Lecture No 03

Title:
TH Design of Fuel Assemblies with Variable Thermal Conductivity

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Outline of the Lecture

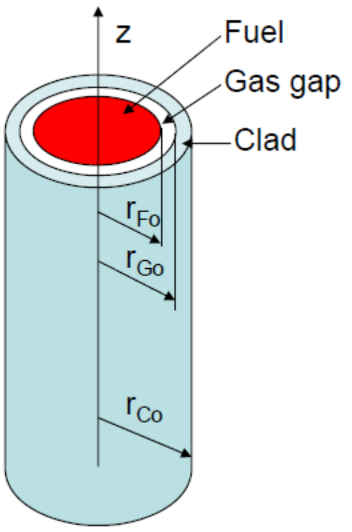
- Heat conduction and temperature distribution in fuel rods with temperature-dependent thermal conductivity
- Fuel restructuring
- Fuel-cladding gap behavior
- Cladding thermal analysis
- Coolant-to-cladding heat transfer with crud deposition

Fuel Thermal Analysis

- The following assumptions are made in the thermal analysis:
  - radial distribution of heat sources in a pin is uniform (no neutronic self-shielding within a pin)
  - heat conduction in the axial direction is small and can be neglected

- With theses assumptions, the conduction equation in a fuel pin is as follows:

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda_F \frac{dT_F(r)}{dr}\right) = -q'''(z)$$



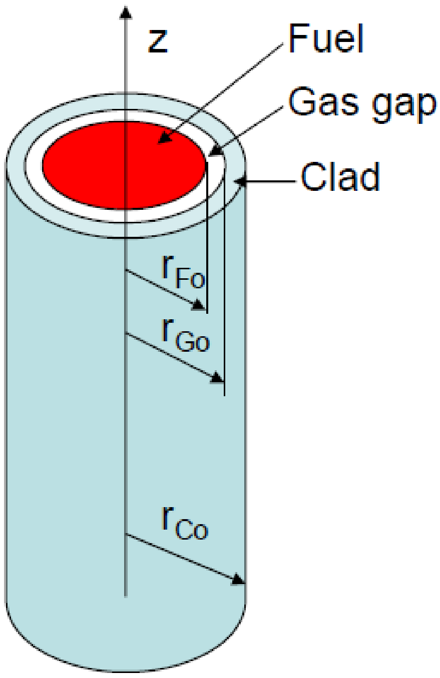
Fuel element

Fuel Thermal Analysis

- The following two boundary conditions are needed:

(1) $$\left.\frac{dT_F}{dr}\right|_{r=0} = 0$$ Symmetry at the centerline

(2) $$\left.T_F\right|_{r=r_{Fo}} = T_{Fo}$$ Constant temperature at pellet surface



Fuel element

- Integration of the equation yields

$$r\lambda_F \frac{dT_F(r)}{dr} + q'''(z)\frac{r^2}{2} = C_1$$

and from boundary condition (1), $C_1 = 0$.

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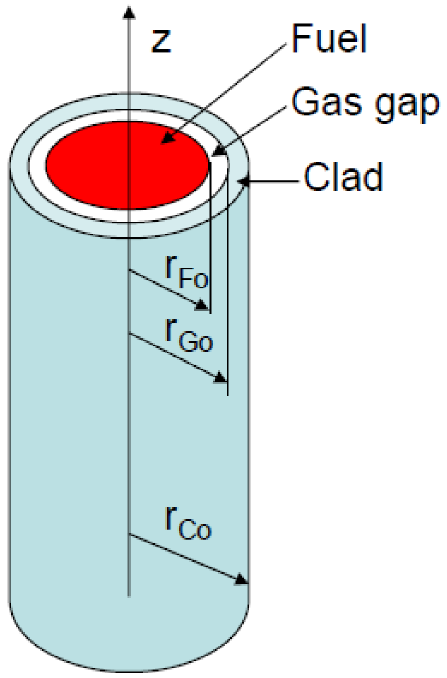
Fuel Thermal Analysis

- The second integration yields:

$$\int_{T(r)}^{T_{Fo}} \lambda_F dT_F + q'''(z)\int_{r}^{r_{Fo}} \frac{r}{2} dr = 0$$

- We can see that if the thermal conducti-vity as a function of temperature is known and the temperature at the pellet surface is given, temperature at any radius r can be found from the following equation:

$$\int_{T_{Fo}}^{T(r)} \lambda_F dT_F = \frac{q'''(z)}{4}\left(r_{Fo}^2 - r^2\right)$$



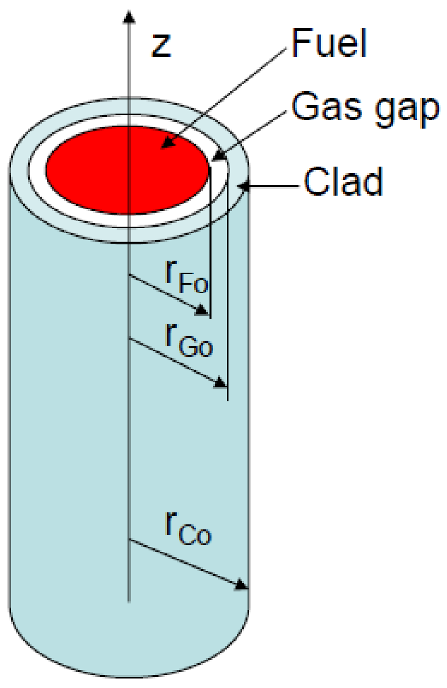
Fuel element

Fuel Thermal Analysis

- In particular, for r = 0 (at the centerline) we get:

$$\int_{T_{Fo}}^{T_{Fc}} \lambda_F dT_F = \frac{q'''(z) r_{Fo}^2}{4}$$

T_{Fc} – temperature at the centerline



Fuel element

- We can introduce into this equation the linear power q’, which is related to the power density q’’’ as follows:

$$q'(z) = q'''(z) \pi r_{Fo}^2$$

- Thus:

$$q'(z) = 4\pi \int_{T_{Fo}}^{T_{Fc}} \lambda_F dT_F$$

Fuel Thermal Analysis

- We obtained a relationship between the linear power and the fuel maximum temperature (at centerline):

$$q'(z) = 4\pi \int_{T_{Fo}}^{T_{Fc}} \lambda_F \, dT_F$$

- However, to find the value of q', we need to perform the integration. For that, we need to know the function $\lambda_F(T_F)$

- For various types of fuels, this function is given in analytical form based on experimental data

Fuel Thermal Analysis

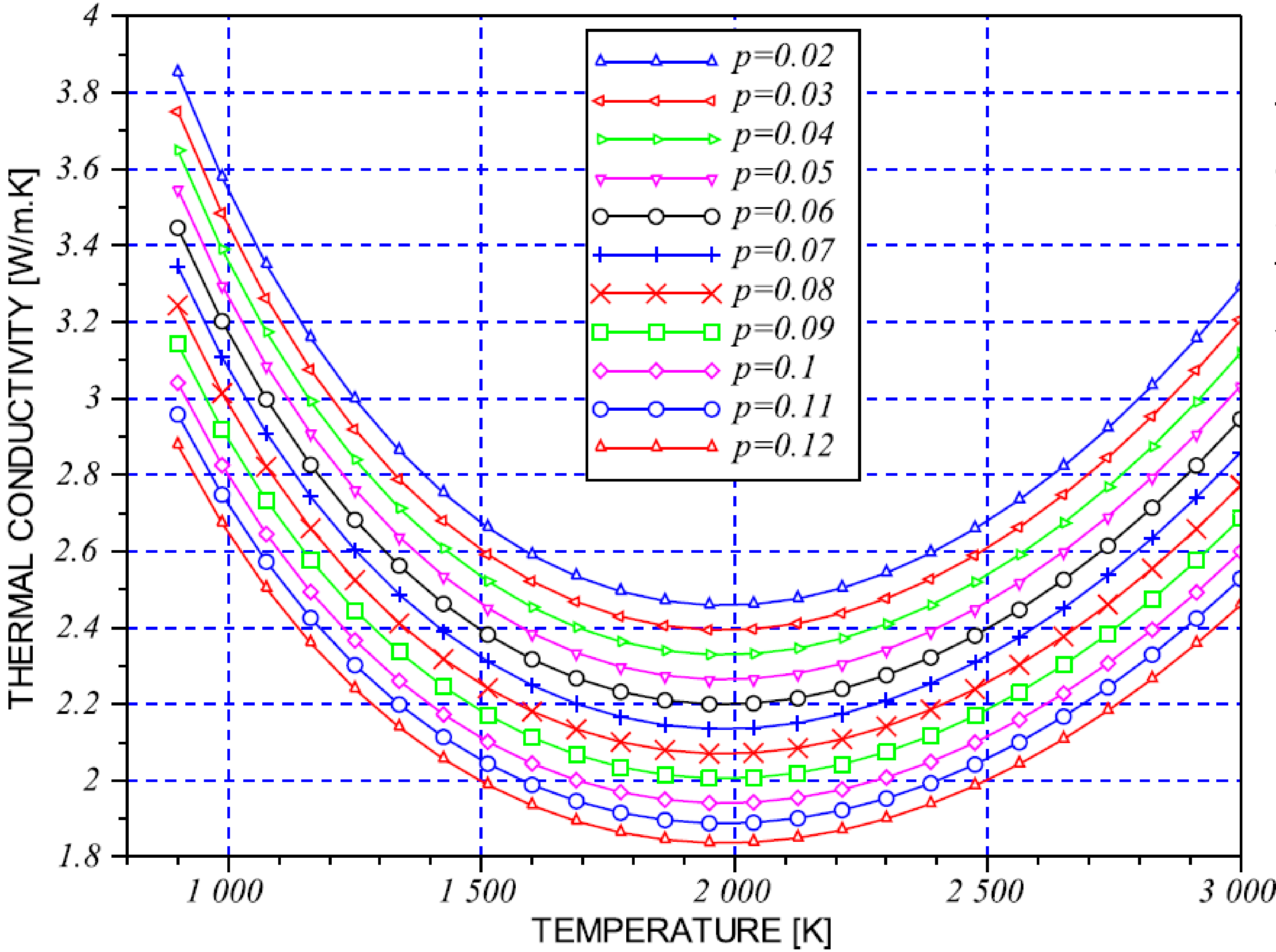
- For mixed oxide fuel (80% U, 20% Pu) at 95% theoretical density and O/M (oxigen/metal)=2.0, the fuel thermal conductivity can be given as (Washington, 1973)

$$\lambda_F(T) = \left(0.042 + 2.71\times10^{-4}T\right)^{-1} + 6.9\times10^{-11}T^3$$

where λ_F is in W/m·K and T in K. For porosity different from 5%, the thermal conductivity is found as:

$$\lambda_{Fp}(T) = \begin{cases} \lambda_F(T)\dfrac{1-2.5p}{0.875} & p \le 0.1 \\ \lambda_F(T)\dfrac{1-p}{0.875(1+2p)} & p > 0.1 \end{cases}$$

Fuel Thermal Analysis



Thermal conductivity of 80% U+20% Pu for various porosities p, with O/M=2.0

Fuel Thermal Analysis

- Typical design tasks for fuel pin can be as follows:

  (1) given linear power and pellet surface temperature, find the pellet maximum temperature at the centerline

  (2) knowing the pellet surface temperature and the fuel melting temperature, calculate the maximum allowed linear power before fuel starts melting

- To solve (1), we use a so-called **conductivity integral**, I_C:

$$I_C(T) \equiv \int_{T_{ref}}^{T} \lambda_F \, dT_F$$

T_{ref} – arbitrary reference temperature

Fuel Thermal Analysis

- Using the conductivity integral, we have:

$$q'(z) = 4\pi \int_{T_{Fo}}^{T_{Fc}} \lambda_F dT_F = 4\pi \left(\int_{T_{ref}}^{T_{Fc}} \lambda_F dT_F + \int_{T_{Fo}}^{T_{ref}} \lambda_F dT_F \right) =$$

$$4\pi \left(\int_{T_{ref}}^{T_{Fc}} \lambda_F dT_F - \int_{T_{ref}}^{T_{Fo}} \lambda_F dT_F \right) = 4\pi \left[I_C(T_{Fc}) - I_C(T_{Fo}) \right]$$

Fuel Thermal Analysis

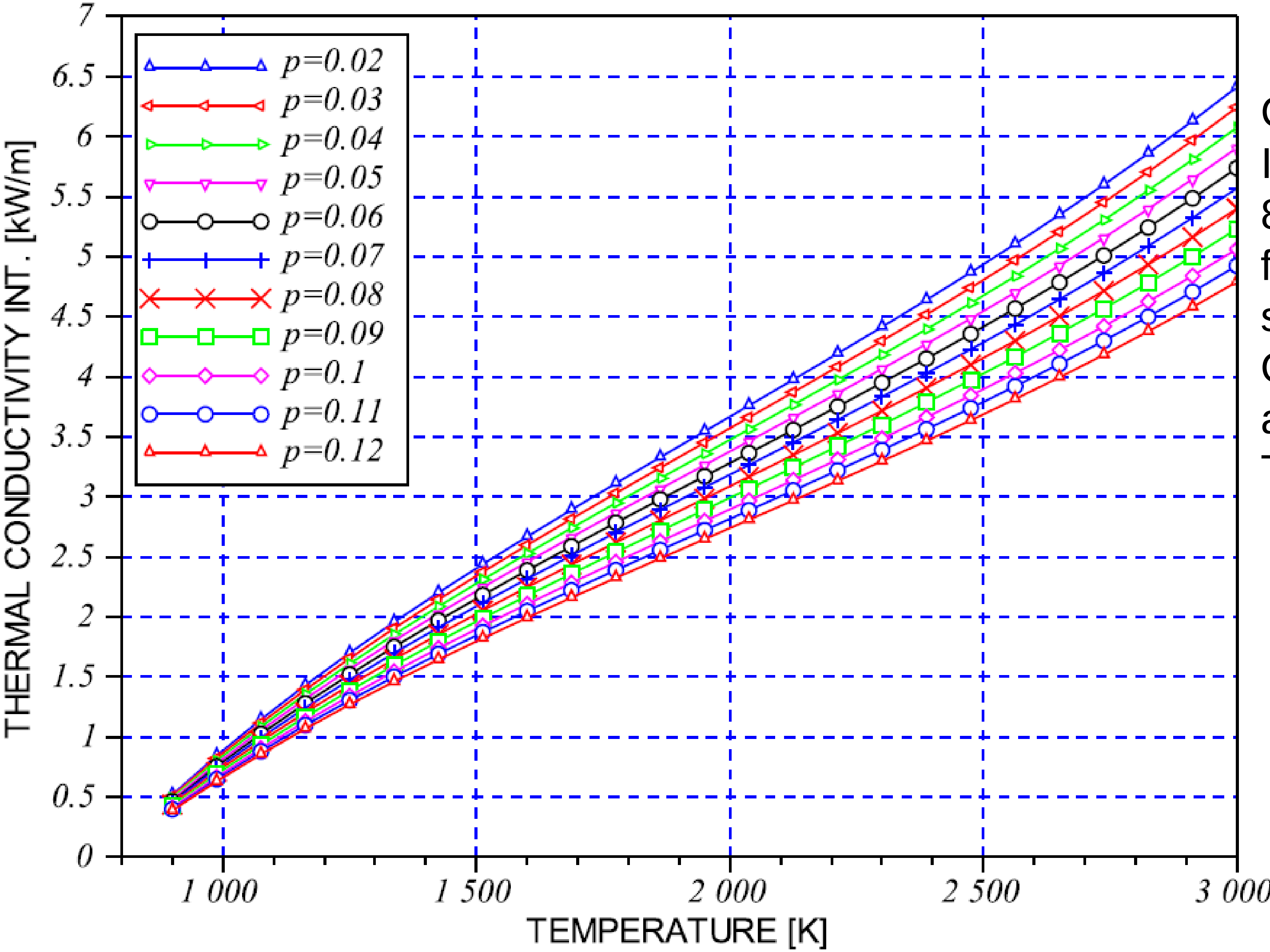
- The conductivity integral can be obtained in an analytical form (for 80%U+20%Pu with porosity 5%) as follows:

$$I_C(T) \equiv \int_{T_{ref}}^{T} \lambda_F dT_F = \frac{1}{2.71\times10^{-4}} \ln\left(0.042 + 2.71\times10^{-4} T\right) + \frac{6.9\times10^{-11}}{4} T^4$$

$$-\frac{1}{2.71\times10^{-4}} \ln\left(0.042 + 2.71\times10^{-4} T_{ref}\right) - \frac{6.9\times10^{-11}}{4} T_{ref}^4$$

This function is often represented in a graph (for various porosities)

Fuel Thermal Analysis



Conductivity Integral ***I*$_c$** of 80% U+20% Pu for various porosities p, with O/M=2.0 and assuming T_{ref} = 773 K

Fuel Thermal Analysis

- For solid UO_2 with 95% density the recommended equation for the thermal conductivity is

$$\lambda_F(T) = \frac{100}{7.5408 + 17.692t + 3.6142t^2} + \frac{6400}{t^{5/2}} \exp\left(-\frac{16.35}{t}\right)$$

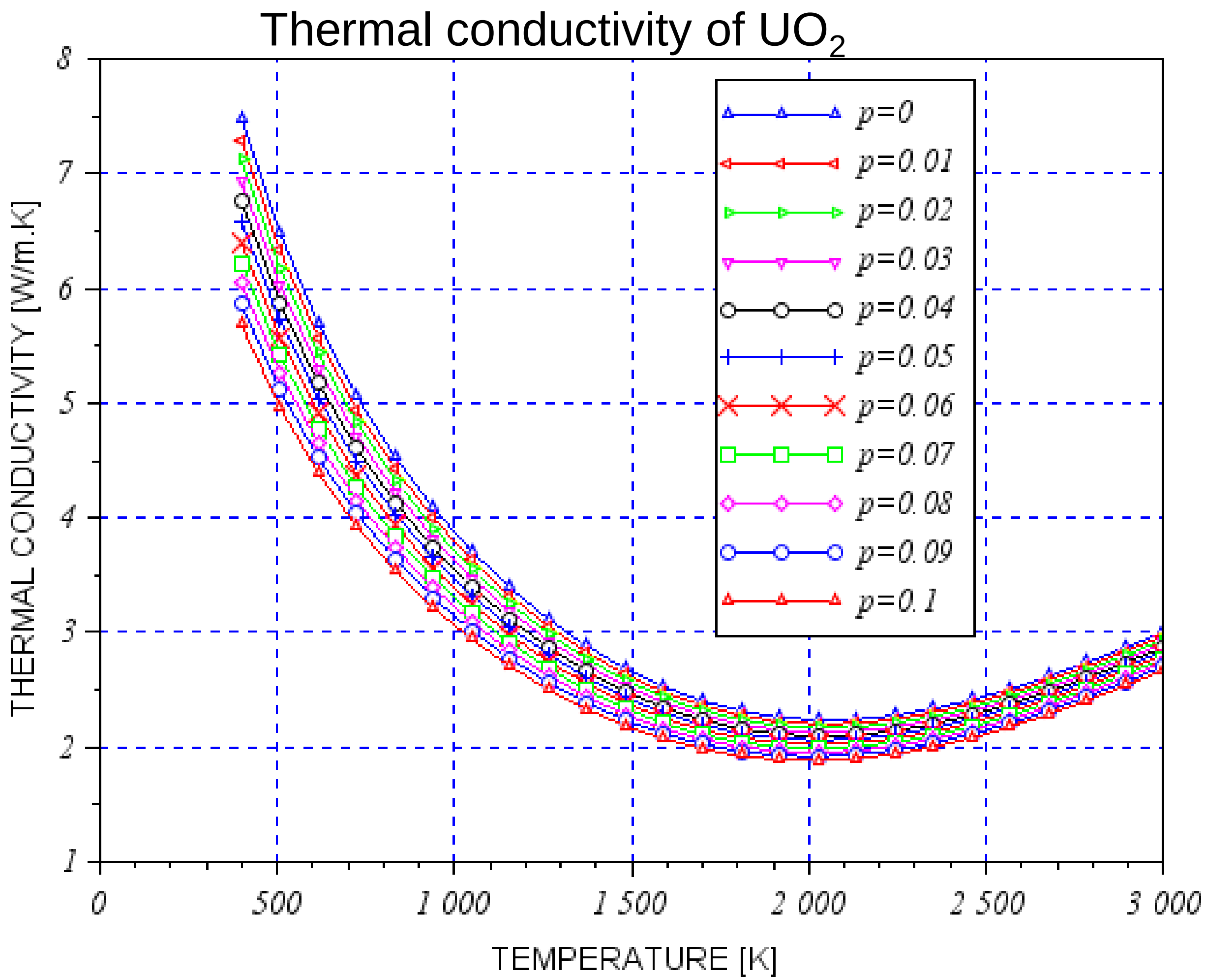
- where λ_F is in W/m·K , t = T/1000 and *T* is temperature in K. For porosity different from 5%, the thermal conductivity is found as:

$$\lambda_0 = \frac{\lambda_p}{1-(2.6-0.5t)p}$$

Here λ_0 is the thermal conductivity of fully dense UO_2 (that is p = 0) and λ_p is the thermal conductivity of UO_2 with porosity p.

$$\lambda_p = \lambda_0\left[1-(2.6-0.5t)\,p\right] = \lambda_F(T)\frac{1-(2.6-0.5t)\,p}{1-(2.6-0.5t)0.05}$$

Fuel Thermal Analysis



Fuel Thermal Analysis

Conductivity integral to melt (CIM) is defined as

$$CIM = \int_{773K}^{Tm} \lambda_F(T)dT$$

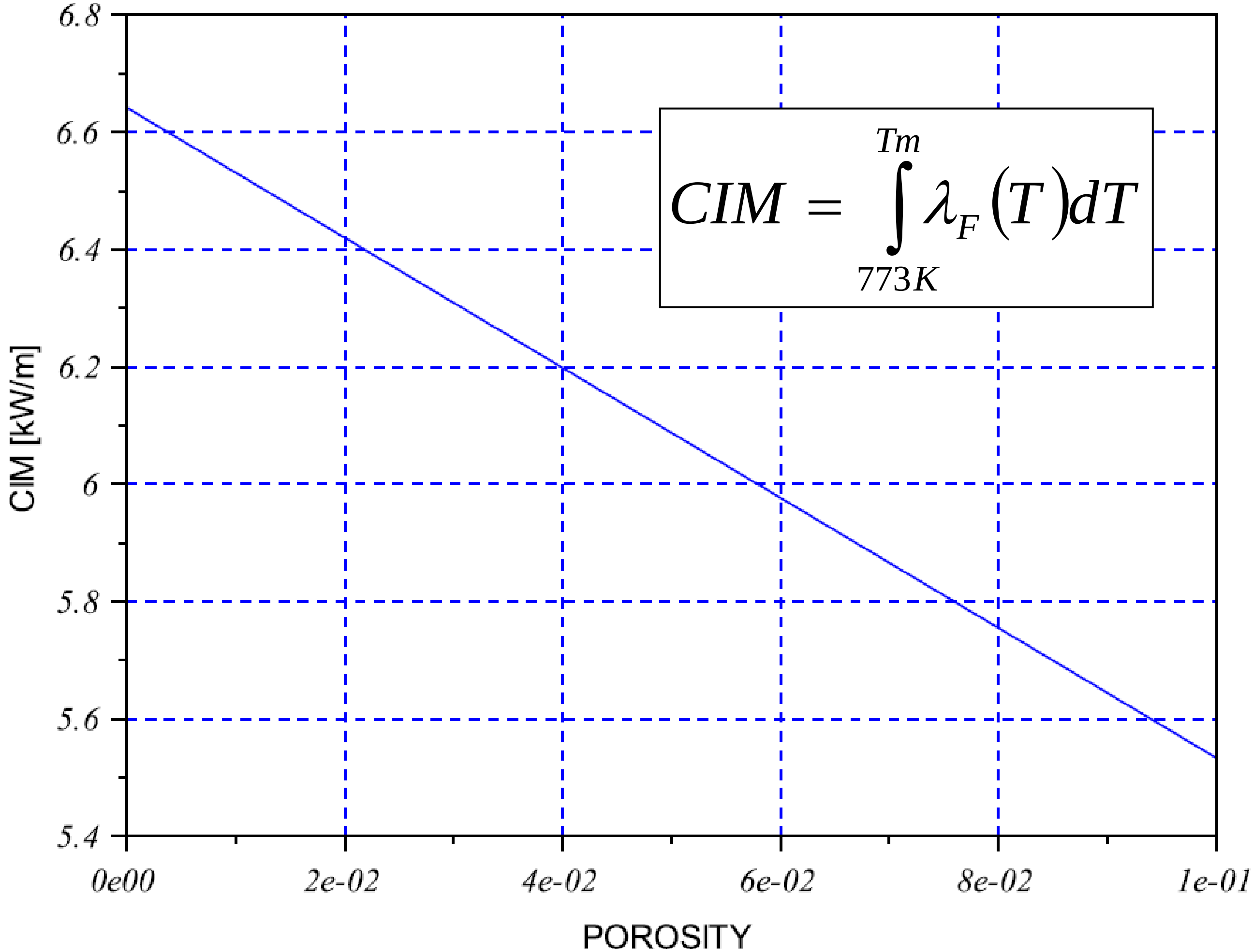
Here T_m is the melting temperature of UO_2 (3120 K ±30K)

The linear power density at which fuel will start melting is thus related to CIM as follows

$$q'_m(z) = 4\pi\int_{T_{Fo}}^{T_m} \lambda_F dT_F = 4\pi\left(\int_{773K}^{T_m} \lambda_F dT_F - \int_{773K}^{T_{Fo}} \lambda_F dT_F\right) =$$

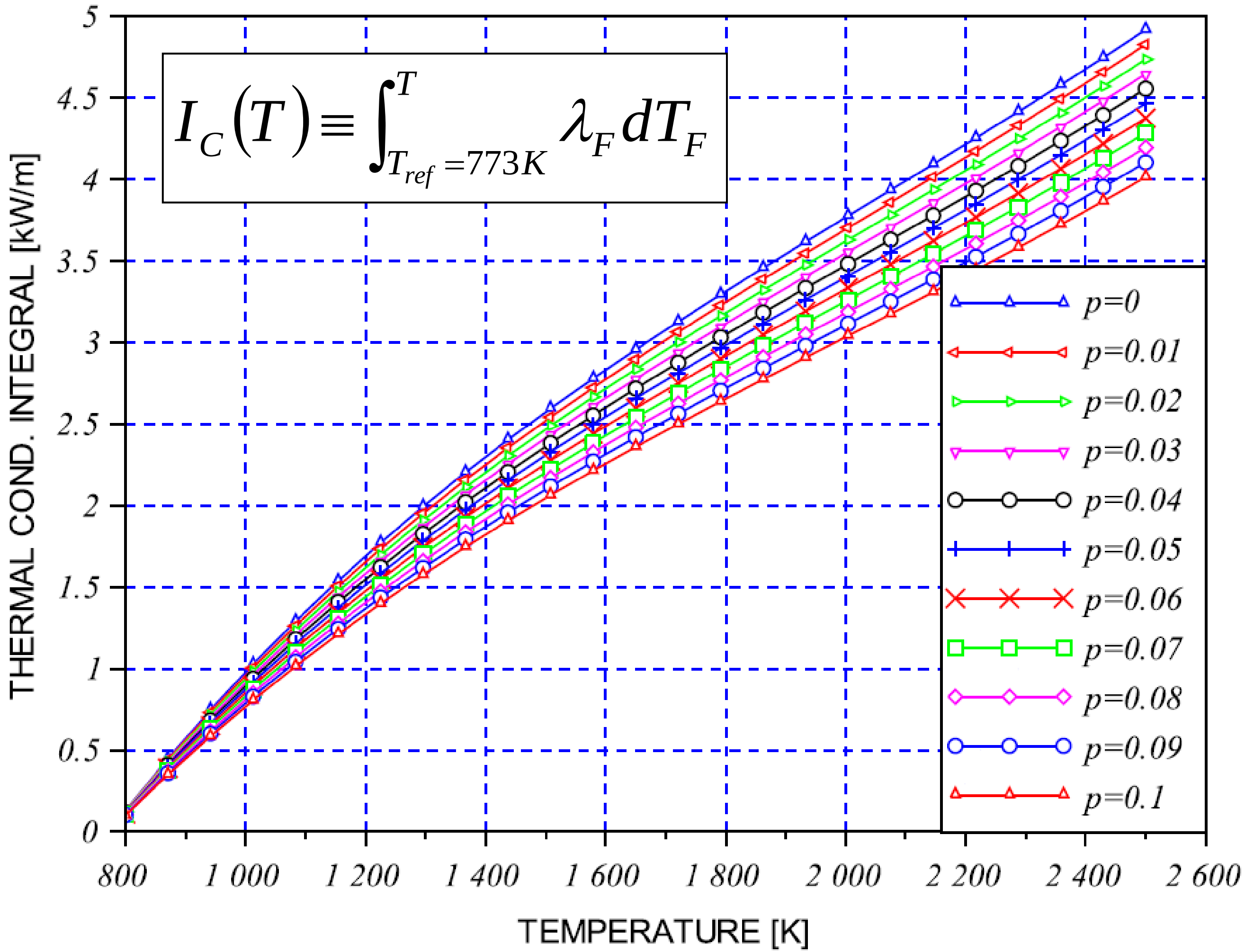
$$4\pi\left(CIM - \int_{773K}^{T_{Fo}} \lambda_F dT_F\right) = 4\pi\left[CIM - I_C(T_{Fo})\right]$$

CIM for UO_2

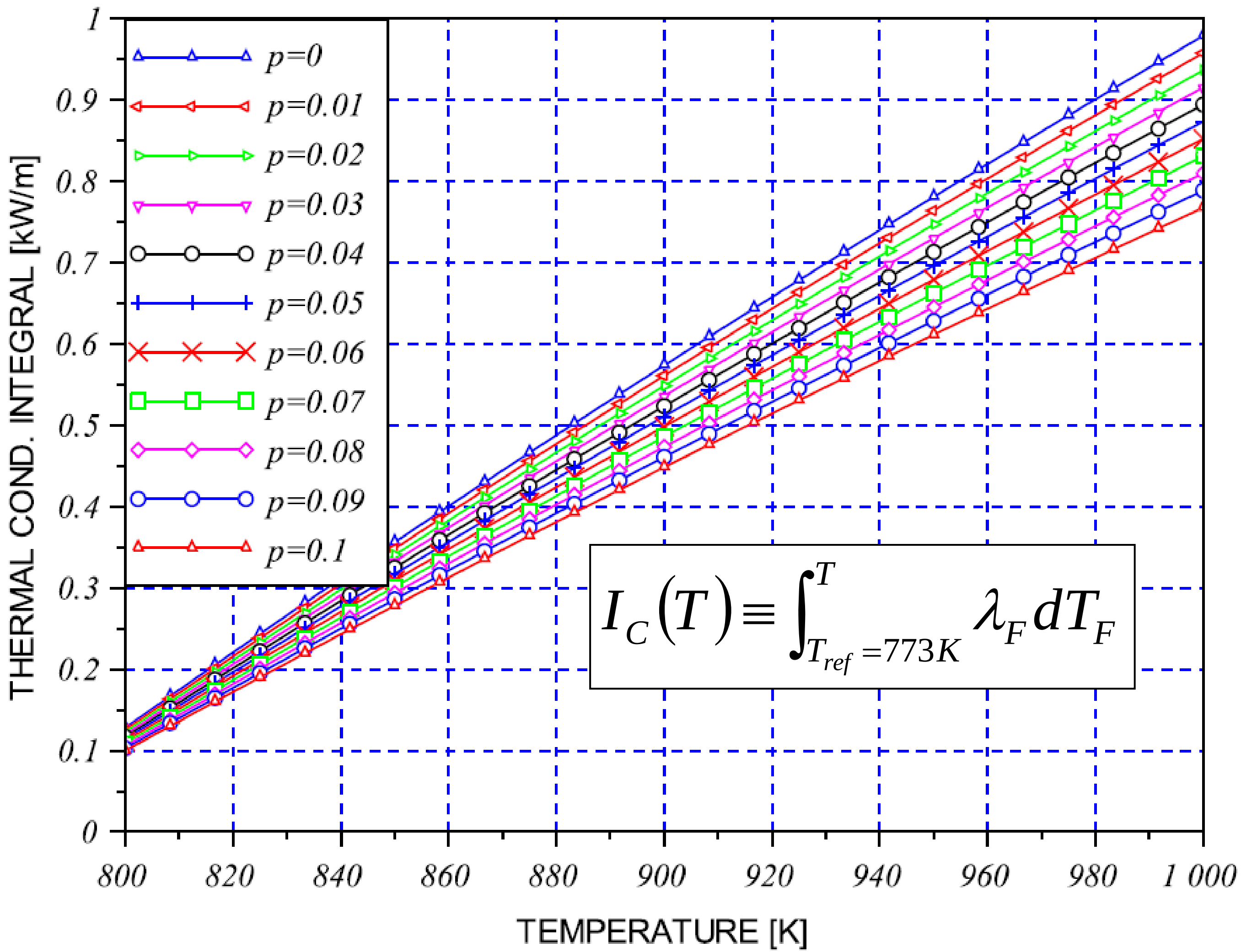


6.8
6.6
6.4
6.2
6
5.8
5.6
5.4
CIM [kW/m]
0e00
2e-02
4e-02
6e-02
8e-02
1e-01
POROSITY
$CIM = \int_{773K}^{Tm} \lambda_F(T)dT$

Conductivity Integral for UO_2



Conductivity Integral for UO_2



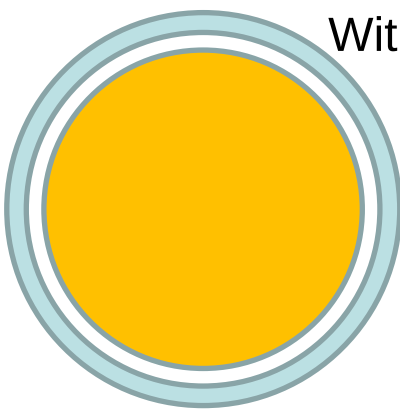
$$I_C(T) \equiv \int_{T_{ref}=773K}^{T} \lambda_F \, dT_F$$

Fuel Smeared Density

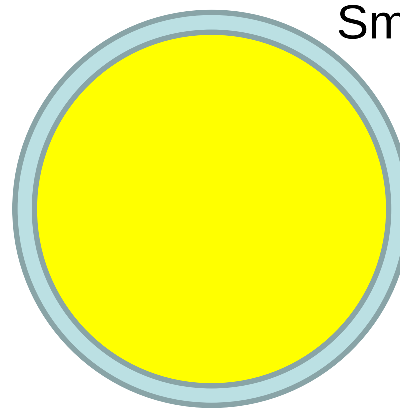
- A parameter called **smear density** is used in calculating **atom densities for the fuel**

- The smeared density is the density of the fuel if it were uniformly spread or smeared throughout the inside of the cladding

Fuel as manufactured
With a gas gap



Smeared fuel



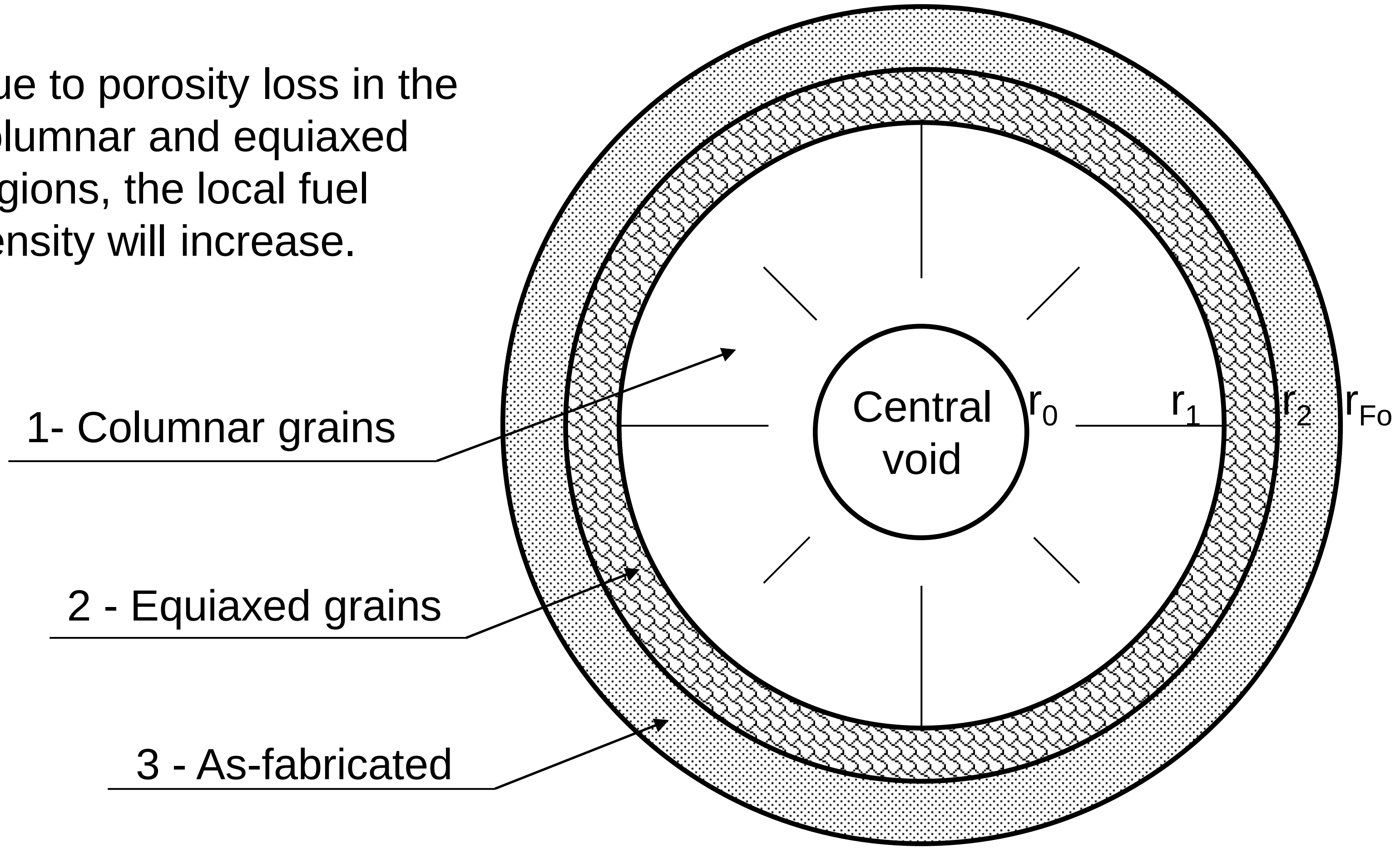
Effect of Fuel Restructuring

- Metal, carbide and nitride fuels do not restructure and the previously discussed procedure holds
- Mixed oxide fuels do restructure at high temperatures and this must be taken into account in the thermal analysis, especially for fast reactors, where fuel may have higher temperature near the center

- The most important effects are creation of central void, changes in thermal conductivity, density and volumetric heat generation rates in the columnar and equiaxed regions (see next slide)

Effect of Fuel Restructuring

Due to porosity loss in the columnar and equiaxed regions, the local fuel density will increase.



Effect of Fuel Restructuring

- Density change in regions 1 and 2 are causing changes of the volumetric power density

- Since the linear power is not affected by fuel restructuring, the volumetric power densities in regions 1 through 3 are as follows:

$$q_3''' = q''' = \frac{q'}{\pi r_{Fo}^2} \qquad \frac{q_1'''}{q'''} = \frac{\rho_1}{\rho_3} \Rightarrow q_1''' = \frac{q'}{\pi r_{Fo}^2}\frac{\rho_1}{\rho_3} \qquad q_2''' = \frac{q'}{\pi r_{Fo}^2}\frac{\rho_2}{\rho_3}$$

- With these new values, conductivity equation can be solved in each region

Effect of Fuel Restructuring

- The radii and the densities in the restructured regions are related according to the following mass conservation equation

$$\rho_1\left(r_1^2 - r_0^2\right) + \rho_2\left(r_2^2 - r_1^2\right) = \rho_3 r_2^2$$

- Using this, one can solve conductivity equations in all regions. In region 3 (unrestructured fuel) we get:

$$\lambda_{F3}\left.\frac{dT}{dr}\right|_{r_{Fo}} = -\left.\frac{q'''r}{2}\right|_{r_{Fo}} + \left.\frac{C}{r}\right|_{r_{Fo}} = -\frac{q'''r_{Fo}}{2} \Rightarrow C = 0 \quad \Rightarrow \quad \int_{T_{Fo}}^{T_2} \lambda_{F3}\, dT = \frac{q'}{4\pi}\left[1 - \left(\frac{r_2}{r_{Fo}}\right)^2\right]$$

here T_2 is the temperature at $r=r_2$

Effect of Fuel Restructuring

- In region 2 (equiaxed fuel) we get:

$$\int_{T_2}^{T_1} \lambda_{F2} dT = \frac{q'}{4\pi} \frac{\rho_2}{\rho_3} \left(\frac{r_2}{r_{Fo}} \right)^2 \left[1 - \left(\frac{r_1}{r_2} \right)^2 - 2\left(1 - \frac{\rho_3}{\rho_2} \right) \ln \frac{r_2}{r_1} \right]$$

here T_1 is the temperature at $r=r_1$

- In region 1 (columnar fuel) we have:

$$\int_{T_1}^{T_0} \lambda_{F1} dT = \frac{q'}{4\pi} \frac{\rho_1}{\rho_3} \left(\frac{r_1}{r_{Fo}} \right)^2 \left[1 - \left(\frac{r_0}{r_1} \right)^2 - 2\left(\frac{r_0}{r_1} \right)^2 \ln \frac{r_1}{r_0} \right]$$

here T_0 is the fuel temperature at $r=r_0$

Effect of Fuel Restructuring

- T_0 is the fuel maximum temperature, which should be found from the known linear power density and fuel outer surface temperature

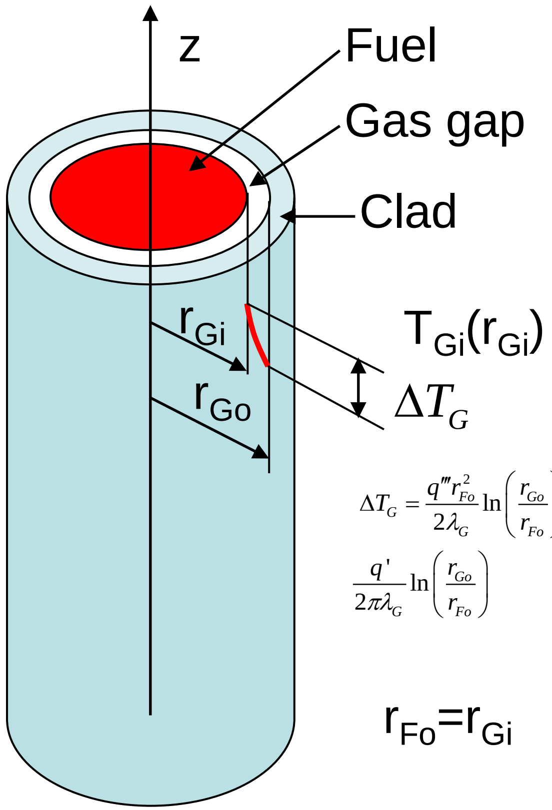
- The procedure is as follows:
  - knowing q' and T_{Fo}, find T_2 from $\int_{T_{Fo}}^{T_2} \lambda_{F3} dT = \frac{q'}{4\pi}\left[1-\left(\frac{r_2}{r_{Fo}}\right)^2\right]$ using graph
  - in a similar manner find T_1 from $\int_{T_2}^{T_1} \lambda_{F2} dT = \frac{q'}{4\pi}\frac{\rho_2}{\rho_3}\left(\frac{r_2}{r_{Fo}}\right)^2\left[1-\left(\frac{r_1}{r_2}\right)^2 - 2\left(1-\frac{\rho_3}{\rho_2}\right)\ln\frac{r_2}{r_1}\right]$
  - and finally T_0 from $\int_{T_1}^{T_0} \lambda_{F1} dT = \frac{q'}{4\pi}\frac{\rho_1}{\rho_3}\left(\frac{r_1}{r_{Fo}}\right)^2\left[1-\left(\frac{r_0}{r_1}\right)^2 - 2\left(\frac{r_0}{r_1}\right)^2\ln\frac{r_1}{r_0}\right]$

Fuel-Cladding Gap

- To determine fuel temperature at the center it is necessary to know the fuel pellet outer temperature T_{Fo}
- This temperature can be determined through a consideration of heat transfer through the fuel-cladding gap, through cladding and liquid film to the bulk coolant
- Of these three heat transfer barriers, the gap provides the greatest resistance to heat flow
- Initially, in unirradiated fuel, the gap is open and temperature drop through it is twice as big as two others

Fuel-Cladding Gap

- After short period of irradiation the gap closes due to fuel swelling
- Assuming first the gap as-fabricated and filled with a stationary gas mixt-ture, the temperature drop in the gap can be calculated from a solution of the conduction equation
- We then assume that the thermal conductivity λ_G of gas mixture is constant and known



$$\Delta T_G = \frac{q''' r_{Fo}^2}{2\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right) = \frac{q'}{2\pi\lambda_G} \ln\left(\frac{r_{Go}}{r_{Fo}}\right)$$

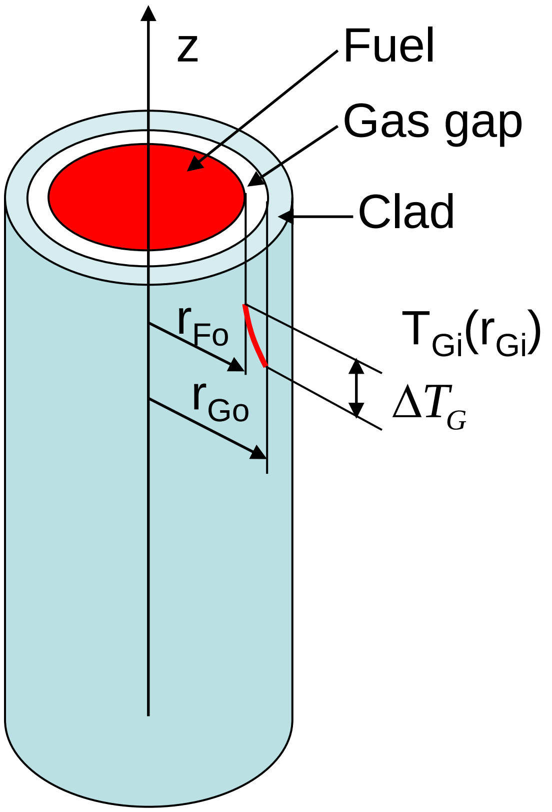
$r_{Fo} = r_{Gi}$

Fuel element

Fuel-Cladding Gap

- The temperature drop through the fuel-cladding gap is then

$$\Delta T_G = \frac{q'}{2\pi\lambda_G}\ln\left(\frac{r_{Go}}{r_{Fo}}\right)$$



Fuel element

Fuel-Cladding Gap

- In calculations, an equivalent **gap heat transfer coefficient (called gap conductance)** h_G is introduced:

$$q' = h_G 2\pi r_{Fo} \Delta T_G$$

- In this equation the curvature effects are neglected and the gap conductance for **open gap** is approximated as:

$$h_G \approx \frac{\lambda_G}{\delta_G}$$

- Here $\delta_G = r_{Go} - r_{Fo}$ is the gap thickness

Fuel-Cladding Gap

- More exact expressions for open gap conductance take into account the wall roughness

- Reported wall roughness for both cladding and fuel vary in a range from 10^{-4} to 10^{-2} mm

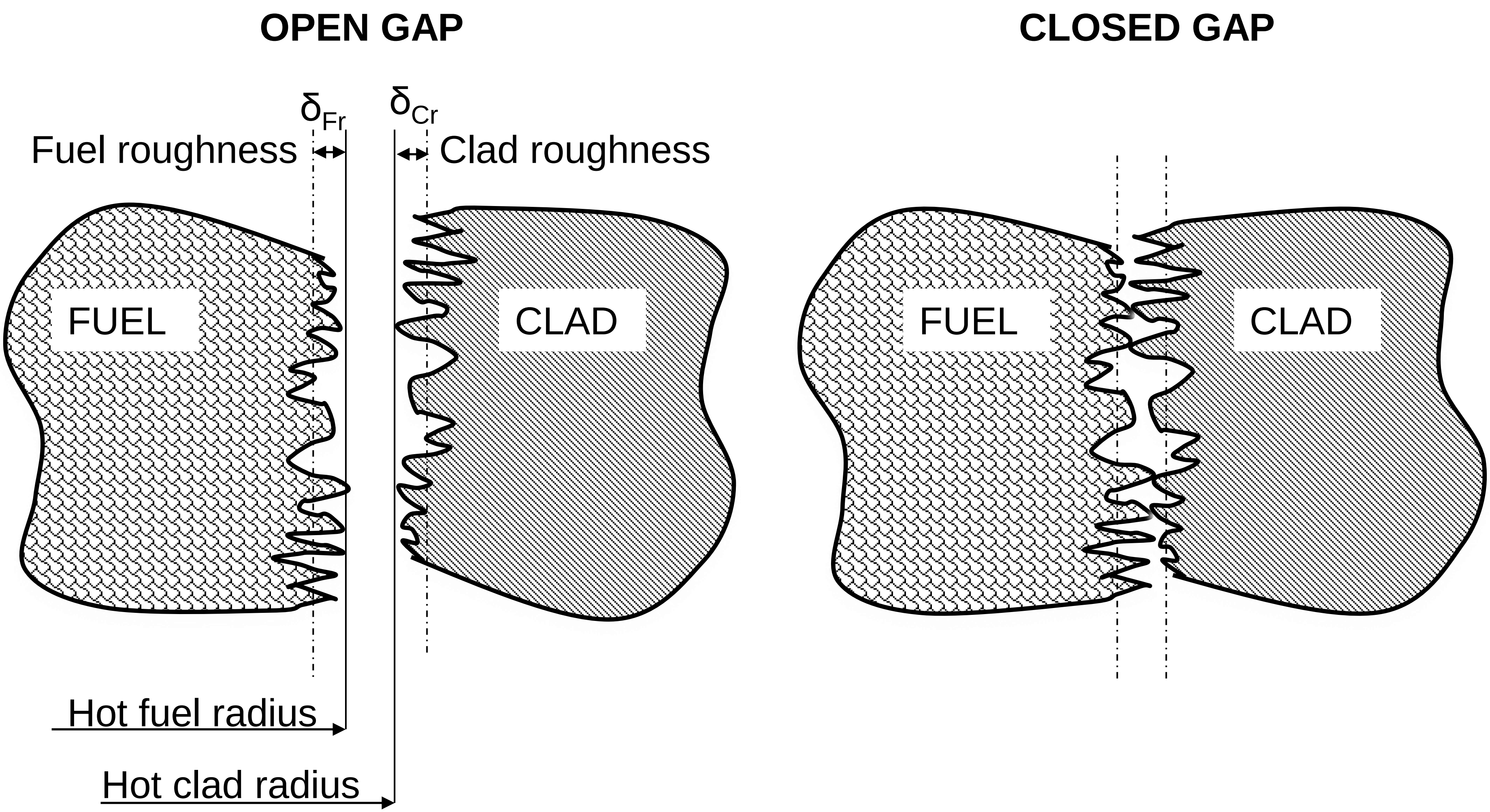
- This can be compared with the initial hot gap of about 0.1 mm

- For such conditions, the gap conductance is

$$h_G = \lambda_G / \left(\delta_G + \delta_{Fr} + \delta_{Cr} \right)$$

δ_{Fr} – fuel roughness, δ_{Cr} – clad roughness

Fuel-Cladding Gap



Fuel-Cladding Gap

- For closed gap, the heat transfer resistance is not reduced to zero due to roughness

- The gap conductance is now a sum of two parts:
  - direct contact between clad and fuel
  - gas layer due to roughness

- The gap htc is now:

$$h_G = \frac{C \cdot k_s \cdot P_{FC}}{H\sqrt{\delta_{EFFr}}} + \frac{\lambda_G}{\delta_{Fr} + \delta_{Cr}}$$

δ_{Fr} – fuel roughness, δ_{Cr} – clad roughness, C – empirical constant, k_s – effective conductivity fuel-clad, H – Mayer hardness of the softer material, δ_{EFFr} – effective roughness, P_{FC} – contact pressure

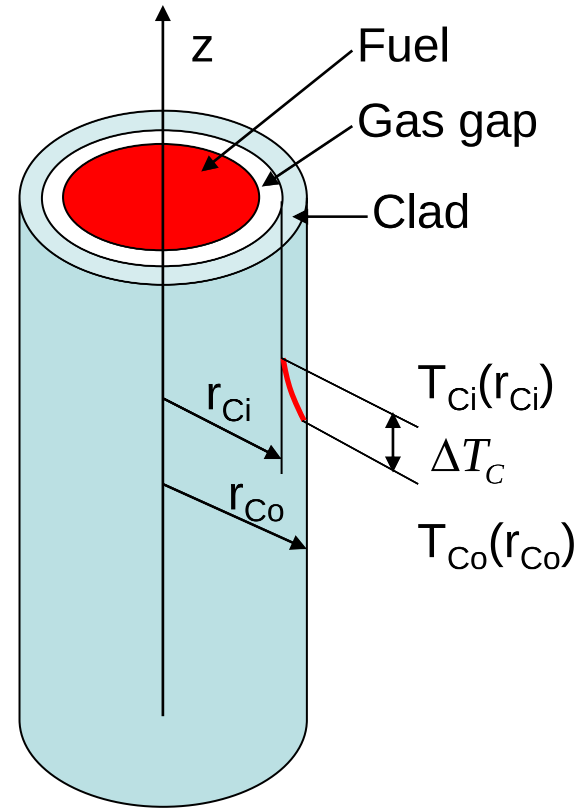
$$\delta_{EFFr} = \sqrt{\left(\delta_{Fr}^2 + \delta_{Cr}^2\right)/2}$$

Cladding Thermal Analysis

- Assuming a constant thermal conductivity of the clad material λ_C, the temperature drop in cladding is found as follows

$$\Delta T_C = \frac{q'}{2\pi\lambda_C} \ln\left(\frac{r_{Co}}{r_{Ci}}\right)$$

- Thermal conductivity of actual cladding materials is a function of temperature, however



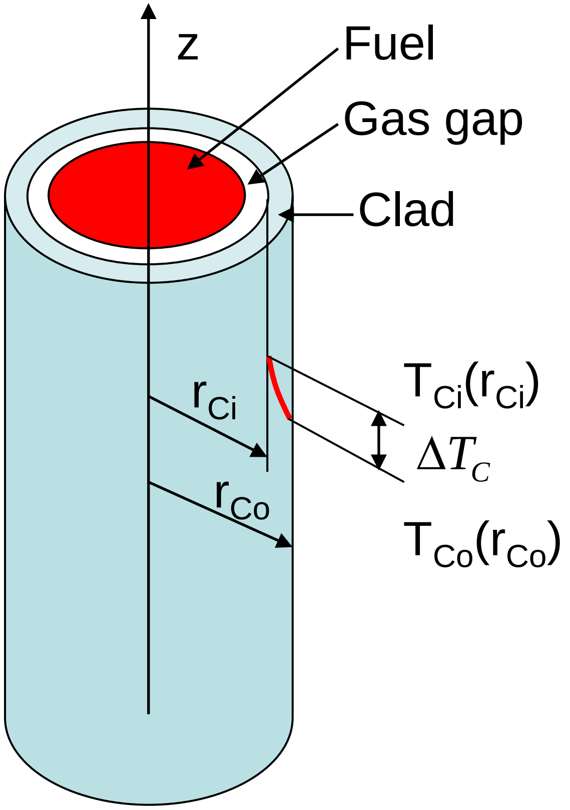
Fuel element

Cladding Thermal Analysis

- For Zircaloy-2 and Zircaloy-4 (α-phase), the thermal conductivity can be found as

$$\lambda_C = 12.6 + 0.0118T$$

  - here: T [°C] – temperature, λ_C [W/mK] – thermal conductivity
  - valid for 20 < T < 800 °C
  - uncertainty ±1.01 W/mK



Fuel element

Cladding Thermal Analysis

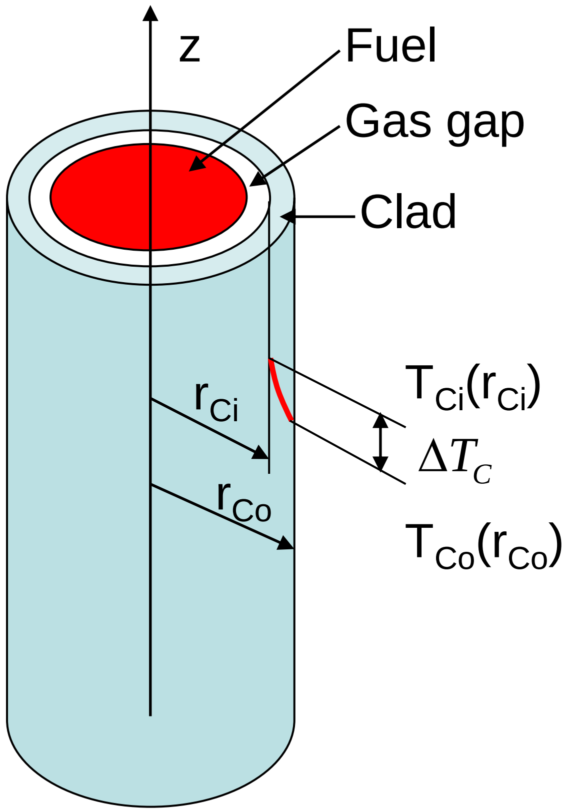
- The heat conduction equation in the clad with variable thermal conductivity is

$$\frac{1}{r}\frac{d}{dr}\left(r\lambda_C\frac{dT_C}{dr}\right)=0$$

  - with condition

$$-\lambda_C\frac{dT_C}{dr}\bigg|_{r=r_{Co}}=q''_{Co}$$

  - Here q''_{Co} is the heat flux at clad outer surface, which can be found as $q''_{Co}=q'/2\pi r_{Co}$



Fuel element

Cladding Thermal Analysis

- Integration of the conduction equation gives

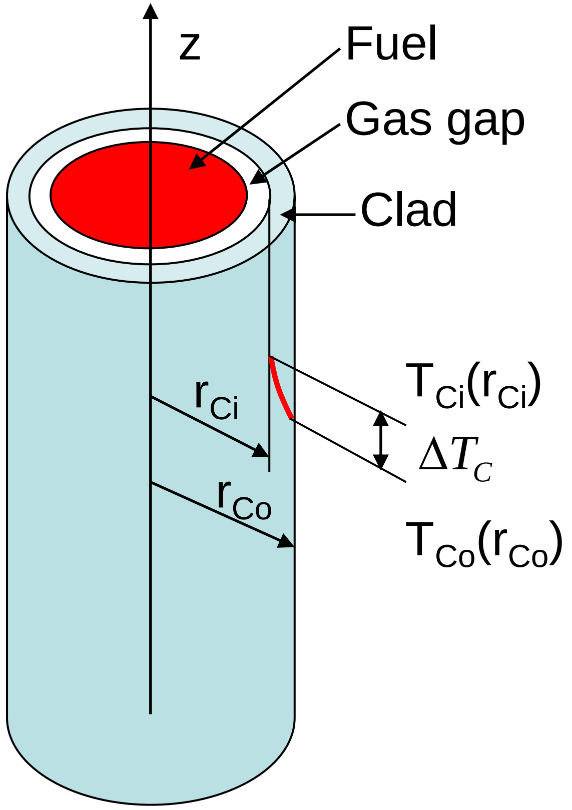
$$r\lambda_C \frac{dT_C}{dr} = C$$

- Applying the boundary condition

$$-r_{Co}\lambda_C \left.\frac{dT_C}{dr}\right|_{r=r_{Co}} = -C = \frac{q'}{2\pi}$$

- Thus we have the following equation

$$r\lambda_C \frac{dT_C}{dr} + \frac{q'}{2\pi} = 0$$



Fuel element

Cladding Thermal Analysis

- We integrate the equation over clad

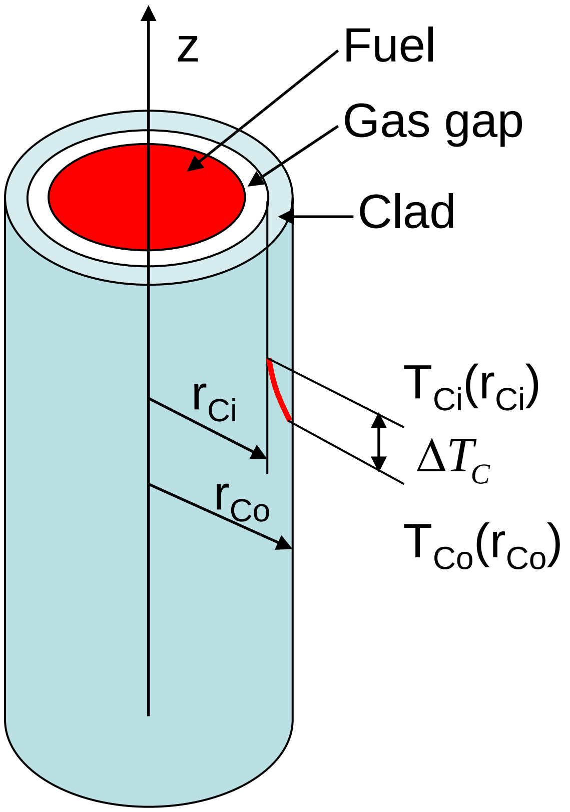
$$\int_{T_{Ci}}^{T_{Co}} \lambda_C dT_C + \frac{q'}{2\pi}\int_{r_{Ci}}^{r_{Co}} \frac{dr}{r} = 0$$

  - Assuming the thermal conductivity as

$$\lambda_C = a + bT$$

  - We get

$$a\left(T_{Co} - T_{Ci}\right) + \frac{b}{2}\left(T_{Co}^2 - T_{Ci}^2\right) + \frac{q'}{2\pi}\ln\frac{r_{Co}}{r_{Ci}} = 0$$

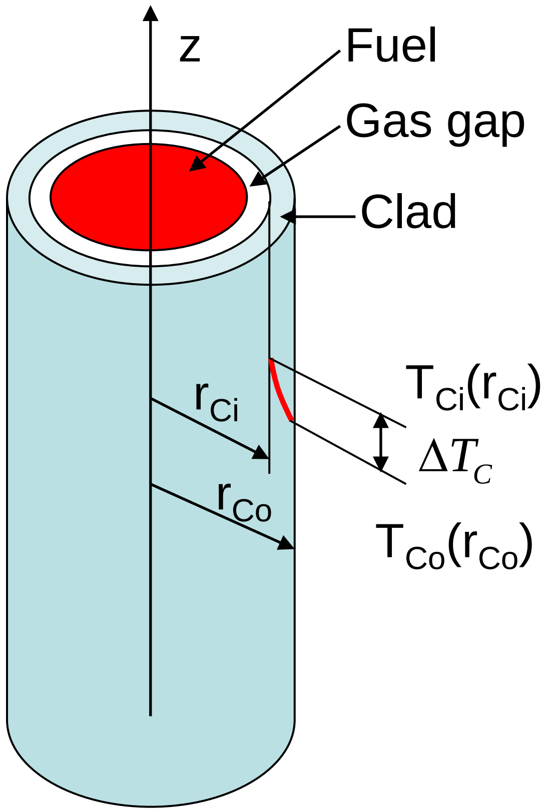


Fuel element

Cladding Thermal Analysis

- The clad inner temperature can be now found as

$$T_{Ci} = \frac{1}{b}\left(\sqrt{\left(a + bT_{Co}\right)^2 + \frac{bq'}{\pi}\ln\frac{r_{Co}}{r_{Ci}}} - a\right)$$



Fuel element

Exercise: show that for *b* = 0, the above solution gets the following form

Hint: use l'Hôspital's rule once taking b→0

$$T_{Ci} = T_{Co} + \frac{q'}{2\pi a}\ln\frac{r_{Co}}{r_{Ci}}$$

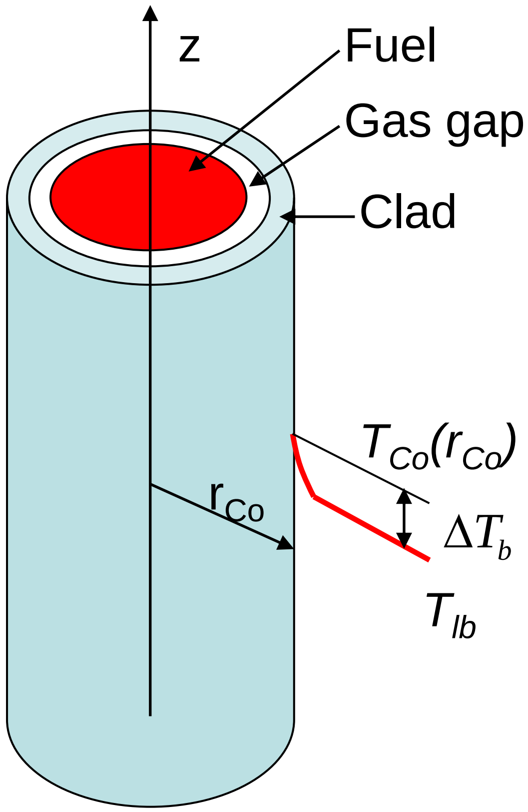
Coolant to Cladding Heat Transfer

- The temperature drop in the thermal boundary layer in coolant can be found from the Newton equation for the convective heat transfer:

$$q''\big|_{r_{Co}} = h \cdot \left(T_{Co} - T_b\right) = h \cdot \Delta T_b$$

since $$q''\big|_{r_{Co}} \cdot 2\pi r_{Co} \cdot dz = q' \cdot dz \Rightarrow q''\big|_{r_{Co}} = \frac{q'}{2\pi r_{Co}}$$

thus $$\Delta T_b = \frac{q'}{2\pi r_{Co} h}$$



Fuel element

Coolant to Cladding Heat Transfer

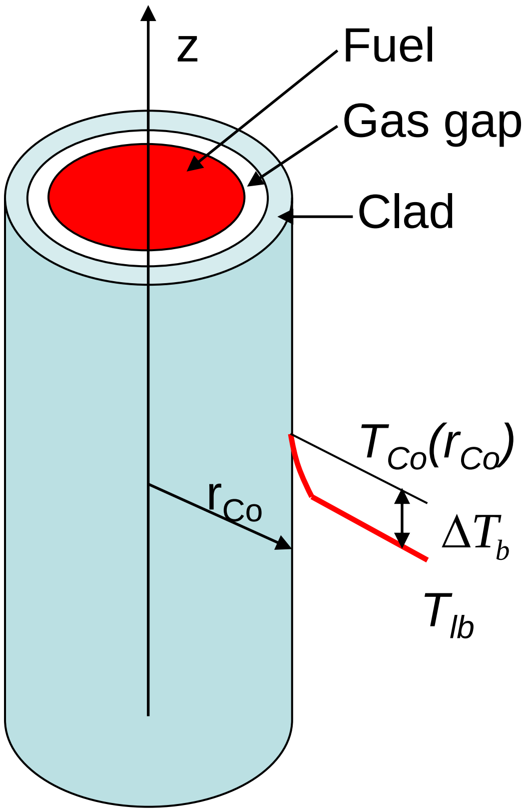
- The clad outer temperature T_{Co} at any axial location z is found as

$$T_{Co}(z) = T_{lb}(z) + \Delta T_b(z) + \Delta T_{ox}(z) + \Delta T_{crud}(z)$$

where the last term accounts for additional heat transfer resistance due to crud:

$$\Delta T_{crud} = \frac{q'}{2\pi\lambda_{cr}} \ln\left(\frac{r_{Co} + S + S_{cr}}{r_{Co} + S}\right)$$

S – the oxide layer thickness, S_{cr} – crud layer thickness (~2.4μm in PWR), λ_{cr} – crud thermal conductivity (= 0.5 W/mK for BWR and 0.865 W/mK for PWR)



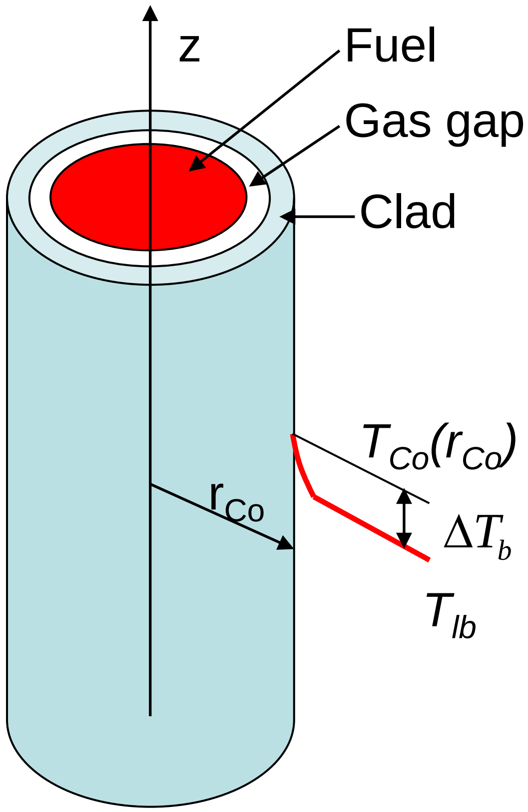
Fuel element

Coolant to Cladding Heat Transfer

- The temperature drop over the oxide layer is as follows

$$\Delta T_{ox} = \frac{q'}{2\pi\lambda_{ox}} \ln\left(\frac{r_{Co} + S}{r_{Co}}\right)$$

S – the oxide layer thickness, λ_{ox} – oxide thermal conductivity (=1.56 W/mK for BWR and 2.0 W/mK for PWR)



Fuel element

Oxide Layer

- PWR
  - Oxidation kinetics correlations are used to predict the oxide layer thickness
  - After some initial transition time, the oxide layer thickness is assumed to grow linearly with time as $$\frac{dS}{dt} = c\exp\left(-Q/RT\right)$$ where *c*, *Q* –constants, *T* – temp. *R* – gas constant
- BWR
  - Corrosion consists of athermal oxidation, which is linearly varying with the time, and thermal oxidation, which in addition depends on the metal/oxide surface temperature

Crud Deposition

- PWR
  - Crud sampling measurements performed in nuclear reactors indicate no correlation between crud deposition and the irradiation time. It can be assumed that independent of time crud thickness is about 2.4 μm.
- BWR
  - It is assumed that the crud deposition rate is constant:

$$\frac{dS_{cr}}{dt} = 2 \times 10^{-4} \left(\mu\mathrm{m/h} \right)$$