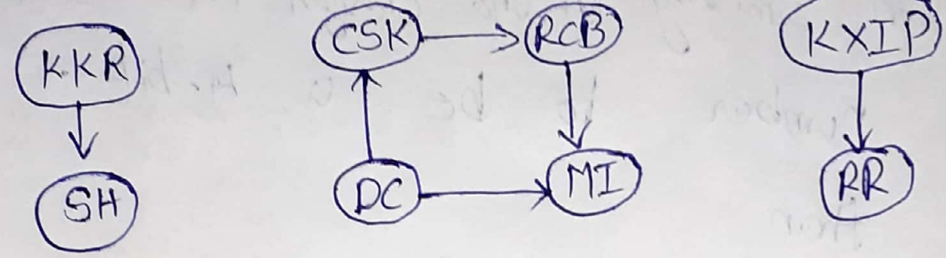
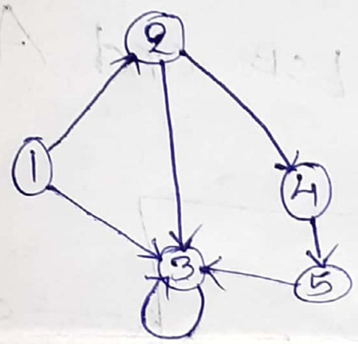
Discrete Mathematics Assignment - 2

Q)
1.a)



b)



Adjacency matrix:

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 1 & 0 & 0 \\
2 & 0 & 0 & 1 & 1 & 0 \\
3 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 1 \\
5 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

The paths of length 2 from any vertex to any vertex are:

$1 \to 2 \to 4$

$2 \to 4 \to 5$

$4 \to 5 \to 3$

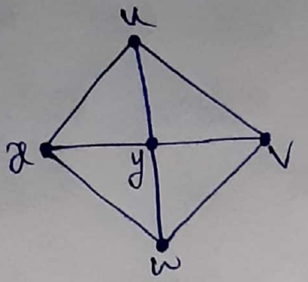
$3 \to 3 \to 3$

$1 \to 3 \to 3$

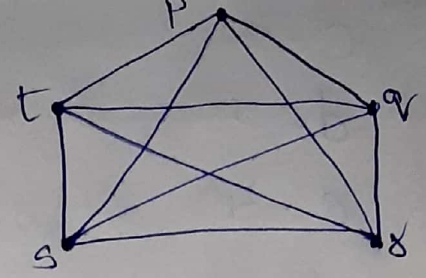
$5 \to 3 \to 3$

$2 \to 3 \to 3$.

2) a)



(G)



(H)

Degrees of vertices

y = 4
u = 3
v = 3
w = 3
x = 3

Degrees of vertices.

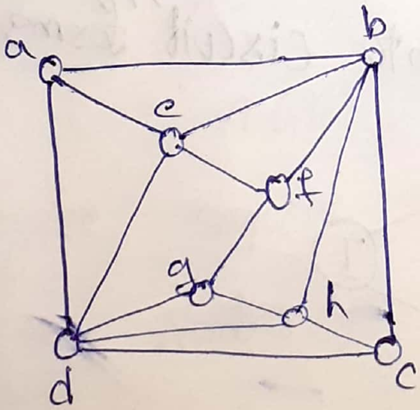
p = 4
t = 3
s = 3
q = 3
r = 3

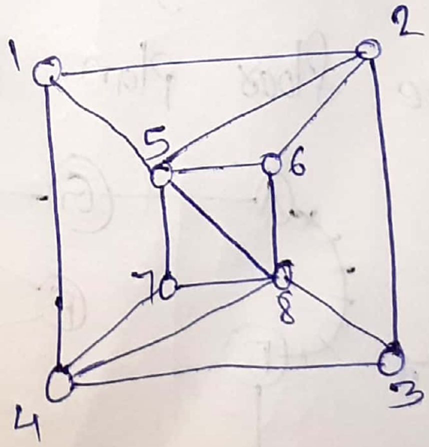
Degrees are equal & number of vertices

Therefore: They are isomorphic.

Mapping :- y → p
u → t
x → s
w → r
v → q

b)





Degrees of vertices

a - 3
b - 5

1 - 3
2 - 4
3 - 3

Scanned with CamScanner

c - 3 4 - 4

d - 5 5 - 5

e - 4 6 - 3

f - 3 7 - 3

g - 3 8 - 5.

h - 4.

Degrees are equal & no. of vertices are equal but still they are not isomorphic because -

in Graph 1 :-

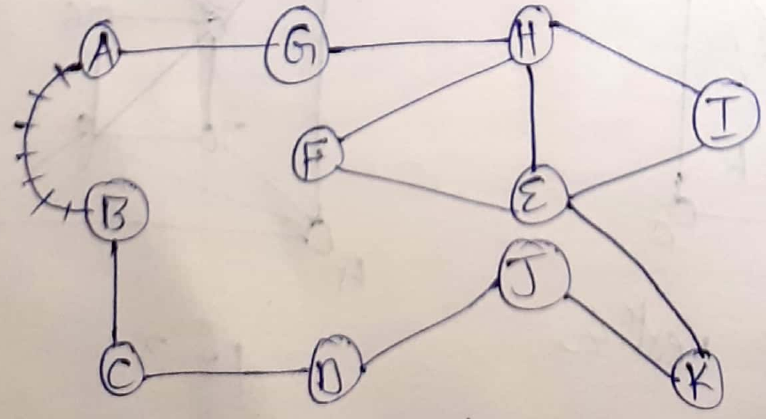
vertices of degree 5 are not connected directly but

in Graph 2 :-

vertices of degree 5 are connected directly..

∴ they are not isomorphic. //

a) The floor plan converted into circuit forms.



ǂ - Exterior doorway/Path

| - Interior doorway/Path

A simple path that starts from one vertex and ends at another vertex and passes through each and every edge once is known as "EULERIAN PATH". Also at most there have to be 2 vertices with odd degree.

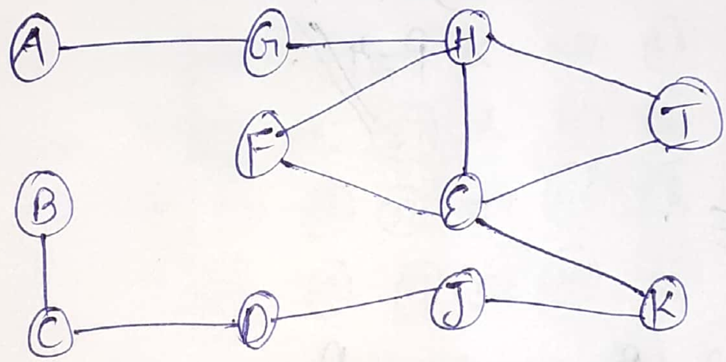
In this case degrees of vertices.

A = 1   C = 2   E = 4   G = 2   I = 2   K = 2

B = 2   D = 2   F = 2   H = 3   J = 2

∴ The vertices A & H have odd degree.

A Euler path can be formed.



The path is

A – G – H – F – E – H – I – E – K – J – D – C – B.

∴ AGHFEHIEKJDCB.

b) Degree sequence $\{5, 4, 2, 2, 1\}$.

Number of vertices = 5 (V)

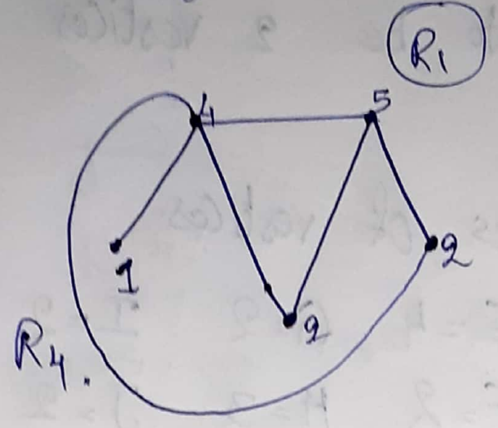
$$\text{Number of Edges} = \frac{5+4+2+2+1}{2}$$

$$= 14/2$$

$$= 7 \ (E).$$

since there are only two vertices with odd degree and each of the degree sequence is $\leq$ no of vertices

$\therefore$ planar graph can be drawn.



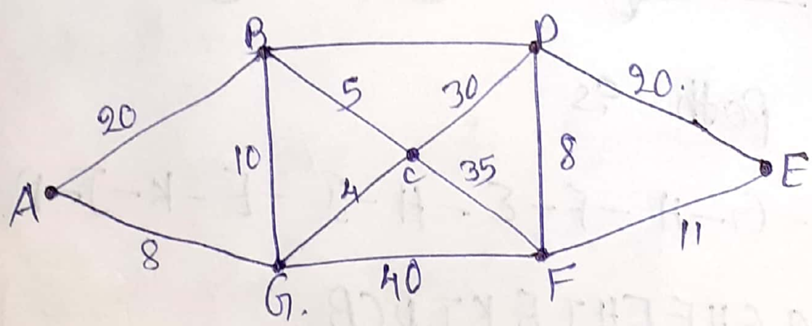
There are 4 regions/planes (P)

Proof :- $V - E + P = 2$.

$5 - 7 + P = 2$.

$P = 4$ //.

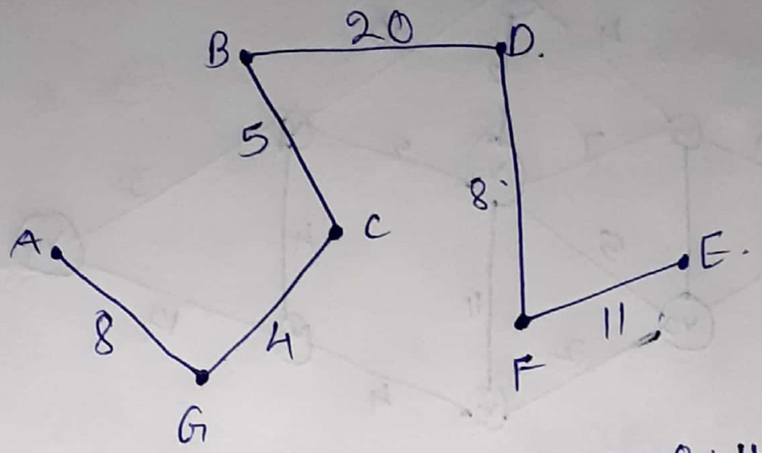
4) i, from to E.



	B	C	D	E	F	G
A	20	α	α	α	α	8
G	18	12	α	α	48	8
C	17	12	42	α	47	8
B	17	12	37	α	47	8
D	17	12	37	57	45	8
F	17	12	37	56	45	8

Scanned with CamScanner

The path with minimum cost is



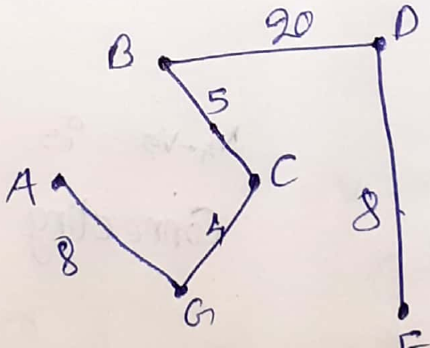
Minimum Cost = 8+4+5+20+8+11

= 56 //.

ii, From A to F.

	B	C	D	E	F	G
A	20	α	α	α	α	8
G	18	12	α	α	48	8
C	17	12	42	α	47	8
B	17	12	37	α	47	8
D	17	12	37	57	45	8
F	17	12	37	56	45	8

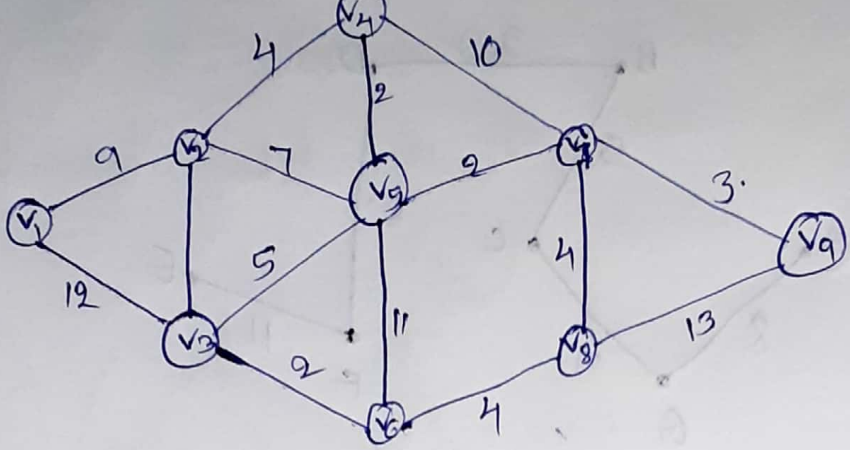
The path with minimum Cost is.



Minimum Cost = 8 + 4 + 5 + 20 + 8

= 45 //.

5)



Edges:

$V_1 - V_2 = 9$

$V_1 - V_3 = 12$

$V_2 - V_3 = 8$

$V_2 - V_4 = 4$

$V_2 - V_5 = 7$

$V_3 - V_5 = 5$

$V_3 - V_6 = 2$

$V_4 - V_5 = 2$

$V_5 - V_6 = 11$

$V_4 - V_7 = 10$

$V_5 - V_2 = 2$

$V_6 - V_8 = 4$

$V_7 - V_8 = 4$

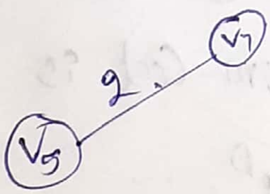
$V_7 - V_9 = 3$

$V_8 - V_9 = 13$

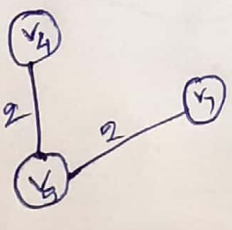
minimum weighted chosen edge $V_5 - V_7$.

steps:

i)

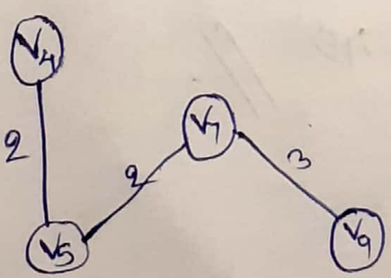


ii)



$V_4 - V_5$ is the shortest edge. Connecting either V_5 or V_7.

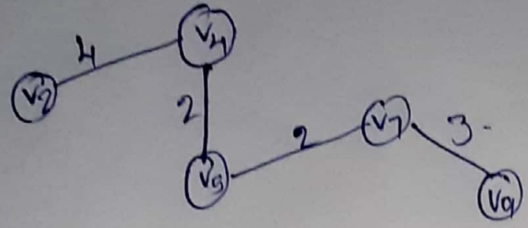
iii)



$V_7 - V_9$ is the shortest edge Connecting either V_4 or V_5 or V_7

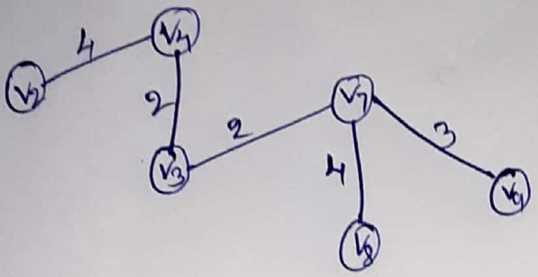
Scanned with CamScanner

iv,



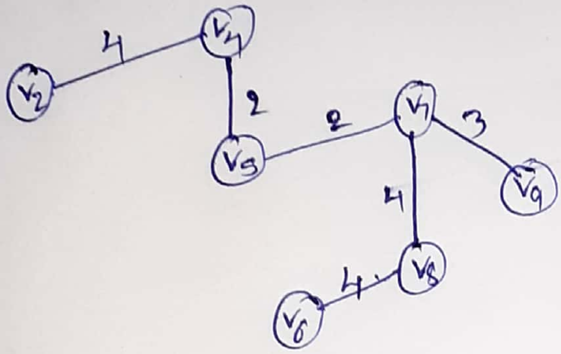
$V_4 - V_2$ is the shortest edge Connecting either V_4 or V_5 or V_7 or V_9.

v,



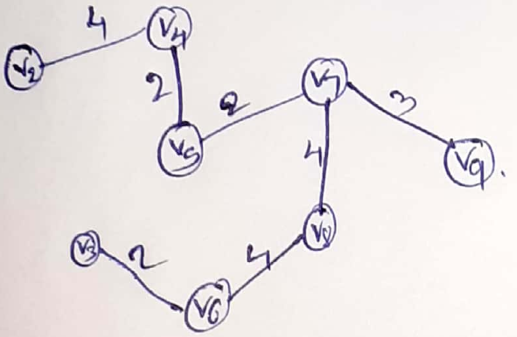
$V_7 - V_8$ is the shortest edge Connecting either V_2 or V_4 or V_5 or V_7 or V_9.

vi,



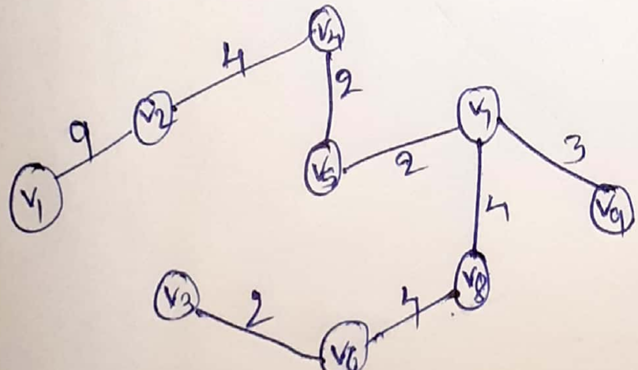
$V_8 - V_6$ is the shortest edge Connecting either V_2 or V_4 or V_5 or V_7 or V_8 or V_9.

vii,



$V_3 - V_6$ is the shortest edge Connecting either V_2 or V_4 or V_5 or V_6 o V_7 or V_8 or V_9.

viii,



$V_1 - V_2$ is the shortest edge Connecting either V_2 or V_3 or V_4 or V_5 or V_6 or V_7 or V_8 or V_9.

$\therefore$ Minimum Cost $= 9+4+2+2+4+3+4+2$

$= 30$