Sustainable Energy Transformation Technologies, SH2706

Lecture No 8

Title:

Energy transformation and degradation in single-phase flows

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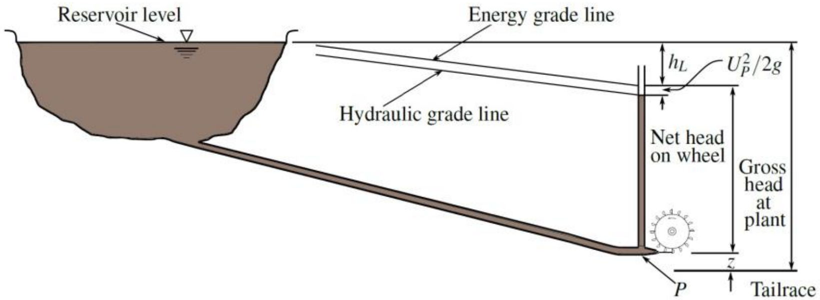
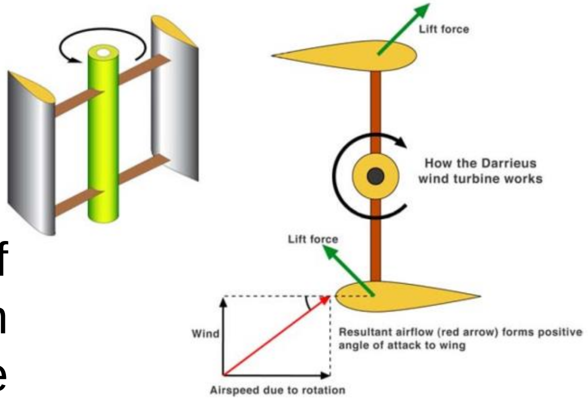
Outline

- Energy transformation and degradation in fluid mechanics
  - Pressure drops and losses in internal flows
  - Drag and lift forces in external flows



Losses in penstock of a hydropower plant



Transformation of kinetic energy of fluid into kinetic energy of a rotor in a wind turbine

Pressure Change in Pipe D=const

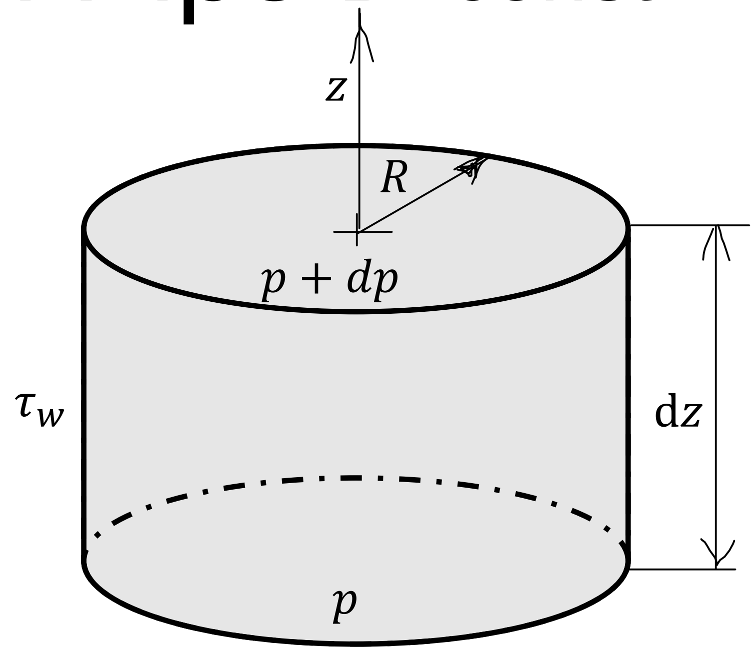
- Pressure change along a pipe with constant D depends on the shear stress and gravity
- We can write the following force balance for the fluid element shown to the right

$$\underbrace{p\pi R^2}_{\text{pressure force on bottom}} - \underbrace{(p+dp)\pi R^2}_{\text{pressure force on top}} - \underbrace{\tau_w dz 2\pi R}_{\text{downward shear force on wall}} + \underbrace{\pi R^2 dz \rho g_z}_{\text{gravity force}} = 0$$

- or

$$-\frac{dp}{dz} = \frac{2\tau_w}{R} - \rho g_z$$

  - here *p* is the cross-section averaged pressure in the pipe

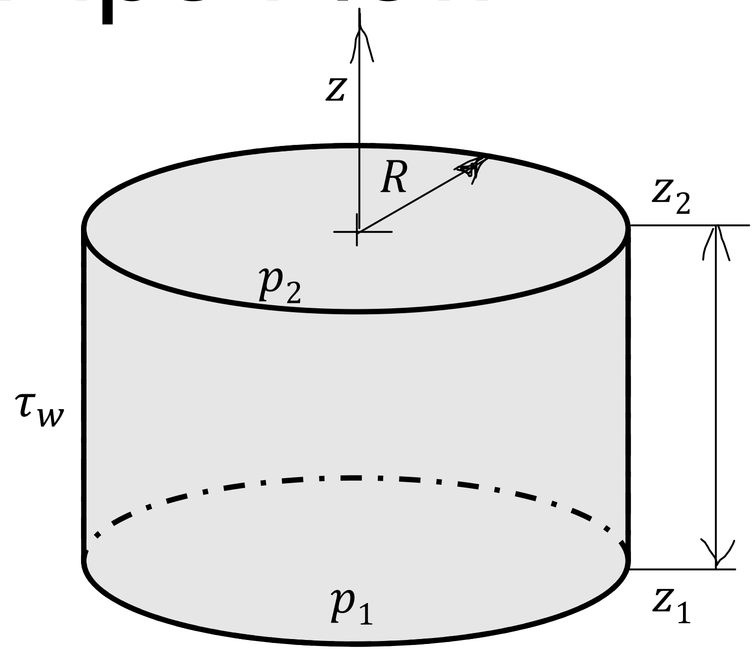


ρ – fluid mass density
g_z – gravity acceleration projected on the pipe axis
τ_w – wall shear stress

Gravity Head in Pipe Flow

- When there is no flow (stagnant fluid), only the gravity head is present, since τ_w=0:

$$\frac{dp}{dz} = \rho g_z$$



- For vertical up-flow (as in a reactor core), g_z = -g and:

$$\frac{dp}{dz} = -\rho g \Rightarrow dp = -\rho g dz$$

- Integration from (1) to (2) yields

$$\int_1^2 dp = -\rho g \int_1^2 dz \Rightarrow$$
$$p_2 - p_1 = -\rho g \left(z_2 - z_1 \right)$$

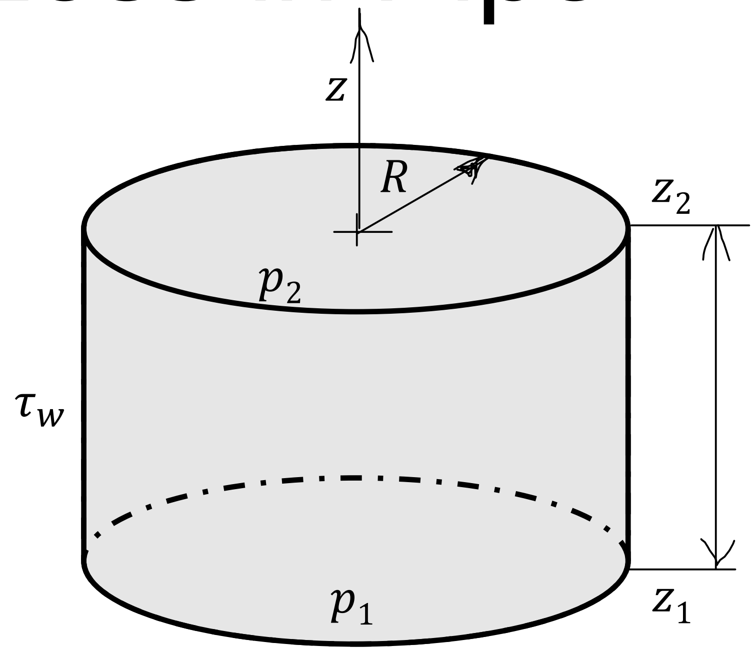
Friction Pressure Loss in Pipe

- When there is no gravity term (g_z=0 for horizontal flows), only the friction pressure loss is present, since τ_w>0:

$$-\frac{dp}{dz}=\frac{2\tau_w}{R}$$

- For flow in channel with constant R and τ_w, the integral pressure drop is:

$$-\int_1^2 dp = p_1 - p_2 = \frac{2\tau_w}{R}\int_1^2 dz = \frac{2\tau_w}{R}\left(z_2 - z_1\right)$$



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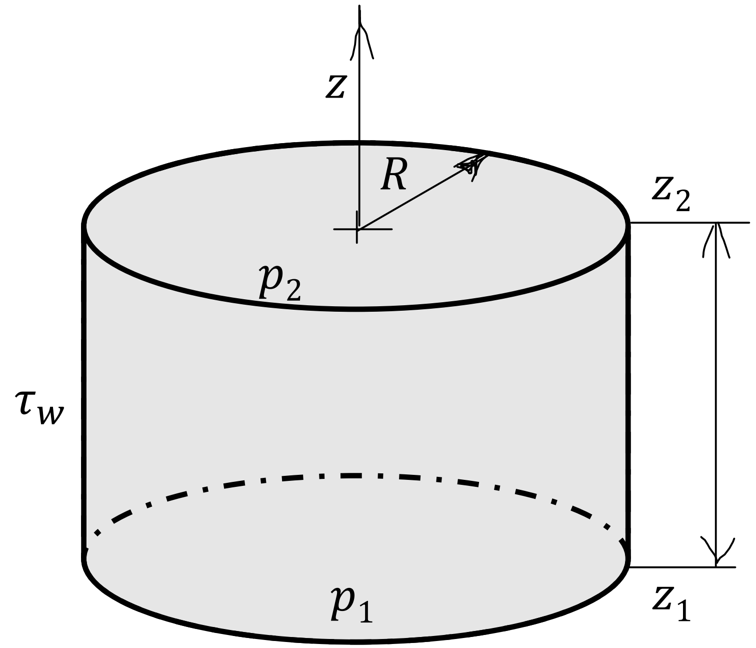
Wall Shear

- We notice that to find pressure loss in a pipe due to friction, we have to know the value of the wall shear stress τ_w,



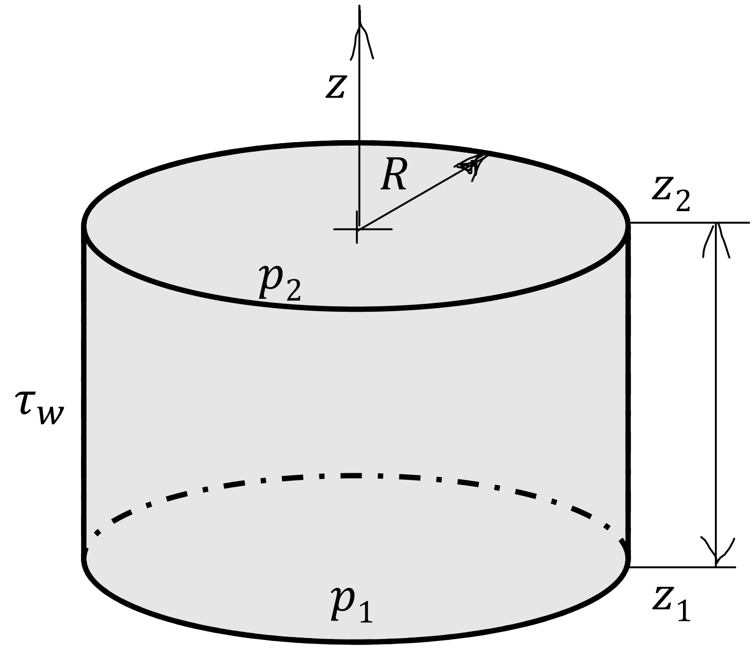
- Unfortunately the wall shear stress can be found analytically only for laminar flows. For laminar flow in circular pipe we have:

$$\tau_w = \frac{4\mu U}{R}$$

here μ is the dynamic viscosity of the fluid: μ=ρ* v

Friction Factors

- For turbulent flows, τ_w, can be obtained only from correlations
- The correlations are frequently expressed in terms of friction factors
- Two different definitions for friction factors are used:



$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2}$$

Fanning friction factor

$$\lambda \equiv \frac{4\tau_w}{\frac{1}{2}\rho U^2}$$

Darcy-Weisbach friction factor

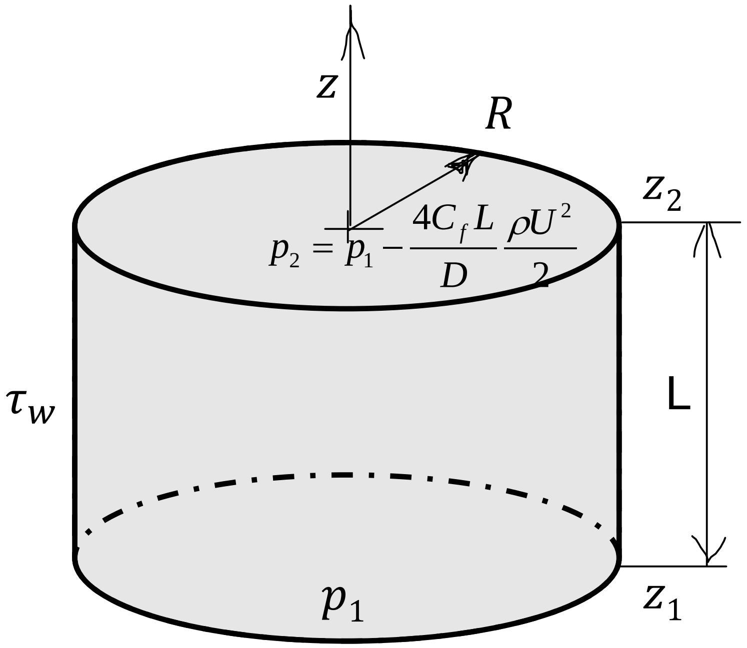
Blasius' Correlation

- One of the early correlations of friction factor valid for turbulent flow in **smooth pipes** was given by Blasius:

$$C_f = \frac{0.0791}{\mathrm{Re}^{0.25}}$$

- This correlation is applicable for smooth-wall pipes and the Reynolds number in a range:

$$4000 < \mathrm{Re} < 10^5$$



$$p_1 - p_2 = \frac{2\tau_w}{R}\left(z_2 - z_1\right) =$$

$$\frac{2C_f}{R}\frac{\rho U^2}{2} L = \frac{4C_f L}{D}\frac{\rho U^2}{2}$$

Colebrook's Correlation

- For turbulent flow in **rough pipes** Colebrook proposed the following correlation for the Darcy-Weisbach friction factor:

$$\frac{1}{\sqrt{\lambda}} = -2.0\log_{10}\left(\frac{k/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

- This correlation is transcendental, so iteration is needed
  - here *D* is the diameter and *k* is the wall roughness (in units of length), with typical values as shown in table

Effective surface roughnesses

Surface	**k (mm)**
Concrete	0.3 – 3.0
Coast iron	0.25
Galvanized iron	0.15
Commercial steel	0.046
Drawn tubing	0.0015

Source: J.O. Wilkes, Fluid Mechanics for Chemical Engineers, Prentice Hall 2010, p. 136.

Haaland's Correlation

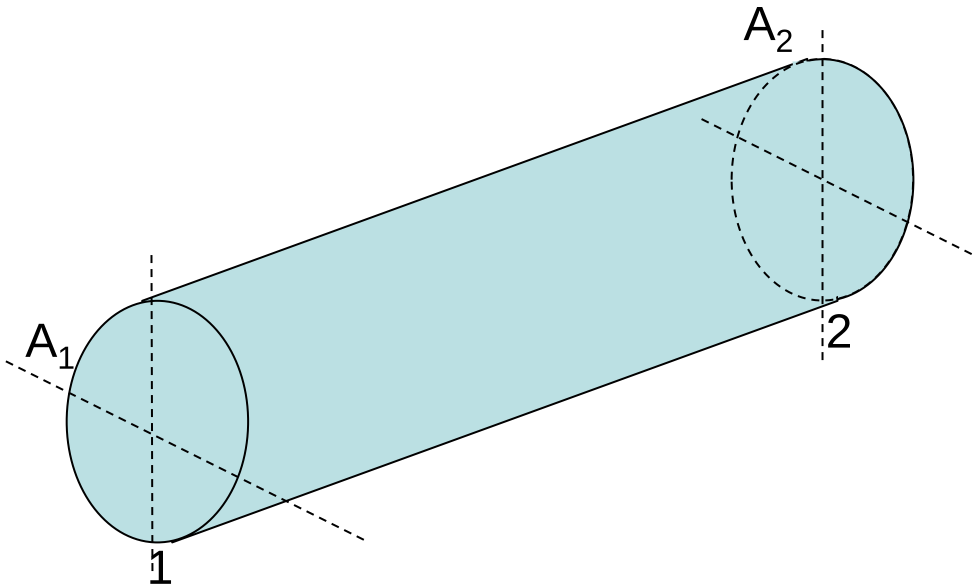
- To avoid iteration, Haaland proposed the following **explicit** correlation for the Darcy-Weisbach friction factor

$$\frac{1}{\sqrt{\lambda}} = -1.8\log_{10}\left[\left(\frac{k/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

- This correlation agrees within 1% with the Colebrook's correlation
  - *k*, *D* and Re have the same meaning as in the Colebrok's correlation

Mass Conservation in Pipe

- For steady-state flow of incompressible fluid in a channel, the mass conservation requires that $W_1 = W_2$



$$\rho U_1 A_1 = \rho U_2 A_2$$

$$U_1 A_1 = U_2 A_2 = Q_1 = Q_2$$

$$G_1 = \rho U_1 = \frac{\rho U_2 A_2}{A_1} = G_2 \frac{A_2}{A_1}$$

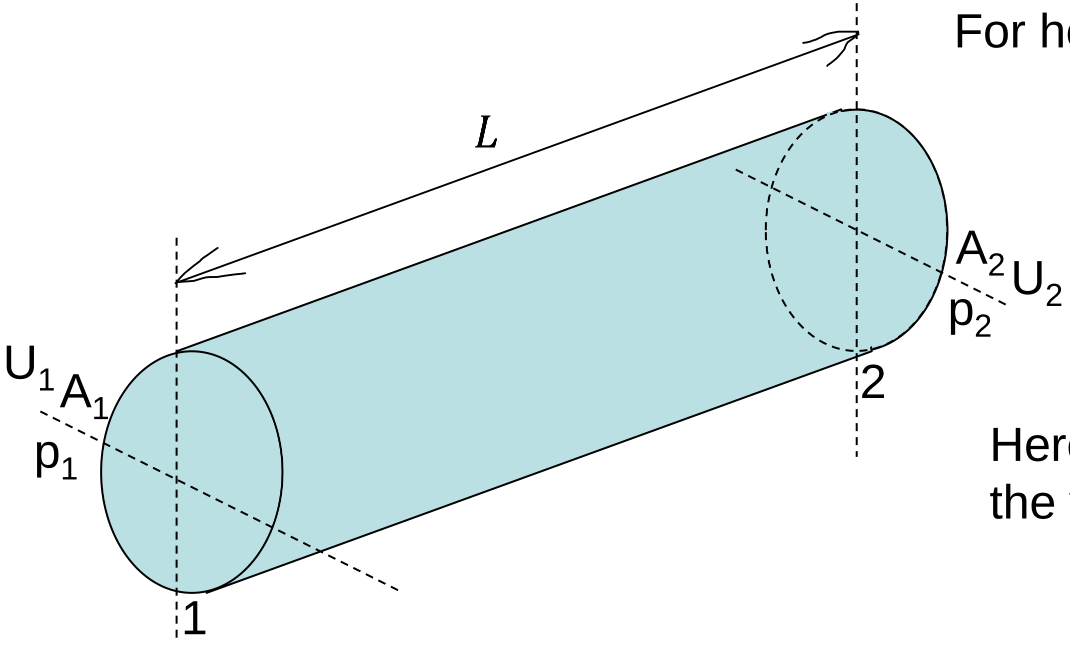
$$U_1 = U_2 \frac{A_2}{A_1}$$

Thus mass flux G and mean velocity U change along a channel with variable cross-section area A

Momentum Conservation in Pipe

- For steady-state flow of incompressible fluid in a channel, the momentum conservation between cross sections (1) and (2) gives:

$$p_1 A_1 + W_1 U_1 - p_2 A_2 - W_2 U_2 + \rho V g_z - \tau_w A_w = 0$$



For horizontal flow g_z = 0 and we have:

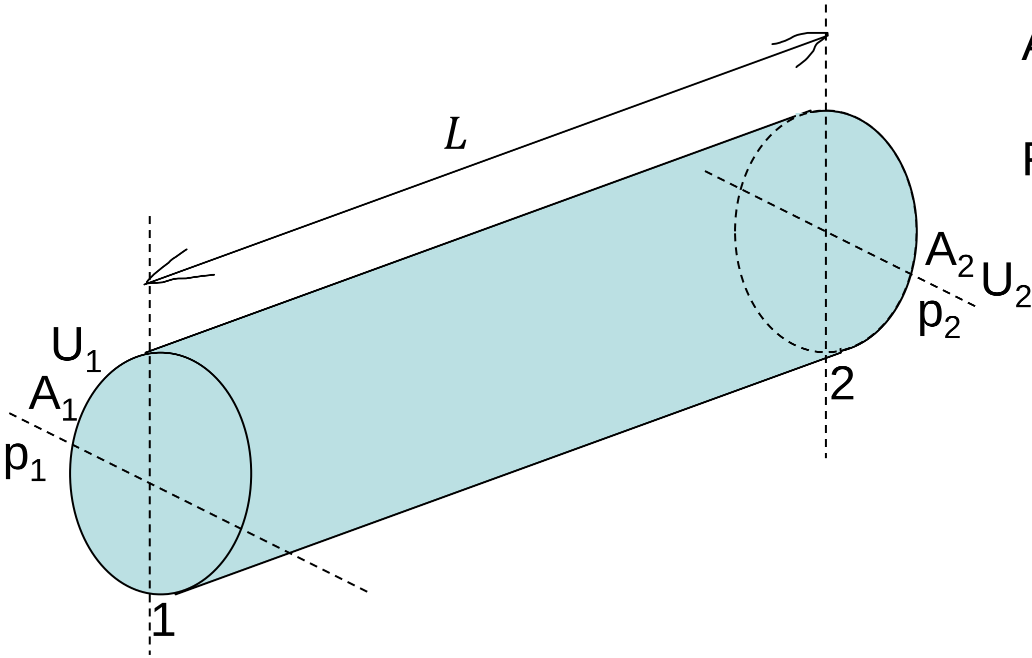
$$p_1 A_1 + \rho U_1^2 A_1$$
$$- p_2 A_2 - \rho U_2^2 A_2 - \tau_w A_w = 0$$

Here A_w is the wall total area and τ_w is the wall shear stress:

Energy Conservation in Pipe (1)

- For steady-state flow of fluid in a channel, the energy conservation between cross sections (1) and (2) gives:

$$\frac{dE_T}{dt} = q - N + W_1 e_{T1} - W_2 e_{T2} = 0$$



Assuming adiabatic flow, we have q=0

Assuming no work performed, N = 0

For steady-state flow, $dE_T/dt = 0$

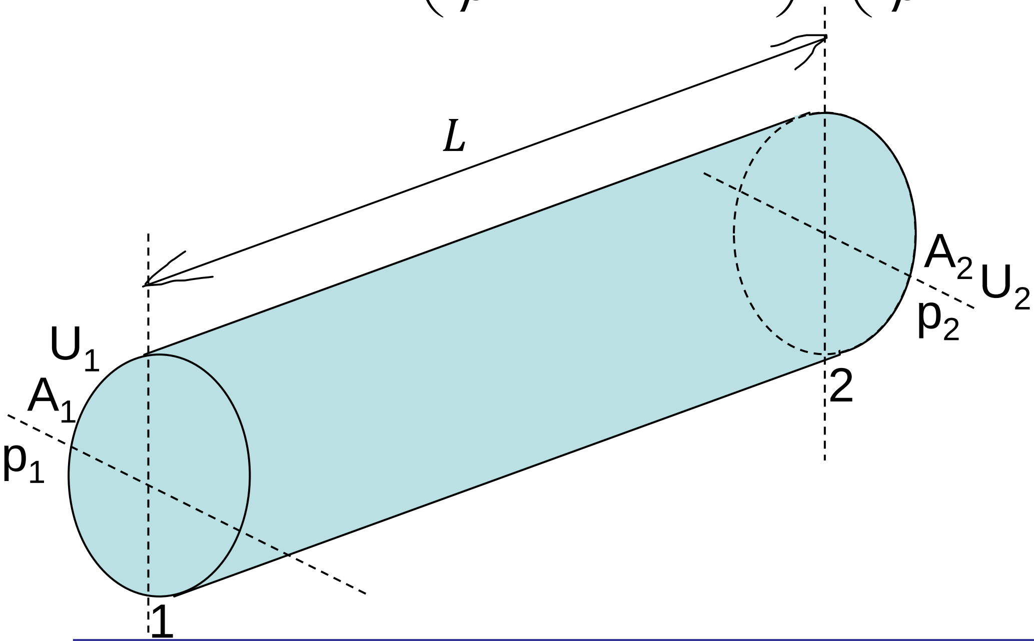
Thus, the energy equation becomes

$$W_1\left(i_1 + \frac{U_1^2}{2} + gH_1\right) - W_2\left(i_2 + \frac{U_2^2}{2} + gH_2\right) = 0$$

Energy Conservation in Pipe (2)

- For flows with shear loss and no heat transfer terms, the internal energy increases due to friction along the pipe. Since mass conservation gives $W_1 = W_2 = const$, we have :

$$\left(\frac{p_1}{\rho} + \frac{U_1^2}{2} + gH_1\right) - \left(\frac{p_2}{\rho} + \frac{U_2^2}{2} + gH_2\right) - e_{loss,1-2} = 0$$



Here $e_{loss,1\text{-}2}$ is the specific energy loss along the pipe due to various irreversible processes, such as wall friction and turbulence.

Bernoulli Equation (1)

- For adiabatic channels with no losses the energy conservation equation can be thus written as follows:

$$\left(\frac{p_1}{\rho}+\frac{U_1^2}{2}+gH_1\right)=\left(\frac{p_2}{\rho}+\frac{U_2^2}{2}+gH_2\right)$$

- Thus:

$$\frac{p}{\rho}+\frac{U^2}{2}+gH=const$$

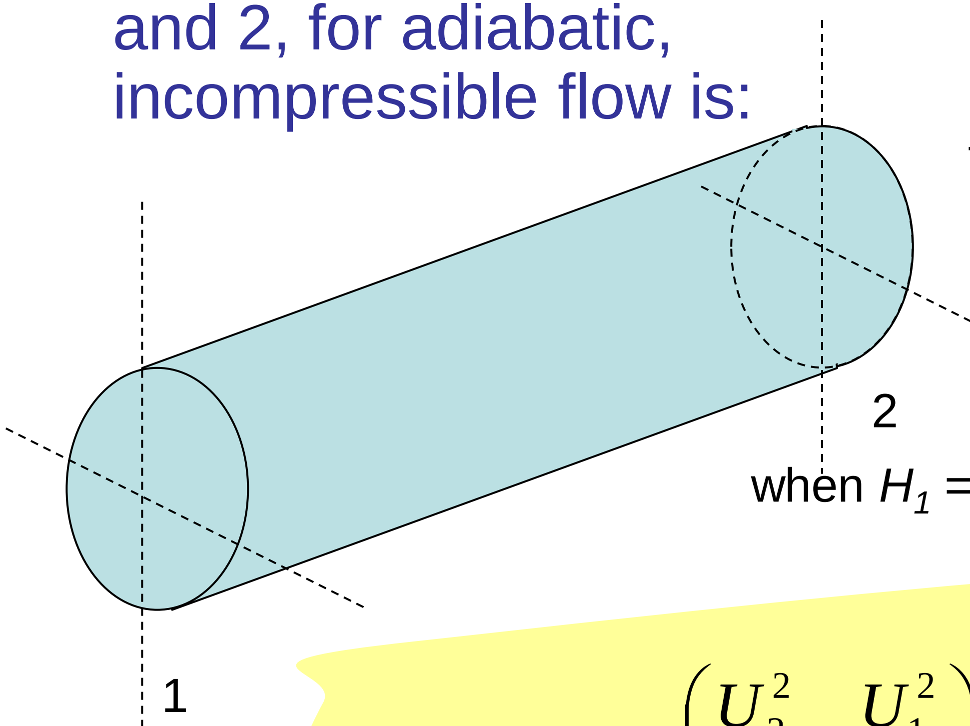
p – pressure
U – velocity
H – elevation
g – gravity
ρ - density

- This is the **Bernoulli equation**, which states that the sum of kinetic energy, static pressure divided by density and the potential energy in an adiabatic channel without friction is constant.

Bernoulli Equation (2)

- Bernoulli equation applied between two cross-sections: 1 and 2, for adiabatic, incompressible flow is:

$$\frac{p_1}{\rho}+\frac{U_1^2}{2}+gH_1=\frac{p_2}{\rho}+\frac{U_2^2}{2}+gH_2$$



when $H_1 = H_2$ and ρ = const

$$p_1-p_2=\rho\left(\frac{U_2^2}{2}-\frac{U_1^2}{2}\right)$$

Known velocity change between two cross-sections yields a pressure drop (Bernoulli effect)

Bernoulli Equation (3)

- Since velocity distribution in channels is not uniform, the kinetic energy convected in the channel should be calculated as:

$$\int_A \frac{u^2}{2}\rho u dA = \alpha \int_A \frac{U^2}{2}\rho U dA = \alpha W \frac{U^2}{2}$$

- Where $\alpha = \dfrac{\int_A \rho u^3 dA}{W U^2}$ is the **kinetic energy coefficient**

- Thus the Bernoulli equation for channels (including losses) should read:

$$\left(\frac{p_1}{\rho} + \frac{\alpha_1 U_1^2}{2} + gH_1\right) - \left(\frac{p_2}{\rho} + \frac{\alpha_2 U_2^2}{2} + gH_2\right) = e_{loss}$$

Here e_{loss} is the mechanical energy loss between points 1 and 2

Bernoulli Equation (4)

- For laminar flows in pipes the kinetic energy coefficient is α = 2

- For turbulent flows, the velocity profile is quite flat and can be represented by the empirical power-law equation

$$\frac{u(r)}{u_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

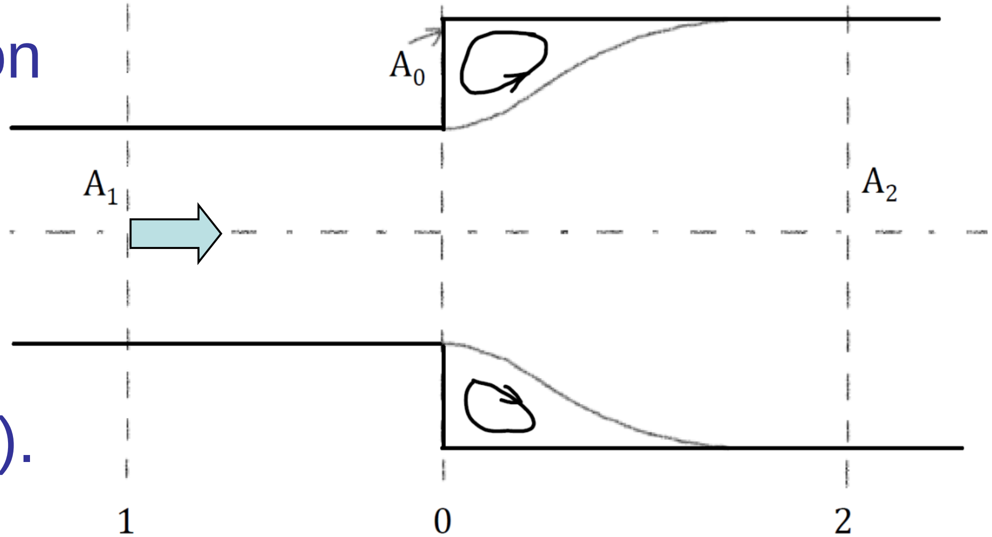
Exponent *n* varies with the Reynolds number. For fully developed turbulent flow *n* = 7 is used.

- The kinetic energy coefficient is then: $$\alpha = \left(\frac{u_c}{U}\right)^3 \frac{2n^2}{(3+n)(3+2n)}$$
- Where $$\frac{U}{u_c} = \frac{2n^2}{(1+n)(1+2n)}$$

u_c – velocity at the centerline

Local Pressure Losses

- Additional pressure losses occur due to local obstacles, (e.g. flow area and/or flow direction changes)
- These losses are termed as *local (or minor) losses*.
- One of the common flow obstacles is a sudden cross-sectional area change.
- When flow has a direction from the smaller to the larger cross-sectional area, the obstacle is called a sudden expansion (enlargement).

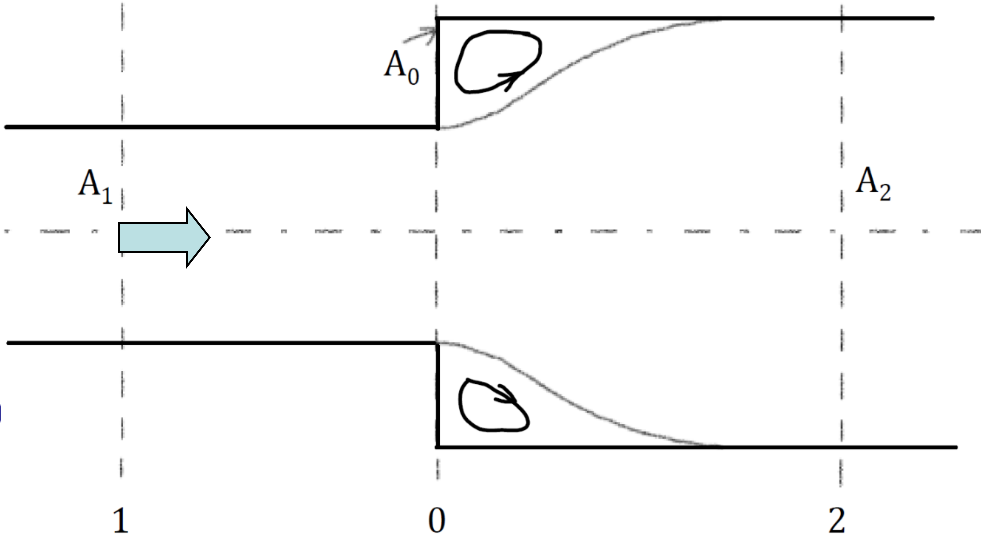


Sudden Expansion

- The total pressure change over the sudden expansion is as follows:

$$\Delta p = \Delta p_R + \Delta p_I \quad \text{where} \quad \Delta p_R = \rho\left(\frac{U_1^2}{2} - \frac{U_2^2}{2}\right) > 0 \quad \Delta p_I = -\rho\frac{U_1^2}{2}\left[1 - \left(\frac{A_1}{A_2}\right)\right]^2 < 0$$

- We note here that the reversible pressure term is positive (gain), but the irreversible term is negative (pressure loss)



$$-\Delta p_I = \rho\frac{U_1^2}{2}\left[1 - \left(\frac{A_1}{A_2}\right)\right]^2 = \xi_{enl}\rho\frac{U_1^2}{2} = \xi_{enl}\frac{G_1^2}{2\rho} = \xi_{enl}\frac{W^2}{2\rho A_1^2}$$

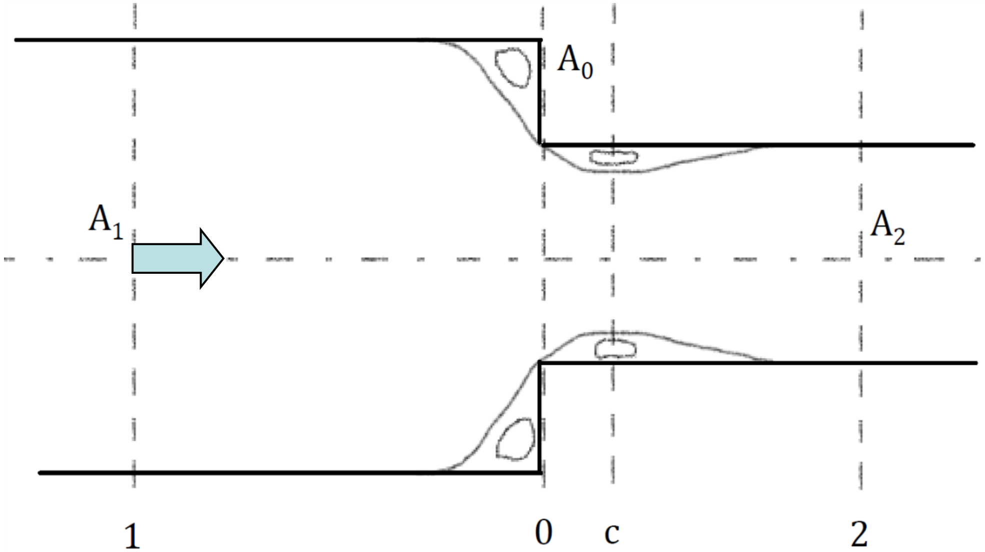
Sudden Contraction

- The total pressure change at sudden contraction is:

$$\Delta p = p_2 - p_1 =$$

$$\underbrace{\rho\left(\frac{U_1^2}{2} - \frac{U_2^2}{2}\right)}_{\Delta p_R < 0} \underbrace{-\left(\frac{A_2}{A_c} - 1\right)^2 \cdot \frac{G_2^2}{2\rho}}_{\Delta p_I < 0} =$$

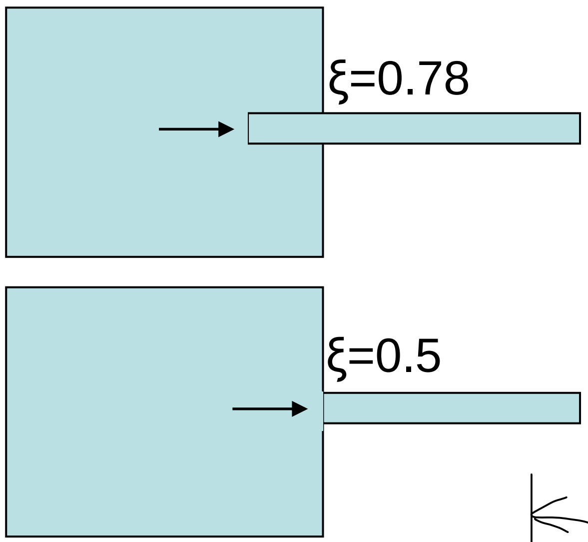
$$\Delta p_R + \Delta p_I$$



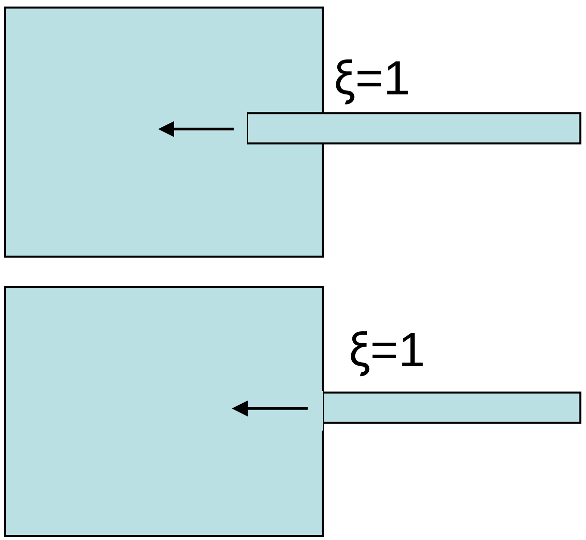
$$-\Delta p_I = \xi_{cont} \cdot \frac{G_2^2}{2\rho}; \quad \xi_{cont} = \left(\frac{A_2}{A_c} - 1\right)^2 \qquad \frac{A_c}{A_2} = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$$

Pressure Loss Coefficients for Inlets/Outlets

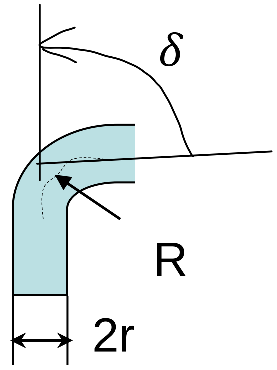
Pipe entrance



Pipe exit



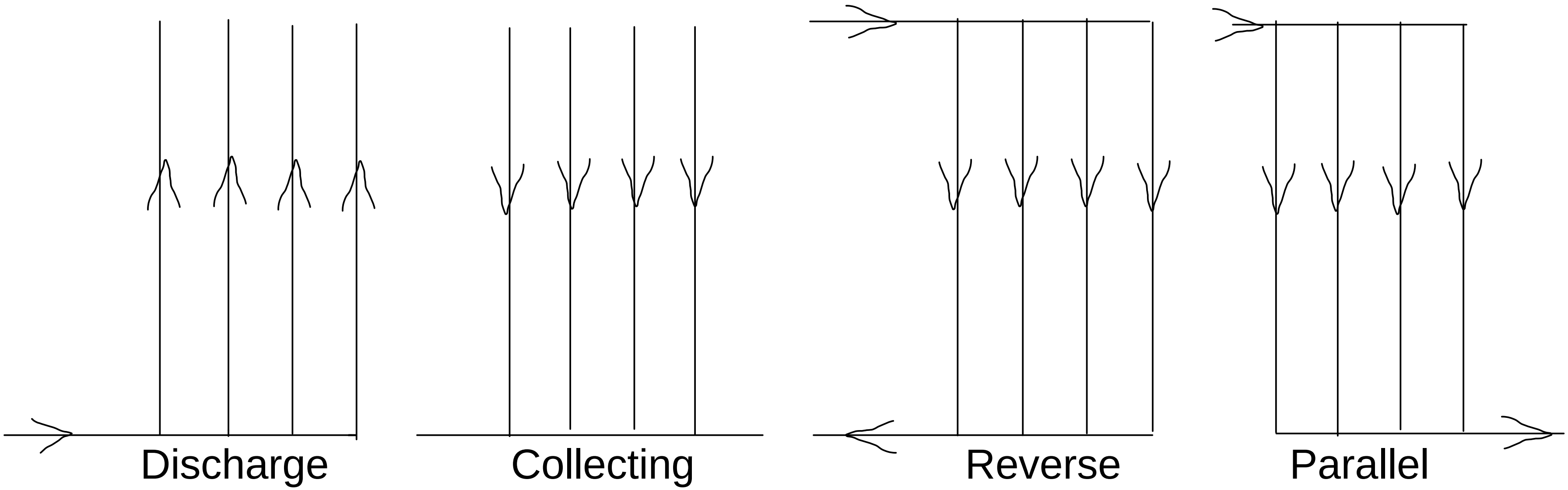
Elbow



$$\xi = \frac{\delta}{90^\circ}\left[0.131 + 1.847\left(\frac{r}{R}\right)^{3.5}\right]$$

Manifolds

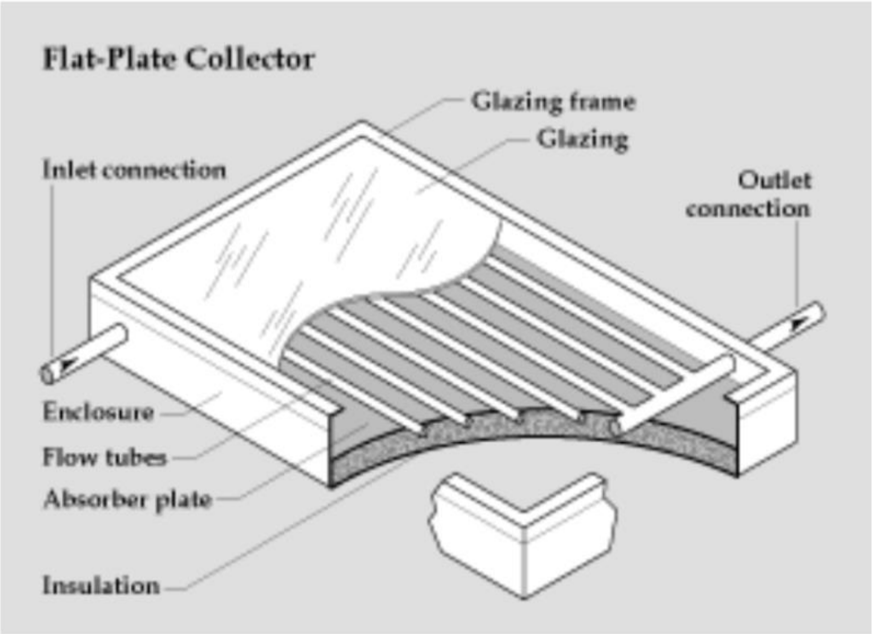
- In many energy transformation systems manifolds are used in pipelines
- Their main function is to distribute or collect flows



- Pressure losses in such systems are determined in both experimental and analytical ways

Parallel Manifold in Solar Power

- Flat-plate collector is an example of parallel manifold applied in solar power
- The design task is to calculate the mass flow rates through all branches
- For that purpose mass, momentum and energy (Bernoulli) equations have to be solved.
- Proper local and friction losses have to be applied



Flat plate collector. Credit: US Dept of Energy.

Total Pressure Drop (1)

- For a channel with length *L*, momentum equation can be integrated from *z* = 0 to *z* = *L* to obtain the total irreversible pressure loss between the inlet and the outlet of the channel:

$$\left(p_1 - p_2\right)_I = \frac{4\tau_w}{D_h} L + \Delta p_{loc} = \left(\frac{4L}{D_h} C_f + \xi\right) \frac{\rho U^2}{2}$$

- or

$$\left(p_1 - p_2\right)_I = \left(\frac{4L}{D_h} C_f + \xi\right) \frac{W^2}{2\rho A^2}$$

- with obstacles, and piecewise constant A, the equation becomes

$$-\Delta p_I = \left(p_1 - p_2\right)_I = \frac{W^2}{2\rho}\left[\sum_k \left(\frac{4L_k}{D_{h,k}} C_{f,k}\right) \frac{1}{A_k^2} + \sum_j \xi_j \frac{1}{A_{j,\min}^2}\right]$$

Total Pressure Drop (2)

- The Bernoulli equation for whole path containing irreversible losses from point 1 to 2 is as follows

$$\frac{W^2}{2\rho A_1^2} + p_1 + \rho g H_1 = \frac{W^2}{2\rho A_2^2} + p_2 + \rho g H_2 + \Delta p_I$$

or

$$-\Delta p = p_1 - p_2 = \frac{W^2}{2\rho}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + \rho g\left(H_2 - H_1\right) + \Delta p_I$$

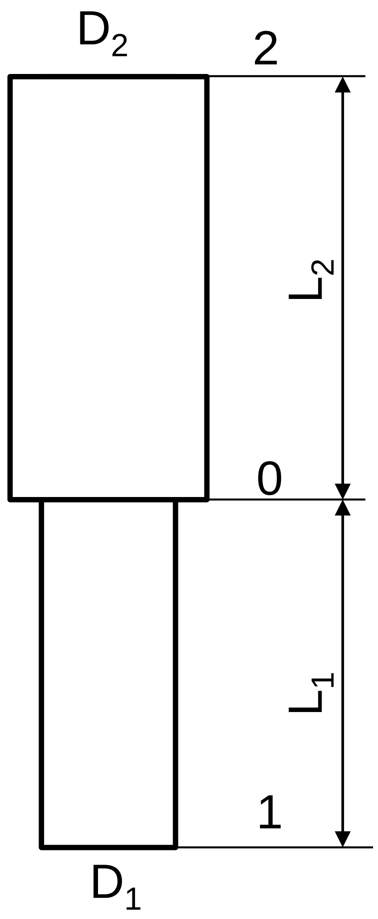
combining with the expression for Δp_I yields

$$\underbrace{p_1 - p_2}_{total\ pressure\ drop} = \frac{W^2}{2\rho}\left[\underbrace{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)}_{reversible\ velocity\ head} + \underbrace{\sum_k \left(\frac{4L_k}{D_{h,k}} C_{f,k}\right)\frac{1}{A_k^2}}_{irreversible\ friction\ loss} + \underbrace{\sum_j \xi_j \frac{1}{A_{j,\min}^2}}_{irreversible\ local\ loss}\right] + \underbrace{\rho g\left(H_2 - H_1\right)}_{reversible\ gravity\ head}$$

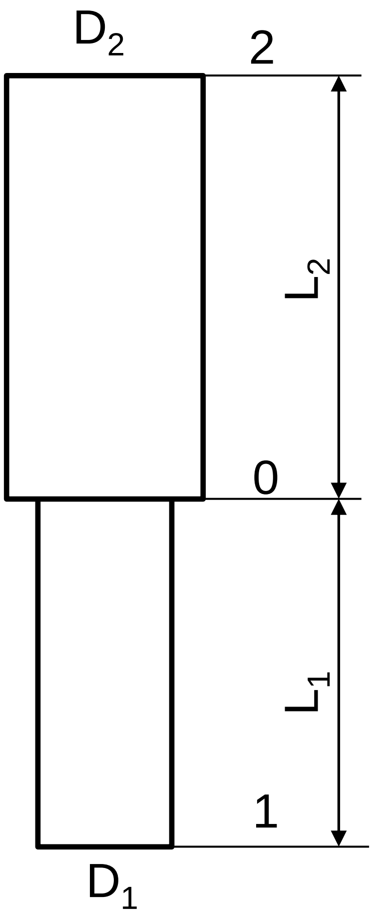
Example

- Consider vertical up-flow of steam at pressure p and temperature T in a pipe with a sudden enlargement. Mass flow rate is W. Assume wall roughness k (same for both pipes)

- Calculate the total pressure drop between cross-sections 1 and 2 neglecting any inlet and outlet losses at those cross-sections



Example



- We use the general equation as:

$$p_1 - p_2 = \frac{W^2}{2\rho}\left[\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)\right.$$

$$+\left(\frac{4L_1}{D_{h,1}}C_{f,1}\right)\frac{1}{A_1^2} + \left(\frac{4L_2}{D_{h,2}}C_{f,2}\right)\frac{1}{A_2^2}$$

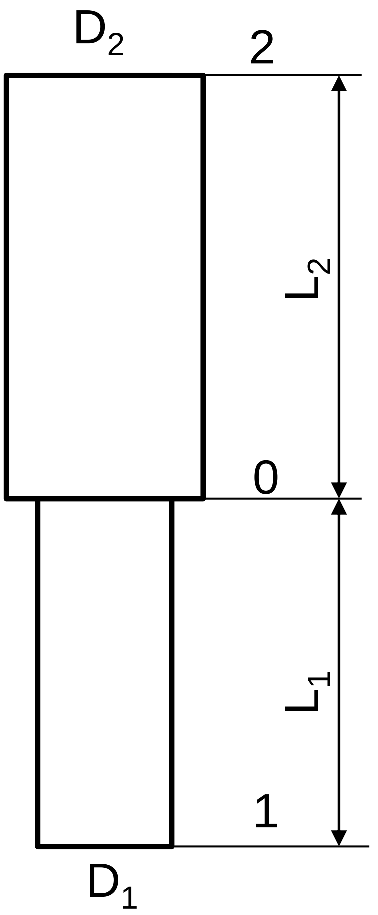
$$\left. + \frac{\xi_{enl}}{A_1^2}\right] \qquad \xi_{enl} \equiv \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2$$

$$\rho g\left(H_2 - H_1\right) \qquad H_2 - H_1 = L_1 + L_2$$

- where

$$\frac{1}{\sqrt{\lambda_j}} = -1.8\log_{10}\left[\left(\frac{k/D_j}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}_j}\right] \qquad C_{f,j} = \frac{\lambda_j}{4} \qquad \mathrm{Re}_j = \frac{D_j W}{\mu A_j} \qquad j=1,2$$

Example



- We need to know the steam density ρ and the dynamic viscosity μ. For that purpose any water/steam property tables/functions can be used
- In this course we use XSteam functions (see Canvas for Matlab and Excel versions of these functions to download)
- For example, in Matlab the steam density for pressure p and temperature T is found as **`rho=XSteam('rho_pT',p,T)`**. Similarly, the dynamic viscosity is found as **`my=XSteam('my_pT',p,T)`**

Drag and Lift Forces

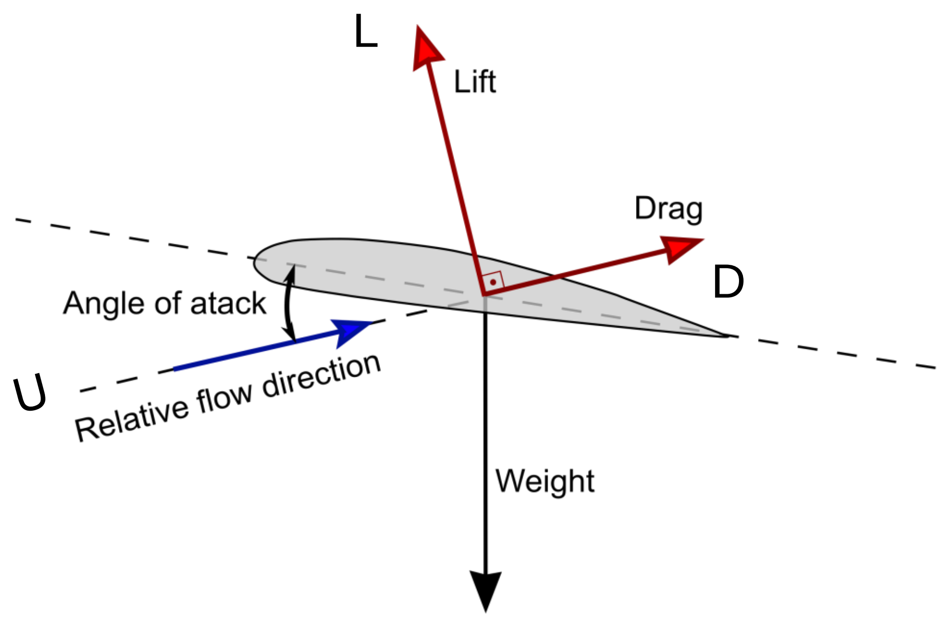
Drag and lift forces appear when a body is in a **relative motion** against the fluid surrounding it

Drag force (D) is the component of the force acting in the direction of the relative velocity

Lift force (L) is the component of the force acting in the direction perpendicular to the relative velocity

The forces depend on:

- angle of attack
- relative velocity *U*
- fluid density *ρ*
- reference area *A*



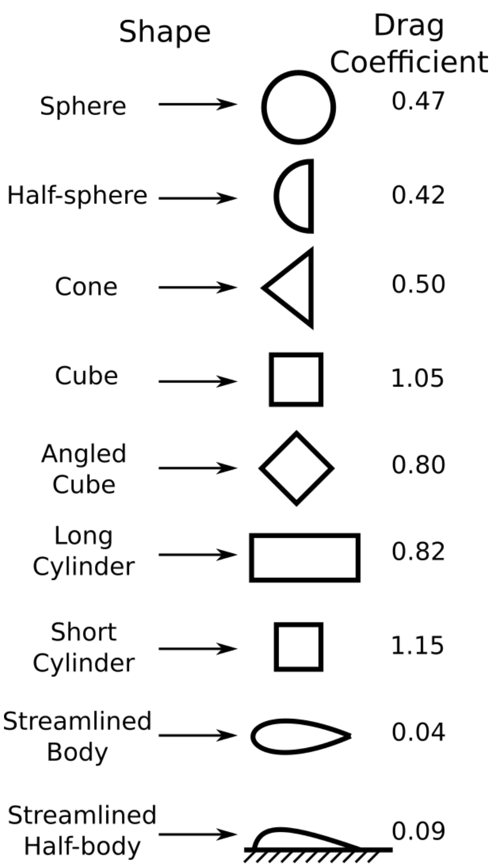
Drag force coefficient is defined as:

$$C_D = \frac{D}{\left(\rho U^2/2\right)A}$$

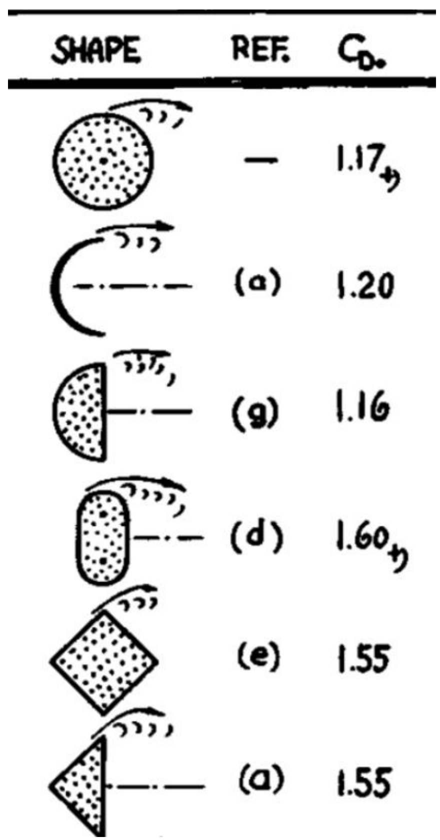
Lift force coefficient is defined as:

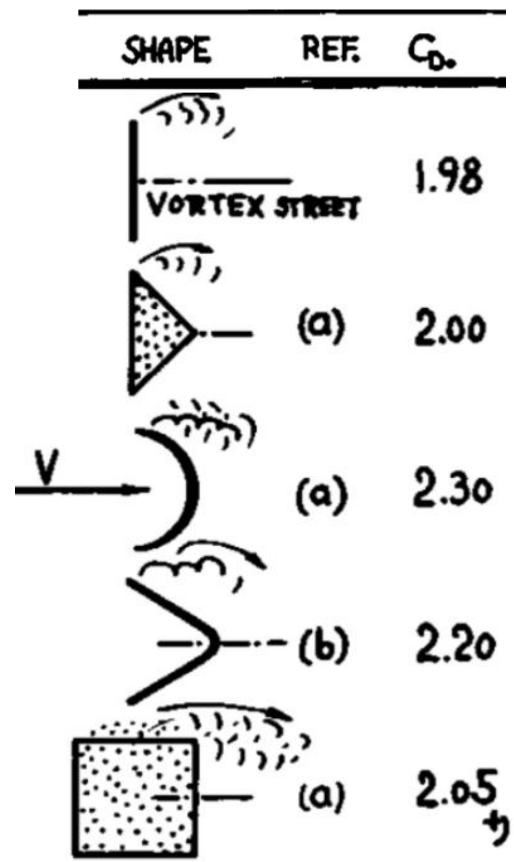
$$C_L = \frac{L}{\left(\rho U^2/2\right)A}$$

Example Drag Coefficient Values



Measured Drag Coefficients

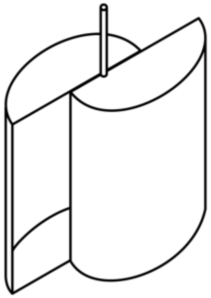


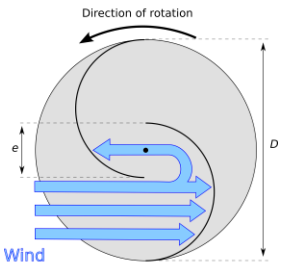


Ref: Sighard Hoerner, Fluid Dynamics Drag

Lift versus Drag Wind Turbine

- Wind turbines extract energy from the wind through aerodynamic forces, drag and lift
- Savonius wind turbine is a drag-based machine





- Darrieus wind turbine and conventional horizontal axis wind turbines are lift-based machines