Sustainable Energy Transformation Technologies, SH2706

Lecture No 5

Title:
Work Extraction From Thermal Energy

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Autumn 2022

Introduction

- The primary energy available in the natural environment must be transformed (or converted) to useful forms of energy

- As an intermediate step, the primary energy is transformed into an energy carrier, which can be easily distributed, stored and consumed

- With a development of our civilization, the energy carriers and useful energy forms are changing

- Since over a century, electricity has evolved as the most desirable energy carrier

- Still the dominant chain of transformations to obtain electricity includes thermodynamic processes

Introduction
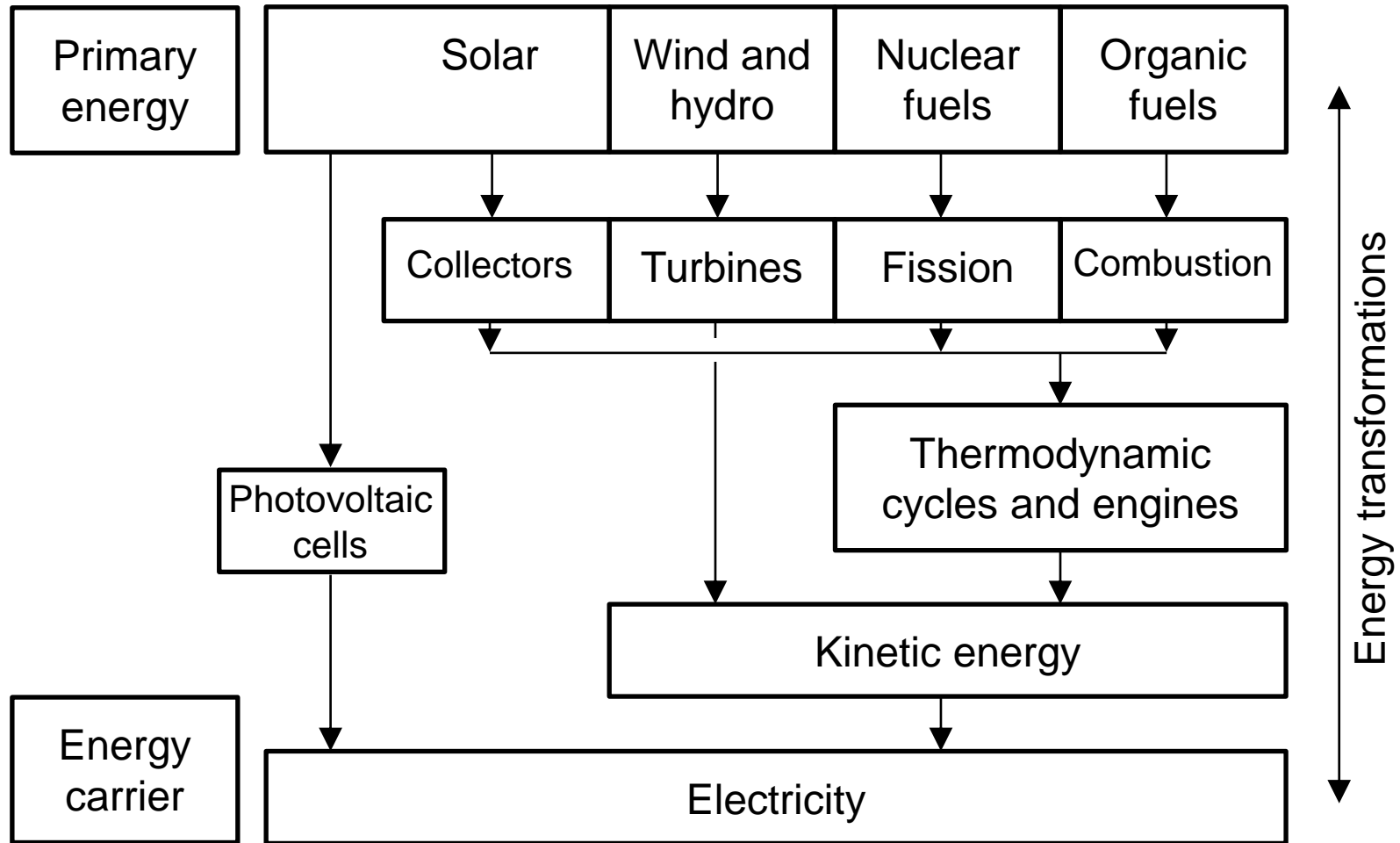
Outline

- Thermodynamic cycles with work extraction
  - Carnot cycle and Stirling cycle
  - Rankine cycle
  - Brayton cycle
  - Kalina cycle
  - Organic Rankine cycle
  - Combined cycles, Integrated solar combined cycle

Work along a path

- We will compare work in various processes with identical conditions at starting and ending points (**a** and **b** in the figure below)

- Our goal is to find how the work magnitude depends on the path of the process

- We will consider processes starting at **a** and ending at **b**:
  - a-c-b and a-d-b (isobaric+isochoric)
  - a-2-b (linear temperature)
  - a-1-b (adiabatic)
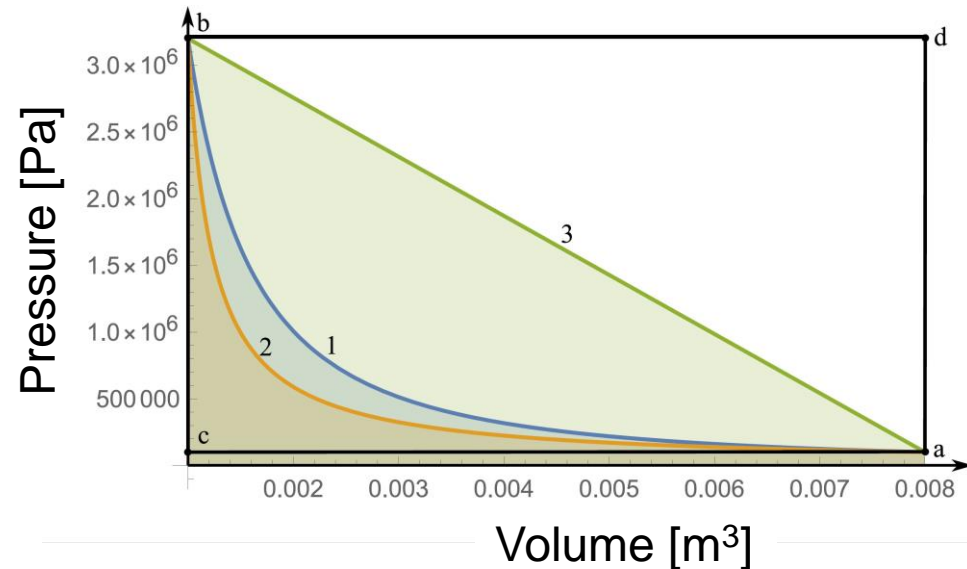 - a-3-b (linear pressure)
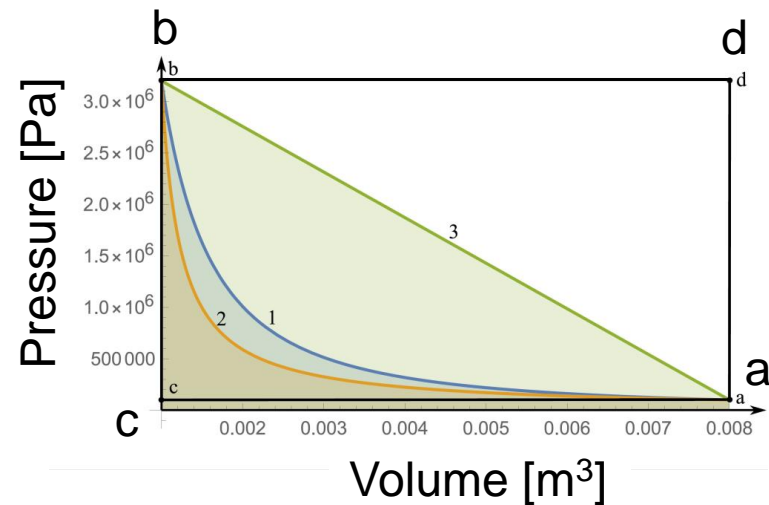
Work along adiabatic path

- We start with the adiabatic process a-1-b, starting at **a** (V=0.008 m^3, p = 0.1 MPa) and ending at **b** (V=0.001 m^3, p = 3.2 MPa)

- We assume an ideal monoatomic gas (κ=5/3)

- For adiabatic process: $pV^\kappa = const = p_a V_a^\kappa = p_b V_b^\kappa$ and the work is

$$L_1 = \int_{V_a}^{V_b} pdV = p_a V_a^\kappa \int_{V_a}^{V_b} \frac{dV}{V^\kappa} =$$

$$10^5 \cdot 0.001^{5/3} \int_{0.001}^{0.008} \frac{dV}{V^{5/3}} = -3.6 \text{ kJ}$$

The work is negative: it is performed on gas to compress it

Work along adiabatic path

- From the first law we obtain: $\Delta E_{I1} = \Delta Q_1 - \Delta L_1$
  but for adiabatic process $\Delta Q_1 = 0$
  Thus $\Delta E_{I1} = -\Delta L_1 = 3.6\ \text{kJ}$

- The internal energy of gas increases, and its temperature increases as well

- During this process, the pressure increases partly due to volume reduction and partly due to increasing temperature
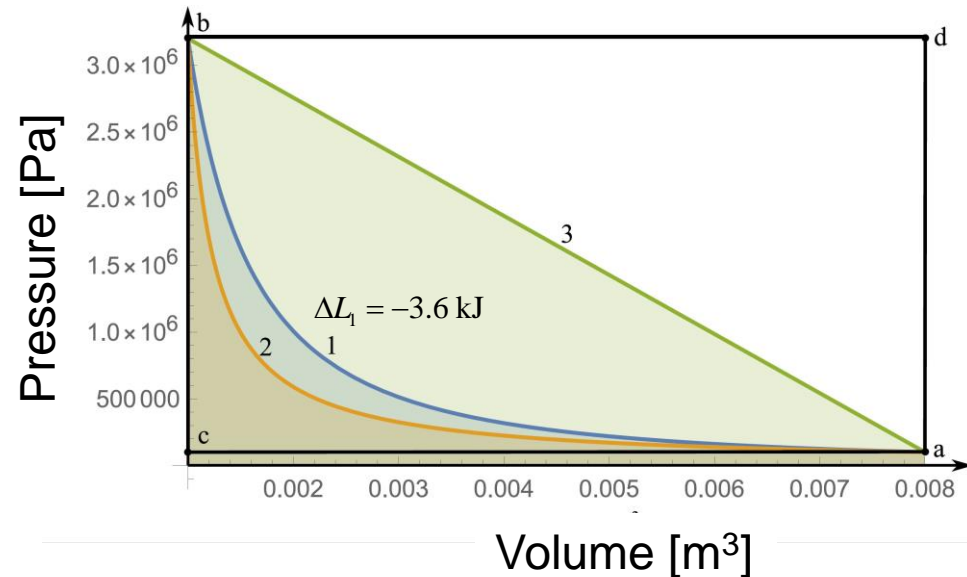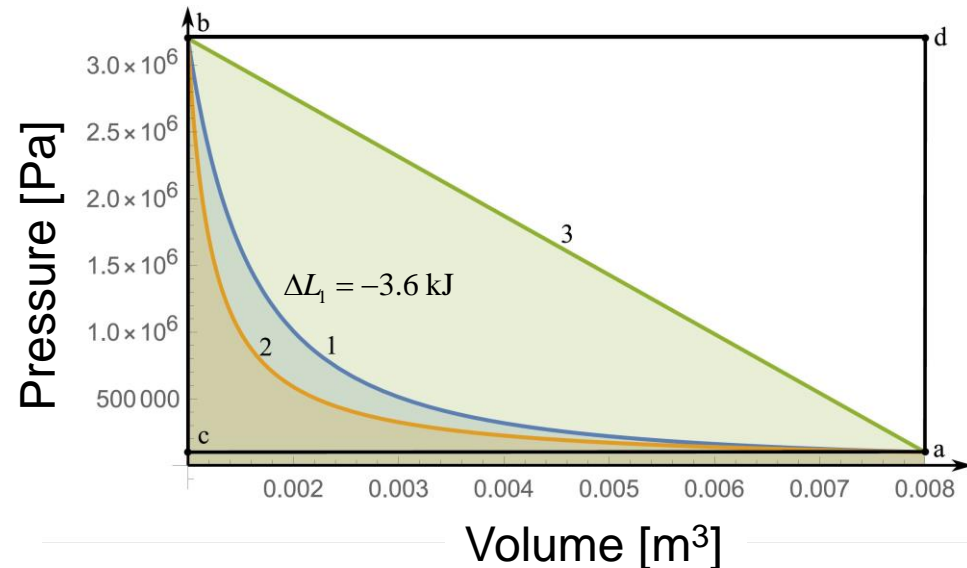
Work along linear temperature

- Assume now that heat transfer with gas is possible, and it is controlled in such a way that the pressure linearly increases with the temperature

- What are values of work and heat during this process?

- We have for the pressure

$$p = p_a + \frac{p_b - p_a}{T_b - T_a}(T - T_a)$$

where

$$T_a = \frac{p_a V_a}{mR} \quad T_b = \frac{p_b V_b}{mR} \quad T = \frac{pV}{mR}$$

Work along linear temperature

- Combining the equations and substituting given data (note that the term mR cancels out) we get the following pressure-versus-volume function for this process
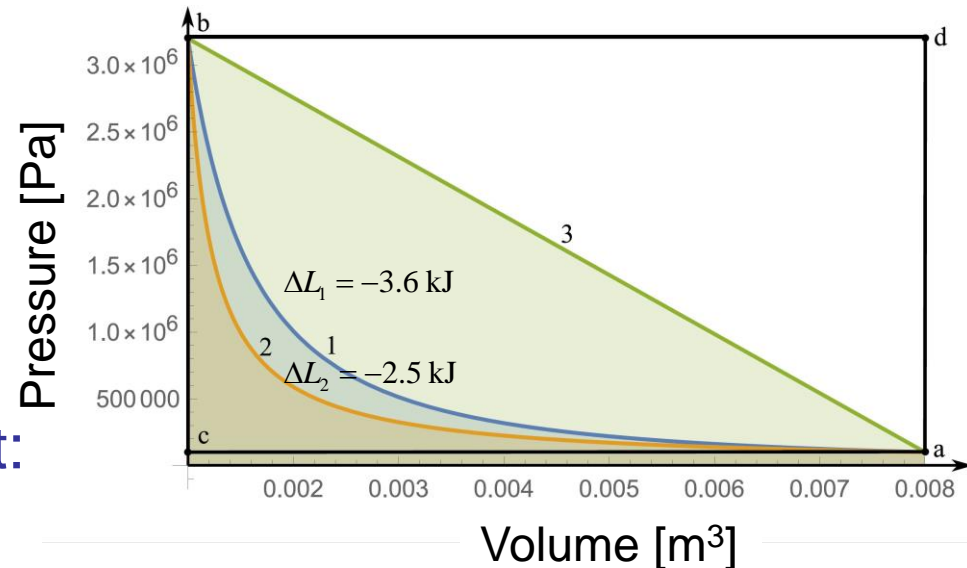
$$p = \frac{9.33 \cdot 10^5}{1291.7V - 1} [\text{Pa}] \quad \text{here } V \text{ is in m}^3$$

- Thus we obtain the work:

$$L_2 = \int_{V_a}^{V_b} p\, dV = -2.5 \text{ kJ}$$

and from 1$^{\text{st}}$ Law, the heat:

$$\Delta Q_2 = \Delta E_{I1} + \Delta L_2 = 1.1 \text{ kJ}$$

Work along linear temperature

- We see that process along path 2 requires less work, which can be replaced by a corresponding amount of heat (since the final internal energy should increase equally in both processes)

- However, the process requires a sophisticated system, that will give linear increase of temperature with pressure (can be too expensive: exergy analysis is required)

$\Delta L_1 = -3.6 \text{ kJ}$

$\Delta L_2 = -2.5 \text{ kJ}$

Pressure [Pa]
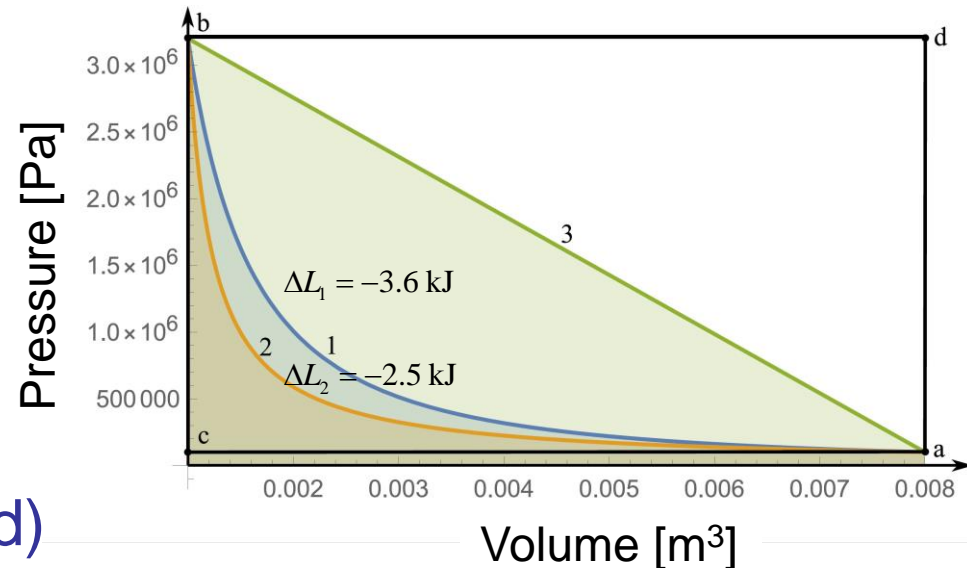
Volume [m³]

Work along path a-c-b

- Let us now turn to two-step process:
  - first reducing the gas volume from 0.008 to 0.001 m^3 keeping pressure constant (here we will need to extract heat from, or cool, the gas to keep pressure constant)
  - next heat the gas to increase the pressure from 0.1 to 3.2 MPa

- Again, we find the required work and heat exchanged

- The work along a-c is:

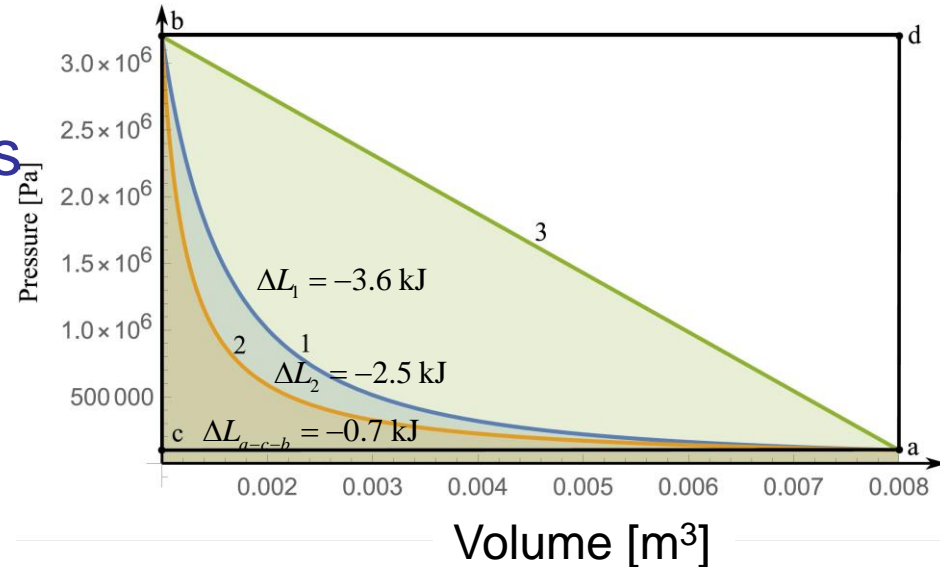$$L_{a-c} = \int_{V_a}^{V_c} p_a dV = p_a \left(V_c - V_a \right) = -0.7 \text{ kJ}$$
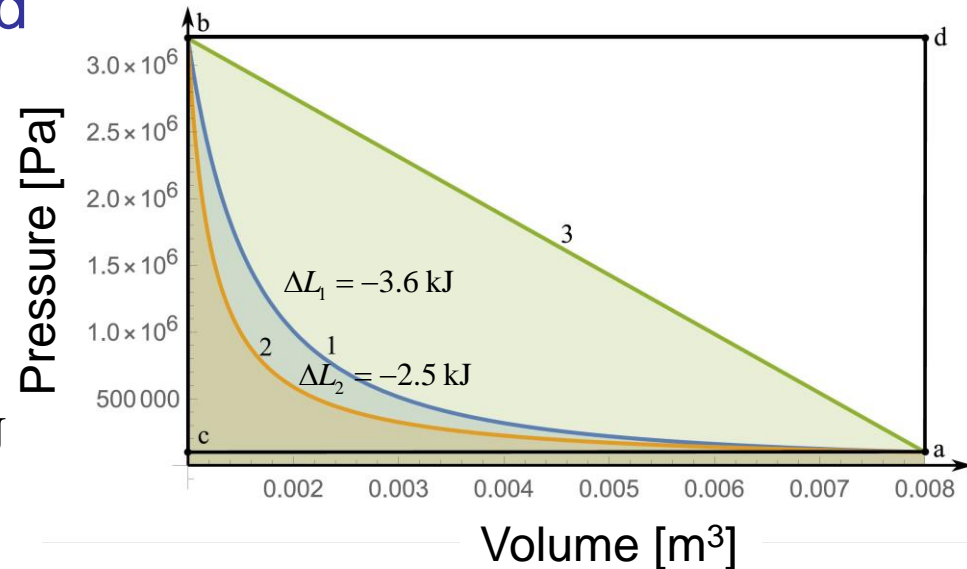
Work along path a-c-b

- Since the work along c-b is zero, the total work for this process is -0.7 kJ $\quad L_{a-c-b} = -0.7 \text{ kJ}$

- From the 1st law we get the total heat that needs to be added to gas (gas heating) as $\Delta Q_{a-c-b} = \Delta E_{I1} + \Delta L_{a-c-b} = 2.9 \text{ kJ}$

- So we see that this process requires least work to be done (we reduce gas volume at low pressure)

Work along path a-d-b

- We find similarly the work and heat for path a-d-b as:

$$L_{a-d-b} = p_b\left(V_b - V_d\right) = -22.4 \text{ kJ} \qquad \Delta Q_{a-d-b} = \Delta E_{I1} + \Delta L_{a-d-b} = -18.8 \text{ kJ}$$
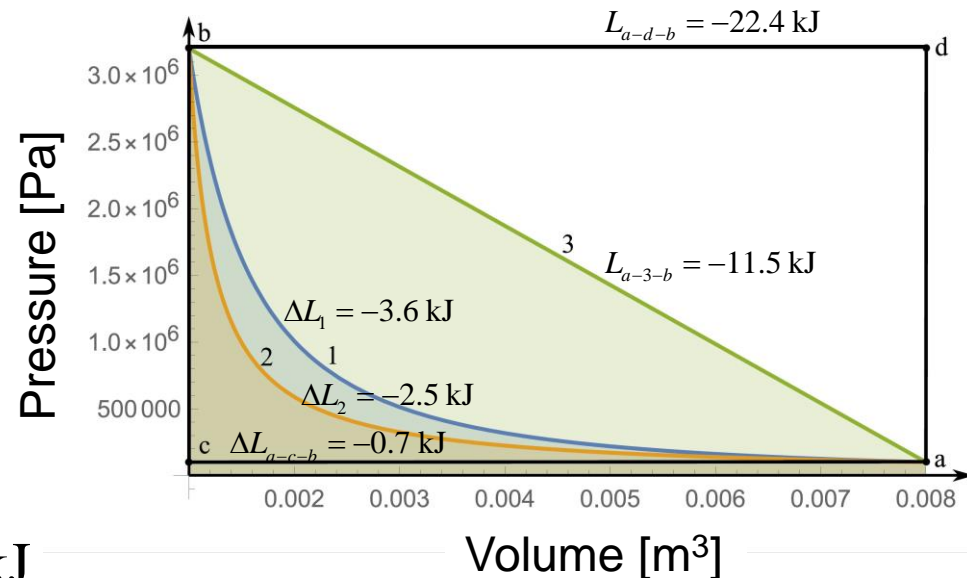
- So we see that this process requires maximum work to be done (we reduce gas volume at high pressure) and a lot of cooling

- The work for line 3 is

$$L_{a-3-b} = \frac{p_b + p_a}{2}\left(V_b - V_d\right) = -11.5 \text{ kJ}$$
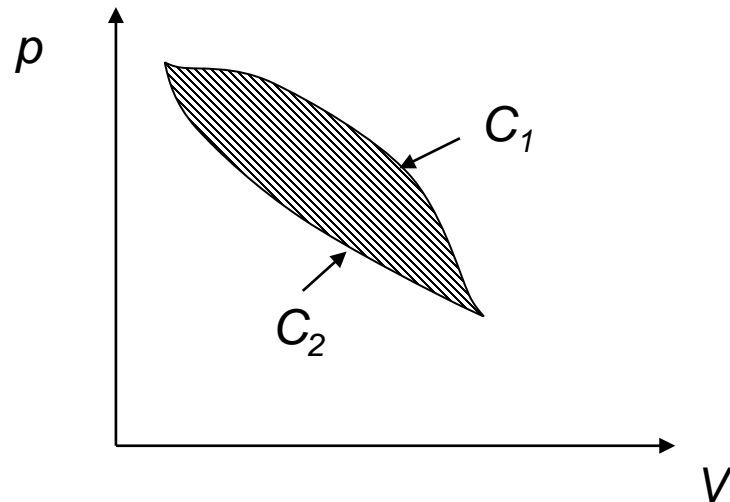
and heat

$$\Delta Q_{a-3-b} = \Delta E_{I1} + \Delta L_{a-3-b} = -7.9 \text{ kJ}$$

Cycle processes

- A cyclic process is a thermodynamic process which begins from and finishes at the same thermostatic state.

- It is a closed loop on a *p-V* diagram and the area enclosed by the loop is the work done by the process,

$$L = \oint p\, dV$$

Carnot Cycle (1)

- The most efficient heat engine cycle is the Carnot cycle, consisting of two isothermal processes and two adiabatic processes.

- The Carnot cycle can be thought of as the most efficient heat engine cycle allowed by physical laws.

- When the second law of thermodynamics states that not all the supplied heat in a heat engine can be used to do work, the Carnot efficiency sets the limiting value on the fraction of the heat which can be so used.
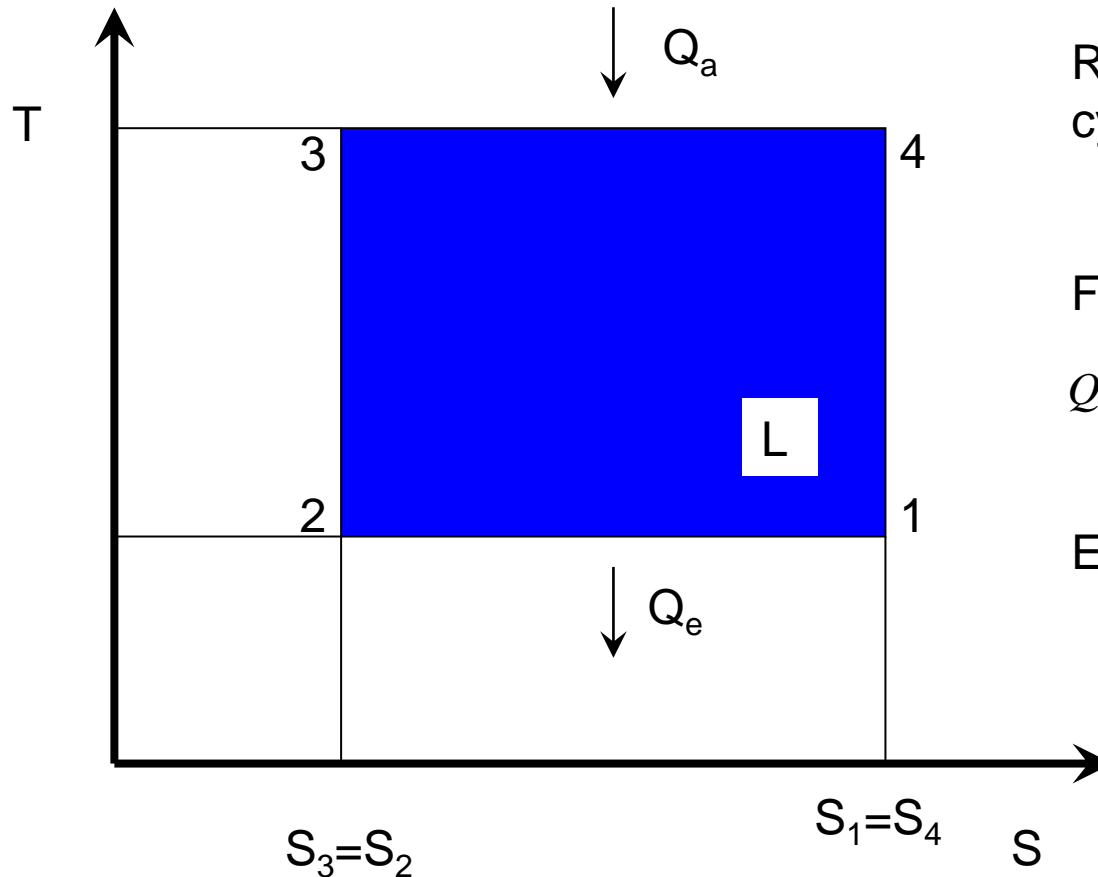
Carnot Cycle (2)

- In order to approach the Carnot efficiency, the processes involved in the heat engine cycle must be reversible and should not incur change in entropy.

- This means that the reversible Carnot cycle is an idealization, since no real engine processes are reversible and all real physical processes involve some increase in entropy.

Carnot Cycle (3)

- The **Carnot cycle** plays an important role in thermodynamics as a theoretical reference cycle

- In this cycle the heat Q_a is added to the system at a constant temperature T_a (=T_3=T_4) and heat Q_e is extracted from the system at constant temperature T_e (=T_1=T_2).
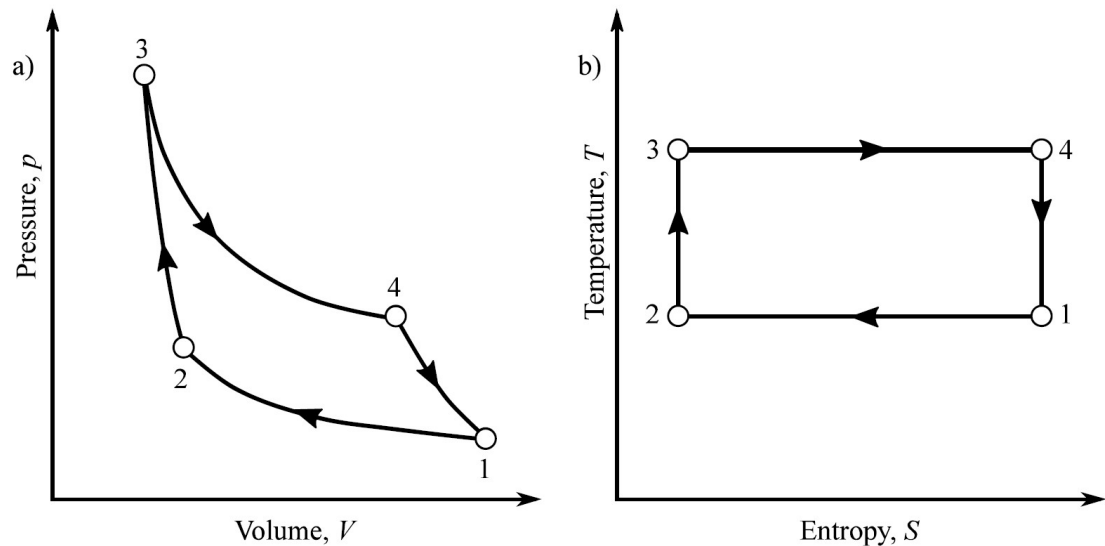
Carnot Cycle (4)

Reversible Carnot cycle on the *TS* plane

From 1st Law

$$Q_a - |Q_e| = L$$

Efficiency of the cycle

$$\eta_C = \frac{L}{Q_a} = 1 - \frac{|Q_e|}{Q_a} =$$

$$1 - \frac{T_e (S_1 - S_2)}{T_a (S_4 - S_3)} = 1 - \frac{T_e}{T_a}$$

Carnot Cycle (5)

- Thus, the efficiency of the Carnot cycle is as follows,

$$\eta_C = 1 - \frac{T_e}{T_a}$$

- Example:
  - Calculate the efficiency of the reversible Carnot cycle if heat is supplied at temperature T_a = 700 °C and extracted at temperature T_e = 20 °C.
  - SOLUTION: the efficiency of the cycle is found as follows, .

$$\eta_C = 1 - T_e/T_a = 1 - (20 + 273)/(700 + 273) = 0.698$$

Carnot Cycle (6)

- The presented derivation does not show how the work is extracted in the cycle

- Let us analyze process-by-process to find out what happens during the cycle

- We take 1 mole of an ideal gas as our model working fluid

- For isothermal processes we have T = const and $dE_I = 0$ (gas internal energy is not changed), and $\delta Q = \delta L = pdV$

Carnot Cycle (7)

- For isothermal heating 3-4 we have

$$Q_a = L_{3-4} = \int_{V_3}^{V_4} p\, dV = \int_{V_3}^{V_4} \frac{RT_a}{V}\, dV =$$

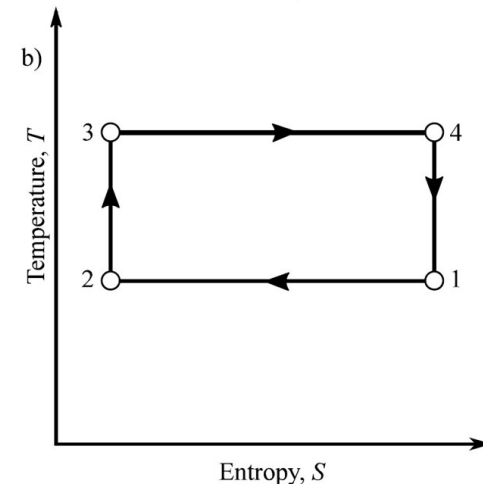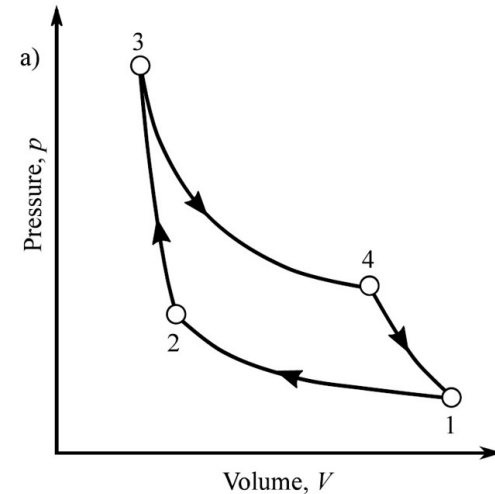$$RT_a \ln \frac{V_4}{V_3} \quad (>0)$$

Note that $Q_a > 0$, as it should be

- Similarly, for isothermal cooling 1-2:

$$Q_e = L_{1-2} = \int_{V_1}^{V_2} p\, dV = \int_{V_1}^{V_2} \frac{RT_e}{V}\, dV =$$

$$RT_e \ln \frac{V_2}{V_1} \quad (<0)$$

Note that $Q_e < 0$, as it should be

Carnot Cycle (8)

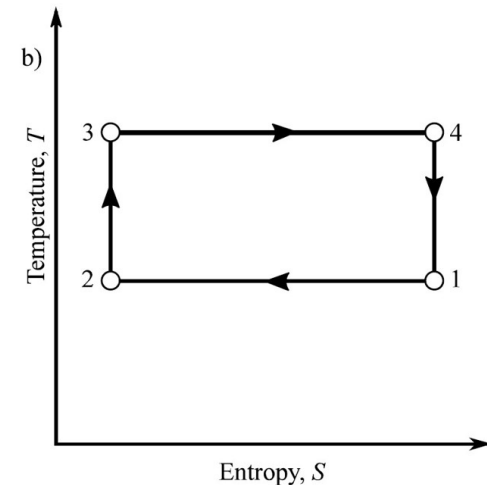- For adiabatic compression 2-3 we have δQ=0, thus from 1st Law $dE_I = -δL$, and

$$c_v dT = -pdV = -\frac{RT}{V}dV$$

$$c_v \int_{T_2}^{T_3} \frac{dT}{T} = -R \int_{V_2}^{V_3} \frac{dV}{V}$$

$$c_v \ln \frac{T_3}{T_2} = -R \ln \frac{V_3}{V_2} \quad \Longrightarrow \quad \left(\frac{T_3}{T_2}\right)^{\frac{c_v}{R}} = \left(\frac{V_3}{V_2}\right)^{-1}$$

- Similarly, for adiabatic expansion 4-1:

$$c_v \ln \frac{T_1}{T_4} = -R \ln \frac{V_1}{V_4} \quad \Longrightarrow \quad \left(\frac{T_1}{T_4}\right)^{\frac{c_v}{R}} = \left(\frac{V_1}{V_4}\right)^{-1}$$

The ratios of volumes in adiabatic expansion and compression are equal!

$$\frac{V_2}{V_3} = \frac{V_1}{V_4}$$

b)

Temperature, T — Entropy, S

Carnot Cycle (9)

- In summary: we have obtained the following expressions for the added and extracted heat
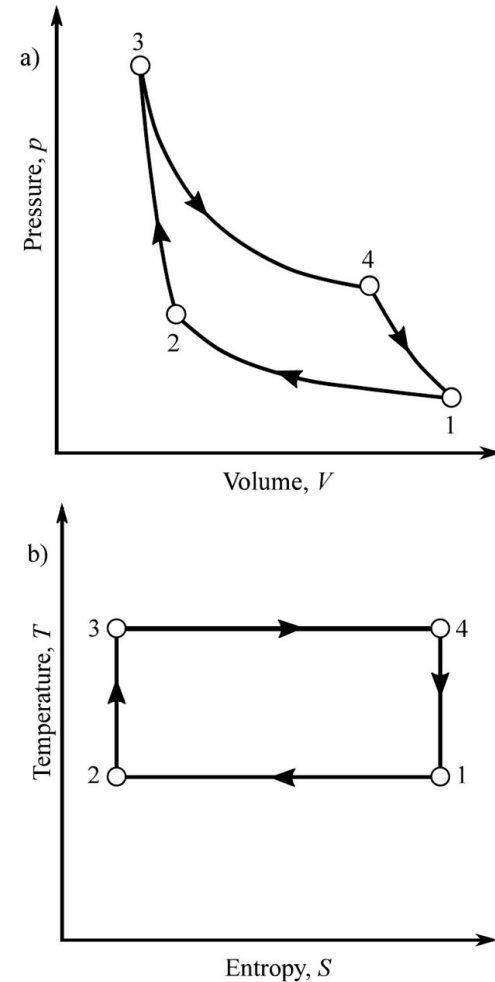
$$Q_a = RT_a \ln \frac{V_4}{V_3} \qquad Q_e = RT_e \ln \frac{V_2}{V_1}$$

- And we found that: $\dfrac{V_2}{V_3} = \dfrac{V_1}{V_4} \quad \Rightarrow \quad \dfrac{V_4}{V_3} = \dfrac{V_1}{V_2}$

- Combining the equations yields

$$\frac{Q_a}{T_a} + \frac{Q_e}{T_e} = 0$$

Since $T_a > 0$, $T_e > 0$ and $Q_a > 0$, this equation can be true only when $Q_e < 0$ (but we know it is!)

Carnot Cycle (10)

- Now we can define the cycle efficiency as a ratio of useful work divided by the added heat

- This is so-called energy efficiency, telling us how much added heat is transformed into work

- From the definition, we have

$$\eta \triangleq \frac{L}{Q_a} = \frac{Q_a - |Q_e|}{Q_a} = 1 - \frac{|Q_e|}{Q_a}$$

- But

$$\frac{Q_a}{T_a} + \frac{Q_e}{T_e} = 0 \Rightarrow Q_e = -Q_a \frac{T_e}{T_a} \Rightarrow |Q_e| = Q_a \frac{T_e}{T_a}$$

$$\eta = 1 - \frac{|Q_e|}{Q_a} = 1 - \frac{T_e}{T_a}$$

We derived an expression for energy efficiency of Carnot cycle without mentioning entropy! But we obtained an interesting relationship: $\frac{Q_a}{T_a} + \frac{Q_e}{T_e} = 0$
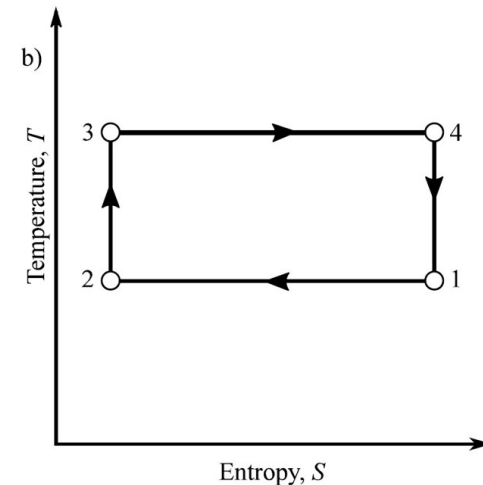
Carnot Cycle (11)

- The relationship $\dfrac{Q_a}{T_a} + \dfrac{Q_e}{T_e} = 0$ can be

  generalized as $\oint \dfrac{\delta Q}{T} = 0$ and δQ/T is
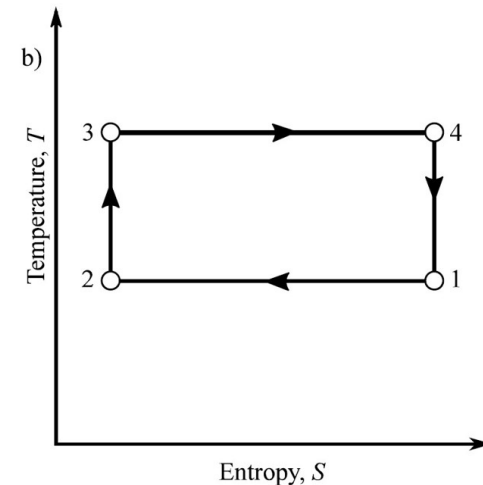 called entropy

- For irreversible processes we will have

$$\oint \frac{\delta Q}{T} < 0$$

- In general, the Clausius inequality is valid

$$\oint \frac{\delta Q}{T} \leq 0$$

The Clausius inequality must be true for any cyclic process that is realizable, reversible or not

Stirling Cycle (1)

- The Stirling cycle contains the following four processes:

  - Isothermal compression, with heat extraction and work input at low temperature T_e
  - Isochoric heating from low temperature T_e to high temperature T_a with heat addition to the system and no work done
  - Isothermal expansion at high temperature T_a with heat addition and work output
  - Isochoric cooling from T_a to T_e with heat extraction and no work done

- The theoretical efficiency of the Stirling cycle is equal to the Carnot efficiency

$$\eta_{Stirling} = \eta_C = 1 - \frac{T_e}{T_a}$$

Stirling Cycle (2)
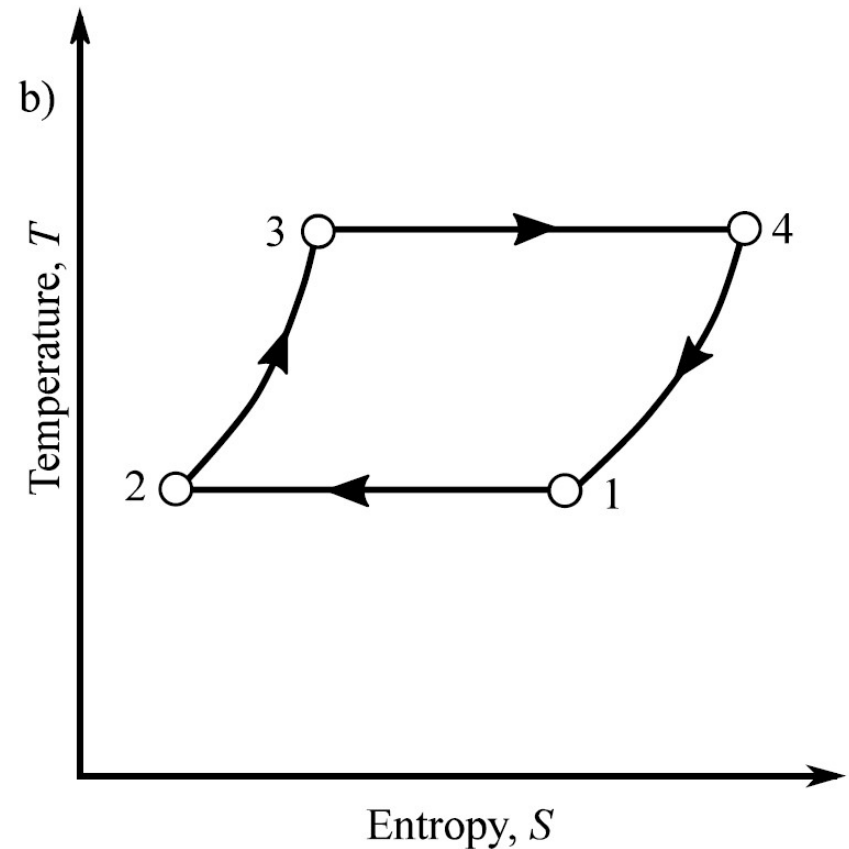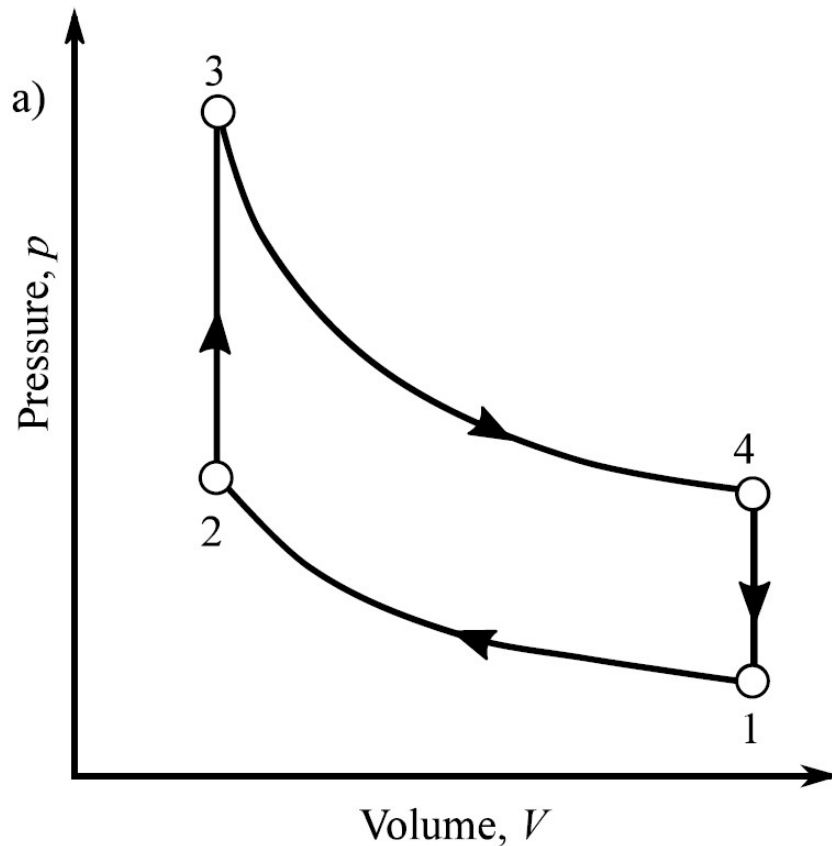
Stirling Cycle (3)

- The advantage of Stirling engine is that it can deliver more work when operating in the same range of pressure and volume, as compared to the Carnot cycle
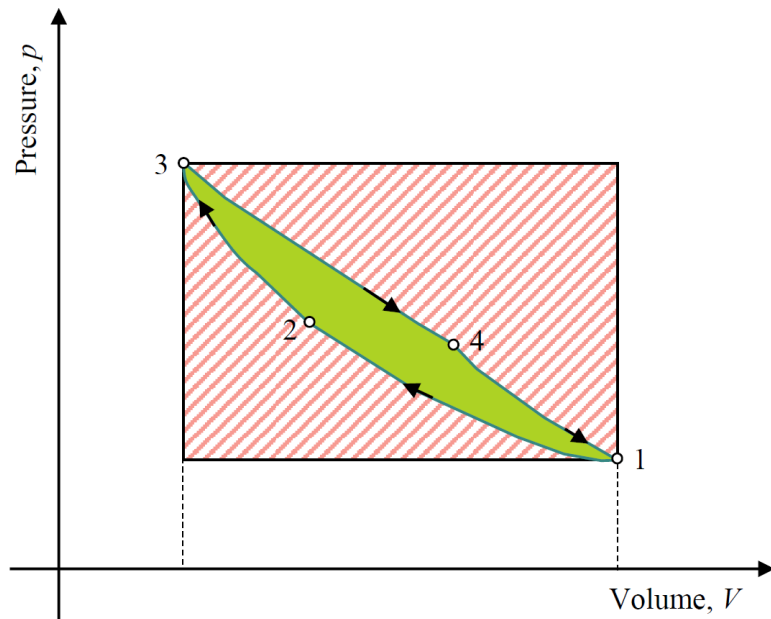

Carnot cycle: green – extracted work
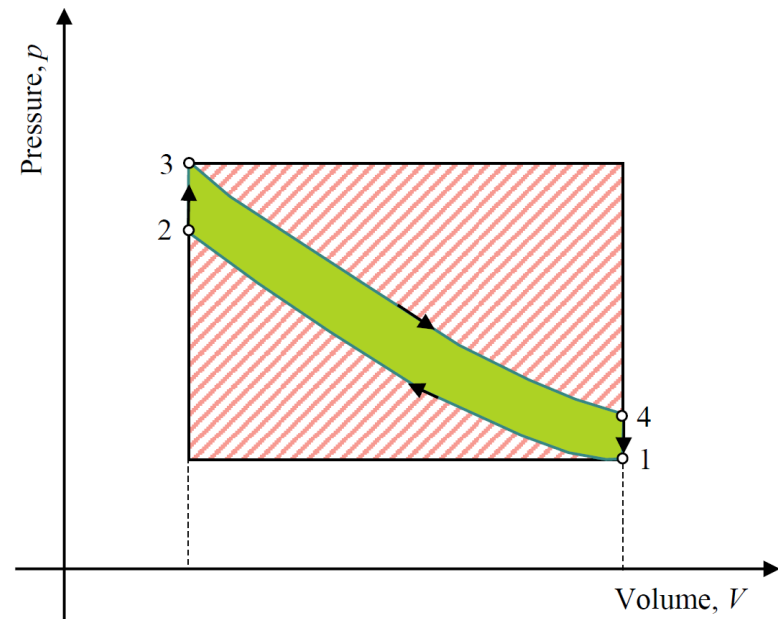
Stirling cycle: green – extracted work

Rankine Cycle System

Exhaust

Fuel

q_a

Boiler

N_e – net power of the cycle
N_P – pumping power
q_a – added thermal power
q_e – extracted thermal power in condenser

4

Turbine

N_{eff} $N_e = N_{eff} - N_P$

Main processes of the working fluid:
4-5 – expansion in the turbine
5-1 – heat rejection in the condenser
1-2 – pressure increase in the pump
2-4 – heat supply in the boiler

N_P

5

q_e

Condenser

2   1

Pump
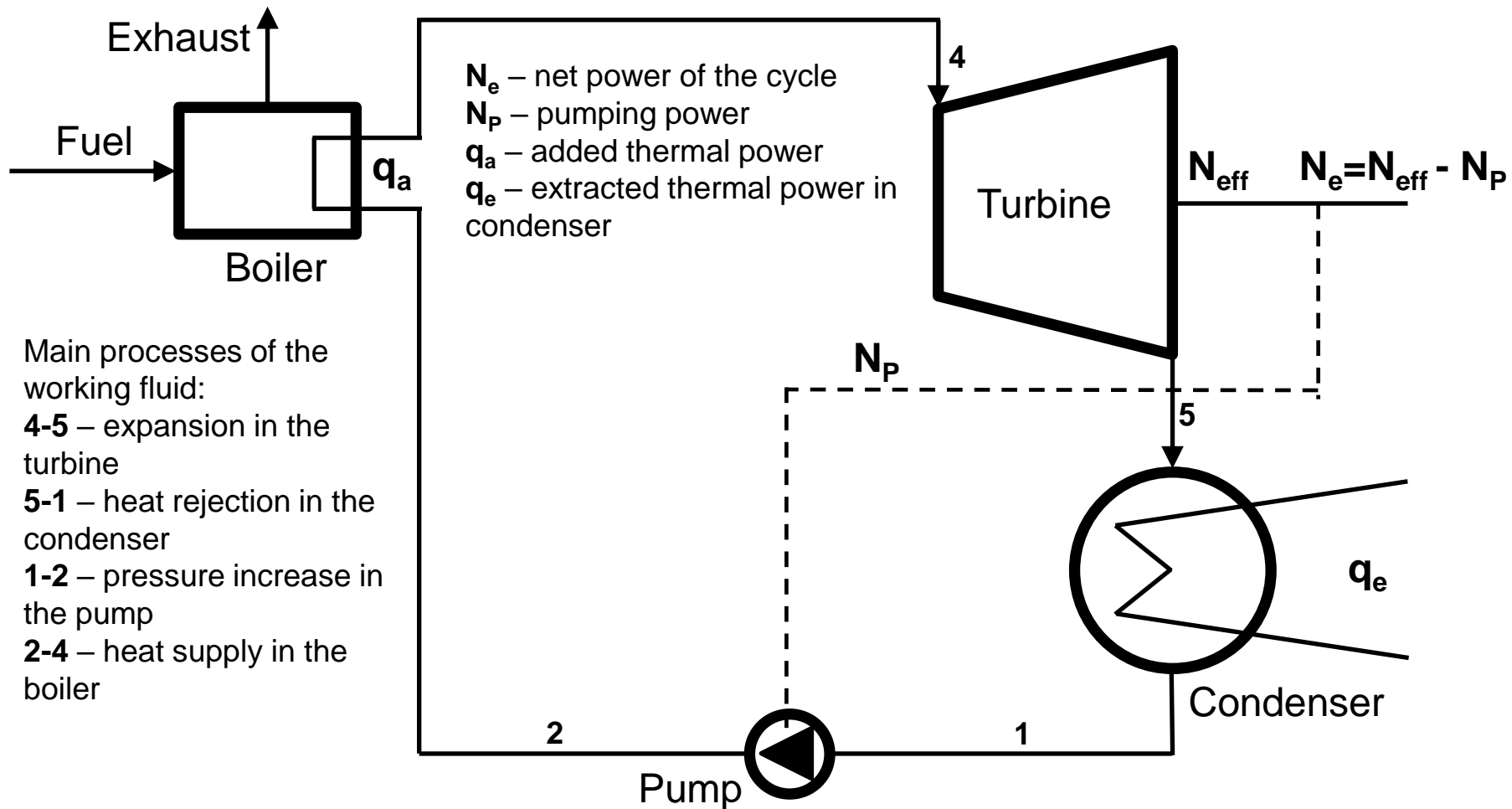
Ideal Rankine Cycle (1)

- The Rankine cycle describes the operation of steam heat engines that are most widespread in the power generation plants.

  - About 85% of electricity is generated by using systems based on Rankine cycle

- This cycle takes advantage of phase change of the working fluid to add and extract heat in an almost isothermal process.

- The schematics of the Rankine cycle is usually shown using the *T-s* plane.

Ideal Rankine Cycle (2)
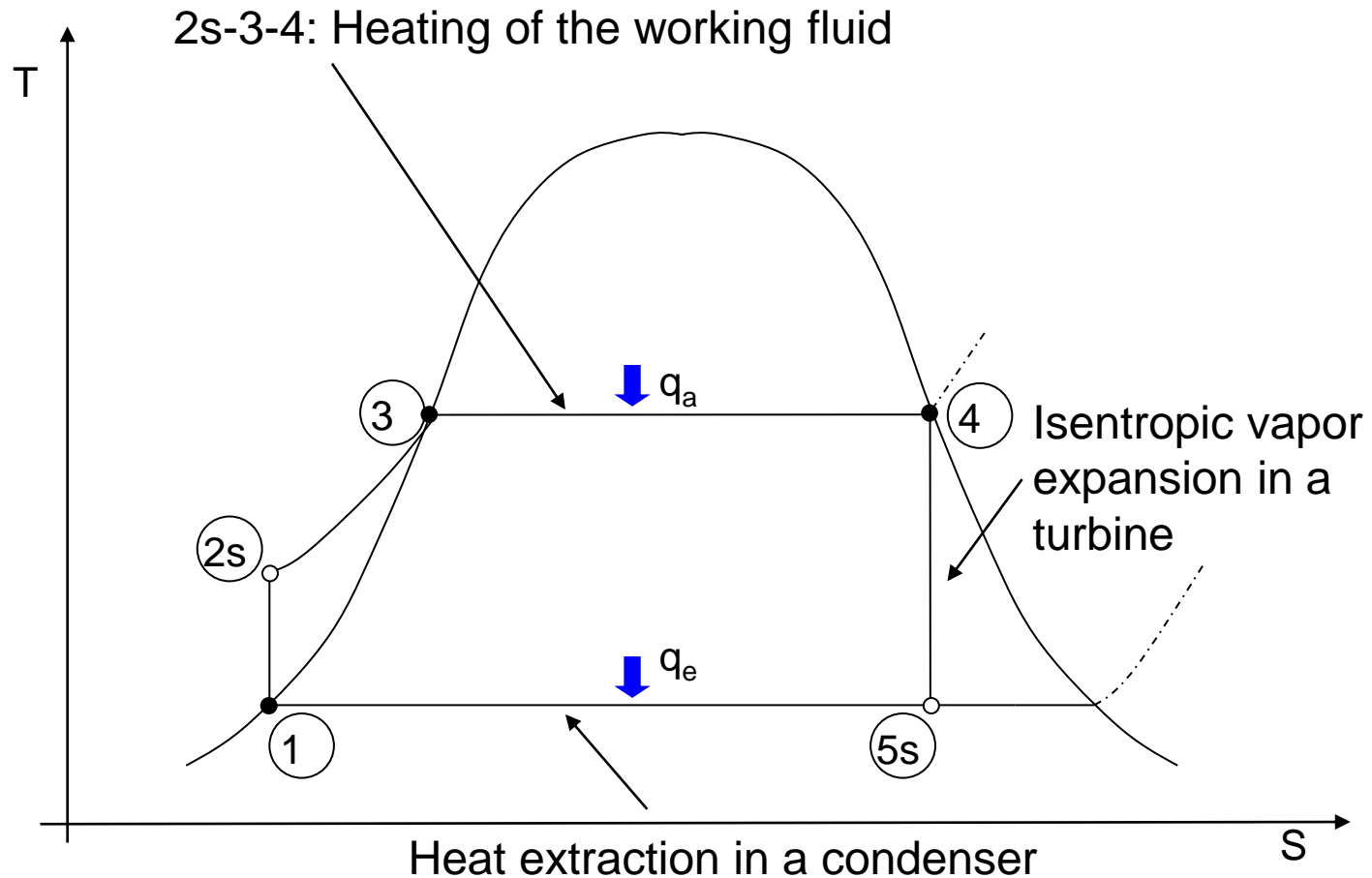
- Process 1-2s represents pumping of the working fluid from low to high pressure. Point 2s represents the cases without losses (isentropic process)
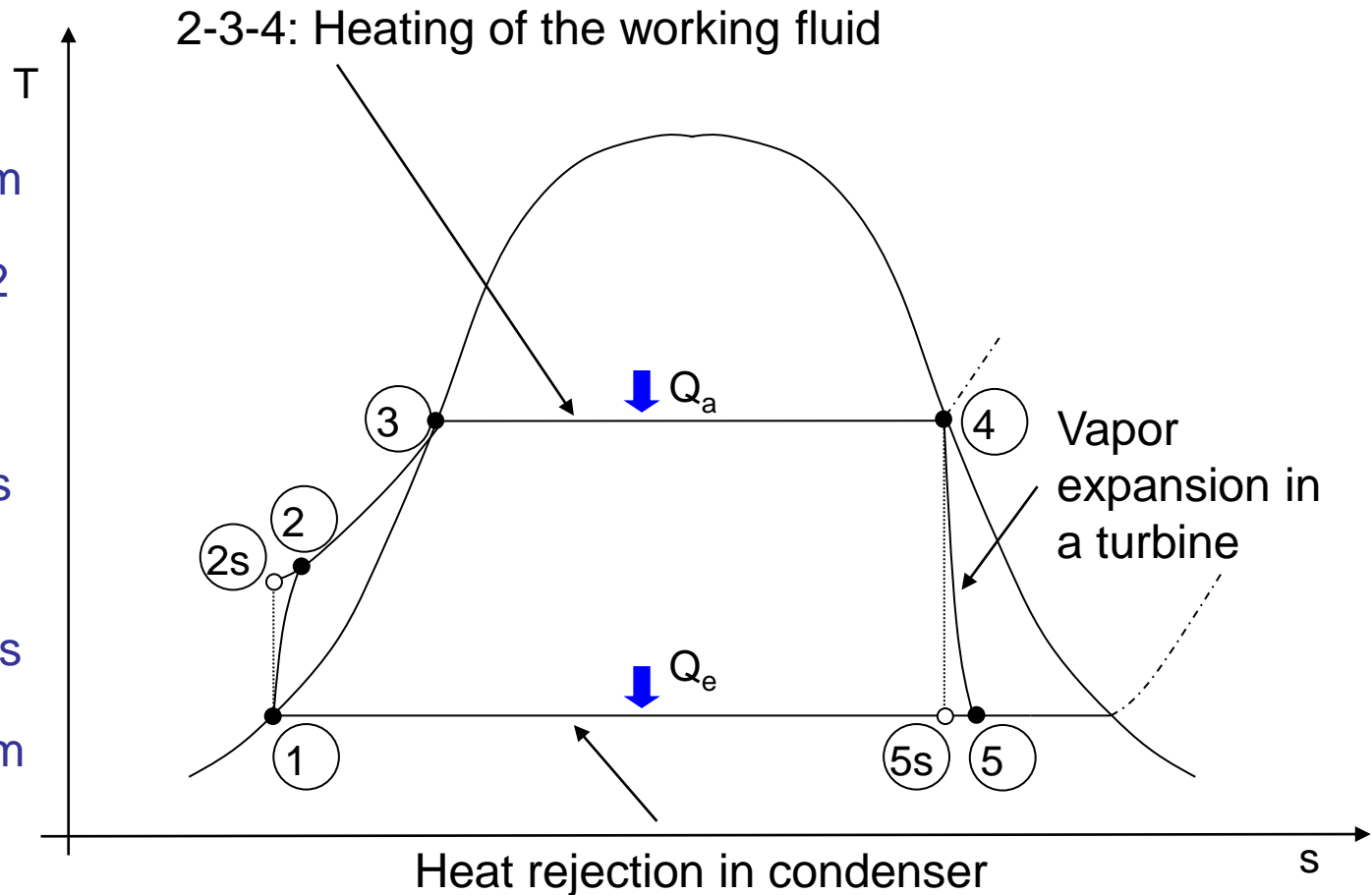

2s-3-4: Heating of the working fluid

q_a

Isentropic vapor expansion in a turbine

q_e

Heat extraction in a condenser

Ideal Rankine Cycle (3)

- Process 1-2: pumping power $\quad \dot{L}_{pump} \equiv N_{pump} = W(i_1 - i_{2s}) < 0$

- Process 2-4: heat added $\quad \dot{Q}_a \equiv q_a = W(i_4 - i_{2s}) > 0$

- Process 4-5: turbine power $\quad \dot{L}_{turbine} \equiv N_{turbine} = W\left(i_4 - i_{5s}\right) > 0$

  also called a **theoretical** turbine power, N_{th}

- Process 5-1: heat extracted $\quad \dot{Q}_e \equiv q_e = W(i_{5s} - i_1)$

- Energy efficiency of an ideal Rankine cycle

$$\eta_{EIR} = \frac{N_{turbine} + N_{pump}}{q_a} = \frac{\left(i_4 - i_{5s}\right) + \left(i_1 - i_{2s}\right)}{\left(i_4 - i_{2s}\right)}$$

- Sometimes it is assumed $\quad i_{2s} \approx i_1 \quad$ and $\quad \eta_{EIR} \approx \frac{i_4 - i_{5s}}{i_4 - i_1} \approx \frac{N_{th}}{q_a}$

Real Rankine Cycle (1)

- Process 1-2 represents real (irreversible) pumping of the working fluid from low to high pressure. Point 2 indicates conditions when losses in the pumping process are taken into account.

- Similarly, process 4-5 represents irreversible steam expansion in turbine

2-3-4: Heating of the working fluid

Vapor expansion in a turbine

Q_a

Q_e

Heat rejection in condenser

T

s

3   4

2
2s

1

5s 5

Real Rankine Cycle (2)

- Process 1-2: pumping power $\quad \dot{L}_{pump} \equiv N_{pump} = W(i_1 - i_2) < 0$

- Process 2-4: heat added $\quad \dot{Q}_a \equiv q_a = W(i_4 - i_2) > 0$

- Process 4-5: turbine power $\quad \dot{L}_{turbine} \equiv N_{turbine} = W(i_4 - i_5) > 0$

- Process 5-1: heat extracted $\quad \dot{Q}_e \equiv q_e = W(i_5 - i_1)$

- Energy efficiency of the real Rankine cycle $\quad \eta_{ERR} = \dfrac{N_{turbine} + N_{pump}}{q_a} = \dfrac{(i_4 - i_5) + (i_1 - i_2)}{(i_4 - i_2)}$

- Sometimes it is assumed $\quad i_1 \approx i_2 \quad$ and $\quad \eta_{ERR} = \dfrac{(i_4 - i_5)}{(i_4 - i_1)}$

Real Rankine Cycle (3)

- Isentropic (or internal) turbine efficiency is defined as

$$\eta_i = \frac{i_4 - i_5}{i_4 - i_{5s}}$$

- It is a measure of the irreversibility of the thermodynamic processes taking place in the turbine

- Cycle power $W(i_4 - i_{5s})$ is called the cycle **theoretical** power (N_{th})

Real Rankine Cycle (4)

- Turbine power $W(i_4 - i_5)$ is called the turbine **internal** power (N_i)

- An **effective** turbine power (that is the power that can be measured on the turbine shaft) is less than the internal power due to mechanical losses (friction on bearings, power losses for control, lubrication, etc)

- The ratio of the effective turbine power N_e to the internal turbine power N_i is called the turbine **mechanical efficiency**

$$\eta_m = \frac{N_e}{N_i}$$

Real Rankine Cycle (5)

- Energy efficiency of a real Rankine cycle can be now written as

$$\eta_{ERR} = \frac{N_i}{q_a} = \frac{N_{th}}{q_a}\frac{N_i}{N_{th}} = \eta_{EIR}\eta_i$$

- Here:
  - N_{th} – theoretical turbine power (assuming isentropic steam expansion in the turbine)
  - N_i – internal turbine power (assuming real steam expansion in the turbine)
  - q_a – heat added

- We can note that the energy efficiency of the real Rankine cycle is equal to the energy efficiency of the ideal Rankine cycle times the turbine internal efficiency
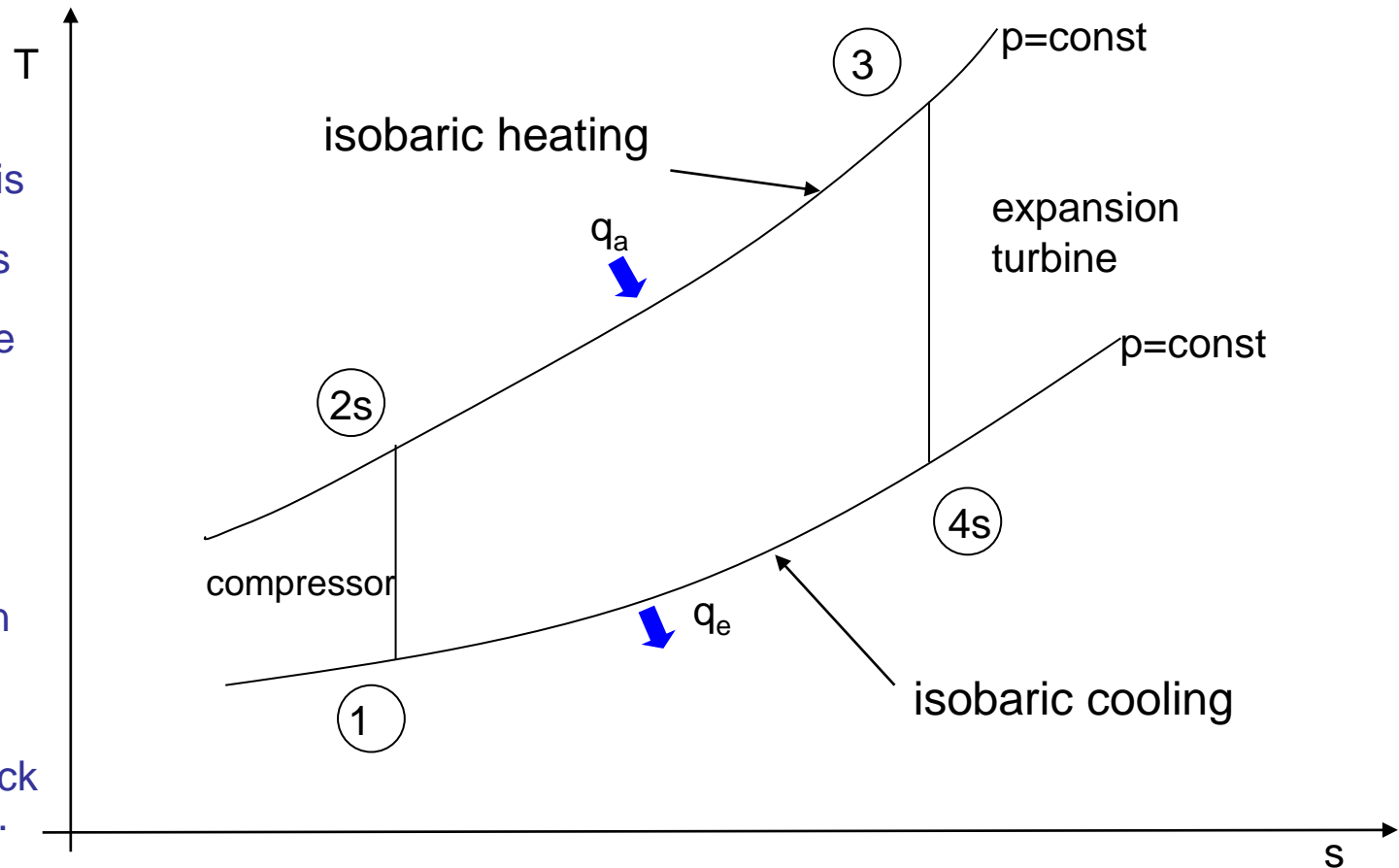
Rankine Cycle with Superheat
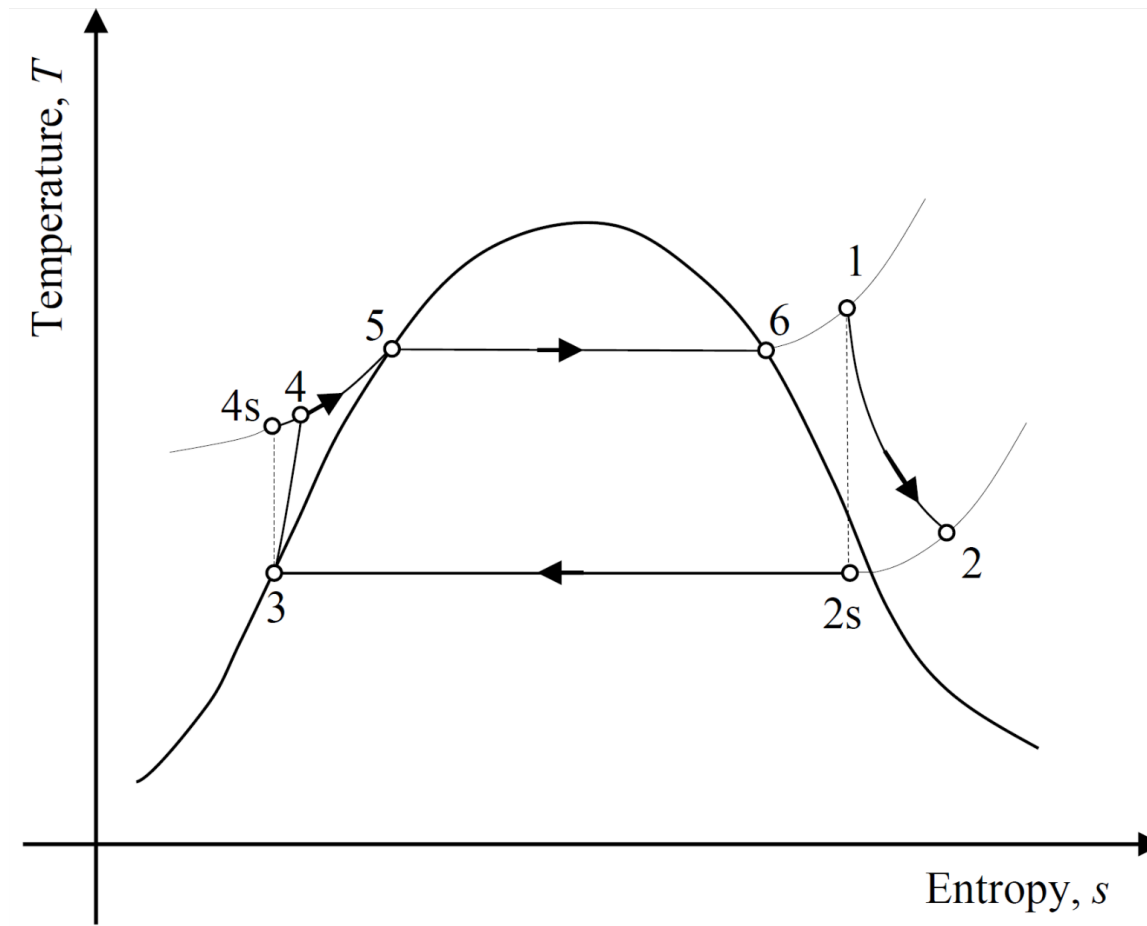
Efficiency of Steam Turbines

Typical values of the internal and mechanical efficiencies of turbines are given in the table below

Efficiency	Symbol	Values depending on power			Maximum value
		Low	Medium	High	
Internal	η_i	0.6÷0.7	0.7÷0.78	0.78÷0.85	0.88
Mechanical	η_m	0.95÷0.97	0.96÷0.98	0.98÷0.99	0.99

Ideal Brayton Cycle (1)

- The schematic of the Brayton cycle is shown in fig to the right. Process 1-2s corresponds to a compression of the working gas in a compressor. In process 2s-3 the working gas is heated and in process 3-4s the gas is expanded in a turbine. After isobaric cooling in process 4s-1, the gas is supplied back to the compressor.


T

isobaric heating

p=const

③ 3

q_a

expansion turbine

p=const

② 2s

④ 4s

compressor

q_e

① 1

isobaric cooling

s

Ideal Brayton Cycle (2)

- 1-2: Compression power:

$$N_{compress} = W(i_1 - i_{2s}) < 0$$

- 2-3: heat added:

$$q_a = W(i_3 - i_{2s}) > 0$$

- 3-4: turbine isentropic power:

$$N_{turbine} = W(i_3 - i_{4s}) > 0$$

- 4-1: heat extracted:

$$q_e = W(i_{4s} - i_1)$$

- Cycle energy efficiency:

$$\eta_{EIB} = \frac{N_{turbine} + N_{compress}}{q_a} = \frac{(i_3 - i_{4s}) + (i_1 - i_{2s})}{(i_3 - i_{2s})}$$

Real Brayton Cycle (1)

- The schematic of the Brayton cycle is shown in fig to the right. Process 1-2 corresponds to a compression of the working gas in a compressor. In process 2-3 the working gas is heated in a nuclear reactor core and in process 3-4 the gas is expanded in a turbine. After isobaric cooling in process 4-1, the gas is supplied back to the compressor.
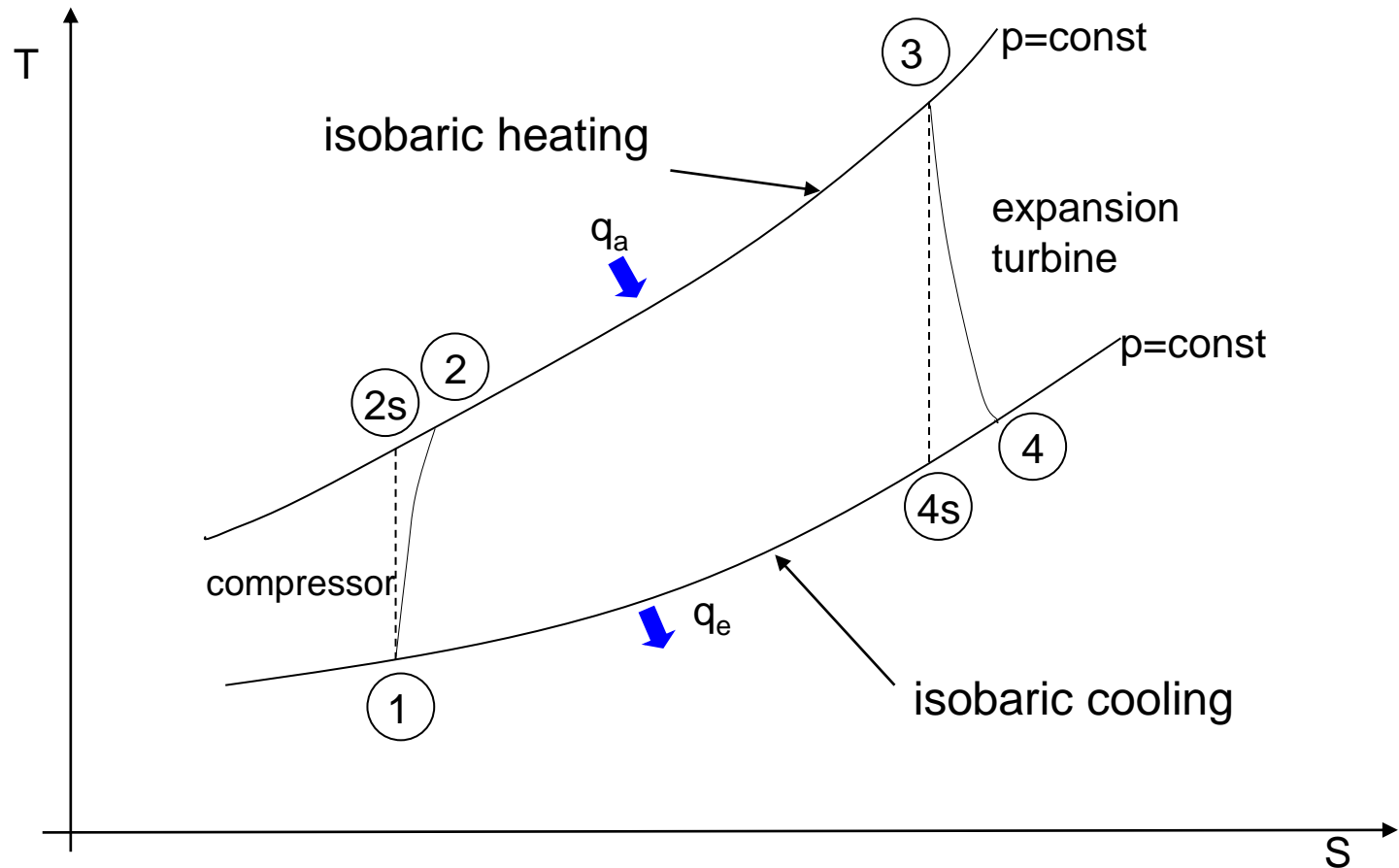
Real Brayton Cycle (2)

- 1-2: Compression power: $N_{compress} = W(i_1 - i_2)$

- 2-3: heat added: $q_a = W(i_3 - i_2)$

- 3-4: turbine internal power: $N_{turbine} = W(i_3 - i_4)$

- 4-1: heat extracted: $q_e = W(i_4 - i_1)$

- Cycle energy efficiency: $\eta_{ERB} = \dfrac{N_{turbine} + N_{compress}}{q_a} = \dfrac{(i_3 - i_4) + (i_1 - i_2)}{(i_3 - i_2)}$

Kalina Cycle

- Kalina cycle uses a solution of 2 fluids with different boiling points as a working fluid.

- Due to that
  - the solution boils over a range of temperatures
  - more heat can be extracted from the source than with a pure working fluid
  - the efficiency is comparable to the combined cycle with less complexity
  - the boiling point can be adjusted to suit the heat input temperature
  - water-ammonia is the most widely used combination

- Used in transformation of geothermal heat, solar heat, industrial process heat, waste heat from power plants

Kalina Cycle
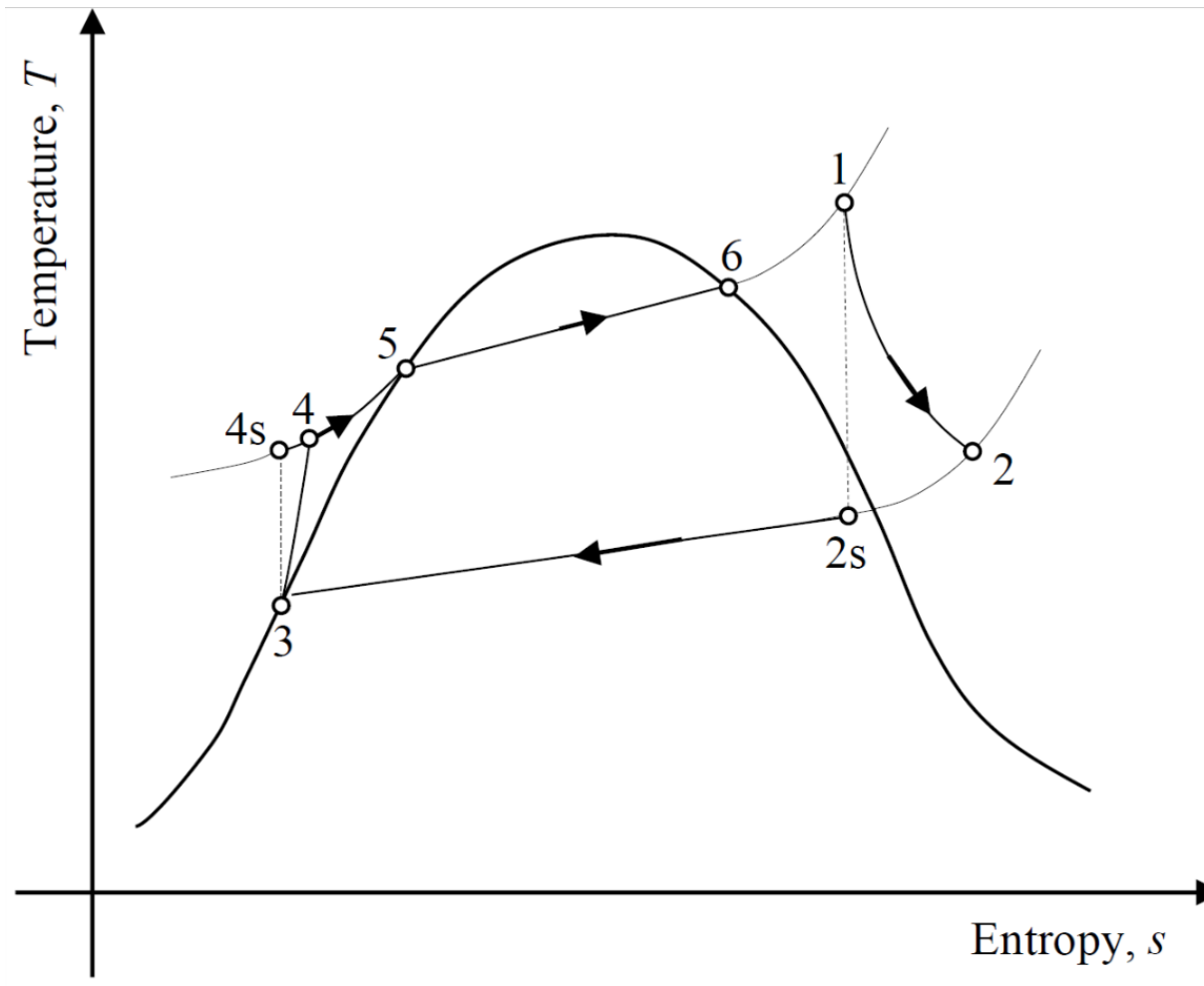
Organic Rankine Cycle (1)

- Organic Rankine Cycle (ORC) refers to use of organic, high molecular mass fluid with liquid-vapor phase change (boiling point) occurring at a lower temperature than the water-steam phase change

- The fluid allows Rankine cycle heat recovery from lower temperature sources such as
  - biomass combustion
  - industrial waste heat
  - geothermal heat
  - solar heat

- Allows to transform low temperature heat into electricity

Organic Rankine Cycle (1)
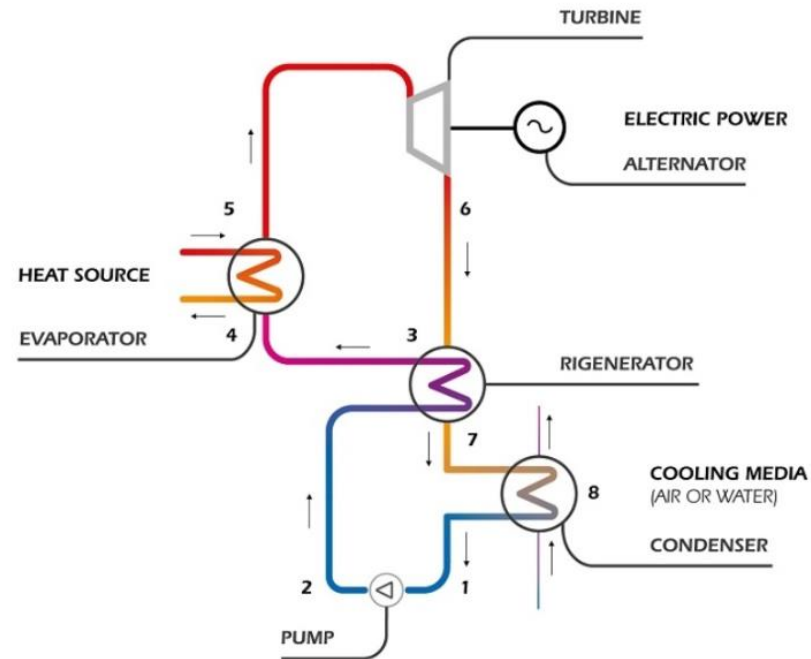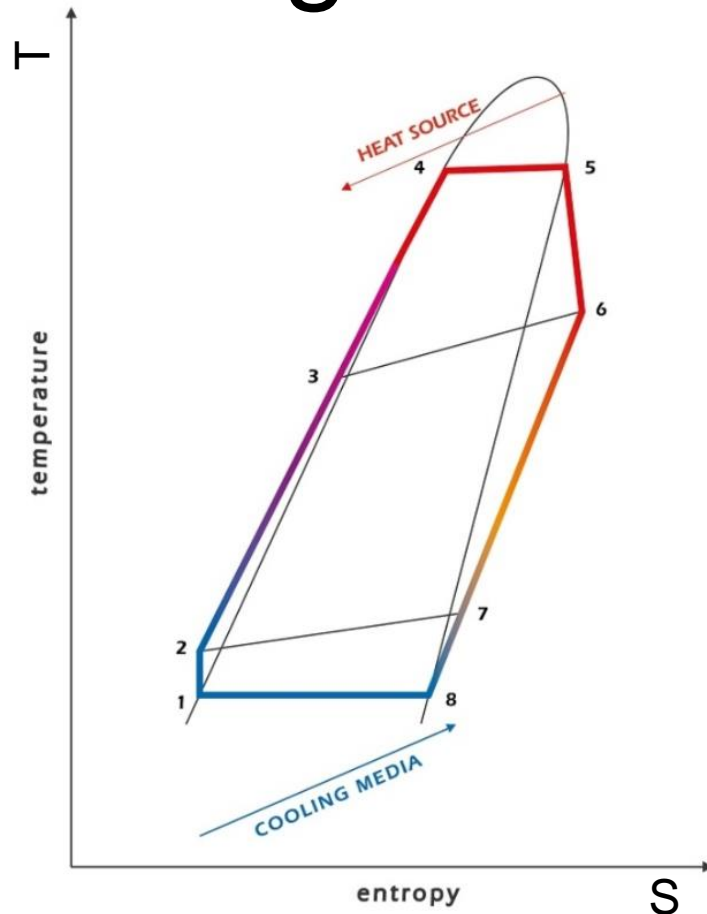
3-4-5 Heating/evaporation of fluid
5-6 Expansion in turbine
6-7 Heat regeneration
7-8-1 Cooling/condensation
1-2 Pumping; 2-3 Regenerative heating

Combined Cycles (1)

- A combined cycle consists of (usually) two heat engines both using the same heat source

- The engines operate in series:
  - after the first engine the temperature of the working fluid is high enough that a second engine may extract energy from the waste heat
  - the efficiency can be increased from e.g. ~30% to over ~60%

- In stationary plants usually a gas turbine (operating in Brayton cycle) is followed by a steam turbine (operating in Rankine cycle) – so called Combined Cycle Gas Turbine (CCGT)
  - can achieve 62% efficiency in base load operation

Combined Cycles (2)
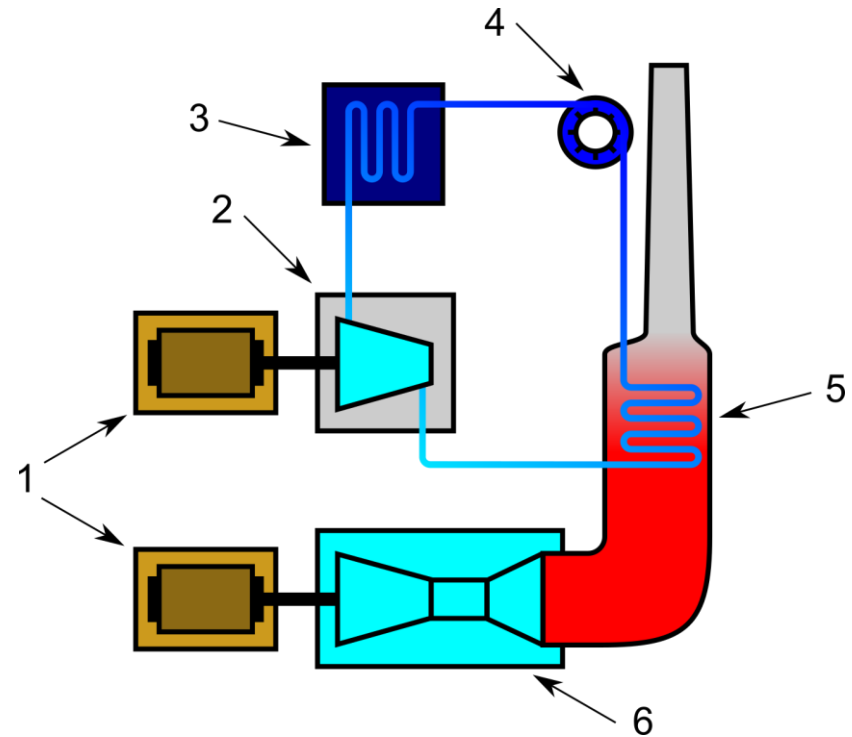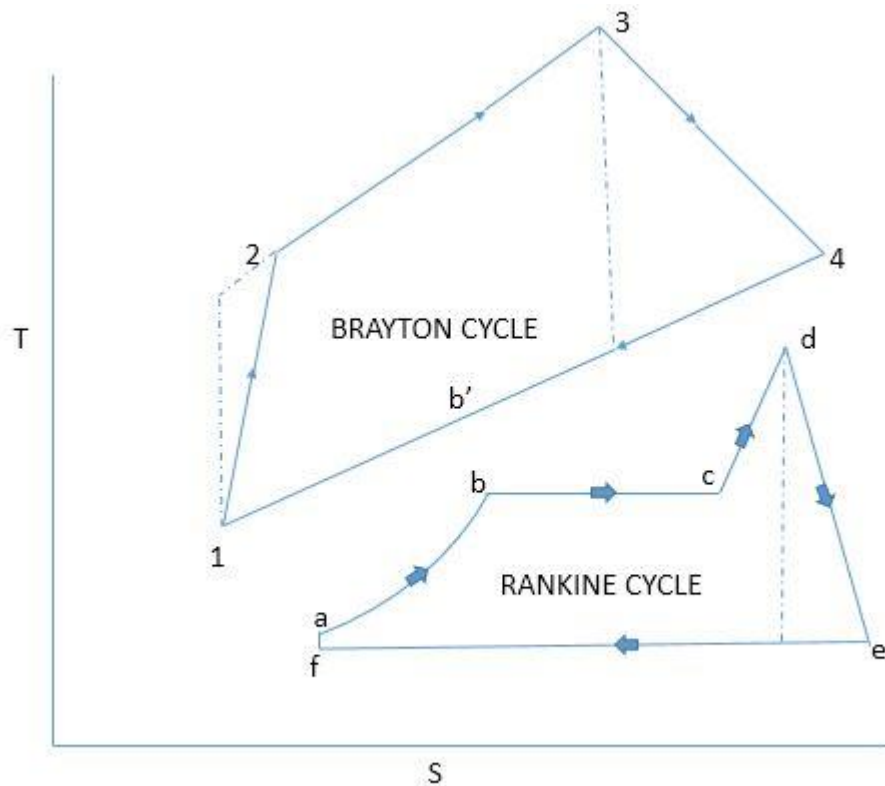
1 – generators, 2- steam turbine, 3 – condenser, 4 – pump, 5 – boiler, 6 – gas turbine

Integrated Solar Combined Cycle

- Integrated Solar Combined Cycle (ISCC) is a hybrid technology in which a solar thermal field is integrated within a combined cycle plant

- In ISCC plant, solar energy is used as an auxiliary heat supply supporting the steam cycle

- This results in increased generation capacity and/or reduced fossil fuel usage

- In comparison to purely solar thermal plant:
  - daily steam turbine start-up losses are eliminated

Summary

- Work extracted/added in a thermodynamic process between two points depends on the path that is followed between the points

- For any cyclic process $\sum \Delta E_{l,i} = 0$, so $\sum \Delta L_i = \sum \Delta Q_i$ (1st Law), but $\sum \Delta Q_i / T_i \leq 0$ (Clausius inequality)

- Carnot and Stirling cycles have the highest efficiency, but they are impractical or difficult to employ

- Cycles of practical importance are: Rankine cycle (using phase-change), Brayton cycle (using gas)

- Other solutions: combined cycle, organic Rankine, etc