3.2\times10^{-11}\mathrm{J})\frac{0.6022\times10^{24}}{235.0439}\times582.2\times10^{-24}\mathrm{cm}^2\right]$$

$$= 4.77m_F g_F(T)\overline{\phi_0}\times10^{-14}\mathrm{MW}.$$

$$\frac{\overline{\phi_0}}{\overline{\phi_T}} = \frac{\sqrt{\pi}}{2}\left(\frac{T_0}{T}\right)^{\frac{1}{2}} \quad , \qquad \overline{\phi_0} = \frac{\sqrt{\pi}}{2}\left(\frac{T_0}{T}\right)^{\frac{1}{2}}\overline{\phi_T}$$

$$\mathbf{P = m_F g_F(T)\frac{\sqrt{\pi}}{2}\left(\frac{T_0}{T}\right)^{\frac{1}{2}}\overline{\phi_T}\left[E_R\frac{N_F\sigma_F}{M_F}\right]}$$

$$\mathbf{= m_F \times 10^{-3} \times g_F(T)\overline{\phi_T}\left[E_R\frac{\sqrt{\pi}}{2}T^{-\frac{1}{2}}(293.61)^{\frac{1}{2}}\frac{N_A\sigma_F}{M_F}\right]}$$

$$\mathbf{= 7.25 m_F g_F(T)\overline{\phi_T}T^{-\frac{1}{2}} \times 10^{-13} MW}$$

13. Solve the following equations:

$$3x + 4y + 7z = 16,$$
$$x - 6y + z = 2,$$
$$2x + 3y + 3z = 12.$$

[sol] $\mathbf{D= \begin{vmatrix} 3 & 4 & 7 \\ 1 & -6 & 1 \\ 2 & 3 & 3 \end{vmatrix} = 3\begin{vmatrix} -6 & 1 \\ 3 & 3 \end{vmatrix} - \begin{vmatrix} 4 & 7 \\ 3 & 3 \end{vmatrix} + 2\begin{vmatrix} 4 & 7 \\ -6 & 1 \end{vmatrix} = 38}$

$$\mathbf{x=\frac{1}{D}\begin{vmatrix} 16 & 4 & 7 \\ 2 & -6 & 1 \\ 12 & 3 & 3 \end{vmatrix} = 6.157894737}$$

$$\mathbf{y = \frac{1}{D}\begin{vmatrix} 3 & 16 & 7 \\ 1 & 2 & 1 \\ 2 & 12 & 3 \end{vmatrix} = 0.578947368}$$

$$\mathbf{z = \frac{1}{D}\begin{vmatrix} 3 & 4 & 16 \\ 1 & -6 & 2 \\ 2 & 3 & 12 \end{vmatrix} = -0.684210526}$$

$$\mathbf{\therefore (x, y, z) = (6.157894737, 0.578947368, -0.684210526)}$$

14. Solve the following equations:

$$3.1x + 4.0y + 7.2z = 0,$$
$$x - 5.0y - 9.0z = 0,$$
$$7x + 4.5y + 8.1z = 0.$$

[sol] D=0

(Trivial solution) : (x,y,z)=(0,0,0)

x-5.0y=9.0z

7x+4.5y=-8.1z

$$\mathbf{D' = \begin{vmatrix} 1 & -5.0 \\ 7 & 4.5 \end{vmatrix} = 39.5}$$

$$\mathbf{x = \frac{1}{D'}\begin{vmatrix} 9.0z & -5.0 \\ -8.1z & 4.5 \end{vmatrix} = 0}$$

$$\mathbf{y = \frac{1}{D'}\begin{vmatrix} 1 & 9.0z \\ 7 & -8.1z \end{vmatrix} = -1.8z}$$

$$\mathbf{\therefore (x, y, z) = (0, -1.8z, z \text{ is any number})}$$

15. A homogeneous solution of 235U and H_2O; contains 10 grams of 235U per liter of solution. Compute (a) the atom density of 235U and the molecular density of H_2O; (b) the thermal utilization; (c) the thermal diffusion area and length; (d) the infinite multiplication factor.

[sol]

$$\mathbf{\rho(U-235) = \frac{10gm}{l} = 0.01 gm/cm^3}$$

(a)

N(U-235) $= \frac{0.01 \times 0.6022 \times 10^{24}}{235}$ **= 2.56 x** $\mathbf{10^{19}}$ **atoms / cm**^{3}

N(H_2O**)** = $\frac{1 \times 0.6022 \times 10^{24}}{18} = \mathbf{3.35 \times 10^{22}}$ **molecules / cm**^{3}

(b)

$$\mathbf{Z = \frac{N_F\sigma_{aF}}{N_M\sigma_{aM}} = \frac{N_F g_{aF(20℃)}\sigma_{aF}(E_0)}{Nm\sigma_{aM}(E_0)} = \frac{2.56 \times 10^{-5} \times 0.978 \times 680.8}{3.35 \times 10^{-3} \times 0.03343} = 15.22}$$

$$\mathbf{\therefore f = \frac{Z}{Z+1} = \frac{15.22}{16.22} = 0.938}$$

(c)
$\mathbf{L_T^2 = (1-f)L_{TM}^2}$ **= (1-0.938) x 8.1 = 0.5cm^2**
$\mathbf{L_T = 0.71cm}$

(d)

$\mathbf{k_\infty = \eta_T f = 2.065 \times 0.938 = 1.937}$

16. Compute the thermal diffusion length for homogeneous mixtures of 235U and the following moderators at the given fuel concentrations and temperatures. Graphite: N (25) / N (e) = 4.7 x 1 0-6; T = 200DC. Beryllium: N (25) / N (Be) 1 .3 x 1 0-5 ; T = 1 00DC. D20: N (25)/N(D20) 1 .4 x 1 0-6; T = 20DC. H20: N (25) / N (H20) = 9.2 x 1 0-4; T = 20DC.

[sol]
(a)
$$\mathbf{Z = \frac{N_F\sigma_{aF}}{N_M\sigma_{aM}} = \frac{N_F}{N_M}\cdot\frac{g_{aF}(T)\sigma_{aF}(E_0)\sigma_{aF}(E_0)}{\sigma_{aM}(E_0)} = 4.7 \times 10^{-6} \times \frac{0.9457x680.8}{0.0034} = 0.89}$$
$$\mathbf{f = \frac{Z}{Z+1} = \frac{0.89}{1.89} = 0.471}$$

$$\mathbf{L_T^2 = (1-f)L_{TM}^2 = (1-0.471)x3500 = 1851.5\ cm^2}$$

$$\mathbf{\therefore\ L_T = 0.71cm}$$

(b)
$$\mathbf{Z = 1.3 \times 10^{-3} x\frac{0.9610x680.8}{0.0092} = 0.925}$$
$$\mathbf{f = \frac{Z}{Z+1} = 0.481}$$

$$\mathbf{L_T^2 = (1-0.481)\ x\ 480 = 2491.}$$

$$\mathbf{\therefore\ L_T = 15.8cm}$$

(c)
$$\mathbf{Z = 1.4 \times 10^{-6} x\ \frac{0.9780x680.}{0.00133} = 0.701}$$
$$\mathbf{f = \frac{0.701}{1.701} = 0.412}$$

$$\mathbf{L_T^2 = (1-f)L_{TM}^2 = (1-0.412)x3x10^4 = 1.76x10^4\ cm^2}$$
$$\mathbf{\therefore\ L_T = 133cm}$$

(d) $\mathbf{Z = 9.2 \times 10^{-4} x\frac{0.9780x680.8}{0.664} = 0.923}$
$$\mathbf{f = \frac{0.923}{1+0.923} = 0.48}$$

$$\mathbf{L_T^2 = (1-f)L_{TM}^2 = (1-0.48)x8.1 = 4.21\ cm^2}$$

$$\mathbf{\therefore\ L_T = 2.05cm}$$

17. Consider a critical bare slab reactor 200 cm thick consisting of a homogeneous mixture of 235U and graphite. The maximum thermal flux is 5 X 1 012 neutrons/cm2-sec. Using modified one-group theory, calculate: (a) the buckling of the reactor; (b) the critical atomic concentration of uranium; (c) the thermal diffusion area; (d) the value of k_∞ ; (e) the thermal flux and current throughout the slab; (f) the thermal power produced per cm2 of this slab.

[sol]
(a)

$\mathbf{B^2 = \left(\frac{\pi}{a}\right)^2 = \left(\frac{\pi}{200}\right)^2 = 2.47x10^{-4}cm^{-2}}$

(b)

$$\mathbf{Z = \frac{N_F\sigma_{aF}}{N_M\sigma_{aM}} = \frac{1+\xi^2(L_{TM}^2-\xi_{TM})}{\eta_T-1+B^2\xi_{TM}}}$$

$$\mathbf{L_T^2 = 3500cm^2, \eta_T = 2.056, \xi_{TM} = 368\ cm^2}$$

$$\mathbf{Z = \frac{1+\xi^2(L_{TM}^2-\xi_{TM})}{2.056-1+2.47\ x\ 10^{-4}x368} = 1.705}$$

$$\mathbf{\frac{A_F}{\rho_M} = Z\frac{M_F\sigma_{aF}}{M_M\sigma_{aM}} = \frac{M_F\sigma_{aM}(E_0)}{M_m g_{af}(20°C)\sigma_{aF}}}$$

$$\mathbf{\therefore\ \rho_F = 1.6\ x\ 1.705\ x\ \frac{235\ x\ 0.0034}{12\ x\ 0.978\ x\ 680.8} = 2.73\ x\ 10^{-4}gm/cm^3}$$

(c)

$$\mathbf{L_T^2 = (1-f)L_{TM}^2 = \frac{L_{TM}^2}{(Z+1)} = \frac{3500}{1.705+1} = 1.293\ x\ 10^3cm}$$

(d) $\mathbf{k_\infty = \eta_T f = \eta_T x(\frac{Z}{Z+1}) = 2.056\ x\ \frac{1.705}{2.705} = 1.296}$

(e) $\mathbf{\emptyset_{th}(x) = \emptyset_{max}\cos\left(\frac{\pi x}{a}\right) = 5x10^{12}\cos\left(\frac{\pi x}{a}\right)\ \#/cm^2sec}$

$\mathbf{J(x) = -D\frac{d\emptyset}{dx} = \frac{\pi D}{a}x5x10^{22}\sigma f(\frac{\pi x}{a})}$

(f) $\mathbf{A = \frac{1.57p}{aE_R\Sigma_f},\quad P = \frac{AqE_R\Sigma_f}{1.57}}$

$\mathbf{\Sigma_f = \Sigma_{fF}+\Sigma_{fM} = \frac{2.73\ x\ 10^{-4}x6.022x10^{23}}{235}x582.2\ x\ 10^{-24}\ x\ 0.9759 = 3.97x10^{-4}\ cm^{-1}}$

$$\mathbf{P = \frac{5x10^{12}\#/cm^2\ sec\ x\ 200cm\ x\ 3.2\ x\ 10^{-17}MJ\ x\ 3.97\ x\ 10^{-4}}{1.57} = 8.1x10^{-6}\ MW/cm^2 = 8.1\ W/cm^2}$$

18. The binding energy of the last neutron in 13C is 4.95 Me V. Estimate the recoverable energy per fission in a large graphite-moderated, 235U-fueled reactor from which there is little or no leakage of neutrons or y-rays.

[sol]

Recoverable energy = 200-4.95 = 195.05 MeV = 3.12x$\mathbf{10^{-11}}$Joules

19. Calculate the concentrations in grams per liter of (1) 235U, (2) 233U, and (3) 239pu required for criticality of infinite homogeneous mixtures of these fuels and the following moderators: (a) H20, (b) D20, (c) Be, (d) graphite.

[sol]

$$\mathbf{Z = \frac{N_F\sigma_{aF}}{N_M\sigma_{aM}} = \frac{1+B^2(L_{TM}^2-\xi_{TM})}{\eta_T-1+B^2\xi_{TM}} = \frac{1}{\eta_T-1}}$$

$\mathbf{\eta_T}$(235) = 2.065 $\mathbf{\eta_T}$(233) = 2.284 $\mathbf{\eta_T}$(239) = 2.035
Z(235) = 0.739, Z(233) = 0.779, Z(239) = 0.966
Ga(235) = 0.9780, ga(233) = 0.9983, ga(239) = 1.0723
$\mathbf{\sigma_a(235) = 680.8b, \sigma_a(233) = 578.8b, \sigma_a(239) = 1011.3b}$

For $\mathbf{U^{235}}$

$$\mathbf{A = 0.939\ x\ \frac{235}{0.918x680.8} = 0.331}$$

(a) $\mathbf{H_2O}$

$$\mathbf{\rho_F = 0.331\ x\ 1\ x\ \frac{0.664}{18} = 1.22\ x\ 10^{-2}\ gm/cm^3 = 12.2gm/l}$$

(b)$\mathbf{D_2O}$

$\rho_F = 0.331 \text{ x } 1 \text{ x } 1.105\text{x } \frac{0.00133}{20} = 0.024\text{gm/l}$

(c)Be

$\rho_F = 0.331 \text{ x } 1.85\text{x } \frac{0.0092}{9} = 0.63\text{gm/l}$

(d)C

$\rho_F = 0.331 \text{ x } 1.6\text{x } \frac{0.0034}{12} = 0.15\text{gm/l}$

For U^{233}

$A = 0.779 \text{ x } \frac{233}{0.9983\text{x}578.8} = 0.314$

(a) H_2O

$\rho_F = 0.314 \text{ x } 1 \text{ x } \frac{0.664}{18} = 11.6\text{gm/l}$

(b)D_2O

$\rho_F = 0.314 \text{ x } 1 \text{ x } 1.105\text{x } \frac{0.00133}{20} = 0.023\text{gm/l}$

(c)Be

$\rho_F = 0.314 \text{ x } 1.85\text{x } \frac{0.0092}{9} = 0.59\text{gm/l}$

(d)C

$\rho_F = 0.314 \text{ x } 1.6\text{x } \frac{0.0034}{12} = 0.14\text{gm/l}$

For Pu^{239}

$A = 0.966 \text{ x } \frac{239}{1.0123\text{x}1011.3} = 0.213$

(a) H_2O

$\rho_F = 0.213 \text{ x } 1 \text{ x } \frac{0.664}{18} = 7.86\text{gm/l}$

(b)D_2O

$\rho_F = 0.213 \text{ x } 1 \text{ x } 1.105\text{x } \frac{0.00133}{20} = 0.015\text{gm/l}$

(c)Be

$\rho_F = 0.213 \text{ x } 1.85\text{x } \frac{0.0092}{9} = 0.4\text{gm/l}$

(d)C

$\rho_F = 0.213 \text{ x } 1.6\text{x } \frac{0.0034}{12} = 0.0\text{gm/l}$

20. A bare-spherical reactor 50 cm in radius is composed of a homogeneous mixture of 235U and beryllium. The reactor operates at a power level of 50 thermal kilowatts. Using modified one-group theory, compute: (a) the critical mass of 235U; (b) the thermal flux throughout the reactor; (c) the leakage of neutrons from the reactor; (d) the rate of consumption of 235U.

[sol]

(a) $B^2 = \left(\frac{\pi}{R}\right)^2 = 3.95 \text{ x } 10^{-3}$

$x_T = 2.065$, $\gamma_{TH} = 102 \text{ cm}^2$, $L_{TM}^2 = 480\text{cm}^{-1}$

$Z = \frac{1+3.95\text{x}10^{-3}(480+102)}{2.065-1-3.95\text{x}10^{-3}\text{x}102} = 4.98$

$M_m = \frac{4}{3}\pi(50)^3\text{x}1.85 = 9.69 \text{ x } 10^3\text{gm}$

$\sigma_{aF}(E_0) = 680.8\text{b}, \sigma_{aM} = 0.0092\text{b}, g_{aF}(T) = 0.978$

$\therefore M_F = Z\frac{\sigma_{aF}M_m}{g_{aF}(T)\sigma_{aF}(E_0)M_n}M_m = 4.98\text{x}\frac{0.0092\text{x}235\text{x}9.69\text{x}10^5}{0.978\text{x}680.8\text{x}9} = 1741\text{gm}$

(b)

$\sigma_{aF}(E_0)582\text{b}$, $g_F(T) = 0.9759$ at 20℃

$\sum_f = \frac{m_F N_A}{M_F V}\frac{\sqrt{\pi}}{2}\sigma_{aF}(E_0)g_F(T) = \frac{1741x0.6022}{235x\frac{4}{3}\pi(50)^3}x\frac{\sqrt{\pi}}{2}x0.9759x582 = 4.288x10^{-3}cm^{-1}$

$\therefore\ \emptyset(r) = A\frac{\sin Br}{r} = \frac{P}{4R^2E_R\sum_f}\frac{\sin Br}{r} = 3.64x10^{13}\frac{\sin Br}{r}$

(c)

$J(R) = -D\frac{d\emptyset}{dr}\Big]_{r=R} = -0.5x3.64x10^{13}x\frac{0.063x50\cos(0.063x50)-\sin(0.063x50)}{50^2} = 2.29x10^{10}\ \text{neutrons / cm}^2\text{sec}$

$\therefore$ **total leakage** $= 4\pi R^2 J(R) = 4\pi(50)^2x2.29x10^{10} = 7.19x10^{14}\ \text{neutrons/sec}$

(d) consumption rate

= 1.05(1+α)P gm/day = 1.05 x (1+0.169)x50x$10^{-3} = 0.0614\text{gm/day}$

21. The flux in a bare-finite cylindrical reactor of radius r and height H is given by ¢ = Alo (□2.405r) cos (rriiz) Find A if the reactor is operated at a power of P watts.

[sol]

$\rho(U^{235}) = \rho(UO_2SO_4)\frac{M(u)}{M(UO_2SO_4)} = 30x\frac{235}{235+16x6+32} = 19.42 = 0.01942\text{gm/cm}^3$

$\frac{N_F}{N_M} = \frac{\rho_F M_M}{\rho_M M_F} = \frac{0.019+2x18}{1x235} = 1.49x10^{-3}$

$Z = \frac{N_F\sigma_{aF}}{N_M\sigma_{aM}} = 1.49x10^{-3}\frac{0.978x680.8}{0.589} = 1.69$

$f = \frac{Z}{Z+1} = 0.628$

$k_\infty = \eta_T f = 2.065\ x\ 0.628 = 1.297$

$L_T^2 = (1-f)L_{TM}^2 = (1\text{-}0.628)\ x\ 8.1 = 3.013cm^2$

$M_T{}^2 = L_T^2 + \gamma_T = 3.013 + 27 = 30.01\ cm^2$

$B^2 = \frac{k_\infty - 1}{M_T} = \frac{0.297}{30.01} = 9.897x10^{-3}$

i) For Spherical tank

$B^2 = 9.897\ x\ 10^{-3} = \left(\frac{\pi}{R}\right)^2,\ \therefore R = 31.58\ cm$

ii) For cylindrical tank

$B^2 = 9.897\ x\ 10^{-3} = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \frac{8.763}{R},$

$\therefore R = 29.76\ cm\ \ H = 54.16\ cm$

22. It is proposed to store H2O solutions of fully enriched uranyl sulfate (235UO2So4) with a concentration of 30 g of this chemical per liter. Is this a safe procedure when using a tank of unspecified size?

[sol] $\rho(^{235}U) = \rho(UO_2SO_4)\frac{M(U)}{M(UO_2SO_4)} = 30 * \frac{235}{235+16*6/32} = 19.42\frac{g}{l} = 0.01942\ g/cm^3$

$\frac{N_f}{N_m} = \frac{\rho_f M_m}{\rho_M M_F} = \frac{0.01942*18}{235} = 1.49 * 10^{-3}$

$Z = \frac{N_f\hat{\sigma}_{af}}{N_m\hat{\sigma}_{am}} = 1.49 * 10^{-3} * \frac{0.978*680.8}{0.589} = 1.69$

$\hat{\sigma}_{af} = g_{af}(20)\hat{\sigma}_{af}(E_0)$

$\hat{\sigma}_{am} = \frac{\hat{Z}_{am}}{N_m} = \frac{0.0197}{0.03343*10^{24}} = 0.589$

$f = \frac{Z}{1+Z} = 0.628$

$k_\infty = \eta_T f = 2.065 * 0.628 = 1.297$

$$\mathbf{L_T^2 = (1-f)L_{TM}^2 = (1-0.628)*8.1 = 3.013cm2}$$
$$\mathbf{M_T^2 = L_T^2 + \tau_T = 3.013 + 27 = 30.01\ cm2}$$
$$\mathbf{B^2 = \frac{k_\infty - 1}{M_T^2} = \frac{0.297}{30.01} = 9.897 * 10^{-3}\ cm2}$$

For spherical tank

$$\mathbf{B^2 = 9.897 * 10^{-3} = (\frac{\pi}{R})^2, R = 31.58\ cm}$$

For cylindrical tank

$$\mathbf{B^2 = 9.897 * 10^{-3} = (\frac{2.405}{R})^2 + (\frac{\pi}{H})^2 = \frac{8.762}{R}}$$
$$\mathbf{R = 29.76cm, H = 54.16\ cm}$$

23. Show that the flux in a bare-cubical reactor of side a is

$$\emptyset = A(\cos\frac{\pi}{\tilde{a}}x)^3$$

[sol] 여기에 수식을 입력하십시오.

24. If the reactor in Problem 6.23 is operating at P watts, show that the constant A is

$$A = \frac{P}{E_R \sum f \frac{8}{\pi^3}\tilde{a}^3(\sin\frac{\pi a}{2\tilde{a}})^3}$$

[sol] 여기에 수식을 입력하십시오.

25. A bare-thermal reactor in the shape of a cube consists of a homogeneous mixture of 235U and graphite. The ratio of atom densities is Nf/Nm = 1.0*10-5 and the fuel temperature is 250℃. Using modified one-group theory, calculate: (a) the critical dimensions; (b) the critical mass; (c) the maximum thermal flux when the reactor operates at a power of 1 kW.

[sol] $\mathbf{(a) g_a(250) = 0.9416}$

$$\mathbf{Z = \frac{N_f\hat{\sigma}_{af}}{N_m\hat{\sigma}_{am}} = 1.0 * 10^{-5} * \frac{0.9416*680.8}{0.0034} = 1.89}$$
$$\mathbf{f = \frac{Z}{1+Z} = 0.65}$$
$$\mathbf{k_\infty = \eta_T f = 2.065 * 0.65 = 1.35}$$
$$\mathbf{B^2 = \frac{k_\infty - 1}{M_T^2} = \frac{k_\infty - 1}{L_T^2 + \tau_T} = \frac{B_\infty - 1}{(1-f)L_T^2 + \tau_T} = \frac{0.35}{(1-0.65)*3500+368} = 2.2 * 10^{-7}\ cm^{-2}}$$
$$\mathbf{For\ cube, B^2 = 2.2 * 10^{-4} = 3(\frac{\pi}{a})^2,\ \ a = 367\ cm}$$
$$\mathbf{(b)\rho_F = \rho_M \frac{N_f}{N_m}\frac{M_f}{M_M} = 1.6 * 1.0 * 10^{-5} * \frac{235}{12} = 3.13 * 10^{-4}\ g/cm3}$$
$$\mathbf{m_m = V\rho_F = (367) * 3.13 * 10^{-4} = 1.55 * 10^4 g = 15.5\ kg}$$
$$\mathbf{(c) \sum f = \frac{\rho_F N_A}{M_f}\bar{\sigma}_{Ff} = \frac{3.87*10^3}{(367)*3.2*10^{-11}*2.81*10^{-2}} = 8.7 * 10^9\ \#/cm2}$$

26. The origin version of the Brookhaven Research reactor consisted of a cube of graphite that contained a regular array of natural uranium rods, each of which was located in an air channel through the graphite. When the reactor was operated at a thermal power level of 22 MW, the average fuel temperature was approximately 300℃ and the maximum thermal flux was 5*1012 neutrons/cm2-sec. The average values of L_T^2 and τ_T were 325 cm2 and 396 cm2, respectively, and k_∞= 1.0735. (a) calculate the critical dimensions of the reactor; (b) What was the total amount of natural uranium in the reactor?

[sol] $\mathbf{B^2 = \frac{k_\infty - 1}{M_T^2} = \frac{k_\infty - 1}{L_T^2 + \tau_T} = \frac{1.0735-1}{325+396} = 1.02 * 10^{-4} cm^{-2}}$

$$\mathbf{B^2 = 1.02 * 10^{-4} = 3(\frac{\pi}{a})^2, \qquad a = 538.8cm}$$
$$\mathbf{b)\ f = \frac{k_\infty}{\eta_T} = \frac{1.0735}{2.055} = 0.522}$$
$$\mathbf{Z = \frac{f}{1-f} = 1.092 = \frac{N_f\hat{\sigma}_{af}}{N_m\hat{\sigma}_{am}} = \frac{N_F}{N_M}\frac{0.9376*680.8}{0.0034},\ \ \frac{N_F}{N_M} = 5.82 * 10^{-6}}$$
$$\mathbf{\rho_F = \rho_M \frac{N_f}{N_m}\frac{M_f}{M_M} = 1.6 * 5.82 * 10^{-6} * \frac{235}{12} = 1.82 * 10^{-4}\ g/cm^3}$$
$$\mathbf{\frac{N(235-U)}{N(U)} = 0.0072 = \frac{\rho(235)M(U)}{\rho(U)M(235)}}$$
$$\mathbf{\rho(u) = 1.82 * 10^{-4} * \frac{238.03}{235} * \frac{1}{0.0072} = 0.0256\frac{g}{cm^3}, the\ total\ amount\ of\ natural\ uraniu}$$

$\mathbf{m = V\rho = (538.8)^3 * 0.0256 = 4.01 * 10^6 g}$

$\mathbf{(c) k_{eff} = k_{\infty P_{NL} = k_{\infty} \frac{1}{(1+B^2L_T^2)(1+B^2\tau_t)})}}$

$\mathbf{when\ critical, k_{\infty} = 1, k_{\infty} = (1 + B^2L_T^2)(1 + B^2\tau_t), 1.0735 = (1 + 325B^2)(1 + 396B^2)}$

$\mathbf{B^2 = 1 * 10^{-4} cm^{-2}, 3(\frac{\pi}{a})^2 = 1 * 10^{-4}, a = 544cm}$

27. Using one-group theory, derive expressions for the flux and the condition for criticality for the following reactors: (a) an infinite slab of thickness a, infinite reflector on both sides; (b) an infinite slab of thickness a, reflectors of thickness n on sides; (c) a sphere of radius R, reflector of thickness b.

[sol] One group theory

$< core >$

$\mathbf{-D\tau^2\emptyset_c(r) + \sum_a \emptyset_c(r) = \frac{1}{k_{eff}} \upsilon \sum_f \emptyset_c(r)}$

$\mathbf{\nabla^2\emptyset_c(r) + B_c^2\emptyset_c(r) = 0}$

$< reflector >$

$\mathbf{-D\tau^2\emptyset_c(r) + \sum_a \emptyset_c(r) = 0}$

$\mathbf{\nabla^2\emptyset_R(r) - \frac{1}{L^2}\emptyset_R(r) = 0, where\ L^2 = \frac{D}{\Sigma_a}}$

a)

1) core region

$\mathbf{\frac{d^2}{d\tau^2}\emptyset_c(x) + B_c^2\emptyset_c(x) = 0, \emptyset_c(x) = AcosB_cx + CsinB_cx}$

$\mathbf{B.C, \frac{d\emptyset_c(x)}{dx}]_{x=0} = 0, ==>} C = 0$

$\mathbf{\therefore \emptyset_c = AcosB_cx}$

2) reflector rwgion

$\mathbf{\frac{d^2}{d\tau^2}\emptyset_R(x) - \frac{1}{L^2}\emptyset_R(x) = 0, \quad \emptyset_R(x) = A^{`} e^{-x\lambda} + C^{`} e^{\frac{x}{l}}}$

$\mathbf{B.C, \lim_{x\to\infty} \emptyset_R(x) = 0, \ d = 0}$

$\mathbf{\therefore \emptyset_R(x) = A^{`} e^{-x/L}}$

3) interface condition

$\mathbf{\emptyset_c\left(\frac{a}{2}\right) = \emptyset_R\left(\frac{a}{2}\right) \dots\dots (1), \ J_c\left(\frac{a}{2}\right) = J_R\left(\frac{a}{2}\right) \dots\dots (2), AcosB_c\frac{a}{2} = A^{`} e^{-\frac{a}{2L}} \dots\dots (3)}$

$\mathbf{J_c\left(\frac{a}{2}\right) = -D_cAB_csinB_cx]_{x=\frac{a}{2}} = AD_cB_csin\frac{B_ca}{2} \dots\dots (4), J_R\left(\frac{a}{2}\right) = \frac{D_RA^{`}}{L}e^{-x/L}]_{x=\frac{a}{2}} = \frac{D_RA^{`}}{L}e^{-a/2L}}$

$$\mathbf{\begin{bmatrix} cosB_c\frac{a}{2} & -e^{-\frac{a}{2L}} \\ D_cB_csin\frac{B_ca}{2} & -\frac{D_R}{L}e^{-\frac{a}{2L}} \end{bmatrix} \begin{bmatrix} A \\ A^{`} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

For non − trivial solutions

$\mathbf{\frac{D_R}{L}e^{-\frac{a}{2L}}cos\frac{B_ca}{2} = B_cD_ce^{-\frac{a}{2L}}sinB_c\frac{a}{2}}$

$\mathbf{\therefore D_cB_ctan\frac{B_ca}{2} = \frac{D_R}{L}, \quad critical\ condition}$

$\mathbf{\emptyset_c(x) = AcosB_cx}$

$\mathbf{\emptyset_R(x) = AcosB_c\frac{a}{2}e^{\frac{a}{2L}}e^{-\frac{x}{L}}}$

b)

1) core region

$\mathbf{\emptyset_c(x) = AcosB_cx}$

2) reflector region

$\mathbf{\frac{d^2}{dx^2}\emptyset_R(x) - \frac{1}{L^2}\emptyset_R(x) = 0}$

$\mathbf{\emptyset_R(x) = A^{`} e^{-\frac{x}{L}} + C^{`} e^{\frac{x}{L}}}$

$\mathbf{= A^{``} sinh\frac{x}{L} + C^{``} cosh\frac{x}{L} = Esinh(\frac{F-x}{L})}$

$\mathbf{B.C, \emptyset_R\left(\frac{a}{2} + b\right) = 0}$

$\mathbf{E\ sinh\left(\frac{F-\frac{a}{2}-b}{L}\right) = 0, ====>} F = \mathbf{\frac{a}{2} + b}$

$\mathbf{\therefore \emptyset_R(x) = Esinh\left(\frac{\frac{a}{2}+b-x}{L}\right)}$

2) Interface condition

$$\emptyset_c\left(\frac{a}{2}\right) = \emptyset_R\left(\frac{a}{2}\right), \qquad A\cos\frac{B_c a}{2} = E\sinh\frac{b}{L}$$
$$J_c\left(\frac{a}{2}\right) = J_R\left(\frac{a}{2}\right), [D_c A B_c \sin B_c x = E\frac{D_R}{L}\cosh(\frac{\frac{a}{2}+b-x}{L})]_{x=\frac{a}{2}}$$
$$AB_c D_c \sin B_c\frac{a}{2} = E\frac{D_c}{L}\cosh\frac{b}{L}$$
$$\begin{bmatrix} \cos\frac{B_c a}{2} & -\sinh\frac{b}{L} \\ B_c D_c \sin\frac{B_c a}{2} & -\frac{D_R}{L}\cosh\frac{b}{L} \end{bmatrix}\begin{bmatrix} A \\ E \end{bmatrix} = 0$$
$$\frac{D_R}{L}\cos\frac{B_c a}{2}\cosh\frac{b}{L} = B_c D_c \sin\frac{B_c a}{2}\sinh\frac{b}{L}$$
$$\therefore B_c D_c \tan\frac{B_c a}{2} = \frac{D_R}{L}\coth\frac{b}{L}, \quad \text{critical condition}$$
$$\left[\emptyset_c(x) = A\cos B_c x, \qquad \emptyset_R(x) = A\cos\frac{B_c a}{2}\frac{\sinh\left(\frac{\frac{a}{2}+b-x}{L}\right)}{\sinh\frac{b}{L}}\right]$$

c)

1) core region

$$\nabla^2\emptyset_c(r) + B_c^2\emptyset_c(r) = 0$$
$$\frac{1}{r^2}\frac{d}{dx}r^2\frac{d\emptyset_c(r)}{dr} + B_c^2\emptyset_c(r) = 0$$
$$\text{Let}, \emptyset_c(r) = \frac{w(r)}{r}$$
$$r^2\emptyset_c'(r) = w'r - w$$
$$=> \frac{1}{r^2}\frac{d}{dx}(r^2\frac{d\emptyset_c(r)}{dr}) = \frac{1}{r}\frac{d^2w}{dr^2}$$
$$\frac{d^2w(r)}{dr^2} + B_c w(r) = 0, \; w(r) = A\cos B_c r + \cosh B_c r$$
$$\therefore \emptyset_r(r) = A\frac{\cos B_c r}{r} + C\frac{\sin B_c r}{r}$$
$$B.C, \lim_{r\to\infty}\emptyset_c(r) = \text{finite}, \quad A = 0$$
$$\emptyset_c(r) = C\frac{\sin B_c r}{r}$$

2) reflector region

$$\frac{1}{r^2}\frac{d}{dx}r^2\frac{d\emptyset_R(r)}{dr} + \frac{1}{L^2}\emptyset_R(r) = 0$$
$$\emptyset_R(r) = A'^{\frac{\sinh\frac{C'-L}{L}}{r}}$$
$$B.C, \emptyset_R(R+b) = A'^{\frac{1}{R+b}}\sinh\frac{C'-R-b}{L} = 0, C' = R+b$$
$$\therefore \emptyset_R(r) = A'^{\frac{1}{r}}\sinh\left(\frac{R+b-x}{L}\right)$$

3) interface condition

$$\emptyset_c(R) = \emptyset_R(R), \; C\frac{\sin B_c R}{R} = A'^{\frac{\sinh\frac{b}{L}}{R}}$$
$$J_{c(R)} = J_R(R)$$
$$-\frac{D_c}{R^2}[-B_c R\cos B_c R + \sin B_c R]C = \frac{D_R}{R^2}\left[\frac{R}{L}\cosh\frac{b}{L} + \sinh\frac{b}{L}\right]A'$$
$$\begin{bmatrix} \frac{\sin B_r R}{R} & -\frac{\sinh\frac{b}{L}}{R} \\ \frac{D_c}{R^2}[-B_c R\cos B_c R + \sin B_c R] & -\frac{D_R}{R^2}\left[\frac{R}{L}\cosh\frac{b}{L} + \sinh\frac{b}{L}\right] \end{bmatrix}\begin{bmatrix} C \\ A' \end{bmatrix} = 0$$
$$\therefore \frac{D_R}{R^3}\sin B_c R\left[\frac{R}{L}\cosh\frac{b}{L} + \sinh\frac{b}{L}\right] = \frac{D_c}{R^3}\sinh\frac{b}{L}[-B_c R\cos B_c R + \sin B_c R]$$
$$\therefore D_c[-B_c R\cot B_c R + 1] = D_R\left[\frac{R}{L}\coth\frac{b}{L} + 1\right], \text{criticality condition}$$
$$\emptyset_c(r) = C\frac{\sin B_c r}{r}$$
$$\emptyset_R(r) = C\sin B_c R\frac{\sinh\left(\frac{R+b-r}{L}\right)}{\sinh\left(\frac{b}{L}\right)r}$$

28. The core of a spherical reactor consists of a homogeneous mixture of 235U and graphite with a fuel-moderator atom ration Nf/Nm = 6.8 * 10-6 The core is surrounded by an infinite graphite reflector. The reactor operates at a thermal power of 100 kW. Calculate the: (a) value of k∞; (b) critical core radius; (c) critical mass; (d) reflector savings; (e)thermal flux throughout the reactor; (f) maximum-to-average flux ratio.

[sol]

$$\frac{N_F}{N_M} = 6.8 * 10^{-6}, P_{th} = 100\ kW$$

$$a)\ Z = \frac{N_F \hat{\sigma}_{aF}}{N_M \hat{\sigma}_{aM}} = 6.8 * 10^{-6} * \frac{0.978*630.8}{0.0034} = 1.332$$

$$f = \frac{Z+1}{Z} = 0.571$$

$$\eta_T = 2.065$$

$$\therefore k_\infty = \eta_T f = 1.179$$

$$b)\ L_{Tc}^2 = (1-f)^2 L_{TM}^2 = (1-0.571) * 3500 = 1.502 * 10^3 cm^{-2}$$

$$B^2 = \frac{k_\infty - 1}{L_{Tc}^2} = \frac{1.179-1}{1.502*10^3} = 1.192 * 10^{-4} cm^{-2}$$

moderator and reflector are the same, so

$$BcotBR = -\frac{1}{L_{Tr}},\ \ cotBR = -\frac{1}{BL_{Tr}} = \frac{-1}{(1.192*10^{-4})^{0.5}*59} = -1.552,\ \ BR = 2.565$$

$$\therefore R = \frac{2.565}{(1.192*10^{-4})^{0.5}} = 2.349 * 10^2 cm$$

$$c)\rho_F = \rho_M \left(\frac{N_F}{N_M}\right)\left(\frac{M_F}{M_M}\right) = 1.6 * 6.8 * 10^{-6} * \frac{235}{12} = 2.131 * 10^{-4}\ g/cm^3$$

$$m_F = v\rho_F = \frac{4}{3}\pi R^3 \rho_F = \frac{4}{3}\pi * (2.349 * 10^2)^3 * 2.131 * 10^{-4} = 11.57\ kg$$

$$d)\ B = \frac{\pi}{R_0},\ \ R_0 = 2.877 * 10^2 cm$$

$$\therefore \text{reflector savings}, \delta = R_0 - R = 52.79cm$$

$$e)\ A = \frac{PB^2}{4\pi*3.2*10^{-14}*3.103*10^{-4}*[g_n(2.565)-2.565cos2.565]} = 3.544 * 10^{13} cm^{-1}$$

$$\text{where}, \bar{\Sigma}_f = \frac{\rho_F N_A}{M_{235}} \hat{\sigma}_F = \frac{2.131*10^{-4}*6.022*10^{23}}{235} * 0.9579 * 582.2 * 10^{-24} = 3.103 * 10^{-4} cm^{-1}$$

$$E_R = 3.2 * 10^{-14} KJ, BR = 2.565$$

$$\therefore \emptyset_c(r) = 3.544 * 10^{13} \frac{sinBR}{r}$$

$$f)\ \emptyset_{max} = \lim_{r\to 0} \emptyset(r) = 3.544 * 10^{13} * B = 3.87 * 10^{11} \frac{neutrons}{cm^2 sec}$$

$$\emptyset_{av} = \frac{1}{v}\int_v \emptyset_c dv = \frac{P}{vE_R \Sigma_f} = \frac{100}{\frac{4}{3}\pi(2.349*10^2)^3*3.2*10^{-14}*3.103*10^{-4}} = 1.855 * 10^{11}\ neutrons/cm^2 sec$$

$$\therefore \Omega = \frac{\emptyset_{max}}{\emptyset_{av}} = \frac{3.87}{1.855} = 2.086$$

29. Estimate the new critical radius and critical mass of a reactor with the same composition as described in problem 6.20 when the core is surrounded by an infinite beryllium reflector.

[Sol]

$\rho_F = m_F/V = 1741/\frac{4\pi}{3}(50)^3 = 3.325 \times 10^{-3} g/cm^3$

Z=4.98

f=Z/Z+1=0.083

$k\infty = \eta_T f = 2.065 \times 0.833 = 1.73$

$L_{TC}^2 = (1-f)\ L_{TM}^2 = (1-0.833)480 = 80.16cm^2$

$B^2 = k\infty - 1/\ L_{TC}^2 = 0.72/80.16 = 8.892 \cdot 10^{-3} \cdot cm^{-2}$

$B = 0.0948cm^{-1}$

Sjince the moderater is the same as reflector,

$BcotBR = -1/\ L_{Tr}$

cotBR=-1/0.0948·21=-0.502

BR=2.23

R=2.23/B=2.23/0.0948=23.5cm

Critical mass

$m_F = \frac{4\pi}{3}(23.5cm)^3 \cdot 3.325 \cdot 10^{-3} g/cm^3$

=181.7g

30. Show that the solution to the one-speed diffusion equation in an infinite cylinder made of a nonmultiplying media as $\emptyset = AKo + CIo$

Where Ko and J1 are modified Bessel functions.

31. An infinite-cylinder reactor core of radius R is surrounded by an infinitely thick reflector. Using one-group theory, (a) find the expressions for the fluxes in the core and reflector; (b) show that the condition for criticality is

DcB x {J1(BR)/Jo(BR)= Dr/Lr x {K1(R/Lr)/Ko(R/Lr)}

Where Jo and J1 are ordinary Bessel functions and Ko and K1 are modified Bessel functions.

[Sol]

a) infinite cylindrical reactor

i) In the core region

$$-D_c\nabla^2\emptyset(r) +\sum ac\ \emptyset(r)-\frac{1}{Keff}\nu\sum f\ \emptyset(r)$$

$$\nabla^2\emptyset(r) +B^2\emptyset(r)=0, \quad \text{where,} \quad B^2=(1/k_{eff}\cdot \nu\sum f^c-\sum a^c)/D_c$$

$$1/r\cdot \frac{d}{dr}r\frac{d}{dr}\emptyset_c(r) + B^2\emptyset(r)=0$$

$$\emptyset_c(r)=AcJo(BR)+CcYo(BR)$$

B.C $\lim_{r-0}\emptyset c(r)(\text{finite}) \quad \therefore\ Cc=0$

$\therefore\ \emptyset c(r)= AcJo(BR)$

ii) for the reflector region

$$-Dc\nabla^2\emptyset(r) +\sum ar\ \emptyset_r(r)=0$$

$$\nabla^2\emptyset(r) -(1/Lr^2)\emptyset_r(r)=0$$

$$1/r\cdot \frac{d}{dr}r\frac{d}{dr}\emptyset_c(r) - (1/Lr)\emptyset_r(r)=0$$

$\therefore\ \emptyset_r(r)=ArIo(r/Lr)+Crko(r/Lr)$

B.C $\lim_{r-0}\emptyset r(r)=0 \quad \rightarrow Ar=0$

$\therefore\ \emptyset_r(r)= CrKo(r/Lr)$

iii) Interface condition

$\Phi c(R)= \Phi r(R)$

$-Dc\ \frac{d}{dr}\emptyset_c(r)\ [r=R]=-Dr\ \frac{d}{dr}\emptyset_r(r)\ [r=R]$

AcJo(BR)+ CrKo(B/Lr) ……………①

$-DcAcBJ_1(BR)$—drcr91/Lr)$K_1(R/Lr)$…………………②

From ①

$Cr=AcJo(BR)/Ko(R/Lr)$

$\therefore\ \Phi c(R)= AcJo(Br)$

$\Phi r(R)= AcJo(Br)/(Ko(R/Lr))\cdot Ko(r/Lr)$

$P=K\sum f\int_1^R \Phi c(r)2\pi rdr$

$= 2\pi k\sum f\ Ac\cdot(R/B)\cdot J_1(BR)$

$Ac=PB/2\pi k\sum f\ R\ J_1(BR)$

$\therefore\ \emptyset r(r)= PB/2\pi k\sum f\ R\ J_1(BR)\cdot Jo(Br)$

$\emptyset r(r)= PB/2\pi k\sum f\ R\ J_1(BR)\cdot Jo(Br)/Ko(R/Lr)\cdot Ko(r/Lr)$

b) From ①, ②

Jo(Br)	Ko(R/Lr)	$\rightarrow 0$
$-DcBJ_1(BR)$	$-Dr(1/Lr)K_1(R/Lr)$	

Critical condition

$DcB\ J_1(BR)/\ Jo(Br)/ = Dr/Lr\cdot K_1(R/Lr) / Ko(R/Lr)$

32. The core of an infinite planar thermal reactor consists of a solution of ^{239}Pu and H_2O with a plutonium concentration of 8.5 g/liter. The core is reflected on both faces by infinitely thick H_2 reflectors. (a) reflector savings, (b) critical thickness of the core, (c) critical mass in g/cm^2.

[Sol]

$Z=N_F\cdot\sigma_{aF} / N_M\cdot\sigma_{aM} =\rho_F M_M g_{aF}(20°C)\ \sigma_{aF}(Eo)/ \rho_M M_F\sigma_{am}(Eo)\ \ =8.5\cdot10^{-3}\cdot18\cdot1.0723\cdot1011.3/1\cdot239\cdot0.664$

$=1.045$

$f=Z/Z+1=0.511$

$L_T^2=(1-f)\ L_{TM}^2=(1-0.511)8.1=3.96cm^2$

$\tau_T=27\ cm^2$

$M_T^2=3.96+27=30.96\ cm^2$

Reflector savings

$\delta=7.2+0.1 \times (M_T^2-40.0)=6.296cm$

b) $\delta=0.5(a_o-a)\quad\rightarrow a= a_o-2\delta$

$B_1^2=(\pi/ a_o)^2=k\infty-1/ M_T^2=$

$a_o=\pi\sqrt{}\ M_T^2/ k\infty-1= \pi\sqrt{}\ M_T^2/ \eta_T f-1=\pi\sqrt{}30.96/(2.035\cdot0.511-1)$

$=87.4cm$

$\therefore\ a=87.4-2\cdot6.296=74.8cm$

c) $m_F=\rho_F\cdot a= (N_F\cdot\sigma_{aM}\ M_F / N_M\ M_M)a$

$= 6.4\cdot10_{-4}\cdot(1\cdot239/18)\cdot748$

$=0.636g/cm^2$

33. The core of a thermal reactor consists of a sphere, 50cm in radius, that contains a homogeneous mixture of ^{235}U and ordinary H_2O. This core is surrounded by infinite thick H_2O reflector. (a) What is the reflector savings ? (b) What is the critical mass? (c) If the maximum thermal flux is 1 x 10^{13} neutrons/cm2-sec, at what power is the reactor operating? [Hint: compute the M_T^2 for core assuming the reactor is bare and of radius 50 cm. Use this value of M_T^2 to estimate δ and then compute new M_T^2 assuming reactor is bare and of radius 50 + δ. Interate until convergence is obtained.]

[Sol]

a)

$\eta_T=2.065$, $L_T^2=8.1cm^2$, $\tau_{TM}=27cm^2$

$B^2=(\frac{\pi}{R})^2=(\frac{\pi}{50})^2=3.948\cdot10^{-3}cm^{-2}$

$Z=1+ B^2(L_T^2+\tau_{TM})/ \eta_T-1- B^2\cdot\tau_{TM}=[1+3.948\cdot10^{-3}(8.1+27)/(2.065-1-3.948\cdot10^{-3}\cdot27)=1.188$

$f=Z/Z+1=5.430\cdot10^{-1}$

$L_T^2=(1-f)\ L_{TM}^2=(1-5.430\cdot10^{-1})8.1=3.702cm^2$

$MT^2= L_T^2 + \tau_{TM} = 3.702+27=30.702\ cm^2$

Reactor savings

$\delta=7.2+0.10(L_T^2\ -40.0)$

$=7.25+0.10((30-40.0)$

$=6.270$

b)

$R=50+ \delta=56.270$

$B^2=(\pi/56.270)^2=3.117 \times 10^{-3}cm$

$=[1+3.117\cdot10^{-3}(8.1+27)/(2.065-1-3.117\cdot10^{-3}\cdot27)=1.131$

$f=Z/Z+1=5.308\cdot10^{-1}$

$L_T^2=(1-f)\ L_{TM}^2=(1-5.308\cdot10^{-1})8.1=3.801$

$MT^2=3.801+27=30.801\ cm^2$

$\rho_F=\rho_M\cdot Z\cdot(\ M_F\sigma_{am}(Eo))/ M_M g_{aF}(20°C)\ g_{aF}(Eo)$

$= 1 \times 1.131 \times 235 \times 0.589/(18 \times 0.978 \times 680.8)=1.306 \times 10^{-2}g/cm^3$

$\eta_T=\nu\cdot \rho_F=4/3\cdot\pi R3\cdot \rho_F=4/3\cdot\pi(50)\cdot\cdot\ \rho_F$

$= 6.839kg$

c)

$\emptyset(r)=A\frac{\sin BR}{r}$, $B=\pi/R=5.583 \times 10^{-2}cm^{-1}$

$\emptyset max=\lim_{r-0} \emptyset(r)=AB$

$A=\emptyset max/B=1\cdot10^{3}/5.583\cdot10^{-2}=1.791\cdot10^{14}$ neutrons/cm·sec

$P=4\pi AE_R\sum_f(\text{SinBR-BRcosBR})/B^2$

$=4\pi\cdot1.79 \times 10^{14} \times 3.2\cdot10^{-11} \times 1.901 \times 10^{-2}(\text{Sin}2.792-2.792\cos2.792)/3.117 \times 10^{-3}$

$=1.303 \times 10^{6}W=1.303MW$

34. A spherical-breeder reactor consists of a core of radius R surrounded by a breeding blanket of thickness b. The infinite multiplication factor for the core k∞ is greater than unity, whereas that for the blanket k∞ is less than unity. Using one-group theory, derive the critical conditions and expressions for the flux throughout the reactor.

[Sol]

At the core region

$-Dc\nabla^2\emptyset c +\sum a\emptyset c=K\infty c\sum a\emptyset c$

$\nabla^2\emptyset c+B^2\emptyset c=0$ where, $K\infty - 1/Lc^2= B^2$

$\emptyset c(r)=\frac{A}{r}\sin B_cr+\frac{A'}{r}\cos B_cr$

$\lim_{r-0} \emptyset c(r)=\text{finite} \rightarrow A'=0$ $\emptyset c(r)=\frac{A}{r}\sin B_cr$

At the blanket

$-Db\nabla^2\emptyset b +\sum ab\emptyset b= K\infty b\sum b\emptyset b$

$\nabla^2\emptyset b+B^2\emptyset b=0$ where $B^2b=-(K\infty b-1/Lb^2)$

$\emptyset_b(r)=\frac{C1}{r}\sin B_br+\frac{C2}{r}\cos B_br$

B.C

$\emptyset_b(R+b)=0=\emptyset_b(r)=\frac{C1}{R+b}\sin B_b(R+b)+\frac{C2}{R+b}\cos B_b(R+b)$

$C_2=-\tanh[B_b(R+b)]$

$\emptyset_b(r)= \frac{C1}{r}\sin B_br - \frac{C1}{r}\tanh(B_b(R+b) \cos B_br$

$=\frac{C1}{r}\text{sech}[B_b(R9b)][\sinh B_br \cosh B_b(R+b)-\cosh B_br \cosh B_br$

$=\frac{C1}{r}\text{sech } B_b(R+b)\cdot\sinh B_b(r-R-b)$

Interface condition

i) $\emptyset_b(R)= \emptyset_c(R)$

$A/R\cdot\sin BcR=C_1/R\cdot\text{sech } B_b(R+b)\sinh(-B_bb)$

$= -C_1/R\cdot\text{sech } B_b(R+b)\sinh(B_bb)$………………. ①

ii) $\emptyset'_c(r)=A/r^2\cdot(B_cr\cdot\cos B_cr-\sin B_cr)$

$\emptyset'_b(r)=C_1/r^2\text{sech } B_b(R+b)[B_cr\cosh B_b(r-R-b)-\sinh B_b(r-R-b)]$

$Dc\emptyset'_c(r)=D_b'(R)$

$D_cA[B_cR\cos B_cr-\sin B_cr]-C_1\text{sech}B_b(R+b)[B_cr\cosh B_b(r-R-b)-\sin B_b(r-R-b)$…………. ②

From the eq. ①,②, the criticality condition is

$$\frac{\sin BcR}{Dc[BcR\cos B_cr-\sin B_cr]} \quad \frac{-\text{sech}B_b(R+b)\sinh B_bb}{\text{sech}B_b(R+b)[B_cr\cosh B_b(r-R-b)-\sin B_b(r-R-b)}$$

∴ $(B_cR\cot B_b b+1)=D_c/D_b\cdot[1-B_cR\cot B_cR]$ critical condition
$C1=-A\cdot\sin B_cR/[\mathrm{sech}B_b(R+b)\cdot\sinh B_b b]$

$\emptyset_c(r)=A/r\cdot\sin B_c r$
$\emptyset_b(r)=-A/r \times \sin B_cR \div [\mathrm{sech}B_b(R+b)\cdot\sinh B_b b] \times \mathrm{sech}B_b(R+b)\sinh B_b(r-R-b)$
$=-A/r \times [\sin B_cR/\sinh B_c b] \times \sinh B_b(r-R-b)$

$P=E_R\sum_f\int_0^R \emptyset c(r)dv= E_R\sum_f\int_0^R A/r\cdot\sin B_c r\cdot 4\pi r^2 dr$
$= 4\, E_R\sum_f AR^2$
∴ $A=p/4\, E_R\sum_f R^2$
∴ $\emptyset_c(r)= p/4\, E_R\sum_f R^2 \times \sin B_c r/r$
$\emptyset_b(r)=-p/4\, E_R\sum_f R^2 \times \sin B_cR/\sinh B_b b \times \sinh B_b(r-R-b)/r$

35. Plate-type fuel elements for an experimental reactor consist of sandwiches of uranium and aluminum. Each sandwich is 7.25 cm wide and 0.16 cm thick. The cladding is aluminum that is 0.050 cm thick. The meat is an alloy of fully enriched uranium and aluminum which has 20 w/o uranium and a density of approximately 3.4 g/cm^3. (a) Estimate the mean free path of thermal neutrons in the meat and cladding in this fuel element. (b) Is a reactor fueled with these elements quasi-homogeneous or heterogeneous?

[sol] meat : 20 w/o uranium + 80 w/o aluminum
ρ of meat : 3.4 g/cm^3
(a) mean free path $\lambda = \frac{1}{\sum t}$
i) in cladding
$N = \frac{\rho N_A}{M} = \frac{2.6999 \times 0.6022045 \times 10^{24}}{26.9815}$ <from Table II.3>
$= 6.02595 \times 10^{22}$ atoms/cm^3
$\sigma_t = \sigma_a + \sigma_s = 0.230 + 1.49 = 1.72$ b $= 1.72 \times 10^{-24}$ cm^2 <from Table II.3>
$\lambda_c = \frac{1}{6.02595 \times 10^{22} \times 1.72 \times 10^{-24}\ \mathrm{cm}^{-1}} = 9.64819 \fallingdotseq 9.65$ cm
ii) in meat
$N(Al) = 0.8 \times \frac{3.4 \times 0.6022045 \times 10^{24}}{26.9815}$ <from Table II.3>
$= 6.07081 \times 10^{22}$ atoms/cm^3
$N(U) = 0.2 \times \frac{3.4 \times 0.6022045 \times 10^{24}}{235.0439}$ <from atom.kaeri.kr>
1.74222×10^{21} atoms/cm^3
$\sigma_t(Al) = 1.72 \times 10^{-24}$ cm^2
$\sigma_t(U) = 698.2 \times 10^{-24}$ cm^2 <from atom.kaeri.kr>
$\lambda_m = \frac{1}{6.07081 \times 10^{22} \times 1.72 \times 10^{-24} + 1.74222 \times 10^{21} \times 698.2 \times 10^{-24}\ \mathrm{cm}^{-1}} = 0.757096 \fallingdotseq 0.76$ cm
(b) $\lambda = \lambda_c + \lambda_m = 10.41$ cm
λ >> thickness of a fuel rod(0.61cm) <from 310 page>
➔ quasi-homogeneous

36. The fuel sandwiches in the reactor in the preceding problem were braised into aluminum holders and placed in a uniform array in an ordinary water moderator. The (total) metal-water volume ratio is 0.73, and there are 120 atoms of aluminum per atom of uranium. Calculate: (a) the thermal utilization; (b) k∞; (c) the thermal diffusion length. [*Note:* In part (c), compute the value of $\overline{D}$ for the homogeneous mixture by cladding together the macroscopic transport cross-sections of the aluminum and water. For Σ_{tr} of water, use $\frac{1}{2}\overline{D}$, where $\overline{D}$ is the experimental value of $\overline{D}$ for water, corrected of course for density.]

[sol] (a) $V_{metal} + V_{water} = V$
$\frac{V_{metal}}{V_{water}} = 0.73$
$V_{water} = 0.578V$, $V_{metal} = 0.422V$
$N_{water} = \frac{\rho N_A \times V_{water}}{M_{water} \times V} = \frac{1 \times N_A \times 0.578}{18} = 1.934 \times 10^{-2} \times 10^{24}$ atoms/cm^3

$N_{metal} = 0.422 \times \frac{N_A}{18} = 1.412 \times 10^{-2} \times 10^{24}$ atoms/cm^3

$N_{Al} + N_U = N_{metal}$

$\frac{N_{Al}}{N_U} = 120$

$N_U = \frac{1}{121} N_{metal} = 1.17 \times 10^{-4} \times 10^{24}$ atoms/cm^3

$N_{Al} = \frac{120}{121} N_{metal} = 1.4 \times 10^{-2} \times 10^{24}$ atoms/cm^3

$f = \Sigma_{aF}/(\Sigma_{ametal} + \Sigma_{awater}) = N_U\sigma_{aU}/(N_U\sigma_{aU} + N_{Al}\sigma_{aAl} + N_{water}\sigma awater)$

$= N_U g_{aU}(T)\sigma_{aU}/(N_U g_{aU}(T)\sigma_{aU} + N_{Al}\sigma_{Al} + N_{water}\sigma_{awater})$

$= 1.17 \times 10^{-4} \times 0.978 \times 681 / (1.17 \times 10^{-4} \times 0.978 \times 681 + 1.4 \times 10^{-2} \times 0.23 + 1.934 \times 10^{-2} \times 0.664)$ <from Table 3.2 and Table II.3>

$= 0.829$

(b) $k\infty = \eta_T \times f$ <Eq. 6.77>

$= 2.065 \times 0.829 = 1.712$ <from Table 6.3>

(c) $D = \lambda_{tr}/3 = 1/3\Sigma_{tr}$ <Eq. 5.10>

$L_T^2 = D/\Sigma_a$ <Eq. 5.62>

$\Sigma_{tr} = \frac{\sqrt{\pi}}{2}(N_{water}\sigma_{trwater} + N_U\sigma_{trU} + N_{Al}\sigma_{trAl})$

$\Sigma_{trwater} = 0.58 \times 1/3D = 0.58 \times 1/(3 \times 0.16) = 2.083 \times 0.58 = 1.208$ cm

$\Sigma_{tr} = \frac{\sqrt{\pi}}{2}(1.17 \times 10^{-4} \times 6.8 + 1.4 \times 10^{-2} \times 3.1) + 1.208$ <from Table 6.1>

$= 1.25$ cm^2

$D = 1/(3 \times 1.25) = 0.27$ cm

$\Sigma_a = \frac{\sqrt{\pi}}{2}(N_{water}\sigma_{awater} + N_U\sigma_{aU} + N_{Al}\sigma_{aAl})$

$= \frac{\sqrt{\pi}}{2}(1.934 \times 10^{-2} \times 0.664 + 1.17 \times 10^{-4} \times 0.978 \times 681 + 1.4 \times 10^{-2} \times 0.23)$

<from Table II.3>

$= 0.0833$ cm^{-1}

$L_T^2 = 0.27/0.08333 = 3.24$ cm^2

$L_T = 1.8$ cm

37. A heterogeneous uranium-water lattice consists of a square array of natural uranium rods 1.50 cm in diameter with a pitch of 2.80 cm. Calculate the: (a) radius of the equivalent cell; (b) uranium-water volume ratio V_F/V_M; (c) thermal utilization; (d) resonance escape probability; (e) fast fission factor (from Fig. 6.10); (f) k∞

[sol] $a = d/2 = 0.75$ cm

(a) $\pi b^2 = 2.80^2$

$b = 1.58$ cm

(b) $\frac{V_F}{V_M} = \frac{\pi(\frac{d}{2})^2}{\pi b^2 - \pi(\frac{d}{2})^2} = 0.291$

(c) $\frac{1}{f} = \frac{\sum aM}{\sum aF}\frac{V_F}{V_M} F + E$

$F(x) = \frac{xI_0(x)}{2I_1(x)} \fallingdotseq 1 + \frac{1}{2}(\frac{x}{2})^2 - \frac{1}{12}(\frac{x}{2})^4 + \frac{1}{48}(\frac{x}{2})^6 - \ldots\ldots$

$E(y, z) = \frac{z^2 - y^2}{2y}\left[\frac{I_0(y)K_1(z) + K_0(y)I_1(z)}{I_1(z)K_1(y) - K_1(z)I_1(y)}\right]$

$L_F = 1.55$ cm, $L_M = 2.85$ cm

$x = a/L_F = 0.75/1.55 = 0.484$

$y = a/L_M = 0.75/2.85 = 0.263$

$z = b/L_M = 1.58/2.85 = 0.554$

$F(x) = 1 + \frac{1}{2}(\frac{0.484}{2})^2 - \frac{1}{12}(\frac{0.484}{2})^4 + \frac{1}{48}(\frac{0.484}{2})^6$

$= 1.029$

$E(y, z) = 1 + \frac{z^2}{2}\left[\frac{z^2}{z^2 - y^2}\ln\frac{z}{y} - \frac{2}{4} + \frac{y^2}{4z^2}\right]$

$= 1.0411$

$\Sigma_{aM} = N_M\frac{\sqrt{\pi}}{2}\sigma_{aM}(E_0)(\frac{T_0}{T})^{1/2}$

$\Sigma_{aF} = N_F\frac{\sqrt{\pi}}{2}g_{aF}(T)\sigma_{aM}(E_0)(\frac{T_1}{T})^{1/2}$

$N_M = \frac{\rho_M N_A}{M_M}\frac{V_M}{V} = \frac{1}{18}N_A(\frac{\pi b^2 - \pi a^2}{\pi b^2})$

$= 2.59 \times 10^{-2} \times 10^{24}$ **atoms/cm^3**

$N_F = \frac{\rho_F N_A}{M_F}\frac{V_F}{V} = \frac{19.1}{238} N_A \frac{\pi a^2}{\pi b^2}$

$= 1.089 \times 10^{-2} \times 10^{24}$ **atoms/cm^3**

$N_{25} = 0.0072 N_F = 7.84 \times 10^{-5} \times 10^{24}$ **atoms/cm^3**

$N_{28} = 0.9928 N_F = 1.081 \times 10^{-2} \times 10^{24}$ **atoms/cm^3**

At 20°C, $g_{a25}(T) = 0.9780$, $g_{a28}(T) = 1.0017$

$\sigma_{aM} = 0.664$ b, $\sigma_{a25} = 681$ b, $\sigma_{a28} = 7.59$ b

$\frac{1}{f} = \frac{N_M \sigma_{aM}(E_0)}{N_{25} g_{a25}(T)\sigma_{a25} + N_{28} g_{a28}(T)\sigma_{a28}} \frac{V_M}{V_F} F + E$

$= 1.496$

$f = 0.669$

(d) $P = \exp[-\frac{N_F V_F I}{g_M \sum SMV_M}]$

$A = 2.8$, $C = 38.3$, $g_M \Sigma_{SM} = 1.46$, $a = 0.75$ cm, $\rho = 19.1$ g/cm3

$I = A + C/\sqrt{a\rho} = 2B + 38.3/\sqrt{0.75 \times 19.1} = 12.92$

$P = 0.9723$

(e) $\varepsilon = 1.2$ <From Fig. 6.10>

(f) $k\infty = \eta_T f P \varepsilon = 2.065 \times 0.669 \times 0.9723 \times 1.2$

$= 1.612$

38. Repeat the calculations of Problem 6.37 for the case in which the uranium is enriched to 2.5 w/o in ^{235}U.

[sol] (a) $b = 1.58$ cm

(b) $\frac{V_F}{V_M} = 3.44$

(c) $x = 0.484$, $y = 0.263$, $z = 0.554$

$F = 1.029$, $E = 1.0411$

$N_M = 2.59 \times 10^{-2} \times 10^{24}$ **atoms/cm^3**

$N_F = 1.089 \times 10^{-2} \times 10^{24}$ **atoms/cm^3**

a/o $= [(2.5/235 + 97.5/238) \times 0.01]^{-0.1}/235 ≒ 2.5$

$N_{25} = 0.025 N_F = 2.7 \times 10^{-4} \times 10^{24}$ **atoms/cm^3**

$N_{28} = 0.995 N_F = 1.058 \times 10^{-2} \times 10^{24}$ **atoms/cm^3**

$\frac{1}{f} = \frac{2.59 \times 10^{-2} \times 0.664}{2.7 \times 10^{-4} \times 0.978 \times 681 + 1.058 \times 10^{-2} \times 1.0017 \times 7.59} 3.44 \times 1.029 + 1.0411$

$= 1.275$

$f = 0.784$

(d) $P = \exp[-\frac{N_F V_F I}{g_M \sum SMV_M}] = 0.9723$

(e) $\varepsilon = 1.2$

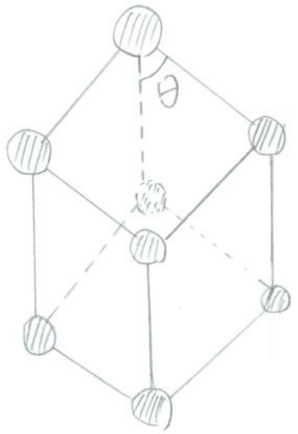
(f) $k\infty = \eta_T f P \varepsilon = 2.065 \times 0.784 \times 0.9723 \times 1.2$

$= 1.36$

39. In a hexagonal lattice (also called a *triangular lattice*), each fuel rod is surrounded by six nearest neighbor rods, equally spaced at a distance equal to the pitch s of the lattice. Show that the radius of the equivalent cell in this lattice is given by

$$b = 0.525 \text{ s.}$$

[sol]



$\theta = 60°$

$\pi b^2 = 2 \times \frac{1}{2} s^2 \sin\theta$

$= s^2 \frac{\sqrt{3}}{2}$

$b \fallingdotseq 0.525\ s$

40. Calculate k∞ for a hexagonal lattice of 1.4 cm radius natural uranium rods and graphite if the lattice pitch is 20 cm. [*Note:* The fast fission factor for this lattice is 1.03.]

[sol] Hexagonal lattice = natural uranium + graphite

a = 1.4 cm, s = 20 cm, $\varepsilon = 1.03$, $\eta_T = 2.065$ <from Table 6.3>

i) $\frac{1}{f} = (\sum_{aM} V_M / \sum_{aF} V_F) F(x) + E(y,z)$

$x = a / L_F = 1.4 / 1.55 = 0.903$

$y = a / L_M = 1.4 / 55 = 0.0255$

$z = b / L_M = 0.5253 / L_M = 10.5 / 55 = 0.191$

$F(x) = x I_0(x) / 2 I_1(x)$

$I_0(0.903) \fallingdotseq 1.22$

$I_1(0.903) \fallingdotseq 0.498$

$F(0.903) = (0.903 \times 1.22) / (2 \times 0.498) = 1.106$

$I_0(0.0255) \fallingdotseq 1.0006$, $I_1(0.0255) \fallingdotseq 0.013$

$I_1(0.191) \fallingdotseq 0.1$, $K_0(0.0255) \fallingdotseq 10$

$K_0(0.0255) \fallingdotseq 40$, $K_1(0.191) \fallingdotseq 4 \sim 9$

$E(y, z) = (z^2 - y^2) / (2y)[(I_0(y) K_1(z) + K_0(y) I_1(z))/(I_1(z)K_1(y) - K_1(z)I_1(y))]$

$E(0.0255, 0.191) = 1.654$

$\sum_{aM} / \sum_{aF} = N_M \sigma_{aM}(E0) / N_F \sigma_{aF}(T) \sigma_{aF}(E0)$

$N_M = \rho_M N_A V_M / M_M V = 3.286 \times 10^{-2} \times 10^{24}$ atoms/cm^3

$N_F = \rho_F N_A V_F / M_F V = 8.592 \times 10^{-4} \times 10^{24}$ atoms/cm^3

$N_{25} = 0.0072 N_F = 6.186 \times 10^{-6} \times 10^{24}$ atoms/cm^3

$N_{28} = 0.9928 N_F = 8.53 \times 10^{-4} \times 10^{24}$ atoms/cm^3

$g_{a25} = 0.978$, $g_{a28} = 1.0017$

$\sigma_{aM}(E_0) = 0.0034$ b, $\sigma_{a25}(E_0) = 681$ b, $\sigma_{a28}(E_0) = 7.59$ b

$\sum_{aM} / \sum_{aF} = N_M \sigma_{aM}(E0) / N_F \sigma_{aF}(T) \sigma_{aF}(E0)$

$= 0.0105$

$V_F / V_M = a^2 / (b^2 - a^2) = 0.0181$

$\frac{1}{f} = (0.0105 / 0.0181) \times 1.106 + 1.054 = 1.696$

$f = 0.59$

ii) $A = 2.8$, $C = 38.3$, $a = 1.4$, $\rho = 19.1$, gm$\sum_{SM} = 0.0608$

$$I = A + C / \sqrt{a\rho} = 2.8 + 38.3 / \sqrt{1.4 \times 19.1)} = 10.207$$

$$P = \exp[-\frac{N_F V_F I}{g_M \sum SMV_M}] = 0.997$$

$$k\infty = \eta_T f P \varepsilon = 2.065 \times 0.59 \times 0.997 \times 1.03$$
$$= 1.25$$

Chapter 7

The Time-Dependent Reactor

1. Compute the prompt neutron lifetime for an infinite critical thermal reactor consisting of a homogeneous mixture of ^{235}U and (a)D_2O, (b) Be, (c) graphite.

[sol] Assume room temperature T=20°C

Table 6.3

$\eta_T(^{235}U, 20°C) = 2.065$

Table 7.1

$D_2O : 4.3 \times 10^{-2}sec$, $Be : 3.9 \times 10^{-3}sec$, $Graphite : 0.017sec$

(a) D_2O

$t_d = t_{dM}(1-f) = (4.3 \times 10^{-2})(1-0.484)sec = 0.022sec$

(b) Be

$t_d = t_{dM}(1-f) = (3.9 \times 10^{-3})(1-0.484)sec = 0.0020sec$

(c) Graphite

$t_d = t_{dM}(1-f) = (0.017)(1-0.484)sec = 0.0088sec$

2. Calculate the thermal neutron diffusion time in water having a density of $44lb/ft^3$.

[sol] Table II.3

$M_w = 18.0153$, $\sigma_a = 0.664 \times 10^{-24}cm^2$

$$N=\frac{\rho N_A}{M} = \frac{(0.705g/cm^3)(0.6022\times10^{24})}{18.0153} = 2.357 \times 10^{22}$$

$\Sigma_a(E_0) = N_w\sigma_a$, $v_0 = 2200m/s$

$$t_d = \frac{1}{\Sigma_a(E_0)v_0} = \frac{1}{(2.357\times10^{22})(0.664\times10^{-24})(220000cm/s)} = 0.00029sec$$

3. Express the following reactivities of a ^{239}Pu-fueled thermal reactor in dollars (a)0.001, (b) 4%, (c) -0.01

[sol] Table 7.2

$^{239}Pu \rightarrow \beta(\text{Thermal neutron fission}) = 0.0021$

(a) $\rho = \frac{0.001}{0.0021} = 0.476dollars$

(b) $\rho = \frac{0.04}{0.0021} = 19.048dollars$

(c) $\rho = \frac{-0.01}{0.0021} = -4.762dollars$

4. Express the following reactivities of a ^{235}U-fueled thermal reactor in percent: (a) 0.001, (b) \$2, (c) -50cents

[sol] Table 7.2

$\beta = 0.0065$

(a) 0.001=0.1%

(b) $dollars = \frac{\rho}{\beta}$, $\therefore \rho = (\$2)(0.0065) = 0.013 = 1.3\%$

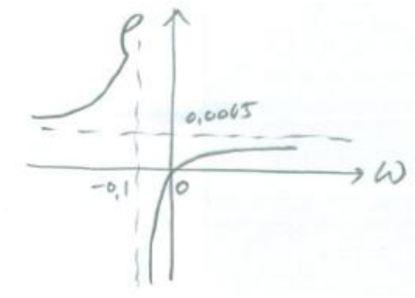
(c) -50cents=-0.5dollars=$\frac{\rho}{\beta}$, $\therefore \rho = (-0.5)(0.0065) = -0.00325 = -0.325\%$

5. Plot the reactivity equation for one-delayed-neutron group for a ^{235}U-fueled thermal reactor and prompt neutron lifetimes of (a) 0sec, (b) 10^{-4}sec, (c) 10^{-3}sec. Take $\beta = 0.0065$ and $\lambda = 0.1sec^{-1}$.

[sol] $\rho = \frac{wl_p}{1+wl_p} + \frac{w}{1+wl_p}\frac{\beta}{w+\lambda} = \frac{wl_p}{1+wl_p} + \frac{w}{1+wl_p}\frac{0.0065}{w+0.1}$

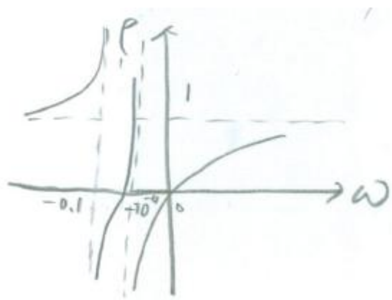
(a) $l_p = 0sec$

$$\rho = \frac{w}{1}\frac{0.0065}{w+0.1} = 0.0065 - \frac{0.0065}{w+0.1}$$



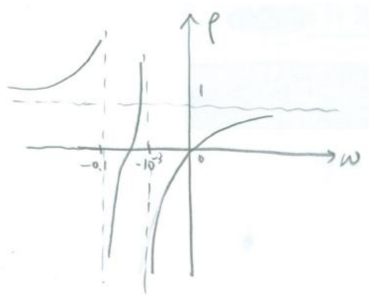
(b) $\mathbf{l_p = 10^{-4}sec}$

$$\boldsymbol{\rho = \frac{10^{-4}w}{1+10^{-4}w} + \frac{w}{1+10^{-4}w}\frac{0.0065}{w+0.1} = \frac{1}{1+10^{-4}w}\left[-1 + \frac{0.0065w}{w+0.1}\right] + 1}$$



(c) $\mathbf{l_p = 10^{-3}sec}$

$$\boldsymbol{\rho = \frac{10^{-3}w}{1+10^{-3}w} + \frac{w}{1+10^{-3}w}\frac{0.0065}{w+0.1} = \frac{1}{1+10^{-3}w}\left[-1 + \frac{0.0065w}{w+0.1}\right] + 1}$$



6. From the plot in Problem 7.5, determine the periods of ^{235}U-fueled thermal reactors with $l_P = 10^{-3}$sec and reactivities of (a) +0.1%, (b) -10cents, (c) +\$.50, (d)+\$1.00.

[sol] $\boldsymbol{\beta = 0.0065}$

$$\boldsymbol{l_p = 10^{-3}\ sec \ \rightarrow\ \rho = \frac{w}{1+10^{-3}w}\left[10^{-3} + \frac{0.0065}{w+0.1}\right]}$$

(a) $\boldsymbol{\rho = 0.1\% = 0.001}$

$$\boldsymbol{0.001 = \frac{w}{1+10^{-3}w}\left[10^{-3} + \frac{0.0065}{w+0.1}\right] \ \rightarrow\ w = 0.0178\ ,\ \ T = \frac{1}{w} = \frac{1}{0.0178} = 56.18s}$$

(b) $\boldsymbol{\rho = -10cents = -0.1dollars = \frac{x}{0.0065} \ \rightarrow\ x = -0.00065}$

$$\boldsymbol{-0.00065 = \frac{w}{1+10^{-3}w}\left[10^{-3} + \frac{0.0065}{w+0.1}\right] \ \rightarrow\ w = -0.00898\ ,\ \ T = \frac{1}{w} = \frac{1}{-0.00898} = -111.36s}$$

(c) $\rho = \$0.50 = 0.5\text{dollars} = \frac{x}{0.0065} \quad \rightarrow \quad x = 0.00325$

$$0.00325 = \frac{w}{1+10^{-3}w}\left[10^{-3} + \frac{0.0065}{w+0.1}\right] \quad \rightarrow \quad w = 0.0944,\ T = \frac{1}{w} = \frac{1}{0.0944} = 10.593s$$

(d) $\rho = \$1.00 = 1\text{dollars} = \frac{x}{0.0065} \quad \rightarrow \quad x = 0.0065$

$$0.0065 = \frac{w}{1+10^{-3}w}\left[10^{-3} + \frac{0.0065}{w+0.1}\right] \quad \rightarrow \quad w = 0.7604,\ T = \frac{1}{w} = \frac{1}{0.7604} = 1.315s$$

7. Compare the answers to Problem 7.6, which are based on the one-delayed-group model, with periods determined from Fig.7.2.

[sol] (a) +0.1% , $l_p = 0.001s$

$\rho = 0.001, \qquad \text{Fig. 7.2} \rightarrow T = \frac{1}{w} \approx 60s$

(b) -10cents

$\rho = -0.00065 \quad$ **Fig.7.2** $\rightarrow T = \frac{1}{w} \approx -110s$ (**negative activity**)

(c)$0.50

$\rho = 0.00325s \quad \text{Fig. 7.2} \rightarrow T = \frac{1}{w} \approx 10s$

(d) $1.00

$\rho = 0.0065 \quad \text{Fig. 7.2} \rightarrow T = \frac{1}{w} \approx 1.3s$

8. A ^{235}U-fueled reactor originally operating at a constant power of 1 milliwatt is placed on a positive 10-minute period. At what time wil the reactor power level reach 1 megawatt?

[sol] T=10minutes=10× 60s=600s

$$P = P_0 e^{t/T},\ e^{t/T} = \frac{P}{P_0}\ ,\ \frac{t}{T} = \ln\left(\frac{P}{P_0}\right)$$

$$\therefore t = T\ln\left(\frac{P}{P_0}\right) = 600\ln\left(\frac{1\times10^{6}}{1\times10^{-3}}\right)s = 12434s$$

9. Calculate the ratio of the concentration of the 22.7-sec precursor to the thermal neutron density in a critical infinite thermal reactor consisting of a mixture of 235U and ordinary water, given that the thermal flux is 1013 neutrins/cm2-sec.[Hint: place dC/dt =0 in eq. 7.21]

[Sol]

dc/dt=[βK∞∑a$\emptyset_T$/P]-λC , C= βK∞∑a$\emptyset_T$/Pλ

β=0.0065, K∞=η_Tf=1, $\emptyset_T$=[α/√π]·η·v_T ,$v_{T=}$2200m/sec

f=∑ aF/∑ a=∑ aF/∑ aF+∑ aM → ∑ aF=[f/1-f]· ∑ aM

f=1/η_T=1/2.065=0.484

∑ aF=[f/1-f]· ∑ aM==[0.484/1-0.484]·0.0197=1.85 x 10^{-2}cm^{-1}

∑a=1.85 x 10^{-2}+0.0197=3.82 x 10^{-2} cm^{-1}

Λ=ln2/22.7=3.054 x 10^{-2}sec^{-1} P=1

C=(0.0065 x 1 x 3.82 x 10^{-2} x (α/√π) η x 2200 x 10^{2}/(1 x 0.054 x 10^{-2})

∴ C/ η=2.018 x 10^{3}

10. The first term on the right hand side of Eq.(7.19) gives the number of prompt neutrons slowing down per cm3/sec while the second term gives this number for the delayed neutrons. Compare the magnitude of these two terms in a critical reactor.

[Sol]

First term $(1-\beta) K\infty \sum a \emptyset_T$

Second term $P\lambda C$

From eq. 7.20

$dC/dt= [\beta K\infty \sum a\emptyset_T/P]- \lambda C$

$\lambda C= \beta K\infty \sum a\emptyset_T/P$

$(1-\beta) K\infty \sum a\emptyset_T - P\lambda C=(r\beta) K\infty \sum a\emptyset_T - \beta K\infty \sum a\emptyset_T$

$=(1-2\beta) \beta K\infty \sum a\emptyset_T > 0 \qquad \therefore$ **first term** $>$ **Second term**

11. Fifty cents in reactivity is suddenly introduced into a critical fast reactor fueled with 235U. What is the period of the reactor?

[Sol]

$\rho=50\text{cents}=0.5 \times 0.0064=3.2 \times 10^{-3}$

$\therefore \quad T=0.0848/3.2 \times 10^{-3}\text{sec}$

12. The reactor in problem 7.8 is scrammed by the instantaneous insertion of 5 dollars of negative reactivity after having reached a constant power level of 1 megawatt. Approximately how long does it take the power level to drop to 1 milliwatt?

[Sol]

$Po=1 \times 10^6\text{watt}, P=1 \times 10^{-3}\text{watt}$

$\rho=-5\$=-0.0325$

Prompt drop $P_J=Po\cdot\beta(1-\rho)/(\beta-\rho)=[1 \times 10^6 \times \beta(1+5 \times 0.0065)]/(\beta+5\beta)=0.1721$ **Mwatt**

$P= P_J e^{-t/80}$

$1 \times 10^{-3}=0.1721 \times 10^6 e^{-t/80}$

$T= -80\ln(1 \times 10^{-3})/(0.1721 \times 10^6)=1517\text{sec}$

13. When a certain research reactor operating at a constant power of 2.7 megawatts is scrammed, it is observed that the power drops to a level of 1 watt in 15 minutes. How much reactivity was inserted when the reactor was scarmmed?

[Sol]

$P_2=P_1 e^{-t/T}$

$P= P_2 e^{t/T}=1 \times \exp(15 \times 60/80) =7.688 \times 10^4\text{Watt}$

$P_1= P_o \cdot\beta(1-\rho)/(\beta-\rho)$

$P_1(\beta-\rho)= \beta(1-\rho) P_o$

$\beta (P_1- P_o)= \rho(P_1- \beta P_o), \qquad \rho= \beta (P_1- P_o)/ P_1- \beta P_o=0.0065(7.688 \times 10^4-2.7 \times 10^6)$

$=-0.2874$

14. An infinite reactor consists of a homogeneous mixture of 235U and H2O. The fuel concentration is 5% smaller than that required for criticality. What is the reactivity of the system?

[Sol]

From the problem(7-9)

$\sum_{aF}=0.0185\text{cm}^{-2}$

Fuel concentration is 5% smaller than criticality

$\sum_{aF}=0.0185 \times 0.95=0.0176\text{cm}^{-2}$

∴ $f=\sum_{aF}/\sum_{aF}+\sum_{aF}=0.0176/(0.0176+\frac{\pi}{\alpha}0.0222)=0.472$

$K\infty-\eta_f f=2.065 \times 0.472=0.975$

∴ $\rho= K\infty - 1/ K\infty=0.975-1/0.975=-0.0256=-2.56$percent.

15. One cent worth of either positive or negative reactivity places a235U-fueled reactor on what period?

[Sol]

ρ=1cents=0.01 x 0.0065=6.5 x 10^{-5}

$T=1/\rho \times \sum_i \beta_i t_i=0.0848/6.5 \times 10^{-5}$sec=1304.6sec

16. It is desired to double the power level of a 235U-fueled reactor in 20 minutes. If lp = 10^{-4} sec, (a) on what stable period should the reactor be placed? (b) How much reactivity should be introduced?

[Sol]

$2P_0=P_0e^{20/T}$, $l_p=10^{-4}$sec

a) 20/T=ln2

T=20/ln2=28.85 min=1731sec

b)Using Fig.7.2

l_p=0.0001sec

T=1731sec

ρ→cannot be found very small reactivity

Using Eq7.40

$T=\frac{1}{\rho}(l_p+\sum_i\beta_i t_i)$

∴ $\rho=\frac{1}{T}(l_p+\sum_i\beta_i t_i)= \frac{1}{1731}(10^4+0.0848)=4.905 \times 10^{-5}=0.007546\beta$

∴ 0.7546cents

17. During test-out procedures, a 239Pu-fueled thermal reactor is operated for a time at a power of 1 megawatt. The power is then to be increased to 1 00 megawatts in 8 hours. (a) On what stable period should the reactor be placed? (b) What reactivity insertion is required?

[sol]

(a)

$$\mathbf{P = P_0 e^{x/t} \quad l_p = 10^{-4}}$$

$$\mathbf{T = t/\ln(P/P_0) = \frac{8}{\ln\left(\frac{100}{1}\right)} = 1.737Pr}$$

(b)

$$\mathbf{T=\frac{l_p}{(1-\beta)k_\infty-1} \quad => \ k_\infty = \frac{1}{1-\beta}[1+\frac{l}{t}]}$$

$$\mathbf{l_p = 10^{-4} \quad k_\infty = \frac{1}{1-\beta} = \frac{1}{1-0.0021}=1.0021}$$

$$\mathbf{\therefore \ \rho = \frac{k_\infty-1}{k_\infty} = 2.096x10^{-3}}$$

19. An experimental reactor facility is a bare square cylinder 100 cm high, composed of small beryllium blocks with thin foils of 235U placed in between, so that the system can be considered to be a homogeneous mixture of Be and 235U. The reactor is to be controlled with a single black control rod 2.5 cm in radius and located along the axis of the system. (a) If the reactor is critical with the rod fully withdrawn, how much negative reactivity is introduced into the system when the rod is fully inserted? (b) Assuming that the rod moves into the reactor instantaneously, on what period does the reactor go?

[sol]

(a)

$B_0^2 = (\frac{2.405}{R})^2 + \left(\frac{F}{H}\right)^2 = \left(\frac{2.405}{50}\right)^2 + \left(\frac{\pi}{H}\right)^2 = 3.3x10^{-3}cm^{-2}$

f = $\frac{1+B_0^2(L_{Tm}^2+\gamma_{TH})}{\eta_T+B_0^2L_{Tm}^2} = \frac{1+3.3x10^{-3}x(480+102)=0.8}{2.065+3.3x10^{-3}x480}$

$M_T{}^2 = L_T^2 + \gamma_T = (1-f)L_{TM}^2 + \gamma_T = (1-0.8)x480 + 102 = 198cm^2$

d = 2.131x0.5x$\frac{2.5x0.76+0.9354}{2.5x0.76+0.5098} = 2.413cm$

where $\sum_t = \sum_a + \sum_s = 0.00137 + 0.7589 = 0.16cm^{-1}$

$\therefore \rho_0 = \frac{7.13M_T{}^2}{(1+B_0^2M_T{}^2)R^2}[0.116 + \ln(\frac{R}{2.4050}) + \frac{2.413}{2.5}]^{-1}$**=0.1112**

(b)

$k_\infty = \frac{1}{1-\rho} = \frac{1}{1-0.1112} = 1.125$

$\therefore$ **T =** $\frac{l_p}{(1-\beta)k_\infty-1} = \frac{3.9x10^{-3}}{1-0.0065)x1.123-1}$ **= 3.3 x** 10^{-2}**sec**

20. Suppose it is desired to control the reactor described in Example 7.7 with one central rod having a worth of 1 0%. How big should the rod be? [Hint: Plot Pw versus a.]

[sol]

$\rho_\infty = \frac{7.43M_T{}^2}{(1+B_0^2M_T{}^2)R^2}[0.116 + \ln\left(\frac{R}{2.405a}\right) + \frac{d}{a}]^{-1} = \frac{7.43x30.4}{(1+6.74x10^{-3}x30.4)x(35)^2}[0.116 + \ln\left(\frac{35}{2.405a}\right) + \frac{d}{a}]^{-1}$

= 0.163[0.116+ln($\frac{14.553}{a}$**)** $+ \frac{d}{a}]^{-1}$ **= 0.1**

ln($\frac{14.553}{a}$**)** $+ \frac{d}{a}$ **= 1.414**

d= 2.131x0.16x$\frac{ax3.443+09354}{ax3.443+0.5098} = \frac{1.174a+0.319}{1.174a+0.174}$

ln($\frac{14.553}{a}$**)** $+ \frac{1.174a+0.319}{1.174a^2+0.174}$**=1.414**

$\frac{a+0.272}{a^2+0.148a} - \ln a + 1.2638 = 0$

$\therefore$ **a = 4.3 cm**

21 . If the reactor i n Problem 7. 1 9 i s controlled b y an array o f 2 5 more or less uniformly distributed black rods 0.3 in. in diameter, what is the total worth of the rods?

[sol]

a = 0.381cm

$\rho_\infty = \frac{f_R}{1-f_R}, \ \frac{1}{f_R} = \frac{(z^2-y^2)d}{2a} + E(y.z)$

25 x $\pi R_c^2 = \pi(50)^2$ **=>** R_c**=10cm**

$L_T^2 = (1-f)L_{TM}^2 = 96cm^2 \ \ L_T = 9.8cm$

d = 2.413cm

y = 0.381/98 = 0.0389

z = 10/98 = 1.03

E(y,z) = $\frac{Z^2-y^2}{2y}\left[\frac{I_0(y)K_1(z)+K_0(y)I_1(z)}{I_1(z)K_0(y)-K_1(z)I_0(y)}\right]$

$\mathbf{I_0(0.0389) = 1.0008 \quad I_1(0.0389) = 0.02, I_1(1.02) = 1.29}$,

$\mathbf{k_0(0.0389) = 10, K * 0.0389) = 40, K(.102) = 0.6}$

$\therefore \mathbf{E(y,z) = 3.495}$ **=>** $\frac{1}{f_R}$ **= 6.785 =>** $\mathbf{f_R}$**= 0.147**

$\therefore\ \rho_\infty = \frac{0.147}{1-0.147} = \mathbf{0.1723 = 17.23}$ %

22. A certain pressurized-water reactor is to be controlled by 61 cluster control assemblies, each assembly containing 20 black rods 1 . 1 5 cm in diameter. The reactor core is a cylinder 320 cm in diameter. The average thermal diffusion length in the core is 1 .38cm, D = 0.2 1 cm, and Lt in the core material is approximately 2.6 cm- I Calculate the total worth of the rods.

[sol]

$\mathbf{\pi R_c^2 = \pi\left(\frac{320}{2}\right)^2 \div (61x20) => R_c}$ **= 4.581 cm**

a = $\frac{1.15}{2} = \mathbf{0.575cm}$, $\mathbf{L_T = 1.38cm}$, $\mathbf{D = 0.21\ cm}$, $\mathbf{\sum_t = 2.6\ cm^{-1}}$, $\mathbf{d = 0.5425}$

y = a/$\mathbf{L_T}$ **= 0.417, Z =** $\mathbf{R_c/L_T}$ **= 3.320**
E(y,z) = 2.625

=> $\frac{1}{f_R} = \frac{(z^2-y^2)d}{2a} +$ **E =>** $\frac{(3.32^2 - 0.417^2)}{2 \times 0.575}$ **x 0.5425 + 2.625 = 7.742** $\mathbf{f_R}$**= 0.1292**

$\therefore\ \rho_\infty = \frac{0.1292}{1-0.1292} = \mathbf{0.1484 = 14.84}$ %

23. Consider the problem of detennining the worth of a cruciform control rod in the onedimensional geometry shown in Fig. 7. 1 0. First suppose that qT fission neutrons slow down to thermal energies per cm3/sec. The quantity qT can be treated as a source term in the thermal diffusion equation,
(a) Show that the solution to this equation, subject to the boundary conditions,
(b) Compute JR by multiplying the neutron current density at the blade surface by the area of the blades in the cell and then dividing by the total number of neutrons thennalizing per second in the cell

[sol]
(a)
$$\mathbf{D\frac{d^2\emptyset_T}{dx^2} - \sum_a \emptyset_T = -q_T}$$

$$\mathbf{\emptyset(x) = C_1 sinh\frac{x}{L_T} + C_2 cosh\frac{x}{L_T} + \frac{{L_T}^2 q_T}{D}}$$

$$\mathbf{\left.\frac{d\emptyset_T}{dx}\right|_{x=0} = 0\ ,\ \left.\frac{d\emptyset_T}{dx}\right|_{x=0} = \frac{1}{L_T}[C_1 cosh\frac{x}{L_T} + C_2 sin\frac{x}{L_T}]_{x=0} = \frac{C_1}{L_T} = 0}$$

$$\mathbf{\therefore\ \emptyset_T = C_2 cosh\frac{x}{L_T} + \frac{{L_T}^2 q_T}{D}}$$

$$\mathbf{\frac{1}{\emptyset_T}\left.\frac{d\emptyset_T}{dx}\right|_{x=\frac{m}{2}-a} = -\frac{1}{d}}$$

$$=> C_2 = \frac{-L_T{}^2 q_T}{\frac{d}{L_T \sinh(\frac{m-2a}{2L_T}) + \cosh(\frac{m-2a}{2L_T})}}$$

$$\therefore \ \emptyset(x) = \frac{L_T{}^2 q_T}{D}\left[1 - \frac{\cos\frac{h}{L_T}}{d/L_T \sinh(\frac{m-2a}{2L_T}) + \cosh(\frac{m-2a}{2L_T})}\right] = \frac{q_T}{\Sigma a}\left[1 - \frac{\cos\frac{h}{L_T}}{d/L_T \sinh(\frac{m-2a}{2L_T}) + \cosh(\frac{m-2a}{2L_T})}\right]$$

(b)

$$J(m/2 - a) = -D \left.\frac{d\emptyset_T}{dx}\right|_{x=\frac{m}{2}-a} = \frac{1}{L_T}\frac{D}{\Sigma a}\frac{q_T}{d/L_T + \cosh(\frac{m-2a}{2L_T})} = g_T L_T \frac{1}{1/L_T \sinh(\frac{m-2a}{2L_T}) + \cosh(\frac{m-2a}{2L_T})}$$

$$f_R = \frac{J\left(\frac{m}{2} - a\right) x4(f-a)}{(m-2a)} = \frac{4(l-a)L_T}{(m/2 - a)^2}\frac{1}{\frac{d}{L} + \cosh(m - 2a\ 2t)}$$

24. It is proposed to use the crucifonn rods described in Example 7.9 to control the PWR described in Problem 7.22. How far apart should these rods be placed to provide the same total worth as the cluster control rods in that problem?

[sol]

R = 320/2 = 160 cm, L_T **= 1.38cm D = 0.21cm ,** $\sum_t = 2.6\ cm^{-1}$ **,** $d = 2.131$**, D = 0.4475** ρ_w**= 0.1723**

$f_R = \frac{\rho_w}{1+\rho_w} = \frac{0.1723}{1.1723}$ **= 0.147,** **a = 0.312 x 2.54 ÷ 2 = 0.396 cm ,** **l = 9.75 x 2.54/2 = 12.38cm**

$$f_R = \frac{4(l-a)L_T}{(m-2a)^2} = \frac{1}{\frac{d}{L_T} + \coth(\frac{m-2a}{25})} = \frac{4(12.38-0.396) x\ 1.38}{(m-2x0.396)}\frac{1}{\frac{0..4475}{1.38} + \coth[\frac{m-2x0.396}{2x1.38}]} = 0.147$$

$$0.147 = \frac{66.15}{(m-0.792)} \ x \ \frac{1}{0.324 + \coth(\frac{m-0.792}{276})} = (m - 0.792)^2(0.324 + \coth\frac{m-0.792}{2.76}) = 450$$

=> m = 192cm

25. Suppose the fast reactor described in Example 6.3 is controlled with 50 rods, each rod containing approximately 500 g of natural boron. Estimate the total worth of the rods.

[sol]

fast reactor, R = 48.5 cm

controlled with 50 rods, each rod contain 500g of natural boron

$$\rho_w = \frac{\Sigma_{aB}}{\Sigma_{aF} + \Sigma_{aC}}$$

$$\sum_{aF} = 0.00395 * 2.11 = 8.33 * 10^{-3} cm^{-1}$$

$$\sum_{aC} = 0.0234 * 0.0008 = 1.9 * 10^{-5} cm^{-1}$$

total mass of boron, $500 * 50 = 25000\ g$

total number of boron in reactor, $\frac{25000}{10.811} * N_A = 1.39 * 10^{27}$ **atoms**

$$\therefore N_B = \frac{1.39*10^{27}}{\frac{4}{3}\pi(48.5)^2} = 2.91 * 10^{-3} * 24\ \# \frac{}{cm^3}$$

$$\sigma_{aB} \approx 0.276$$

$$\therefore \rho_w = 9.41\%$$

26. The core of a fast reactor is a square cylinder 77.5 cm in diameter. The composition of the core by volume is as follows; 25% fuel; 25% stainless steel cladding and structural material; 50% liquid sodium. The fuel consists of a mixture of 238U and 239Pu having a density of 193.1 g/cm3, the plutonium making up 20 w/o of mixture. It is desired to provide 5.5% reactivity control in 10 control rods containing B4C. How much B4C is required per rod?

[sol]

$$N(^{238}u) = \frac{0.8*19.1*0.6022*10^{24}}{238} * 0.25 = 9.665 * 10^{21}\ \#/cm^3$$

$$N(^{239}Pu) = \frac{0.2*19.1*0.6022*10^{24}}{239} * 0.25 = 2.406 * 10^{21}\ \# \frac{}{cm^3}$$

$$N_{Fe} = 0.08487 * 10^{24} * 0.25 = 0.02122 * 10^{24}\ \#\frac{}{cm^3}$$
$$N_{Na} = 0.02549 * 10^{24} * 0.5 = 0.0127 * 10^{24}\ \#\frac{}{cm^3}$$
$$\Sigma_{aF} = \Sigma_a(238) + \Sigma_a(239) = 9.665 * 10^{-3} * 0.2556 + 2.405 * 10^{-3} * 2.11 = 7.54 * 10^{-3}cm^{-1}$$
$$\Sigma_{aC} = \Sigma_a(Fe) + \Sigma_a(Na) = 0.02122 * 0.006 + 0.0127 * 0.0008 = 1.375 * 10^{-4}cm^{-1}$$
$$\therefore \rho_w = \frac{\Sigma_{aB}}{\Sigma_{aF} \quad + \Sigma_{aC}} = \frac{N_B * 0.27 * 10}{7.54 * 10^{-3} + 1.375 * 10^{-4}} = 351.4N_B = 0.055$$
$$\therefore N_B = 1.564 * 10^{-4}\ \#\frac{}{cm^{-3}}$$
$$\text{molecular weight of B} = 1.564 * 10^{-4} * \rho(B) * 22400 = 8.0577\ g$$
$$\text{weight of } B_4C, \frac{M(B_4C)}{M(B_4)} * 8.0577 = 10.296\ g$$

27. A control rod 100cm long is worth 50 cents when totally inserted. (a) How much reactivity is introduced into the reactor when the rod is pulled one-quarter of the way out? (b) At what rate is reactivity introduced at this point per cm motion of the rod?

[sol]

$$\rho_w(H) = 50\ cm$$
$$a)\ \rho_w(x) = \rho_w(H)\left[\frac{x}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi x}{H}\right)\right]$$
$$\rho_w\left(\frac{3}{4}H\right) = 0.5\left[\frac{75}{100} - \frac{1}{2\pi}\sin\left(\frac{2\pi * 75}{100}\right)\right] = 0.455\$, \quad 45.5 cents$$
$$b)\ \frac{1}{\rho_w(H)}\frac{d\rho_w(x)}{dx} = \frac{1}{H}\left[1 - \cos\left(\frac{2\pi x}{H}\right)\right]$$
$$\frac{d}{dx}\rho_w\left(\frac{3}{4}H\right) = \frac{\rho_w(H)}{H}\left[1 - \cos\left(\frac{2\pi\frac{3}{4}H}{H}\right)\right] = 0.5\ cents/c$$

28. Suppose that, at some time during its operating history, the reactor described in Example 7.7 is critical with the rod withdrawn one-half of its full length. If the rod is now suddenly withdrawn another 10 cm. (a) hoe much reactivity is introduced? (b) on what period does the reactor power rise?

[sol]

$$a)\ \rho_w\left(\frac{H}{2}\right) = 0.065, \qquad \rho_w(H) = 2\rho_w\left(\frac{H}{2}\right) = 0.13$$
$$\therefore \rho_w(45) = \rho_w(H)\left[\frac{45}{70} - \frac{1}{2\pi}\sin\left(\frac{2\pi * 45}{70}\right)\right] = 0.0997, \quad 9.97\%$$
$$T = \frac{1}{\rho}\sum_i \beta_i t_i = \frac{0.0848}{0.0997} = 0.85 sec$$

29. What concentration of boron in ppm will give the same worth as the single control rod in Example 7.7?

[sol]

$$\rho_w = 0.065, \quad f_0 = 0.583$$
$$\rho_w^{A_2BO_3} = 1.905C(1 - f_0) * 10^{-3}$$
$$\therefore C = \frac{0.065}{1.905(1 - 0.583) * 10^{-3}} = 81.82\ ppm$$

30. An infinite 235U-fueled, water-moderated reactor contains 20% more 235U than required to become critical. What concentration of (a)boron in ppm or (b) boric acid in g/liter is required to hold down the excess reactivity of the system?

[sol]

$$\text{At steady} - \text{state},\ k_\infty = \eta_T f = 1, \quad f = \frac{1}{2.065} = 0.484$$
$$f = \frac{\Sigma_{aF}}{\Sigma_{aF} \quad + \Sigma_{aM}} = 0.484$$
$$\Sigma_{aM} = \frac{\sqrt{\pi}}{2} * 0.0222 = 0.019674$$
$$\therefore \Sigma_{aF} = \frac{f}{1-f}\Sigma_{aM} = \frac{0.484}{1 - 0.484} * 0.019674 = 0.018454$$
$$\text{At this state}, 20\% \text{ of more 235U than critical condition, so}$$
$$f = \frac{\Sigma_{aF}}{\Sigma_{aF} \quad + \Sigma_{aM}} = \frac{0.018454 * (1 + 0.2)}{0.018454 * (1 + 0.2) + 0.019674} = 0.52954$$

$$\mathbf{k_\infty = \eta_T f = 2.065 * 0.52954 = 1.0935}$$
$$\boldsymbol{\rho_w = \frac{k_\infty - 1}{k_\infty} = 0.0855}$$
$$\therefore \mathbf{C} = \frac{\rho_w}{1.92(1-f)*10^{-3}} = \frac{0.085}{1.92(1-0.52954)*10^{-3}} = \mathbf{94.655\ ppm}$$
$$\mathbf{b)}\ \frac{M(H_3BO_3)}{M(B)} * \mathbf{94.665} = \frac{1*3+10.8+16*3}{10.8} = \mathbf{94.655 = 0.542\ g/l}$$

31. A reactor is operating at constant power. Suddenly the temperature of the incoming coolant drops below its previous value. Discuss qualitatively the subsequent behavior of the reactor in the two cases; (a) αT is positive, (b) αT is negative. Show, in particular, that the reactor is unstable in (a) and stable in (b).

[sol]

여기에 수식을 입력하십시오.

32. Calculate the prompt temperature coefficient at room temperature of a reactor lattice consisting of as assembly of 1-in diameter natural uranium rods in a heavy-water moderator, in which the moderator volume to fuel volume ration is equal to 30.

[sol]

$$\boldsymbol{\alpha_{prompt}} = -\frac{\beta_I}{2\sqrt{T}} \ln\left[\frac{1}{P(300)}\right] = -\frac{N_F V_F I(300)}{\xi_M \Sigma_{aM} V_M} \frac{\beta_I}{2\sqrt{T}}$$
$$\frac{V_M}{V_F} = \mathbf{30}, \quad \boldsymbol{\xi_M \Sigma_{aM} = 0.178}$$
$$\boldsymbol{\beta_I} = \mathbf{A'} + \frac{C'}{a\rho} = \mathbf{48 * 10^{-4}} + \frac{1.28*10^{-2}}{0.5*2.54*19.1} = \mathbf{5.3277 * 10^{-3}}$$
at room temp.,
$$\mathbf{I(293) = I(300)\left[r\beta_I\left(\sqrt{293} - \sqrt{300}\right)\right] = I(300) * 0.9989 = 10.565}$$
$$\mathbf{where, I(300) = A} + \frac{C}{\sqrt{a\rho}} = \mathbf{2.8} + \frac{35.3}{\sqrt{0.5*2.54*19.1}} = \mathbf{10.5764}$$
$$\boldsymbol{\alpha_{prompt}} = -\frac{0.04833*10.565}{0.178*30} * \frac{5.3277*10^{-3}}{2\sqrt{293}} = \mathbf{-1.488 * 10^{-5}}\ /℃$$

33. A $C0_2$-cooled, graphite-moderated reactor fueled with rods of slightly enriched uranium dioxide 1.5 cm in diameter has a resonance escape probability of 0.9 1 2 at 300K. What is the value of p at the fuel operating temperature of 665°C?

[sol] $$\mathbf{p = exp}\left[-\frac{N_F V_F I}{\zeta_M \Sigma_{sM} V_M}\right]$$
$$\mathbf{I(T) = I(300)\left[1 + \beta_I\left(\sqrt{T} - \sqrt{300}\right)\right]}$$
$$\boldsymbol{\beta_I = A' + C'/\rho a}$$
$$\mathbf{CO_2, A' = 61 \times 10^{-4}, C' = 0.94 \times 10^{-2}}$$
$$\mathbf{a} = \frac{1.5}{2} = \mathbf{0.75cm}, \boldsymbol{\rho} = \mathbf{10.5gm/cm^3}$$
$$\boldsymbol{\beta_I} = \mathbf{61 \times 10^{-4} + 0.94 \times 10^{-2}/(0.75 \times 10.5) = 0.0073}$$
$$\mathbf{p(300) = exp}\left[-\frac{N_F V_{FI} I(300)}{\zeta_M \Sigma_{sM} V_M}\right] = \mathbf{0.912}$$
$$\mathbf{I(938) = I(300)\left[1 + 0.00073\left(\sqrt{938} - \sqrt{300}\right)\right] = 1.097 \times I(300)}$$
$$\mathbf{p(938) = exp}\left[-\frac{N_F V_{FI} I(938)}{\zeta_M \Sigma_{sM} V_M}\right] = \mathbf{exp}\left[-\frac{N_F V_{FI} \times 1.097}{\zeta_M \Sigma_{sM} V_M}\right] = \mathbf{[p(300)]^{1.097} = (0.912)^{1.097} = 0.904}$$

34. The overall temperature coefficient of a ^{235}U-fueled reactor is -2×10^{-5} per ℃ and is independent of temperature. By how much does the reactivity of the system drop when its temperature is increased from room temperature (about 70°F) to the operating temperature of 550°F? Give your answer in percent and in dollars. [Note: The decrease in reactivity calculated in this problem is called the temperature defect of the reactor.]

[sol] $$\frac{d}{dT}(\ln k) = \frac{1}{k}\frac{dk}{dT}(\ln p) = \frac{1}{p}\frac{dp}{dT} = \propto_{prompt}$$
$$\mathbf{d(lnk)} = \boldsymbol{\alpha_{prompt}}\mathbf{dT} => ln\mathbf{k_1 - lnk_2} = \boldsymbol{\alpha_{prompt}}\mathbf{(dT_1 - dT_2)}$$
$$\mathbf{ln}\frac{k_1}{k_2} = \mathbf{-2 \times 10^{-5}(287.77 - 21) = -5.33 \times 10^{-3}}$$

$\frac{k_1}{k_2} = \exp(-5.33 \times 10^{-3})$=0.994684
$\Delta k = 0.005344k_1 = \$0.872k_1$

35. Show that the temperature coefficient of B^2 is given by

$$\alpha_T(B^2) = -\frac{2}{3}\beta,$$

where β is the coefficient o f volume expansion o f the reactor structure. [Hint: Recall that β is equal to three times the coefficient of linear expansion.]

[sol] $\boldsymbol{\alpha_T(\beta^2) = \frac{1}{B^2}\frac{\partial B^2}{\partial T}}$

$\boldsymbol{\beta = \frac{1}{V}\frac{dV}{dT}, \alpha = \frac{1}{L}\frac{dL}{dT}}$ $=> \beta = 3\boldsymbol{\alpha}$

1) For spherical reactor

$\boldsymbol{B^2 = \left(\frac{\pi}{R}\right)^2}$

$\boldsymbol{\frac{\partial B^2}{\partial T} = \pi^2\frac{\partial(-R^2)}{\partial T} = \pi^2(-2)R^{-3}\frac{\partial R}{\partial T} = -2\left(\frac{\pi}{R}\right)^2\frac{1}{R}\frac{\partial R}{\partial T} = -2R^{-3}\alpha = -\frac{3}{2}\beta B^2}$

$\boldsymbol{\therefore \alpha_T(B^2) = -\frac{2}{3}\beta}$

2) For cylindrical reactor

$\boldsymbol{B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2}$

$\boldsymbol{\frac{\partial B^2}{\partial T} = (2.405)^2\frac{\partial(R^2)}{\partial T} + \pi^2\frac{\partial(H^{-2})}{\partial T} = -2\frac{(2.405)^2}{R^3}\frac{\partial R}{\partial T} - 2\pi^2\frac{1}{H^3}\frac{\partial H}{\partial T}}$

$\boldsymbol{= -2\frac{1}{R}\frac{\partial R}{\partial T}\left(\frac{2.405}{R^3}\right)^2 - 2\frac{1}{H}\frac{\partial H}{\partial T}\left(\frac{\pi}{H}\right)^2 = -2R\left(\frac{2.405}{R^3}\right)^2 - 2\alpha\left(\frac{\pi}{H}\right)^2 = -2\alpha B^2 = -\frac{2}{3}\alpha B^2}$

3) For cubic reactor

$\boldsymbol{B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = \pi^2\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)}$

$\boldsymbol{\frac{\partial B^2}{\partial T} = \pi^2\left\{-2\frac{1}{a^3}\frac{da}{dT} - 2\frac{1}{b^3}\frac{db}{dT} - 2\frac{1}{c^3}\frac{dc}{dT}\right\} = \pi^2(-2a)\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) = -2\alpha B^2 = -\frac{2}{3}\alpha B^2}$

$\boldsymbol{\therefore \alpha_T(B^2) = -\frac{2}{3}\beta}$

36. A pressurized water reactor fueled with stainless steel-clad fuel elements is contained in a stainless steel vessel that is a cylinder 6 ft in diameter and 8 ft high. Water having an average temperature of 575°F occupies approximately one-half of the reactor volume. How much water is expelled from the reactor vessel if the average temperature of the system is increased by 10°F? [Note: The volume coefficients of expansion of water and stainless steel at 300°C are 3×10^{-3} per °C and 4.5×10^{-5} per °C, respectively.]

[sol] $\boldsymbol{V'(H_2O) = \frac{V}{2}[1 + 3\times10^{-3}\times(307.22 - 301.66)] = 0.50834V}$

$\boldsymbol{V'(fuel) = \frac{V}{2}\left[1 + 4.5\times10^{-5}\times(307.22 - 301.66)\right] = 0.5000128V}$

$\boldsymbol{\Delta V = V'(H_2O) - V'(fuel) = 8.215\times10^{-3}V}$

$\boldsymbol{\Delta m = \Delta V\times\rho = 8.215\times10^{-3}\times\pi\times3^2\times8\times2.832\times10^4 = 52.62kg}$

37. The thermal utilization of the reactor in Problem 7.36 is 0.682 at 575°F. Using the results of that problem and ignoring the presence of structural material in the core, estimate $\alpha_T(f)$ at approximately 575°F.

[sol] $\boldsymbol{T = 575°F = 301.67°C, f = \frac{\Sigma_{aF}}{\Sigma_{aF}+\Sigma_{aM}} = 0.682}$

$\boldsymbol{\Sigma_{aM} = \frac{\sqrt{\pi}}{2}\Sigma_{aM}(E_0)\left(\frac{T_0}{T}\right)^{\frac{1}{2}} = \frac{\sqrt{\pi}}{2}\times0.0222\times\left(\frac{273+20}{273+301.67}\right)^{\frac{1}{2}} = 0.01405cm^{-1}}$

$\boldsymbol{\Sigma_{aF} = \frac{f}{1-f}\Sigma_{aF} = \frac{0.682}{1-0.682}\times0.01405 = 0.03013cm^{-1}}$

$\boldsymbol{V(H_2O) = 0.50834V = 0.50834\times(\pi\times3^2\times8\times2.832\times10^2) = 3.256\times10^6cm^3}$

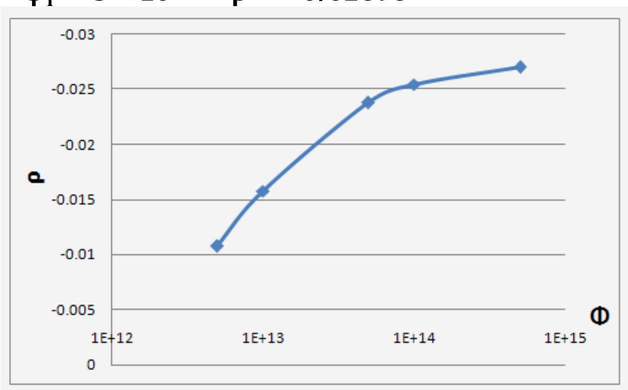
$$\rho(585°F) = \frac{M_{H2O}}{V} = \frac{1}{2\times 0.50834} = 0.984 g/cm^3$$
$$N_{H2O} = \frac{0.984\times 0.6088\times 10^{24}}{18} = 0.0329\times 10^{24}\#/cm^3$$
$$\Sigma_{aM} = \frac{\sqrt{\pi}}{2}\times 0.0014\times 0.664\times\left(\frac{273+20}{273+301}\right)^{\frac{1}{2}} = 0.01376 cm^{-1}$$
$$f = \frac{\Sigma_{aF}}{\Sigma_{aF}+\Sigma_{aM}} = \frac{0.03013}{0.03013+0.01376} = 0.6865$$
$$\propto_T = \frac{1}{f}\frac{df}{dT} = \frac{1}{0.682}\frac{\{0.6865-0.682\}}{(307.22-301.67} = 1.1846\times 10^{-3}/℃$$

38. What is the effective half-life of ^{135}Xe in a thermal flux of 10^{14} neutrons/cm^2-sec at a temperature of 800°C?

[sol] $$T_{eff} = \frac{0.693}{\lambda_X+\sigma_{aX}\phi_T} = \frac{0.693}{2.09\times 10^{-5}+\frac{\sqrt{\pi}}{2}\times 0.9887\times 24.5\times\left(\frac{293}{1073}\right)^{1/2}\times 10^4} = 6.18\times 10^{-16} sec$$

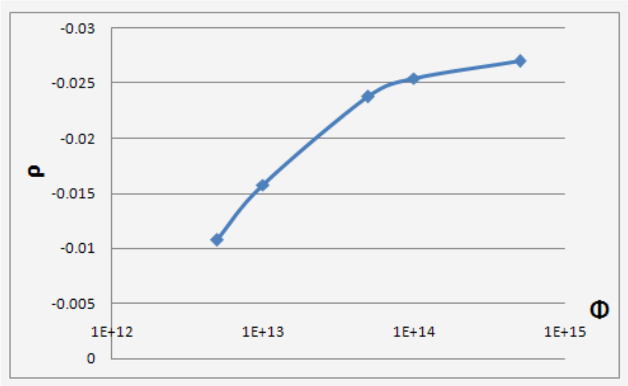
39. Compute and plot the equilibrium xenon reactivity as a function of thermal flux from $\Phi_T = 5\times 10^{12}$ to $\Phi_T = 5\times 10^{14}$

[sol] $$I_\infty = \frac{\gamma_I\Sigma_f\phi_T}{\lambda_I}$$
$$X_\infty = \frac{\lambda_I I_\infty+\gamma_X\Sigma_f\phi_T}{\lambda_X+\sigma_{aX}\phi_T} = \frac{(\gamma_I+\gamma_X)\Sigma_f\phi_T}{\lambda_X+\sigma_{aX}\phi_T}$$
$$\Sigma_{aX} = X_\infty\sigma_{aX} = \frac{(\gamma_I+\gamma_X)\Sigma_f\sigma_{aX}\phi_T}{\lambda_X+\sigma_{aX}\phi_T} = \frac{(\gamma_I+\gamma_X)\Sigma_f\phi_T}{\frac{\lambda_X}{\sigma_{aX}}+\phi_T}$$
$$\rho = -\frac{\Sigma_a/\Sigma_f}{\nu p\epsilon} = -\frac{(\gamma_I+\gamma_X)}{\nu p\epsilon}\frac{\phi_T}{\phi_X+\phi_T}$$
$$\phi_X = \frac{\lambda_X}{\sigma_{aX}} = \frac{2.09\times 10^5}{2.65\times 10^6\times 10^{-24}} = 0.770\times 10^{13}$$
$$\gamma_I+\gamma_X = 0.0639+0.00237 = 0.06627$$
For u-235, $\nu = 2.42, P = \epsilon = 1.0$
$$\rho = -0.02738\times\frac{\phi_T}{\phi_X+0.770\times 10^{23}}$$
$$\phi_T = 5\times 10^{22}\rightarrow\rho = -0.01078$$
$$\phi_T = 1\times 10^{23}\rightarrow\rho = -0.01547$$
$$\phi_T = 5\times 10^{13}\rightarrow\rho = -0.02373$$
$$\phi_T = 1\times 10^{14}\rightarrow\rho = -0.02542$$
$$\phi_T = 5\times 10^{24}\rightarrow\rho = -0.02696$$



40. Using Fig. 7. 14, plot the maximum post-shutdown xenon reactivity as a function of thermal flux from $\Phi_T = 5\times 10^{13}$ to $\Phi_T = 5\times 10^{14}$

[sol] $$\phi_T = 10^{13}\rightarrow\rho = 0.01338$$
$$\phi_T = 5\times 10^{13}\rightarrow\rho = 0.06$$
$$\phi_T = 10^{14}\rightarrow\rho = 0.233$$
$$\phi_T = 5\times 10^{14}\rightarrow\rho = 0.533$$



41. A ^{235}U-fueled reactor operating at a thermal flux of 5 $\times$ 10^{13} neutrons/cm^2-sec is scrammed at a time when the reactor has 5% in reverse reactivity. Compute the time to the onset of the deadtime and its duration.

[sol]

$$\rho = -\frac{1}{\nu p \epsilon}\left[\frac{(\gamma_I + \gamma_X)\phi_T}{\phi_X + \phi_T}e^{-\lambda_X t} + \frac{\gamma_I \phi_T}{\phi_I - \phi_X}(e^{-\lambda_X t} - e^{-\lambda_I t})\right]$$

<from Eq. 7.103>

v = 2.42
pε = 1
$\gamma_I + \gamma_X$ = 0.066 <from Table 7.5>
γ_I = 0.0639 <from Table 7.5>
$\emptyset_T$ = 5 $\times$ 10^{13}
$\emptyset_X$ = 0.770 $\times$ 10^{13} <from Eq. 7.97>
$\emptyset_I$ = 1.055 $\times$ 10^{13} <from Eq. 7.104>
λ_X = 0.0753 hr-1 <from Table 7.6>
λ_I = 0.1035 hr-1 <from Table 7.6>
$\rho = -0.486877e^{-0.0753t} + 0.463244e^{-0.1035t}$
= -0.05
t = 3.20253 and 19.7751
the onset of the deadtime = 3.20253 hr
duration = 16.5726 hr

42. Calculate the equilibrium concentration, in atoms/cm$_3$, of ^{135}Xe and ^{149}Sm in an infinite critical ^{235}U-fueled, water-moderated thermal reactor operating at a temperature of 200℃ and a thermal flux of 10^{13} neutrons/cm^2-sec.

[sol]

43. How much reactivity is tied up in ^{135}Xe and ^{149}Sm in the reactor described in Problem 7.42?

[sol]

44. Gadolinium-157 is a stable nuclide having an absorption cross-section at 0.0253 eV of 240,000 b. It is formed from the decay of the fission product ^{157}Sm according to the following chain:

$$\begin{array}{c} ^{157}Sm \xrightarrow[0.5m]{\beta^-} {}^{157}Eu \xrightarrow[15.2h]{\beta^-} {}^{157}Gd \\ \uparrow \\ \text{Fission} \end{array}$$

Neither ^{157}Sm nor ^{157}Eu absorbs neutrons to a significant extent. The ^{235}U fission yield of ^{157}Sm is 7 $\times$ 10^{-5} atoms per fission. (a) What is the equilibrium reactivity tied up in ^{157}Gd in a reactor having an average thermal flux of 2.5 $\times$ 10^{13} neutrons/cm^2-sec? (b) What is the maximum reactivity due to this nuclide after the shutdown of the reactor in part (a)?

[sol] (a) $\frac{ds}{dt} = r_s\sum_f \emptyset_T - \lambda_s S \Rightarrow S_\infty = r_s\sum_f \emptyset_T / \lambda_s$

$\frac{dE}{dt} = \lambda_s S - \lambda_E E \Rightarrow E_\infty = \lambda_s S_\infty / \lambda_E = r_s\sum_f \emptyset_T / \lambda_E$

$\frac{dG}{dt} = \lambda_E E - \sigma_{aG}\emptyset_T G$

$\Rightarrow G_\infty = \lambda_E E_\infty / \sigma_{aG}\emptyset_T = r_s\sum_f \emptyset_T / \sigma_{aG}\emptyset_T = r_s\sum_f / \sigma_{aG}$

$\rho = -(\sum_{aG} / \sum_f) / vp\varepsilon = -(G_\infty\sigma_{aG} / \sum_f) / vp\varepsilon = -r_s / vp\varepsilon = -(7 \times 10^{-5}) / 2.418$

$= -2.895 \times 10^{-5}$

$= -2.895 \times 10^{-3}$ **%**

(b) $G(t) = G_0 + E_0(1 - e^{-\lambda t})$

If G_d and E_u are equilibrium concentraties prior to shutdown

$G_0 = G_\infty, E_0 = E_\infty$

$\rho(t) = -(\sum_{aG} / \sum_f) / vp\varepsilon = -(\sigma_{aG}G(t) / \sum_f) / vp\varepsilon = -(1 / vp\varepsilon)(r_s + (r_s\sigma_{aG}\emptyset_T / \lambda_E)(1 - e^{-\lambda Et}))$

$= -(r_s / vp\varepsilon)(1 + (\emptyset_T / \emptyset_E)(1 - e^{-\lambda Et}))$

$P_{max} = \lim_{t\to\infty} \rho(t) = -(r_s / vp\varepsilon)(1 + (\emptyset_T / \emptyset_E))$

$\emptyset_E = \lambda_E / \sigma_{aG} = (\ln 2 / (1.52 \times 3600)) / (\sqrt{\pi}/2 \times 240000 \times 10^{-24})$

$= 5.955 \times 10^{13}$ **atoms/cm²sec**

45. A nonradioactive fission produce has an absorption cross-section of 75 b. Should this be considered a permanent poison in a reactor having a thermal flux of 3×10^{13} neutrons/cm²-sec?

[sol]

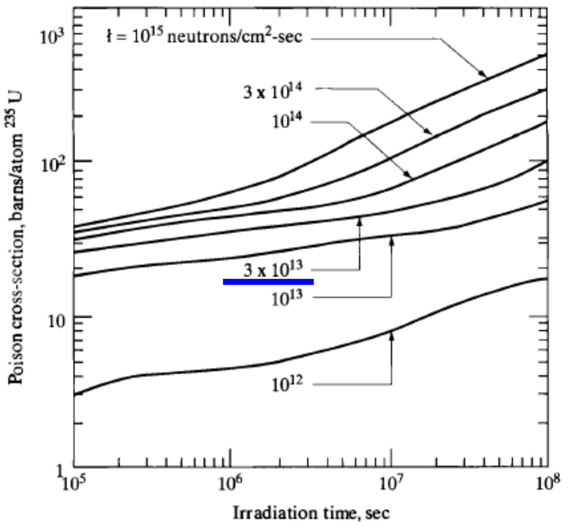


Figure 7.18 Buildup of fission product poisons in barns of absorption cross-section per original atom of ²³⁵U. (From ANL-5800, 2nd ed., 1963.)

t ≒ 6.2 × 10⁷ s

This is considered a permanent poison.

46. What fraction of the poisoning in Example 7.14 is due to ¹³⁵Xe?

[sol] $\rho_{xe} = -\frac{r_I + r_X}{vp\epsilon}\emptyset_T/(\emptyset_X + \emptyset_T) = -\frac{(0.0639+0.00237)\times 10^{12}}{2.418 \times (7.684 \times 10^{12} + 10^{12})}$

$= -3.156 \times 10^{-3}$

in $\emptyset_X = \lambda_X / \sigma_{aX} = \frac{2.09 \times 10^{-5}}{\frac{\sqrt{\pi}}{2} \times 1.581 \times 2.65 \times 10^6 \times 10^{24}}$

$= 7.684 \times 10^{12}$ **atoms/cm²sec**

From Example 7.14

$\rho = -0.00849$

fraction of the poisoning $= \frac{\rho}{\rho_{xe}} = \frac{-3.156 \times 10-3}{-0.00849} = 37.17$ **%**

47. An infinite reactor containing no fertile material operates at constant power over its lifetime. (a) Show that the atom density of the fuel decreases according to the relation

$$N_F(t) = N_F(0)[1 - \bar{\sigma}_{aF}\phi_T(0)t],$$

Where $N_F(0)$ and $\phi_T(0)$ are, respectively, the fuel atom density and thermal flux at startup. (b) Find an expression for the flux as a function of time.

[sol] (a) $N_F(t) = N_F(0) - \int_0^t \Sigma_{aF}(0)\phi_T(0)dt$

$= N_F(0) - N_F(0)\,\bar{\sigma}_{aF}\phi_T(0)t$

$= N_F(0)[1 - \bar{\sigma}_{aF}\phi_T(0)t]$

(b) $\Sigma_{aF}(t)\phi_T(t) = \Sigma_{aF}(0)\phi_T(0) = \text{Constant}$

$\Sigma_{aF}(t)\phi_T(t) = \bar{\sigma}_{aF}N_F(0)[1 - \bar{\sigma}_{aF}\phi_T(0)t]\,\phi_T(t)$

$= \Sigma_{aF}(0)[1 - \bar{\sigma}_{aF}\phi_T(0)t]\,\phi_T(t)$

$= \Sigma_{aF}(0)\phi_T(0)$

$\phi_T(t) = \phi_T(0) / (1 - \bar{\sigma}_{aF}\phi_T(0)t)$

48. An infinite 139Pu-fueled fast breeder has the breeding gain G and is operated at constant power before refueling. Derive expressions for the concentration of 239Pu and the (one-group) fast flux as a function of time after startup.

[sol] $Pu^{239} + {}^1_0n \Rightarrow$ **fission and** Pu^{240}

Concentration of ^{239}Pu

$\frac{dN_{29}}{dt} = -N_{29}\sigma_f\phi(r, t) - N_{29}\sigma_r\phi(r, t)$

$= -N_{29}(\sigma_f + \sigma_r)\phi(r, t)$

where, σ_f **= fission cross section**

σ_r **= radioactive capture cross section**

Chapter 8

Heat Removal from Nuclear Reactors

1. The nuclear ship Savannah was powered by a PWR that operated at a pressure of 1,750 psia. The coolant water entered the reactor vessel at a temperature of 497°F, exited at 519°F, and passed through the vessel at a rate of 9.4 × 1 06 lb/hr. What was the thermal power output of this reactor?

[sol] $\mathbf{q = w(h_{out} - h_{in}), w = 9.4 \times 10^6 lb/hr}$

$\mathbf{h_{out} = 511.5Btu/lb, h_{in} = 484.2Btu/Ib\ (from\ Table\ IV.1)}$

$\mathbf{q = 9.4 \times 10^6 \times (511.5 - 484.3) = 2.57 \times \frac{10^8 Btu}{hr} = 75.21MW}$

2. An experimental LMFBR operates at 750 MW. Sodium enters the core at 400°C and leaves at 560°C. At what rate must the sodium be pumped through the core?

[sol] $\mathbf{T_o = 1040°F, h_{out} = 468.5Btu/Ib}$

$\mathbf{T_F = 497°F, h_{in} = 381.4Btu/Ib}$

$\mathbf{q = 750MW = 2.559 \times 10^9 Btu/hr}$

$\mathbf{w = \frac{q}{h_{out} - h_{in}} = \frac{2.559 \times 10^9}{468.5 - 381.4} = 2.938 \times 10^7 Ib/hr}$

3. Show that if water passes through the core of a BWR at the rate of w lb/hr, then w_g lb/hr of steam will be produced, where

$$w_g = \frac{q - w(h_f - h_{in})}{h_{fg}};$$

q is the thermal power of the reactor; h_f and h_{in} are, respectively, the specific enthalpies of the saturated water leaving the core and the water entering the core; and h_{fg} is the heat of vaporization per pound.

[sol] $\mathbf{q - w\int_{T_{in}}^{T_{out}} c_p(T)dT = w\int_{T_{in}}^{T_{sat}} c_p(T)dT + w_g\int_{T_{sat}}^{T_{out}} c_p(T)dT = w(h_f - h_{in}) + w_g h_{fg}}$

$\mathbf{\therefore w_g = \frac{q - w(h_f - h_{in})}{h_{fg}}}$

4. A BWR operates at a thermal power of 1,593 MW. Water enters the bottom of the core at 526°F and passes through the core at a rate of 48 x 1 06 lb/hr. The reactor pressure is 1,035 psia. Using the result of the previous problem, compute the rate in lb/hr at which steam is produced in this reactor.

[sol] $\mathbf{q = 1593MW = 5.44 \times 10^9 Btu/hr}$

$\mathbf{w = 48 \times 10^6 lb/hr}$

$\mathbf{h_{in} = 519.02Btu/lb}$

$\mathbf{h_f = 547.65Btu/Ib}$

$\mathbf{h_{fg} = 643.35Btu/Ib}$

$\mathbf{w_g = \frac{q - w(h_f - h_{in})}{h_{fg}} = \frac{5.44 \times 10^9 - 48 \times 10^6(547.65 - 519.02)}{643.35} = 6.32 \times 10^6 Ib/hr}$

5. Starting with the value of the specific enthalpy of saturated water at 350°F given in Table IV.1, calculate the specific enthalpy of saturated water at 600°F. [Hint: Integrate the specific heat using Simpson's rule.]

[sol] $\mathbf{h(600°F) = h(350°F) + \int_{350}^{600} c_p(T)dT = h(350°F) + c_{p(350)} \times 10 + \int_{350}^{600} c_p(T)dT}$

$\mathbf{\int_{350}^{600} c_p(T)dT = \frac{h}{3}\left[1 \times c_p(350) + 4 \times c_p(380) + 2 \times c_p(400)..+1 \times c_p(600)\right]}$

$\mathbf{h = \frac{600 - 350}{12} = 20}$

$\mathbf{\int_{350}^{600} c_p(T)dT = \frac{20}{3} \times 43.12 = 287.47Btu/Ib}$

$\mathbf{\therefore h(600°F) = 321.8 + \left(\frac{1.046 + 1.056}{2}\right) \times 10 + 287.47 = 615.6Btu/Ib}$

6. The decommissioned Fort St. Vrain HTGR was designed to produce 330 MW of electricity at an efficiency of 39.23%. Helium at a pressure of 710 psia enters the core of the reactor at 760°F and exits at 1,430°F. What is the

helium flow rate through the core?

[sol] $P_{th} = 330 \times \frac{100}{39.23} = 841.193\text{MW} = 2.87 \times 10^9\text{Btu/hr}$

$h_{out} = 5480.79\frac{T}{g} \times \frac{g}{2.205\times10^{-3}\text{Ib}} \times \frac{9.478\times10^{-4}\text{Btu}}{J} = 2355.87\text{Btu/Ib}$

$h_{in} = 3548.71\frac{J}{g} \times \frac{g}{2.205\times10^{-3}\text{Ib}} \times \frac{9.478\times10^{-4}\text{Btu}}{J} = 1525.38\text{Btu/Ib}$

$\dot{Q} = \frac{q}{h_{out}-h_{in}} = \frac{2.87\times10^9\text{Btu/hr}}{(2355.87-1523.38)\text{Btu/Ib}} = 3.456 \times 10^6\text{Ib/hr}$

7. A small PWR plant operates at a power of 485 MWt. The core, which is approximately 75.4 in. in diameter and 91.9 in high, consists of a square lattice of 23,142 fuel tubes of thickness 0.02 1 in and inner diameter of 0.298 in on a 0.422-in pitch. The tubes are filled with 3.40 w/o-enriched UO_2. The core is cooled by water, which enters at the bottom at 496°F and passes through the core at a rate of 34×10^6 lb/hr at 2,015 psia. Compute (a) the average temperature of the water leaving the core; (b) the average power density in kW/liter; (c) the maximum heat production rate, assuming the reactor core is bare.

[sol] a) $q = 485 \times 10^3\text{kW} = 1.635 \times 10^9\text{Btu/hr}$

$h_{in}(496°\text{F}) = 483.02\text{Btu/Ib}$

$w = 34 \times 10^6\text{Ib/hr}$

$h_{out} = q/w + h_{in} = 1.635 \times 10^9/34 \times 10^6 + 483.02 = 531.1\text{Btu/Ib}$

b) $q''_{av} = \frac{q}{w} = \frac{q}{\pi a^2 H\times23142} = \frac{485\times10^3\text{kW}}{\pi\left(\frac{0.298}{2}\right)^2\times91.9\times0.01639\times23142} = 199.5\text{kW/liter}$

c) $q(0) = \frac{2.32PE_l}{HE_R} = \frac{2.32\times485\times180}{23142\times200} = 0.044\text{MW} = 1.5 \times 10^8\text{Btu/hr}\cdot\text{hr}$

$q''_{max} = \frac{1}{2Ha^2}q(0) = \frac{1}{2\times(75.4/2\times1/12)\times(0.298/2\times1/12)} \times 1.5 \times 10^5 = 1.55 \times 10^8\text{Btu/hr}\cdot\text{ft}^3$

8. The core of a BWR consists of 764 fuel assemblies, each containing a square array of 49 fuel rods on a 0.738-in pitch. The fuel rods are 175 in long, but contain fuel over only 144 in of their length. The outside diameter of the fuel rods is 0.563 in, the cladding (fuel tube) is 0.032 in thick, and the UO_2 fuel pellets are 0.487 in diameter. This leaves a gap of (0.563 - 2 × 0.032 - 0.487) / 2 = 0.006 in between the pellets and the cladding. The UO_2 has an average density of approximately 10.3 g/cm^3. The radius of the core is 93.6 in, and the reactor is designed to operate at 3,293 MW. The peak-to-average power density is 2.62. Calculate for this reactor: (a) the total weight of U02 and uranium in the core; (b) the specific power in kW/kg U; (c) the average power density in kW/liter; (d) the average linear rod power q''_{av} in kW/ft; (e) the maximum heat production rate.

[sol] $n = 764 \times 49 = 37436\text{fual rods}, \rho(UO_2) = 10.3\text{g/cm}^3,\ \Omega = 2.62$

a) $V_F = 37436 \times \left\{144 \times \pi \times \left(\frac{0.487}{2}\right)^2\right\} \times \left(\frac{5.787\times10^4\text{ft}^3}{\text{in}^3}\right) = 581.11\text{ft}^3 = 1.65 \times 10^7\text{cm}^3$

$m(UO_2) = \rho V_F = 10.3 \times 1.65 \times 10^7 = 1.7 \times 10^8\text{gm} = 3.7485 \times 10^5\text{Ib}$

$M(U) = \left(\frac{0.03}{235} + \frac{0.97}{238}\right)^{-1} = 237.9 \Rightarrow \frac{M(U)}{M(UO_2)} = \frac{237.9}{237.9+16\times2} = 0.8814$

$\therefore m(U) = 37485 \times 10^5 \times 0.8814 = 3.304 \times 10^5\text{Ib} = 1.5 \times 10^8\text{gm}$

b) $\text{Specifit power} = 3293 \times 10^3\text{kW}/1.5 \times 10^5\text{kg} = 21.95\text{kW/kg}$

c) $q''_{av} = \frac{P}{V_F} = \frac{3293\times10^3\text{kW}}{1.65\times10^7\text{cm}^3} \times \frac{10^3\text{cm}^3}{\text{liter}} = 199.6\text{kW/liter}$

d) $q'_{r,av} = \frac{3293\times10^3\text{kW}}{37436\times175\times1/12\text{ft}} = 6.032\text{kW/ft}$

e) $q_r(0) = \frac{2.32PE_l}{nE_R} = \frac{2.32\times3293\times10^3\times180}{37436\times200} \times 3412 = 6.267 \times 10^5\text{Btu/hr}$

$q''_{max} = \frac{1}{2Ha}q_r(0) = \frac{6.267\times10^5\text{Btu/hr}}{2\times144\times\frac{0.487}{2}\text{in}^3\times\frac{5.781\times10^{-4}}{\text{in}^3}\text{ft}^3} = 6.342 \times 10^7\text{Btu/hr}\cdot\text{ft}^3$

9. The core of an LMFBR consists of a square lattice of 13,104 fuel rods 0.158 in in diameter, 30.5 in long, on a 0.210-in pitch. The fuel rods are 26 *w/o*-enriched uranium clad in 0.005-in stainless steel. Liquid sodium enters the core at approximately 300℃ and passes through the core at an average speed of 31.2 ft/sec. The core produces 270 MW of thermal power, with a maximum-to-average power density of 1.79. Calculate: (a) the

maximum heat production rate; (b) the maximum neutron flux.

[sol] (a) $q''' = 1.16PE_d / Ha^2nE_R$

$$= (1.16 \times 270 \times 10^3 \times 3412 <\frac{Btu/Hr}{kW}> \times 180<MeV>)$$

$$/ [30.5<in> \times (0.158/2)^2<in^2> \times 0.000579<\frac{ft^3}{in^3}> \times 13104 \times 200<MeV>]$$

$$= 6.663 \times 10^8 \text{ Btu/hr-ft}^3$$

(b) $\rho(U) = 19.1 \text{ g/cm}^3$

$$N(U^{235}) = (0.26 \times 19.1 \times 0.6022 \times 10^{24}) / 235 = 0.0127 \times 10^{24} \text{ atoms/cm}^3$$

$$\sum_{fr} = 0.0127 \times 582.2 = 7.41 \text{ cm}^{-1}$$

$$\overline{\sum fr} = (\sqrt{\pi} / 2) \times 0.931 \times 7.41 \times (\frac{293}{273+300})^{1/2} = 4.37 \text{ cm}^{-1}$$

$$\emptyset_{max} = \emptyset(r=0, z=0) = q'''_{max} / (\overline{\sum fr} \times E_d)$$

$$= (6.663 \times 10^8<Btu/hr\text{-}ft^3> \times 6.461 \times 10^7<\frac{MeV/sec-cm3}{Btu/hr-ft3}>) / (4.37<cm^{-1}> \times 180<MeV>)$$

$$= 5.47 \times 10^{13} \text{ neutrons/cm}^2\text{-sec}$$

10. The variation of the neutron flux and/or heat production rate along the z direction in the core of a reflected reactor is often approximated by the function

$$\emptyset = \text{constant} \times \cos(\frac{\pi z}{\tilde{H}}),$$

where $\tilde{H}$ is the distance between the extrapolated boundaries and is somewhat larger than H, the actual height of the core. Show that the maximum-to-average heat production ratio in the z direction Ω_z is given by

$$\Omega_z = \frac{\pi H/2\tilde{H}}{\sin(\pi H/2\tilde{H})}.$$

[Note: A similar approximation can be made for $\emptyset$ and/or q^m in the radial direction by writing $\emptyset$ = constant $\times$ $J_0(2.405r / \tilde{R})$.]

[sol] $\emptyset_{max} = \emptyset(z=0) = C$

$$\emptyset_{av} = \int_{-H/2}^{H/2} \emptyset dz \;/\; \int_{-H/2}^{H/2} dz = \frac{C}{H}\int_{-H/2}^{H/2} \cos(\frac{\pi z}{\tilde{H}})dz$$

$$= \frac{2C\tilde{H}}{\pi H}\sin(\frac{\pi H}{2\tilde{H}})$$

$$\Omega_z = \emptyset_{max} / \emptyset_{av} = C / \frac{2C\tilde{H}}{\pi H}\sin(\frac{\pi H}{2\tilde{H}}) = \frac{\pi H/2\tilde{H}}{\sin(\pi H/2\tilde{H})}$$

11. Show that the maximum-to-average flux and/or power distribution ratios in the axial and radial direction are related to the overall ratio by

$$\Omega_z\Omega_r = \Omega,$$

Where the notation is that of Problem 8.10.

[sol] $\emptyset = CJ_0(\frac{2.405r}{\acute{R}})$

$$\Omega_r = \frac{CJ_0(0)}{\frac{C}{\pi R^2}\int_0^R J_0(\frac{2.405r}{\acute{R}})2\pi r dr} = \frac{R^2}{2\int_0^R J_0(\frac{2.405r}{\acute{R}})r dr}$$

In general,

$$\emptyset(r, z) = CJ_0(\frac{2.405r}{\acute{R}})\cos(\frac{\pi z}{\acute{R}})$$

$$\emptyset_{av} = \frac{C}{V}\int_V J_0(\frac{2.405r}{\acute{R}})\cos(\frac{\pi z}{\acute{R}})$$

$$\emptyset_{max} = CJ_0(0)\cos(0) = C$$

$$\Omega = \emptyset_{max} / \emptyset_{av} = \frac{C}{\frac{C}{V}\int_V J_0(\frac{2.405r}{\acute{R}})\cos(\frac{\pi z}{\acute{R}})}$$

$$= \frac{1}{(\frac{1}{\pi R^2}\int_0^R J_0(\frac{2.405r}{\acute{R}})2\pi r dr) \times (\frac{1}{H}\int_{-H/2}^{H/2}\cos(\frac{\pi z}{\tilde{H}})dz)}$$

$$= \frac{\mathbf{R^2}}{2\int_0^R J_0(\frac{2.405r}{R})rdr} \times \frac{\mathbf{H}}{\int_{-H/2}^{H/2}\cos(\frac{\pi z}{H})dz)}$$

$= \mathbf{\Omega_r\ \Omega_z}$

12. The core of a fast reactor is a cylinder 38.8 cm in radius and 77.5 cm high. Two dimensional (r, z) multigroup calculations show that the power density distribution in the core can be represented approximately by the expression

$$P(r, z) = P_0[1 - (\frac{r}{51})^2]\cos(\frac{\pi z}{109}),$$

Where P_0 is a constant and r and z are the distances in centimeters from the axis and the midplane of the core, respectively. (a) Evaluate P_0 in terms of the total core power P. (b) What is the maximum-to-average power ratio in the core? (c) Calculate the maximum-to-average power ratios in the radial and axial directions.

[sol] (a) $\mathbf{P = \int_v P(r,z)dV}$

$= 2\pi P_0\int_{-H/2}^{H/2}\int_0^R[1 - (\frac{r}{51})^2]\cos(\frac{\pi z}{109})rdrdz$

$= \pi P_0\int_{-H/2}^{H/2}[r^2 - \frac{r^4}{2\times 51^2}]_0^R\cos(\frac{\pi z}{109})dz$

$= \pi P_0[R^2 - \frac{1}{2}(\frac{R^2}{51})^2] \times [\frac{109}{\pi}\sin(\frac{\pi z}{109})]_{-H/2}^{H/2}$

$= 109P_0[38.8^2 - \frac{1}{2}(\frac{38.8^2}{51})^2] \times 2\sin(\frac{77.5\pi}{109\times 2})$

$= 2.096 \times 10^5 P_0$

$P_0 = \frac{P}{2.096\times 10^5} = 4.771 \times 10^{-6}P$

(b) $P_{max} = P(r, z)_{r=0,\ z=0} = P_0$

$P_{av} = \frac{1}{\pi R^2 H}\int_v P(r,z)dV$

$= \frac{2.096\times 10^5 P_0}{\pi\times 38.8^2\times 77.5}$

$= 0.572P_0$

$\Omega = \frac{P_{max}}{P_{av}} = \frac{P_0}{0.572P_0} = 1.748$

(c) $P(r) = P_0[1 - (\frac{r}{51})^2]$

$P(r)_{max} = P_{r0}$

$P(r)_{av} = \frac{\int P(r)dA}{\int dA}$

$= \frac{\pi P_{r0}[R^2 - \frac{1}{2}(\frac{R^2}{51})^2]}{\pi R^2}$

$= P_{r0}[1 - \frac{1}{2}(\frac{R}{51})^2]$

$= 0.7106P_{r0}$

$\Omega_r = P(r)_{max} / P_{r0} = \frac{P_{r0}}{0.7106P_{r0}} = 1.407$

$\Omega = \Omega_r\ \Omega_z$

$\Omega_z = \Omega / \Omega_r = 1.748 / 1.407 = 1.242$

13. A BWR power plant operating at an efficiency of 34% has an electrical output of 1,011 MW. (a) What is the maximum fission product decay energy in the reactor at shutdown? (b) What is the decay energy 6 months after shutdown?

[sol] (a) $P_0 = 1011 / 0.34 = 2973.53$ **MW**

At shutdown, the shortest time in Fig. 8.3 is 10^{-1} sec and the longest time in Fig. 8.3 is 10^9 sec.

$\frac{P_{max}}{P_0} = \frac{P(t_s)}{P_0} - \frac{P(t_0 + t_s)}{P_0}$

$= \frac{P(10^{-1})}{P_0} - \frac{P(10^9)}{P_0}$

$= 0.07 - 3 \times 10^{-4}$

$= 0.0697$

$P_{max} = 0.0697 \times P_0 = 207.255$ **MW**

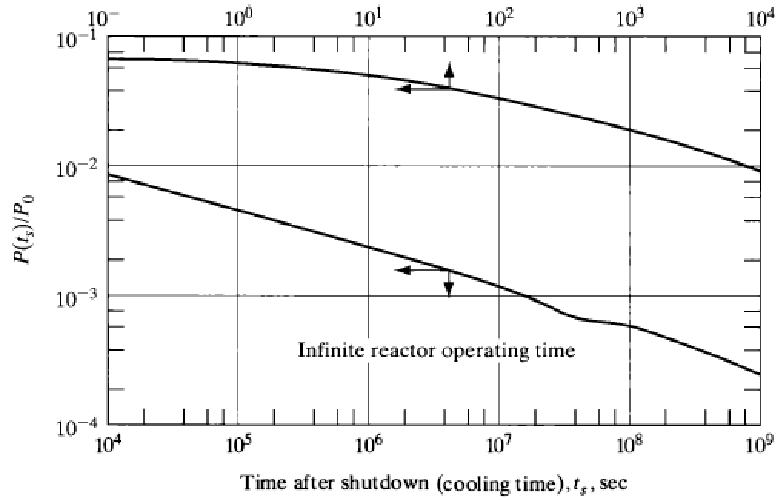


Figure 8.3 The ratio $P(t_s)/P_0$ of the fission product decay power to the reactor operating power as a function of time t_s after shutdown. (Subcommittee ANS-5 of the American Nuclear Society, 1968.)

(b) t_s = 6 months = 1.58 $\times$ 10^7 sec

If the reactor operates for one year, t_0 = 3.16 $\times$ 10^7 sec

$$\frac{P}{P_0} = \frac{P(1.58\times10^7)}{P_0} - \frac{P(1.58\times10^7+3.16\times10^7)}{P_0}$$

= 0.001 – 0.0007 = 0.0003

P = 0.0003 $\times$ 2973.53 = 0.892 MW

14. A 1,000-MW reactor is operated at full power for 1 year, then at 10% power for 1 month, and the shut down. Calculate the fission product decay heat at shutdown and for 1 month after shutdown.

[sol] (a) at shutdown

(i) For 1,000 MW

t_s = 1 month = 2.6 $\times$ 10^6 sec

t_0 = 1 year = 3.16 $\times$ 10^7 sec

$$\frac{P}{P_0} = \frac{P(2.6\times10^6)}{P_0} - \frac{P(2.6\times10^6+3.16\times10^7)}{P_0}$$

= 1.8 $\times$ 10^{-3} – 6.8 $\times$ 10^{-4} = 1.12 $\times$ 10^{-3} <From Figure 8.3>

P = 1,000 $\times$ 1.12 $\times$ 10^{-3} = 1.12 MW

(ii) For 100 MW

t_s = 10^{-1} sec

t_0 = 1 month = 2.6 $\times$ 10^6 sec

$$\frac{P}{P_0} = \frac{P(10^{-1})}{P_0} - \frac{P(2.6\times10^6+10^{-1})}{P_0}$$

= 0.07 – 1.8 $\times$ 10^{-3} = 0.0682 <From Figure 8.3>

P = 100 $\times$ 0.0682 = 6.82 MW

the fission product decay heat at shutdown

= 1.12 MW + 6.82 MW = 7.94 MW

(b) for 1 month after shutdown

(i) For 1,000 MW

t_s = 2 month = 5.2 $\times$ 10^6 sec

t_0 = 1 year = 3.16 $\times$ 10^7 sec

$$\frac{P}{P_0} = \frac{P(5.2\times10^6)}{P_0} - \frac{P(2.6\times10^6+5.2\times10^7)}{P_0}$$

= 1.5 $\times$ 10^{-3} – 7.2 $\times$ 10^{-4} = 7.8 $\times$ 10^{-4} <From Figure 8.3>

P = 1,000 $\times$ 7.8 $\times$ 10^{-4} = 0.78 MW

(ii) For 100 MW

t_s = 1 month = 2.6 $\times$ 10^6 sec

t_0 = 1 month = 2.6 $\times$ 10^6 sec

$\frac{P}{P_0} = \frac{P(2.6\times10^6)}{P_0} - \frac{P(2.6\times10^6+2.6\times10^6)}{P_0}$

= 1.8 × 10^{-3} – 1.5 × 10^{-3} = 3 × 10^{-4} <From Figure 8.3>

P = 100 × 3 × 10^{-4} = 0.02 MW

the fission product decay heat at shutdown

= 0.78 MW + 0.02 MW = 0.8 MW

15. A power reactor fueled with ^{235}U operates at the power P_0 for 1 year and is then scrammed by the insertion of $9.63 in negative reactivity. (a) Calculate the level to which the fission power of the reactor immediately drops. (b) To what power level does the reactor actually drop? (c) Compute and plot the reactor power from 0.1 sec to 0.5 hr after shutdown.

[sol] (a) ρ = $9.63 = 9.63 × 0.0065 = 0.06259

The fission power drops suddenly t the level P_1, where

$P_1 = \frac{\beta(1-\rho)}{\beta-\rho}P_0$

$= \frac{0.0065(1+0.06259)}{0.0065+0.06259}P_0 = 0.09997P_0$

(b) The fission produce decay heat

$\frac{P(t_0,t_s)}{P_0} = \frac{P(t_s)}{P_0} - \frac{P(t_0+t_s)}{P_0} = \frac{P(10^{-1})}{P_0} - \frac{P(3.16\times10^7+10^{-1})}{P_0}$

= 0.07 – 0.0007 = 0.0693 <From Figure 8.3>

P_a = 0.0693P0

Since the actual power are produced by the fission within the reactor and the decay of radioactive fission products, the actual power is given by

$P = 0.09997P_0 + 0.0693P_0 = 0.16927P_0$

(c) By fission

$P_f = P_1e^{-t/T} = 0.09997P_0e^{-t_s/80}$

Where T = 80 sec

By the decay of fission product

$P_d = (\frac{P(t_s)}{P_0} - \frac{P(t_0+t_s)}{P_0})P_0$

$P = 0.09997P_0e^{-t_s/80} + (\frac{P(t_s)}{P_0} - \frac{P(t_0+t_s)}{P_0})P_0$

(i) t_s = 0.1 sec

$P = 0.09997P_0e^{-0.1/80} + (0.07 - 0.0007)P_0$

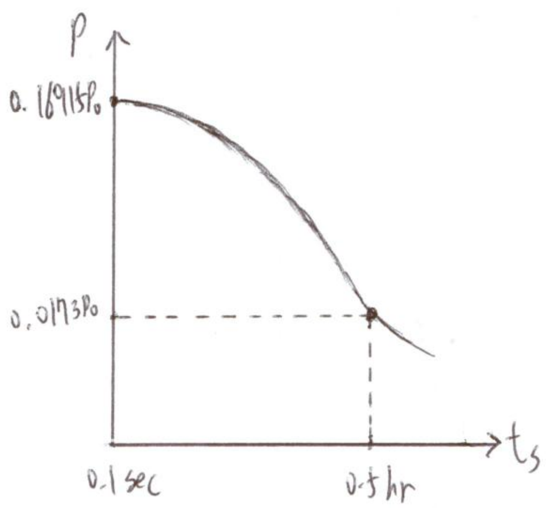
$= 0.16915P_0$

(ii) t_s = 0.5 hr = 1,800 sec

$P = 0.09997P_0e^{-1800/80} + (\frac{P(1800)}{P_0} - \frac{P(3.16\times10^7+1800)}{P_0})P_0$

$= 1.69 \times 10^{-4}P_0 + (1.8 \times 10^{-2} - 0.0007)P_0$

$= 0.0173P_0$



16. A thermal reactor fueled with slightly enriched ^{235}U is operated at a power of 2,800 MW for a period of 2 years and then shut down for refueling. The reactor has a conversion factor of 0.82. (a) What is the fission product decay energy at shutdown and 1 month later? (b) How much decay heat is due to ^{239}U and ^{239}Np? (c) What is the activity of the fission products and the ^{239}U and ^{23P}Np at the prior times?

[sol] (a) (i) at shutdown

$$\frac{P}{P_0} = \frac{P(10^{-1})}{P_0} - \frac{P(6.32\times10^{7})}{P_0}$$

= 0.07 – 6.5 × 10^{-4} <From Figure 8.3>

= 0.06935

P = 2800 × 0.06935 = 194.18 MW

(ii) one month later

$$\frac{P}{P_0} = \frac{P(2.6\times10^{6})}{P_0} - \frac{P(2.6\times10^{6}+6.32\times10^{7})}{P_0}$$

= 1.8 × 10^{-4} – 6.45 × 10^{-4} <From Figure 8.3>

= 1.155 × 10^{-3}

P = 2800 × 1.155 × 10^{-3} = 3.234 MW

(b) (i) For ^{239}U

a. at shutdown $t_s = 0$

$$\frac{P_{29}}{P_0} = 2.28\times10^{-3}\times\left(\frac{\sigma_{a25}}{\sigma_{f25}}\right)[1 - e^{-4.91\times10^{-4}t_s}]\ e^{-4.91\times10^{-4}t_s}$$

$$= 2.28\times10^{-3}\times0.82\times\left(\frac{6.81}{582}\right)$$

$$= 2.187\times10^{-3}$$

P_{29} = 2800 × 2.187 × 10^{-3} = 6.12 MW

b. one month later

P_{29} ≒ 0

(ii) For ^{239}Np

a. at shutdown $t_s = 0$

$$\frac{P_{29}}{P_0} = 2.17\times10^{-3}\times\left(\frac{\sigma_{a25}}{\sigma_{f25}}\right)\{[1 - e^{-3.14\times10^{-6}t_s}]\ e^{-3.41\times10^{-6}t_s}$$

$$- 7.0\times10^{-3}(1 - e^{-4.91\times10^{-4}t_s})\ e^{-4.91\times10^{-4}t_s}\}$$

$$= 2.17\times10^{-3}\times0.82\times\left(\frac{6.81}{582}\right)$$

$$= 2.082\times10^{-3}$$

P_{29} = 2800 × 2.082 × 10^{-3} = 5.83 MW

b. one month later

$$\frac{P_{29}}{P_0} = 2.082\times10^{-3}\times e^{(-3.41\times10^{-6}\times2.6\times10^{6})} = 2.94\times10^{-7}$$

P_{29} = 2800 × 2.94 × 10^{-7} = 8.3 × 10^{-4} MW

(c) (i) fission product activity

$$= 1.4\times10^{6}P[t_s^{-0.2} - (t_s + t_0)^{-0.2}]$$

a. at shutdown, t_s = 0.1 sec = 1.16 × 10^{-6} day

fission product activity

$$= 1.4\times10^{6}P[(1.16\times10^{-6})^{-0.2} - (2\times365)^{-0.2}]$$

$$= 2.11\times10^{7}\times2800$$

$$= 5.91\times10^{10}$$

b. one month later, t_s =30 day

fission product activity

$$= 1.4\times10^{6}P[(30)^{-0.2} - (30 + 2\times365)^{-0.2}]$$

$$= 3.38\times10^{5}\times2800$$

$$= 9.45\times10^{8}$$

(ii) For ^{239}U

a. t_s = 0.1 sec

fission product activity

$$= 5.91\times10^{10}\times\frac{6.12}{194.18} = 1.86\times10^{9}$$

b. t_s = 30 day

fission product activity

$$= 9.45\times10^{8}\times\frac{6.12}{194.18} = 2.98\times10^{7}$$

(iii) For ^{239}Np

a. $\mathbf{t_s = 0.1}$ **sec**

fission product activity

$= 5.91 \times 10^{10} \times \frac{5.83}{194.18} = 1.77 \times 10^{9}$

b. $\mathbf{t_s = 30}$ **day**

fission product activity

$= 9.45 \times 10^{8} \times \frac{5.83}{194.18} = 2.84 \times 10^{7}$

17. The reactor core in Problem 8. 1 2 produces 400 MW, of which 8 MW is due to y-ray heating of the coolant. The total heat transfer area of the fuel is 1 ,580 ft2 Compute (a) the average power density in the core in kWniter and kW/ft3 ; (b) the average heat flux; (c) the maximum heat flux.

[sol]

(a)

$$P_{av} = \frac{1}{V}\int_V P(r,z)dv = \frac{1}{\pi R^2 H}\int_V P(r,z)dv = \frac{392}{\pi(388)^2(77.5)} = 1.07x10^{-3}MW/cm^3$$

$= 1.07mw/l = 1070kw/l$, **where** $P(r,z) \Rightarrow [MW/cm^3] = 30.3MW/ft^3$

(b)

$\pi(388)^2(77.5)x3.531x10^{-5}ft^3 = 1580\,x\,a$

$\therefore$ **total heat transfer length** $a = 8.2\,x\,10^{-3}$ **ft**

$\therefore q''_{av} = 30.3\ MW/ft^3\ x\ 8.2\ x\ 10^{-3}\ ft = 0.248\ MW/ft^2$

(c) Maximum power P_{max}

$= \int_V P(r=0,z=0)dv = \pi R^2 H P_0 = 4.771\,x\,10^{-6} P \pi R^2 H$

$\therefore q''_{max} = \frac{4.771\,x\,10^{-6} x 392 x \pi x (38.8)^2 x (77.5)}{1580\,x\,929} = 3.92\ x\ 10^{-3}\ MW/cm^2 = 0.364\ MW/ft^2$

18. The plate-type fuel element described in Example 6. 1 0 is placed in a test reactor in a region where the thermal flux is 5 X 1 013 neutrons/cm2-sec. Calculate: (a) the heat production rate in the fuel; (b) the heat flux at the cladding surface; (c) the difference in temperature between the center of the fuel and the cladding surface.

[sol]

(a)

$q'' = E_d \Sigma_f \phi_T$, $N = 2.86x10^{-4}x10^{24}\ \#/cm^3$

$\sigma_{f25} = 582.2b$

$\therefore \Sigma_{fr} = \frac{\sqrt{\pi}}{2} x0.9759x2.85x10^{-4}x582.2 = 0.144cm^{-1}$

$q'' = 180Mev\ x\ 0.144cm^{-1}x5x10^{23}\ \#/cm^2\ sec = 1.3\ x\ 10^{15}\ MeV/cm^2 sec = 2.06x10^7\ Btu/kr - ft^3$

$\therefore$ **total rate of heat production**

$q = q''Aa = 3.25\,x\,10^{13}\ AMeV/sec = 7.8\ ABtu\ /Pr$

(b)

$q'' = \frac{q}{A} = \frac{3.25x10^{13}}{A} = 3.25x10^{13}\ MeV / sec\ cm^2 = 1.65\ x\ 10^4 Btu / Pr\text{-}ft^3$

(c)

$k_c = \frac{1}{150} x20 + 10.6 = 10.7 \frac{Btu}{Pr} - ft - F$

$T_m - T_c = q(\frac{a}{2k_f A} + \frac{b}{k_c}) = 7.8A(\frac{0.05x0.03281}{2x10.7xA} + \frac{0.05x0.03281}{11A}) = 0.0004F \approx 0$

$\therefore T_m = T_c$

19. The temperature at the center of the fuel rod described in Problem 8.9, where q" is the largest, is 1 ,220°F. Calculate the temperatures at the fuel-cladding interface at the outer surface of the cladding.

[sol]

$\mathbf{q''' = 6.663 \text{ x } 10^{8} Btu / Pr\text{-}ft^{3}}$, $\mathbf{q = \pi a^{2} H q''' = \pi \left(0.079 \text{ x } \frac{1}{12}\right) x\, 30.5 \text{ x } \frac{1}{12} \text{ x } 6.663 \text{ x } 10^{8} = 2.306x10^{5}}$ **Btu/Pr**

$\mathbf{R_f = 1.47 \text{ x } 10^{3}}$ **F-hr/Btu** , $\mathbf{R_c = 3.84x10^{-4} F - hr/Btu}$

i) $\mathbf{q = \frac{T_m - T_s}{R_f}}$ => $\mathbf{T_s = 1220 - 2.306 \text{ x } 10^{5} \text{ x } 1.47 \text{ x } 10^{3} = 881.5\ F}$

ii) $\mathbf{q = \frac{T_m - T_s}{R_f + R_c}}$

=> $\mathbf{T_c = 1220 - 2.306x10^{5} x (1.47x10^{-3} + 3.34x10^{-4}) = 793F}$

20. Derive the general solution to Eq. (8.44). [Hint: Write the first two tenns of the equation as $\frac{1}{r}\frac{d}{dr} r \frac{dT}{dr}$,and then perform the integration.]

[sol]

$\mathbf{\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q'''}{R_f} = 0}$ => $\mathbf{\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}T\right) + \frac{q''}{R_f} = 0}$, $\mathbf{\frac{d}{dr}\left(r\frac{d}{dr}T\right) = -\frac{q''}{Rf} r}$

$\mathbf{\frac{dT}{dr} = -\frac{q''}{2R_f} r + \frac{C_1}{r}}$, => $\mathbf{T(r) = -\frac{q''}{4R_f} r^2 + C l n r + C}$

BC r =0 , T=$\mathbf{T_m}$**,** $\mathbf{C_1 = 0}$ **C =** $\mathbf{T_r}$ **,**

∴ $\mathbf{T = T_m - \frac{q'' r^2}{4R_f}}$

21. Show that Eq. (8.47) can be written as where q' is the heat generated per unit length of a fuel rod. [Note: This result shows that the temperature difference across a fuel rod is directly proportional to the linear power density q' The quantity q' is an important parameter in reactor design.]

[sol]

$\mathbf{T_m - T_e = \frac{q}{4\pi H R_f} = \frac{q/H}{4\pi R_f} = \frac{q'}{4\pi R_f}}$

22. The gap between UO_2 pellets and the inside of a fuel tube can be taken into account in calculations of heat flow out of the fuel by defining a gap conductance h_{gap} by the relation

$$q'' = h_{gap}\ \Delta T,$$

where q" is the heat flux and ΔT is the temperature difference across the gap. Values of h_{gap} of about 1 ,000 Btu/hr-ft^2-F are typical for PWR and BWR fuel. Show that the thermal resistance of a gap is where A is the area of the fuel.

$R_{gap} = \frac{1}{h_{gap}A}$, where A is the area of the fuel

[sol]

$\mathbf{q = q''A = h_{gap}\Delta T A = \Delta T,/(1/h_{gap}A) = \frac{\Delta T}{R_{gap}}}$

∴ $\mathbf{R_{gap} = \frac{1}{h_{gap}A}}$,

23. At one point in the reactor described in Problem 8.8, the heat flux is 280,000 Btulhr-ft2, and the outer temperature of the cladding is 563°F. Compute and plot the temperature distribution in the fuel rod at this location (a) ignoring the presence of the gap between the pellets and the cladding; (b) using the gap conductance (see Problem 8.22) of 1 ,000 Btu/hr-ft^2 − F.

[sol]

(a) fuel region

$\sigma^2 T + \frac{q'''}{R_f} = 0$, $\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}T\right) + \frac{q''}{R_f} = 0$

T(r) = $-\frac{q'''}{4R_f} r^2 + c_1 \ln r + c_2$

B.C

$C_1 = 0$, $T_m - T_c = q(R_f + R_c) = 2\pi r_c H_c q''(R_f + R_c)$

$T_m = 563 + 2\pi x 0.0235 x 14.583 x 280000 x (6.03x10^{-3} + 5.153x10^{-4}) = 4509$ °F

$q = 2\pi x 0.0235 \text{ x } 14.583 \text{ x } 280000 = 6.029x10^5$ BTU/Pr

$T_f = T_M - 8R_f = 4509 - 6.029x10^5 x 604 x 10^{-3} = 873.5$°F

(b) If gap exist

1) fuel region

$q = \frac{T_m - T_f}{R_f}$

2) gap region

$q'' = \frac{\Delta T}{1/h_{gap}} \Rightarrow q = \frac{\Delta T}{1/2\pi\gamma_s H h_{gap}} - \frac{T_s - T_s}{R_g}$

3) cladding region

$\nabla^2 T = 0 \Rightarrow T = C_1 \ln + C_2$

B.C i) $T(r_0) = T_s$

$T_s = C_1 \ln r + C_2$

ii) $T(r_c) = T_c$

$T_c = C\ln r_c + C_2$

$\therefore C_1 = \frac{T_s - T_c}{\ln(r_s/r_c)}, C_2 = T_c - \frac{T_s - T_c}{\ln(\frac{r_s}{r_c})} \ln r_c$

$\therefore T - T_c = \frac{T_s - T_c}{\ln(\frac{r_s}{r_c})} \ln(\frac{r}{r_c})$

$q = -2\pi r_c H R_c \left.\frac{dT}{dr}\right|_{r=r_c} = 2\pi r_c H R_c [\frac{T_s - T_c}{\ln(\frac{r_s}{r_c})}\frac{1}{r_c}] = \frac{T_s - T_c}{\ln(r_c/r_s)} = \frac{T_s - T_c}{R_c}$

$q = \frac{T_m - T_c}{R_f + R_c + R_g}$

$\Rightarrow T_m - T_c = 2\pi r_c H q''(R_f + R_c + R_g)$

$R_s = 6.03 \text{ x } 10^{-3}$ F $- R_r$/BTU, $R_g = 5.247x10^{-4}$F $- R_r$/BTU , $R_c = 4.291 \text{ x } 10^{-4}$F $- R_r$/BTU

$T_m = 563 + 2\pi x 0.0235 x 14.583 x 280000 (6.03x10^{-3} + 5.247x10^{-4} + 4.291x10^{-4}) = 4774$°F

$T_f = 4774 - 6.03x10^{-2} x 6.029x10^5 = 1139$°F

$T_s = 1139 - 5.247x10^{-4} \text{ x } 6.029 \text{ x } 10^5 = 822$°F

24. The fuel rod in the reactor described in Problem 8.8 with the highest heat flux contains uranium enriched to 2.28 w / o. (a) Calculate the maximum heat-generation rate in this rod when the reactor is operating at design power. (b) What is the maximum thermal neutron flux in this rod? [Note: In part (b), use the average fuel temperature from Problem 8.23.]

[sol]

(a) $q = \frac{T_m - T_c}{R_f + R_g + R_c} \Rightarrow q'' = \frac{T_m - T_c}{\pi T_f^2 H(R_f + R_g + R_c)}$

$\therefore q''' = \frac{4129.8 - 563}{\pi(0.0203)^2 x 14.583 x (4.96x10^{-3} + 5.247x10^{-4}) + 4.297x10^{-4}} = 3.19x10^7$ Btu / Pr-ft^3

(b) $(T_f)_{av} = \frac{\int Tdv}{\int dv} = \frac{\int_0^{r_f}(T_m - \frac{\iota}{4\Gamma_f}\gamma^2)\pi H dr}{\pi \Gamma_f^2 H} = \frac{2}{\Gamma_f^2}[\frac{T_m}{2} r^2 - \frac{q''}{16R_f} r^2]_0^{r_f}$

$= \mathbf{T_m - \frac{q''}{fR_s} r_f^2}$

= 4128.8 -3.19x10⁷ → $= \mathbf{4128.8 - 3.19x10^7 \; x \; 0.0203^2 x \frac{1}{8x1.1}}$ **= 2634.97°F= 1719 K = 1456℃**

From problem 8.8 v(u) = 1.65 x $\mathbf{10^7 \; cm^3}$**, ρ(u) =** $\mathbf{\frac{1.5x10^3}{1.55x10^7} = 9.091 \; gm/cm^3}$

$\therefore \mathbf{N(U^{235}) = \frac{0.0228x9.091x6.022x10^{23}}{235.0439} = 5.31 \; x \; 10^{23} \; \#/cm^3}$

$\mathbf{\Sigma_f = \frac{\sqrt{\pi}}{2} x0.88x5.31x10^{20}x582.2x10^{-24}x(\frac{293}{1719})^{1/2}}$ **= 9.954x10⁻²** $\mathbf{cm^{-1}}$

$\mathbf{q'''_{max} = (180MeV \; x \; 9.954x10^{-2}cm^{-1})^{-1}x3.19 \; x10^7}$ **Btu / Pr-ft³x** $\mathbf{\frac{6.40x10^{23}MeV/seccm^3}{Btu \, / \, Pr-ft^3}}$

= $\mathbf{1.15x10^{14}}$ **neutrons /** $\mathbf{cm^2}$ **sec**

25. A slab of iron 5 cm thick, forming part of the thermal shield of an LWR, is exposed to γ-rays having an average energy of 5 MeV and incident intensity of 1.5 × 10^{14} γ-rays/cm^2-sec. The side on which the radiation is incident is held at 585°F; the opposite side is at 505°F. (a) At what rate is energy deposited per ft^2 of the slab? (b) compute and plot the temperature distribution within the slab. (c) Calculate the heat fluxes at both faces of the slab. [Note: For simplicity, assume that the γ-ray absorption is exponential with μ = 0.18cm$_{-1}$.]

[sol]

a) $\mathbf{q''=\emptyset_r \cdot Er \cdot Ua = e^{-UaX} \cdot = \emptyset_{rx} \cdot Er \cdot Ua}$

$\mathbf{=e^{-0.18\cdot 5} \; x \; 1.5 \; x \; 10^{15}\#/cm^2 \cdot sec \; x \; 3MeV \; x \; 0.18cm^{-1} \; x \; [(1.548 \; x \; 10^{-8}Btu/hr\text{-}ft^3)/(MeV/cm^3 \cdot sec)]}$

$\mathbf{= 5.098 \; x \; 10^5 \; Btu/hr\text{-}ft^2}$

b) $\mathbf{J^2T=0}$

$\mathbf{d^2T/dx^2=0 \quad \rightarrow T=C_1X+C_2}$

B.C $\mathbf{T(0)=To \qquad \therefore C_2=T_2}$

$\mathbf{T(x\text{-}a)=Ta \qquad \therefore C_1=T(x\text{-}a)/a}$

$\mathbf{\therefore T=X \cdot (Ta = To)/a+To}$

c) $\mathbf{q''=-k\frac{dT}{dx}=k(To\text{-}Tx)/a}$

From the Table Ⅳ.6, k=≅10.5/hr-ft·°F

$\mathbf{\therefore q''=10.5(585\text{-}505)/5 \cdot 0.03281=5.12 \; x \; 103 \; Btu\text{-}ft^2}$

26. Verify that Eq. (8.59) is a solution to Eq. (8.58) and satisfies the given boundary conditions.

[sol]

$\mathbf{d^2T/dx^2+\frac{S}{K} \cdot e^{-UX}=0}$

$\mathbf{dT/dx= C_1+\frac{S}{Ku} \cdot e^{-UX}=0}$

$\mathbf{\rightarrow T= C_1X+C_2 - \frac{S}{Ku2} \cdot e^{-UX}}$

B.C i) $\mathbf{T(0)=T_1 \qquad T_1= C_2 - \frac{S}{Ku2}}$

ii) $\mathbf{T(a)=T_2 \qquad T_2= C_1 \, a+ C_2 - \frac{S}{Ku2} \cdot e^{-UX}}$

$\mathbf{T_1\text{-} T_{2=}\text{-} \frac{S}{Ku2} - C_1 \, a + \frac{S}{Ku2} \cdot e^{-UX}=\frac{S}{Ku2}(e^{-UX}\text{-}1)\text{-} C_1 \, a}$

$\mathbf{\therefore C_1=\frac{S}{Ku2a}(e^{-UX}\text{-}1) \div (T_2 - T_1)/a}$

$\mathbf{C_2 = T_1+\frac{S}{Ku2}}$

$\mathbf{\therefore T=[\frac{S}{Ku2a}(e^{-UX}\text{-}1) \div (T_2 - T_1)/a]x + T_1 + \frac{S}{Ku2} - \frac{S}{Ku2} \cdot e^{-UX}}$

27. γ-radiation is incident on a slab of thickness a. (a) Derive an expression giving the condition under which the temperature in the slab passes through a maximum. (b) In the special case where the two sides of slab are held at

the same temperature, show that the maximum temperature occurs at

$$x_m = \frac{a}{2}\left(1-\frac{\mu a}{12}\right)$$

Provided $\mu a \ll 1$.

[sol]

a) $\mathbf{d^2T/dx^2+\frac{S}{K}\cdot e^{-UX}=0}$

$\mathbf{dT/dx= C_1+\frac{S}{Ku}\cdot e^{-UX}}$ ……………………. ①

B.C. $\mathbf{dT/dx[x=x_m]=0}$ $\rightarrow \mathbf{C_1= -\frac{S}{Ku}\cdot e^{-UXm}}$

From ①

$\mathbf{T= C_1X+C_2 - \frac{S}{Ku2}\cdot e^{-UX}}$

B.C.

T(x=0) = $\mathbf{T_1}$ $\rightarrow \mathbf{C_2=\frac{S}{Ku2}}$

$\therefore \mathbf{T=-\frac{S}{Ku}\cdot e^{-UXm}\cdot X + T_1+\frac{S}{Ku2}(1- e^{-UX})}$

b) $\mathbf{T_1=T_2}$

$\mathbf{T(a)= T_2=T_{1=} T_1 \ \frac{S}{Ku2}(1- uxe^{-UXm}- e^{-UX})}$

$\mathbf{(1- uxe^{-UXm}- e^{-UX})=0}$

$\mathbf{e^{-UX}=1-ua+0.5(ua)^2-1/6\cdot(ua)^3}$

$\mathbf{1- e^{-UX}= ua-0.5(ua)^2+1/6\cdot(ua)^3}$

$\mathbf{uxe^{-UXm}=1- e^{-UX}= ua-0.5(ua)^2+1/6\cdot(ua)^3}$

$\mathbf{e^{-UXm}=1-ua+0.5(ua)-1/6\cdot(ua)^2}$

$\mathbf{-UXm=\ln[1-ua+0.5(ua)-1/6\cdot(ua)^2]}$

From $\mathbf{\ln(1+X)=X-X^2/2!+2X^3/3!\cdots\cdots}$

$\mathbf{\ln[1-ua+0.5(ua)-1/6\cdot(ua)^2]= -0.5(ua)+1/6\cdot(ua)^2 -0.5[-0.5(ua)+1/6\cdot(ua)^2]^2}$

$\mathbf{= -0.5(ua)+1/6\cdot(ua)^2-1/8\cdot(ua)^2}$

$\mathbf{= -0.5(ua) -1/24\cdot(ua)^2}$

$\therefore \mathbf{Xm=-1/u[-0.5(ua) -1/24\cdot(ua)^2]=a/2(1-ua/12)}$

28. The velocity profile 1,000-psia water flowing through an insulated 8-in I.D.(inner diameter) coolant pipe of a research reactor can be fit by the expression

$$V = 60-540r^2$$

Where T is in °F. (a) Compute the bulk temperature of the water. (b) compute the average velocity of the water in the pipe. (c) Compute the value of the Reynolds number. (d) How much water, in lb/hr, is flowing through the pipe? The water density is constant across the pipe at 61.8 lb/ft^3 and μ = 1.270 lb/hr-ft.

[sol]

a) If ρ, Cp are constans

$\mathbf{Tb= \int \rho\, Cp\, v\, T\, dAc \div \int \rho\, Cp\, v\ dAc=\int v\, T\, dAc \div \int v\ dAc}$

$\mathbf{= \int_0^{rc} 2\pi rvTdr/\int_0^{rc} 2\pi rvdr}$ $\mathbf{(rc=8/2\pi\cdot ft/12\pi=0.333ft)}$

$\mathbf{=\int_0^{0.333} r(50-540r2)(155-720r2)rdr \div \int_0^{0.333} r(50-540r2)dr}$

$\mathbf{=\int_0^{0.333}(1-9r2)(155-720r2)rdr \div \int_0^{rc0.333}(1-9r2)dr}$

$\mathbf{=\int_0^{0.333}(155r-245r3+6480r5)dr) \div \int_0^{rc0.333}(1-9r3)dr}$

$\mathbf{=[155/2r^2-245/4(0.333)^2+6480(0.333)^4]\div[1/2-9/4(0.333)^2}$

$\mathbf{= 128.33°F}$

b) $\mathbf{V= \int v\, dAc \div \int dAc=\int_0^{0.333}(50-540r2)rdr \div \int_0^{0.333} r\, dr}$

$\mathbf{=[30r2-135r4]_0^{0.333} \div [1/2r2]_0^{0.333}=30ft/sec=108000\ ft/hr}$

c) u=1.270 lb/hr-ft

ρ=61.8lb/ft^3

$D_2=4\times\frac{\pi r2}{2\pi rc}$=2rc=0.667ft

Re=$\frac{\rho vDe}{u}=\frac{61.8\cdot108000\cdot0.667}{1.270}$=3503622

d) w= ρ v Ac =π ρ v rc^2

= π x 61.8 x 108000 x (0.333)^2

= 2.33 x 10^{6}lb/hr

29.compute and plot the temperatures of the water, the outer surface of the cladding, and the fuel center as a function of distance along the hottest channel of the reactor described in examples 8.3 and 8.5 under the conditions given in Example 8.7.

[sol]

H= 12ft

r_f=0.42/2 x 1/12=0.0175ft

r_c= (0.42+0.024)/2 x (1/2)=0.0185ft

Tm=3970°F

Tt=543°F

Kf=1.1 Btu/Hr-ft-°F

Kc=3.0 Btu/Hr-ft-°F

Tm-Tf=qR_f $\quad$ $R_f=1/4\pi HK_f)$

Tf-Tc=qR_c $\quad$ $R_c=\ln(r_c/r_f)/2\pi HK_c)$

Tc-Tb=qR_b $\quad$ $R_b=1/2\pi K_b r_b H)$

∴ Tm-Tb=q($R_f-R_c-R_b$)

R_f=6.0286 x 10^{-3}°F-hr/Btu

R_c=2.4567 x 10^{-3}°F-hr/Btu

q$^{”}$=4.66 x 10^7 Btu/hr-ft^3

q= πr_f^2H q$^{”}$ =π(0.0175)^2 x 12 x 4.66 x 10^7=5.38 x 10^{5}Btu/hr

3970-543=5.38 x 10^5 x (6.0286 x 10^{-3}+2.4567 x 10^{-2}+R_b)

∴ R_b=9.5618 x 10^{-5} °F-hr/Btu

h_b=1(1/2π x 0.0185 x 12 x 9.5618 x 10^{-3})=7.4977 x 103Btu/hr-ft^2

Tc=Tb+qRo=543+5.38 x 10^5 x 9.5618 x 10^{-5}

=594.4°F

Tf=Tc+qRc=594.4+5.38 x 10^5 x 2.4567 x 10^{-4}=726.6°F

30. For the hottest channel in the reactor described in problem 8.9, Calculate the (a) exit temperature of the sodium; (b) maximum temperatures of the fuel and the cladding surface. [Note: Take the heat transfer coefficient to be 35,000 Btu/hr-ft^2-°F.]

[sol]

Q=m△h=△H-ρAc△ZCp△Tb

Q-(dQ/dt)=q= m△h

= ρAc(△Z/△t)Cp△Tb= ρ v Ac Cp △Tb

q=q”AfdZ=q’$_f$dz=mCp△Tb $\quad$ ∴mCp(dTb/dz)=q’

If $\quad$ q’(z)= q’$_{max}$cos($\frac{\pi z}{H}$) $\quad$, $\quad$ mCp($\frac{d\pi z}{dH}$)= q’$_{max}$cos($\frac{\pi z}{H}$)

$T_b(z)-T_{bm}=\int_{-H/2}^{z}$ $(q'_{max}/mCp)\cos(\frac{\pi z}{H})dz$

$\therefore$ $T_b(z)= T_{bm,}+(H\cdot q'_{max}/mCp\cdot\pi)[\sin(\frac{\pi z}{H})+1]$ (m=ω)

$T_b(z)= T_{bm}+(H\cdot q'_{max}/\ \omega\ Cp\cdot\pi)[\sin(\frac{\pi z}{H})+1]$

a)$T_b = T_{b\ max}=T_b(\frac{H}{2})=T_{bm}+2q''_{max}V_f/\ \pi\ \omega\ Cp$

$r_f=(0.158/2)\cdot 1/12=6.583 \times 10^{-3}$ ft

H=30.5/12=2.542ft

$\therefore V_f= \pi\ r_f^2\ H= \pi(6.583 \times 10^{-3})^2 \times 2.542=3.46 \times 10^{-4}ft^3$

$r_c=(0.158/12)/0.005\cdot(1/12)=7 \times 10^{-3}ft$

$Ac=(0.21/12)^2 - \pi(7 \times 10^{-3})=1.523 \times 10^{-4}\ ft^2$

ρ(Na)=55.06 lb/ ft^3

V=31.2ft/sec

∵ ω= ρ VAc=55.06 x 31.2 x 1.523 x 10^{-4}=0.262lb/swc=943.2lb/hr(per channel)

Cp=0.3116 Btu/lb-°F from table Ⅳ.5

$q''_{max}=6.663 \times 10^8$Btu/hr- t^3

$T_{bm,}$=572°F

$\therefore T_b=572\div(2 \times\ 6.66310^8 \times 3.46 \times 10^{-4})(\ \pi \times 943.2 \times 0.3116)=1071.4$°F

b) $T_{c,max}= T_{bm}+ q''_{max}\ V_fR_h[(1+\sqrt{1+\alpha 2})/\ \alpha]=$

$R_h=1/hA=1/(35000 \times 2\pi \times 7 \times 10^{-3} \times 2.542)=2.556 \times 10^{-4}$°F-hr/Btu

$\alpha= \pi\ \omega\ C_p\ R_h=\pi \times 943.2 \times 0.3116 \times 2.556 \times 10^{-4}=0.236$

$T_{c\ max}=572+6.663 \times 10^8 \times 3.46^{-4} \times 2.556 \times 10^{-4}[(1+\sqrt{}1+2.036^2)/2.036]=1078.2$°F

$T_{m\ max}= T_{bm}+ q''_{max}\ V_fR_h[(1+\sqrt{1+\beta 2})/\ \beta]$

$R_f=1/4\ \pi H\ k_f=1/(4\pi \times 2.542 \times 20)=1.565 \times 10^{-3}$°F-hr/Btu

$R_c=\ln(r_c\div\ r_f)\div 2\pi H\ k_f= \ln(7\div\ 6.583)\div 2\pi \times 2.542 \times 10=3.845 \times 10^{-4}$°F-hr/Btu

$\therefore R=R_f+R_c+R_h=2.205 \times 10^{-3}$°F-hr/Btu, $\beta= \pi\ \omega\ C_p\ R= \pi \times 943.2 \times 0.3116 \times 2.205 \times 10^{-3}=2.036$

$\therefore T_{m,\ max}=572+6.663 \times 10^8 \times 3.46 \times 10^{-4} \times 2.205 \times 10^{-3}[(1+\sqrt{}1+2.036^2)/2.036]=1388$°F

31. For the hottest channel in the reactor described in problem 8.7, Calculate, calculate, assuming the reactor is bare, the (a) exit temperature of the water; (b) maximum temperatures of the fuel and the cladding surface. [Note: Take h = 7,600 Btu/hr- ft^2-°F.]

[sol]

a) $T_{b\ max}= T_{bm,}+ 2q''_{max}\ V_f\ /\ \pi\ \omega\ C_p$

$T_{bm,}$=496°F $q''_{max} = 1.3 \times 10^9$ Btu/hr-ft^3

H=91.9in=7.658ft

r_f=0.298/2 in =1.242 x 10^2ft , Vf= π x (1.242 x 10^2) x 7.658=3.709 x 10^5ft^3

ω=3.4 x 10^6lb/hr ÷23142=1469.19 lb/hr (per channel)

Cp=1.15 Btu/lb°F

∴) $T_{b\ max}=496+(2 \times 1.3 \times 10^7 \times 3.709 \times 10^{-3})/(\ \pi \times 1469.19 \times 1.15)=514.2$°F

b) $T_{c\ max}= T_{bm,}+ + q''_{max}\ V_fR_h[(1+\sqrt{1+\alpha 2})/\ \alpha]$

$R_h=1/hA=1/[7600 \times 2\pi \times (0.298/2+0.021)\div\ 12 \times 7.658=1.93 \times 10^{-4}$°F-hr/Btu

$\alpha= \pi\ \omega\ C_p\ R_h= \pi \times 1469.19 \times 1.15 \times 1.93 \times 10^{-4}=1.025$

$T_{c\ max}=496+1.3 \times 10^7 \times 3.709 \times 10^{-3} \times 1.93 \times 10^{-4}[(1+\sqrt{}1+1.025^2)/1.025]=518.1$°F

c) $T_{m\,max} = T_{bm,+} + q''_{max} V_f R[(1+\sqrt{1+\beta 2})/\beta]$

$R_f = 1/4\pi H k_f = 1/(4\pi \times 7.658 \times 2.5) = 4.157 \times 10^{-3}$ °F-hr/Btu

$R_c = \ln(r_c \div r_f) \div 2\pi H k_f = \ln\{[[(0.298/2)+0.021] \div (0.298/2)]\} \div 2\pi \times 7.658 \times 10 = 2.74 \times 10^{-4}$ °F-hr/Btu

$K_f = 2.5$ **Btu/hr-ft-°F**

$K_c = 10$ **Btu/hr-ft-°F(for zircaloy)→ Assuming cladding is zircaloy**

$\therefore\ R = R_f + R_c + R_h = 4.624 \times 10^{-3}$ °F-hr/Btu

$\beta = \pi\,\omega\,C_p\,R = \pi \times 1469.19 \times 1.15 \times 4.624 \times 10^{-3} = 24.54$

$\therefore\ T_{c\,max} = 496 + 1.3 \times 10^7 \times 3.709 \times 10^{-3} \times 1.93 \times 10^{-4}[(1+\sqrt{1+24.54^2})/24.54] = 728.2$°F

32. The heat flux at the surface of fuel rods in an experimental reactor varies with position as indicated in Fig. 8.15. The fuel rods are 0.5 in. in diameter and 6 ft long. Water enters the reactor core at a temperature of 150F and flows through the core at a rate of 350 lb/hr per rod. The heat transfer coefficient is 1,000 Btu/hr-ft2-F. Calculate the temperatures of the coolant, fuel rod surface, and fuel rod center at the entrance of the channel, 3 ft up the channel, and at its exit.

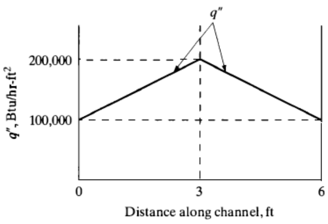


Figure 8.15 Heat flux v. distance for the fuel rod in Prob. 8.32.

[sol]

$$T_{bFa} = 150\text{°F}, w = 350\frac{lb}{hr}\ \text{per rod}, h = 1000\ Btu/hrft^2\text{°F}$$
$$v_f = \pi(\frac{0.5}{2} * \frac{1}{12})^2 * 6 = 8.181 * 10^3 ft^3$$
$$g = \pi a H g'''$$

33. Water at about 2,000 psia and 613F leaves a PWR vessel vis a 29-in I.D. primary coolant pipe with an average flow velocity of 4935 ft/sec. (a) What is the mass flow rate this pipe in lb/hr? (b) What is the Reynolds number for the water in the pipe? (c) If the water returns to the reactor vessel at the same pressure and velocity but at 555F, how large should the return pipe be?

[sol]

$$R = \frac{29}{2} * \frac{1}{12} = 1.208\ ft$$
$$a)\ \rho_{at\ 613\text{°F}} = 41.63\ lb/ft^3$$
$$\therefore w = \rho v A = 41.63 * 49.5 * \pi(1.208)^2 = 9452.24\frac{lb}{sec} = 3.404 * 10^7\ lb/hr$$
$$b)\ D_e = 4 * \frac{\pi R^2}{2\pi R} = 2R = 2.417\ ft$$
$$\mu = 0.206\frac{lb}{hrft}, \qquad \therefore Re = \frac{D_e v \rho}{\mu} = 8.704 * 10^7$$
$$c)\ \rho_{at\ 555\text{°F}} = 46.37\ lb/ft^3$$
$$x = 27.48\ in\ I.D.$$

34. Convert each of the factors in the computation on the Reynolds number given in Example 8.8 under the conditions given in that example.

[sol]

$$Pitch = 1.524 * 10^{-2}\ m$$

$$\mathbf{radius\ of\ the\ fuel\ rod = 0.21 + 0.024 = 0.234\ in = 5.94 * 10^{-3} m}$$
$$\therefore \mathbf{D_e = 0.013m}$$
$$\mathbf{v = 15.6\frac{ft}{sec} = 4.754\frac{m}{sec}, \mu = 8.764 * \frac{10^{-5}kg}{msec}, \rho = 42.9\frac{lb}{ft^3} = 687.\frac{127kg}{m^3}},$$
$$\therefore \mathbf{Re = 485000}$$

35. Calculate the mass flow rate in lb/hr at which the coolant passes up a single channel of the lattice described in Example 8.8 under the conditions given in that example.

[sol]

$$\mathbf{v = 5616.\frac{0ft}{hr}}$$
$$\mathbf{A_s = (\frac{0.6}{12})^2 - \pi(\frac{0.234}{12})^2 = 0.015665ft^2}$$
$$\mathbf{\rho = 42.9\frac{lb}{ft^3}}$$
$$\mathbf{w = 37824\ lb/hr}$$

36. For the reactor in Problem 8.7, compute the (a) average flow velocity in ft/sec; (b) equivalent channel diameter; (c) Reynolds number entering and leaving the hottest channel; (d) heat transfer coefficient at the locations in part (c).

[sol]

$$\mathbf{a)\ A_s = 15.7753ft^2}$$
$$\therefore \mathbf{v = \frac{w}{\rho A_s} = \frac{34*10^6}{50*15.77} = 43105\ ft/hr}$$
$$\mathbf{b)\ D_B = 4 * \frac{S^2 - \pi R^2}{2\pi r} = 4 * \frac{6.8167*10^{-2}}{2\pi*0.1595*\frac{1}{12}} = 0.03265ft}$$
$$\mathbf{c) Re = 264400}$$
$$\mathbf{d)\ P_r = 0.851,\ C{=}0.0311}$$
$$\therefore \mathbf{h = C\left(\frac{k}{D_e}\right) Re^m Pr^n = 7034\ Btu/hrft^2}{}^\circ\mathrm{F}$$

37. For the LMFBR core described in Problem 8.9, calculate the; (a) equivalent diameter of a coolant channel; (b) total coolant flow area; (c) sodium flow rate in lb/hr; (d) average sodium exit temperature; (e) Reynolds number for the sodium entering and leaving (at the temperature in part [d]) the core; (f) average heat transfer coefficient for the sodium entering and leaving the core.

[sol]

$$\mathbf{a)\ D_e = 0.01645\ ft}$$
$$\mathbf{b) A_T = 2.229ft^2}$$
$$\mathbf{c)\ w = \rho v A_T = 55.06 * 112320 * 2.229 = 13.785 * 10^6 lb/hr}$$
$$\mathbf{d)\ h_{exit} = 399.6\frac{Btu}{lb},\ (T_{exit})_{av} = 812}{}^\circ\mathrm{F}$$
$$\mathbf{e)\ Re = 121835}$$
$$\mathbf{f)\ k = 43.75, Pr = 0.0059, C = 0.03119, \therefore h = 124676\ Btu/hrft^2}{}^\circ\mathrm{F}$$

38. Water enters the core of the BWR described in Problem 8.8 at 525F and flows through the core at a rate of $106.5*10^6$ lb/hr. The heat production rate in the fuel along the hottest channel can be approximated as

$$q'''(z) = q'''_{max}\cos(\pi z/165)$$

where z is in inches. The reactor pressure is 1,020 psia. Compute the (a) maximum-to-average power density in the z direction; (b) total flow area in the core; (c) average water velocity near the bottom of the core; (d) water/fuel volume ratio; (e) equivalent diameter of a coolant channel; (f) Reynolds number near the entrance to the hottest channel; (g) heat transfer coefficient for the convective part of the hottest channel; (h) location of the onset of local boiling.

[sol]

$$\mathbf{a)\ \frac{q'''_{max}}{q'''_{av}} = 1.399}$$
$$\mathbf{b)\ (A_c)_T = 76.873ft^2}$$

c) $\mathbf{V = \frac{w}{\rho A_{cT}} = 2.91 * 10^4 ft/hr}$
d) $\mathbf{\frac{v_w}{v_F} = 1.188}$
e) $\mathbf{D_e = 0.0469\ ft}$
f) $\mathbf{Re = 267454}$
g) $\mathbf{Pr = 0.875, C = 0.0308,\ \therefore h = 4678\ Btu/hrft^2}$°F

39. Referring to the PWR described in Examples 8.3 through 8.9: (a) Does boiling of any type occur in this reactor? (b) Does bulk boiling occur? (c) Does boiling occur in a channel 30 in from the axis of the reactor?

[sol] (a) PWR →150bar=2176psia

$$\frac{2200-2176}{2200-2100} = \frac{649.64-x}{649.64-642.95}, \qquad \therefore T_{sat} = 642.95°F$$

$$T_{b,max} = 626°F, \qquad T_{c,max} = 649°F$$

$\therefore T_{b,max} < T_{sat} < T_{c,max}$ → **Boiling occurs partially.**

(b) $T_{LB} = T_{sat} + (T_c - T_{sat}) - \frac{q"}{h}$

←(eq. 8.5) $q" = 3.66 \times 10^5 Btu/hr \cdot ft^2$

(eq. 8/9) $h = 7436 Btu/hr \cdot ft^2 \cdot °F$

$$= 642.95°F + \left(649°F - 642.95°F\right) - \frac{3.66\times10^5 Btu/hr\cdot ft^2}{7436 Btu/hr\cdot ft^2\cdot °F} = 599.78°F$$

$\therefore T_{LB} < T_{sat}$ → **No bulk boiling**

(c) $q"(z) = \frac{a^2}{2(a+b)} q'''(z)$

$$\leftarrow J_o\left(\frac{2.405r}{\tilde{R}}\right) = J_o\left(\frac{2.405\times30}{67}\right) = 0.7616$$

$$q'''(30in) = (4.66\times10^7 Btu/hr\cdot ft^3)(0.7616) = 3.549\times10^7 Btu/hr\cdot ft^3$$

$$q"(30in) = \frac{0.0175^2}{2(0.0175+0.002)} 3.549\times10^7 Btu/hr\cdot ft^3$$

$$= 0.028\times10^7 Btu/hr\cdot ft^3$$

$$T_{LB} = T_{sat} + (T_c - T_{sat}) - \frac{q"}{h}$$

$$= 642.95°F + \left(649°F - 642.95°F\right) - \frac{0.028\times10^7 Btu/hr\cdot ft^3}{7436 Btu/hr\cdot ft^2\cdot °F}$$

$$= 611.35°F$$

$\therefore T_{LB.30in} < T_{sat}$ → **The boiling in a channel 30in from the axis of the reactor doesn't occur.**

40. Starting with the curves of q" and T_b shown in Fig. 8.12, calculate the curves T_c and $T_c - T_b$.

[sol] $q" = h(T_c - T_b)$

$$\therefore T_c = T_b + \frac{q"}{h}, \qquad (T_c - T_b) = \frac{q"}{h}$$

41. (a) Using the results of Problem 8.31 and the Bernath correlation, compute and plot q_c" as a function of position along the hottest channel of the reactor in Problem 8.7. (b) Compute and plot the DNBR for this channel. (c) What is the minimum DNBR?

[sol] $T_{wc} = 102.6 \ln P - \frac{97.2P}{P+15} - 0.45v + 32$

←PWR 150bar≈ 2176psia

V=11.97ft/s

$$= 718.58°F$$

$$h_c = 10890\left(\frac{D_e}{D_e+D_i}\right) + \frac{48v}{D_e^{0.6}}$$

$$\leftarrow D_e = 2\times\frac{s^2-\pi a^2}{\pi a}\ ,\quad a = 0.149\text{in} + 0.021\text{in} = 0.17\text{in}, s = 0.422\text{in}$$
$$= 0.3269\text{in} = 0.027\text{ft}$$
$$D_i = 2a = 0.34\text{in} = 0.028\text{ft}$$
$$= 10363.84\text{Btu/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$q_c'' = h(T_c - T_b)$$
$$\leftarrow T_b = T_{b0} + \frac{q'''_{max}V_f}{\pi w C_p}\left[1+\sin\left(\frac{\pi z}{\tilde{H}}\right)\right]$$
$$= 505.284 + 9.084\sin(0.0342z)^\circ\text{F}$$
$$= 2132007.8 - 94145.12\sin(0.0342z)\ \text{Btu/hr}\cdot\text{ft}^2$$

(b) $\text{DNBR} = \frac{q_c''}{q''_{actual}},\ q''_{actual} = \frac{q''_{av}A_f}{C_c}$

$$\leftarrow q''_{actual} = 1.923\times10^7\text{Btu/hr}\cdot\text{ft}^3$$
$$A_f = 0.000484\text{ft}^2$$
$$C_c = 0.078\text{ft}$$
$$\therefore\ q''_{actual} = 119324.62\text{Btu/hr}\cdot\text{ft}^2$$
$$\text{DNBR} = 17.87 - 0.79\sin(0.0342z)$$

(c) $\text{DNBR}_{min}\ \rightarrow z = 45.95$

$\therefore \text{DNBR}_{min} = 17$

42. Figure 8.16 shows the flow quality as a function of position along the hot channel of the reactor in Fig. 8.12. The mass flux for this channel is $2.3\times10^6\text{lb/hrft}^2$. (a) Compute and plot q_c'' and the DNBR along the channel. (b) What is the minimum DNBR?

[sol] (a) $\chi_1 = 0.197 - 0.108\left(\frac{G}{10^6}\right) = -0.0514$

$$\chi_2 = 0.254 - 0.026\left(\frac{G}{10^6}\right) = 0.1942$$
$$\therefore \chi_1 < \chi < \chi_2\quad\rightarrow \text{eq.}(8.104)$$

$$\frac{q_c''}{10^6} = 1.364 - 0.270\left(\frac{G}{10^6}\right) - 4.710\chi$$
$$q_c'' = 10^6[1.013 - 4.710]\chi$$
$$\leftarrow \text{Fig. 8.16},\ \chi = -0.01 + 0.08[1-\cos(0.02z)]$$

$$\therefore \text{DNBR} = \frac{10^6\left[1.013 - 4.710[-0.01+0.08[1-\cos(0.02z)]]\right]}{\left[\frac{q'''_{max}A_f}{C_c}\right]\cos\left(\frac{\pi z}{\tilde{H}}\right)}$$

(b) $\text{DNBR}_{min} \rightarrow z: 140\ (\text{max value})$

$$\therefore \text{DNBR}_{min} = \frac{0.375\times10^6}{\left[\frac{q'''_{max}A_f}{C_c}\right]\cos\left(\frac{\pi z}{\tilde{H}}\right)}$$

43. The maximum heat flux in the HTGR described in Problem 8.6 is $120{,}000\text{Btu/hrft}^2$ and the hot channel factor is 2.67. (a) What is the average heat flux? (b) What is the total heat transfer area?

[sol] (a) $F = 2.67 = \frac{q''_{max}}{q''_{av}},\qquad \therefore q''_{av} = 44943.82\text{Btu/hr}\cdot\text{ft}^2$

(b) $q''_{av} = \frac{P}{A}$

$$\leftarrow (\text{Problem 8.6})P_e = 330\text{MW},\quad \text{efficiency} = 0.3923\ (39.23\%)$$

$$P_t = \frac{330MW}{0.3923} = 2.87 \times 10^9 Btu/hr$$
$$\therefore A = \frac{P}{q''_{av}} = 63857.50ft^2$$

44. The hot channel factor for the LMFBR in Problem 8.9 is 1.85. Calculate the: (a) total heat transfer area in the core; (b) average heat flux; (c) maximum heat flux.

[sol] (a) $P_t = 270MW = 9.22 \times 10^8 Btu/hr$

$A = \pi DL \times (number) = \pi \times (0.158in) \times (30.5in) \times 13104 = 1377.68ft^2$

(b) $\therefore q''_{av} = \frac{P}{A} = 669241.04 Btu/hr \cdot ft^2$

(c) $\therefore F = \frac{q''_{max}}{q''_{av}}, \quad q''_{max} = F \cdot q''_{av} = 1.24 \times 10^6 Btu/hr \cdot ft^2$

45. The power distribution in the axial direction of the 12ft long fuel rods of a PWR is well represented by

$$q''' = constant \times \cos(\pi z/144),$$

where z is in inches. The minimum DNBR is 2.1 and occurs 90 in up the hottest channel, where the critical heat flux is computed to be $1.8 \times 10^6 Btu/hrft^2$. The hot channel factor for the reactor is 2.78, and the total heat transfer area is 48,000ft^2. Calculate (a) q_{max}'''; (b) q_{av}''; (c) the operating power of the reactor.

[sol] (a) $DNBR = \frac{q''_c}{q''_{actual}} \quad \rightarrow q''_{actual} = q''_c \times DNBR = 2.268 \times 10^6 Btu/hr \cdot ft^2$

(b) $q''_{av} = \frac{q''_{max}}{F} = 0.82 \times 10^6 Btu/hr \cdot ft^2$

(c) $P = q''_{av} \times A = (0.82 \times 10^6 Btu/hr \cdot ft^2)(48000ft^2) = 3.92 \times 10^{10} Btu/h$