(1) $R = 2\%$, $P = 100$, $T = 20yrs$, $A = ?$

$P(0) = 100 + ((2\% \text{ of } 100) - 3)$

$P(n+1) = P(n) + (2\% \text{ of } P(n) - 3)$

$P(n+1) = 100 + [(2\% \text{ of } 100) - 3]$

$P(n+1) = 1.02P(n) - 3$

$a = 1.02$, $C = -3$

$$P(n) = a^n p(0) + C \cdot \frac{a^n - 1}{a - 1}$$

$$= (1.02)^{20}(100) - 3\,\frac{(1.02)^{20} - 1}{(1.02 - 1)}$$

$$= 1.485(100) - 3\,\frac{(1.485) - 1}{0.02}$$

$$\boxed{P(20) = 75.75}$$

(2) Let y_n is the no. of plant at n^{th} years

$y_2 = 8y_1$; $y_3 = 8y_2 + 18y_1$

$y_{n+1} = 8y_n + 18(y_1 + y_2 + \cdots y_{n+1})$ —— (1)

$y_{n+2} = 8y_{n+1} = 18(y_1 + y_2 + \cdots + y_n)$ —— (2)

eqn(1) - eqn(2)

$\Rightarrow y_{n+2} - 9y_{n+1} - 10y_n = 0$

let $y_n = r^n$, $y_{n+1} = r^{n+1}$

$y_{n+2} = r^{n+2}$

Substitute

$\Rightarrow r^{n+2} - 9r^{n+1} - 10r^n = 0$

Scanned with CamScanner

$\Rightarrow r^2 - 9r - 10 = 0$

$\Rightarrow r = \dfrac{9 \pm \sqrt{81+40}}{2} = \dfrac{9 \pm 11}{2}$

$\boxed{r = 10, -1}$ $\qquad \boxed{\therefore y_n = C_1 10^n + C_2(-1)^n}$

(3) Given

T_0 (Initial temp) $= 45°F$ $\quad T_1$ (after 1min) $= 50°F$

T_2 (after 2min) $= 54°F$ $\quad T_s$ (Temperature of room) $= ?$

Newton's law of cooling

$$\boxed{T_{n+1} - T_n = K(T_n - T_s)}$$

$t = 25 min$, $T_{25} = ?$

$\underline{n=0}$

$$T_1 - T_0 = K(T_0 - T_s)$$

$$5 = K(45 - T_s) \quad \text{——— (1)}$$

$\underline{n=1}$ $\quad T_2 - T_1 = K(T_1 - T_s)$

$$4 = K(50 - T_s) \quad \text{——— (2)}$$

By eq $\frac{(1)}{(2)} \Rightarrow \frac{5}{4} = \frac{45 - T_s}{50 - T_s}$

$$250 - 5T_s = 180 - 4T_s$$

$$\boxed{T_s = 70}$$

from (2)

$$4 = -K(20)5$$

$$K = -1/5 = -0.2 \qquad \boxed{K = -0.2}$$

$$T_{n+1} - T_n = -1/5 (T_n - 70)$$

$$T_{n+1} - T_s = -T_n/5 + 14$$

$$T_{n+1} - \frac{4T_n}{5} = 14$$

$$5T_{n+1} - 4T_n = 70$$

$$T_n = \alpha^n$$

$$T_{n+1} = 0.8T_n + 14$$

$$\therefore T_{n+1} = 0.8\,T_n + 0.2(70)$$

$$T_{n+1} = 0.8T_n + 14$$

$$T_n \text{ i.e, } T(n) = a^n T(0) + c \cdot \frac{a^n - 1}{a-1}$$

$$T(25) = (0.8)^{25}(45) + 14\left(\frac{1-(0.8)^{25}}{1-0.8}\right)$$

$$= 0.17 + 69.79$$

$$\boxed{T(25) = 69.906F^\circ}$$

(4) $\{y_{\delta K}\} = \{a^K - 0.2^K\}$, $a > 0$

$a = 0.3$ $\quad a = 1.1$

Impulse sequence $\{y_{\delta K}\}$

For step response

$$\{y_K\} = z^{-1}\{y(z)\} = z^{-1}\{y_\delta(z)\, u(z)\}$$

$$= z^{-1}\{G(z)\, U(z)\}$$

$$G(z) = y_\delta(z) = z\{y_{\delta K}\} = z\{a^K - 0.2^K\}$$

$$= \frac{z}{z-a} - \frac{z}{z-0.2}$$

$U(z) = z\{U_K\}$ $\quad U_K$ = unit step sequence

$$z\{U_K\} = U(z) = \frac{z}{z-1}$$

$$y(z) = G(z)\, U(z) = \left(\frac{z}{z-a} - \frac{z}{z-0.2}\right)\frac{z}{z-1}$$

$$\Rightarrow \frac{a}{a-1}\,\frac{z}{z-a} - \frac{z}{z-0.2} + \left(-5 + \frac{1}{1-a}\right)\frac{z}{z-1}$$

$\Rightarrow y_K = \left\{ \frac{a}{a-1} a^K - (0.2)^K + \left(-5 + \frac{1}{1-a}\right) 1^K \right\}$

i.e $y_K = \left\{ \frac{a}{a-1} a^K - (0.2)^K + \left(-5 + \frac{1}{1-a}\right) \right\}$

a) If $a = 0.3$ as $K \to \infty$

$0.3^K \to 0$ & $0.2^K \to 0$

So, $y_x \to$ a constant term

$= -5 + \frac{1}{1-0.3} = -5 + \frac{1}{-0.7}$

$= -3.571$

b) $a = 1.1$ as $K \to \infty$, $1.1^K \to \infty$.

Output sequence is unbounded & again the system "blows up".

⑤ Radium decays 1% every 25 years

$x_{n+1} = x_n - \frac{1}{100} x_n$

$x_{n+1} = 0.99 x_n$

$n = 1, 2, 3 \ldots$

$x_n = \alpha^n x_0$

$x_n = (0.99)^n x_0$

If we consider x_0 as initial medium x_n is the amount of radium after 25 years

$x_4 = (0.99)^4 x_0$

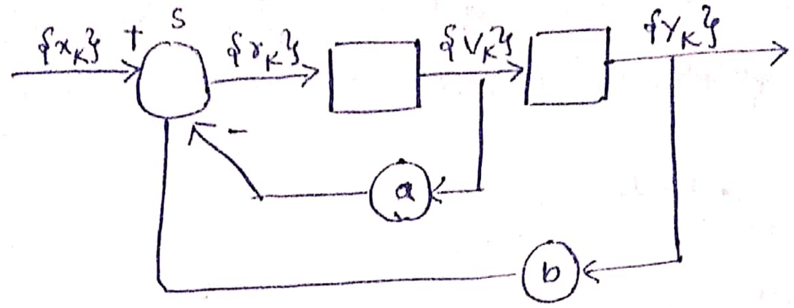
$= 0.9609 x_0$

$\boxed{\therefore x_4 = 0.9606 x_0}$

the amount of radium after 100 years is 0.9606 times initial one

⑥ ~~a=2, b=8~~

$a=2,\ b=-8,\ y_0=0,\ y_1=1,\ x_K=K\cdot 5$



$\{y_{K+1}\}=\{V_K\},\quad \{V_{K+1}\}=r_K$

$r_K = x_K - aV_K + by_K$

$y_{K+2}+ay_{K+1}-by_K = x_K$

$y_{K+2}+2y_{K+1}+8y_K = K\cdot 5^K$ ——— ①

$\Rightarrow Z\{K+2\} = z^2y(z)-z^2y_0-zy_1 = z^2y(z)-z$

$Z\{y_{K+1}\} = zy(z)-zy_0 = zy(z)$

from ①

$Z\{K\cdot 5^K\} = \left(-z\frac{d}{dz}\right)' X(z)$

$= -z\frac{d}{dz}\left(\frac{z}{z-5}\right) = -z\left(\frac{(z-5)-z}{(z-5)^2}\right)$

$= \frac{5z}{(z-5)^2}$

$z^2y(z)-z+2zy(z)+8y(z) = \frac{5z}{(z-5)^2}+z$

$y(z) = \frac{5z}{(z-5)^2(z^2+z+8)}$

$\frac{y(z)}{z} = \frac{z^2+10z+30}{(z-5)^2(z-(-1+\sqrt{7}i))(z-(-1+\sqrt{7}i))}$

$= \frac{A}{z-5}+\frac{B}{(z-5)^2}+\frac{C}{z-(-1+\sqrt{7}i)}+\frac{D}{z-(-1+\sqrt{7}i)}$

Solve for A, B, C, D

$$y(z) = 0.14\,(5)^k + (-1.17)\,(5)^n - \frac{1}{9}\,k \cdot 5^{-k}$$

$$A = -0.14548, \quad B = -1.1759$$

7) $$G(z) = \frac{7z-1}{z^2+7z-9}, \quad n = 2$$

$$\frac{y(z)}{u(z)} = \frac{7z-1}{z^2+7z-9}$$

$$(z^2+7z-9)\,y(z) = (7z-1)\,u(z)$$

$$y_r(z) = \frac{7z-1}{z^2+7z-9}$$

Let $\{r_k\}$ $\quad Z\{r_k\} = R(z)$

$$(z^2+7z-9)\,R(z) = u(z) \quad \text{—— } ①$$

$$7z(z^2+7z-9)\,R(z) = 7z\,u(z) \quad \text{—— } ②$$

② − ①

$$(7z-1)(z^2+7z-9)\,R(z) = (7z-1)\,u(z)$$

$\Rightarrow$ so, $$(z^2+7z-9)\,R(z) = u(z) \quad \text{—— } ④$$

By comparing ③ & ④

$$(7z-1)\,R(z) = y(z)$$

$$y(z) = 7z\,R(z) - R(z) \quad \text{—— } ⑤$$

④ $$z^2R(z) + 7z\,R(z) + (-9R(z)) = u(z)$$

$$z^2R(z) = u(z) - 7z\,R(z) + 9R(z)$$

⑤ $$7z\,R(z) - R(z) = y(z)$$

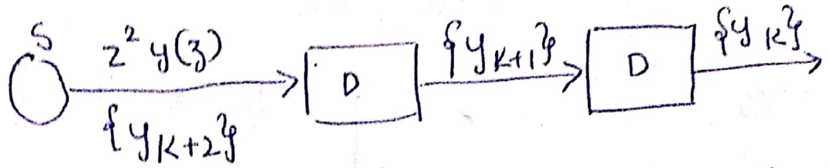
$$7z\,R(z) = y(z) + R(z)$$

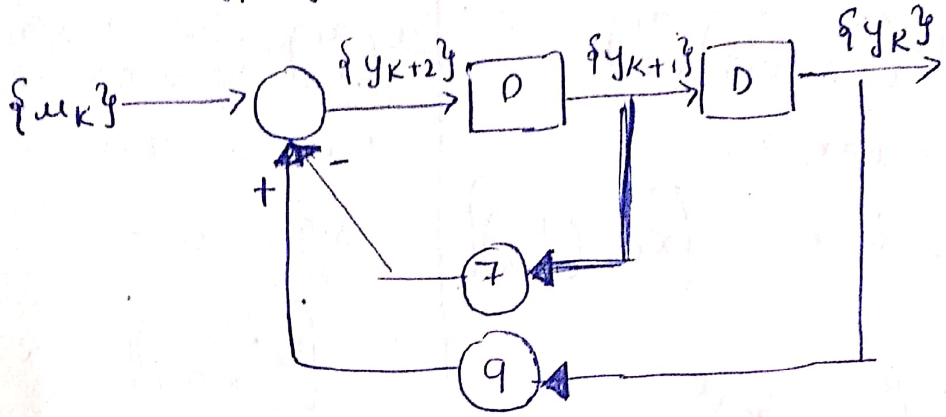
Scanned with CamScanner

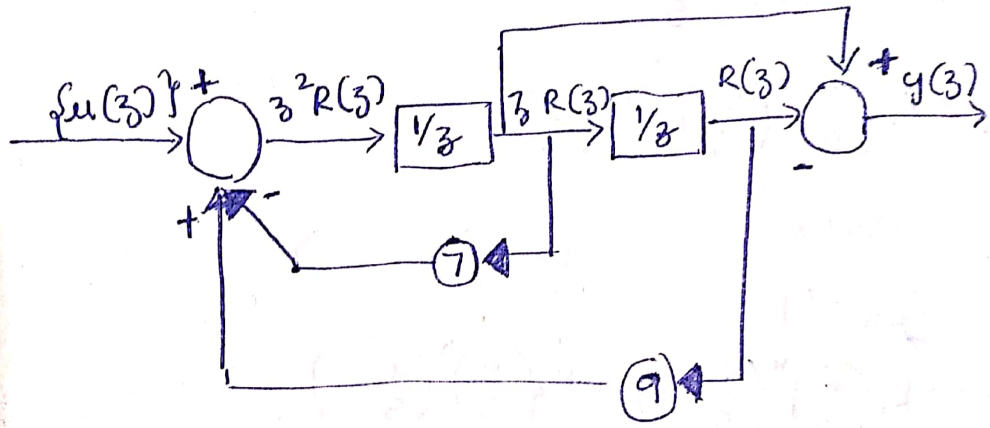
from ④ & ⑤

$$y_{K+2} = u(z) - 7y_{K+1} + 9y_K$$

$$\boxed{\therefore y_{K+2} + 7y_{K+1} - 9y_K = 7u_{K+1} - u_K}$$







⑧ $C_{K+1} = 1.5C_K - E_K$

$E_{K+1} = 0.21C_K + 0.5_K$

Let $y[n] = \begin{bmatrix} C_n \\ E_n \end{bmatrix}$

$$y_{n+1} = \begin{bmatrix} 1.5 & -1 \\ 0.21 & 0.5 \end{bmatrix} y_n$$

Let $y_n = X_\alpha^{\,n}$

$\boxed{\alpha X = 4X}$ $\qquad A \to \begin{bmatrix} 1.5 & -1 \\ 0.21 & 0.5 \end{bmatrix}$

↳ eigen value

$$\begin{vmatrix} 1.5-\alpha & -1 \\ 0.21 & 0.5-\alpha \end{vmatrix} \Rightarrow (1.5-\alpha)(0.5-\alpha)+0.21=0$$

$$\Rightarrow 0.75-2\alpha+\alpha^2+0.21=0$$

$$\alpha^2-2\alpha+0.96=0$$

$$\alpha=6/5,\ 4/5$$

for $\alpha=6/5$

$$\frac{6}{5}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}1.5 & -1\\0.21 & 0.5\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$\frac{6}{5}x_1 = 1.5x_1-x_2$$

$$\Rightarrow x_2=0.3x_1 \quad \boxed{X_1=\begin{pmatrix}1\\0.3\end{pmatrix}}$$

for $\alpha=4/5$

$$\frac{4}{5}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}1.5 & -1\\0.21 & 0.5\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$1.5x_1-x_2=4/5x_1$$

$$\frac{3.5}{5}x_1-x_2=0$$

$$\boxed{X_2=\begin{pmatrix}1\\0.7\end{pmatrix}}$$

(a) $C_K=K_1\left(6/5\right)^K+K_2\left(4/5\right)^K$

$$C_{K+1}=K_1\left(\frac{6}{5}\right)^K\left(\frac{6}{5}\right)+K_2\left(\frac{4}{5}\right)^K\left(\frac{4}{5}\right)$$

$$=X+C_K$$

$$X=C_{K+1}-C_K$$

$$=K_1\left(\frac{6}{5}\right)^n\left(\frac{6}{5}-1\right)+K_2\left(\frac{4}{5}\right)^n\left(\frac{4}{5}-1\right)$$

$$\left(K_1\left(\frac{6}{5}\right)^n-K_2\left(\frac{4}{5}\right)^n\right)1/5$$

$$=\frac{20}{100}\left(K_1\left(6/5\right)^n-K_2\left(4/5\right)^n\right)1/5$$

$=20\%$ Increases in growth in C_K in long term

(b) $C_1=6000$, $E_1=3720$

$$K_1\left(\frac{6}{5}\right)+K_2\left(\frac{4}{5}\right)=6000 \quad (1)$$

$$K_1(0.3)\left(\frac{6}{5}\right)+K_2(0.7)\left(\frac{4}{5}\right)=3720 \quad (2)$$

$$K_1(0.3)\left(6/5\right)+K(0.7\left(4/5\right)=3720$$

$$0.4K_2=1720 \quad \boxed{K_2=4800}$$

Scanned with CamScanner

$$K_1\left(\frac{6}{5}\right)+4800\times\frac{4}{5}=6000$$

$$C_K=1800\left(\frac{6}{5}\right)^K+4800\left(\frac{4}{5}\right)^K$$

$$\frac{dC_K}{dK}=0$$

$$\Rightarrow 1800\left(\frac{6}{5}\right)\ln\left(\frac{6}{5}\right)+4800\left(\frac{4}{5}\right)^K\ln(4/5)=0$$

$$\left(\frac{3}{2}\right)^K=\frac{-4800\ln(4/5)}{1800\ln(6/5)}$$

$$\left(\frac{3}{2}\right)^K=3.2637$$

$$K=2.9173 \quad (\text{Approx } 3 \text{ years})$$

$$C_3=1800\left(\frac{6}{5}\right)^3+4800\left(\frac{4}{5}\right)^3$$

$$=\frac{15552}{5}+\frac{12288}{5}=5568,$$

$$K=\frac{\log 0.0875}{\log(0.8/1.2)}=6.007$$

The nearest integer is $K=6$, corresponding seven year is

$$\boxed{C_6=4841}$$

(9) $y_{K+2}-5y_{K+1}+6y_K=u_K$ —— (1)

$u_K=1$

$y_0=0,\ y_1=0$

$$Z^+\{y_{K+2}\}=z^2y(z)-z^2y_0-zy_1$$

$$=z^2(y(z))$$

$$Z^+\{y_{K+1}\}=zy(z)-zy_0=zy(z)$$

Scanned with CamScanner

from ①

$$z^2 y(z) - 5z\, y(z) + 6y(z) = \frac{z}{z-1}$$

$$y(z)\,[z^2 - 5z + 6] = \frac{z}{z-1}$$

$$y(z) = \frac{z}{z^2 - 5z + 6(z-1)}$$

$$y(z) = \frac{z}{z^2 - 2z - 3z + 6\,(z-1)} = \frac{z}{(z-2)(z-3)(z-1)}$$

$$\frac{y(z)}{z} = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-3)}$$

$$\frac{z}{(z-1)(z-2)(z-3)} = \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-3)}$$

$\Rightarrow z = A(z-2)(z-3) + B(z-1)(z-3) + C(z-1)(z-2)$

$z = A(z^2 - 5z + 6) + B(z^2 - 4z + 3) + C(z^2 - 3z + 2)$

$\Rightarrow$ "z^2" coeff $\Rightarrow A + B + C = 0$ ——— ①

$\Rightarrow$ "z" Coeff

$-5A - 4B - 3C = 1$ ——— ②

$\Rightarrow$ Constant coeff

$6A + 3B + 2C = 0$ ——— ③

$$\boxed{C = -A - B}$$

$\Rightarrow -5A - 4B - 3C = 1$

$\Rightarrow -5A - 4B - 3(-A - B) = 1$

$\Rightarrow -5A - 4B + 3A + 3B = 1$

$\Rightarrow -2A - B = 1$ ——— ④

from ③

$$6A + 3B + 2(-A-B) = 0$$

$$6A + 3B - 2A - 2B = 0$$

$$4A + B = 0 \quad \text{——} ⑤$$

from ④ & ⑤

$$-2A - \cancel{B} = 1$$
$$4A + \cancel{B} = 0$$
$$2A = 1$$

$$\boxed{A = 1/2}, \quad \boxed{B = -2}, \quad \boxed{C = 3/2}$$

Applying Inverse z-transform on both sides

$$z^{-1}\{y(z)\} = y_K = \frac{1}{2}(1)^K - 2(2)^K + \frac{3}{2}(3)^K$$

$$K \geq 0$$

$$\Rightarrow \boxed{y_K = \frac{1}{2} - 2(2)^K + \frac{3}{2}(3)^K, K \geq 0}$$

⑩ $f(t) = e^{-2\omega t} H(t)$

$$\{f(KT)\} = \{1, e^{-2\omega T}, e^{2(-2\omega t)}, e^{3(-2\omega t)} \ldots e^{n(-2\omega t)}$$

$$Z\{e^{-2\omega KT}\} = \sum_{K=0}^{\infty} \frac{(e^{-2\omega t})^{x}}{z^K} = \frac{z}{z - e^{-2\omega T}}$$

$$|z| > e^{-2\omega T}$$

⑪ $X_{n+2} + X_n = -2\sin\left(\frac{n\pi}{2}\right)$ —— ①

$$r^{n+2} + r^n = 0$$

$$r^n(r^2+1) = 0$$

$$r = \pm i$$

$$\alpha = 0 \quad \beta = 1 \qquad P = \sqrt{\alpha^2 + \beta^2} = 1$$

$P = \tan^{-1}\left(\frac{1}{3}\right) = \pi/2$

$x_n = C_1 \cos\left(\frac{n\pi}{2}\right) + C_2 \sin\left(\frac{n\pi}{2}\right)$

$x_p(n) = A\cos\left(\frac{n\pi}{2}\right) + B\sin\left(\frac{n\pi}{2}\right)$ —— ①

$x_p(n+1) = -A_n \sin\left(\frac{n\pi}{2}\right) + B_n\cos\left(\frac{n\pi}{2}\right)$ —— ②

$x_p(n+2) = -A_n^2\left(\frac{n\pi}{2}\right) - B_n^2\cos\left(\frac{n\pi}{2}\right)$ —— ③

Substitution ①, ②, ③

$-An^2\cos\left(\frac{n\pi}{2}\right) - Bn^2\cos\left(\frac{n\pi}{2}\right) + A\cos\left(\frac{n\pi}{2}\right) + B\sin\left(\frac{n\pi}{2}\right)$

$= 2\sin\left(\frac{n\pi}{2}\right) + 0\cos\left(\frac{n\pi}{2}\right)$

for cos

$-n^2[A] + A = 0$

$1 - n^2 = 0 \qquad n = \pm 1, A = 0$

for sin

$-n^2 B + B = -2$

$B(1-n^2) = -2 \qquad B = -2$

$1 - n^2 = -2$

$x(n) = x_c(n) + x_p(n)$

$\Rightarrow \boxed{C_1\cos\left(\frac{n\pi}{2}\right) + C_n\sin\left(\frac{n\pi}{2}\right) - 4\sin\left(\frac{n\pi}{2}\right)}$