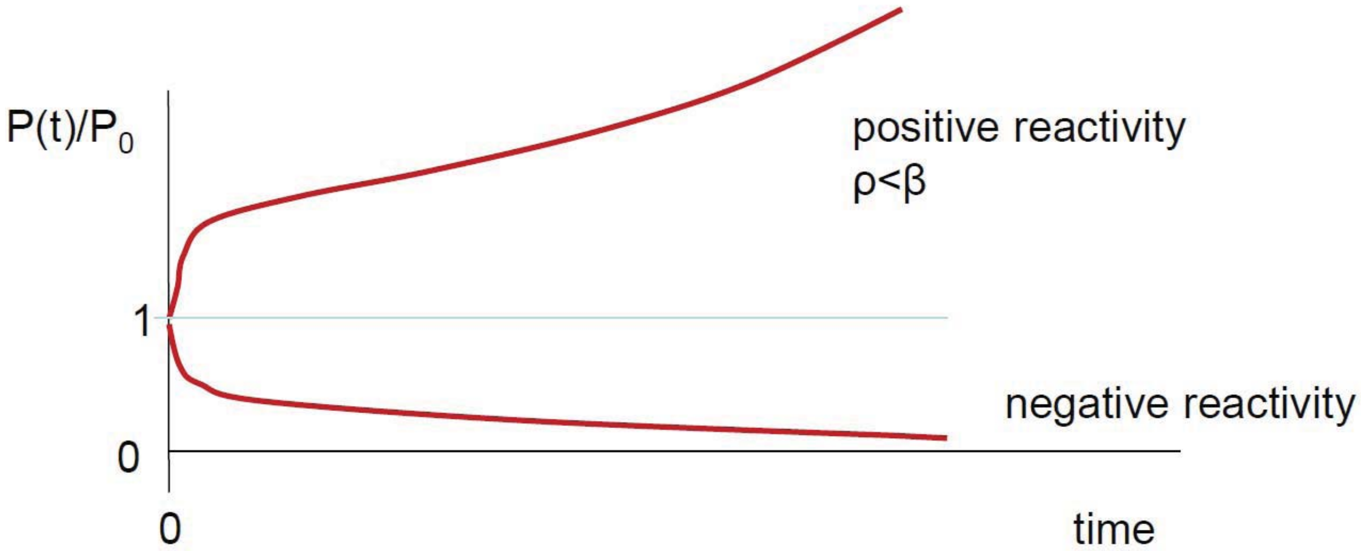
SH2706
Sustainable Energy Transformation Technologies

Exercise Session 04

E04_P01

The reactivity in a steady-state thermal reactor with no external neutron sources, in which the neutron generation time is 1.0e-3 (s), is suddenly made positive and equal to 0.0022. Assuming one group of delayed neutrons (Lambda = 0.08 1/s, Beta = 6.5e-3), determine the reactor power increase in percent of the initial power after time 1 (s).

If the positive reactivity is equal to 0.0075, what is the power increase?



Reactor Kinetics

- Reactor kinetics
  - Reactivity is only a function of time (affected by control system, noise or disturbances)
  - Reactivity will change from zero if:
    - boron concentration in coolant is changed
    - control rod position is changed
    - density of moderator is changed
  - Reactivity is changing steadily during reactor operation due to:
    - fuel burn-up (less and less fissile materials are in the core)
    - burnable absorbers (poisons)
    - reactor poisoning (accumulation of Xenon-135 and Samarium-149)

Point Reactor Kinetics

Point Reactor Kinetics Model

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \sum_{i=1}^{6} \lambda_i C_i + S$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i C_i , \quad i = 1, \ldots ,6$$

Neutron balance equation: *n* – neutron concentration in reactor

Balance equation for concentration of precursors of delayed neutrons: C_i – precursor-*i* concentration

ρ – reactivity, β_i – yield of precursor *i*, λ_i – decay constant of precursor *i*
Λ – average neutron generation time,
S – neutron sources

$$\beta = \sum_{i=1}^{6} \beta_i$$

$$\frac{\beta}{\lambda} = \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} \Rightarrow \lambda = \frac{\beta}{\sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}}$$

One group approximation

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda C + S$$

$$\frac{dC}{dt} = \frac{\beta}{\Lambda} n - \lambda C$$

One Group Approximation

One group approximation has an exact analytical solution for a step change of reactivity!

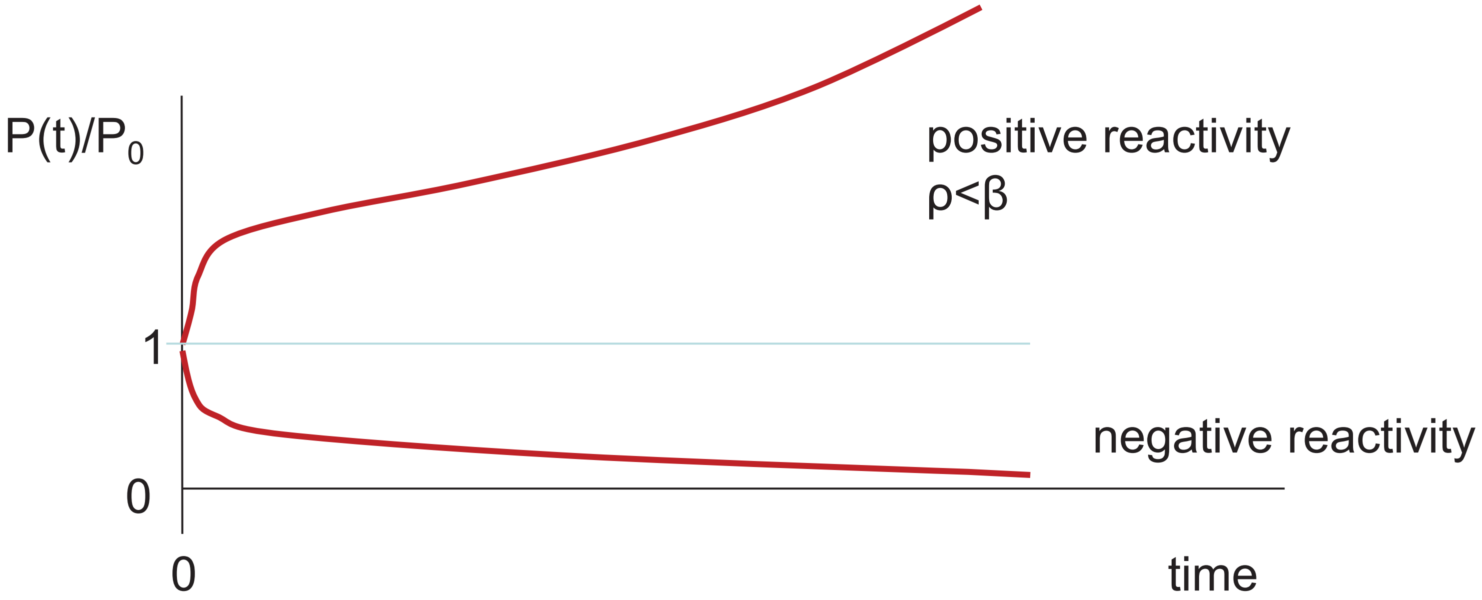
$$x(t) \equiv \frac{n(t) - n_0}{n_0} = \frac{\rho_0}{\Lambda}\left[\frac{\lambda}{s_1 s_2} + \frac{s_1 + \lambda}{s_1(s_1 - s_2)} e^{s_1 t} + \frac{s_2 + \lambda}{s_2(s_2 - s_1)} e^{s_2 t}\right]$$

where $$s_{1,2} = \frac{-\left(\frac{\beta}{\Lambda} - \frac{\rho_0}{\Lambda} + \lambda\right) \pm \sqrt{\left(\frac{\beta}{\Lambda} - \frac{\rho_0}{\Lambda} + \lambda\right)^2 + 4\frac{\lambda \rho_0}{\Lambda}}}{2}$$

n_0 – neutron concentration at time t = 0, ρ_0 – step change of reactivity at time 0 ; *x(t)* – relative change of neutron concentration (or power) in reactor

Reactor Power Change

- Reactor power change:
  - Reactor power change P(t) after step change of reactivity is as shown below (P_0 – initial power)



E04_P01

The reactivity in a steady-state thermal reactor with no external neutron sources, in which the neutron generation time is 1.0e-3 (s), is suddenly made positive and equal to 0.0022. Assuming one group of delayed neutrons (Lambda = 0.08 1/s, Beta = 6.5e-3), determine the reactor power increase in percent of the initial power after time 1 (s).

If the positive reactivity is equal to 0.0075, what is the power increase?

One group approximation

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda C + S$$

$$\frac{dC}{dt} = \frac{\beta}{\Lambda} n - \lambda C$$

Solution

s1 = 0.0398

s2 = -4.4198

x = 53.89%

One group approximation has an exact analytical solution for a step change of reactivity!

$$x(t) \equiv \frac{n(t) - n_0}{n_0} = \frac{\rho_0}{\Lambda}\left[\frac{\lambda}{s_1 s_2} + \frac{s_1 + \lambda}{s_1(s_1 - s_2)} e^{s_1 t} + \frac{s_2 + \lambda}{s_2(s_2 - s_1)} e^{s_2 t}\right]$$

where $$s_{1,2} = \frac{-\left(\frac{\beta}{\Lambda} - \frac{\rho_0}{\Lambda} + \lambda\right) \pm \sqrt{\left(\frac{\beta}{\Lambda} - \frac{\rho_0}{\Lambda} + \lambda\right)^2 + 4\frac{\lambda \rho_0}{\Lambda}}}{2}$$

If rho = 0.0075

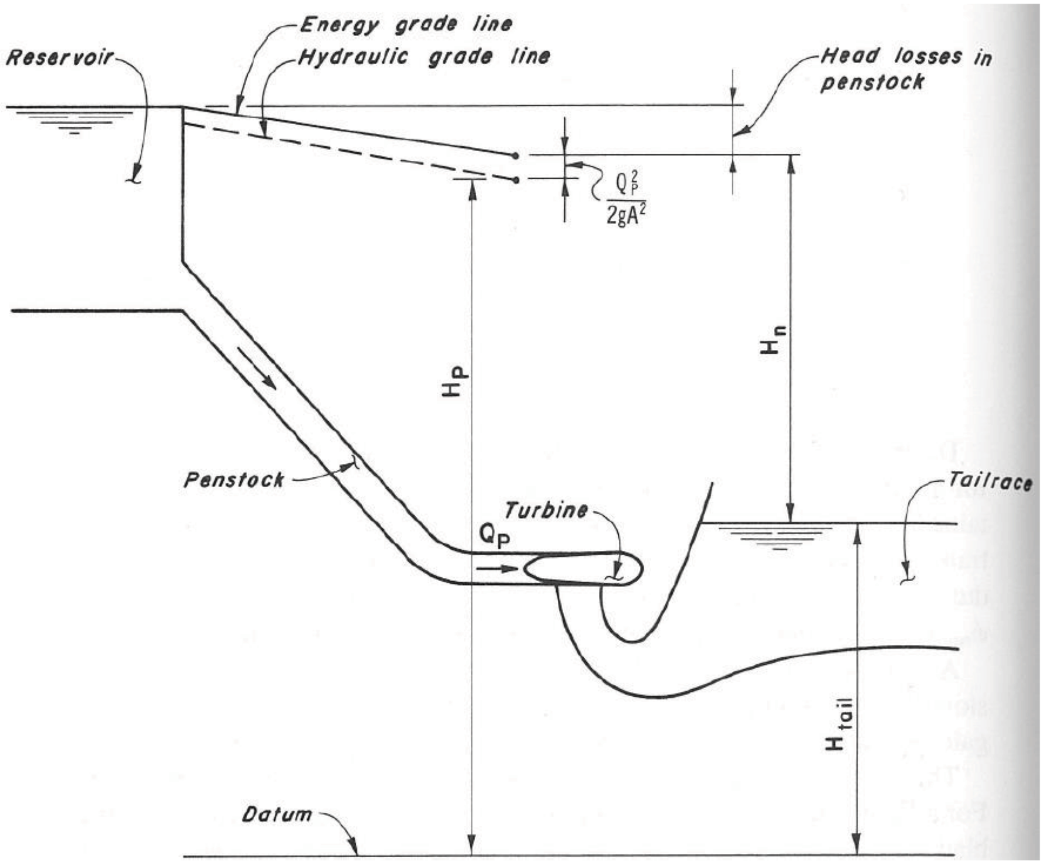
s1 = 1.3609

s2 = -0.4409

x = 14.

E04_P02

A hydraulic turbine with known efficiency 88.5% is installed in a hydropower plant with penstock cross-section area 2.25 (m^2). The net head of the hydropower is 20 (m) and the water flow velocity in the penstock is 1 (m/s). Calculate the mechanical power output of the turbine. Assume water density rho = 1000 kg/m^3.



Introduction (2)

- Hydro energy is utilised in hydro power stations that capture water flowing from a height, either in rivers, water falls or artificial dams
- The power that can be captured (hydraulic power) is:

$$N_h = \rho g Q H$$

  - where: ρ – water density (kg/m^3), g = 9.81 m/s^2, Q – volumetric flow rate of water (m^3/s) and H is the drop height (m)

- Natural ranges of Q and H are wide:
  - Niagara Falls: Q = 1420 m^3/s, H = 52 m
  - Val Strem: Q = 0.71 m^3/s, H = 216 m

Turbine Efficiency (1)

For hydraulic turbines the efficiency is defined as:

$$\eta_t \equiv \frac{N_m}{N_h} = \frac{\omega T_s}{\rho Q g H}$$

Here $\mathbf{N_h}$ is the so-called **hydraulic power** determined from the first law of thermodynamics, and ρ is the fluid density.
Correspondingly, $\mathbf{N_m}$ is called a mechanical power, that is the actual power measured on the turbine shaft. $\mathbf{N_m}$ is less than $\mathbf{N_h}$ due to friction and flow losses within turbine. Thus $\eta_t < 1$.
H in the efficiency equation is always the net head of the system

Large turbines can achieve efficiencies of 96% or greater

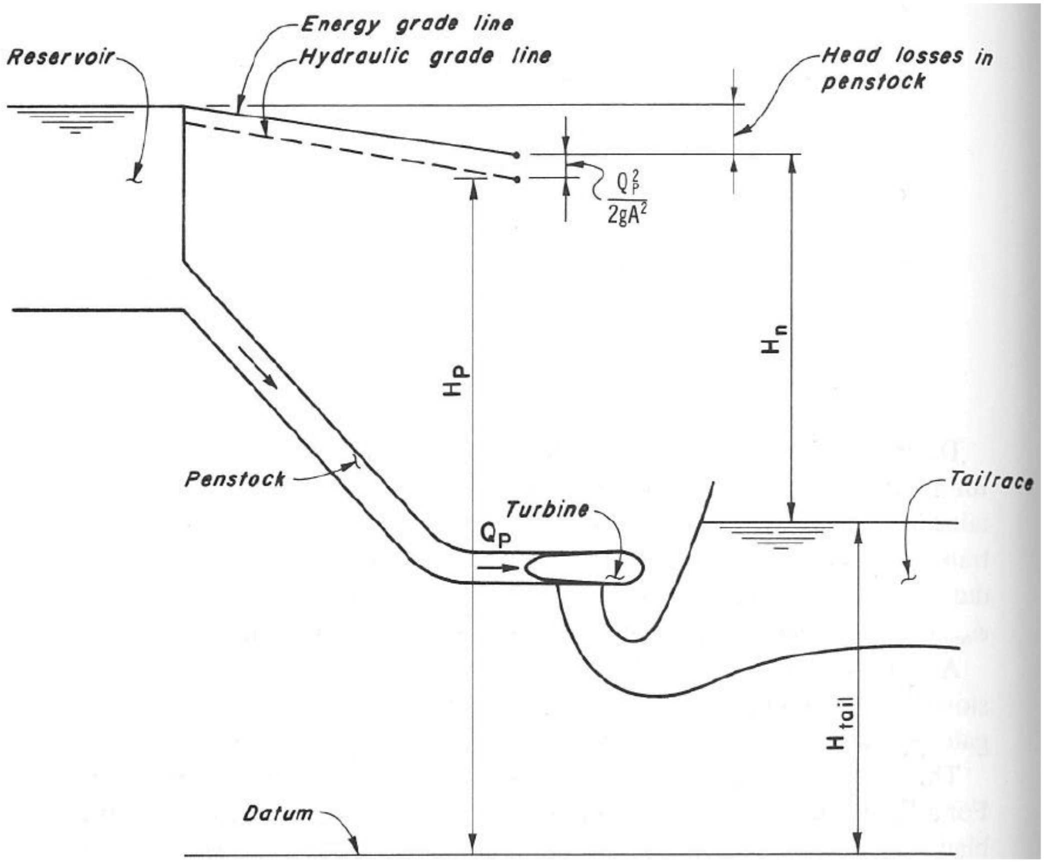
E04_P02

A hydraulic turbine with known efficiency 88.5% is installed in a hydropower plant with penstock cross-section area 2.25 (m^2). The net head of the hydropower is 20 (m) and the water flow velocity in the penstock is 1 (m/s). Calculate the mechanical power output of the turbine. Assume water density rho = 1000 kg/m^3.

Solution

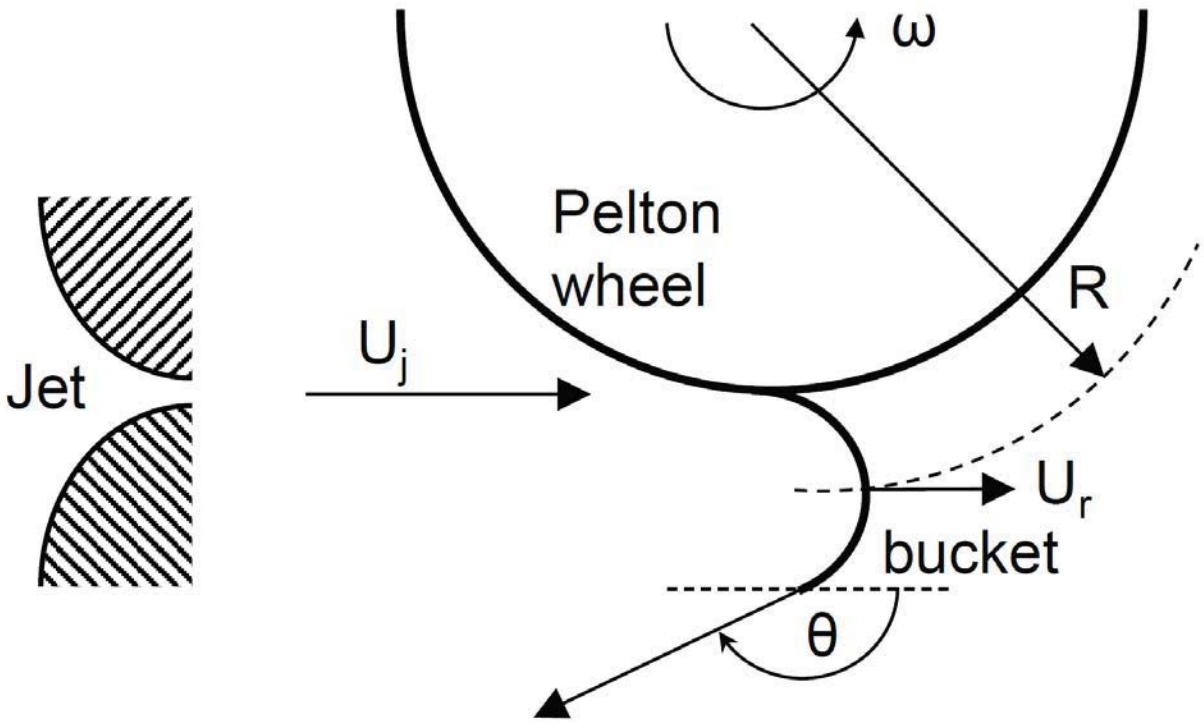
Q = Ap*Up = 2.25 (m³/s)

Nm = eta_t*rho*g*H*Q = 390.7 (kW)



E04_P03

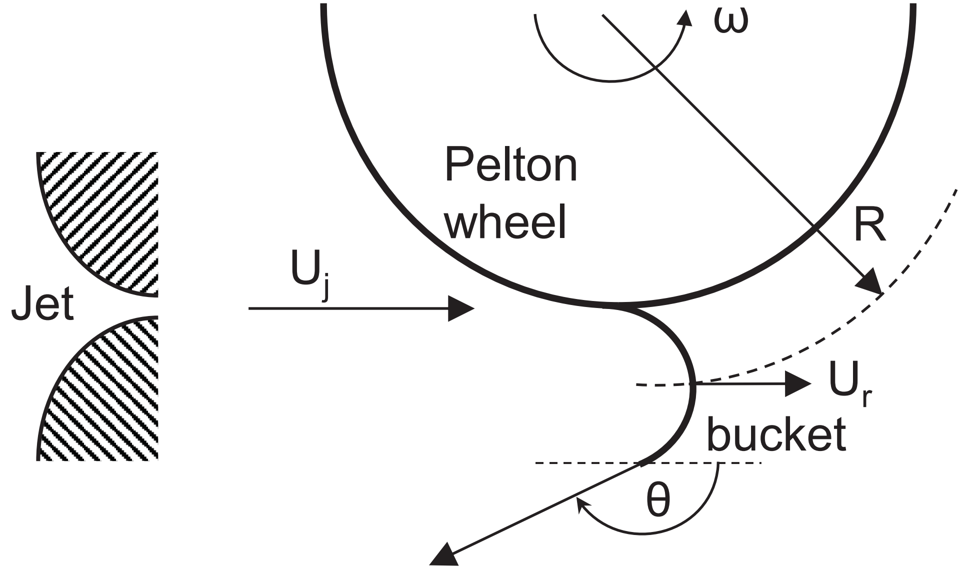
Pelton turbine operates with volumetric water flow 1 (m^3/s). The rotor has a mean radius 0.8 (m) and rotate with speed 160 (rpm). The blade angle is 165 (deg) and the water jet radius is 0.15 (m). Calculate the turbine output power assuming water density rho = 1000 (kg/m^3).



Optimum Speed for Impulse Turbine (1)

Assumptions:

Neglect torque due to surface forces
Neglect torque due to body forces
Neglect mass of water on wheel
Steady flow
All jet water acts on the bucket
Bucket height is small compared to wheel, hence $r_1 \sim r_2 \sim R$
No change in jet speed relative to bucket



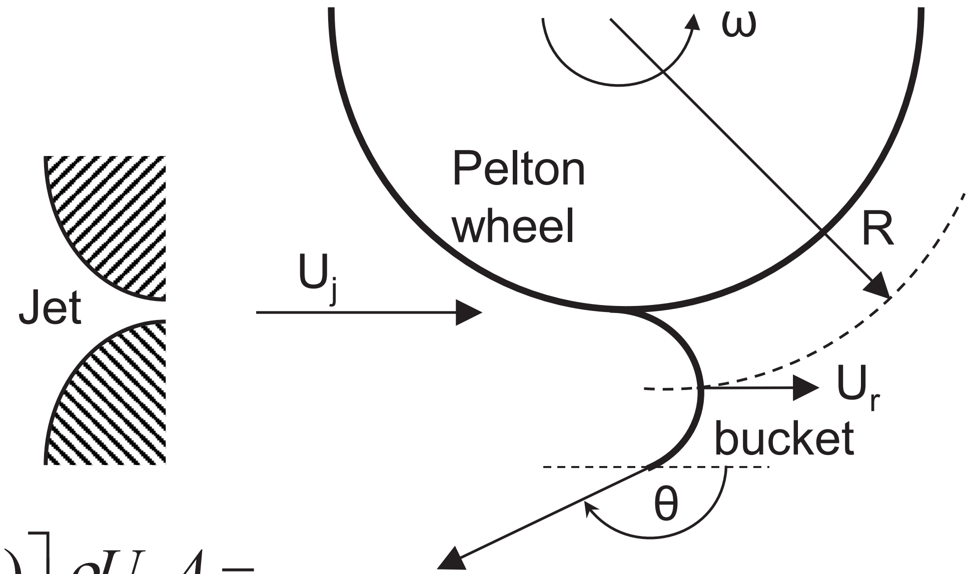
The angular momentum principle gives:

$$T_s = \left(r_2 U_{t2} - r_1 U_{t1}\right) W = \left[R(U_j - U_r)\cos\theta - R(U_j - U_r) \right] \rho U_j A =$$
$$R(U_j - U_r)\rho U_j A(\cos\theta - 1) = \rho Q R(U_j - U_r)(\cos\theta - 1)$$

Optimum Speed for Impulse Turbine (2)

Thus, the angular momentum principle gives: Since:

$$T_s = \left(r_2 U_{t2} - r_1 U_{t1}\right) W$$



we have:

$$T_s = \left[R(U_j - U_r)\cos\theta - R(U_j - U_r) \right] \rho U_j A =$$

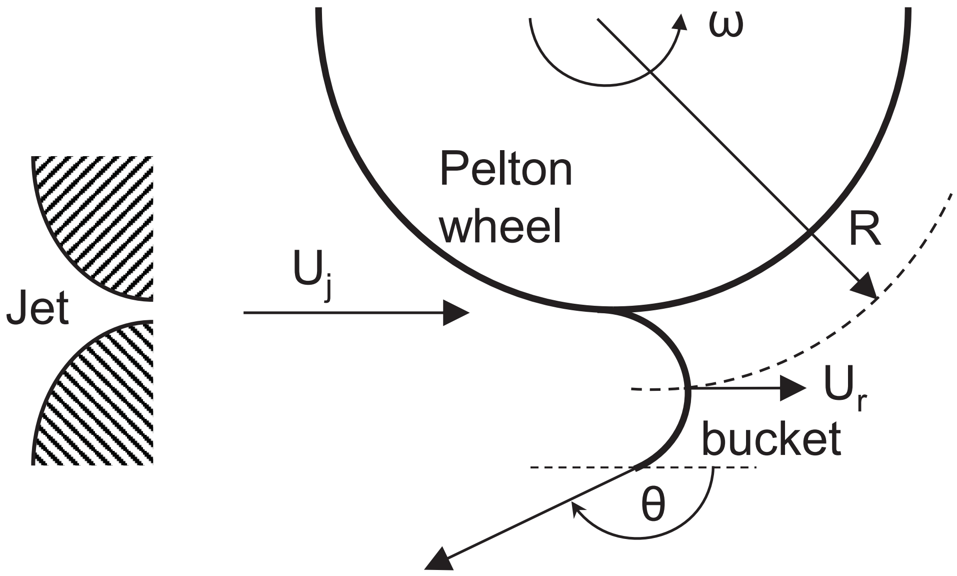
$$R(U_j - U_r)\rho U_j A(\cos\theta - 1) = \rho Q R(U_j - U_r)(\cos\theta - 1)$$

This equation gives us a relationship for the torque T_s in terms of main parameters, such as volumetric flow rate Q, wheel radius *R*, jet velocity U_j, etc

Optimum Speed for Impulse Turbine (3)

Thus, the torque exerted by the shaft on water is:

$$T_s = \rho QR(U_j - U_r)(\cos\theta - 1)$$



The output torque exerted by water on the wheel is just equal and opposite:

$$T_{out} = -T_s = \rho QR(U_j - U_r)(1 - \cos\theta)$$

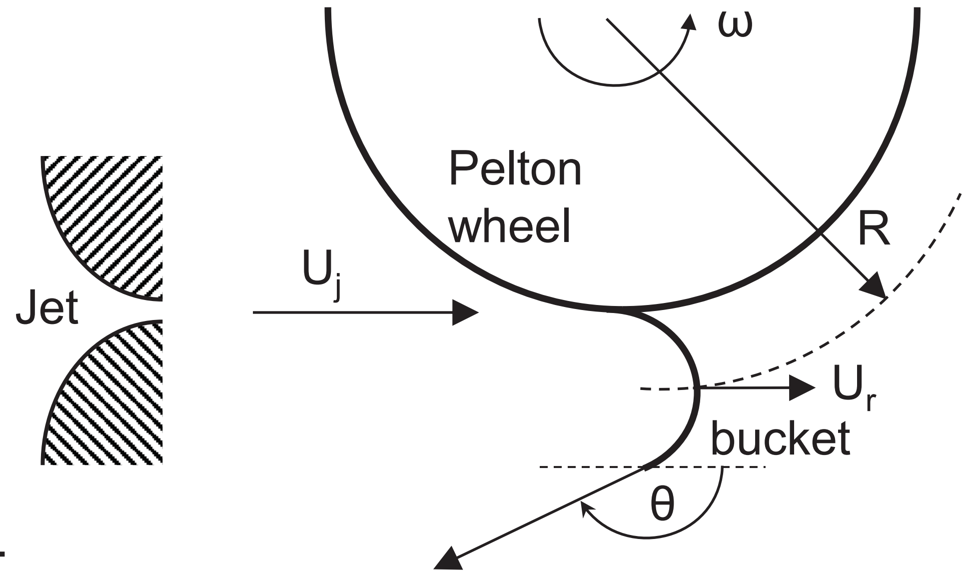
The corresponding hydraulic power output is:

$$N_{out} = \omega T_{out} = \rho QR\omega(U_j - U_r)(1 - \cos\theta) = \rho QU_r(U_j - U_r)(1 - \cos\theta)$$

Optimum Speed for Impulse Turbine (4)

The maximum power can be found from: $\dfrac{dN_{out}}{dU_r} = 0$

Which gives $U_r = U_j/2$, and the corresponding maximum power is



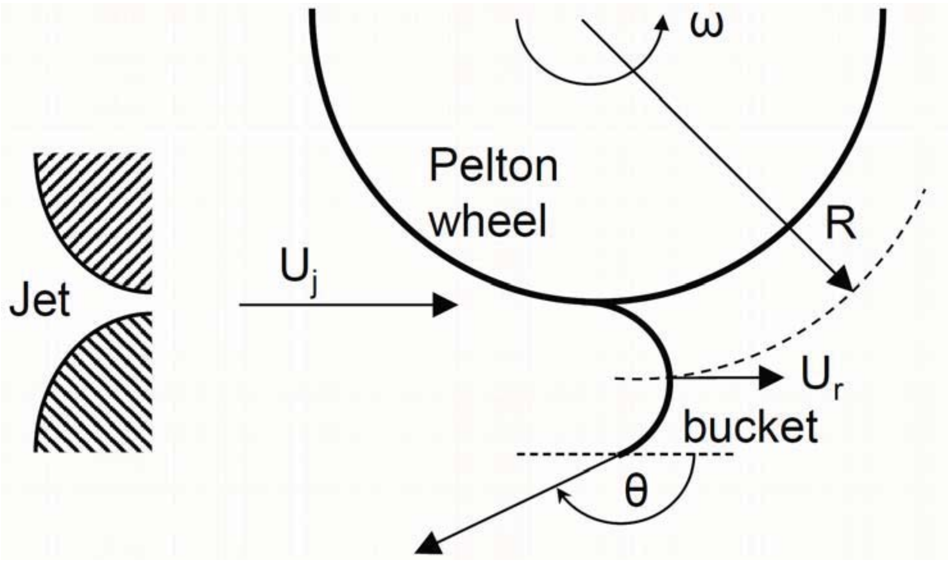
$$N_{out,\max} = \rho Q U_j^2 (1 - \cos\theta)/4$$

Note that the power decreases with decreasing angle θ. The absolute maximum is when θ = 180° and then $N_{out,max} = \rho^* Q U^2/2$.
In practice, 180° angle is not achievable and it can be up to 165°. Then [1-cos(165°)] = 1.97, which gives about 1.5% below the absolute maximum power.

E04_P03

Pelton turbine operates with volumetric water flow 1 (m^3/s). The rotor has a mean radius 0.8 (m) and rotate with speed 160 (rpm). The blade angle is 165 (deg) and the water jet radius is 0.15 (m). Calculate the turbine output power assuming water density rho = 1000 (kg/m^3).

Solution



Rotor mean speed is

Ur = R*n*pi/30 = 13.4 (m/s)

Jet mean speed is

Aj = pi*Rj^2 = 0.07 (m2)

Uj = Q/Aj = 14.147 (m/s)

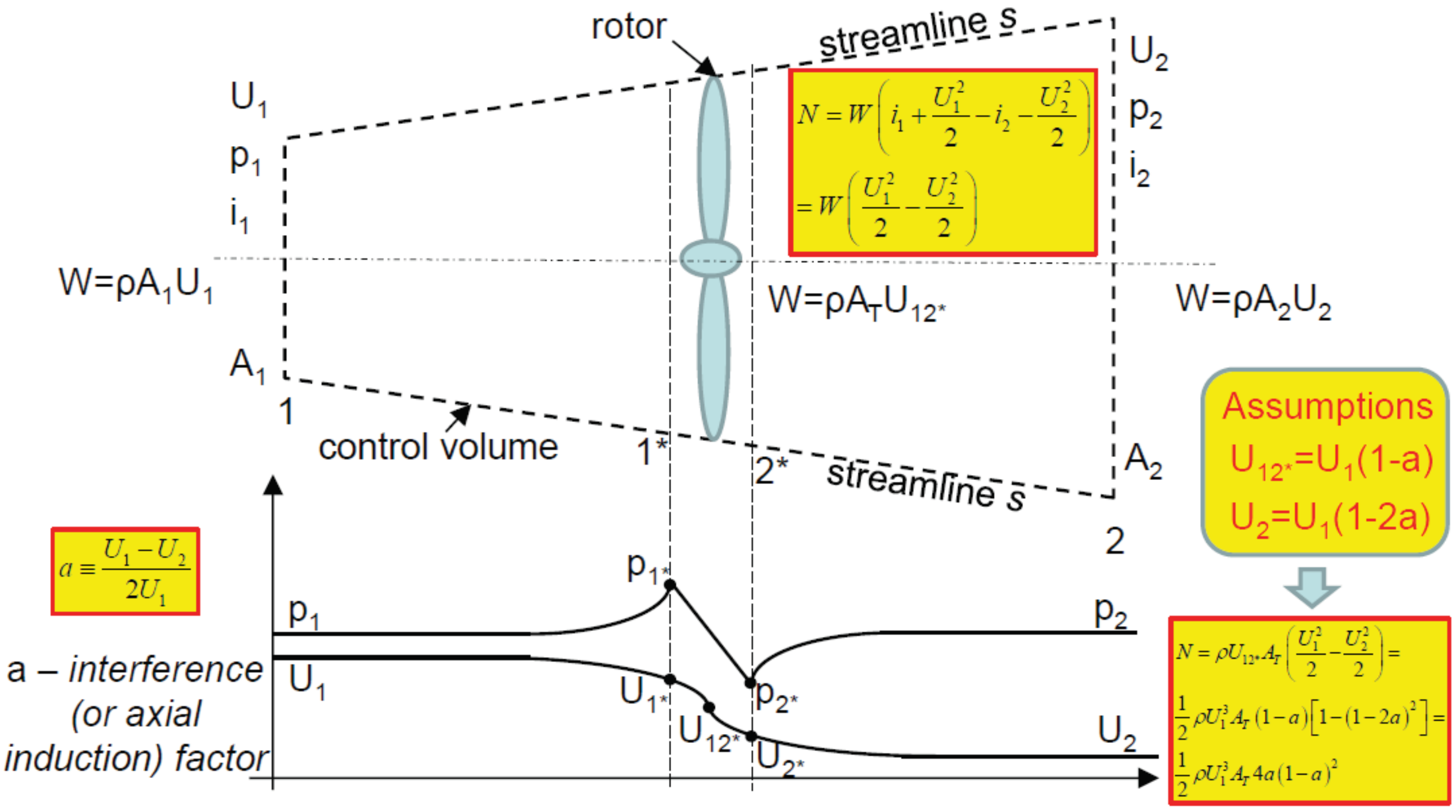
Turbine output power is

Nto_W = rho*Q*Ur*(Uj-Ur)*(1-cos(theta*pi/180)) = 19579 (W)

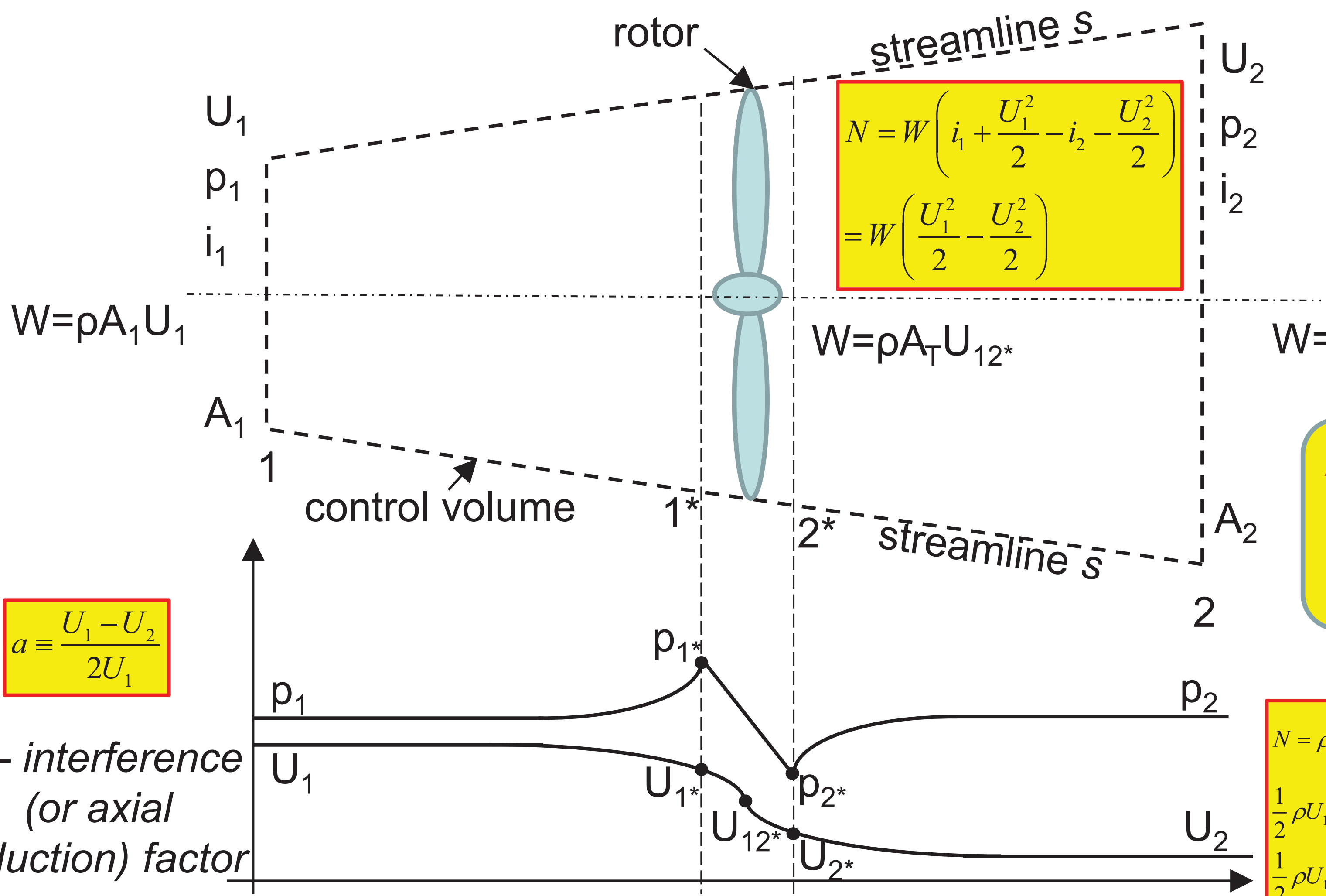
$$N_{out} = \omega T_{out} = \rho Q R \omega (U_j - U_r)(1 - \cos\theta) =$$
$$\rho Q U_r (U_j - U_r)(1 - \cos\theta)$$

E04_P04

Calculate the specific energy (energy per unit air mass flowing through the area occupied by a single wind turbine rotor) for a wind turbine operating with the maximum efficiency given by the Betz limit, when the unperturbed wind speed is 10 m/s.



Maximum Power of WT



$$N = W\left(i_1 + \frac{U_1^2}{2} - i_2 - \frac{U_2^2}{2}\right) = W\left(\frac{U_1^2}{2} - \frac{U_2^2}{2}\right)$$

$$a \equiv \frac{U_1 - U_2}{2U_1}$$

a – interference (or axial induction) factor

Assumptions

$U_{12*}=U_1(1-a)$

$U_2=U_1(1-2a)$

$$N = \rho U_{12*} A_T \left(\frac{U_1^2}{2} - \frac{U_2^2}{2}\right) = \frac{1}{2}\rho U_1^3 A_T (1-a)\left[1-(1-2a)^2\right] = \frac{1}{2}\rho U_1^3 A_T 4a(1-a)^2$$

Maximum Power of WT

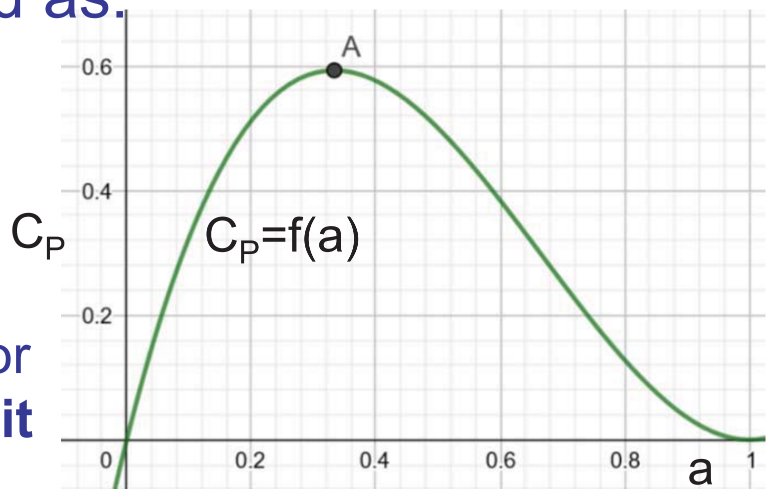
- Thus the turbine power, for given U_1, A_T and ρ, is the following function of *a*:

$$N(a) = \frac{1}{2}\rho U_1^3 A_T 4a(1-a)^2$$

- The power coefficient is defined as:

$$C_P \equiv \frac{N(a)}{1/2\rho U_1^3 A_T} = 4a(1-a)^2$$

  - with maximum C_P = 16/27≈0.59 for a = 1/3. This is so called **Betz limit**



E04_P04

Calculate the specific energy (energy per unit air mass flowing through the area occupied by a single wind turbine rotor) for a wind turbine operating with the maximum efficiency given by the Betz limit, when the unperturbed wind speed is 10 m/s.

Solution

We find this ratio as N/W, where N is the maximum wind mill power

$$N(a) = \frac{1}{2}\rho U_1^3 A_T 4a(1-a)^2$$

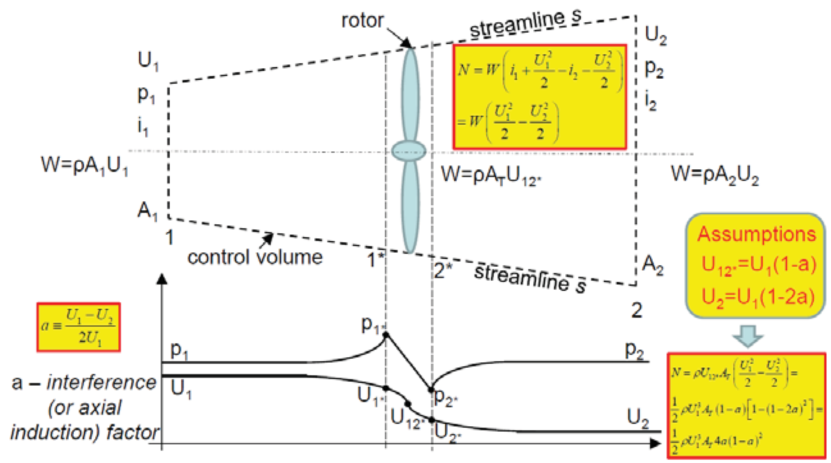
and

$W=\rho A_T U_{12*}$ $\quad$ $U_{12*}=U_1(1-a)$

here W is mass flow rate, a is the interference factor

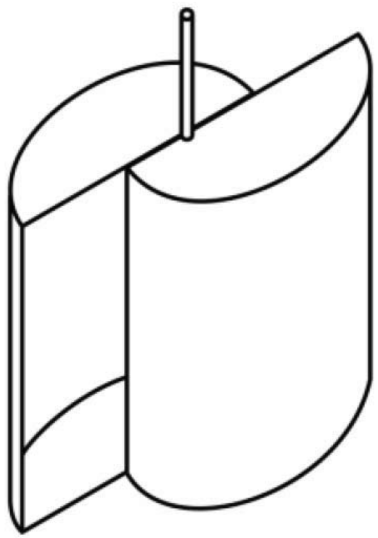
a=1/3; when interference factor at maximum power.

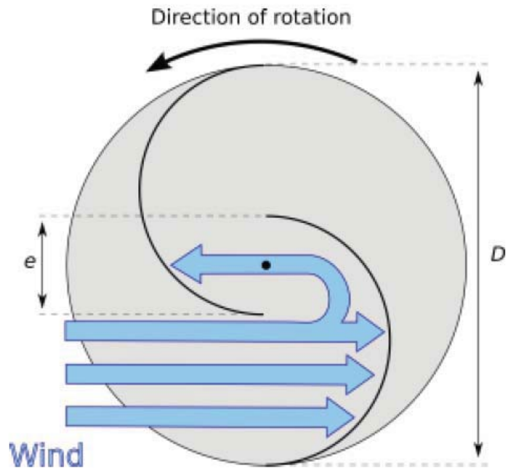
ratio = 0.5*U*U*4*a*(1-a) = 44.44 (J/kg)



E04_P05

Calculate the maximum power that can be generated by the Savonius vertical-axis wind turbine rotor with radius 2 (m) and height 3 (m) if wind speed is 15 (m/s) and air density is 1.25 (kg/m^3).

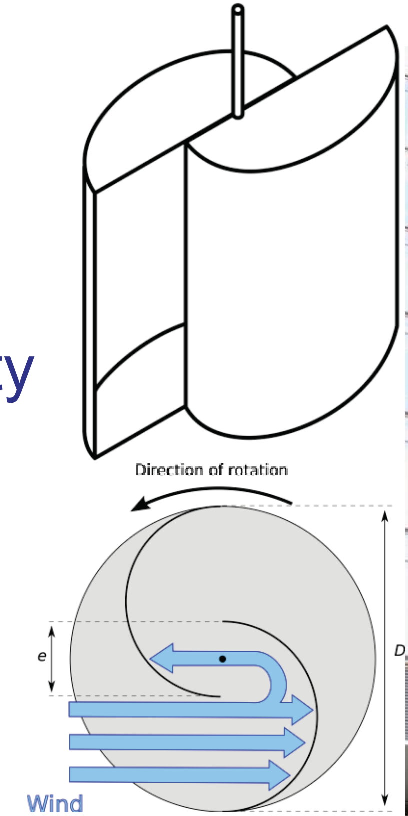




Savonius Vertical-Axis WT

- These are drag-type devices with two (or more) scoops that are used in anemometers and some high-reliability low efficiency power turbines

- They are always self-starting if there are at least three scoops



Two-scoop design

Savonius Vertical-Axis WT

- According to Betz's law, the maximum power that can be extracted from a rotor is N_{max}, where ρ is density of air, *h* and *r* are the height and radius of the rotor and *U* is the wind speed.
- The angular frequency of a rotor is ω, where λ is a dimensionless factor (tip-speed ratio)≈1

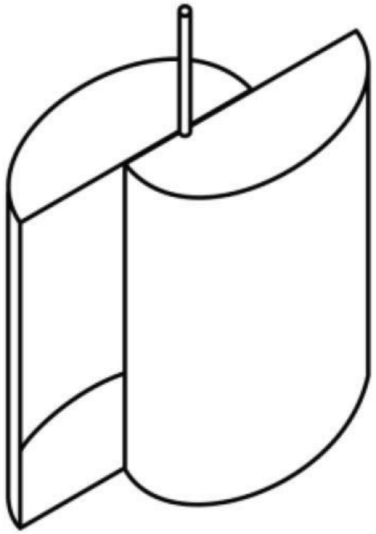
$$N_{\max} = \frac{16}{27}\rho \cdot r \cdot h \cdot U^3$$

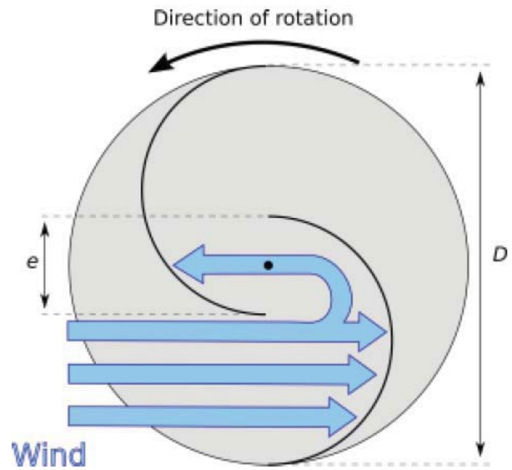
$$\omega = \frac{\lambda \cdot U}{r}$$

For example, the maximum power generated by the Savonius rotor with height h=1m and radius r=0.5m, when U=10m/s, is about 180 W, and the angular speed is 20 rad/s

E04_P05

Calculate the maximum power that can be generated by the Savonius vertical-axis wind turbine rotor with radius 2 (m) and height 3 (m) if wind speed is 15 (m/s) and air density is 1.25 (kg/m^3).





Solution

According to Betz limit,

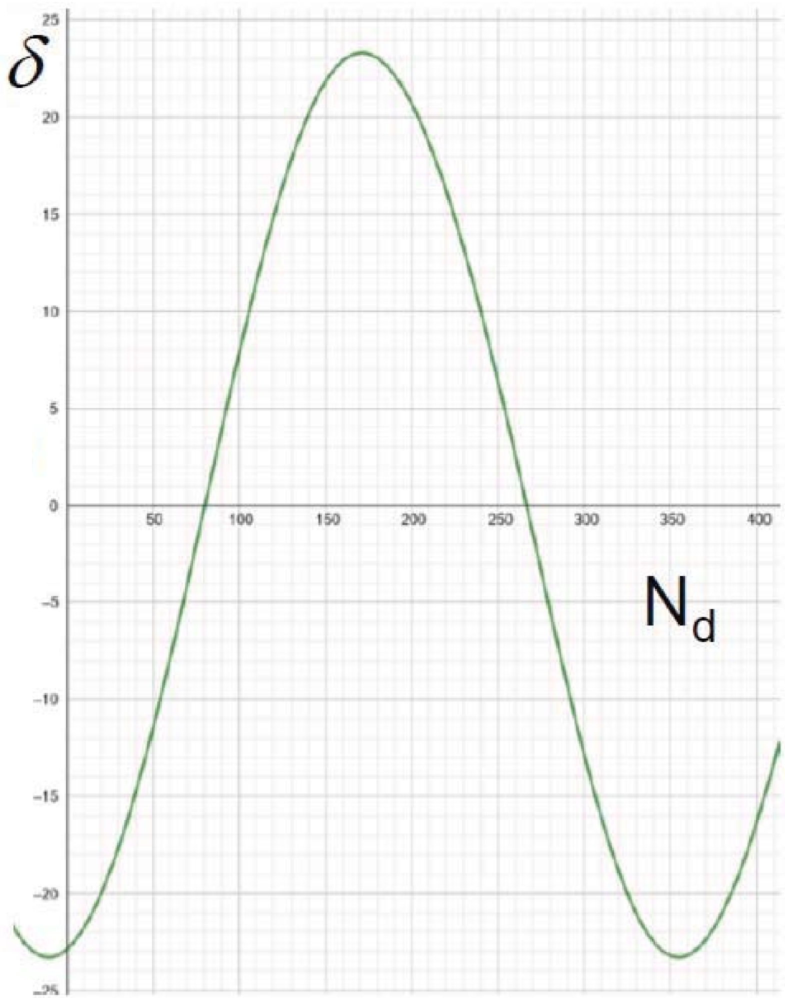
Cp_max = 16/27

$$N_{\max} = \frac{16}{27} \rho \cdot r \cdot h \cdot U^3$$

PowerMax = 15 (kW)

E04_P06

Calculate declination of the Sun on June 21. Give answer in angle degrees. Assume that the year has 365 days.



Position of the Sun

- Position of the Sun can be found in three steps
  - calculate the Sun's position in the ecliptic coordinate system
  - convert to the equatorial coordinate system
  - convert to the horizontal coordinate system (at given time and location on Earth)

- In solar energy applications, we usually use correlations to determine the Sun coordinates in the sky:
  - the altitude of the Sun (ψ)
  - the azimuth of the Sun (α)
- These coordinates are functions of the local longitude and latitude, the solar declination and the hour angle

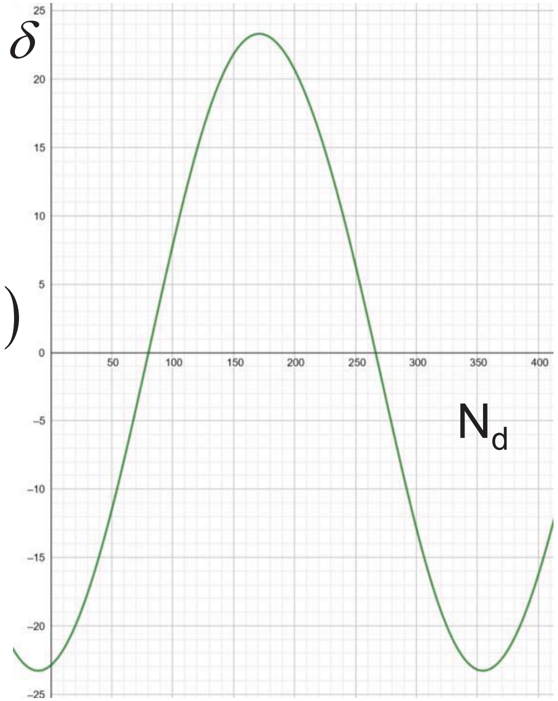
The Declination of the Sun

- The declination of the Sun is the angle between the ray of the Sun and the plane of the Earth's equator. It varies from -23.45° (December 20) to 23.45° (June 21)

- The Sun's declination is calculated as:

$$\delta = 0.3948 - 23.2559\cos\left(N'_d + 9.5°\right) - 0.3915\cos\left(2N'_d + 5.4°\right) - 0.1764\cos\left(3N'_d + 105.2°\right)$$

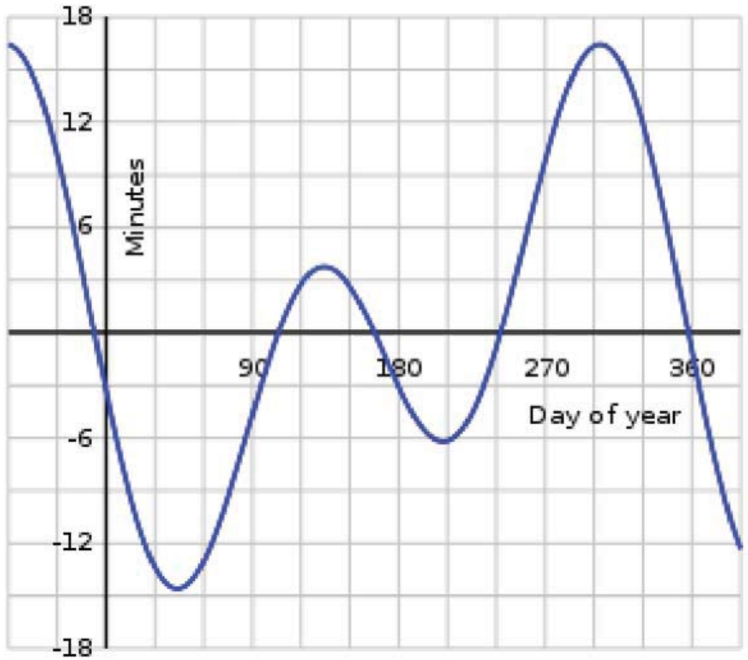
  - here $N'_d = 360° N_d / 365$; for leap year $N'_d = 360° N_d / 366$
  - N_d is the day number during the year, e.g. N_d = 1 for January 1



Equation of Time

- The equation of time ET describes the discrepancy between two kinds of solar time: the apparent solar time (direct tracked Sun motion) and the mean solar time (with noon 24 hours apart)
- It can be calculated as (in min):

$$\mathrm{ET} = 0.0066 + 7.3525\cos\left(N'_d + 85.9°\right) +$$
$$9.9359\cos\left(2N'_d + 108.9°\right) +$$
$$0.3387\cos\left(3N'_d + 105.2°\right)$$



- here $N'_d = 360° N_d / 365$
- or $N'_d = 360° N_d / 366$

N_d - day number, e.g. for January 1st N_d=1

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Solar Time

- Solar time is a measure of the passage of time based on the position of the Sun in the sky

- It can be found based on the known local time LT (e.g. Central European Time – CET, in most of EU countries) as:

$$\mathrm{ST} = \mathrm{LT} - 4\left(15^\circ - \lambda\right)\frac{\mathrm{min}}{\circ} + \mathrm{ET}$$

  - here ET is the Equation of Time [min] correction and λ is the local geographic longitude

Hour Angle

- The hour angle of a point is the angle between two planes: one containing the Earth's axis and the zenith (the meridian plane), and the other containing the Earth's axis and the given point

- It can be calculated as:

$$\omega = \left(12.00\,\mathrm{h} - \mathrm{ST}\right)\frac{15^\circ}{\mathrm{h}}$$

- here ST is the solar time

Hour angle is an angle "distance" to the noon.

When ST = 12.00h, $\omega=0$

$\omega > 0$ before the noon, and $\omega<0$ in the afternoon

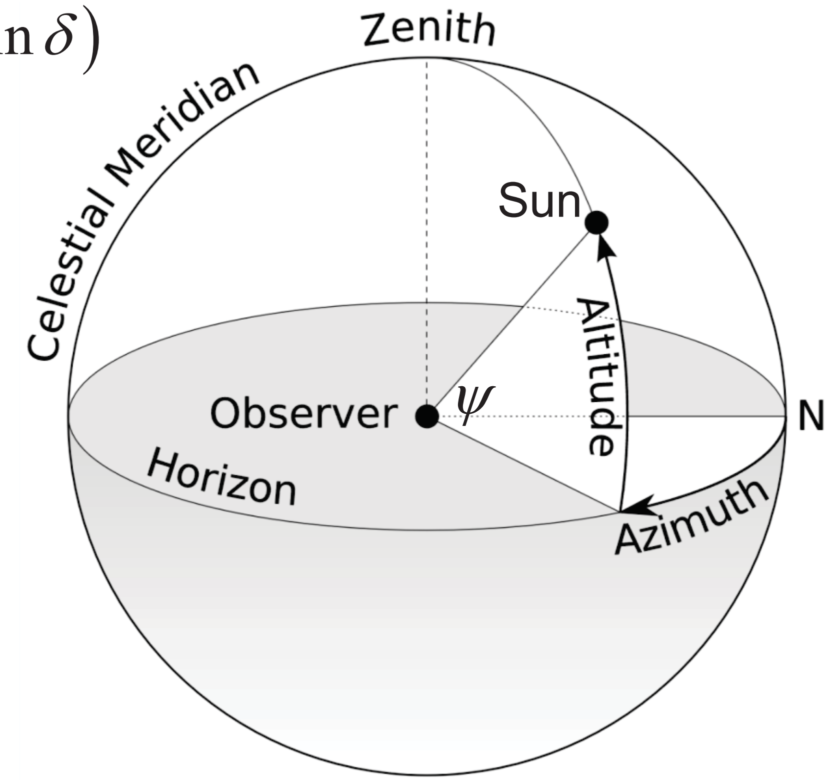
$\omega = 15^\circ$ corresponds to 1 h

Altitude of the Sun

- The altitude of the Sun can be found as

$$\psi = \arcsin\left(\cos\omega\cos\varphi\cos\delta + \sin\varphi\sin\delta\right)$$

  - here ω is the hour angle,
    φ is the local latitude, and
    δ is the declination of the Sun



Azimuth of the Sun

- The azimuth of the Sun can be found as

$$\alpha = \begin{cases} 180^\circ - \arccos\left(\dfrac{\sin\psi \sin\varphi - \sin\delta}{\cos\psi \cos\varphi}\right) & \text{for} \quad \mathrm{ST} \le 12.00\,\mathrm{h} \\ 180^\circ + \arccos\left(\dfrac{\sin\psi \sin\varphi - \sin\delta}{\cos\psi \cos\varphi}\right) & \text{for} \quad \mathrm{ST} > 12.00\,\mathrm{h} \end{cases}$$

  - here ψ is the altitude of the Sun, φ is the local latitude, and δ is the declination of the Sun.
  - ST is the local solar time

E04_P06

Calculate declination of the Sun on June 21. Give answer in angle degrees. Assume that the year has 365 days.

Solution

$$\delta = 0.3948 - 23.2559\cos\left(N'_d + 9.5°\right) -$$
$$0.3915\cos\left(2N'_d + 5.4°\right) - 0.1764\cos\left(3N'_d + 105.2°\right)$$

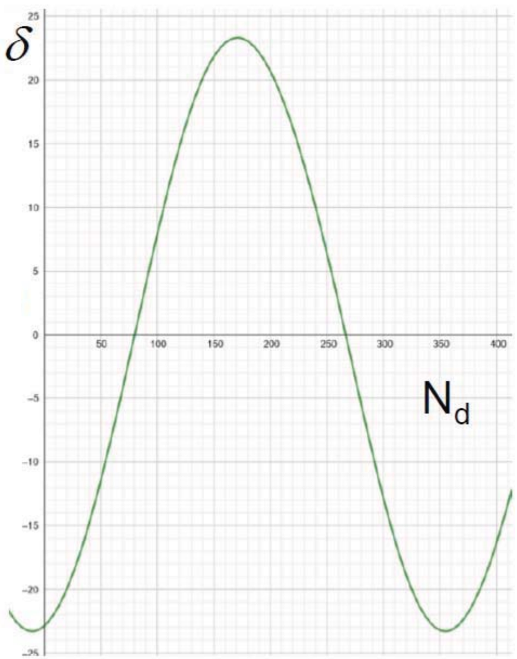
We find the day number

here $N'_d = 360° N_d / 365$;
for leap year $N'_d = 360° N_d / 366$
N_d is the day number during the year,
e.g. N_d = 1 for January 1

Nd = 3*31+28+30+21;

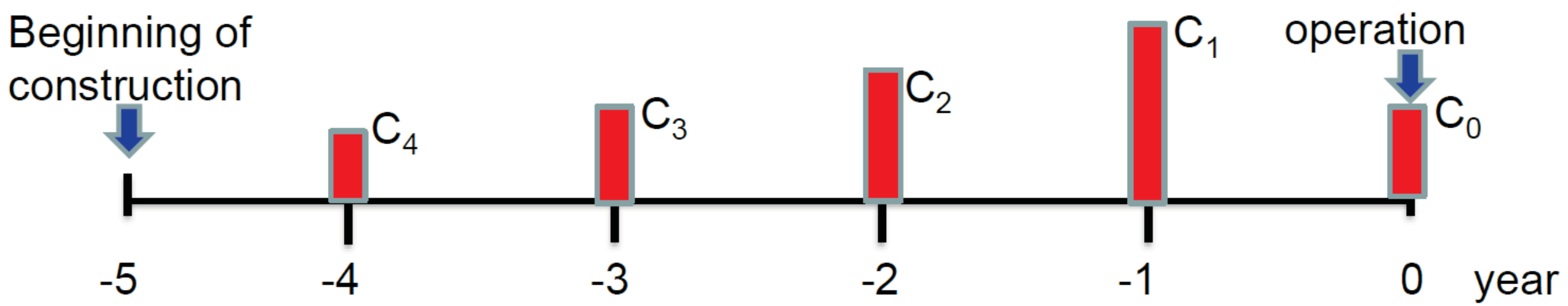
Declination of the Sun

d= 23.3187 (deg)



E04_P07

Electricity generating plant is typically built within 5 years with uniformly distributed expenditures per each year. A new construction schedule is considered in which the plant will be built within 4 years with uniformly distributed expenditures. The sum of expenditures in both cases will be the same. Calculate the relative change of capital cost using the original schedule as a reference. The interest rate in both cases is the same and equal to 5%. Use the end of construction as the reference time point, and assume that each payment is made at the end of the year.



Time Value of Money

- Money is a valuable asset with a variable time value determined by an **interest rate *i***
- The total amount of money that has to be paid back at the end of the year, when borrowing amount ***C*** at the beginning of the year with interest rate ***i*** is

$$\text{Pay back} = \underbrace{C}_{\text{Principal}} + \underbrace{iC}_{\text{Interest}} = C\left(1+i\right)$$

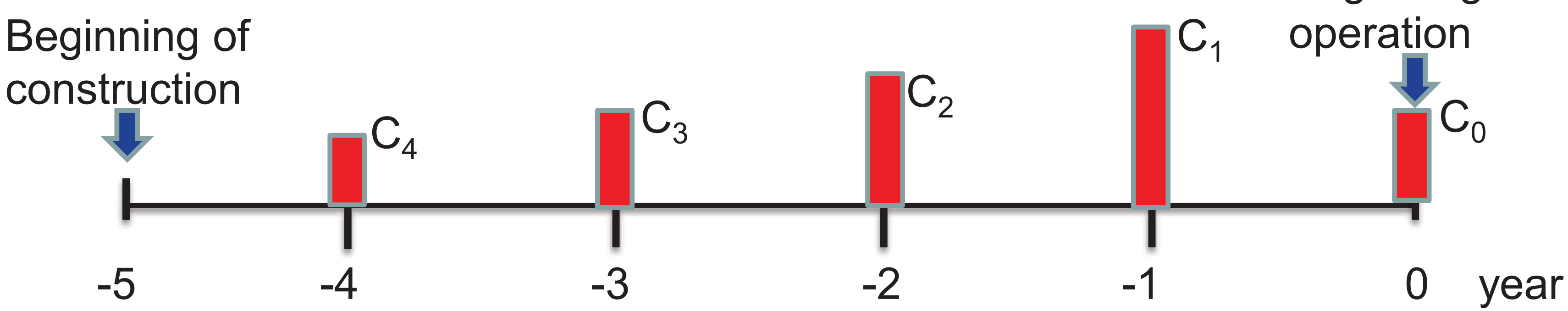
- If we expect a cost ***C*** at the end of a year, we can invest amount ***C'*** at the beginning of the year with interest rate ***i*** such that ***C'(1+i) = C***. Thus $C' = \dfrac{C}{1+i}$ C' is a current value of an expense C

Present Value Concept

- With **present value concept** we can find an amount of money in any year that is equivalent to an amount in any other year
- This concept is very useful when analysing economics of an electricity generating plant (which is usually costly enterprise)
- It is useful to determine the current value of the past cost or the cost that will occur in the future
  - if we invested amount ***C*** two years ago, its current value is $C(1+i)^2$
  - thus if we had an expenditure ***C*** two years ago, its current value is $C(1+i)^2$. In general, expense ***C*** that occurred ***n*** years ago has present value $C(1+i)^n$. Similarly $C' = \frac{C}{(1+i)^n}$ is a present value of an expense *C* in *n* years

Capital Cost - Construction

- Plant construction occurs over a certain period of time and involves nonuniform expenditures
- The complete capital cost is determined by both the magnitude and timing of these expenditures



- The total capital cost is the sum of present value of all expenditures. Using year 0 as the reference time point:

$$C = C_0 + C_1\left(1+i\right) + C_2\left(1+i\right)^2 + C_3\left(1+i\right)^3 + C_4\left(1+i\right)^4 = \sum_{k=0}^{4} C_k\left(1+i\right)^k$$

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Solution

We assume annual expenditure in original schedule:

Ex_o = 1;

Then expenditures in new schedule will be:

Ex_n = 5*Ex_o/4;

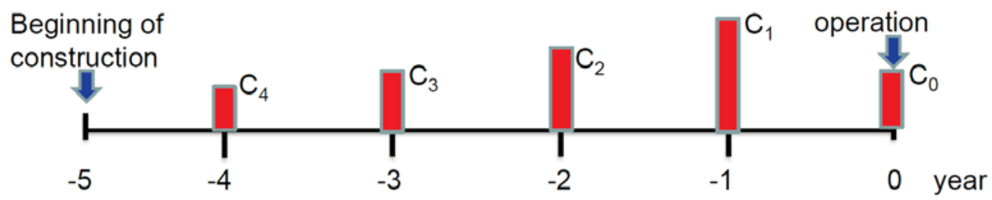
Capital cost for original schedule is now:

Co = Ex_o*(1+(1+i)+(1+i)^2+(1+i)^3+(1+i)^4);

and for the new one

Cn = Ex_n*(1+(1+i)+(1+i)^2+(1+i)^3);

answer = (Cn-Co)/Co = -2.5%



$$C = C_0 + C_1(1+i) + C_2(1+i)^2 + C_3(1+i)^3 + C_4(1+i)^4 = \sum_{k=0}^{4} C_k (1+i)^k$$