Measurements in which neutrons are scattered from nuclei show that, to a first approximation, the nucleus may be considered to be a sphere with a radius given by the following formula

$R = 1.25 \text{ fm } \times A^{1/3}$

Using this equation do the following:

(a) Estimate the radius in cm of the nucleus ^{238}U;

(b) Roughly, what volume fraction of the ^{238}U atom is taken up by the nucleus if a measurement gave the atomic radius of uranium atoms to be 175 pm?

Note that

- 1 fm = 10^{-15} m = 10^{-13} cm;
- 1 pm = 10^{-12} m = 10^{-10} cm.

Answers have a margin of error of 1%.

(a) Radius of ^{238}U, R =

cm

(b) Fraction of nucleus, f =

We have, $A = 238$; $R_{atm} = 175\text{pm}$; $R_{nuc} = 1.25 \times A^{(1/3)}$ fm.

The fraction in question is then

$$f = \frac{V_{\text{nuc}}}{V_{\text{atm}}} = \left(\frac{R_{\text{nuc}}}{R_{\text{atm}}}\right)^3 = \left(\frac{1.25 \times A^{1/3} \text{fm}}{175 \times 10^3 \text{fm}}\right)^3 = \left(\frac{1.25}{175}\right)^3 \frac{A}{10^9}$$

An electron starting from rest is accelerated across a potential difference of 5 million volts.

(a) What is its final kinetic energy in MeV?

(b) What is its total final energy in MeV?

(c) What is its final mass in grams?

(d) What is the mass ratio, i.e. final mass/ rest mass?

Given: potential difference, $U = 5\times10^6$ V.

Data that might be useful:

- Speed of light, c = 299 792 458 m/s (exact);
- Electron rest mass, $m_0 = 9.10938356\times10^{-31}$ kg;
- Electron charge, $e = 1.60217662\times10^{-19}$ C.

Answers have a margin of error of 1%.

(a) Final kinetic energy, $K =$

MeV;

(b) Total final energy, $E_{tot} =$

MeV;

(c) Final mass i grams, $m =$

g

(d) Mass ratio, $m/m_0 =$

Preamble

A huge potential difference of 5 million volts suggests that we cannot treat the electron under consideration in terms of classical mechanics rather relativistic mechanics is suitable here. Let m_0 be the electron mass at rest and m be the electron relativistic mass. They are related to each other as

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

The total energy of an electron (as well as any other physical object) at rest is

$$E_{\text{rest}} = m_0c^2$$

The total enery of a moving electron (as well as any other moving physical object) is

$$E_{\text{tot}} = mc^2$$

The relativistic kinetic energy is found as

$$K = E_{\text{tot}} - E_{\text{rest}}$$

Solution (b)

The work expended by the electric field while accelerating the electron is given by

$$W = e \times U$$

By definition, it is simply, W = 5 MeV. Clearly, the total electron energy becomes

$$E_{\text{tot}} = E_{\text{rest}} + W$$

Solution (a)

The kinetic energy becomes

$$K = E_{\text{tot}} - E_{\text{rest}} = W$$

Solution (c)

The mass of the moving electron is found by

$$E_{\text{tot}} = mc^2 \longrightarrow m = E_{\text{tot}}/c^2 = m_0 + W/c^2$$

Solution (d)

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-v^2/c^2}}$$

Calculate:

(a) The speed in m/s of a 4-MeV electron, one with a kinetic energy of 4 MeV;

(b) The speed of 4-MeV electron relative to the speed of light, i.e. the fraction v/c.

Given: K = 4 MeV (kinetic energy).

Data that might be useful:

- Speed of light, c = 299 792 458 m/s (exact);
- Electron rest mass, $m_0 = 9.10938356\times10^{-31}$ kg;
- Electron charge, $e = 1.60217662\times10^{-19}$ C.

Answers have a margin of error of 1%.

(a) The absolute speed is, v =

m/s

(b) The relative speed is, v/c =

Preamble

A very big kinetic energy of 4 Mev suggests that we cannot treat the electron under consideration in terms of classical mechanics rather relativistic mechanics is suitable here. Let m_0 be the electron mass at rest and m be the electron relativistic mass. They are related to each other as

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

The total energy of an electron (as well as any other physical object) at rest is

$$E_{\text{rest}} = m_0c^2$$

The total enery of a moving electron (as well as any other moving physical object) is

$$E_{\text{tot}} = mc^2$$

The relativistic kinetic energy is found as

$$K = E_{\text{tot}} - E_{\text{rest}}$$

Given

Kinetic energy, K = 4 Mev.

Solution

The relation between the rest and relativistic masses reads as

$$\frac{m_0}{m} = \frac{m_0c^2}{mc^2} = \frac{E_{\text{rest}}}{E_{\text{tot}}} = \sqrt{1-v^2/c^2}$$

Squaring and rearranging the last equation finally gives

$$v = c\sqrt{1-E_{\text{rest}}^2/E_{\text{tot}}^2}$$

The total energy is easily found as

$$E_{\text{tot}} = E_{\text{rest}} + K$$

A glass of water is known to contain 6.6×10^{24} atoms of hydrogen. The inside diameter is 7.5 cm. How high does the water stand in the glass?

Given

- Number of hydrogen atoms, $N_H = 6.6\times10^{24}$;
- Diameter, $D = 7.5$ cm.

Data that might be useful

- Nominal water density, $\rho = 1$ g/cm^3;
- Molecular weight of water, $M = 18.015694$ g/mol;
- Avogado number, $N_A = 6.02214076\times10^{23}$ (exact).

Answers have a margin of error of 1%.

The height is, $H =$

If V is the water volume the beaker, it is related to the diameter D and the height H as

$$V = \frac{\pi D^2 H}{4} \longrightarrow H = \frac{4V}{\pi D^2}$$

In turn, the water volume V expresses through the water mass m in the beaker as

$$V = m/\rho$$

Next, the water mass m is given by

$$m = n \cdot M$$

Here, n is the number of moles of water in the beaker, which can further be expressed through the total number of water molecules, N_{tot}, in the beaker as

$$n = N_{tot}/N_A$$

Finally, the number of water molecules, N_{tot}, is half the number of hydrogen atoms, N_H, due to the chemical formula of water, H_2O,

$$N_{tot} = N_H/2$$

Doing numerical calculations in the reverse order gives the water height H in the end.

Two beams of 0.9-eV neutrons intersect at an angle of 50°. The density of neutrons in both beams is 2.9×10^8 neutrons/cm^3.

(a) Calculate the intensity of each beam.

(b) What is the neutron flux where the two beams intersect?

Given

- Neutron kinetic energy, $E = 0.9$ eV;
- Neutron density, $n = 2.9 \times 10^8$ n/cm^3;
- Angle between the beams, $\theta = 50$ °;

Data that might be useful

- 1 eV = $1.602176565 \times 10^{-19}$ J (NIST);
- Neutron mass, $m_n = 1.67492749804 \times 10^{-27}$ kg (NIST);

Answers have a margin of error of 0.1%.

(a) Intensity, $I =$

n/(cm^{2}s)

(b) Neutron flux, $\phi =$

The intensity of a beam is defined by $I = n \cdot v$.

The neutron speed is found as

$$E = \frac{m_n v^2}{2} \longmapsto v = \sqrt{\frac{2E}{m_n}}$$

Solution (a)

The intensity of each beam is, $I_1 = I_2 = I = n \cdot v$

Solution (b)

The neutron flux at the point of the intersection is

$$\phi = I_1 + I_2 = 2I$$

Stainless steel, type 304 having a density of 7.76 g/cm^3, has been used in some reactors. The nominal composition by weight of this material is as follows: carbon, 0.08%; chromium, 24%; nickel, 13%; iron, the remainder. Calculate the macroscopic absorption cross-section of SS-304 at 0.0253 eV.

Given

- Density, $\rho = 7.76$ g/cm^3;
- Weight fraction of carbon, $w_C = 0.08\%$;
- Weight fraction of chromium, $w_{Cr} = 24\%$;
- Weight fraction of nickel, $w_{Ni} = 13\%$;
- Weight fraction of iron, $w_{Fe} = 100\% - w_C - w_{Cr} - w_{Ni}$.

Data that might be useful

- Avogadro's number, $N_A = 6.02214076 \times 10^{23}$ a/mol (exact);
- Carbon molar mass, $M_C = 12.01115$ g/mol (Table II.3);
- Chromium molar mass, $M_{Cr} = 51.996$ g/mol (Table II.3);
- Nickel molar mass, $M_{Ni} = 58.71$ g/mol (Table II.3);
- Iron molar mass, $M_{Fe} = 55.847$ g/mol (Table II.3);
- Microscopic absorption cross-sections at 0.0253 eV:
  - Carbon: $\sigma_{a,C} = 0.0034$ b (Table II.3);
  - Chromium, $\sigma_{a,Cr} = 3.1$ b (Table II.3);
  - Nickel, $\sigma_{a,Ni} = 4.43$ b (Table II.3);
  - Iron, $\sigma_{a,Fe} = 2.55$ b (Table II.3).

Answers have a margin of error of 0.1%.

The macroscopic absorption cross-section, $\Sigma_{a,SS} =$

Let a subscript i runs over the set of chemical elements

$i \in \{C, Cr, Ni, Fe\}$

Then the macroscopic absorption cross-section of stainless steel, SS, is given by

$$\Sigma_{a,SS} = \Sigma_{a,C} + \Sigma_{a,Cr} + \Sigma_{a,Ni} + \Sigma_{a,Fe} = \sum_i \Sigma_{a,i}$$

Individual cross-sections are found as

$$\Sigma_{a,i} = \sigma_{a,i} \times N_i \text{ here } N_i = \frac{\rho_i N_A}{M_i} \text{ and } \rho_i = w_i \rho.$$

There are no resonances in the total cross-section of ^{12}C from 0.01 eV to cover 1 MeV. If the radiative capture cross-section of this nuclide at 0.05 eV is 0.002405 b, what is the value of σ_γ at 10 eV?

Answers have a margin of error of 0.1%.

$\sigma_\gamma(E) =$

In the low energy region, σ_γ varies as $1/v$, i.e.

$$\sigma_\gamma(E) = \frac{C}{v} \longmapsto \frac{\sigma_\gamma(E)}{\sigma_\gamma(E_0)} = \frac{v_0}{v} = \sqrt{\frac{E_0}{E}}$$

Thus, we have

$$\sigma_\gamma(E) = \sigma_\gamma(E_0)\sqrt{E_0/E}$$

What is the probability that a neutron can move one mean free path without interacting in a medium?

The probability that a neutron travels distance x in a homogeneous medium without having a collision is

$$P = e^{-\Sigma_t \cdot x}$$

The mean free path is given by

$$\lambda_t = 1/\Sigma_t$$

Setting $x = \lambda_t$, we arrive at

$$P = e^{-1}$$

What value of the breeding gain is necessary for a fast breeder operating on the ^{238}U–^{239}Pu cycle to have an exponential doubling time of 10 years if the specific power for this type of reactor is 0.57 megawatts per kilogram of ^{239}Pu?

Given

- Specific power, $\beta = 0.57$ MW/kgPu;
- Exponential doubling time, $t_{De} = 10$ yr.

Data that might be useful

- Julian year, 1 yr = 365.25 days;
- Avogadro's number, $N_A = 6.02214076\times10^{23}$;
- Recoverable energy, $E_R = 190$ MeV/fis;
- 1 eV = 1.60217662×10^{-19} J
- Molar mass for ^{239}Pu, $M_9 = 239.0521634$ g/mol;
- Capture-to-fission ratio in the fast region for ^{239}Pu, $\alpha = 0.065$.

Answers are required with a margin of error of 0.1%.

The breeding gain must be, $G =$

The reactor power which can be produced from a given fuel mass, m_f, is proportional to the mass – that is $P = \beta m_f$. Here, β is the specific power which is known. The exponential doubling time is given by

$$t_{De} = \frac{\ln 2}{G \cdot w \cdot \beta}$$

Here, $G = C-1$, is the gain factor and w is the fuel consumption rate, i.e. the consumed fuel mass per thermal megawatt-day energy. Let a "unit" energy amount be, $E = 1$ MWd. It requires to fission $N_{fis} = E/E_R$ fuel atoms. The number of fuel atoms consumed is greater because the relationship between the number of absorptions by fuel atoms and the number of fissions is $N_{fis} = P_f N_{abs}$. Here, the probability for fission upon absorption is

$$P_f = \frac{\sigma_f}{\sigma_f + \sigma_c} = \frac{1}{1+\sigma_c/\sigma_f} = \frac{1}{1+\alpha}$$

Hence, the number of the consumed fuel atoms, N_f, that have undergone absorptions to generate 1 MWd of thermal energy is found as

$$N_f = N_{abs} = (1+\alpha)N_{fis}$$

It is equivalent to consuming a fuel mass

$$w = m_f = \frac{N_f M_f}{N_A}$$

Here $M_f = M_9$ is the molar mass of the fuel atoms. Finally, we arrive at

$$G = \frac{\ln 2}{w \cdot \beta \cdot t_{De}}$$

A certain fossil fuelled generating station operates at a power of 1000 MWe at an overall efficiency of 39% and an average capacity factor of 0.7.

(a) How many tons of 13,000 Btu per pound of coal does the plant consume in 1 year?

(b) If an average coal-carrying railroad car carries 100 tons of coal, how many car loads must be delivered to the plant on an average day?

(c) If the coal contains 1.5% by weight sulphur and in the combustion process this all goes up the stack as SO_2, how much SO_2 does the plant produce in 1 year?

Note: it is an assignment for working with the US customary system of measurements.

Definitions that might be useful

- Pound, 1 lb = 453.59237 g;
- Ton, 1 ton = 2000 lb;
- Btu, British thermal unit, is a traditional unit of heat. It is defined as the amount of heat required to raise the temperature of one pound of water by one degree Fahrenheit;
- Fahrenheit temperature scale, $^{\circ}\mathrm{C} = (^{\circ}\mathrm{F} - 32) \times \frac{5}{9} \longleftrightarrow {}^{\circ}\mathrm{F} = {}^{\circ}\mathrm{C} \times \frac{9}{5} + 32$;
- The capacity factor is a ratio of an actual electrical energy output over a given period of time to the maximum possible (nominal) electrical energy output.

Data that might be useful

- Specific heat capacity of water, c = 4.186 J/(g×°C);
- Sulphur molar weight, M_S= 32.065 g/mol;
- Oxygen molar weight, M_O= 15.999 g/mol;
- Year (Julian), 1 yr = 365.25 days.

Answers must be given in US customary units. Answers are required with a margin of error of 1%.

(a) Mass of coal in tons per year, m =

US tons

(b) Number of car loads per day, Ncar =

(c) Mass of SO_2 released per year, m_S =

We begin first by defining some of the US customary units:

- lb = 453.59237 g;
- ton = 2000*lb g;
- fa = 5/9 °C;
- Btu = c×lb×fa [J];
- h_v= 13000×Btu/lb [J/g];

Solution (a)

The nominal electrical energy produced in 1 year is, $E_{nom} = P_0$×1yr. The actual electrical energy produced in 1 year is, $E_{el} = c_f$×E_{nom}; the actual thermal energy generated at the station is $E = E_{el}/\varepsilon$. The mass of coal, m, needed to generate this amount of heat is found as

$E = h_v \times m \longmapsto m = E/h_v$

Solution (b)

The mass of coal needed per day is simply given by, $m_1 = m/365.25$. The number of railroad cars needed per day clearly evaluates to, $N_1 = m_1/100\mathrm{ton}$, (US tons!).

Solution (c)

The mass of sulphur, m_S, which contains in m, is $m_S = w \times m$. The number of sulphur atoms evaluates as

$$N_S = \frac{m_S N_A}{M_S}$$

Under combustion, the sulphur forms the same number of sulphur dioxide molecules, SO_2, the molecular weight of which is, $M = M_S + 2M_O$. Thus, the total mass of sulphur dioxide released in the atmosphere per year is

$$m = \frac{N_S M}{N_A} = \frac{N_S M_S}{N_A}\frac{M}{M_S} = m_S\left(1 + 2\frac{M_O}{M_S}\right)$$

The table below describes a nominal LWR once-through fuel cycle. Mass flows are given in kg's per 0.75 GWe-yr. Compute the following.

(a) The specific burnup of the fuel in MWd/t;

(b) The fractional burnup of the fuel;

(c) The enrichment of the fresh fuel;

(d) The enrichment (^{235}U) of the spent fuel;

(e) The fraction of the power originating in fissions in ^{235}U and plutonium, respectively;

The plant operates at an overall efficiency of 33.4%.

Given

- Nominal energy, E_{el} = 0.75 GWe-yr;
- Thermal efficiency, ε = 33.4%.

	^{235}U	U	Pu-fis	Pu	THM	FP
BOC:	813	26977			26977	
EOC:	220	25858	178	246	26104	873

Notes

- BOC = beginning of refuelling cycle;
- EOC = end of refuelling cycle;
- U = total uranium;
- Pu-fis = ^{239}Pu + ^{241}Pu;
- Pu = total plutonium;
- THM = total heavy metal;
- FP = fission products.

Data that might be useful

- Julian year, 1 yr = 365.25 days;
- Avogadro's number, $N_A = 6.02214076\times10^{23}$;
- 1 eV = 1.60217662×10^{-19} J;
- Recoverable energy, E_R = 200 MeV/fis;
- Molar mass of ^{235}U, M_{25} = 235.0439299 g/mol;
- Molar mass of ^{238}U, M_{28} = 238.05078826 g/mol;
- Thermal cross-sections for ^{235}U, $\sigma_{c,U5}$= 97.83 b, $\sigma_{f,U5}$= 571.4 b.

Answers are required with a margin of error of 1%. Note the required units.

(a) Specific burnup, B =

MWd/tHM

(b) Fractional burnup, β =

Solution

Let m with a suitable subscript be a given mass of a heavy metal, HM, at the beginning of cycle, BOC, and $\bar{m}$ be the mass of the same heavy metal found at the end of cycle, EOC.

Thus, we are given

- BOC:
  - Total mass of uranium, m_U;
  - Total mass of ^{235}U, m_{U5};
- EOC:
  - Total mass of uranium, $\bar{m}_{\mathrm{U}}$;
  - Total mass of ^{235}U, $\bar{m}_{\mathrm{U5}}$;
  - Total mass of HM, $\bar{m}_{\mathrm{HM}}$.

Solution (a)

The initial amount of fuel is m_U. If the overall efficiency is ε, the nominal thermal energy is

$E_{th} = E_{el}/\varepsilon$

Hence the specific burnup is found as

$$B = E_{\mathrm{th}}/m_{\mathrm{U}}$$

Solution (b)

By definition, the fractional burnup is

$$\beta \equiv \frac{\text{Number of fissions}}{\text{Number of initial HM atoms}}$$

The number of fissions is found as

$$N_{\mathrm{fis}} = E_{\mathrm{th}}/E_{\mathrm{R}}$$

The fresh fuel is a mixture of ^{235}U and ^{238}U. Thus, the number of HM atoms evaluates as

$$N_{\mathrm{U}} = N_{\mathrm{U5}} + N_{\mathrm{U8}} = \frac{m_{\mathrm{U5}}N_A}{M_{\mathrm{U5}}} + \frac{m_{\mathrm{U8}}N_A}{M_{\mathrm{U8}}}$$

Hence, we arrive at

$$\beta = N_{\mathrm{fis}}/N_{\mathrm{U}}$$

Solution (c)

Clearly, the enrichment of the fresh fuel is

$$e = m_{\mathrm{U5}}/m_{\mathrm{U}}$$

Solution (d)

At the end of cycle, the total mass of heavy metal, essentially uranium and plutonium, is $\bar{m}_{\mathrm{HM}}$.

Hence the enrichment at EOC in weight percent is

$$\bar{e} = \bar{m}_{\mathrm{U5}}/\bar{m}_{\mathrm{HM}}$$

Solution (e)

At the end of cycle, the mass of ^{235}U consumed evaluates as

$$m_{\mathrm{U5}}^{(c)} = m_{\mathrm{U5}} - \bar{m}_{\mathrm{U5}}$$

The mass of ^{235}U that underwent fission is less because of the relationship between the number of absorptions by fuel atoms and the number of fissions

$$N_{\mathrm{fis}} = P_f N_{\mathrm{abs}}$$

Here, the probability for fission upon absorption is

$$P_f = \frac{\sigma_f}{\sigma_f+\sigma_c} = \frac{1}{1+\sigma_c/\sigma_f} = \frac{1}{1+\alpha}$$

Hence, the mass of ^{235}U that underwent fission becomes

$$m_{\mathrm{U5}}^{(f)} = \frac{m_{\mathrm{U5}}^{(c)}}{1+\alpha}$$

Then the number of fissions in ^{235}U evaluates as

$$N_{\mathrm{U5}}^{(f)} = \frac{m_{\mathrm{U5}}^{(f)}N_A}{M_{\mathrm{U5}}}$$

Assuming, that the recoverable energy per fission is the same for any fissile isotope, gives the fraction of the power originating in fissions in ^{235}U as

$$f_{\mathrm{U5}} = N_{\mathrm{U5}}^{(f)}/N_{\mathrm{fis}}$$

The rest is essentially due to fissions in the fissile plutonium. The contribution from higher actinides such as Am, Cm, Cf etc. is assumed negligible.

$$f_{\mathrm{Pu}} = 1 - f_{\mathrm{U5}}$$

It is found that, in a certain thermal reactor, fuelled with partially enriched uranium, 13% of the fission neutrons are absorbed in resonances of ^{238}U and 3% leak out of the reactor, both while these neutrons are slowing down; 5% of the neutrons that slowdown in the reactor subsequently leak out; of those slow neutrons that do not leak out, 82% are absorbed in fuel, 74% of these in ^{235}U.

(a) What is the multiplication factor of this reactor?

(b) What is its conversion ratio?

Given

- Absorption in resonances of ^{238}U, $a_8 = 0.13$;
- Leakage of fast neutrons (while slowing down), $l_{fa} = 0.03$;
- Leakage of thermal neutrons, $l_{th} = 0.05$;
- Absorption in uranium, $a_u = 0{,}82$;
- Absorption in ^{235}U, $a_5 = 0.74$

Data that might be useful

- Reproduction factor of ^{235}U, $\eta = 2.068$;
- Average number of neutrons per fission in ^{235}U, $\nu = 2.42$.

Answers are required with a margin of error of 0.1%.

(a) Multiplication factor, $k =$

(b) Conversion factor, $C =$

The six-factor formula gives the multiplication factor as

$k = \eta \cdot \epsilon \cdot p \cdot f \cdot P_{FNL} \cdot P_{TNL}$

The problem says nothing about fast fission, thus assuming, $\varepsilon = 1$. The product of the resonance escape probability, p, and the fast non-leakage probability, P_{FNL} , is readily given as

$p \cdot P_{FNL} = 1 - a_8 - l_F$

Next, $P_{TNL} = 1 - l_T$. The thermal utilisation is found as

$f = a_U \cdot a_5$, which completes solution (a).

Solution (b)

By definition, the conversion ratio is

$C \equiv \frac{\text{Number of fissile atoms produced}}{\text{Number of fissile atoms consumed}}$

The following items should be taken into consideration:

- In our case, ^{235}U atoms are consumed and ^{239}Pu atoms are produced;
- Each absorption of a neutron by ^{238}U, whether at resonance or thermal energies, produces an atom of ^{239}Pu;
- Each time a neutron is absorbed by a ^{235}U nucleus, that nucleus is consumed;
- When a neutron is absorbed by a ^{235}U nucleus, η fission neutrons are born on average.

Per one fission neutron a_8 neutrons are absorbed in resonances of ^{238}U that leads in the end to the production of a_8 atoms of ^{239}Pu. Further, $(1 - a_8 - l_F)$, neutrons reach thermal energies out of which l_T neutrons leak out the system hence, $(1 - a_8 - l_F) \times (1 - l_T)$ are still in the system, of which a_U neutrons are absorbed in the uranium thus giving $(1 - a_8 - l_F) \times (1 - l_T) \times a_U$ neutrons absorbed in uranium fuel at thermal energies. Next, a_5 neutrons are absorbed in ^{235}U. Consequently, $(1 - a_5)$ are absorbed in ^{238}U thus producing another, per one fission neutron, $(1 - a_8 - l_F) \times (1 - l_T) \times a_U \times (1 - a_5)$ nuclei of ^{239}Pu. Recalling that the above calculations are made per one fission neutron whereas η fission neutrons are born per one atom of ^{235}U consumed, we arrive at

$C = \frac{a_8 + (1 - a_8 - l_F) \times (1 - l_T) \times a_U \times (1 - a_5)}{\text{1 fiss neutron}} \times \frac{\eta \text{ fiss neutrons}}{\text{1 atom of } ^{235}\text{U}}$

A point source emits S neutrons per second isotropically in an infinite vacuum. Let a coordinate system be centred at the neutron source and $\vec{r}$ be the radius-vector of a point of interest. What is the neutron flux at some point $\vec{r}$? Mark all correct answers.

If $\vec{n} = \vec{r}/r$ is a normal vector in the radial direction, what is the neutron current at some point $\vec{r}$? Mark all correct answers.

In vacuum, neutrons travel at a constant speed without absorption and without scattering. In our case, all neutrons travel along radial lines. Clearly, the neutron flux depends only on distance r. Neutron balance says, $(4\pi r^2)\phi(r) = S$, which gives $\phi(r) = \frac{S}{4\pi r^2}$. In this particular situation, neutron current coincides with neutron flux with the only difference that neutron current is a vector quantity as contrast to neutron flux thus $\vec{\mathrm{J}}(\vec{\mathrm{r}}) = \phi(r)\vec{\mathrm{n}} = \frac{S}{4\pi r^2}\vec{\mathrm{n}}$.

An infinite moderator, characterised by the diffusion coefficient D and the macroscopic absorption cross-section Σ_a, contains uniformly distributed isotropic neutron sources emitting S n/(cm^{3}s). Determine the steady-state flux and current at any point in the medium.

(a) Neutron flux is

(b) Neutron current is

The general equation of continuity reads as

$$\frac{\partial n}{\partial t} = S - \Sigma_a \phi - div(\vec{\mathrm{J}})$$

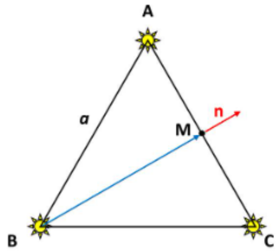
In steady state, the time derivative is zero; flux is constant and current is zero due to symmetry. Hence

$$\frac{\partial n}{\partial t} = 0; \quad div(\vec{\mathrm{J}}) = 0.$$

it follows then

$$0 = S - \Sigma_a \phi \longrightarrow \phi = S/\Sigma_a$$

Three isotropic neutron sources, each emitting S neutrons per second, are located in an infinite vacuum at the three corners of an equilateral triangle of side a. Find the flux and current at the midpoint of one side. Let $\vec{n}$ be the outward normal vector at the midpoint.



1) Neutron flux is a scalar quantity.

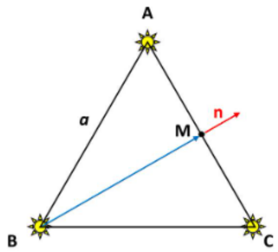
?

2) Neutron current is a vector quantity.

?

3) Mark the correct answer.

4) Mark the correct answer.



Let $\vec{n}$ be a unit outward vector at the midpoint as shown in the figure. Let also ϕ_M be the neutron flux at **M** and ϕ_A,$\phi_B$and ϕ_C be neutron fluxes at **M** coming from points **A**, **B** and **C**.

Obviously, $\phi_M = \phi_A + \phi_B + \phi_C$ and $\phi_A = \phi_C$

The neutron flux at distance r from a point source emitting S n/s is given by

$\phi(r) = \frac{S}{4\pi r^2}$

Clearly, $AM = \frac{1}{2}a$ and $BM = \frac{\sqrt{3}}{2}a$

This leads to

$\phi_M = 2\phi_A + \phi_B = 2\frac{S}{4\pi(a/2)^2} + \frac{S}{4\pi\left(\sqrt{3}a/2\right)^2}$

Collecting similar terms gives

$\phi_M = \frac{7S}{3\pi a^2}$

The neutron current from **A** and **C** compensate each other at **M**. Thus

$\vec{J} = \phi_B \cdot \vec{n} = \frac{S}{3\pi a^2} \cdot \vec{n}$

Isotropic point source emits S n/s in an infinite moderator characterised by a diffusion coefficient, D, and a macroscopic absorption cross-section, Σ_a. Compute the following:
(a) The net number of neutrons passing per second through a spherical surface of radius r centred on the source.
(b) The number of neutrons absorbed per second within the sphere.
(c) The sum of your results in (a) and (b).

(a) The escape rate, $R_E(r)$, is

(b) The absorption rate, $R_a(r)$, is

(c) The sum of (a) and (b) is

(a) The net number of neutrons escaping a volume V is given by

$R_E = \int_A (\vec{J} \cdot \vec{n}) dA$

Here, $A = \partial V$, is the external surface of volume V and $\vec{n}$ is the outward unit vector on A. In case of a point neutron source, the neutron current depends only on the distance r, is orthogonal to the spherical surface and its magnitude is constant on the spherical surface. Hence the escape rate simplifies to

$R_E = 4\pi r^2 J(r)$

The neutron current may be evaluated through Fick's law, $\vec{J} = -D\nabla\phi$. The neutron flux from a point source is given by

$\phi(r) = \frac{S}{4\pi D}\frac{e^{-r/L}}{r}$

Here the diffusion length is defined as, $L = \sqrt{D/\Sigma_a}$. The magnitude of neutron current thus evaluates as

$J(r) = -D\phi'(r) = \frac{S}{4\pi}\frac{e^{-r/L}}{r^2}(r/L+1)$

It gives the escape rate as

$R_E(r) = 4\pi r^2 J(r) = Se^{-r/L}\,(r/L+1)$

(b) The number of neutrons absorbed per second within a volume V is given by

$R_a(V) = \int_V R_a(\vec{r})dV$

The absorption rate density, $R_a(\vec{r})$, is found as $R_a(\vec{r}) = \Sigma_a(\vec{r})\phi(\vec{r})$. In our case

$\Sigma_a(\vec{r}) = \Sigma_a = const$ and $\phi(\vec{r}) = \phi(r)$

Spherical system of coordinates, (ρ, θ, φ), clearly simplifies calculations:

$R_a(r) = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \Sigma_a\phi(\rho)\rho^2\sin(\theta)d\rho d\theta d\varphi$

It leads to

$R_a(r) = \Sigma_a\int_0^{2\pi} d\varphi \int_0^{\pi}\sin(\theta)d\theta \int_0^{r}\phi(\rho)\rho^2 d\rho$

All the involved integrals are easily found

$\int_0^{2\pi} d\varphi = 2\pi$ and $\int_0^{\pi}\sin(\theta)d\theta = 2$ and $\int_0^{r}\phi(\rho)\rho^2 d\rho = \frac{S}{4\pi D}L^2\left[1 - e^{-r/L}(1 + r/L)\right]$

Taking into account that, $L^2 = D/\Sigma_a$, the answer to question (b) becomes

$R_a(r) = S\left[1 - e^{-r/L}(1 + r/L)\right]$

(c) We easily verify that it holds within radius r

$R_E(r) + R_a(r) = S$

A point source emitting S n/s is placed at the centre of a sphere of moderator of radius R. The moderator is characterised by the diffusion coefficient D and the macroscopic absorption cross-section Σ_a. Determine the flux in the sphere assuming the extrapolation length d.

Let the extrapolation radius be $\tilde{R} = R + d$

The steady-state diffusion equation reads as

$$D\nabla^2\phi - \Sigma_a\phi + S = 0$$

If the source point is taken to be at the centre of a spherical coordinate system, the flux obviously only depends on r. When $r \neq 0$, the diffusion equation becomes

$$D\nabla^2\phi - \Sigma_a\phi = 0 \longmapsto \nabla^2\phi - \frac{1}{L^2}\phi = 0$$

Here $L = \sqrt{D/\Sigma_a}$

The boundary condition requires $\phi(\tilde{R}) = 0$.

The source condition is given by $\lim_{r\to 0} 4\pi r^2 J(r) = S$

In spherical coordinates, the equation reads

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\phi}{\mathrm{d}r}\right) - \frac{1}{L^2}\phi = 0$$

Introducing a new unknown function

$$w(r) \equiv r\phi(r) \longmapsto \phi(r) = \frac{w(r)}{r} \longmapsto \phi'(r) = \frac{w'(r)r - w(r)}{r^2}$$

The diffusion equation becomes

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{w'(r)r - w(r)}{r^2}\right) - \frac{1}{L^2}\frac{w(r)}{r} = 0$$

It further simplifies to

$$w''(r) - \frac{1}{L^2}w(r) = 0 \longmapsto w(r) = A'e^{-r/L} + Be^{r/L}$$

Hence,

$$\phi(r) = \frac{1}{r}w(r) = \frac{1}{r}\left(A'e^{-r/L} + Be^{r/L}\right)$$

BC: $\phi(\tilde{R}) = \frac{1}{\tilde{R}}\left(A'e^{-\tilde{R}/L} + Be^{\tilde{R}/L}\right) = 0 \longmapsto B = -A'e^{-2\tilde{R}/L}$

It is convenient to redefine,

$$A' = Ae^{\tilde{R}/L} \longmapsto B = -A'e^{-2\tilde{R}/L} = -Ae^{-\tilde{R}/L}$$

Finally, the solution simplifies to

$$\phi(r) = \frac{1}{r}\left(A'e^{-r/L} + Be^{r/L}\right) = \frac{A}{r}\left(e^{(\tilde{R}-r)/L} - e^{(r-\tilde{R})/L}\right) = \frac{2A}{r}\sinh\left((\tilde{R}-r)/L\right)$$

The neutron flux is found through Fick's law

$$J = -D\phi' = -2AD\frac{\mathrm{d}}{\mathrm{d}r}\frac{\sinh\left[(\tilde{R}-r)/L\right]}{r}$$

The derivative evaluates in a straight forward manner

$$\frac{\mathrm{d}}{\mathrm{d}r}\frac{\sinh\left[(\tilde{R}-r)/L\right]}{r} = \frac{-\frac{1}{L}\cosh\left[(\tilde{R}-r)/L\right]\cdot r - \sinh\left[(\tilde{R}-r)/L\right]}{r^2}$$

The source condition transforms then to

$$4\pi r^2 J(r) = -4\pi r^2 2AD\frac{-\frac{1}{L}\cosh\left[(\tilde{R}-r)/L\right]\cdot r - \sinh\left[(\tilde{R}-r)/L\right]}{r^2}$$

It further simplifies to

$$4\pi r^2 J(r) = 4\pi 2AD\left(\frac{1}{L}\cosh\left[(\tilde{R}-r)/L\right]\cdot r + \sinh\left[(\tilde{R}-r)/L\right]\right)$$

Clearly,

$$4\pi r^2 J(r) \to 4\pi 2AD\sinh\left(\tilde{R}/L\right) = S$$

The constants then evaluates to

$$2A = \frac{S}{4\pi D\sinh\left(\tilde{R}/L\right)}$$

Finaly the solution becomes

$$\phi(r) = \frac{S}{4\pi D\sinh\left(\tilde{R}/L\right)}\frac{\sinh\left[(\tilde{R}-r)/L\right]}{r} = \frac{S}{4\pi D\sinh[(R+d)/L]}\frac{\sinh[(R+d-r)/L]}{r}$$

The three-group fluxes for a bare spherical fast reactor of radius R = 42 cm are given by the following expressions

$$\phi_1(r) = \frac{3\times10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$

$$\phi_2(r) = \frac{2\times10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$

$$\phi_3(r) = \frac{1\times10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$

The three-group coefficients are D_1 = 2.6 cm, D_2 = 1.5 cm and D_3 = 1.07 cm.

Calculate the total leakage of neutrons from the reactor. Ignore the extrapolated distance.

The answer is required in neutrons per second, and it is required with a margin of error of 1%.

Total escape rate R_E =

Given R = 42 cm, D_1 = 2.6 cm, D_2 = 1.5 cm, D_3 = 1.07 cm.

Let f_1 = 3x10^{15} n/(cm*s), f_2 = 2x10^{15} n/(cm*s) and f_3 = 1x10^{15} n/(cm*s).

The fluxes can be written as

$$\phi_i(r) = \frac{f_i}{r}\sin\left(\frac{\pi r}{R}\right)$$

The escape rate from the reactor in group i is given by

$$R_{E,i} = 4\pi R^2 J_i(R)$$

The neutron current in group i is found through Fick's law

$$J_i(r) = -D_i\frac{\mathrm{d}}{\mathrm{d}r}\phi_i(r) = -D_i f_i\frac{r\frac{\pi}{R}\cos\left(\frac{\pi r}{R}\right)-\sin\left(\frac{\pi r}{R}\right)}{r^2}$$

At the external surface, the current evaluates to

$$J_i(r) = D_i f_i\frac{\pi}{R^2}$$

The escape rate in group i then becomes

$$R_{E,i} = 4\pi R^2 J_i(R) = 4\pi D_i f_i$$

The total escape rate is

$$R_E = R_{E,1} + R_{E,2} + R_{E,3}$$

The thermal flux in the centre of a beam tube of a certain reactor is 3.2×10^{13} n/(cm^{2}s). The temperature in this region is 148 °C. Calculate:

(a) The thermal neutron density.

(b) The energy E_T in electron-volt.

(c) The 2200 meter-per-second flux.

Data that might useful:

- $N_A = 6.02214076 \times 10^{23}$ mol^{-1};
- k_B = 1.380649×10^{-23} J/K;
- $m_n = 1.67492749804 \times 10^{-27}$kg;
- 0°C = 273.15 K

Answers are required with a margin of error of 0.1%.

(a) The thermal neutron density, n =

n/cm^3

(b) The thermal energy, E_T =

eV

(c) The 2200 m/s pseudo flux, ϕ_0 =

(a) The thermal flux, ϕ_T, and thermal neutron density, n, are related as

$$\phi_T = \frac{2n}{\sqrt{\pi}} \left(\frac{2kT}{m_n}\right)^{1/2}$$

Hence the thermal neutron density is given by

$$n = \frac{\sqrt{\pi}}{2} \left(\frac{m_n}{2kT}\right)^{1/2} \phi_T$$

(b) The thermal energy and temperature are related as

$$E_T = kT$$

(c) The pseudo-flux, ϕ_0, and thermal flux, ϕ_T, and are related as

$$\frac{\phi_0}{\phi_T} = \frac{\sqrt{\pi}}{2} \frac{v_0}{v_T} \text{ thus}$$

$$\phi_0 = \frac{\sqrt{\pi}}{2} \frac{v_0}{v_T} \phi_T$$

Here, $v_0 = 2200$m/s and v_T is found through

$$\frac{m_n v_T^2}{2} = E_T$$

The neutron flux in a bare spherical reactor of radius, R = 42 cm, is given by

$\phi = A\frac{\sin(k\cdot r)}{r}$ $\frac{\text{n}}{\text{cm}^2\cdot\text{s}}$ where r is measured from the centre of the reactor.

The constants are $A = 4\times10^{13}$ n/(cm×s) and k = π/42 cm^{-1}.

The diffusion coefficient is D = 0.7 cm.

(a) What is the maximum value of the flux in the reactor?

(b) How many neutrons escape from the reactor per second?

Answers are required with a margin of error of 0.1%.

(a) $\phi\text{max} =$

$\frac{\text{n}}{\text{cm}^2\cdot\text{s}}$

(b) Escape rate =

Solution (a)

On the physical grounds, the maximum neutron density is found in the centre of the reactor. One can also refer to a mathematical statement that the $\sin(x)/x$ function monotonically decreases on [0, 1]. A straightforfard evaluation of $\phi(0)$ runs into a mathematical uncertainty 0/0. A better way of evaluating $\phi(0)$ is based on the identity

$\lim_{x\to0}\frac{\sin x}{x} = 1$ Thus, we proceed

$$\phi\text{max} = \lim_{r\to0}\phi(r) = Ak\lim_{r\to0}\frac{\sin(kr)}{kr} = kA$$

Solution (b)

The number on neutrons escaping the reactor is given by

$R_E = J(R)\cdot 4\pi R^2$

The neutron curren as function of r is found through Fick's law as

$J(r) = -D\phi'(r) = -AD\frac{rk\cos(kr)-\sin(kr)\cdot 1}{r^2}$

Recalling $kR = \pi$, the neutron current density at the reactor periphery thus evaluates as

$J(R) = -AD\frac{kR\cos(kR)-\sin(kR)}{R^2} = -AD\frac{\pi\cos(\pi)-\sin(\pi)}{R^2} = \frac{\pi AD}{R^2}$

Finally

$R_E = J(R)\cdot 4\pi R^2 = \frac{\pi AD}{R^2}\cdot 4\pi R^2 = 4\pi^2 AD$

A large research reactor consists of a cubical array of natural uranium rods in a graphite moderator. The reactor is 750 cm on a side and operates at a power of 16 MW. The average value of $\Sigma_{f,ave}$ is 2.9×10^{-3} cm^{-1}.

a) Calculate the buckling B in inverse cm. Since the system is very large the extrapolation distance can be neglected.

b) What is the maximum value of the thermal flux, ϕ_{max}, in units of n/(cm^{2}s)?

c) What is the average value of the thermal flux, ϕ_{ave}, in units of n/(cm^{2}s)?

d) At what rate, m_F (F for fuel), in milligrams per second is ^{235}U consumed in the reactor?

Given:

- Side a = 750 cm;
- Power P = 16 MW;
- Average $\Sigma_{f,ave} = 2.9\times10^{-3}$ cm^{-1}.

Data that might be useful

- Avogadro's number $N_A = 6.02214076\times10^{23}$ mol^{-1} (exact);
- Electron-volt, 1eV = 1.60217662×10^{-19} J (exact);
- Recoverable energy E_R = 200 MeV/fiss;
- Molar weight of ^{235}U M_5 = 235.044 g/mol.

Answers are required with a margin of error of 0.1%.

a) Buckling, $B =$

cm^{-1}

b) Value of $\phi_{max} =$

n/(cm^{2}s)

c) Value of $\phi_{ave} =$

n/(cm^{2}s)

d) Consumption, $m_F =$

Known

$V = a^3$ (volume of the reactor)

$$\Omega = \frac{\phi_{\max}}{\phi_{ave}} = \left(\frac{\pi}{2}\right)^3 = 3.88 \text{ (Table 6.2)}$$

Let a rectangular parallelepiped be

$-\frac{a}{2} \le x \le \frac{a}{2},\ -\frac{b}{2} \le y \le \frac{b}{2},\ -\frac{c}{2} \le z \le \frac{c}{2}.$

In our case, $a = b = c$. One-group diffusion theory requires knowledge of the extrapolated side,

$\tilde{a} = a + d.$

The extrapolation length,

$d = 2.13D,$

is typically of order of 1 cm and may be negleted, $\tilde{a} \approx a$.

Solution a)

The buckling for a rectangular parallelepiped is given by

$$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = 3\left(\frac{\pi}{a}\right)^2 \rightarrow B = \sqrt{3}\frac{\pi}{a}$$

Solution b)

In one-group diffusion theory, the flux in a rectangular parallelepiped, is found as

$$\phi(x, y, z) = A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$$

The constant A may be expressed in terms of power as (Table 6.2)

$$A = \left(\frac{\pi}{2}\right)^3 \frac{P}{E_R \Sigma_f V} = \frac{\Omega P}{E_R \Sigma_f V} = \frac{3.88\ P}{E_R \Sigma_f V}$$

Obviously, the neutron flux reaches its maximum value in the centre.

$\phi_{\max} = \phi(0, 0, 0) = A$

Solution c)

The average flux is easily found through

$$\Omega = \frac{\phi_{\max}}{\phi_{ave}} = \left(\frac{\pi}{2}\right)^3 \rightarrow \phi_{ave} = \frac{1}{\Omega}\phi_{\max}$$

Solution d)

The average fission rate is found as $R_f = \Sigma_f \phi_{ave}$.

Hence $N_f = \Sigma_f \phi_{ave} V$ fission events are ocurring in the whole reactor per second.

Every fission is the destruction of an ^{235}U atom, whose mass is $m_5 = \frac{M_5}{N_A}$

Thus the uranium mass consumption, N_F (F for fuel) is found as

$$M_F = m_5 N_f = \frac{M_5}{N_A}\Sigma_f \phi_{ave} V = \frac{M_5 \Sigma_f \phi_{ave} V}{N_A}$$

A bare-spherical reactor 80 cm in radius is composed of a homogeneous mixture of ^{235}U and ^{9}Be. The reactor operates at a power level of 45 thermal kilowatts. Assume the recoverable energy to be E_R = 184 MeV. Using modified one-group theory, compute:

(a) The critical mass of ^{235}U in kilograms;

(b) The leakage of neutrons from the reactor in neutrons per second;

(c) The rate of consumption of ^{235}U in milligrams per day.

Note that in the homogeneous type of reactor, very little fissile material is necessary to reach criticality therefore the concentration of the mixture may be assumed to be that of the moderator. Furthermore, the scattering cross-sections of fissile atoms are not significantly larger than those of the ordinary moderators. It is possible, therefore, to ignore the presence of the fuel when evaluating quantities such as the moderator density or the neutron age.

Given

- Radius of a bare-spherical reactor, R = 80 cm;
- Homogeneous mixture of ^{235}U and ^{9}Be;
- Thermal power, P = 45 kW;
- Fuel = ^{235}U, labelled with F;
- Moderator = ^{9}Be, labelled with M;

Required

(a) CM - critical mass of Fuel in kilograms.

(b) R_{esc} - total neutron escape rate;

(c) CoF - consumption of fuel in milligrams per day;

Use the following data.

- Avogadro's number, $N_A = 6.02214076 \times 10^{23}$ mol^{-1} (exact);
- Recoverable energy, E_R = 184 MeV;
- Fuel is ^{235}U, non-1/v absorber:
  - Molar weight, M_F = 235.0439 g/mol;
  - Fission reproduction factor, η_T = 2.065;
  - Fission cross-section, at 0.0253 eV, σ_f = 587 b ;
  - Absorption cross-section at 0.0253 eV, $\sigma_{a,F}$ = 687 b;
  - Non-1/v fission factor, g_f = 0.9759;
  - Non-1/v absorption factor, g_a = 0.9780;
- Moderator is ^{9}Be, 1/v-absorber:
  - Molar weight, M_M = 9.0122 g/mol
  - Mass density, ρ_M = 1.85 g/cm^3;
  - Absorption cross-section at 0.0253 eV, $\sigma_{a,M}$ = 0.0092 b;
  - Scattering cross-section in thermal region, $\sigma_{s,M}$ = 6.1400 b;
  - Neutron age in beryllium, $\tau_{T,M}$ = 102 cm^2

Answers are required with a margin of error of 0.1%.

(a) Critical mass, CM =

[kg]

(b) Escape rate, R_{esc} =

[n/s]

(c) Consumption, CoF =

Solution (a)

Let N_F and N_M be atom densities of Fuel and Moderator respectively.

Average macroscopic cross-sections, XS, are defined through average microscopic cross-sections, xs, as

Fission in Fuel: $\overline{\Sigma}_f = \bar{\sigma}_f N_{\mathrm{F}}$.

Absorption in Fuel: $\overline{\Sigma}_{a,\mathrm{F}} = \bar{\sigma}_{a,\mathrm{F}} N_{\mathrm{F}}$

Absorption in Moderator: $\overline{\Sigma}_{a,\mathrm{M}} = \bar{\sigma}_{a,\mathrm{M}} N_{\mathrm{M}}$

The average (thermal) microscopic xs are related to the corresponding reference values measured at $E_0 = 0.0253$ eV as

$\bar{\sigma}_f = g_f \sigma_f(E_0) = g_f \sigma_f$

$\bar{\sigma}_{a,\mathrm{F}} = g_a \sigma_{a,\mathrm{F}}(E_0) = g_a \sigma_{a,\mathrm{F}}$ and

$\bar{\sigma}_{a,\mathrm{M}} = 1 \times \sigma_{a,\mathrm{M}}(E_0) = \sigma_{a,\mathrm{M}}$ (1/v absorber).

For a 1/v-absorber, we can simply write

$\overline{\Sigma}_{a,\mathrm{M}} = \bar{\sigma}_{a,\mathrm{M}} N_{\mathrm{M}} = \sigma_{a,\mathrm{M}} N_{\mathrm{M}} = \Sigma_{a,\mathrm{M}}$

Often, it is convenient to define,

$$Z \equiv \frac{\overline{\Sigma}_{a,\mathrm{F}}}{\overline{\Sigma}_{a,\mathrm{M}}}$$

In a critical bare homogeneous reactor, it holds

$$Z = \frac{1+B^2\left(L^2_{T,\mathrm{M}}+\tau_{T,\mathrm{M}}\right)}{\eta_T - 1 - B^2 \tau_{T,\mathrm{M}}}$$

For a sphere, the buckling is

$B = \frac{\pi}{\tilde{R}};\ \tilde{R} = R + d;\ d = 2.13\ D;\ D \approx D_{\mathrm{M}}.$

The diffusion coefficient for the moderator, D_{M}, is found as

$$D_{\mathrm{M}} = \frac{1}{3\Sigma_{tr,\mathrm{M}}};\ \Sigma_{tr,\mathrm{M}} = (1-\overline{\mu})\Sigma_s;\ \overline{\mu} = \frac{2}{3A};\ \ \Sigma_s \approx \Sigma_{s,\mathrm{M}} = \sigma_{s,\mathrm{M}} N_{\mathrm{M}}$$

The thermal diffusion area for Moderator is

$$L^2_{T,\mathrm{M}} = \frac{D_{\mathrm{M}}}{\Sigma_{a,\mathrm{M}}}$$

Hence the parameter Z is uniquely determined now. Next, from the definition of Z,

$$N_{\mathrm{F}} = Z \frac{\bar{\sigma}_{a,\mathrm{M}}}{\bar{\sigma}_{a,\mathrm{F}}} N_{\mathrm{M}}$$

Further,

$$N_{\rm F}=\frac{\rho_{\rm F}N_A}{M_{\rm F}}=Z\frac{\bar{\sigma}_{a,\rm M}}{\bar{\sigma}_{a,\rm F}}N_{\rm M}=Z\frac{\bar{\sigma}_{a,\rm M}}{\bar{\sigma}_{a,\rm F}}\frac{\rho_{\rm M}N_A}{M_{\rm M}}$$

It follows then

$$\rho_{\rm F}=Z\frac{\bar{\sigma}_{a,\rm M}}{\bar{\sigma}_{a,\rm F}}\frac{M_{\rm F}}{M_{\rm M}}\rho_{\rm M}\longrightarrow CM=\rho_{\rm F}V;\;\; V=\frac{4\pi}{3}R^3$$

Solution (b)

The neutron leakage from a reactor can be evaluated through one-group diffusion theory as a surface integral over the external surface of the reactor.

$$R_{\rm esc}=\int_A\left(\vec{J}\cdot\vec{n}\right)dA$$

Here, $\vec{J}$ is neutron current at the periphery and $\vec{n}$ is an external normal. Fick's law gives neutron current by

$\vec{J}=-D\nabla\phi$. We recall that $D\approx D_{\rm M}$.

In a homogeneous bare-spherical reactor, neutron flux depends only on the distance to the centre, r, and is found to be

$$\phi(r)=A\frac{1}{r}\sin\left(\frac{\pi r}{\tilde{R}}\right)$$

When d is small,

$$\tilde{R}=R+d\approx R,$$

The constant A is determined by the total thermal power as

$$A=\frac{P}{4E_{\rm R}\Sigma_f R^2}$$

Neutron current is pointed outward and easily evaluated as

$$J(r)=-D\phi'(r)=-DA\frac{r\frac{\pi}{R}\cos\frac{\pi r}{R}-\sin\frac{\pi r}{R}}{r^2}$$

At the periphery, we have

$$J(r=R)=\pi DA/R^2$$

The total neutron escape rate is thus given by

$$R_{\rm esc}=J(R)\cdot 4\pi R^2=4\pi^2DA=\frac{\pi^2PD}{E_{\rm R}\Sigma_fR^2}$$

Solution (bb)

If R_f is the total fission rate in the whole reactor, i.e. the total number of fissions per one second, then

$$P=E_{\rm R}R_f\longrightarrow R_f=P/E_{\rm R}$$

Every fission event brings us ν fission neutrons while 1 neutron is absorbed thus giving $\nu-1$ net neutron production per one fission. Hence, the total neutron production is $(\nu-1)R_f$ out of which $P_{\rm L}$ fraction will leak out the reactor, where the leakage probability, $P_{\rm L}=1-P_{\rm NL}$. Here, the non-leakage probability is given by the modified one-group theory as

$$P_{\rm NL}=\frac{1}{1+B^2M_{\rm T}^2};\;\; M_{\rm T}^2=L_{\rm T}^2+\tau_{\rm T};\, \tau_{\rm T}\approx\tau_{\rm M}.$$

Here

$$L_{\rm T}^2=\frac{D}{\Sigma_a}\approx\frac{D_{\rm M}}{\Sigma_{a,\rm M}+\Sigma_{a,\rm F}}=\frac{D_{\rm M}/\Sigma_{a,\rm M}}{1+Z}=\frac{L_{\rm T,M}^2}{1+Z}=(1-f)L_{\rm T,M}^2$$

The neutron escape rate, $R_{\rm esc}$, then becomes

$$R_{\rm esc}=(\nu-1)R_fP_{\rm NL}$$

Solution c)

Each absorption event destroys one atom of ^{235}U.

The (conditional) probability of a fission event upon neutron absorption is

$$P_f=\frac{\bar{\sigma}_f}{\bar{\sigma}_{a,\rm F}}=\frac{g_f\sigma_f}{g_a\sigma_{a,\rm F}}$$

The fission and absorption rates are related as

$$R_f=P_fR_a\longrightarrow R_a=R_f/P_f$$

If the mass of a fuel atom is

$$m=M_{\rm F}/N_A$$

the mass consumption rate of fuel, CoF, becomes

$$CoF=mR_a=\frac{M_{\rm F}R_a}{N_A}$$

A fast reactor assembly consisting of a homogeneous mixture of ^{239}Pu and sodium is made in the form of a bare sphere.

The atom densities of these constituents are

$N_F = 0.00395 \times 10^{24}$ a/cm^3 for the plutonium and

$N_S = 0.0235 \times 10^{24}$ a/cm^3 for the sodium.

Suppose this reactor is operated at a power level of 7 kW then what is the total neutron leakage rate, i.e. how many neutrons would escape from the reactor per second?

Data that might be useful

- E_R = 200 MeV/fiss, recoverable energy per fission;
- eV = 1.60217662×10^{-19} J (exact).

Table: Nominal one-group constants for a fast reactor

El./Iso.	σ_γ	σ_f	σ_a	σ_{tr}	ν	η
Na	0.0008	0	0.0008	3.3	–	–
Al	0.002	0	0.002	3.1	–	–
Fe	0.006	0	0.006	2.7	–	–
^{235}U	0.25	1.4	1.65	6.8	2.6	2.2
^{238}U	0.16	0.095	0.255	6.9	2.6	0.97
^{239}Pu	0.26	1.85	2.11	6.8	2.98	2.61

Question 1) If $\vec{J}$ is a neutron current, Fick's law says, $\vec{J} = D\,\nabla\phi$

Question 2) If n is a neutron density and ϕ is a neutron flux, then the neutron leakage, N_L, through a surface A with the outward normal $\vec{n}$ is given by

Question 3) requires an answer in n/s with a margin of error of 0.1%.

Neutron leakage from the reactor, N_L =

The neutron leakage rate through the rector surface, A, is found as, $N_L = \int_A \left(\vec{J} \cdot \vec{n}\right)\,\mathrm{d}A$, here $\vec{n}$ is the outward normal on A. The divergence theorem and Fick's law give

$$N_L = \int_A \left(\vec{J} \cdot \vec{n}\right)\,\mathrm{d}A = \int_V div\,\vec{J}\,\mathrm{d}V = -D\int_V \nabla^2\phi\,\mathrm{d}V = DB^2\int_V \phi\,\mathrm{d}V$$

The total reactor power is found as

$$P = E_R\int_V \Sigma_f\phi\,\mathrm{d}V = E_R\Sigma_f\int_V \phi\,\mathrm{d}V \longmapsto \int_V \phi\,\mathrm{d}V = \frac{P}{E_R\Sigma_f}$$

Thus, $N_L = DB^2\int_V \phi\,\mathrm{d}V = \dfrac{DB^2P}{E_R\Sigma_f}$

$\Sigma_f = \sigma_f N_F$; $D = \dfrac{1}{3\,\Sigma_{tr}}$; $\Sigma_{tr} = \sigma_{tr,F}N_F + \sigma_{tr,S}N_S$.

Next, $B^2 = \dfrac{\nu\Sigma_f - \Sigma_a}{D}$; $\Sigma_a = \sigma_{a,F}N_F + \sigma_{a,S}N_S$; $\sigma_{a,F} = \sigma_f + \sigma_{\gamma,F}$.

Assume a hypothetical spherical fast reactor operates at a thermal power level of 800 MW. It consists of a mixture of liquid sodium and plutonium (^{239}Pu), in which the plutonium is present to 2 w/o. The density of the mixture is 1.4 g/cm^3. The molar masses of sodium and plutonium are

M_S = 22.98977 g/mol (S for sodium) and

M_F = 239.05216 g/mol (F for fuel).

Given:

- Thermal power, P = 800 MW;
- Weight percent for ^{239}Pu, w = 2 w/o;
- Density of the mixture, ρ = 1.4 g/cm^3;

Data that might be useful:

- Recoverable energy, E_R = 200 MeV/fiss;
- Electron-volt, eV = 1.60217662 × 10^{-19} J (exact).

Do the following:

1. Estimate the critical radius of this reactor in cm;
2. Find the maximum value of the flux.
3. Evaluate the probability that a fission neutron will escape from the reactor.

Answers are required with a margin of error of 0.1%. Note the units.

Question 1. Critical radius, R =

cm

Question 2. Maximum flux =

n/(cm^{2}s)

Question 3. Escape Pr =

Solution 1.

The one-group diffusion equation evaluates the multiplication factor as

$$k = \frac{\nu\Sigma_f}{\Sigma_a + DB^2}$$

When the reactor is critical ($k = 1$), the critical buckling is given by

$$B^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{\nu\Sigma_f/\Sigma_a - 1}{D/\Sigma_a} = \frac{k_\infty - 1}{L^2} \equiv B_{\mathrm{m}}^2$$

It is also referred to as the material buckling, B_{m}, because it is defined entirely by the material composition. On the other hand, the criticality condition requires the material buckling be equal to the geometric buckling, B_{g}, (the first eigenvalue), which for a homogeneous sphere is

$$B_{\mathrm{g}}^2 = \left(\frac{\pi}{\tilde{R}}\right)^2$$

here $\tilde{R} = R + d$ is the extrapolated radius and d is the extrapolation length.

Hence the criticality condition becomes

$$B_{\mathrm{m}}^2 \equiv \frac{k_\infty - 1}{L^2} = \left(\frac{\pi}{\tilde{R}}\right)^2 \equiv B_{\mathrm{g}}^2$$

This equation gives the critical extrapolated radius as

$$\tilde{R} = \pi\sqrt{\frac{L^2}{k_\infty - 1}}$$

Clearly, we need to evaluate k_∞ and L. The diffusion length and related constants are found as

$L^2 = D/\Sigma_a$ and $D = 1/(3\Sigma_a)$

Next,

$$\Sigma_a = \Sigma_{a,\mathrm{S}} + \Sigma_{a,\mathrm{F}} = \sigma_{a,\mathrm{S}} N_{\mathrm{S}} + \sigma_{a,\mathrm{F}} N_{\mathrm{F}}$$

Similarly,

$$\Sigma_{tr} = \Sigma_{tr,\mathrm{S}} + \Sigma_{tr,\mathrm{F}} = \sigma_{tr,\mathrm{S}} N_{\mathrm{S}} + \sigma_{tr,\mathrm{F}} N_{\mathrm{F}}$$

Further,

$$N_{\mathrm{S}} = \frac{\rho_{\mathrm{S}} N_A}{M_{\mathrm{S}}};\ \rho_{\mathrm{S}} = (1 - w)\rho \text{ and}$$

$$N_{\mathrm{F}} = \frac{\rho_{\mathrm{F}} N_A}{M_{\mathrm{F}}};\ \rho_{\mathrm{F}} = w\rho$$

We will need also

$$k_\infty = f \cdot \eta = \frac{\Sigma_{a,\mathrm{F}}}{\Sigma_a} \cdot \frac{\nu\Sigma_f}{\Sigma_{a,\mathrm{F}}} = \frac{\nu\Sigma_f}{\Sigma_a}$$

Here, $\Sigma_f = \sigma_f N_{\mathrm{F}}$

The extrapolation length is given by $d = 2.13D$. Collecting everything gives the answer

$$R = \tilde{R} - d$$

Solution 2.

For a homogeneous spherical reactor, one-group diffusion theory gives the neutron flux as

$$\phi(r) = A\frac{\sin \pi r/\tilde{R}}{r}$$

The constant A is related to the total thermal power as

$$A = \frac{P}{4\Sigma_{\mathrm{R}}\Sigma_f R^2} \text{ here } \Sigma_f = \sigma_f N_{\mathrm{F}}$$

The maximum flux is found at the centre, $r = 0$.

Because of the singularity at $r = 0$, it is convenient to define a dimensionless variable

$$x \equiv \pi R/\tilde{R} \longmapsto r = x\tilde{R}/\pi$$

which expresses the flux as

$$\phi(r) = A\frac{\sin x}{x\tilde{R}/\pi} = \frac{\pi}{\tilde{R}}A\frac{\sin x}{x} \xrightarrow[x \to 0]{} \frac{\pi}{\tilde{R}}A = \phi(0)$$

Solution 3.

In a critical reactor,

$$k = \frac{\nu\Sigma_f}{\Sigma_a + DB^2} = \frac{\nu\Sigma_f/\Sigma_a}{1 + B^2 D/\Sigma_a} = \frac{k_\infty}{1 + B^2 L^2} = k_\infty P_{\mathrm{NL}}$$

Here the non-leakage probability is

$$P_{\mathrm{NL}} = \frac{1}{1 + B^2 L^2}$$

which give the leakage probability as

$$P_{\mathrm{L}} = 1 - P_{\mathrm{NL}}$$

The first term on the Right-Hand Side of Eq. (7.19) gives the number of prompt neutrons slowing down per cm^3/s while the second term gives this number for the delayed neutrons. Compare the magnitude of these two terms in a critical thermal reactor for which the delayed neutron fraction was measured to be

$\beta = 0.0024$.

Eq.(7.19) reads

$s_{\mathrm{T}} = (1-\beta)k_{\infty}\overline{\Sigma}_a\phi_{\mathrm{T}} + p\lambda C$

The equation determining the precursor concentration, (7.21), reads

$\frac{dC}{dt} = \frac{\beta k_{\infty}\overline{\Sigma}_a\phi_{\mathrm{T}}}{p} - \lambda C$

Here p is the probability that a delayed neutron escapes resonance capture.

Answer is required within 1% of accuracy.

If n_{p} is the number of prompt neutrons slowing down per cm^3/s and

n_{d} is the number of delayed neutrons slowing down per cm^3/s then

the ratio is $n_{\mathrm{d}}/n_{\mathrm{p}}$ =

The problem states that the number of prompt neutrons slowing down per cm^3/s and the number of delayed neutrons slowing down per cm^3/s are

$n_{\mathrm{p}} = (1-\beta)k_{\infty}\overline{\Sigma}_a\phi_{\mathrm{T}}$ and $n_{\mathrm{d}} = p\lambda C$

Obviously, in a critical reactor, $k_{\infty} = 1$, moreover, time derivatives are zero, i.e. the steady state concentration of delayed neutrons is found as

$\frac{dC}{dt} = \frac{\beta k_{\infty}\overline{\Sigma}_a\phi_{\mathrm{T}}}{p} - \lambda C = 0 \longrightarrow n_{\mathrm{d}} = p\lambda C = \beta\overline{\Sigma}_a\phi_{\mathrm{T}}$

Hence, the ratio between slowing-down delayed and prompt neutrons in a critical reactor is

$\frac{n_{\mathrm{d}}}{n_{\mathrm{p}}} = \frac{\beta\overline{\Sigma}_a\phi_{\mathrm{T}}}{(1-\beta)\overline{\Sigma}_a\phi_{\mathrm{T}}} = \frac{\beta}{1-\beta}$

A critical thermal reactor fueled by ^{235}U has been running at power P_0=6 MW until time t=0s. From time t=0s, its reactivity became maintained at the value $\rho = 34$ pcm. What is the approximate value of the reactor power *P* at time t=2 min?

The value of the power *P* is

The period of the reactor can be approximated for a very small reactivity as

$$T = \left(\Lambda + \sum_i \frac{\beta_{\text{eff}\,i}}{\lambda_i}\right)/\rho$$

where the generation time can be neglected and the term

$$\sum_i \frac{\beta_{\text{eff}\,i}}{\lambda_i}$$

equals about 0.0848 when the reactor is fueled by ^{235}U. So, the period is then

$$T = 0.0848/\rho$$

and the power *P(t)* can be roughly approximated as

$$P(t) = P_0 e^{t/T}$$

During test-out procedures, a ^{233}U-fueled thermal reactor at 20 °C is operated for a time at a power of 1 megawatt. The power is then to be increased to 115 megawatts in 7 hours.

(a) On what stable period should the reactor be placed?

(b) What reactivity insertion in cents is required?

Hint: ignore the prompt neutron lifetime.

Given

- Thermal reactor fuelled with ^{233}U;
- Initial power, $P_0 =$ 1 MW;
- Final power, $P_1 = 115$ MW;
- Power maneuver time, $\Delta t = 7$ hr;

Data that might be useful

Table 7.2 (Lamarsh, p.343) Delayed neutron fractions

Nuclide	β (thermal)	β (fast)
^{232}Th	—	0.0203
^{233}U	0.0026	0.0026
^{235}U	0.0065	0.0064
^{238}U	—	0.0148
^{239}Pu	0.0021	0.0020

Table 7.3 (Lamarsh, p.348) Values of the sum $\sum \beta_i \bar{t}_i$

Nuclide	$\sum \beta_i \bar{t}_i$[s]
^{233}U	0.0479
^{235}U	0.0848
^{239}Pu	0.0324

Answer (a) is required in minutes within 1% of accuracy.

(a) Reactor period, $T =$

min

Answer (b) is required in cents within 1% of accuracy.

(b) Reactivity in cents, $\rho_c =$

Solution (a)

If a reactor is placed on period T, its power level varies as

$P(t) = P_0 e^{t/T}$

Let time t be measured since the maneuver start then we have

$P(\Delta t) = P_0 e^{\Delta t/T} = P_1 \longrightarrow \Delta t/T = \ln P_1/P_0 \longrightarrow T = \frac{\Delta t}{\ln P_1/P_0}$

Solution (b)

The (asymptotic) reactor period is given by the smallest root of the reactivity equation

$T = 1/\omega_1$

The general reactivity equation reads

$\rho = \frac{\omega l_p}{1+\omega l_p} + \frac{\omega}{1+\omega l_p} \sum \frac{\beta_i}{\omega + \lambda_i}$

Here, β_i and λ_i refer to the ith delayed neutron group. Let the mean lifetime be denoted

$\bar{t}_i \equiv 1/\lambda_i$

Small reactivity results in a small value for the first root, ω_1, which gives grounds to neglect ω_1 in the denominators thus leading to

$\rho \approx \omega_1 \left(l_p + \sum \frac{\beta_i}{\lambda_i} \right) = \omega_1 \left(l_p + \sum \beta_i \bar{t}_i \right) \approx \frac{1}{T_1} \sum \beta_i \bar{t}_i$

Express the following reactivities of a ^{239}Pu-fueled thermal reactor in percent:

(a) 1.2 dollars;

(b) 47 cents.

Data that might be useful

Table 7.2 (Lamarsh, p.343) Delayed neutron fractions

Nuclide	β (thermal)	β (fast)
^{232}Th	—	0.0203
^{233}U	0.0026	0.0026
^{235}U	0.0065	0.0064
^{238}U	—	0.0148
^{239}Pu	0.0021	0.0020

Answers have a margin of error of 1%.

(a) 1.2 dollars =

%

(b) 47 cents =

First, we convert the reactivity given in cents, if any, to that in dollars.

$\rho_{\$} = \rho_{\text{cent}}/100$

Second, if β is the delayed neutron fraction, the reactivity is found as

$\rho = \rho_{\$} \times \beta$

Third, the reactivity in percents is given by

$\rho_{\%} = \rho \times 100\%$

Operators want to keep the power of a large thermal power reactor (fueled by ^{235}U) slowly decreasing during a time interval $\Delta t = 4$ min, so that at the end of the time interval the power would be decreased to 95 % of the original power (i.e., the power at the beginning of the time interval). At approximately what reactivity do the operators need to maintain the reactor during this time interval?

The operators need to maintain the reactivity of the reactor at about

Clearly, a small reactivity is needed since the power change is relatively small and the time interval is relatively large. Therefore, we can again use approximation of the reactor period for a very small reactivity,

$$T = \left(\Lambda + \sum_i \frac{\beta_{\mathrm{eff}\,i}}{\lambda_i}\right)/\rho$$

where the generation time can be again neglected and the term

$$\sum_i \frac{\beta_{\mathrm{eff}\,i}}{\lambda_i}$$

equals about 0.0848. So, the period is then

$$T = 0.0848/\rho$$

and the power P at the end of the time interval Δt can be roughly approximated as

$$P = P_0 e^{\Delta t/T} = P_0 e^{\Delta t \times \rho/0.0848}$$

from where we can express the reactivity as

$$\rho = \frac{0.0848 \ln \frac{P}{P_0}}{\Delta t}$$

A ^{235}U-fueled reactor originally operating at a constant power of 1 milliwatt is placed on a positive 50-minute period. At what time will the reactor power level reach 7 megawatt?

Given

- Initial power, $P_0 = 1$ mW;
- Final power, $P_f = 7$ MW;
- Reactor period, $T = 50$ min.

Answer is required with a margin of error of 1%.

The reactor power level reaches 7 MW in $t =$

The reactor power level varies as

$P(t) = P_0 e^{t/T}$

The reactor power level reaches P_f when

$P(t) = P_0 e^{t/T} = P_f \longrightarrow t = T \ln(P_f/P_0)$

A ^{235}U-fueled thermal reactor operating at a constant power of 5 megawatt is scrammed by the instantaneous insertion of 7 dollars of negative reactivity. Approximately how long does it take the power level to drop to 3 milliwatt?

Given

- Original power, P_0 = 5 MW;
- Final power, P_f = 3 mW;
- Inserted reactivity, ρ = -7 dollars.

Data that might be useful

Data that might be useful

Table 5. Decay constants for delayed neutron precursors

Nuclide	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
^{233}U	0.0126	0.0337	0.139	0.325	1.13	2.50
^{235}U	0.0124	0.0305	0.111	0.301	1.14	3.01
^{239}Pu	0.0128	0.0301	0.124	0.325	1.12	2.69

Table 7.2 (Lamarsh, p.343) Delayed neutron fractions

Nuclide	β (thermal)	β (fast)
^{232}Th	—	0.0203
^{233}U	0.0026	0.0026
^{235}U	0.0065	0.0064
^{238}U	—	0.0148
^{239}Pu	0.0021	0.0020

Answer is required in minutes within 1% of accuracy.

The requested time is t =

A solution may be based on the following observations:

- If a reactor operates at a constant power, it is critical, $k = 1$;
- In case of an instantaneous insertion, the Prompt Jump/Drop model is valid;
- The reactor power decreases in two stages
  - A sudden drop to a new power level, P_1;
  - Exponential decay with a reactor period T;
- With large negative reactivity insertions, the value of ω_1 aproaches λ_1, the decay constant for the longest lived precursor.

If $\phi_{T,0}$ is the thermal flux before the drop and $\phi_T$is the thermal flux after the drop, it holds (Eq. 7.39 in Lamarsh)

$$\phi_T = \frac{\beta(1-\rho)}{\beta-\rho}\phi_{T,0}$$

Accordingly, the power level drops at the same pace

$$P_1 = \frac{\beta(1-\rho)}{\beta-\rho}P_0$$

With large negative reactivity insertions, the (negative) reactor period has the value

$$T = \frac{1}{\omega_1} \approx \frac{1}{\lambda_1}$$

The reactor power level then decreases as

$$P(t) = P_1 e^{-t/T}$$

The time needed for the reactor power level to drop to P_f is thus found through the equation

$$P(t) = P_1 e^{-t/T} = P_f \longrightarrow t = -T \ln \frac{P_f}{P_1}$$

A neutron is scattering in an infinite block of ^{238}U at a low temperature. The energy of the neutron is exactly at the peak of one of the resonances of the absorption cross sections of ^{238}U. Assume that the probability that the neutron is absorbed in the next collision with ^{238}U (at its current energy) is 0.9912. (We know that if the temperature of ^{238}U increased then its absorption resonances would broaden so the neutrons in the vicinity of the peaks would be more likely to be absorbed. Nevertheless, we also know that the peak centres would decrease at the same time, so our particular neutron would actually be less likely to be absorbed in the next collision - if the temperature increased.)

Assume now that the temperature of ^{238}U material really did increase, and consequently the value of the absorption cross section of ^{238}U decreased 9 times for the energy of the particular neutron. What is now the probability p_{hot} that the neutron is absorbed in the next collision with ^{238}U at the higher temperature?

Assume that the scattering cross section is not temperature-dependent.

Answers are required with a margin of error of 1%.

p_{hot} =

The probability that the neutron is absorbed in the next collision with ^{238}U at the low temperature (at its current energy) is

$$p = \frac{\sigma_a}{\sigma_a + \sigma_s}$$

where σ_a and σ_s are microscopic cross section for absorption and scattering on ^{238}U at the low temperature. From the above equation, we can derive the scattering cross section as

$$\sigma_s = \frac{1-p}{p}\sigma_a$$

As the temperature of ^{238}U was increased, the value of the absorption cross section of ^{238}U decreased x times for the energy of the particular neutron, so the absorption cross section of ^{238}U at the hight temperature is $\sigma_a^{hot} = \sigma_a/x$, and the probability that the neutron is absorbed in the next collision with ^{238}U at the high temperature (at its current energy) is therefore

$$p_{hot} = \frac{\sigma_a/x}{\sigma_a/x + \sigma_s}$$

where σ_s is not changed and it is given by the formula above, so we can write

$$p_{hot} = \frac{\sigma_a/x}{\sigma_a/x + \frac{1-p}{p}\sigma_a} = \frac{1/x}{1/x + \frac{1-p}{p}}$$

A ^{235}U-fueled reactor operating at a thermal flux of 5×10^{14} n/(cm^{2}s) and a temperature of 200°C is scrammed at a time when the reactor has 5% in reserve reactivity. Determine whether or not the reactor can be restarted at time 49.2 hours after the scram.
Hint: Suppose the reactor contains no resonance absorber or fissionable material other than ^{235}U.

Given:

- Operating temperature, T = 200°C;
- Thermal flux, $\phi_T = 5\times10^{14}$ n/(cm^{2}s)
- Reserve reactivity, $\rho_0 = 5\%$;
- Restart time, $t = 49.2$ hours after the scram.

If a reactor contains no resonance absorber or fissionable material other than ^{235}U, the resonance escape probability, p, and the fast fission factor, ε, may be assumed to be

$p = \varepsilon = 1$

Additionally, we assume that the xenon and iodine had reached equilibrium prior to shutdown than the reactivity due the xenon buildup is (Eq.7.103)

$$\rho(t) = -\frac{1}{\nu}\left[\frac{(\gamma_I+\gamma_X)\phi_T}{\phi_X+\phi_T}e^{-\lambda_X t} + \frac{\gamma_I\phi_T}{\phi_I-\phi_T}\left(e^{-\lambda_X t} - e^{-\lambda_I t}\right)\right]$$

Here, ϕ_I and ϕ_X are the temperature-dependent parameters

$$\phi_I = \frac{\lambda_I}{\bar{\sigma}_{a,X}} \text{ and } \phi_X = \frac{\lambda_X}{\bar{\sigma}_{a,X}}$$

Further, the thermal absorption cross-section of ^{135}Xe evaluates as

$$\bar{\sigma}_{a,Xe} = \frac{\sqrt{\pi}}{2}g_a(T)\sigma_{a,Xe}(E_0)\left(\frac{T_0}{T}\right)^{1/2}$$

Clearly, the reactor can be restarted at time t if the reserve reactivity ρ_0 can compensate the negative reactivity due ^{135}Xe

$$\rho_0 + \rho(t) \geq 0$$

A power reactor is suddenly shut down at time t_0=0s. At that moment, the concentration of ^{135}I in the fuel was I_0 = 4.08E15 atoms/cm^3 and the concentration of ^{135}Xe was X_0= 4.32E14 atoms/cm^3. The concentration of xenon will start to grow after shut down. At what time t will the xenon concentration reach its maximum before it starts to decay? (Margin for error of 5%.)

The value of t=

The concentration of xenon X after shutdown is described by the equation

$$X(t) = X_0 e^{-\lambda_X t} + \frac{\lambda_I I_0}{\lambda_I - \lambda_X}(e^{-\lambda_X t} - e^{-\lambda_I t})$$

So, we need to locate t for the maximum X in the above equation. Let A denote the term

$$A \equiv \frac{\lambda_I I_0}{\lambda_I - \lambda_X}$$

to simplify the equation into

$$X(t) = X_0 e^{-\lambda_X t} + A(e^{-\lambda_X t} - e^{-\lambda_I t})$$

and let's calculate the derivative of X with respect to t,

$$dX(t)/dt = -\lambda_X X_0 e^{-\lambda_X t} + A(-\lambda_X e^{-\lambda_X t} + \lambda_I e^{-\lambda_I t}) = 0$$

Let's modify the above equation into the form

$$(\lambda_X X_0 + A\lambda_X)e^{-\lambda_X t} = A\lambda_I e^{-\lambda_I t}$$

which equals

$$e^{\ln(\lambda_X X_0 + A\lambda_X)} e^{-\lambda_X t} = e^{\ln A\lambda_I} e^{-\lambda_I t}$$

which equals

$$e^{\ln(\lambda_X X_0 + A\lambda_X) - \lambda_X t} = e^{\ln A\lambda_I - \lambda_I t}$$

which equals

$$\ln(\lambda_X X_0 + A\lambda_X) - \lambda_X t = \ln(A\lambda_I) - \lambda_I t$$

which gives

$$t = \frac{\ln\left(\frac{A\lambda_I}{\lambda_X X_0 + A\lambda_X}\right)}{\lambda_I - \lambda_X}$$

Calculate the prompt temperature coefficient of reactivity at temperature 25° C of a reactor lattice consisting of an assembly of natural uranium (metallic) rods with a diameter of 3.4 cm in a heavy water moderator, in which the moderator volume to fuel volume ratio is 30.

Margin of error 1.5%.

α_{prompt} =

The prompt temperature coefficient of reactivity is given by the formula

$$\alpha_{prompt} = -\frac{\beta_I}{2\sqrt{T}} ln \frac{1}{p(300K)}$$

where β_I is given by the empirical formula

$$\beta_I = A' + \frac{C'}{a\rho}$$

Here a is the fuel rod radius in centimeters, i.e. a = 1.5 cm, ρ is the fuel density in g/cm^3, i.e. ρ = 19.1 g/cm^3 (for metallic natural uranium) and the coefficients A´ and C´ are found in the tables for metallic ^{238}U, A´ = 48×10^{-4}, C´ = 1.28×10^{-2} . That gives β_I = 0.0052.

The resonance escape probability is approximated by

$$p = exp\left(-\frac{N_F I}{\xi_M \Sigma_s^M} \frac{V_F}{V_M}\right)$$

The volume ratio is defined in the problem, $V_F/V_M = 1/30$, the moderation power can be found in the tables as $\xi_M \Sigma_s^M$, and the fuel atomic density is evaluated as:

$$N_F = \frac{\rho N_A}{M} = \frac{19.1\times6.022\times10^{23}}{238.03} = 0.0483 \times 10^{24} \frac{\#}{cm^3}$$

The resonance integral, I, is empirically given by

$$I = A + \frac{C}{\sqrt{a\rho}}$$

Here A and C are found in the tables, A = 2.8, C = 38.3 that evaluates the integral as

$$I = 2.8 + \frac{38.3}{\sqrt{1.5\times19.1}}$$

We thus evaluate the resonance probability as

$$p = exp\left(-\frac{0.0483\times9.96}{0.18} \frac{1}{30}\right) = 0.915$$

Finally, we arrive at the answer

$$\alpha_{prompt} = -\frac{0.0052}{2\sqrt{300}} ln \frac{1}{0.915} = -0.004/K$$

Calculate the prompt temperature coefficient at 90 °C of a reactor lattice consisting of an assembly of 5-inch diameter Th-232 dioxide rods in a graphite moderator, in which the moderator volume-to-fuel volume ratio is equal to 70.

Given

- Fuel, Th-232 dioxide;
- Moderator, graphite;
- Fuel rods' diameter $d = 5$ in;
- Working temperature, $T = 90$ °C;
- Moderator-to-fuel volume ratio, $V_M/V_F = 70$;
- Reference temperature, $T = 300$ K.

Data that might be useful

- Celcius to kelvin conversion, 0 °C = 273.15 K;
- Inch to centimeter conversion, 1 in = 2.54 cm;
- Avogadro's number, $N_A = 6.02214086\times10^{23}$ #/mol;
- Parts per cent mille, 1 pcm = 10^{-5}.

Table 6.5 (Lamarsh, p.317) Constants for computing the resonance integral I

Fuel	M [g/mol]	ρ [g/cm^3]	A	C
^{238}U (metal)	238.051	19.1	2.8	38.3
$^{238}UO_2$	270.028	11.0	3.0	39.6
^{232}Th (metal)	232.038	11.7	3.9	20.9
$^{232}ThO_2$	264.037	10.0	3.4	24.5

Table 6.6 (Lamarsh, p.317) Values of $\xi_M\Sigma_{s,M}$

Moderator	$\xi_M\Sigma_{s,M}$
Water	1.46
Heavy water	0.178
Beryllium	0.155
Graphite	0.0608

Answer is required with the correct sign in pcm/K unit within 1% of accuracy.

Prompt temperature coefficient at $T = 90$ °C, α_T =

The prompt temperature coefficient is well approximated by

$$\alpha_T = -\frac{\beta_I}{2\sqrt{T}}\ln\left[\frac{1}{p(300\mathrm{K})}\right]$$

The empirical parameter β_I reads as

$$\beta_I = A' + C'/a\rho$$

Here, A' and C' are measured constants given in Table 7.4, a is the fuel rod radius in cm and ρ is the density of the fuel in g/cm^3.

The resonance escape probability p may be approximated by

$$p = \exp\left(-\frac{N_F V_F I(T)}{\xi_M \Sigma_{s,M} V_M}\right)$$

Here, I is the resonance integral, V_F and V_M are the volumes in a unit cell of fuel and moderator, respectively, N_F is the atom density of the fuel in units of 10^{24}, $\Sigma_{s,M}$ is the macroscopic scattering cross-section of the moderator, and ξ_M is a constant. Values of I at the reference temperature (300K) for cylindrical fuel rods are well represented by the following empirical expression

$$I(300\mathrm{K}) = A + C/\sqrt{a\rho}$$

Here, A and C are measured constants given in Table 6.5, a is the fuel rod radius and ρ is the density of the fuel at the reference temperature. It should be noted, the above formula gives the resonance integral I in units of barns, however, a must be given in cm and ρ in g/cm^3.

A pressurized water reactor fueled with stainless steel-clad fuel elements is contained in a stainless steel vessel that is a cylinder 6 ft in diameter and 8 ft high. Water having an average temperature of 300°C occupies approximately 55% of the reactor volume. What is the volume fraction in percent of the water expelled from the reactor vessel if the average temperature of the system is increased by 7°C? The volume coefficients of expansion of water and stainless steel at 300°C are 3×10^{-3} per °C and 4.5×10^{-5} per °C, respectively.

Given

- Operating temperature, $T = 300$°C;
- Temperature increase, $\Delta T =$ 7°C;
- Volume coefficients of expansion at 300°C:
  - Moderator (water), $\beta_M = 3\times10^{-3}$/°C;
  - Fuel (and vessel) $\beta_F = 4.5\times10^{-5}$/°C;
- Fuel-to-vessel volume fraction, $q = V_F/V = 0.45$

Data that might be useful

- 0°C = 273.15 K;
- Imperial/US unit of foot (feet), 1 ft = 12 in = 0.3048 m.

Answer is required in percent with 1% of accuracy

Volume fraction =

The volume expansion coefficient is defined as

$$\beta \equiv \frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}T}$$

It follows then a small increase in temperature results in a small increase in volume

$$\Delta V \approx \beta V \Delta T$$

Let V_M be the volume of the moderator (water) in the reactor vessel. The fuel is contained in stainless steel elements. The vessel is also made of stainless steel. Therefore, we assume the same coefficient of volumetric expansion, β_F, for both the fuel and the vessel. The geometric volume occupied by the moderator expands at the same rate as the restor vessel, i.e.

$$\Delta V_{\mathrm{M,geom}} = \beta_F V_M \Delta T$$

On contrary, the physical volume of the moderator initially present in the reactor vessel expands at the rate determined by the moderator (water) physical properties, i.e.

$$\Delta V_{\mathrm{M,phys}} = \beta_M V_M \Delta T$$

Clearly, the volume of the moderator (water) expelled from the reactor vessel is given by

$$\Delta V_{\mathrm{M,exp}} = \Delta V_{\mathrm{M,phys}} - \Delta V_{\mathrm{M,geom}} = (\beta_M - \beta_F)\, V_M \Delta T$$

The volume fraction of the expelled water is finally found as

$$\delta \equiv \frac{\Delta V_{\mathrm{M,exp}}}{V_M} = \frac{(\beta_F - \beta_V) V_M \Delta T}{V_M} = (\beta_M - \beta_F)\, \Delta T$$