Sustainable Energy Transformation Technologies, SH2706

Lecture No 3

Title:

Nuclear Energy

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Outline

- Binding energy
- Energy release during nuclear reaction: Q - value
- Nuclear fusion
- Nuclear fission
  - Prompt energy
  - Delayed energy
  - Fission products
  - Neutron emission

Binding Energy of Nucleus

- Binding is the energy required to disassembly a whole into separate parts
- To find a binding energy (BE) of a nucleus, we need to compare the nucleus with its constituents: Z protons and A-Z neutrons:

    Z protons + (A-Z) neutrons -> nucleus + BE
- The binding energy is determined from the change of rest mass between the left- and the right-hand sides

mass of neutron

$$\text{Mass Defect} = \text{BE}/c^2 = Zm_p + (A-Z)m_n - m\left({}^A_Z\text{X}\right)$$

mass of nucleus

mass of proton

Binding Energy of Nucleus

- We use $m\left({}_{Z}^{A}\mathrm{X}\right)$ to indicate the mass of nucleus and $M\left({}_{Z}^{A}\mathrm{X}\right)$ for the atomic mass

- Since the atomic mass is always given in tables (rather than nuclear mass), we use it in the expression for BE:

$$\mathrm{BE}\left({}_{Z}^{A}\mathrm{X}\right) = \left[ZM\left({}_{1}^{1}\mathrm{H}\right) + (A-Z)m_n - M\left({}_{Z}^{A}\mathrm{X}\right)\right]c^2$$

- To compensate for Z subtracted electrons, we replaced the mass of proton with the mass of a hydrogen atom

- We neglect the binding energy of electron in hydrogen

Binding Energy - Example

- Calculate the total binding energy in nucleus of ^{4}He

  (Use data given in Appendix C, § C.1)

- Solution: we find the mass defect for the nucleus as: mass defect = $2*M(^1H) + 2*m_n – M(^4He) = 2*1.007825 + 2*1.0086649 – 4.0026032 = 0.0303766$ u

- Thus the binding energy (MeV) = mass defect (u) * 931.5 (MeV/u) = 28.296 MeV

- The binding energy per nucleon is thus: 28.296/4 = 7.074 MeV

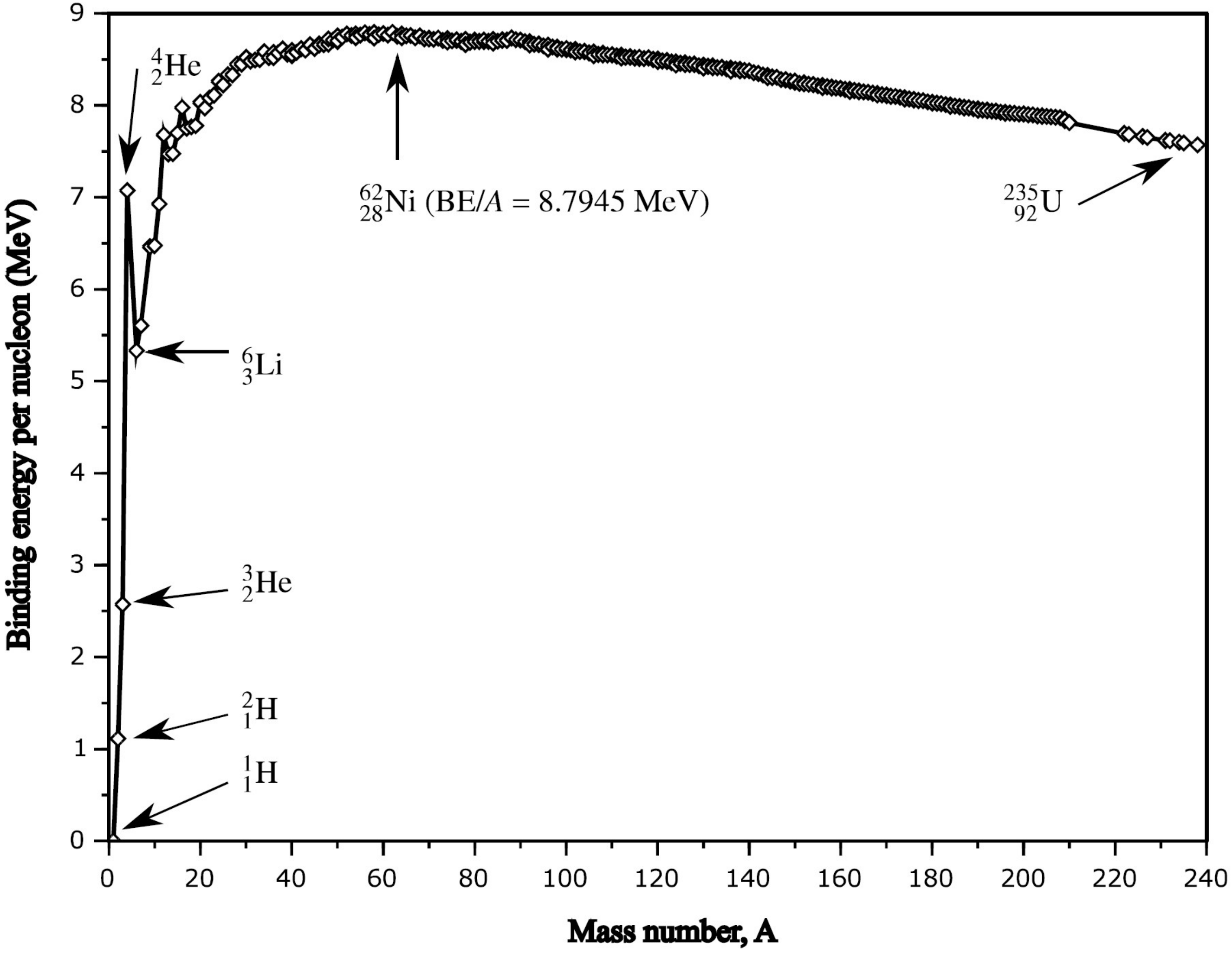
Binding Energy of Nucleus

- The binding energy can be obtained from a semi-empirical mass formula (SEMF) as follows

$$\mathrm{BE}\left({}_{Z}^{A}\mathrm{X}\right) = 15.56A - 17.23A^{2/3} - 0.7\frac{Z^2}{A^{1/3}} - 23.28\frac{(A-2Z)^2}{A} + \frac{12}{A^{1/2}}\delta$$

- Here BE is expressed in MeV, A and Z are the mass number and the atomic number, respectively, and δ is equal to +1 for even N=A-Z and Z, -1 for odd N and Z, and 0 for odd A.

Binding Energy per Nucleon



Energy in Nuclear Reactions

- Almost all energy in the universe originates from nuclear reactions
- We can find the energy released in nuclear reactions using the total energy (kinetic + rest mass energy) conservation principle

$$\sum_{i \in \text{reactants}} \left(E_i + m_i c^2\right) = \sum_{i \in \text{products}} \left(E_i' + m_i' c^2\right)$$

- Q-value of the reaction is defined as a change in the kinetic energy:

$$Q = \sum_{i \in \text{products}} E_i' - \sum_{i \in \text{reactants}} E_i$$

Q-value

- Using the total energy conservation principle, Q-value can be found from the rest mass change during the reaction $$Q = \left(\sum_{i \in \text{reactants}} m_i - \sum_{i \in \text{products}} m'_i \right) c^2.$$
- When Q>0, the reaction is exothermic, and conversely, if Q<0 the reaction is endothermic.
- For binary reaction: $x + X \rightarrow y + Y.$

$$Q = \left[(m_x + m_X) - (m_y + m_Y) \right] c^2$$

m_x, m_X – rest mass of reactants
m_y, m_Y – rest mass of products

We can replace nuclear masses m_i with atomic masses M_i, by adding and subtracting the same number of electrons, and we get:

$$Q = \left[(M_x + M_X) - (M_y + M_Y) \right] c^2$$

Here tabulated atomic masses can be used

Q-value Example

Example 1.4. Calculate the Q value for the reaction ${}_{4}^{9}\mathrm{Be}(\alpha,n){}_{6}^{12}\mathrm{C}$ knowing the following rest masses: $M({}_{4}^{9}\mathrm{Be}) = 9.012182$ u, $M({}_{2}^{4}\mathrm{He}) = 4.002603$ u, $M({}_{6}^{12}\mathrm{C}) = 12.000000$ u and $m_n \equiv m({}_{0}^{1}\mathrm{n}) = 1.008664$ u. Is the reaction endothermic or exothermic?

Solution: we find the difference in rest masses between reactants and products as $M({}_{4}^{9}\mathrm{Be})$ + $M({}_{2}^{4}\mathrm{He})$ - $M({}_{6}^{12}\mathrm{C})$ - $m({}_{0}^{1}\mathrm{n})$ = 0.006121 u. Thus Q = 931.5 MeV/u $\times$ 0.006121 u = 5.702 MeV > 0. The reaction is exothermic. $\square$

Nuclear Fusion

Some possible fusion reactions

$$^2_1\mathrm{H} + ^2_1\mathrm{H} \rightarrow ^3_2\mathrm{He} + ^1_0\mathrm{n}, \; Q = 3.27 \text{ MeV},$$

$$^2_1\mathrm{H} + ^2_1\mathrm{H} \rightarrow ^3_1\mathrm{H} + ^1_1\mathrm{H}, \; Q = 4.03 \text{ MeV},$$

$$^2_1\mathrm{H} + ^3_1\mathrm{H} \rightarrow ^4_2\mathrm{He} + ^1_0\mathrm{n}, \; Q = 17.59 \text{ MeV},$$

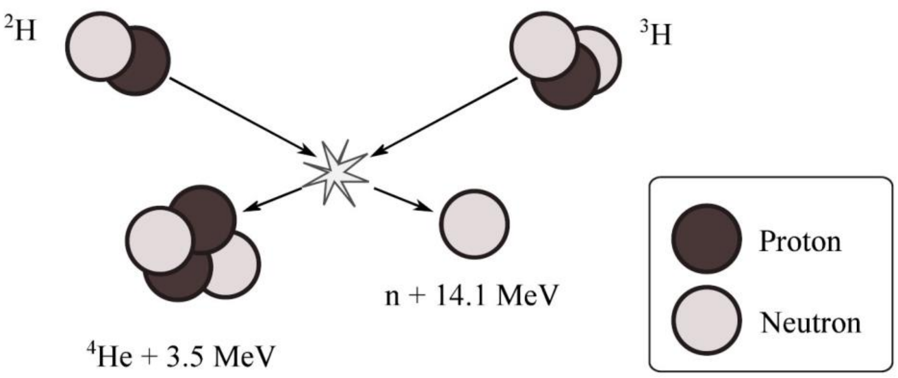
$$^2_1\mathrm{H} + ^3_2\mathrm{He} \rightarrow ^4_2\mathrm{He} + ^1_1\mathrm{H}, \; Q = 18.35 \text{ MeV},$$

$$^3_1\mathrm{H} + ^3_1\mathrm{H} \rightarrow ^4_2\mathrm{He} + 2^1_0\mathrm{n}, \; Q = 11.33 \text{ MeV},$$

$$^1_1\mathrm{H} + ^6_3\mathrm{Li} \rightarrow ^4_2\mathrm{He} + ^3_2\mathrm{He}, \; Q = 4.02 \text{ MeV},$$

$$^1_1\mathrm{H} + ^{11}_5\mathrm{B} \rightarrow 3(^4_2\mathrm{He}), \; Q = 8.08 \text{ MeV},$$

Most promising D+T fusion reaction



Energy Production in Stars

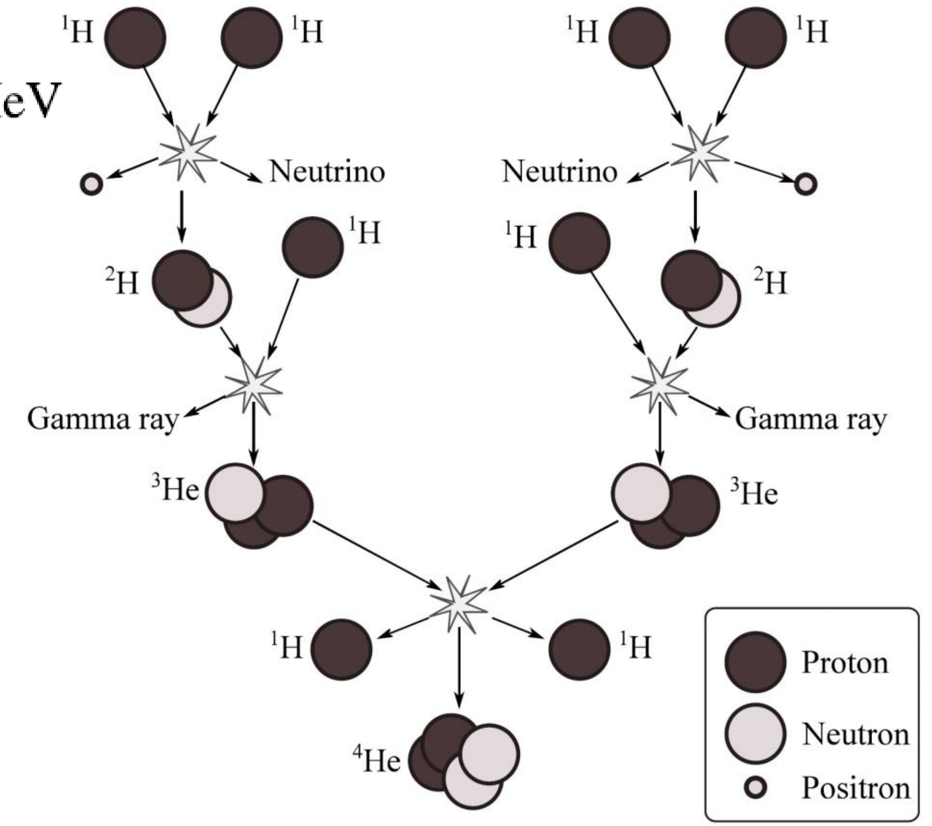
- Early stage: hydrogen is fused into helium

$${}_1^1\mathrm{H} + {}_1^1\mathrm{H} \rightarrow {}_1^2\mathrm{D} + \beta^+ + \nu,\ Q = 0.42\ \mathrm{MeV}$$

- “Burning” of deuterium and helium

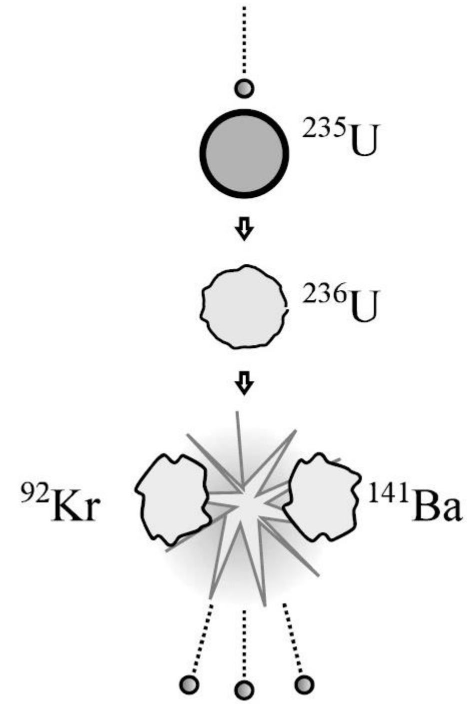
$${}_1^2\mathrm{D} + {}_1^1\mathrm{H} \rightarrow {}_2^3\mathrm{He} + \gamma,\ Q = 5.49\ \mathrm{MeV}$$

$${}_2^3\mathrm{He} + {}_2^3\mathrm{He} \rightarrow {}_2^4\mathrm{He} + 2{}_1^1\mathrm{H} + \gamma,\ Q = 12.86\ \mathrm{MeV}$$



Nuclear Fission

- Nuclear fission are very special type of reactions in which a very heavy nucleus (e.g. ^{235}U) splits into two lighter nuclei



- The reaction can go as follows

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{236}_{92}\mathrm{U}^{*} \rightarrow \mathrm{Y}_{H}^{+n} + \mathrm{Y}_{L}^{+m} + (n+m)\mathrm{e}^{-} $$

Excited compound nucleus

10^{-14} s

Heavy (H) and Light (L) primary fission products

10^{-20} s

10^{-12} s slowing down and transferring kinetic energy to the ambient medium

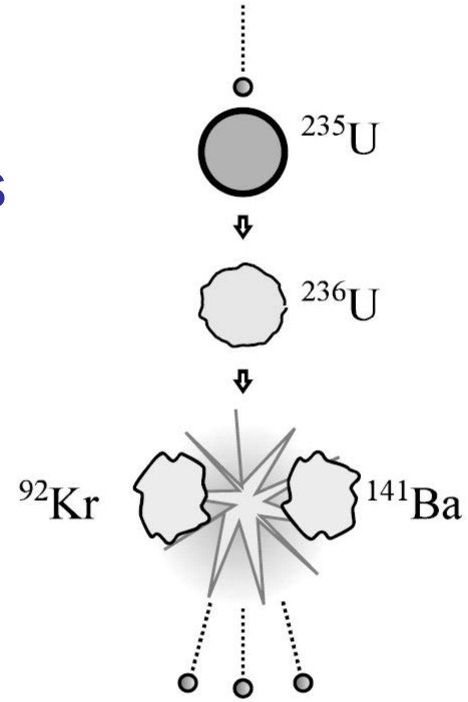
Nuclear Fission

- After slowing down of fission products, they acquire electrons to become neutral atoms

- At this stage, the reaction can be written as

$$ {}_{0}^{1}\mathrm{n} + {}_{92}^{235}\mathrm{U} \rightarrow {}_{92}^{236}\mathrm{U}^{*} \rightarrow \mathrm{Y}_{H} + \mathrm{Y}_{L} + \nu_{p}\left({}_{0}^{1}\mathrm{n}\right) + \gamma_{p} $$

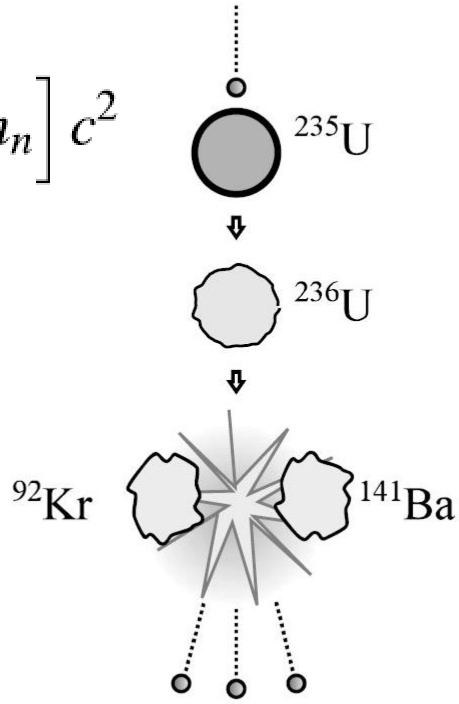
here v_p – number of neutrons emitted from the primary fission fragments within 10^{-17} s after splitting (0 to 8), γ_p – prompt gamma rays emitted from fission fragments within $2\text{x}10^{-14}$ s after splitting



Prompt Energy of Fission

- The prompt energy released during fission is obtained from the mass deficit of the reaction as

$$E_p = \left[M\left({}^{235}_{92}\mathrm{U}\right) + m_n - M\left(\mathrm{Y}_H\right) - M\left(\mathrm{Y}_L\right) - \nu_p m_n\right] c^2$$



Here *M()* - atomic mass, m_n – neutron mass

Prompt Energy - Example

Example 1.5. Calculate the prompt energy release for the following fission reaction $^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{139}_{54}\mathrm{Xe} + ^{95}_{38}\mathrm{Sr} + 2\left(^{1}_{0}\mathrm{n}\right) + 7\left(\gamma_{\mathrm{p}}\right)$. Assuming that two prompt fission neutrons have a total kinetic energy of 5.2 MeV and the prompt gamma rays have a total energy of 6.7 MeV, find the total kinetic energy of initial fission fragments.

Solution: the mass deficit of the reaction is $E_p = M(^{235}_{92}\mathrm{U}) + m_n - M(^{139}_{54}\mathrm{Xe}) - M(^{95}_{38}\mathrm{Sr}) - 2m_n = (235.043923 + 1.008665 - 138.918787 - 94.919358 - 2 \times 1.008665)\mathrm{u} = 0.197113$ u. Thus, $E_p = 931.5$ MeV/u $\times$ 0.197113 u = 183.6 MeV. The total kinetic energy of initial fission fragments can be found as E_{Kff} = 183.6 - 5.2 - 6.7 = 171.7 MeV. □

Delayed Fission Energy

- Majority of fission energy is released as prompt energy, within 10^{-12} s after fission
- However, fission products are not stable and decay after some time to their final stable end-chain nuclei
- For example, ^{139}Xe reaches stable ^{139}La after 3 β^- decays, and ^{95}Sr reaches stable ^{95}Mo after 4 β^- decays

$$\left.{}^{139}_{54}\mathrm{Xe} + {}^{95}_{38}\mathrm{Sr} + 7\left({}^{0}_{-1}\mathrm{e}\right) \rightarrow {}^{139}_{57}\mathrm{La} + {}^{95}_{42}\mathrm{Mo} + \underbrace{7\left({}^{0}_{-1}\beta\right)}_{\text{7 electron acquired by decaying fission products}} + 7\left(\overline{\nu}\right)\right\}$$

This reaction can be used to calculate the delayed fission energy

Fission Product Decay

- Uranium fission was discovered through this decay chain

$$^{140}_{54}\text{Xe} \xrightarrow[16\text{ s}]{\beta^-} {}^{140}_{55}\text{Cs} \xrightarrow[66\text{ s}]{\beta^-} {}^{140}_{56}\text{Ba} \xrightarrow[12.8\text{ d}]{\beta^-} {}^{140}_{57}\text{La} \xrightarrow[40\text{ h}]{\beta^-} {}^{140}_{58}\text{Ce (stable)}$$

- Another important decay chain, containing reactor poison (^{135}Xe)

$$^{135}_{52}\text{Te} \xrightarrow[19\text{ s}]{\beta^-} {}^{135}_{53}\text{I} \xrightarrow[6.57\text{ h}]{\beta^-} {}^{135}_{54}\text{Xe} \xrightarrow[9.10\text{ h}]{\beta^-} {}^{135}_{55}\text{Cs} \xrightarrow[2.3\times 10^6\text{y}]{\beta^-} {}^{135}_{56}\text{Ba (stable)}$$

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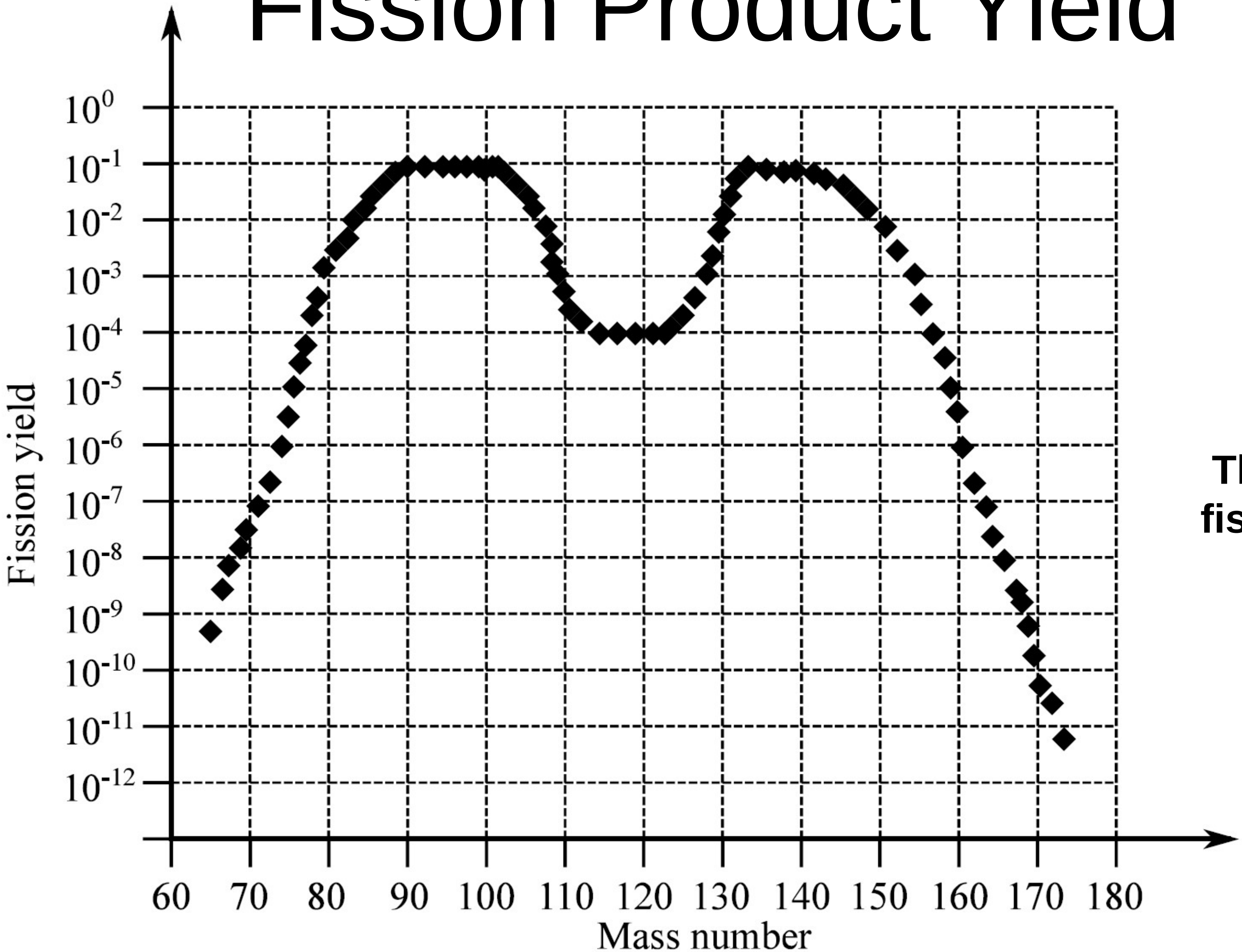
Average Fission Energy

	Energy from Fission (MeV)	Recoverable in Core (MeV)
Prompt :		
kinetic energy of the fission fragments	168	168
kinetic energy of prompt fission neutrons	5	5
fission γ-rays	7	7
γ-rays from neutron capture	-	3-9
Delayed :		
fission product β-decay energy	8	8
fission product γ-decay energy	7	7
neutrino kinetic energy	12	0
Total energy (MeV)	207	198-204

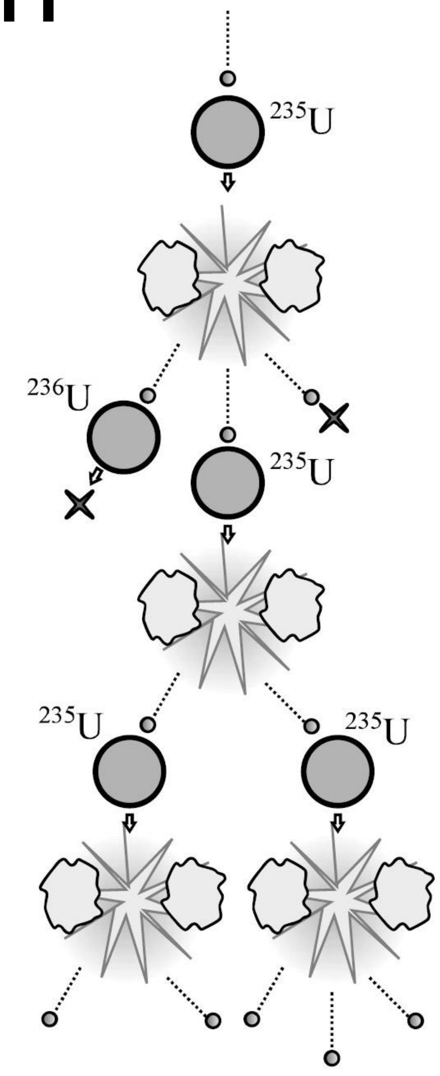
Fission Product Yield



Thermal fission of ^{235}U

Neutron Emission

- The neutron emitted by a fission are of great importance for practical generation of nuclear power
- Neutrons are needed to cause other fission reactions, and to sustain a chain reaction
- Almost all fission neutrons are emitted within 10^{-14} s, and the number of these prompt neutrons ν_p is within 0 and 8, with a mean value around $\overline{\nu_p} \approx 2.5$

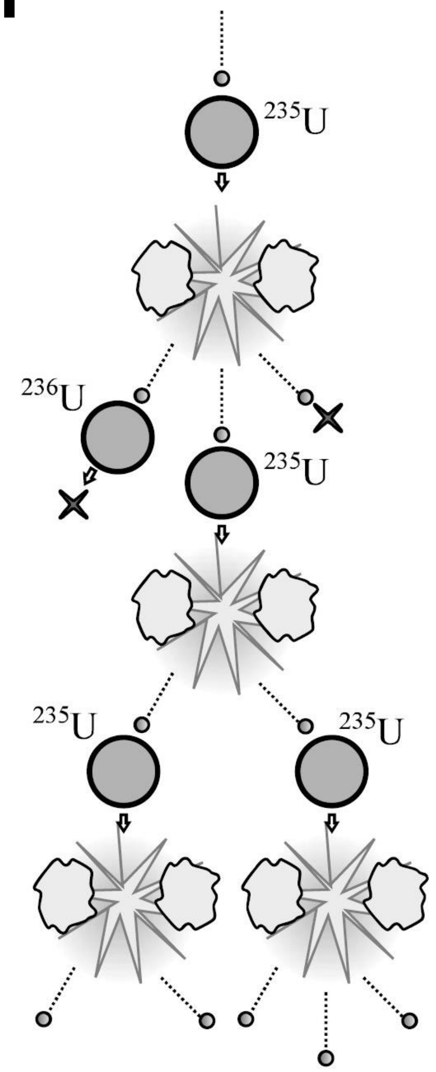


Neutron Emission

- A small fraction (always less than 1% for thermal fission) are emitted as delayed neutrons, with average number $\overline{\nu_d}$
- The total average number of neutrons per fission is thus $\overline{\nu} = \overline{\nu_p} + \overline{\nu_d}$, with the delayed neutron fraction defined as $\beta \equiv \overline{\nu_d}/\overline{\nu}$.
- For example, for ^{235}U, this number is given as

$$\bar{\nu}(E) = \begin{cases} 2.432 + 0.066E & 0 \le E \le 1 \\ 2.348 + 0.150E & E > 1 \end{cases}$$

***E* – neutron energy in MeV**



Total Average Fission Neutrons

	Fast Fission		Thermal Fission	
Nuclide	$\overline{\nu}$	β	$\overline{\nu}$	β
^{235}U	2.57	0.0064	2.43	0.0065
^{233}U	2.62	0.0026	2.48	0.0026
^{239}Pu	3.09	0.0020	2.87	0.0021
^{241}Pu	-	-	3.14	0.0049
^{238}U	2.79	0.0148	-	-
^{232}Th	2.44	0.0203	-	-
^{240}Pu	3.3	0.0026	-	-

What have we learned

- How to calculate the binding energy for any nucleus
- How to calculate Q-value of any reaction
- Fusion reactions: in stars and on Earth
- Energy from nuclear fission