Monte Carlo Methods and Simulations in Nuclear Technology

Home Assignment 01

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Prb: Find the analytical equation giving the probability density function (pdf) of the energy spectrum of fission neutrons of a fissile nuclide of your choice. Use any sources available to you, such as nuclear engineering textbooks (available online via KTH library) or online publications.

Having the pdf of the fission neutron energy, calculate deterministically (either analytically or by numerical integration):

- the most probable energy value,
- the mean value,
- the variance,
- the standard deviation of the energy of the fission neutrons.
- Include a plot of the energy spectrum in your presentation.

Probability Density Function

Probability density function defines the density of the probability that a continuous random variable will lie within a particular range of values. To determine this probability, we integrate the probability density function between two specified points.

Therefore, probability density function of the energy spectrum of fission neutrons of a fissile nuclide shows the continuous distribution of the energy spectrum of the nuclide over a particular range from which the most probable energy, mean value of it or the variance and standard deviation can be calculated.

Let's take distribution for thermal-neutron fission of ^{235}U, whose fission-neutron spectrum is often used as an approximation for other isotopes that are undergoing fission:

$$\chi(\bar{E}) = ae^{-\frac{\bar{E}}{b}}\sinh(\sqrt{c\bar{E}})$$

where,

$$a = 0.5535\,,\ b = 1.0347\ MeV, and\ c = 1.6214\ MeV^{-1}$$

Expectation value, E[X], is the mean of all possible values x weighted according to their probability. The expectation value of a continuous random variable is:

$$E[X] = \int_{-\infty}^{\infty} x f_x(x) dx$$

Therefore, the mean value of the energy of fission neutrons of a fissile nuclide from its energy spectrum is the expectation value of the energy spectrum provided via its density function.

$$E[\bar{E}] = \int_{0}^{\infty} \bar{E}\chi(\bar{E}) dx = \int_{0}^{\infty} a\bar{E}e^{-\frac{E}{b}} \sinh(\sqrt{c\bar{E}}) dx = 2.0\ MeV\ **$$

Moreover, the common measure of the spread is the variance Var[X], i.e., the expected quadratic deviation from the expectation value:

$$Var[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

To evaluate the variance of the energy distribution of the fission neutrons of a fissile nuclide:

$$Var[\bar{E}] = E[\bar{E}^2] - (E[\bar{E}])^2$$

** See Appendix for python (function) code used to find integration values

From the definition of expectation value mentioned previously,

$$E[\bar{E}^2] = \int_0^{\infty} a\bar{E}^2 e^{-\frac{E}{b}} \sinh(\sqrt{c\bar{E}})dx$$

Hence,

$$Var[\bar{E}] = E[\bar{E}^2] - (E[\bar{E}])^2 = 5.0 - (2.0)^2 = 1\ MeV^2 \ast\ast$$

In addition to that, it is useful to measure the spread with the same unit as that of the expectation value; therefore, the standard deviation σ_x has been introduced as:

$$\sigma_x = \sqrt{Var[X]}$$

Therefore, the standard deviation of the energy of the fission neutrons, $\sigma_{\bar{E}}$:

$$\sigma_{\bar{E}} = \sqrt{Var[\bar{E}]} = \sqrt{1} = 1\ MeV$$

Lastly, the most probable energy is the energy that most of the neutrons seem to possess.

$$\bar{E}_M = 0.71\ MeV$$

(Obtained from the peak of the graph which is highlighted at the bottom of the figure)

** See Appendix for python (function) code used to find integration values

Graph:

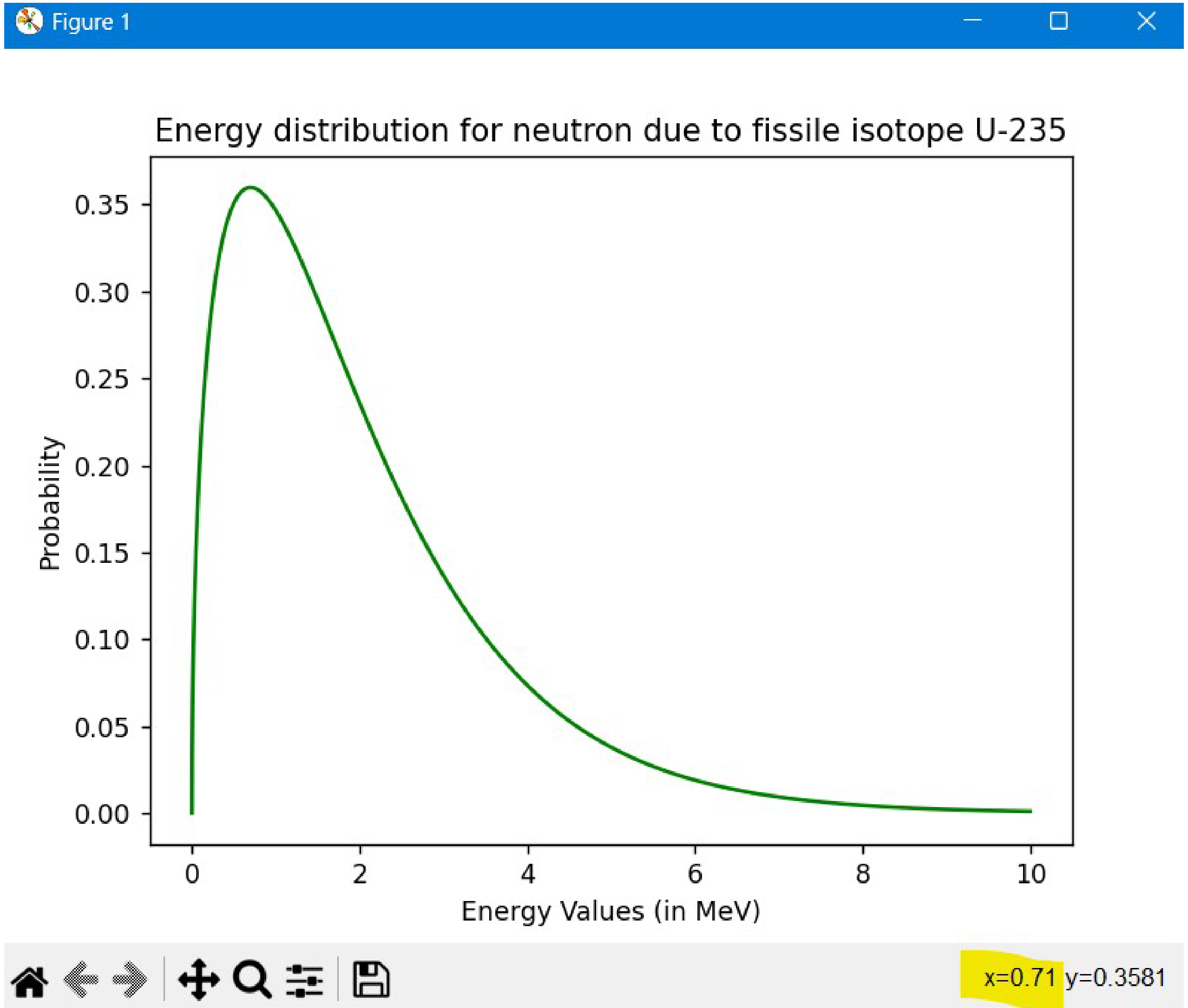

Figure 1
Energy distribution for neutron due to fissile isotope U-235
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Probability
0
2
4
6
8
10
Energy Values (in MeV)
x=0.71 y=0.3581

The vast of the prompt neutrons and even the delayed neutrons are born as fast neutrons (i.e., with kinetic energy higher than > 1 keV). But these two groups of fission neutrons have different energy spectra, contributing to the fission spectrum differently. Since more than 99 percent of the fission neutrons are prompt neutrons, it is obvious that they will dominate the entire spectrum.

Therefore, the fast neutron spectrum can be described by the following points:

- Almost all fission neutrons have energies between 0.1 MeV and 10 MeV.
- The mean neutron energy is about 2 MeV
- The most probable neutron energy is about 0.71 MeV.

On average, the neutrons released during fission with an average energy of 2MeV in a reactor undergo many collisions (inelastic or inelastic) before they are absorbed. As a result of these collisions, they lose energy, so the reactor spectrum is not identical to the fission spectrum, and it is always ‘softer’ than the fission spectrum. The fact is that the fission spectrum is part of the reactor spectrum.

Appendix:

Basically, the integration values are obtained by generating random numbers in the interval of the $\bar{E}$-values with a bounding parameter that the corresponding values of $\bar{E}^2 or \bar{E}\chi(\bar{E})$ residing under the curve are counted against the total number of those values:

$$\frac{points\ lying\ under\ the\ curve}{total\ points\ generated} = \frac{area\ under\ the\ curve\ i.e.the\ integration\ value}{area\ of\ the\ characteristic\ rectangle\ in\ which\ all\ the\ points\ lie}$$

```python
import math
import random
import numpy as np
import matplotlib.pyplot as plt

def func(x):
    a = 0.5535
    b = 1.0347
    c = 1.6214

    return a * np.exp(-x / b) * np.sinh(np.sqrt(c * x))

def RAND_MAX(size):
    """ Generates pseudo-numbers range and returns the max value
    Parameters
    ----------
    size : The length of random number list

    Returns
    -------
    Max of generated numbers
    """
    rands=[random.random() for i in range(size)]
    m=max(rands)

    return m


def MC_ID(a, b, c, d, N):
    area = 0
    count = 0
    x = 0
    y = 0
    RAND_MAX_i=RAND_MAX(N)
    for i in range(0, N + 1):
        x = a+(b-a)*(random.random()/float(RAND_MAX_i))
        y = c+(d-c)*(random.random()/float(RAND_MAX_i))
        if y<=func(x):
            count += 1
            area = (b-a)*(d-c)*(float(count)/float(N))
    return area

MC_ID_Value = MC_ID(0, 1e5, 0, 10, 1000000)

print(MC_ID_Value)
'''
#########################################################################

x2 = np.linspace(0, 10, 100000)

plt.plot(x2, func(x2), 'g')
plt.title("Energy distribution for neutron due to fissile isotope U-235")
plt.xlabel("Energy Values (in MeV)")
plt.ylabel("Probability")
plt.show()
'''
```