# Sustainable Energy Transformation Technologies, SH2706

Lecture No 9

Title:

Energy transformation and degradation in two-phase flows

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# Outline

- Introduction
  - Two-phase flow patterns
  - Notation and flow variables
- Volume fraction (void) prediction
  - Homogeneous Equilibrium Model
  - Drift-Flux Model
- Pressure drop prediction
  - Momentum balance equation
  - Local pressure losses and drops
  - Friction pressure losses
  - Gravity pressure drop
  - Acceleration pressure drop

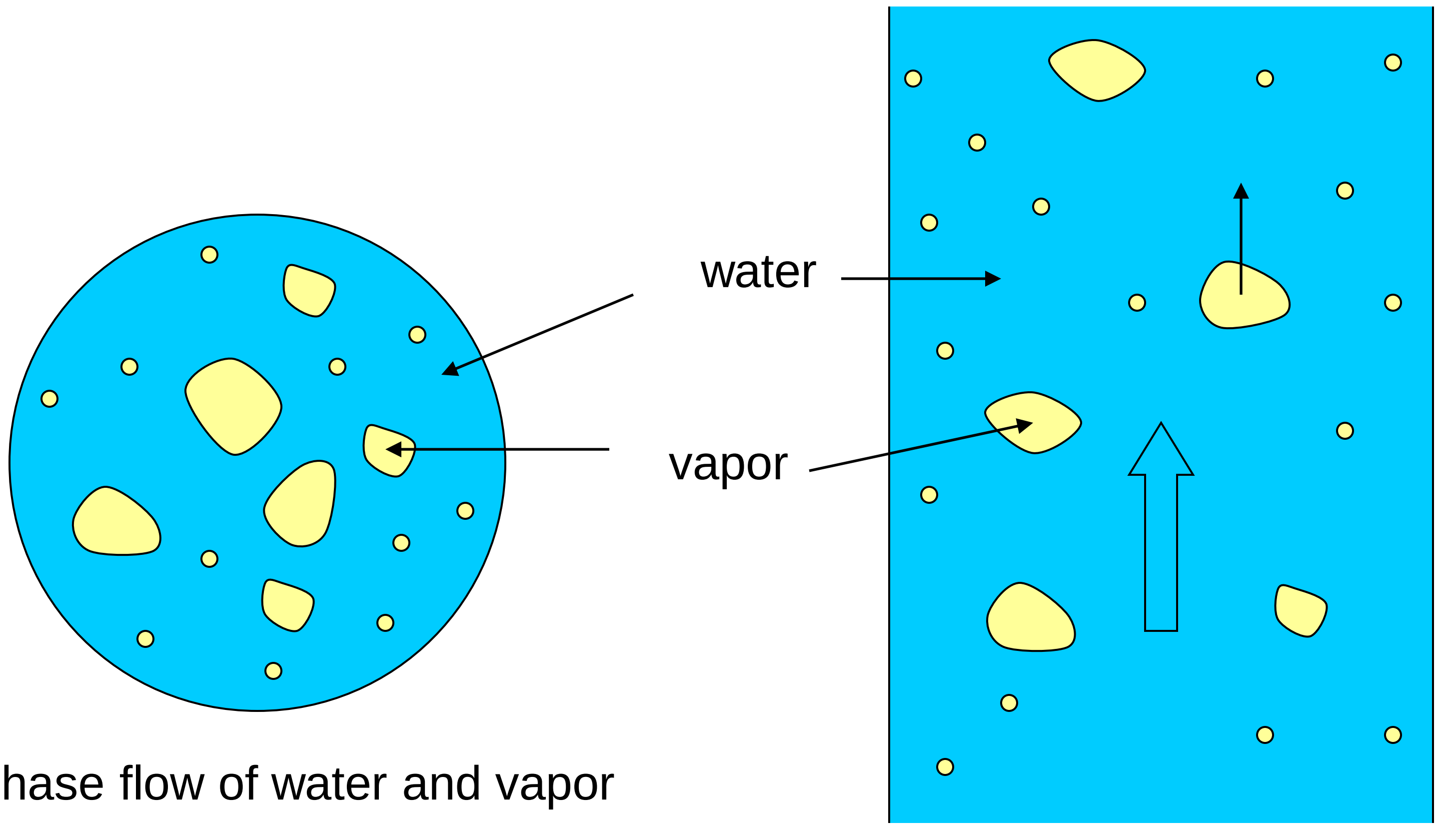
# Introduction (1)

- The term two-phase flow is used to refer to any fluid flow consisting of two phases or components, which are mixed at scales well above the molecular level, with a clear interface between the phases/components
  - phases can be discerned by observation/measurement

- The phases can be of the same species
  - water flowing with water vapor
- Or of different species
  - water flowing with air

# Introduction (2)

## Example:




two-phase flow of water and vapor

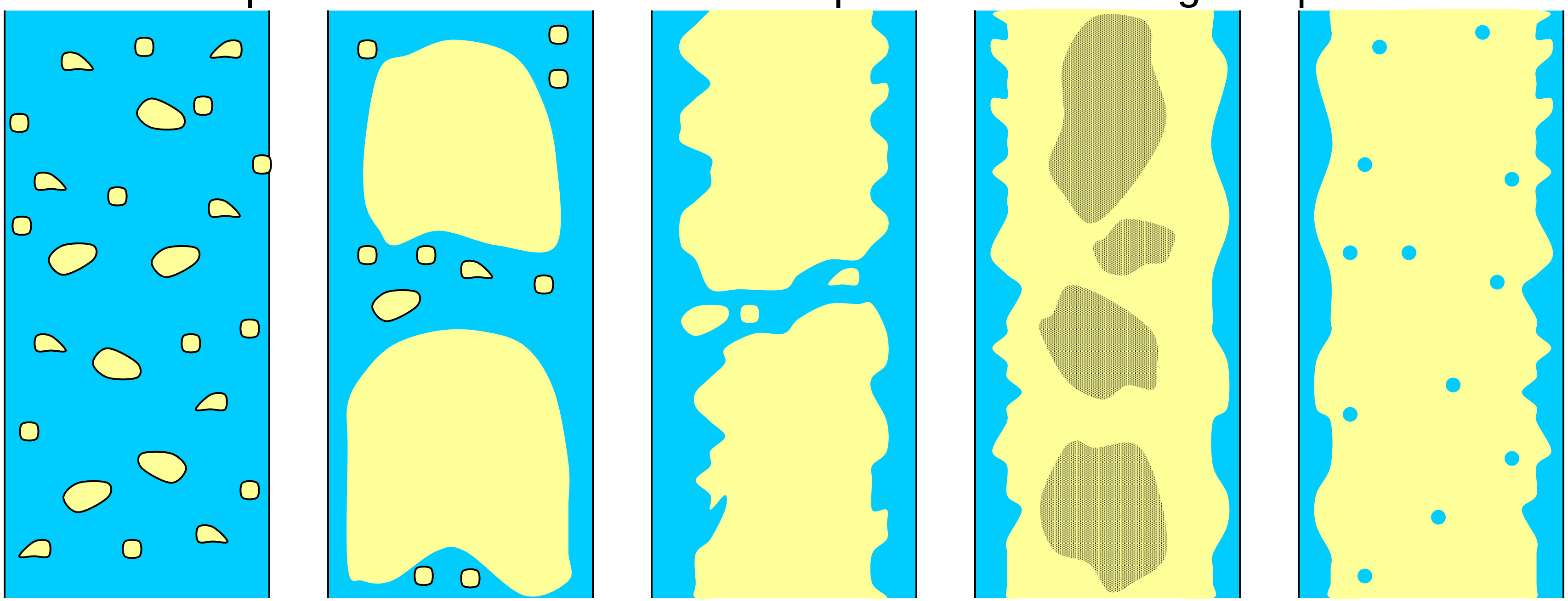
# Introduction (3)

- Scope of this course: in this course we will:
  - focus on two-phase flows dominating in energy technology (liquid/gas)
  - consider one-dimensional, stationary two-phase flow
  - predict void fraction and pressure drop

# Two-Phase Flow Patterns

Various flow patterns (or regimes) are observed in vertical two-phase flows

Flow patterns observed in vertical upward co-current gas-liquid flow



Bubbly flow | Slug flow | Churn flow | Wispy-annular flow | Annular flow

# Phase Indices (1)

- Parameters describing a specific phase will be designated with a subscript
- *k* will be use to indicate phase “k”; e.g. $u_k$ will mean a velocity of phase k
- in two-phase flow (for example water and vapor), the liquid phase will have subscript *l*, and the gas phase *v*
- The same notation will be used for two-component flow (e.g. water and air)

# Phase Indices (2)

- For water-vapor flow at saturation conditions, *f* will be used to indicate the saturated water and *g* the saturated vapor

- subscript *fg* will be used to indicate parameter difference between phases, e.g. $i_{fg} = i_g - i_f$ (latent heat), where *i* [J/kg] is the specific enthalpy

- Similarly we define specific volume ($\upsilon = \frac{1}{\rho}$) difference between phases

$$\upsilon_{fg} \equiv \upsilon_g - \upsilon_f = \frac{1}{\rho_g} - \frac{1}{\rho_f} = \frac{\rho_f - \rho_g}{\rho_f \rho_g}$$

# Volumetric Fluxes

**volumetric fluxes** in two phase flow can be calculated separately for each phase:

$$J_k = \frac{Q_k}{A} = \frac{1}{A}\int_{A_k} u_k dA_k = \frac{A_k}{A}\frac{1}{A_k}\int_{A_k} u_k dA_k = \alpha_k U_k$$

$k = l$ or $v$, for liquid and gas phase, respectively

We introduced here **volume fraction** of phase $k$, defined as $\alpha_k = A_k/A$

Sometimes volumetric flux is referred to as **superficial velocity**

# Mass Fluxes

**mass fluxes** in two phase flow can be calculated separately for each phase:

$$G_k = \frac{W_k}{A} = \frac{1}{A}\int_{A_k} \rho_k u_k dA_k = \frac{A_k}{A}\frac{1}{A_k}\int_{A_k} \rho_k u_k dA_k = \alpha_k \left\langle \rho_k u_k \right\rangle$$

$k = l$ or $v$, for liquid and gas phase, respectively

Assuming constant phasic density, $\rho_k$ = const

$$G_k = \alpha_k \rho_k U_k = \rho_k J_k$$

we use notation

$\langle p \rangle$ to indicate cross-section mean value of variable $p$.

In particular

$U = \langle u \rangle$ and $U_k = \langle u_k \rangle$

# Volume Fraction

**Volume fraction** of phase *k* has been defined as:

$$\alpha_k \equiv \frac{A_k}{A}$$

Assuming liquid/gas two-phase flow:

$$\alpha_l \equiv \frac{A_l}{A} \qquad \alpha_v \equiv \frac{A_v}{A}$$

Since $A_l + A_v = A$

$$\alpha_l + \alpha_v = 1$$

In nuclear applications $\alpha_G$, often just designed as $\alpha$ is referred to as the **void fraction**

# Total Mass Flux

- **Total mass flux** in two phase flow is a sum of the component mass fluxes:

$$G_l + G_v = \alpha_l \rho_l U_l + \alpha_v \rho_v U_v = \rho_l J_l + \rho_v J_v = G$$

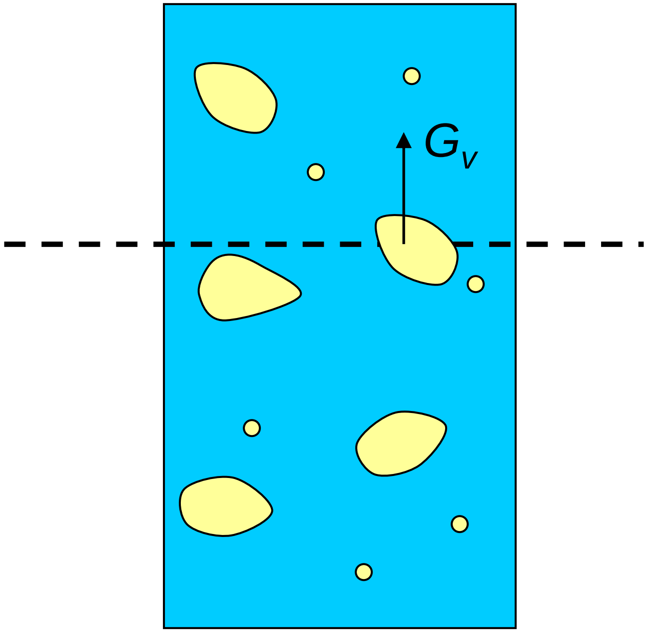
- Also total volumetric flux is a sum of the component volumetric fluxes, since

$$J = \frac{Q}{A} = \frac{Q_l}{A} + \frac{Q_v}{A} = J_l + J_v$$

# Actual (Flow) Quality (1)

- **actual (flow) quality** in two phase flow is defined as the ratio of the mass flux of gas (vapor phase) to the total mass flux:

$$x_a = \frac{G_v}{G} = \frac{\rho_v J_v}{\rho_v J_v + \rho_l J_l}$$



# Actual (Flow) Quality (2)

- Actual quality can be expressed in terms of void fraction as follows:

$$x_a = \frac{\rho_v J_v}{\rho_v J_v + \rho_l J_l} = \frac{\rho_v \alpha U_v}{\rho_v \alpha U_v + \rho_l (1-\alpha) U_l} =$$

$$\boxed{\frac{1}{1 + \frac{(1-\alpha)}{\alpha} \frac{\rho_l}{\rho_v} \frac{U_l}{U_v}}}$$

# Thermodynamic Equilibrium Quality

- The thermodynamic equilibrium quality is defined in terms of a mixture thermodynamic property related to the difference of that property between saturated vapor and saturated liquid condition
- Usually we use the specific enthalpy as the thermodynamic property and then the thermodynamic equilibrium quality is defined as:

$$x_e \equiv \frac{i_m - i_f}{i_g - i_f} = \frac{i_m - i_f}{i_{fg}}$$

$i_m$ – mixture specific enthalpy
$i_f$ – saturated liquid specific enthalpy
$i_g$ - saturated vapor specific enthalpy
$i_{fg}$ – latent heat

Note that $0<x_e<1$ for two phase mixture; $x_e> 1$ for **superheated vapor** and $x_e<0$ for **subcooled liquid**; $x_e=0$ for **saturated liquid** and $x_e=1$ for **saturated vapor**

# Boiling Channel (1)

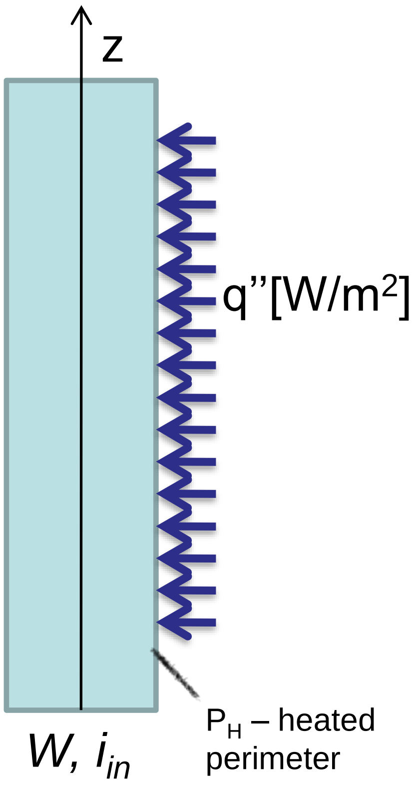
- In a boiling channel two phases co-exist, but the mixture quality, enthalpy, and void fraction are changing along the channel due to the phase change.

- Consider a uniformly heated channel:
  - *W* – inlet mass flow rate, $i_{in}$ – inlet enthalpy
  - q'' – heat per unit area and time (heat flux)
- At a distance *z* from the inlet, the enthalpy will be:

$$W\left(i - i_{in}\right) = q'' \cdot P_H z \qquad i(z) = i_{in} + \frac{q'' \cdot P_H}{W} z$$

- And the thermodynamic equilibrium quality $x_e$:

$$x_e(z) \equiv \frac{i(z) - i_f}{i_{fg}} = \frac{i_{in} - i_f}{i_{fg}} + \frac{q'' \cdot P_H}{W \cdot i_{fg}} z$$



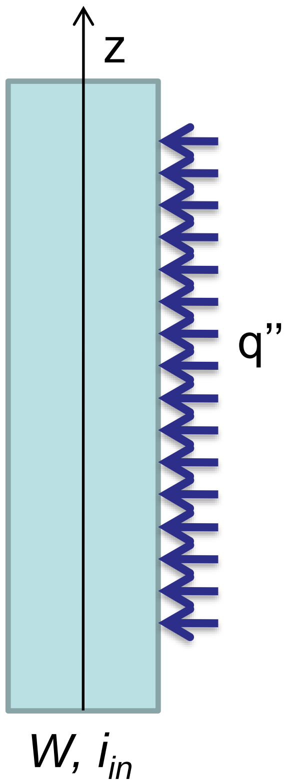
# Boiling Channel (2)

- Thus, for uniformly heated channel, the equilibrium thermodynamic quality changes linearly as:

$$x_e(z) \equiv x_{e,in} + \frac{q'' \cdot P_H}{W \cdot i_{fg}} z$$

- Here $x_{e,in}$ is the inlet equilibrium quality.

- Note that the equilibrium thermodynamic quality can be either negative (subcooled liquid) or positive less than 1 (two-phase mixture) or larger than 1 (superheated vapor)



# Boiling Channel (3)

- If axial distribution of heat flux is non-uniform, the differential energy balance yields:

$$Wdi = q''(z) \cdot P_H dz \quad \Rightarrow \quad di = \frac{P_H}{W} q''(z) \cdot dz$$

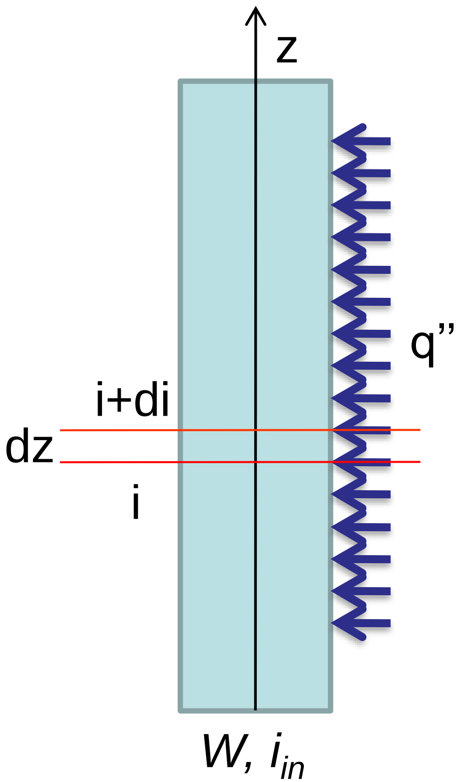
$$\int_0^z di = i(z) - i_{in} = \frac{P_H}{W} \int_0^z q''(z) \cdot dz$$

- Thus the enthalpy distribution is:

$$i(z) = i_{in} + \frac{P_H}{W} \int_0^z q''(z) \cdot dz$$

- And the quality distribution:

$$x_e(z) = x_{e,in} + \frac{P_H}{W i_{fg}} \int_0^z q''(z) \cdot dz$$



# Void-Quality Relationship (1)

- In the same manner, void fraction can be expressed in terms of quality as follows: since $x_a = G_v/G$, we have:

$$x_a\left[\rho_v \alpha U_v + \rho_l (1-\alpha) U_l\right] = \rho_v \alpha U_v$$

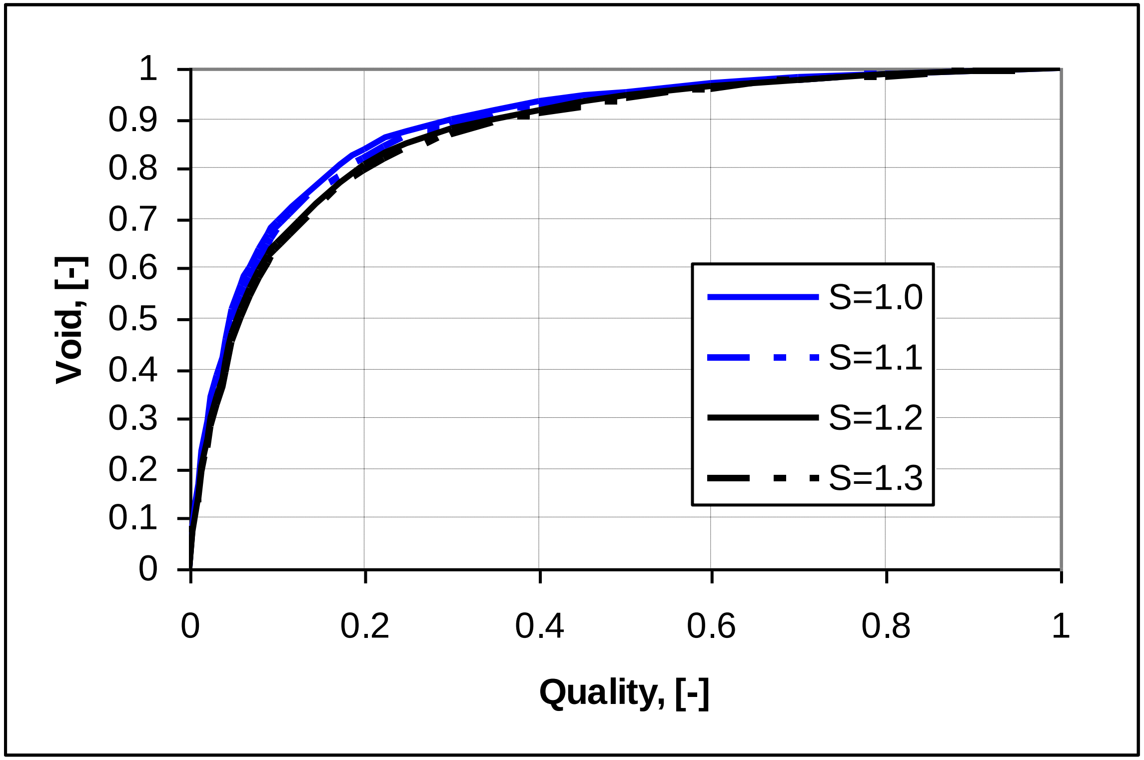
$$x_a \rho_v \alpha U_v - x_a \rho_l \alpha U_l - \rho_v \alpha U_v = -x_a \rho_l U_l$$

$$\alpha = \frac{-x_a \rho_l U_l}{x_a \rho_v U_v - x_a \rho_l U_l - \rho_v U_v} = \boxed{\frac{1}{1 + \frac{1-x_a}{x_a}\frac{\rho_v}{\rho_l}\frac{U_v}{U_l}}}$$

Phases usually move with different velocities, and the ratio $S=U_v/U_l$, called "**slip ratio**" is not equal to 1!

# Void-Quality Relationship (2)

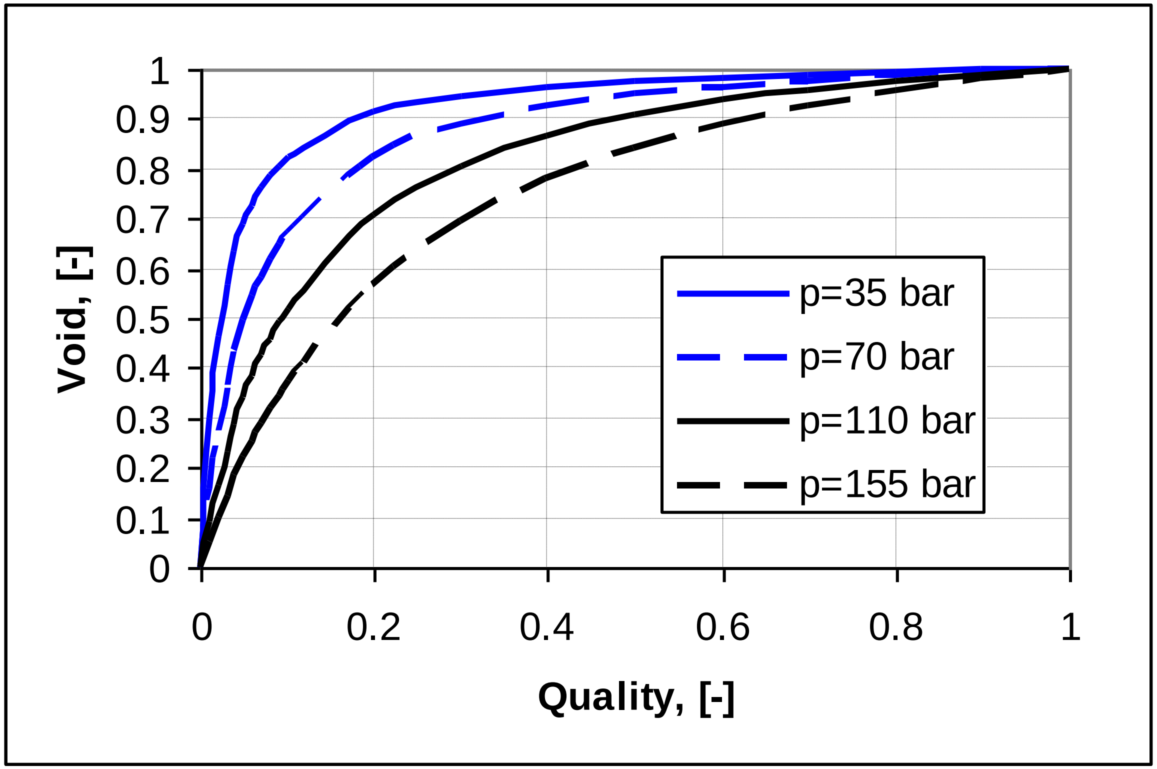
- Void-quality relationship for two-phase water/vapor flow at 7 MPa pressure




Four curves are plotted with various values of the slip ratio

# Void-Quality Relationship (3)

- Void-quality relationship for two-phase water/vapor flow with slip ratio S = 1.1




Four curves are plotted with various values of the system pressure

# Homogeneous Equilibrium Model (1)

- Homogeneous Equilibrium Model (HEM) is the simplest model to predict the two-phase flow behavior
  - the term ***homogeneous flow*** denotes flow with negligible relative motion of phases allowing to treat the two-phases as a homogeneous mixture
  - the term ***equilibrium flow*** denotes flow of phases that are in the thermodynamic equilibrium

# Homogeneous Equilibrium Model (2)

- HEM belongs to the class of **one-fluid models** of two-phase flows
- Another example of one-fluid model is the Homogeneous Relaxation Model (HRM), where the equilibrium assumption is not used
- In this course only HEM will be used

# Homogeneous Equilibrium Model (3)

- In the absence of relative motion and thermodynamic non-equilibrium, the mass, momentum and energy equations for the mixture reduce to the single-phase form

- We will use the formulation derived for the stationary, single phase, one-dimensional flow in a channel

# Homogeneous Equilibrium Model (4)

- Since the phases are treated as a homogeneous mixture, the slip ratio is equal to 1 (phases are moving with the same velocity)

- Thus the void-quality relationship in HEM reduces to:

$$\boxed{\alpha = \frac{1}{1 + \frac{1 - x_a}{x_a} \frac{\rho_v}{\rho_l}}}$$

- Thus to calculate void fraction, it is enough to know actual quality and density ratio

# Homogeneous Equilibrium Model (5)

The mixture density can be obtained as:

$$\rho_m \equiv \frac{m_m}{V_m} = \frac{V_g \rho_g + V_f \rho_f}{V_m} = \alpha \rho_g + (1-\alpha)\rho_f = \rho_f - \alpha(\rho_f - \rho_g)$$

since: $\alpha = \dfrac{1}{1 + \dfrac{1-x_a}{x_a}\dfrac{\rho_g}{\rho_f}}$

$$\rho_m = \frac{\rho_f \rho_g}{x_a \rho_f + (1-x_a)\rho_g} \boxed{= \frac{1}{\upsilon_f + x_a \upsilon_{fg}}}$$

$x_a$ – actual quality

where: $\upsilon_{fg} = \upsilon_g - \upsilon_f$

$\upsilon_f = 1/\rho_f$ and $\upsilon_g = 1/\rho_g$ are specific volumes of liquid and vapor, respectively

# Homogeneous Equilibrium Model (6)

- The mixture enthalpies are defined as:

$$i_m = i_f \left(1 - x_e\right) + i_g x_e$$

$x_e$ – thermodynamic equilibrium quality

# Homogeneous Equilibrium Model (7)

- When phases are in the thermodynamic equilibrium, the actual quality $x_a$ is equal to the thermodynamic equilibrium quality $x_e$:

$$x_a \equiv \frac{G_g}{G} = x_e \equiv \frac{i_m - i_f}{i_{fg}}$$

- Proof:

Combining the mass and energy conservation equations yields:

$$\frac{di_m}{dz} = \frac{q'' \cdot P_H}{GA} \Rightarrow di_m = \frac{q'' \cdot P_H dz}{GA} = \frac{i_{fg} dG_g}{G}$$

That is:

$$\frac{di_m}{dz} = \frac{q'' \cdot P_H}{GA} \Rightarrow \frac{di_m}{i_{fg}} \equiv dx_e = \frac{dG_g}{G} \equiv dx_a$$

($dG_g=q''*P_H*dz/Ai_{fg}$ since all heat is used to generate vapor)

Finally: $x_e = x_a$

# Drift-Flux Model

- The drift-flux void correlation expresses area-averaged void fraction in terms of superficial velocity of vapor and the total superficial velocity.

$$\langle \alpha \rangle = \frac{J_v}{C_0 J + U_{vj}}$$

- Two additional parameters are needed:
  - $C_0$ – distribution parameter
  - $U_{vj}$ – drift velocity
- Both these parameters are flow-regime dependent and need to be known to obtain void fraction.

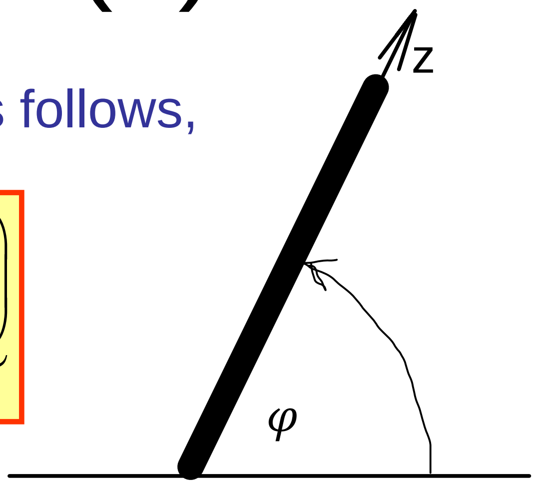
# Drift-Flux Model

- $C_0$ and $U_{vj}$ values ($p_c$ – critical pressure)

| Flow pattern | Distribution parameter | Drift velocity |
|---|---|---|
| bubbly $0 < \alpha \le 0.25$ | $C_0 = \begin{cases} 1 - 0.5\,p/p_c & D \ge 0.05m \\ \left. \begin{array}{ll} 1.2 & p/p_c < 0.5 \\ 1.4 - 0.4\,p/p_c & p/p_c \ge 0.5 \end{array} \right\} & D < 0.05m \end{cases}$ | $U_{vj} = 1.41\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2}\right)^{0.25}$ |
| Slug/churn $0.25 < \alpha \le 0.75$ | $C_0 = 1.15$ | $U_{vj} = 0.35\left(\frac{gD(\rho_l - \rho_v)}{\rho_l}\right)^{0.5}$ |
| Annular $0.75 < \alpha \le 0.95$ | $C_0 = 1.05$ | $U_{vj} = 23\left(\frac{\mu_l J_l}{\rho_v D_h}\right)^{0.5}\frac{(\rho_l - \rho_v)}{\rho_l}$ |
| Mist $0.95 < \alpha < 1$ | $C_0 = 1.0$ | $U_{vj} = 1.53\left(\frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2}\right)^{0.25}$ |

# Momentum Balance (1)

- The two-phase mixture momentum equation is as follows,




$$-\frac{dp}{dz} = \underbrace{\left(\frac{dp}{dz}\right)_w}_{wall\ \ friction} + \underbrace{\rho_m g \sin\varphi}_{gravity} + \underbrace{\frac{1}{A}\frac{d}{dz}\left(\frac{G^2 A}{\rho_M}\right)}_{acceleration}$$

- Where two definitions of mixture density are introduced:
  - Mixture static density $$\rho_m = \sum_k \rho_k \alpha_k$$
  - Mixture dynamic density $$\rho_M = \left(\sum_k \frac{x_k^2}{\rho_k \alpha_k}\right)^{-1}$$

# Mixture Density

- For HEM, we can show that the static and the dynamic densities are equivalent to each other

- The mixture density can be expressed in terms of the quality as,

$$\rho_m = \rho_M = \alpha\rho_v + \left(1-\alpha\right)\rho_l = \frac{\rho_l}{x\left(\dfrac{\rho_l}{\rho_v}-1\right)+1}$$

# Friction Pressure Losses (1)

- Assuming single fluid model of two-phase flow, the friction pressure gradient is given as (*tp* stands for *t*wo-*p*hase),

$$-\left(\frac{dp}{dz}\right)_{w,tp} = \frac{P_w}{A} C_{f,tp} \frac{G^2}{2\rho_m}$$

- Assuming that only liquid flows in the same channel with the same mass flux, the pressure gradient will be (*lo* stands for *l*iquid-*o*nly),

$$-\left(\frac{dp}{dz}\right)_{w,lo} = \frac{P_w}{A} C_{f,lo} \frac{G^2}{2\rho_l}$$

# Friction Pressure Losses (2)

- Dividing the two-pressure gradient expressions with each other, we get,

$$\left(\frac{dp}{dz}\right)_{w,tp} \Big/ \left(\frac{dp}{dz}\right)_{w,lo} = \frac{C_{f,tp}}{C_{f,lo}} \frac{\rho_l}{\rho_m}$$

- The above ratio is called a **two-phase pressure multiplier (with liquid-only flow as a reference)** and is defined as

$$\phi_{lo}^2 \equiv \left(\frac{dp}{dz}\right)_{w,tp} \Big/ \left(\frac{dp}{dz}\right)_{w,lo}$$

# Friction Pressure Losses (3)

- Using the definition of the two-phase pressure multiplier, the two-phase pressure gradient can be expressed as,

$$-\left(\frac{dp}{dz}\right)_{w,tp} = -\phi_{lo}^2\left(\frac{dp}{dz}\right)_{w,lo} = \frac{P_w}{A}\phi_{lo}^2 C_{f,lo}\frac{G^2}{2\rho_l}$$

- The two-phase multiplier can be calculated using, e.g. HEM model as,

$$\phi_{lo}^2 = \frac{C_{f,tp}}{C_{f,lo}}\frac{\rho_l}{\rho_m} = \frac{C_{f,tp}}{C_{f,lo}}\left[1+\left(\frac{\rho_l}{\rho_v}-1\right)x\right]$$

# Friction Pressure Losses (4)

- The friction factors can usually be expressed as functions of the Reynolds number.
- Using the Blasius formula as a prototype of such a function, the friction factors read as follows,

$$C_{f,lo} = A \cdot \mathrm{Re}_{lo}^{-a} = A\left(\frac{GD_h}{\mu_l}\right)^{-a} \qquad C_{f,tp} = B \cdot \mathrm{Re}_{tp}^{-b} = B\left(\frac{GD_h}{\mu_m}\right)^{-b}$$

- Assuming next that coefficients for single-phase and two-phase are equal, that is *A* = *B* and *a* = *b*, we get

$$\phi_{lo}^2 = \left(\frac{\mu_m}{\mu_l}\right)^b \left[1 + \left(\frac{\rho_l}{\rho_v} - 1\right)x\right]$$

# Friction Pressure Losses (5)

- The remaining quantity to be determined is the mixture viscosity.

- The following models are used:

$$\frac{1}{\mu_m} = \frac{x}{\mu_v} + \frac{1-x}{\mu_l}$$

$$\mu_m = x\mu_v + (1-x)\mu_l$$

$$\frac{\mu_m}{\rho_m} = \frac{x\mu_v}{\rho_v} + \frac{(1-x)\mu_l}{\rho_l}$$

# Friction Pressure Losses (6)

- Using the first expression, the following final form of the two-phase multiplier is obtained:

$$\phi_{lo}^2 = \left[1 + \left(\frac{\mu_l}{\mu_v} - 1\right)x\right]^{-0.25}\left[1 + \left(\frac{\rho_l}{\rho_v} - 1\right)x\right]$$

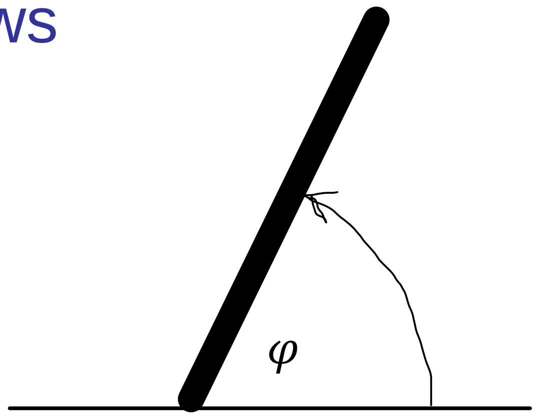
- And the two-phase friction pressure gradient becomes:

$$-\left(\frac{dp}{dz}\right)_{w,tp} = -\phi_{lo}^2\left(\frac{dp}{dz}\right)_{w,lo} = \frac{P_w}{A}\left[1 + \left(\frac{\mu_l}{\mu_v} - 1\right)x\right]^{-0.25}\left[1 + \left(1 - \frac{\rho_l}{\rho_v}\right)x\right]C_{f,lo}\frac{G^2}{2\rho_l}$$

# Gravity Pressure Gradient

- The gravity pressure gradient is as follows

$$-\left(\frac{dp}{dz}\right)_{grav} = \rho_m g \sin\varphi$$




- Here $\sin\varphi$ is equal to +1 for upwards flow in vertical channels, -1 for downwards flow, and to 0 for horizontal channels.
- Since, in general, the mixture density $\rho_m$ can change along channel, the pressure gradient will change accordingly.

# Acceleration Pressure Gradient (1)

- The acceleration pressure gradient can be evaluated as

$$-\left(\frac{dp}{dz}\right)_{acc} = \frac{1}{A}\frac{d}{dz}\left(\frac{G^2 A}{\rho_M}\right)$$

- For constant *G* and *A*, this equation reduces to:

$$-\left(\frac{dp}{dz}\right)_{acc} = G^2 \frac{d}{dz}\left(\frac{1}{\rho_M}\right) = G^2 \frac{d\upsilon_M}{dz}$$

- As can be seen, the pressure gradient is proportional to the gradient of mixture specific volume, $v_M$, multiplied with a square of the mass flux.

# Acceleration Pressure Gradient (2)

- According to the definition, the dynamic mixture density can be expressed in terms of quality and void fraction as follows

$$-\left(\frac{dp}{dz}\right)_{acc} = G^2 \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_f}\right]$$

- For HEM, we have

$$-\left(\frac{dp}{dz}\right)_{acc} = G^2 \frac{d}{dz}\left[\frac{x\left(\frac{\rho_f}{\rho_g}-1\right)+1}{\rho_f}\right] = G^2\left(\underbrace{\frac{1}{\rho_g}-\frac{1}{\rho_f}}_{\upsilon_g-\upsilon_f=\upsilon_{fg}}\right)\frac{dx}{dz} = G^2\upsilon_{fg}\frac{dx}{dz}$$

# Local Pressure Losses (1)

- Using HEM the irreversible pressure loss at sudden expansion is obtained as,

$$-\Delta p_I = \left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]\left(1 - \frac{A_1}{A_2}\right)^2 \frac{G_1^2}{2\rho_l}$$

- This equation can be compared with its equivalent for the single-phase flow through a sudden expansion.
- As can be seen, a new term appears, which can be identified as a two-phase multiplier for the local pressure loss

$$\phi_{lo,d}^2 = \left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]$$

# Local Pressure Losses (2)

- The subscript *lo,d* is used to indicate that the multiplier is valid for local losses, where the viscous effects can be neglected and only the ***d***ensity ratio between the two phases plays any role
- The corresponding irreversible pressure drop for homogeneous two-phase flow through a sudden contraction becomes,

$$-\Delta p_I = \left[1 + x\left(\frac{\rho_l}{\rho_v} - 1\right)\right]\left(\frac{A_2}{A_c} - 1\right)^2 \frac{G_2^2}{2\rho_l}$$

# Local Pressure Losses (3)

- In general, a local irreversible pressure drop for two-phase flows can be expressed as:

$$\Delta p_{I,tp} = \phi_{lo,d}^{2} \Delta p_{I,lo}$$

- where *tp* stands for ***t***wo-***p***hase and *lo* for ***l***iquid ***o***nly.
- As can be seen, the local pressure drop for two-phase flows can be obtained from a multiplication of the corresponding local pressure drop for single-phase flow and a proper local two-phase multiplier.

# Total Integral Pressure Drop (1)

- In practical calculation it is usually required to determine the over-all pressure drop in a channel of a given length and shape.
- The total pressure drop can be readily obtained from the integration of the pressure gradient expression along the channel length as follows

$$-\int_0^L \frac{dp}{dz}dz \equiv -\left[p(L) - p(0)\right] \equiv -\Delta p =$$

$$\int_0^L \left(\frac{dp}{dz}\right)_w dz + \int_0^L \rho_m g \sin\phi dz + \int_0^L \frac{1}{A}\frac{d}{dz}\left(\frac{G^2 A}{\rho_M}\right)dz$$

# Total Integral Pressure Drop (2)

- Assuming that the channel has a constant cross-section area and using expressions for the friction, gravity and acceleration terms, the following expression is obtained,

$$-\Delta p = C_{f,lo}\frac{4}{D_h}\frac{G^2}{2\rho_l}\int_0^L \phi_{lo}^2 dz + g\sin\varphi\int_0^L\left[\alpha\rho_v + (1-\alpha)\rho_l\right]dz +$$

$$G^2\int_0^L\frac{d}{dz}\left[\frac{x^2}{\alpha\rho_v}+\frac{(1-x)^2}{(1-\alpha)\rho_l}\right]dz$$



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# Total Integral Pressure Drop (3)

- It is customary to introduce integral multipliers into the above equations which are defined as follows.

  - The **integral acceleration multiplier**

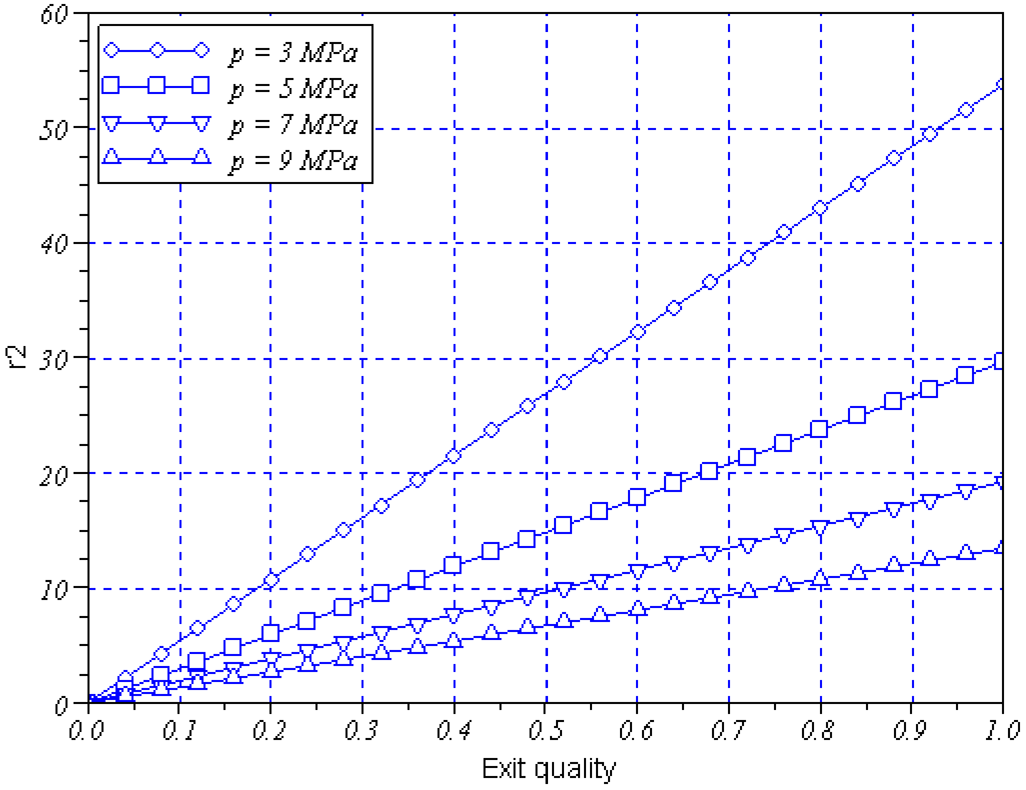
$$r_2 \equiv \rho_l \int_0^L \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_v}+\frac{(1-x)^2}{(1-\alpha)\rho_l}\right]dz = \left[\frac{x^2\rho_l}{\alpha\rho_v}+\frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - \left[\frac{x^2\rho_l}{\alpha\rho_v}+\frac{(1-x)^2}{(1-\alpha)}\right]_{in}$$

  - Here subscripts *ex* and *in* mean that the expression in the rectangular parentheses is evaluated at the channel *ex*it ($z=L$) and at the channel *in*let ($z=0$), respectively.

# Total Integral Pressure Drop (4)

- For heated channel with $x = \alpha = 0$ at the inlet and $x_{ex}$ with $\alpha_{ex}$ at the outlet, the multiplier is as follows,
- $r_2 = \left[\frac{x^2 \rho_f}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - 1$ , or for HEM $r_2 = \rho_f \upsilon_{fg} x_{ex}$
- This multiplier describes the pressure change due to flow acceleration caused by mixture expansion.
- It should be noted that it depends only on inlet and outlet values of void and quality.

# Total Integral Pressure Drop (4)




$r_2$ multiplier as a function of the exit quality, for various reference pressures

It is assumed that quality is 0 at the channel inlet.

# Total Integral Pressure Drop (5)

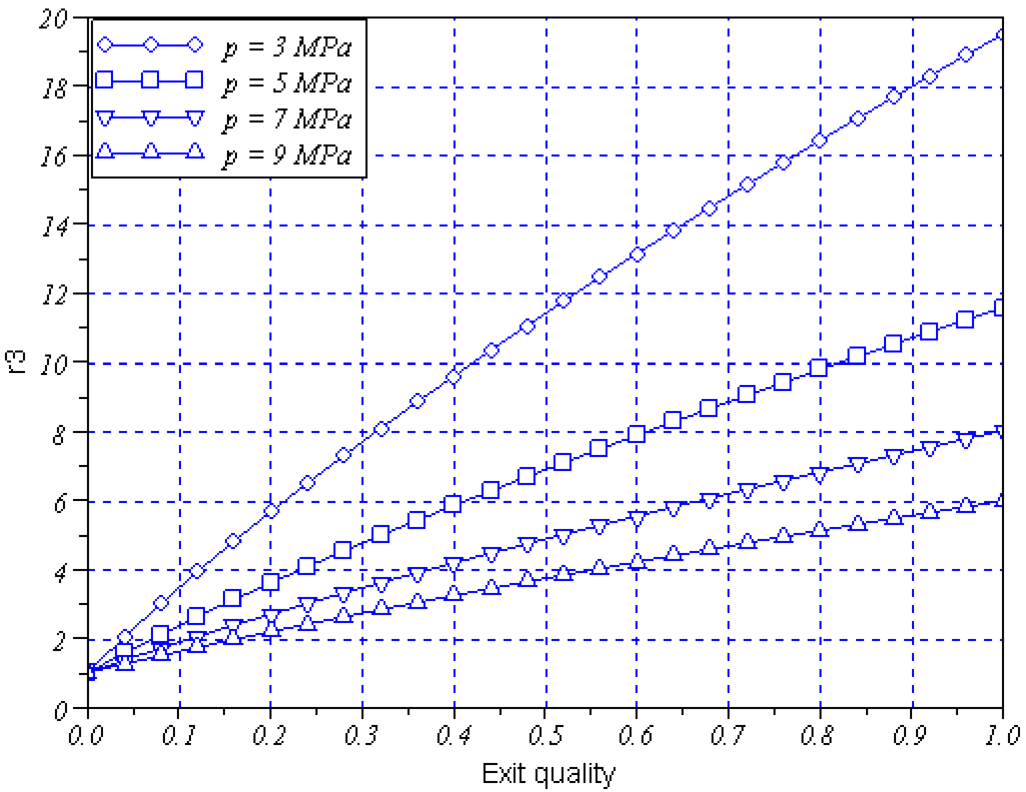
- Integral friction multiplier:

$$r_3 = \frac{1}{L}\int_0^L \varphi_{lo}^2 dz = \frac{1}{L}\int_0^L \left[1+\left(\frac{\mu_f}{\mu_g}-1\right)x\right]^{-0.25}\left[1+\left(\frac{\rho_f}{\rho_g}-1\right)x\right]dz$$

- This multiplier represents the effect of two-phase flow conditions on the friction pressure loss.

- The value of the integral multiplier depends on the values of local multiplier along the channel

# Total Integral Pressure Drop (5)




$r_3$ multiplier as a function of the exit quality, for various reference pressures

It is assumed that quality is 0 at the channel inlet and that the power is distributed uniformly in the channel.

# Total Integral Pressure Drop (6)

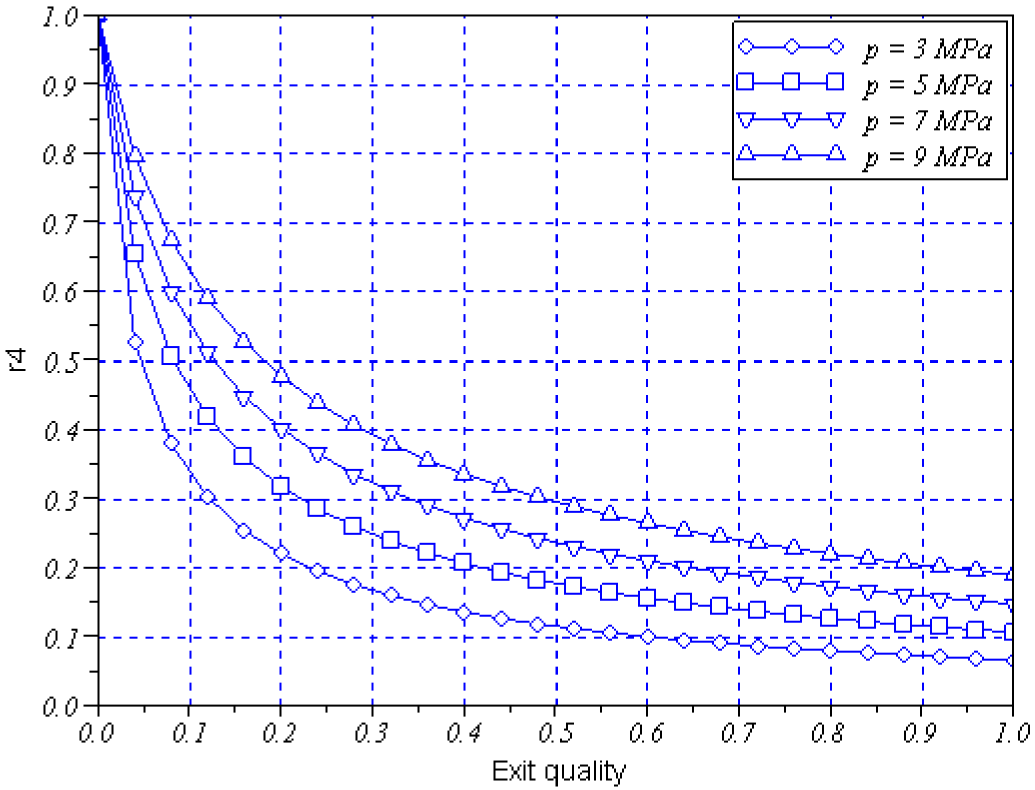
- Integral gravity multiplier:

$$r_4 = \frac{1}{L\rho_l}\int_0^L \left[\alpha\rho_g + (1-\alpha)\rho_f\right]dz = 1 - \frac{\rho_f - \rho_g}{\rho_f}\frac{1}{L}\int_0^L \alpha dz$$

- This multiplier describes the influence of two-phase flow conditions on the gravity pressure drop.

- The value of the friction multiplier depends on the void fraction distribution along the channel.

# Total Integral Pressure Drop (6)




$r_4$ multiplier as a function of the exit quality, for various reference pressures

It is assumed that quality is 0 at the channel inlet and that the power is distributed uniformly in the channel.

# Total Integral Pressure Drop (7)

- **The total channel pressure drop can be then found as,**

$$-\Delta p = r_3 C_{f,lo}\frac{4L}{D}\frac{G^2}{2\rho_f} + r_4 L\rho_f g\sin\varphi + 2r_2\frac{G^2}{2\rho_f} =$$

$$\left(r_3\frac{4C_{f,lo}L}{D} + 2r_2\right)\frac{G^2}{2\rho_f} + r_4 L\rho_f g\sin\varphi$$

- **If the channel contains a number of local losses (*i* = 1, .., *N*), the total pressure drop will be as follows,**

$$-\Delta p = r_3 C_{f,lo}\frac{4L}{D}\frac{G^2}{2\rho_f} + r_4 L\rho_f g\sin\varphi + 2r_2\frac{G^2}{2\rho_f} + \left(\sum_{i=1}^{N}\phi_{lo,di}^2\xi_i\right)\frac{G^2}{2\rho_f} =$$

$$\left[r_3\frac{4C_{f,lo}L}{D} + 2r_2 + \left(\sum_{i=1}^{N}\phi_{lo,di}^2\xi_i\right)\right]\frac{G^2}{2\rho_f} + r_4 L\rho_f g\sin\varphi$$

# Total Integral Pressure Drop (8)

- **NOTE:**

  definitions of the integral multipliers used in this course are slightly different from definitions used in literature. This is due to two reasons:

  - our definitions give non-dimensional values of multipliers

  - with definitions used in this course, the two-phase pressure drop equation is a natural extension of the single-phase equation

$$-\Delta p = \left( r_3 \frac{4C_{f,lo}L}{D} + 2r_2 \right) \frac{G^2}{2\rho_f} + r_4 L \rho_f g \sin\varphi$$

two-phase flow pressure drop

$$-\Delta p = \frac{4C_f L}{D} \frac{G^2}{2\rho_f} + L \rho_f g \sin\varphi$$

single-phase flow pressure drop

$r_2 = 0$, $r_3 = r_4 = 1$