Nuclear Reactor Physics

Reactor Theory II

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2022

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Contents

Solution of one-group reactor equation for slab reactor

Slab reactor

Let's assume critical reactor in form of infinite bare slab (1-D geometry)

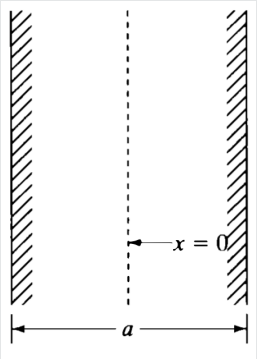


Figure 1: Slab geometry. The center of the slab is set at $x = 0$.

The 1-D reactor equation is

$$\nabla^2\phi + B^2\phi = 0$$

The general solution is

$$\phi(x) = A\cos Bx + C\sin Bx$$

where A, B and C need to be determined from the boundary conditions (and the requirement set on the physical solution).

Slab reactor

What can we say about the C value in the solution?

$$\phi(x) = A\cos Bx + C\sin Bx$$

- The slab is symmetrical with respect to its center axis, so the physical solution must also be symmetrical with respect to the center.
- This requirement eliminates the term $C\sin Bx$, so the solution is now

$$\phi(x) = A\cos Bx$$

How can we set the B value?

- We can apply the void boundary condition.
- The condition of zero flux at the extrapolated distance $x = \tilde{a}/2$ gives

$$\phi\left(\frac{\tilde{a}}{2}\right) = A\cos\left(\frac{B\tilde{a}}{2}\right) = 0$$

Slab reactor

For which A and B parameters is the equation

$$A\cos\left(\frac{B\tilde{a}}{2}\right) = 0$$

satisfied?

- Either for $A = 0$ (trivial solution - that is not useful)
- or for

  $$\cos\left(\frac{B\tilde{a}}{2}\right) = 0$$

  which is satisfied for

  $$B_n = \frac{n\pi}{\tilde{a}}$$

  for $n = 1, 3, 5, \ldots$

A physical solution must be non-negative in the system, and this is satisfied only by $n = 1$, i.e.,

$$\phi(x) = A\cos\left(\frac{\pi x}{\tilde{a}}\right)$$

which is the flux in a critical slab reactor. Note that the buckling decreases as a increases.

Slab reactor

How can we set parameter A in the solution?

$$\phi(x) = A\cos\left(\frac{\pi x}{\tilde{a}}\right)$$

- Note that any A constant will satisfy the reactor equation, since the flux solution is an eigenfunction. An eigenfunction can be scaled, and it still remains the same eigenfunction.
- The A constant must be determined so that the solution corresponds to neutron flux in the reactor of the desired power.
- The power P per cm^2 can be computed as

$$P = E_R \Sigma_f \int_{-a/2}^{a/2} \phi(x) dx$$

  where E_R is a recoverable energy per fission ($E_R = 3.2 \times 10^{-11}$J).
- After integration of the result for ϕ, P can be expressed as

$$P = \frac{2\tilde{a}E_R\Sigma_f A \sin(\frac{\pi a}{2\tilde{a}})}{\pi} \Rightarrow A = \frac{\pi P}{2\tilde{a}E_R\Sigma_f \sin(\frac{\pi a}{2\tilde{a}})}$$

Solutions for other geometries

Geometry	Dimensions	Buckling	Flux
Infinite slab	Thickness, a	$\left(\frac{\pi}{a}\right)^2$	$A\cos\left(\frac{\pi x}{a}\right)$
Parallelepiped	$a \times b \times c$	$\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2+\left(\frac{\pi}{c}\right)^2$	$A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$
Infinite cylinder	R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405\times r}{R}\right)$
Finite cylinder	R, H	$\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405\times r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$
Sphere	R	$\left(\frac{\pi}{R}\right)^2$	$A\frac{1}{r}\sin\left(\frac{\pi r}{R}\right)$

Figure 2: Note: J_0 is the ordinary Bessel function of the first kind.

Solutions for other geometries

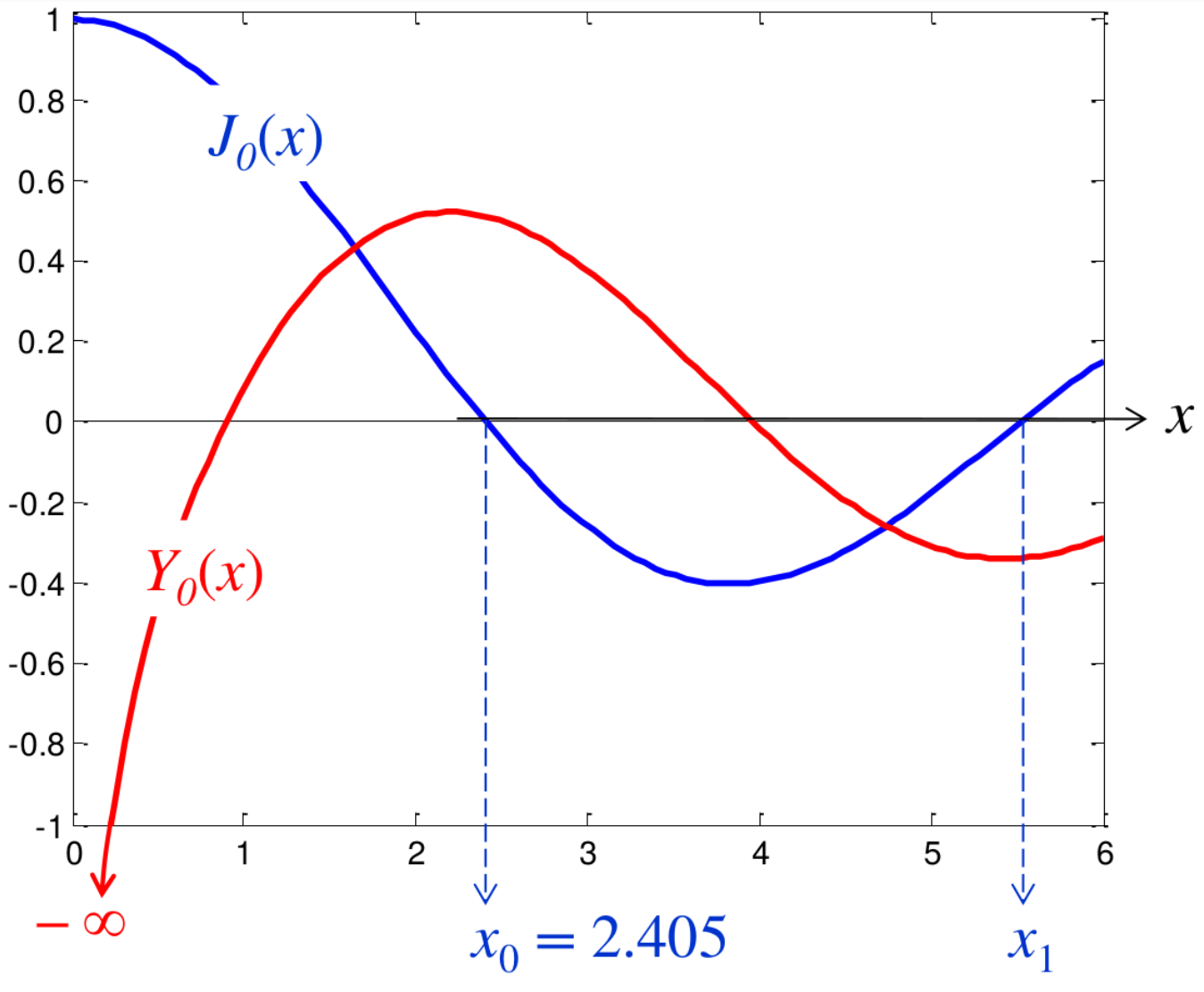


Figure 3: Bessel function

Thermal reactors (homogeneous)

Thermal reactors

Let's now consider thermal reactors that contain fuel, coolant, structural materials, and a moderator to slow down fission neutrons to thermal energies. We need to define new variables.

Define the thermal utilization f

Thermal utilization f is similar to the fuel utilization defined previously, however, it is applied to thermal neutrons only now, i.e.

$$f = \frac{\bar{\Sigma}_{aF}}{\bar{\Sigma}_{a}}$$

where

- $\bar{\Sigma}_a$ is **thermal** mac. abs. XS of the material mix,
- $\bar{\Sigma}_{aF}$ is **thermal** mac.c abs. XS of fuel,
- $\bar{\Sigma}_{aM}$ is **thermal** mac. abs. XS of moderator.

So, f is a fraction of thermal neutrons that is absorbed in the fuel.

For simplicity, we assumed that $\bar{\Sigma}_a = \bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}$

Thermal reactors

Define η_T

η_T is the average number of neutrons emitted per **thermal** neutron absorbed in fuel.

Define the fast fission factor ϵ

- A small fraction of fissions is induced by fast neutrons.
- Fast fission factor ϵ is defined as the **ratio of the total number of fission neutrons produced by both fast and thermal fission to the number of fission neutrons produced by thermal fission alone**.
- The value of ϵ for reactors fueled with natural or slightly enriched uranium ranges from about 1.02 to 1.08.

Define the resonance escape probability p

- In a thermal reactor, most of the neutrons are absorbed after they become thermal.
- Nevertheless, some neutrons do get absorbed while slowing down, mainly by nuclei with absorption resonances at energies above the thermal region.
- Resonance escape probability p is the probability that a fission neutron is not captured during the slowing-down process.

What is the four-factor formula good for?

The four-factor formula gives the k_∞ in thermal reactors as

$$k_\infty = \eta_T \epsilon p f$$

Thermal reactors

How can we specify the non-leakage probability for thermal reactors?

- Previously, we defined the non-leakage probability P_L as the probability that a neutron is absorbed (does not leak out of the system),

$$P_L = \frac{\Sigma_a \phi}{\Sigma_a \phi - D\nabla^2 \phi} = \frac{\Sigma_a}{\Sigma_a + DB^2} = \frac{1}{1 + B^2 L^2}$$

  and we showed that for a critical system it holds that $k_\infty P_L = 1$.
- In thermal reactors, we can factorize P_L into a product of two factors

$$P_L = P_T \times P_F$$

  where $P_T = \frac{1}{1+B^2 L_T^2}$ is the non-leakage probability for thermal neutrons, and $P_F = \frac{1}{1+B^2 \tau_T}$ is the probability that neutrons don't leak out while slowing down.
- Finally, we can write the so-called **six-factor formula** for k:

$$k = \eta_\mathrm{T} \times \epsilon \times p \times f \times P_T \times P_F$$

- Note that P_L may be approximated as $\frac{1}{1+B^2 M_T^2}$ where $M_T^2 = L_T^2 + \tau_T$ is called the thermal migration area.

Reflected reactors

Reflected reactors

The purpose of the reflector around the active core

- The neutron economy can be improved when the reactor core is surrounded by a reflector (a thick unfueled region of moderator).
- Neutrons that otherwise would leak from the bare core pass into the reflector, and some of them diffuse back into the core.
- As a result, the critical size (mass) of the system can be reduced when the reflector is used.

To be able to solve the flux in the core and the reflector, we need to set interface boundary conditions (between the core and reflector). How?

- Continuity of the flux at the interface between the core and reflector, i.e.

$$\phi_c(R) = \phi_r(R)$$

  where c denotes the core, r denotes the reflector, and R denotes the interface position.

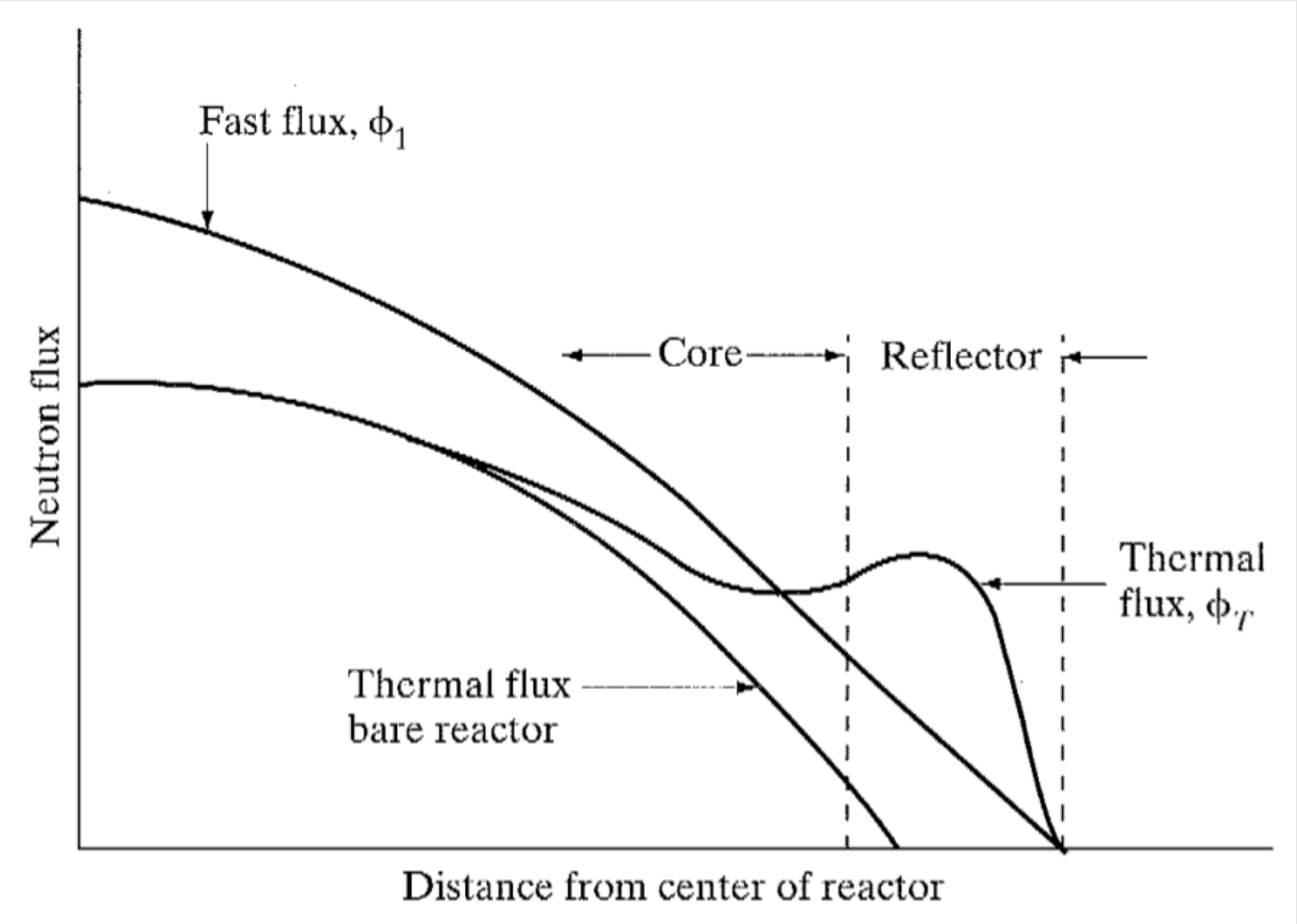
- Continuity of the current at the interface between the core and reflector, i.e.

$$D_c\phi_c'(R) = D_r\phi_r'(R)$$

Reflected reactors

Can we solve the neutron flux correctly in reflected reactors with one-group diffusion approximation?

Criticality calculations for reflected reactors require two-group or multi-group diffusion approximation (or more accurate methods) in order to accurately describe the effect of the reflector on the neutron flux.



Heterogeneous reactors

Is it possible to use the four-factor formula for non-homogeneous reactors?

Yes, it is possible to use the four-factor formula

$$k_\infty = \eta_\mathrm{T} \epsilon p f$$

for non-homogeneous reactors, however, the factors need to reflect the non-homogeneous environment.

How can we compute the value of η_T for UO_2 fuel?

- The term "fuel" then refers to a mixture of ^{235}U, ^{238}U, and oxygen.
- η_T can be calculated as

$$\eta_T = \frac{\nu_{235} \bar{\Sigma}_{f235}}{\bar{\Sigma}_{a235} + \bar{\Sigma}_{a238}}$$

since the absorption cross section of oxygen is practically zero.

How can we compute the thermal utilization in heterogeneous reactors?

- Thermal utilization is a probability that a thermal neutron, if it is absorbed in the core, is in fact absorbed in fuel.
- This equals to the ratio of the absorption rate in fuel to the absorption rate in fuel and moderator,

$$f = \frac{\bar{\Sigma}_{aF}\bar{\phi}_{TF}V_F}{\bar{\Sigma}_{aF}\bar{\phi}_{TF}V_F + \bar{\Sigma}_{aM}\bar{\phi}_{TM}V_M}$$

where

  - $\bar{\Sigma}_{aF}$ and $\bar{\Sigma}_{aM}$ is mac. thermal absorption cross sections of fuel and moderator, resp.
  - $\bar{\phi}_{TF}$ and $\bar{\phi}_{TM}$ is the average thermal flux in fuel and moderator, resp.
  - V_F and V_M is the volume of fuel and moderator, resp.

What is the thermal disadvantage factor in heterogeneous reactors?

Divide the numerator and denominator in

$$f = \frac{\bar{\Sigma}_{aF}\bar{\phi}_{TF}V_F}{\bar{\Sigma}_{aF}\bar{\phi}_{TF}V_F + \bar{\Sigma}_{aM}\bar{\phi}_{TM}V_M}$$

by $\bar{\phi}_{TF}$ to obtain the form

$$f = \frac{\bar{\Sigma}_{aF}V_F}{\bar{\Sigma}_{aF}V_F + \bar{\Sigma}_{aM}V_M\zeta}$$

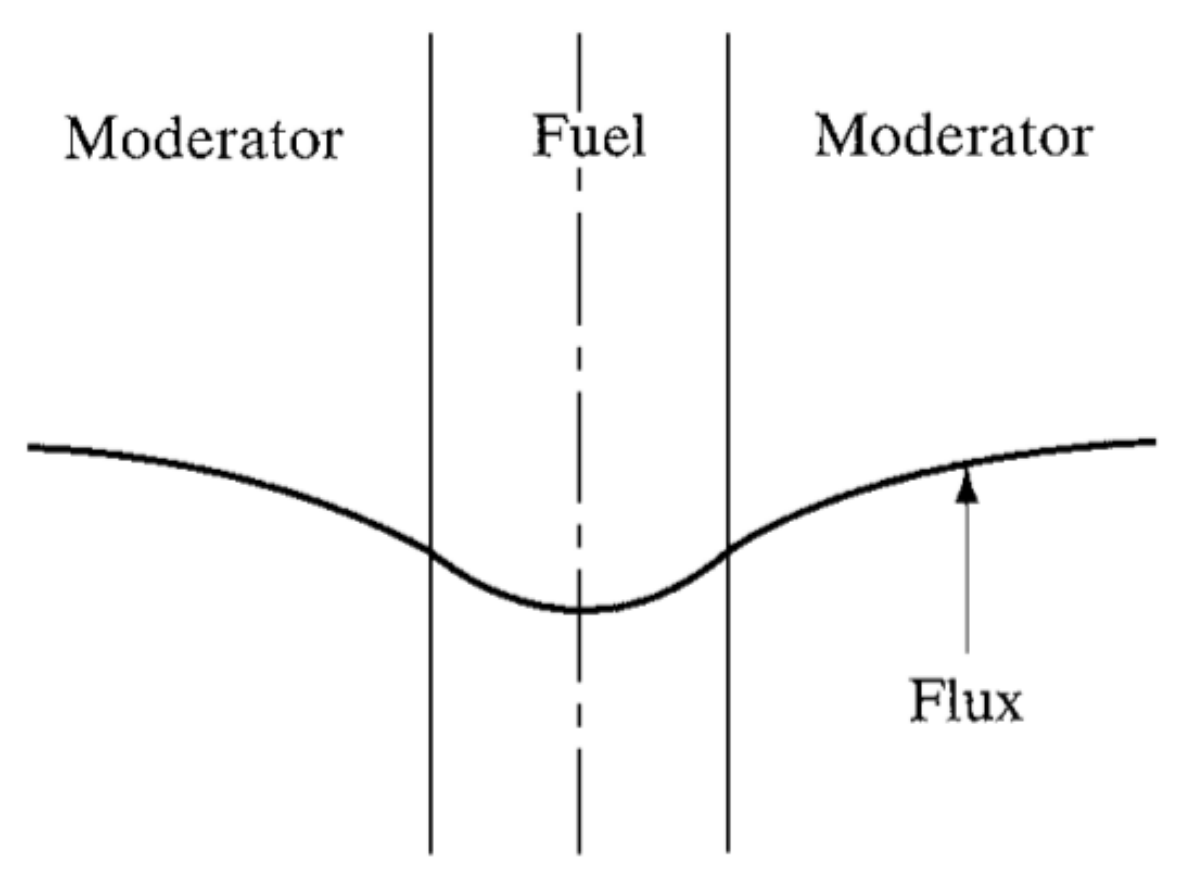
where

$$\zeta = \frac{\bar{\phi}_{TM}}{\bar{\phi}_{TF}}$$

is called the **thermal disadvantage factor**.

Is the thermal disadvantage factor larger or smaller than unity?

- The thermal disadvantage factor is typically larger than unity since the thermal neutron flux in moderator is usually larger than flux in the fuel.
- The depression of flux in fuel is caused by absorption of neutrons near the surface of fuel. The outer layer of fuel shields the interior, which is known as **self-shielding**.



Heterogeneous reactors

Using the same amount of fuel and moderator, is the thermal utilization f larger in heterogeneous or homogeneous reactors?

- Heterogeneous: $f = \frac{\bar{\Sigma}_{aF} V_F}{\bar{\Sigma}_{aF} V_F + \bar{\Sigma}_{aM} V_M \zeta}$
- Homogeneous: $f = \frac{\bar{\Sigma}_{aF} V_F}{\bar{\Sigma}_{aF} V_F + \bar{\Sigma}_{aM} V_M}$
- The only difference is the thermal disadvantage factor ζ that is > 1 in heterogeneous reactors and equal to unity in homogeneous reactors.
- Therefore, f is smaller in heterogeneous reactors than in homogeneous.

How can we compute the resonance escape probability p?

An empirical formula for p has been suggested as

$$p = \exp\left[-\frac{N_F V_F I}{\xi_M \Sigma_{sM} V_M}\right]$$

where

- N_F is the fuel atom density in units of 10^{24}.
- V_F and V_M are volumes of fuel and moderator, resp.
- ξ_M is the average increase in lethargy u per collision in the moderator $[u = \ln(E_M/E)]$ where E_M is an arbitrary energy,

$$\xi_M = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right)$$

- Σ_{sM} is the macroscopic scattering cross section of moderator at resonance energies
- I is the **resonance integral** (next slide).

How can we compute the resonance integral I?

The resonance integral I can be approximated by another empirical formula as

$$I = A + C/\sqrt{a\rho}$$

where

- a is the fuel rod radius in cm
- ρ is the density of the fuel in g/cm^3

Fuel	A	C
^{238}U (metal)	2.8	38.3
$^{238}UO_2$	3.0	39.6
^{232}Th (metal)	3.9	20.9
$^{232}ThO_2$	3.4	24.5

Figure 5: A and C parameters for the resonance integral

Heterogeneous reactors

Is the resonance escape probability p larger in heterogeneous or homogeneous reactors?

The resonance escape probability p for heterogeneous systems is larger than for equivalent homogeneous mixture because resonance neutrons do not collide with fuel while slowing down in the moderator.

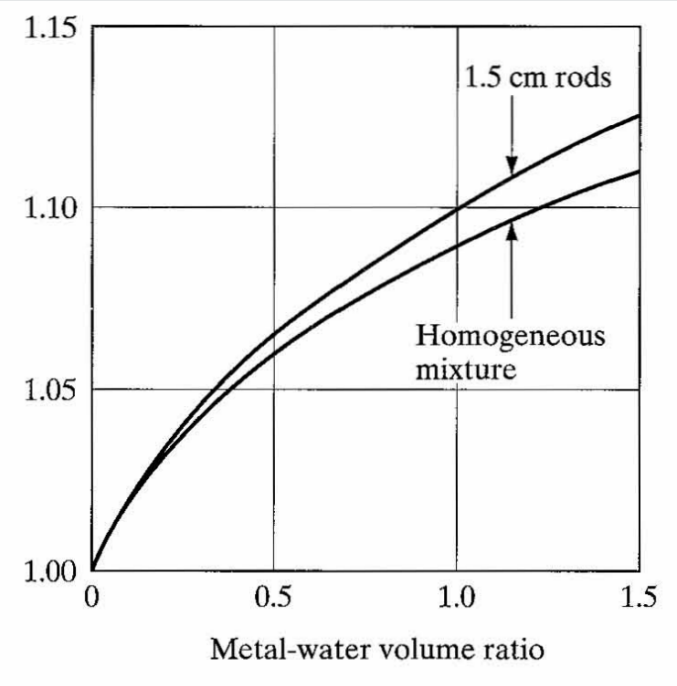
Although f is smaller in heterogeneous reactors than in homogeneous, the decrease is smaller than the corresponding increase in p, i.e.

$$(fp)_{\text{heterogeneous}} > (fp)_{\text{homogeneous}}$$

Is the fast fission factor ϵ larger in heterogeneous or homogeneous reactors?

$$\epsilon_{\text{heterogeneous}} > \epsilon_{\text{homogeneous}}$$

since in heterogenous systems fast neutrons pass through a region of pure fuel before they enter moderator, and fast neutrons are thus more likely to cause fission. Moreover, experiments with slightly enriched uranium rods in water show that the fast fission factor ϵ increases as the uranium to water volume ratio increases.



Heterogeneous reactors

Is the value of k_∞ larger in heterogeneous or homogeneous reactors?

Since

$$(fp)_{\text{heterogeneous}} > (fp)_{\text{homogeneous}}$$

and

$$\epsilon_{\text{heterogeneous}} > \epsilon_{\text{homogeneous}}$$

it follows that

$$(k_\infty)_{\text{heterogeneous}} > (k_\infty)_{\text{homogeneous}}$$

Note that

Max. value of k_∞ for a homogeneous mixture of **natural uranium and graphite** is only 0.85, while a heterogeneous configuration of these materials can bring k_∞ above unity, making it possible to construct critical reactor.