Q2_Ans:

2(a):

We know that in Riemann sum technique, we select the numbers ξ_i that minimize $f(x)$ on every sub-interval $[x_{i-1}, x_i]$

$$\sum_{i=1}^{n} \min_{[x_{i-1},x_i]} f(\xi_i)\,(x_i - x_{i-1}) \leq \int_a^b f(x)dx \leq \sum_{i=1}^{n} \max_{[x_{i-1},x_i]} f(\xi_i)\,(x_i - x_{i-1})$$

Also, we define Reimann Left (L) and Right (R) as:

$$L(f, x_i) = \sum_{i=1}^{n} f(x_{i-1})\,(x_i - x_{i-1})$$

$$R(f, x_i) = \sum_{i=1}^{n} f(x_i)\,(x_i - x_{i-1})$$

Hence, for a monotonically decreasing function, we minimalize the $R(f, x_i)$ and take the maximum of $L(f, x_i)$ to minimize the $f(x)$ on every sub-interval $[x_{i-1}, x_i]$.

Therefore,

$$\min\{L(f, x_i), R(f, x_i)\} \leq \int_a^b f(x)dx \leq \max\{L(f, x_i), R(f, x_i)\}$$

Reduces to

$$R(f, x_i) \leq \int_a^b f(x)dx \leq L(f, x_i)$$

2(b):

$$L(f, x_i) = \sum_{i=1}^{n} f(x_{i-1})\,(x_i - x_{i-1})$$

$$R(f, x_i) = \sum_{i=1}^{n} f(x_i)\,(x_i - x_{i-1})$$

So,

$$L - R = \sum_{i=1}^{n} [f(x_{i-1})\,(x_i - x_{i-1}) - f(x_i)\,(x_i - x_{i-1})]$$

$$= \sum_{i=1}^{n} (x_i - x_{i-1})[f(x_{i-1}) - f(x_i)]$$

We know,

$$h = \max_{1<i\leq n} (x_i - x_{i-1}) = \frac{b-a}{n}$$

and

$$x_i = b \ \ \& \ \ x_{i-1} = a$$

Therefore,

$$L - R = \frac{b-a}{n}[f(a) - f(b)]$$