Nuclear Reactor Physics

Reactor Dynamics II

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2022

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Fission product poisoning

What is the fission product poisoning?

All fission products absorb neutrons to some extent, so their accumulation in the fuel can reduce the multiplication factor.

Is the effect of fission products same in thermal and fast reactors?

Fission products affect mainly thermal reactors since absorption cross sections decrease rapidly with increasing neutron energy.

Fission product poisoning

Which of the factors in the six-factor formula is most sensitive to the fission product poisoning?

- Practically the only effect of fission products on reactivity comes via the **thermal utilization factor**.
- Let's have a critical reactor with no fission products, with multiplication factor $k_0 = 1$.
- Reactivity of the same reactor with fission products is then

$$\rho = \frac{k-1}{k} = \frac{k-k_0}{k}$$

- Since it is mainly the thermal utilization factor that is sensitive to the poisons, we can write

$$\rho = \frac{f-f_0}{f}$$

  where f and f_0 are the thermal utilization of the reactor with and without fission products, resp.

Fission product poisoning

- If we consider a homogeneous reactor then

$$f_0 = \frac{\overline{\Sigma}_{aF}}{\overline{\Sigma}_{aF} + \overline{\Sigma}_{aM}}; \quad f = \frac{\overline{\Sigma}_{aF}}{\overline{\Sigma}_{aF} + \overline{\Sigma}_{aM} + \overline{\Sigma}_{aP}}$$

  where $\overline{\Sigma}_{aP}$ is the macroscopic cross section of the poison, then

$$\rho = \frac{f - f_0}{f} = -\frac{\overline{\Sigma}_{aP}}{\overline{\Sigma}_{aF} + \overline{\Sigma}_{aM}}$$

- Terms $\overline{\Sigma}_{aF}$ or $\overline{\Sigma}_{aM}$ can be taken from the critical reactor,

$$k_0 = \eta_T p \epsilon f = \eta_T p \epsilon \frac{\overline{\Sigma}_{aF}}{\overline{\Sigma}_{aF} + \overline{\Sigma}_{aM}} = \nu p \epsilon \frac{\overline{\Sigma}_{f}}{\overline{\Sigma}_{aF} + \overline{\Sigma}_{aM}} = 1$$

$$\Rightarrow \rho = -\frac{\overline{\Sigma}_{aP}/\overline{\Sigma}_{f}}{\nu p \epsilon}$$

What is the most important fission product in thermal reactors?

The most important fission product is ^{135}Xe with its thermal absorption cross section of 2.65×10^6b.

How is ^{135}Xe formed in the fuel?

- ^{135}Xe is formed from fission and by decay of ^{135}I - another fission product:

$$\begin{array}{ccccccccc} ^{135}\mathrm{Te} & \xrightarrow[11\mathrm{sec}]{\beta-} & ^{135}\mathrm{I} & \xrightarrow[6.7\mathrm{hr}]{\beta-} & ^{135}\mathrm{Xe} & \xrightarrow[9.2\mathrm{hr}]{\beta-} & ^{135}\mathrm{Cs} & \xrightarrow[2.3\times10^6\mathrm{yr}]{\beta-} & ^{135}\mathrm{Ba(stable)} \\ \uparrow & & \uparrow & & \uparrow & & & & \\ \mathrm{Fission} & & \mathrm{Fission} & & \mathrm{Fission} & & & & \end{array}$$

Figure 1: Formation of ^{135}Xe.

- Since tellurium decays rapidly into ^{135}I, we can assume that all ^{135}I comes directly from fission.

Fission product poisoning

Fission yields (atoms per fission) for xenon-135, iodine-135 and promethium-149

Isotope	^{233}U	^{235}U	^{239}Pu
^{135}I	0.0475	0.0639	0.0604
^{135}Xe	0.0107	0.00237	0.0105
^{149}Pm	0.00795	0.01071	0.0121

Figure 2: Fission product yields

Isotope	λ, sec^{-1}	λ, hr^{-1}
^{135}I	2.87×10^{-5}	0.1035
^{135}Xe	2.09×10^{-5}	0.0753
^{149}Pm	3.63×10^{-6}	0.0131

Figure 3: Fission product decay constants

What are the processes by which ^{135}I is generated and removed? Suggest a balance equation for the production ^{135}I

We need to consider a) the production from the fission and b) the direct decay:

$$\frac{dI}{dt} = \gamma_I \overline{\Sigma}_f \phi_T - \lambda_I I$$

where I is the concentration of ^{135}I and γ_I is the effective yield of ^{135}I from fission.

What are the processes by which ^{135}Xe is generated and removed? Suggest a balance equation for the production ^{135}Xe

Xe is generated a) by the decay of ^{135}I and b) from the fission at some yield, and it is destroyed c) by its decay and d) by neutron capture:

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \overline{\Sigma}_f \phi_T - \lambda_X X - \overline{\sigma}_{aX} \phi_T X$$

where X is the concentration of ^{135}Xe, γ_X is the effective yield of ^{135}X from fission, and $\overline{\sigma}_{aX}$ is the thermal absorption cross section of ^{135}Xe.

Fission product poisoning

Saturated concentration of xenon

- When the terms in the right-hand side of the xenon rate equation

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \overline{\Sigma}_f \phi_T - \lambda_X X - \overline{\sigma}_{aX} \phi_T X$$

  balance to zero, the concentration of xenon becomes saturated.

- Saturated concentration of xenon, X_∞, and iodine, I_∞, can be obtained by setting the left-hand sides of the xenon and iodine rate equations to zero,

$$0 = \gamma_I \overline{\Sigma}_f \phi_T - \lambda_I I_\infty$$

$$0 = \lambda_I I_\infty + \gamma_X \overline{\Sigma}_f \phi_T - \lambda_X X_\infty - \overline{\sigma}_{aX} \phi_T X\infty$$

  from where

$$I_\infty = \frac{\gamma_I \overline{\Sigma}_f \phi_T}{\lambda_I}$$

$$X_\infty = \frac{(\gamma_I + \gamma_X)\overline{\Sigma}_f \phi_T}{\lambda_X + \overline{\sigma}_{aX} \phi_T}$$

What time is needed for xenon to reach its saturated level?

Saturated concentration of xenon is approached after about a day **when the neutron flux is fixed**.

How can the reactor keep its operation when its reactivity drops due to presence of ^{135}Xe in the fuel?

- The decrease in the reactivity due to presence of xenon must be compensated externally by adding reactivity by control elements to keep the reactor critical.
- The reactor must have sufficient reactivity reserves to compensate the buildup of ^{135}Xe and the depletion of the fuel.

The amount of reactivity that has to be compensated due to neutron absorption on saturated xenon is

$$\rho = -\frac{\overline{\Sigma}_{aX}/\overline{\Sigma}_f}{\nu p \epsilon}$$

where $\overline{\Sigma}_{aX}$ is macroscopic abs. cross section of saturated xenon,

$$\begin{aligned}\overline{\Sigma}_{aX} = X_\infty \overline{\sigma}_{aX} &= \frac{(\gamma_I + \gamma_X)\overline{\Sigma}_f \phi_T \overline{\sigma}_{aX}}{\lambda_X + \overline{\sigma}_{aX}\phi_T} \\ &= \frac{(\gamma_I + \gamma_X)\overline{\Sigma}_f \phi_T}{\phi_X + \phi_T}\end{aligned}$$

where $\phi_X = \frac{\lambda_X}{\overline{\sigma}_{aX}} = 0.77 \times 10^{13}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$.

$$\Rightarrow \rho = -\frac{\gamma_I + \gamma_X}{\nu p \epsilon} \frac{\phi_T}{\phi_X + \phi_T}$$

What is, approximately, the largest possible decrease of reactivity due to xenon poisoning in ^{235}U-fueled reactors?

For a large neutron flux ϕ_T, when $\phi_T \gg \phi_X$, the reactivity change due to the saturated xenon concentration

$$\rho = -\frac{\gamma_I + \gamma_X}{\nu p \epsilon} \frac{\phi_T}{\phi_X + \phi_T}$$

cannot exceed the limit value

$$\rho = -\frac{\gamma_I + \gamma_X}{\nu p \epsilon}$$

which is about four dollars in ^{235}U-fueled reactors.

Will xenon concentration decrease after reactor shutdown?

When a reactor is shut down the fission production of ^{135}Xe and ^{135}I is stopped; however, at the same time ^{135}Xe is not transmuted by neutron absorption, and moreover ^{135}Xe continues to be produced from the decay of accumulated ^{135}I. So, the concentration of xenon grows right after the reactor shutdown.

How can we simplify the balance equation for the production/decay of ^{135}I and ^{135}Xe after shutdown?

Production rates of ^{135}Xe and ^{135}I after shut down become ($\phi_T = 0$)

$$\frac{dI}{dt} = \cancel{\gamma_I \overline{\Sigma}_f \phi_T} - \lambda_I I$$

$$\frac{dX}{dt} = \lambda_I I + \cancel{\gamma_X \overline{\Sigma}_f \phi_T} - \lambda_X X - \cancel{\overline{\sigma}_{aX} \phi_T X}$$

Solution of the balance equations for ^{135}I and ^{135}Xe after shutdown:

Iodine and xenon concentrations after shutdown are thus given by equations

$$\frac{dI}{dt} = -\lambda_I I$$

$$\frac{dX}{dt} = \lambda_I I - \lambda_X X$$

Solutions are then:

$$I(t) = I_0 e^{-\lambda_I t}$$

$$X(t) = X_0 e^{-\lambda_X t} + \frac{\lambda_I I_0}{\lambda_I - \lambda_X}(e^{-\lambda_X t} - e^{-\lambda_I t})$$

Where I_0 and X_0 are iodine and xenon concentrations at the shutdown moment.

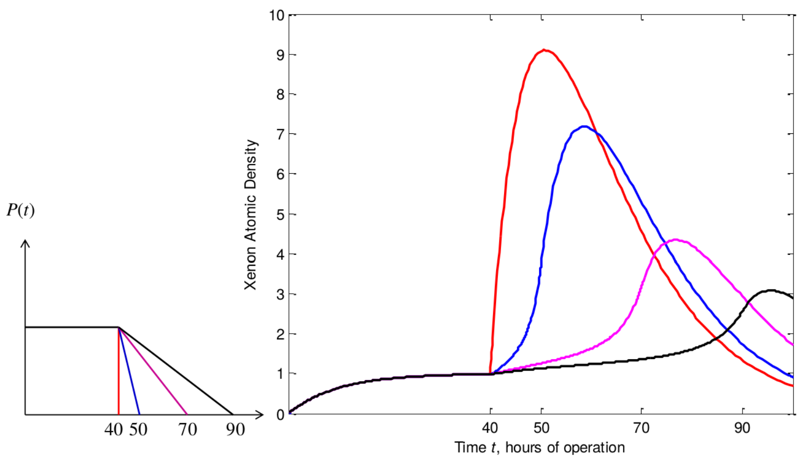

Figure 4: Xenon concentration after reactor shutdown

Fission product poisoning

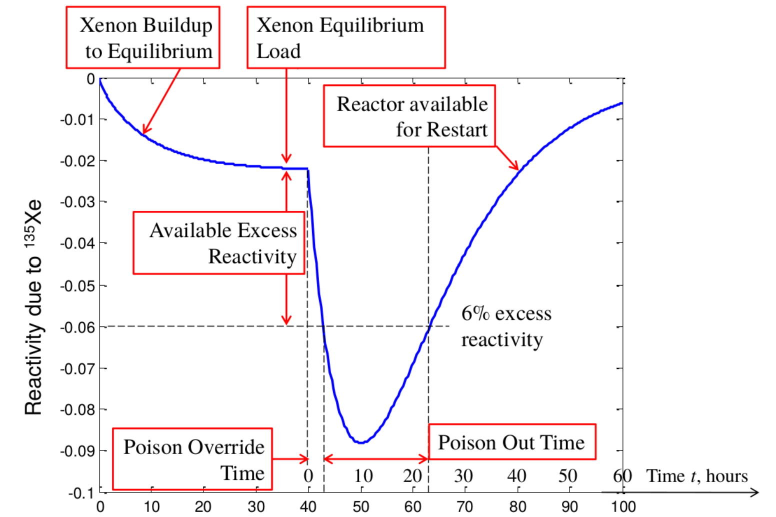

Figure 5: Reactivity change due to xenon after reactor shutdown

Xenon-induced power oscillations

Xenon-induced power oscillations

In addition to the poisoning effect, xenon can also cause localized oscillations of power in large thermal reactors.

These oscillations arise as a consequence of a localized perturbation in the neutron flux. Dynamics of xenon oscillations:

- **In locations with an increased neutron flux** the consumption rate of ^{135}Xe increases.
- The decreasing concentration of ^{135}Xe leads to a further increase of the neutron flux in that location.
- The increased neutron flux increases the production rate of ^{135}I from fission at that place.
- The decay of ^{135}I will eventually increase the concentration of ^{135}Xe at that location.
- The increased concentration of ^{135}Xe will then start reducing the neutron flux, and the power will shift to another place in the reactor.
- ^{135}Xe will eventually decay (and be burned) and power will start to grow locally again, closing one cycle of the oscillation.

Xenon-induced power oscillations

Characteristics of the xenon power oscillations

- The period of the oscillation is about a day (may be less or more, depending on the neutron flux level).
- The oscillations may be damped (stable), undamped, or growing (unstable), depending on the flux level, reactivity feedback, size of the reactor (whether different parts of the core can maintain the chain reaction independently), and other conditions.
- As the xenon-induced oscillation periods are quite long, they can be controlled by adjustments of control rods.
- Neutron flux detectors need to be distributed throughout the reactor core to monitor the local power changes.
- Note that the total power doesn't change during xenon oscillations. An increase of power in one place leads to a reduction of power at another place.

Xenon oscillations may have various spatial modes

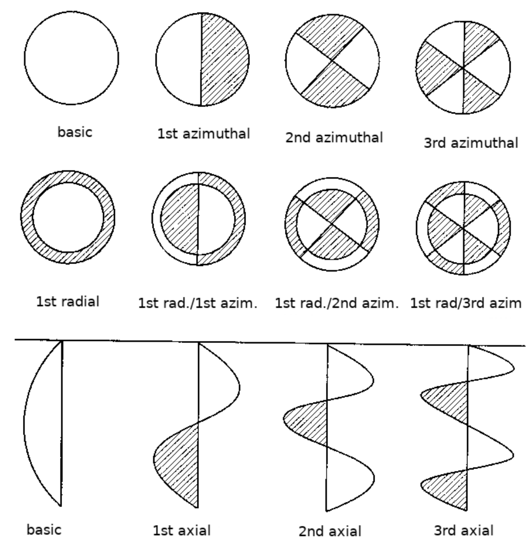

Figure 6: Modes of xenon oscillations

Fuel burnup problems

Fuel burnup problems

The burnup equation

All nuclides (fissile, fertile, fission products, burnable poisons) can be treated in the same way by the burnup equation

$$\frac{dN_i(\vec{r}, t)}{dt} = \text{Formation Rate} - \text{Removal Rate}$$

The formation and removal rates are due to:

- the exposure to the neutron flux (removal by neutron absorption and formation as fission product or transmutation),
- the natural decay of nuclides into other nuclides.

The burnup equation

What is the formation rate?

The formation rate of nuclide N_i is given by a sum of the following terms:

- $\sum_j \overline{\gamma_{ji}\sigma_{f,j}} N_j \overline{\phi}$ - formation as a fission product from various fission nuclides N_j with a yield γ_{ji}. One-group cross sections, yields and flux are considered here and on next slides.
- $\overline{\sigma_{\gamma,i-1}} N_{i-1} \overline{\phi}$ - formation by radiative capture of neutron on nuclide N_{i-1}.
- $\lambda_i' N_i'$ - formation by decay of nuclide N_i'.

where all microscopic cross sections and the neutron flux have one-group values.

What is the removal rate?

The removal rate of nuclide N_i is given by a sum of the following terms:

- $\overline{\sigma}_{f,i} N_i \overline{\phi}$ - destruction by fission.
- $\overline{\sigma}_{\gamma,i} N_i \overline{\phi}$ - destruction by neutron capture.
- $\lambda_i N_i$ - the natural decay rate of the nuclide.

The burnup equation

Hence, the complete burnup equation is

$$\frac{dN_i}{dt} = \sum_j \overline{\gamma_{ji}\sigma_{f,j}} N_j \overline{\phi} + \overline{\sigma_{\gamma,i-1}} N_{i-1} \overline{\phi} + \lambda_i' N_i' - \overline{\sigma}_{f,i} N_i \overline{\phi} - \overline{\sigma}_{\gamma,i} N_i \overline{\phi} - \lambda_i N_i$$

The burnup equation

Vector form of the burnup equation

The burnup equation

$$\frac{dN_i}{dt} = \sum_j \overline{\gamma_{ji}\sigma_{f,j}} N_j \overline{\phi} + \overline{\sigma_{\gamma,i-1}} N_{i-1} \overline{\phi} + \lambda_i' N_i' - \overline{\sigma}_{f,i} N_i \overline{\phi} - \overline{\sigma}_{\gamma,i} N_i \overline{\phi} - \lambda_i N_i$$

can be written in a vector form as

$$\frac{d\vec{N}(\vec{r},t)}{dt} = \mathbf{M}(\overline{\phi}, t)\vec{N}(\vec{r},t),$$

where $\vec{N}(\vec{r},t)$ is the nuclide field (a specific element in this vector gives the concentration of a specific nuclide), and

$$\mathbf{M} = \mathbf{X}\overline{\phi}(\vec{r},t) + \mathbf{D}$$

is the transmutation matrix, where **X** is a matrix that contains all cross-section and fission yield terms, and **D** is a decay matrix.

What is the analytical solution to the burnup equation assuming that the neutron flux is fixed?

Under the condition that the neutron flux is fixed, the solution to the burnup equation

$$\frac{d\vec{N}}{dt} = \mathbf{M}\vec{N}$$

is

$$\vec{N}(t + \Delta t) = e^{\mathbf{M}\Delta t}\vec{N}(t)$$

What is the exponential of a matrix?

The exponential of matrix $\mathbf{M}\Delta t$ is also a matrix that can be computed e.g. as a power series expansion

$$e^{\mathbf{M}\Delta t} = \mathbf{1} + \mathbf{M}\Delta t + \frac{1}{2}(\mathbf{M}\Delta t)^2 + \cdots$$

We know that the neutron flux changes over time due to the nuclide field changes. How can we then solve the burnup equation over a large time period?

- While the nuclide field changes depend on the neutron flux, the neutron flux also depends on the nuclide field, and so the neutron flux changes in time.
- We can split the whole time period into a number of short time steps, and we can assume that when the time step Δt is sufficiently short then the neutron flux does not change much during the step, and we can use the analytical solution formula for the fuel depletion at each time step.
- We have to re-evaluate the neutron flux at each time step. (Note that the spatial distribution of the neutron flux changes over time, and also the total flux grows over time. As the fuel depletes over time, the neutron flux generally must grow in order to ensure the required power.)

Time changes in the power density distribution (left: fresh fuel, right: after burnup of 23GW-days/tonne)