Lectures on Thermal-Hydraulics in Nuclear Energy Engineering

Lecture No 01

Title:

Heat Sources in Nuclear Reactors

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Outline of the Lecture

- Introduction
- Thermal Power of Fission Reactors
- Distribution of Thermal Power in Reactors
- Decay Heat

Introduction

- The operational power of the nuclear reactor core is limited by thermal, not nuclear, considerations

- The main limiting factors, determining the power, are:
  - the rates at which heat can be transported from the reactor core
  - the maximum allowable temperatures in reactor core components, such as:
    - fuel temperature shall be always below the melting temperature
    - cladding temperature shall be below the phase-change temperature in the cladding material

- Temporal and spatial temperature gradients in construction materials are limited by damage risks

Fission Energy

- The thermal energy released in reactors originates from fission of fuel nuclei
- In current reactors, the main fissile material that is used is uranium isotope ^{235}U
- During reactor operation, additional fissile material, isotope of plutonium ^{239}Pu, is produced (transmuted) from fertile material ^{238}U
- Other fissile materials include ^{241}Pu and ^{233}U
- All these isotopes release slightly different amounts of energy after fissions. This energy can be estimated as:

$$e_f = 1.29927 \cdot 10^{-3} Z^2 A^{0.5} + 33.12 \; (\mathrm{MeV} / \; fission)$$

Z- atomic number
A- atomic mass

Fission Energy in LWRs

- In typical LWR only about ½ of all neutrons are absorbed in fissile isotopes and the rest are captured by
  - fertile isotopes
  - control materials
  - coolant/moderator
  - structural materials
- Fresh fuel is typically enriched with ^{235}U to between 3.5 and 5% wt%
- Close to the end of fuel cycle, energy released from fissions of plutonium may reach up to 50% of the total energy

Fission Power and Thermal Power

- Total energy generated in a reactor core due to fission is called the **fission power**, q_f (W)
  - this includes all the energy released from the fission, inelastic scattering and neutron capture

- Not all fission power stays, however, in the reactor core, since 5% is carried away by neutrinos, whereas the rest is deposited in the core and shields

- The fraction of the fission power that is deposited in the core/shields (0.95 q_f) is called the thermal power q_{th} (W)

Fission Power and Thermal Power

- Fission reactor power can be roughly estimated as:

$$q_f = \frac{V_c \overline{\Sigma}_f \overline{\varphi}}{3.1 \cdot 10^{10}} \text{ (W)}$$

Number of fissions per unit core volume and unit time

Number of fissions per second to generate 1 W

  - here V_c is the volume of the reactor core (m^3)
  - q_f is the reactor fission power (W)
  - $\overline{\Sigma}_f$ is the core-mean macroscopic cross section for fission (1/m)
  - $\overline{\varphi}$ is the core-mean neutron flux ($1/m^2 s$)

- The above expression gives the total energy released in fissions, even the energy carried away by neutrinos. Since this energy is lost, the reactor thermal power is found as:

$$q_{th} = 0.95 q_f$$

Power Distribution in Fuel Rod

- The thermal power is distributed between fuel rods, coolant/moderator, shields and construction materials
- Estimates show that in PWRs the amount of heat deposited in fuel material is about 88.4% of q_f, 2.5% in the moderator and 4.1% in the structure (of which 3% in the cladding and the rest in thermal shields)
- We can express the amount of heat deposited to fuel rods q_{FR} (fuel+cladding) in terms of the thermal power as:

$$q_{FR} = 0.914 q_f = 0.914 q_{th} / 0.95 \simeq 0.962 q_{th}$$

Core and Fuel Power Density

- The power density in a reactor core can be calculated in two ways:
  - As a core-mean power density, when the thermal power is divided by the whole core volume (fuel + structure + coolant/moderator):

$$\bar{q}'''_c = q_{th} / V_c = q_{th} / \left(\pi R^2 H \right)$$

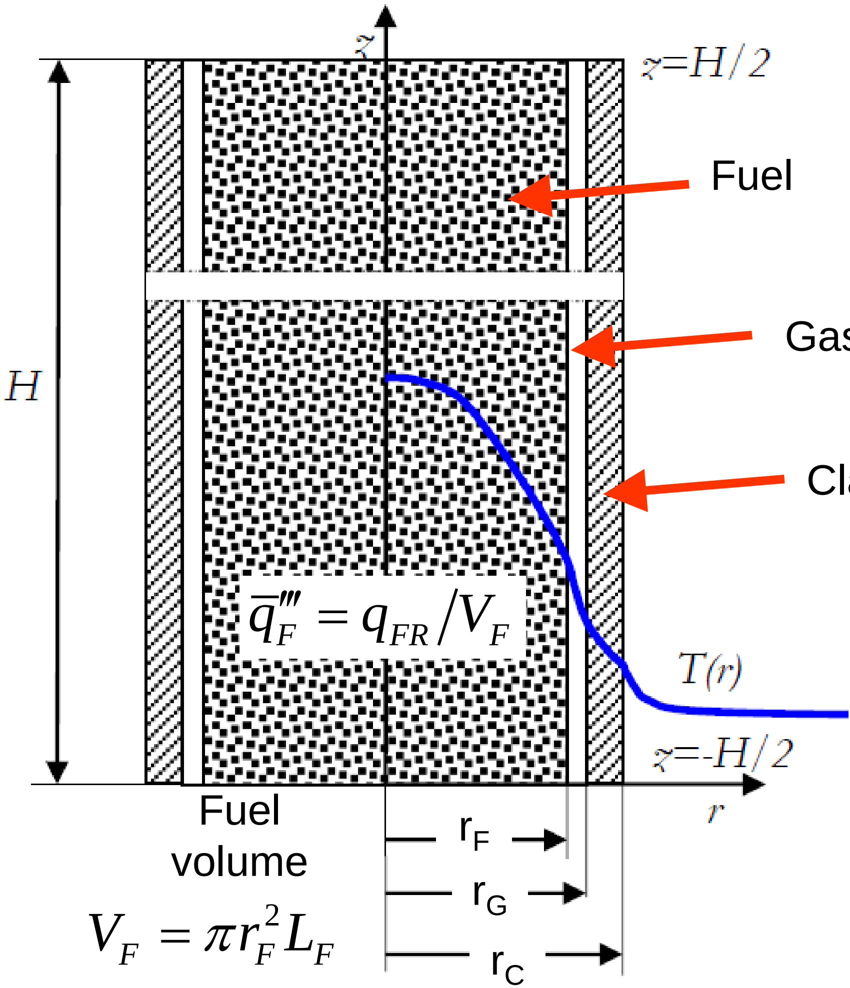
  - As a fuel-mean power density, when the thermal power fraction that stays in fuel rod is divided by the fuel volume only

$$\bar{q}'''_F = q_{FR} / V_F$$

We take a conservative approach and allocate all heat released in fuel rods to fuel pellets

  - where the fuel volume can be found from know fuel pellet radius r_F and the total fuel rod length in the core, L_F: $V_F = \pi r_F^2 L_F$

Core and Fuel Power Density



Power density distribution along a single fuel rod follows the cosine function:

$$q_F'''(z) = q_F'''(0)\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

where $q_F'''(0)$ - is the power density at z =0.

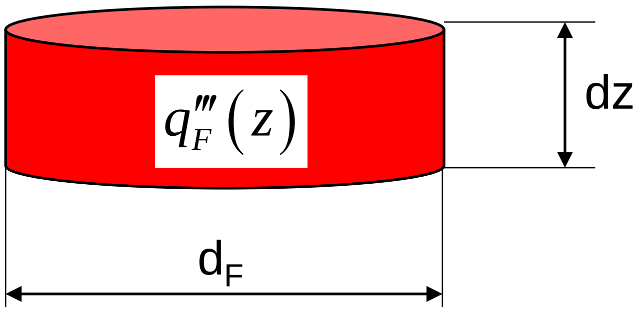
Here $q_F'''(0)$ and $q_F'''(z)$ represent mean power densities in a fuel cross-section area.

More detailed analysis indicates that the fuel power density has a radial distribution in a rod.

Linear Power in Fuel

- Linear power is defined in fuel rods

- It is defined as rod power distribution per its unit length

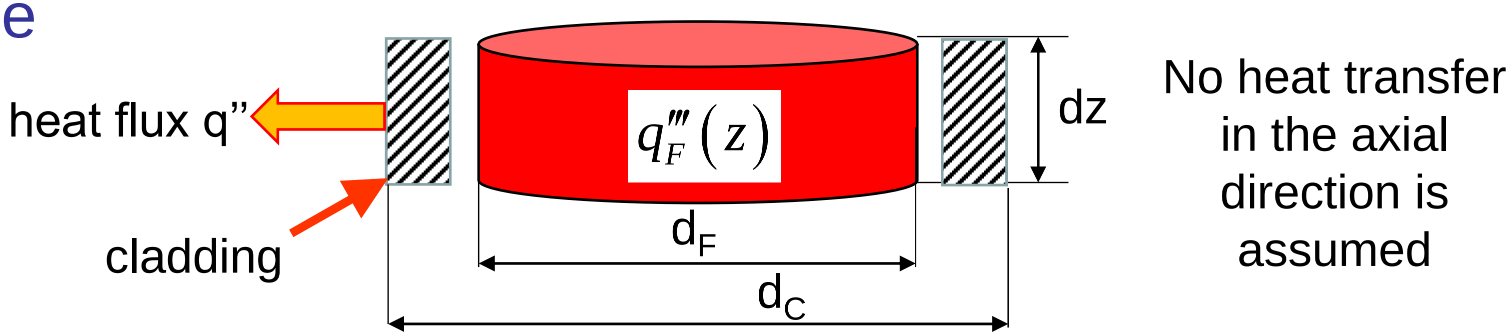


- Assuming $q_F'(z)$ is a linear power density at location *z*, the energy balance for a differential fuel volume gives:

$$q_F'''(z)\cdot dz\cdot \pi\, d_F^2/4 = q_F'(z)\,dz \qquad \text{thus} \qquad q_F'(z) = q_F'''(z)\cdot \pi\, d_F^2/4$$

Heat Flux on Clad

- Heat flux is defined on the outer clad surface
- It is defined as rod power distribution per unit clad outer surface



- Assuming $q''(z)$ is a heat flux on clad outer surface at *z*, the energy balance for a differential fuel volume gives:

$$q_F'''(z) \cdot dz \cdot \pi d_F^2 / 4 = q''(z) \cdot \pi d_C \cdot dz \quad \text{thus} \quad q''(z) = q_F'''(z) \cdot d_F^2 / (4 d_C)$$

Thermal Power Distribution

- Thus, the thermal power ($q_{th} = 0.95 * q_f$) is distributed as follows:

Region	% of q_f	% of q_{th}
Fuel	88.4	93.0
Cladding	3.0	3.2
Moderator	2.5	2.6
Thermal shields	1.1	1.2

Deposited inside reactor core (Fuel, Cladding, Moderator)

Deposited inside reactor pressure vessel (Fuel, Cladding, Moderator, Thermal shields)

- All this thermal power is removed from the reactor pressure vessel by the coolant

Coolant Enthalpy Increase

- In LWRs, 100% of the thermal power is gained by coolant, which is passing through the downcomer, the lower plenum, the core and the upper plenum
- From the energy balance for the reactor pressure vessel, we get the following :

$$W_{RPV}\Delta i_{RPV} = q_{th}$$

- And the coolant enthalpy increase in RPV is:

$$\Delta i_{RPV} = \frac{q_{th}}{W_{RPV}}$$

W_{RPV} – coolant mass flow rate through the reactor pressure vessel (kg/s)

Coolant Enthalpy Increase

- 98.8% of the thermal power is released in the core area (fuel, cladding and moderator)
- From the energy balance for the reactor core, we get the following :

$$W_c \Delta i_c = 0.988 \cdot q_{th} = q_c$$

- And the coolant enthalpy increase in the core is:

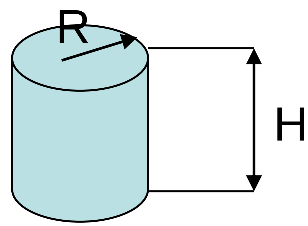
$$\Delta i_c = \frac{0.988 \cdot q_{th}}{W_c} = \frac{q_c}{W_c}$$

W_c – coolant mass flow rate through the reactor core (kg/s)
q_c – thermal power released in the core (W)

Thermal Power Distribution in Fission Reactors

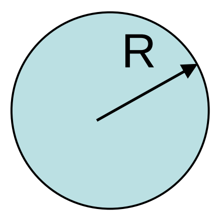
$J_0(x)$ – Bessel function of first kind and zero order. See Compendium, Appendix B

- Distribution of thermal power density in nuclear reactors depends on the shape of the reactor:
  - –Finite cylindrical with radius R and height H:



$$q'''(r,z) = q_0''' J_0\left(\frac{2.405 r}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)$$

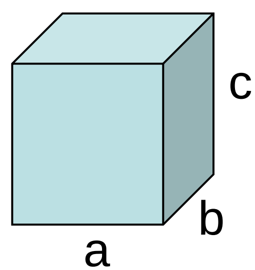
  - –Sphere with radius R:



$$q'''(r) = q_0'''\left(\frac{\tilde{R}}{\pi}\right)\frac{\sin\frac{\pi r}{\tilde{R}}}{r}$$

Note: dimensions with tilde are so-called extrapolated dimensions to avoid zero flux at reactor boundary

  - –Rectangular parallelepiped with sides a, b, c:



$$q'''(x,y,z) = q_0'''\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{b}}\right)\cos\left(\frac{\pi z}{\tilde{c}}\right)$$

q_0''' - power density at the core centre; r=0, z=0

Cylindrical Core (1)

- The overall thermal power distribution in a cylindrical reactor with uniform concentration of fissile material is given as

$$q'''(r,z) = q_0''' J_0\left(\frac{2.405r}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right) \qquad \tilde{R} = R + d \qquad \tilde{H} = H + 2d$$

  - here *d* is the extrapolation length, which for a bare core is $d=0.71\lambda_{tr}$ and λ_{tr} is the neutron transport length in the core material
  - For a reflected cylindrical reactor we can roughly assume:

$$\frac{R}{\tilde{R}} \cong \frac{H}{\tilde{H}} \cong \frac{5}{6}$$

Cylindrical Core (2)

- The average power density in a cylindrical core can be found as:

$$\bar{q}''' = \frac{q_0'''}{\pi R^2 H} \int_0^R J_0\left(\frac{2.405 r}{\tilde{R}}\right) 2\pi r dr \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{\tilde{H}}\right) dz$$

- We use here the following rule for integration of the Bessel function of the first kind and zero order:

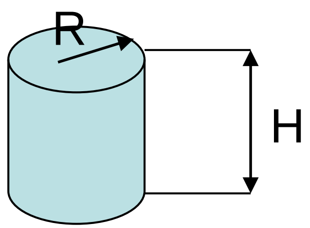
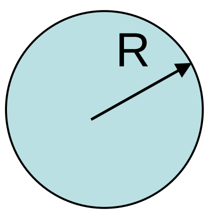
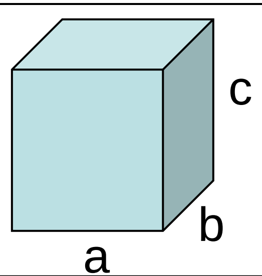
$$\int_0^r x J_0(\alpha x) dx = \frac{r}{\alpha} J_1(\alpha r)$$

α - constant

- and we get

$$\bar{q}''' = q_0''' \frac{2\tilde{R}}{2.405 R} J_1\left(\frac{2.405 R}{\tilde{R}}\right) \frac{2\tilde{H}}{H\pi} \sin\left(\frac{\pi H}{2\tilde{H}}\right)$$

Power Distribution

	Volume	Power density distribution	Mean power density
	$V = \pi R^2 H$	$q'''(r,z) = q_0''' J_0\left(\frac{2.405r}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)$	$\bar{q}''' = q_0''' \frac{2\tilde{R}}{2.405R} J_1\left(\frac{2.405R}{\tilde{R}}\right)\frac{2\tilde{H}}{H\pi}\sin\left(\frac{\pi H}{2\tilde{H}}\right)$
	$V = \frac{4}{3}\pi R^3$	$q'''(r) = q_0'''\left(\frac{\tilde{R}}{\pi}\right)\frac{\sin\frac{\pi r}{\tilde{R}}}{r}$	$\bar{q}''' = 3q_0'''\left(\frac{\tilde{R}}{\pi R}\right)^2\left[\frac{\tilde{R}}{\pi R}\sin\left(\frac{\pi R}{\tilde{R}}\right) - \cos\left(\frac{\pi R}{\tilde{R}}\right)\right]$
	$V = a \cdot b \cdot c$	$q'''(x,y,z) = q_0'''\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{b}}\right)\cos\left(\frac{\pi z}{\tilde{c}}\right)$	$\bar{q}''' = q_0'''\frac{\tilde{a}\tilde{b}\tilde{c}}{abc}\left(\frac{2}{\pi}\right)^3\sin\left(\frac{\pi a}{2\tilde{a}}\right)\sin\left(\frac{\pi b}{2\tilde{b}}\right) \cdot \sin\left(\frac{\pi c}{2\tilde{c}}\right)$

Peaking Factors (1)

- Peaking factor is a ratio of the maximum to average power densities in a reactor core
- Peaking factor can be calculated for the whole core volume:
$$f_V = \frac{q_0'''}{\bar{q}'''} = \frac{q'''(0,0)}{\frac{1}{V}\int_V q''' dV}$$
- In a cylindrical core, we have in addition radial and axial peaking factors:
$$f_R(z_P) = \frac{q'''(0,z_P)}{\frac{1}{\pi R^2}\int_0^R q'''(r,z_P)\,2\pi r dr} \qquad f_A(r_P) = \frac{q'''(r_P,0)}{\frac{1}{H}\int_{-H/2}^{H/2} q'''(r_P,z)\,dz}$$
- Here z_P and r_P are fixed values of the axial and radial coordinates at which peaking factors are defined

Peaking Factors (2)

- For example for a fuel rod located at $r=r_P$ distance from the centreline, the axial peaking factor is found as:

$$f_A(r_P) = \frac{q_0''' J_0\left(\frac{2.405 r_P}{\tilde{R}}\right)\cos(0)}{\frac{1}{H}\int_{-H/2}^{H/2} q_0''' J_0\left(\frac{2.405 r_P}{\tilde{R}}\right)\cos\left(\frac{\pi z}{\tilde{H}}\right)dz} =$$

$$\frac{1}{\frac{1}{H}\int_{-H/2}^{H/2}\cos\left(\frac{\pi z}{\tilde{H}}\right)dz} = \frac{\pi H}{2\tilde{H}\sin\left(\frac{\pi}{2}\cdot\frac{H}{\tilde{H}}\right)}$$

- As can be seen the axial peaking factor does not depend on r_P

Peaking Factors (3)

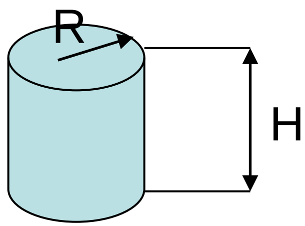
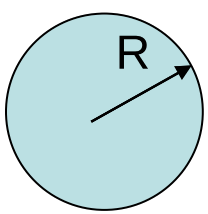
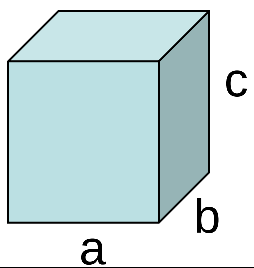
- Similarly for a core cross-section located at $z=z_P$, the radial peaking factor is found as:

$$f_R\left(z_P\right)=\frac{q_0'''J_0\left(0\right)\cos\left(\frac{\pi z_P}{\tilde{H}}\right)}{\frac{1}{\pi R^2}\int_0^R q_0'''J_0\left(\frac{2.405r}{\tilde{R}}\right)2\pi r\cos\left(\frac{\pi z_P}{\tilde{H}}\right)dr}=$$

$$\frac{1}{\frac{1}{\pi R^2}\int_0^R J_0\left(\frac{2.405r}{\tilde{R}}\right)2\pi r dr}=\frac{2.405\cdot R}{2\tilde{R}\cdot J_1\left(\frac{2.405R}{\tilde{R}}\right)}$$

- As can be seen the radial peaking factor does not depend on z_P

Power Distribution – Peaking Factors

	Mean power density	Assuming extrapolation length equal to zero
	$\bar{q}''' = q_0''' \frac{2\tilde{R}}{2.405R} J_1\left(\frac{2.405R}{\tilde{R}}\right)\frac{2\tilde{H}}{H\pi}\sin\left(\frac{\pi H}{2\tilde{H}}\right)$	$\bar{q}''' = 0.274824 q_0'''$ $q_0''' = 3.63869\bar{q}'''$
	$\bar{q}''' = 3q_0'''\left(\frac{\tilde{R}}{\pi R}\right)^2\left[\frac{\tilde{R}}{\pi R}\sin\left(\frac{\pi R}{\tilde{R}}\right)-\cos\left(\frac{\pi R}{\tilde{R}}\right)\right]$	$\bar{q}''' = \frac{3q_0'''}{\pi^2} \approx 0.303964 q'''$ $q_0''' = 3.28986\bar{q}'''$
	$\bar{q}''' = q_0'''\frac{\tilde{a}\tilde{b}\tilde{c}}{abc}\left(\frac{2}{\pi}\right)^3 \sin\left(\frac{\pi a}{2\tilde{a}}\right)\sin\left(\frac{\pi b}{2\tilde{b}}\right)\sin\left(\frac{\pi c}{2\tilde{c}}\right)$	$\bar{q}''' = \frac{8q_0'''}{\pi^3} \approx 0.258012 q'''$ $q_0''' = 3.87579\bar{q}'''$

Decay heat in Fission Reactors (1)

• After reactor scram the fission thermal power is fast decreasing, (e.g. to 1% of full power after 4 s) however, decay heat is still generated in the fuel

•Calculation of the decay heat is important to determine the required cooling that must be provided after reactor scram or normal shutdown

•The decay heat results from gamma and beta radioactive decay of fission products

•There is also contribution from actinides, mainly ^{239}U and ^{239}Np, which is in a range from a few percent to maximum 20%

Decay heat in Fission Reactors (2)

- During reactor operation, the decay heat is corresponding to about 7 to 8 % of the total power

- After reactor shutdown, the decay heat exponentially decreases with time, where the time change is mainly governed by short-lived fission products

- After normal shutdown, the fission power decreases slower, e.g. the power is below 10% after 10 seconds

Decay heat in Fission Reactors (3)

- In a simplified analysis, a one-equation model can be used to approximate the decay power after shutdown

- As an example, using this model for a reactor with 3500 MWt during normal operation, the power after shut down drops to 227.5 MWt – still a considerable thermal power that requires efficient reactor cooling

- According to this model, the decay heat is given as follows:

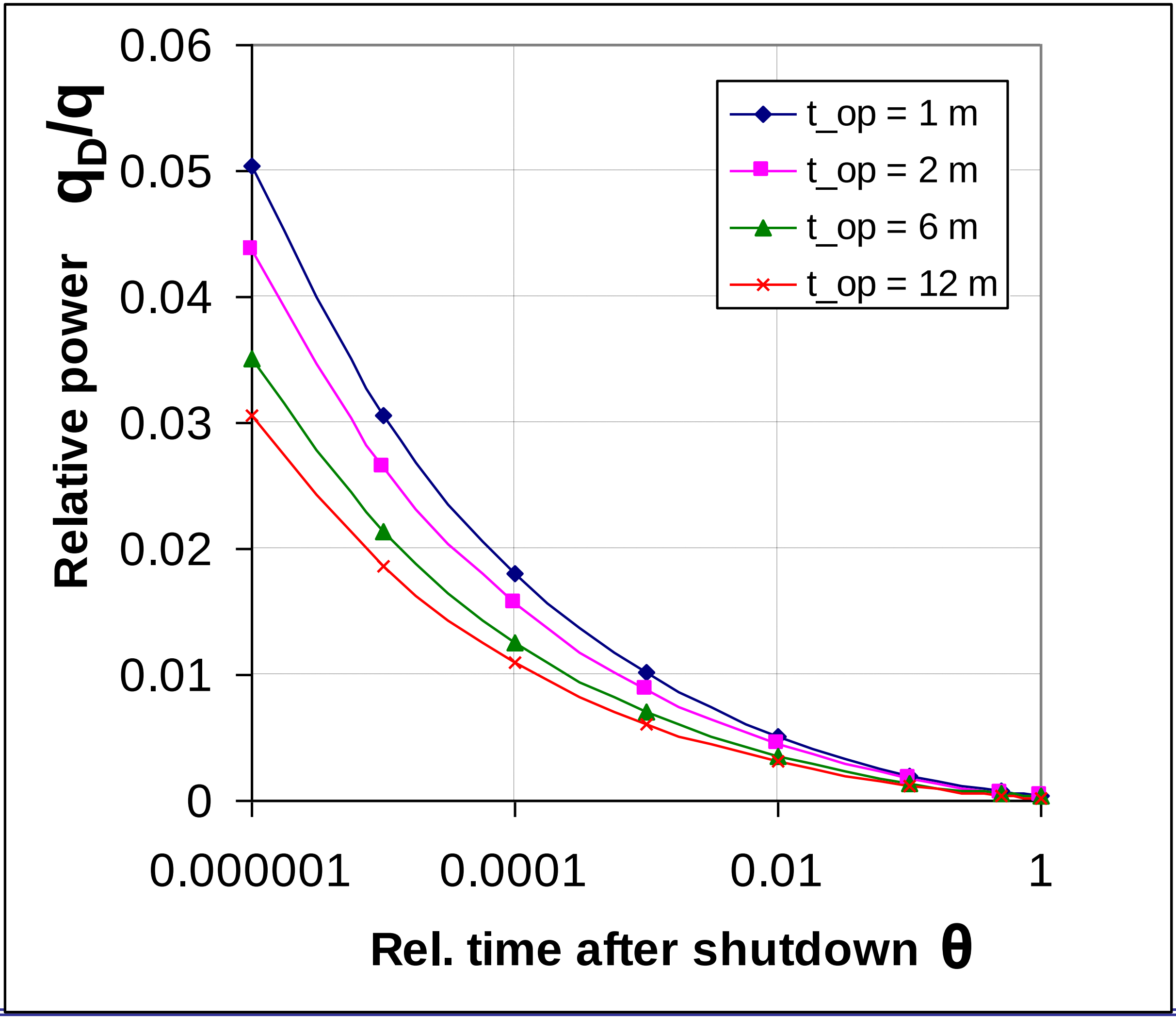
$$\frac{q_D}{q} = \frac{0.065}{t_{op}^{0.2}}\left[\frac{1}{\theta^{0.2}} - \frac{1}{(\theta+1)^{0.2}}\right]$$

t=0 $\quad$ t=t_{op} $\quad$ Shutdown Reactor t>t_{op}

$$\theta = \left(t - t_{op}\right)/t_{op}$$

- Here q_D is the decay heat, q is the reactor thermal power before shut-down, t_{op} is the reactor operation time [s] and t is time after reactor start-up [s]

Decay heat in Fission Reactors (4)



Standard curves of relative decay power after shutdown for various reactor operation periods

Decay heat in Fission Reactors (5)

- In more accurate calculations of the emergency cooling, models developed by ANS (American Nuclear Society) are used

- In ANS-71 algorithm, all fissions resulting from ^{235}U are taken into account and an infinite operation time before shutdown is assumed

- For licensing purposes an uncertainty factor ε is added

Decay heat in Fission Reactors (6)

- The ANS-71 algorithm is as follows: the fission product contribution is found as

$$\frac{q_{D,FP}}{q_0} = \left(1 + \frac{\varepsilon}{100}\right)\sum_{i=1}^{i=11} E_i \cdot e^{-\lambda_i t}$$

ε [%]	t [s]
20	$0 < t < 10^3$
10	$10^3 < t < 10^7$
25	$10^7 < t < 10^9$

$q_{D,FP}$ – fission product decay power,
q_0 – reactor power before shutdown,
t – time after shutdown, s

i	E_i	λ_i [1/s]
1	2.99E-3	1.772E+0
2	8.25E-3	0.577E+0
3	15.50E-3	6.743E-2
4	19.35E-3	6.214E-3
5	11.65E-3	4.739E-4
6	6.45E-3	4.810E-5
7	2.31E-3	5.344E-6
8	1.64E-3	5.726E-7
9	0.85E-3	1.036E-7
10	0.43E-3	2.959E-8
11	0.57E-3	7.585E-10

Decay heat in Fission Reactors (7)

- The contribution due to actinides is found as:

$$\frac{q_{D,ACT}}{q_0} = \frac{1}{e_f}\left[F_{239U}(t,T) + F_{239Np}(t,T)\right]$$

$q_{D,ACT}$ – decay power due to actinides, e_f – energy release per fission , MeV

where

$$F_{239U}(t,T) = E_U \cdot R \cdot \left(1 - e^{-\lambda_U T}\right) e^{-\lambda_U t}$$

$$F_{239Np}(t,T) = E_{Np} \cdot R \cdot \left[\frac{\lambda_U}{\lambda_U - \lambda_{Np}}\left(1 - e^{-\lambda_{Np} T}\right) e^{-\lambda_{Np} t} - \frac{\lambda_{Np}}{\lambda_U - \lambda_{Np}}\left(1 - e^{-\lambda_U T}\right) e^{-\lambda_U t}\right]$$

E_U – avearge decay energy for ^{239}U = 0.474 MeV
λ_U – decay constant for ^{239}U = 4.91E-4 s^{-1}

E_{Np} – avearge decay energy for ^{239}Np = 0.419 MeV
λ_{Np} – decay constant for ^{239}Np = 3.411E-6 s^{-1}

t – time after shutdown,
T – time before shutdown,
R =0.7

Decay heat in Fission Reactors (8)

- The total decay heat is found as:

$$\frac{q_{D,TOT}}{q_0} = \frac{q_{D,FIS}}{q_0} + \frac{q_{D,FP}}{q_0} + \frac{q_{D,ACT}}{q_0}$$

where $\frac{q_{D,FIS}}{q_0}$ is the fission power contribution (due to delayed neutrons) 10 s after reactor shutdown

Decay heat in Fission Reactors (9)

- ANS has also developed more sophisticated models afterwords

- For example, ANS-79 takes into account variable power during reactor operation

- In general ANS-79 gives 6-7 % lower decay power during the first 10^5 s, as compared to ANS-71

- Subsequent standard revisions were issued in 1994 and 2005
  - improved accounting for neutron capture in fission products
  - improve prediction of decay heat from fissionable nuclides ^{235}U, ^{239}Pu, $^{241}Pu_{th}$, $^{238}U_{fast}$
  - include power from additional actinide isotopes (^{239}U, ^{239}Np)
  - extend to longer time 10^{10} s after reactor shutdown

Example

- Example:

  – Calculate the power density at the center of reactor cores with three different shapes, assuming that the volumes and the total powers of the reactor cores are the same.

  – estimate the ratios assuming the extrapolated dimensions equal to the physical dimensions ($R/\tilde{R} = H/\tilde{H} = 1$).

Example

- Solution:
  - –if the total power and the volume in a finite cylinder and a sphere are the same, then the mean power density has to be the same as well.
  - –thus, we can find the ratio of the power density at the centers as follows

$$\bar{q}''' = \underbrace{0.274824 q'''_{0,cyl}}_{\text{cylinder}} = \underbrace{0.303964 q'''_{0,sph}}_{\text{sphere}}$$

$$Ratio = \frac{q'''_{0,cyl}}{q'''_{0,sph}} \approx \frac{0.303964}{0.274824} \cong 1.10603$$

Example

- Solution:
  - in a similar manner, we find the ratio of power density of cylinder to parallelepiped

$$\bar{q}''' = \underbrace{0.274824 q'''_{0,cyl}}_{\text{cylinder}} = \underbrace{0.258012 q'''_{0,par}}_{\text{parallelepiped}}$$

$$Ratio = \frac{q'''_{0,cyl}}{q'''_{0,par}} \approx \frac{0.258012}{0.274824} \cong 0.938828$$

Example

- Conclusions:
  - the ratio of the maximum power density to the mean power density is the smallest one in the spherical shape and is equal to 3.29
  - The same ratio for the cylindrical reactor is 3.64
  - The highest ratio is obtained for the parallelepiped and is 3.88
  - most reactor cores have the cylindrical shape due to construction simplicity as compared with the spherical shape.

Example

- Calculate the total peaking factor in a cylindrical core assuming

$$\frac{R}{\tilde{R}} \cong \frac{H}{\tilde{H}} \cong \frac{5}{6}$$

Example

- Solution:

$$f_A = \frac{\pi H}{2\tilde{H}\sin\left(\frac{\pi}{2}\cdot\frac{H}{\tilde{H}}\right)} = \frac{\pi}{2}\frac{5}{6}\frac{1}{\sin\left(\frac{\pi}{2}\frac{5}{6}\right)} \simeq 1.355$$

$$f_R = \frac{2.405\cdot R}{2\tilde{R}\cdot J_1\left(\frac{2.405R}{\tilde{R}}\right)} = \frac{2.405}{2}\frac{5}{6}\frac{1}{J_1\left(2.405\frac{5}{6}\right)} \simeq 1.738$$

- Answer: the total peaking factor is $f_A * f_R = 2.35578$