Nuclear Reactor Physics

Fundamentals of atomic and nuclear physics

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Questions to discuss

The natural decay of neutrons is not considered in physics of nuclear reactors. Why?

- Neutrons are not stable except when bound into an atomic nucleus.
- A free neutron decays to a proton with the emission of a negative electron (β-decay) and an antineutrino.
- The decay takes on average about 15 minutes.
- Free neutrons in reactors have a very short lifetime, about 10^{-3}s to 10^{-5}s, since they are quickly absorbed in fuel and other materials in the reactor. So, the fact that free neutrons are unstable can be ignored.

Questions to discuss

What is the atomic weight of the neutral ^{12}C atom?

- The **atomic weight**, M, of an atom is defined as the mass of the neutral atom relative to the 1/12 of the mass of neutral ^{12}C atom.
- So, the atomic weight of ^{A}Z is

$$M(^{A}Z) = 12\ \frac{m(^{A}Z)}{m(^{12}C)}$$

  where $m(^{A}Z)$ is the mass of the neutral atom ^{A}Z.

Don't confuse the atomic weight with:

- atomic mass number A - the total number of *nucleons* (protons and neutrons in a nucleus), also known as the nucleon number,
- atomic mass m (mass e.g. in grams),
- atomic number Z - the total number of protons in the nucleus,
- neutron number N - the total number of neutrons in the nucleus,

Questions to discuss

What is the mass of one mole of ^{12}C?

- Mole is the amount of a substance having a mass, in grams, equal to the atomic or molecular weight of the substance.
- Thus, one mole of ^{12}C is exactly 12g of this isotope. Similarly, one mole of O_2 is 31.99876g, etc.

Figure 1: Mole of ^{12}C

Mass of a single atom or molecule

How can we compute the mass of a single atom with atomic weight M?

Since one mole of the substance has a mass of M grams and contains N_A atoms, it follows that the mass of one atom ^{A}Z is

$$m(^{A}Z) = \frac{M(^{A}Z)}{N_A}$$

How is the atomic mass unit u defined?

- The **atomic mass unit** (u) is defined as one twelfth the mass of neutral ^{12}C atom, that is

$$\mathrm{u} = \frac{1}{12} m(^{12}C)$$

- It follows from the definition that

$$\mathrm{u} = \frac{1}{12} \frac{12g}{N_A} = \frac{g}{N_A} = 1.66057 \times 10^{-24} \mathrm{g}$$

- We can therefore also calculate mass of a single atom or molecule using the atomic mass unit as

$$m(^A Z) = \frac{M(^A Z)}{N_A} = M(^A Z) \times \mathrm{u}$$

Atom number density

How can you calculate the atomic concentration N of a substance that has a mass density ρ [g/cm^3] and atomic weight M?

Solution:

- In 1 cm^3 of sodium there is ρ grams of the substance,
- one atom of the substance has a mass

$$m(^AZ) = \frac{M(^AZ)}{N_A}$$

- therefore, 1 cm^3 of the substance must contain

$$N = \frac{\rho}{m(^AZ)} = \frac{\rho N_A}{M(^AZ)}$$

  of atoms.

How is electron volt defined?

In nuclear engineering, energy is often expressed in terms of the unit **electron volt**, denoted by eV. This is defined as the increase in the kinetic energy of an electron when it falls through an electrical potential of one volt.

$$1\mathrm{eV} = 1.60219 \times 10^{-19}\mathrm{J}$$

Other energy units often used are MeV (10^6 eV) and keV (10^3 eV).

Energy of a neutron at motion

Why do we use the classical-mechanics formula (not the relativistic formula) for the kinetic energy of neutrons in reactor physics?

- When $v \ll c$ then the kinetic energy of the particle can be approximated by

$$E = \frac{1}{2} m_0 v^2$$

  which is the classical mechanics formula for kinetic energy.
- As a practical matter, the above classical mechanics approximation is accurate enough for most purposes provided that

$$E < 0.02 E_{rest}$$

- **The rest mass of the neutron is almost 1000 MeV, and** $0.02 E_{rest} = 20$ **MeV.**
- **Neutrons in current nuclear reactors rarely have kinetic energy above 20 MeV; therefore, it is permissible in all nuclear engineering problems to use the classical mechanics formula for the kinetic energy of neutrons.**

Excited states of nuclei

What is the origin of γ photons?

- The nucleons in nuclei may, similarly to electrons, be in ground or excited states.
- Nuclei in excited states may decay to lower states by emitting a photon with the energy equal to the difference between the energies of the initial and final state.
- Such high-energy photons are called γ-rays.

What is the typical energy difference between energy states of a nucleus?

The energies of the excited states and the energies between states are considerably greater for nuclei than for atoms, typically $\sim$ MeV.

Is there another way how a nucleus may de-excite?

An excited nucleus may also lose its excitation energy by **internal conversion** where the energy is transferred into the kinetic energy of the innermost atomic electron which is then ejected from the atom.

Nuclear stability and radioactive decay

Why there are more neutrons than protons in stable nuclides with Z greater than about 20?

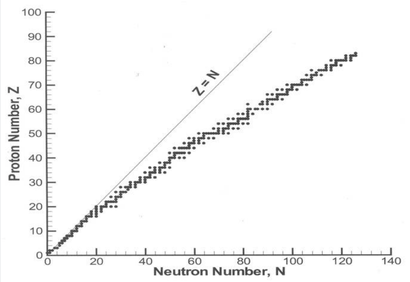


Figure 2: Chart of nuclides

- The extra neutrons are necessary for the stability of heavier nuclei. The neutrons act like glue, holding nucleus together, compensating for the repulsive electrical forces between positively charge protons.
- If there are too many or too few neutrons for a given number of protons, the resulting nucleus is not stable and undergoes a radioactive decay.

In which way a nucleus that lacks neutrons decay?

- Nuclei that are lacking neutrons undergo β^+ decay.
- In this process, one of the protons in the nucleus is transformed into a neutron, and a positron and a neutrino are emitted.
- Such a reaction can be e.g.:

$$^{15}O \xrightarrow{\beta^+} {}^{15}N + \nu$$

where β^+ denotes the emitted positron and ν is the neutrino.

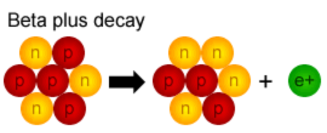


Figure 3: β^+ decay

In which way a nucleus that lacks protons decay?

- Some nuclei that are excessively neutron-rich decay by β^- decay, emitting an electron and an antineutrino, e.g.

$$^{19}O \xrightarrow{\beta^-} {}^{19}F + \bar{\nu}$$

- In this case, a neutron changes into a proton and the atomic number increases by one.
- The atomic mass number remains the same after both β^+ and β^- decays.
- Frequently, the *daughter nucleus*, the nucleus formed in β decay, is also unstable and undergoes β decay.

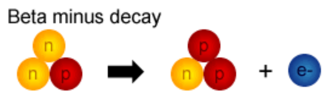


Figure 4: β^- decay

Describe the nuclear decay known as the "electron capture"

- A nucleus that is lacking in neutrons can also **increase its neutron number** by electron capture.
- In this process, an atomic **electron interacts with one of the protons** in the nucleus, and a **neutron is formed** of the union.
- A vacancy is left in the electron cloud, which is filled by another electron, which in turn leads to the **emission of γ-rays**.

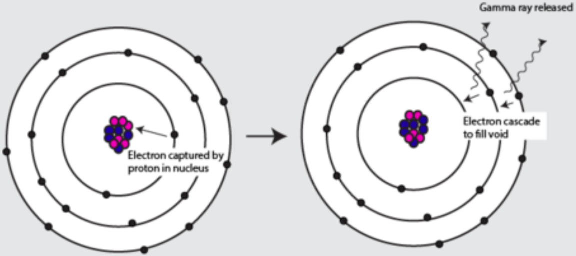


Figure 5: Electron capture

Nuclear decay

In which way uranium decays?

- An unstable nucleus may also undergo a radioactive decay by the emission of an α-particle (the nucleus of ^{4}He, having two protons and two neutrons).
- For instance, the α-decay of uranium-238:

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$$

- Decay by α-decay is rare in nuclides lighter than lead, but it is common for heavier nuclei.
- Uranium-235 has a half-life of about 703.8 million years.
- Uranium-238 has a half-life of about 4.51 billion years.
- α-particles are emitted in a discrete energy spectrum.

Isomeric states

What is an isomeric state of a nucleus?

- The nucleus formed as a result of β decays, electron capture or α-decay is often left in an **excited state**.
- The excited daughter nucleus usually decays by the emission of one of more γ-rays.
- Most nuclei in excited states decay by the emission of γ-rays immediately after these states are formed.
- However, some states appear to be semi-stable. Such long-lived states are called **isomeric states** of the nuclei in question. The decay of such a state is called an **isomeric transition** (IT).
- An example: Protactinium-234m, ${}^{234\mathrm{m}}_{91}\mathrm{Pa}$

What is the physical meaning of the decay constant λ?

- All decay processes are governed by a single fundamental law that states that - **the conditional probability per unit time that a nucleus, that has not decayed until time t, will decay** - is a constant independent of time t.
- This constant is called the **decay constant** and is denoted by λ.
- More accurately, we can say that the conditional probability that an atom, that has not decayed till time t, would decay during the time interval $[t, t + dt]$ is λdt.
- If at time t there are $n(t)$ atoms then the rate at which the atoms decay is $\lambda n(t)$ on average. This decay rate is called **activity** and is denoted by α.
- The SI unit of activity is the becquerel, Bq, which is equal exactly to one atom disintegration per second.

Decay in time

How does the activity of a sample change in time?

- Since $\lambda n(t)dt$ nuclei decay within dt, it follows that the decrease in the number of undecayed nuclei in the sample within dt is

$$dn(t) = -\lambda n(t)dt$$

- This equation can be integrated to give

$$n(t) = n_0 e^{-\lambda t}$$

  where n_0 is the number of atoms at $t = 0$.

- Multiplying the both sides of the above equation by λ gives the activity of the sample

$$\alpha(t) = \alpha_0 e^{-\lambda t}$$

  where α_0 is the activity at $t = 0$.

- **Thus, the activity decreases exponentially with time.**

What is half-life of a nuclide and how it relates to the decay constant λ?

The time during which the activity falls by a factor of two is known as the half-life and is given the symbol $T_{1/2}$.

By substituting $t = T_{1/2}$ and $\alpha(t) = \alpha_0/2$ into the equation

$$\alpha(t) = \alpha_0 e^{-\lambda t}$$

one gets

$$\alpha_0/2 = \alpha_0 e^{-\lambda T_{1/2}}$$

from where it follows that

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

Mass defect

What is the mass defect of a nucleus?

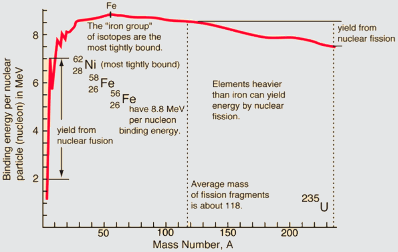
- Masses of nuclei are smaller than the sum of the masses of the neutrons and protons contained in them.
- This **mass defect** for an arbitrary nucleus is the difference

$$\Delta = Zm(p) + Nm(n) - m(a)$$

where $m(a)$ is the mass of the nucleus a.

What is the binding energy of a nucleus?

- When Δ is expressed in energy units, it is equal to the energy that is necessary to break the nucleus into its constituent nucleons.
- This energy is known as the **binding energy** (BE) of the system.
- When a nucleus is produced from a number of nucleons, Δ is equal to the energy released in the process.
- The total binding energy is an increasing function of the atomic mass number.
- However, it does not increase at a constant rate, which can be seen from a plot showing the average binding energy per nucleon, Δ/A versus A.



Binding energy

How does the stability of a nucleus relate to the binding energy?

- The nuclei with high binding energy per nucleon are more stable, and relatively larger amount of energy must be supplied to break them apart.
- However, when such nuclei are formed, a relatively large amount of energy is released.

Q-value of the nuclear reaction

What is the Q-value of the nuclear reaction?

- The conservation of energy law for nuclear reactions can be written for the reaction
$$a + b \to c + d$$
as
$$E_a + E_b + m_0(a)c^2 + m_0(b)c^2 = E_c + E_d + m_0(c)c^2 + m_0(d)c^2$$
where E_a, E_b, .. are the kinetic energies of particles a, b, ...
- The above equation can be rearranged into
$$(E_c + E_d) - (E_a + E_b) = \left[(m_0(a) + m_0(b)) - (m_0(c) + m_0(d))\right] c^2$$
where the lef-hand side represents the change in the kinetic energies of the particles before and after the reaction, and the right-hand side gives the difference in the rest-mass energies of the particles before and after the reaction.

Q-value of the reaction

The equation

$$(E_c + E_d) - (E_a + E_b) = [(m_0(a) + m_0(b)) - (m_0(c) + m_0(d))]c^2$$

can be, for practical purposes, written as

$$(E_c + E_d) - (E_a + E_b) = [(M_a + M_b) - (M_c + M_d)] \times 931\mathrm{MeV}$$

where M_a is the atomic weight of atom a, ..., and 931MeV is the energy corresponding to the mass of one atomic mass unit.

The right-hand side of the above equation is know as the Q-value of the reaction

$$Q = [(M_a + M_b) - (M_c + M_d)] \times 931\mathrm{MeV}$$

How does the Q-value relate to the stability of nuclei before and after the reaction?

- Since $m_0(a) = Zm(p) + Nm(n) - \Delta_a$, the Q-value,

  $$Q = [(m_0(a) + m_0(b)) - (m_0(c) + m_0(d))]c^2$$

  can be written as

  $$Q = [(\Delta_c + \Delta_d) - (\Delta_a + \Delta_b)]c^2$$

- The equation can also be expressed in terms of the binding energy as

  $$Q = [BE(c) + BE(d)] - [BE(a) + BE(b)]$$

  since $BE(a) = \Delta_a c^2$.

- So, whenever it is possible to produce a more stable configuration by combining two less stable nuclei, kinetic energy is released in the process.
- **Q is positive, i.e., the reaction is exothermic, when the total binding energy of the product nuclei is greater than the binding energy of the initial nuclei.**

Exothermic nuclear reactions

Can a fusion reaction be exothermic?

Fusion reactions, in which at least one heavier, more stable, nucleus is produced from **two lighter, less stable nuclei**, are exothermic.

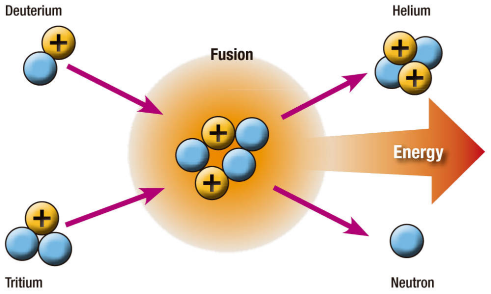


Figure 7: Fusion reaction

Exothermic nuclear reactions

Can a fission reaction be exothermic?

- A more stable configuration is formed when a **heavy nucleus** is split into two parts - the reaction is then exothermic.
- The binding energy per nucleon in ^{238}U, for instance, is 7.5 MeV; whereas it is about 8.4 MeV for A=238/2=119.
- Hence, when ^{238}U divides into two nuclei, there is a gain in the binding energy of about 0.9 MeV per nucleon, which amounts to a total energy release of about $238 \times 0.9 = 214$ MeV.
- **Nuclear fission** is the source of heat in nuclear reactors.

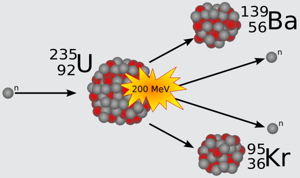


Figure 8: Fission reaction