

Introduction to Dark Matter: History, Evidences & a bit of Cosmology

Astro Particle Physics
WS 2020

Federico Ambrogi (dep. Meteorology, Univie)

Today, 18th November 2020:

- Brief history of Dark Matter
- Astrophysical Evidences
- A bit of Cosmology

Next Lecture(s) (when = ???):

- (Experimental) Searches for Dark Matter
- Gravitational Waves

My Slides:

- Professor Jeitler's website + recordings

Part 1. History & Evidences

Overview

- Why do we talk about dark matter i.e. what are the evidences for Dark Matter existence?
- What do we think Dark matter is?
- What is the role of Dark Matter in the evolution of the Universe?
- How can we search for Dark Matter?
- ...



Introduction: An Astrophysical Puzzle

- Dark Matter is a building block of modern Cosmology and of the LambdaCDM model
- Pioneer of DM studies: the Swiss-American astronomer Fritz Zwicky (1933)
- He studied the velocity dispersion of galaxies inside the Coma cluster (a large cluster of galaxies with more than 1,000 identified galaxies) and found a gravitational anomaly
- He was the first to use the virial theorem to determine the mass of a galaxy cluster
- He estimated the total mass M_{tot} of Coma as
$$M_{tot} = 800 * \langle m \rangle$$



where $\langle m \rangle = 10^9 M_\odot$ is the average mass of the galaxies in a radius of the system equal to 10^6 light-years

- He calculated the potential energy, the average kinetic energy, applied the **virial theorem***, obtained a velocity dispersion of ~80 km/s

!!! The observed average velocity dispersion along the line-of-sight was approximately 1000 km/s >> 80 km/s !!!

<https://arxiv.org/pdf/1605.04909.pdf>

<http://hosting.astro.cornell.edu/academics/courses/astro201/vt.htm>

Estimating the Virial Mass of the Cluster

- **The Virial Theorem**

relates the kinetic and potential energy of a system in equilibrium

where:

$$K = -\frac{1}{2}P$$

Kinetic Energy:

$$K = \frac{1}{2}mv^2$$

Gravitational Potential Energy:

$$P \sim -\frac{1}{2}G \frac{N^2 m^2}{R_{tot}} = -\frac{1}{2}G \frac{M_{tot}^2}{R_{tot}}$$

- Assume the system is composed of N galaxies with average mass $\langle m \rangle = m$
- Apply the Virial Theorem:

$$K_{tot} = \frac{1}{2}mNv^2 = \frac{1}{2}M_{tot}v^2$$

$$\frac{1}{2}M_{tot}v^2 = +\frac{1}{4}G \frac{M_{tot}^2}{R_{tot}} \quad \longrightarrow \quad M_{tot} = 2 \frac{R_{tot}v^2}{G}$$

Idea: the dispersion of the velocity of the galaxies inside the cluster is related to the total mass of the cluster

Introduction: An Astrophysical Puzzle

In conclusion:

- the Virial theorem, applied to the Coma Cluster, gave a velocity dispersion which was much less than what observed:

$$1000 \text{ km/s observed} \gg 80 \text{ km/s calculated}$$

- in the calculation, the total visible mass was used, estimated by observations by Zwicky
- since the velocity dispersion is related to the total mass of the cluster (i.e. visible **and** invisible), lead to think that *a large fraction of the total mass could not be observed*

*“If this would be confirmed, we would get the surprising result that dark matter (**dunkle materie**) is present in much greater amount than luminous matter“*

“ [In order to derive the mass of galaxies from their luminosity] we must know how much dark matter is incorporated in nebulae in the form of cool and cold stars, macroscopic and microscopic solid bodies, and gases“

<https://ned.ipac.caltech.edu/level5/Sept16/Bertone/Bertone3.html>

Mass-to-Light Ratio

- Useful astronomical quantity: Mass to Light ratio, M/L
- It is defined as the ratio of the mass and the luminosity of an object
- Typically use the Solar value $M_{\odot} / L_{\odot} \sim 5100 \text{ kg/W}$
- Segue 1 dwarf spheroidal galaxy: $M/L > 3.400$, contains only a few hundred stars, yet has a large mass
- “Segue 1: The Darkest Galaxy” (*Ideal objects to study DM annihilation with indirect searches*)

Objects	Distance (in kpc)	Luminosity (in sol. lum.)	Mass (in sol. mass)	Mass/Lum. f
Solar Neighborhood	—	—	—	—
Triangulum Nebula, M33	480	1.4×10^9	5×10^9	4
Large Magellanic Cloud	44	1.2×10^9	2×10^9	4
Andromeda Nebula	460	9×10^9	1.4×10^{11}	16
Globular Cluster, M92	11	1.7×10^5	$< 8 \times 10^5$	—
Elliptical Galaxy, NGC 3115	2100	9×10^8	9×10^{10}	100
Elliptical Galaxy, M32	460	1.1×10^8	2.5×10^{10}	200
Average S in Double Gal.	—	1.3×10^9	7×10^{10}	50
Average E in Double Gal.	—	8×10^8	2.6×10^{11}	300
Average in Coma Cluster	25000	5×10^8	4×10^{11}	800

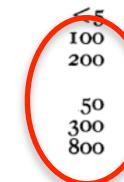


FIG. 1. A snapshot of the dark matter problem in the 1950s: the distance, mass, luminosity, and mass-to-light ratio of several galaxies and clusters of galaxies, as compiled by M. Schwarzschild in 1954 [282].

Several galaxies showed an ‘excess of mass’
wrt the total visible luminous emission



Optical image of an “Ultra faint”
dwarf galaxy

Rotation Velocity and Galaxy Rotation Curve

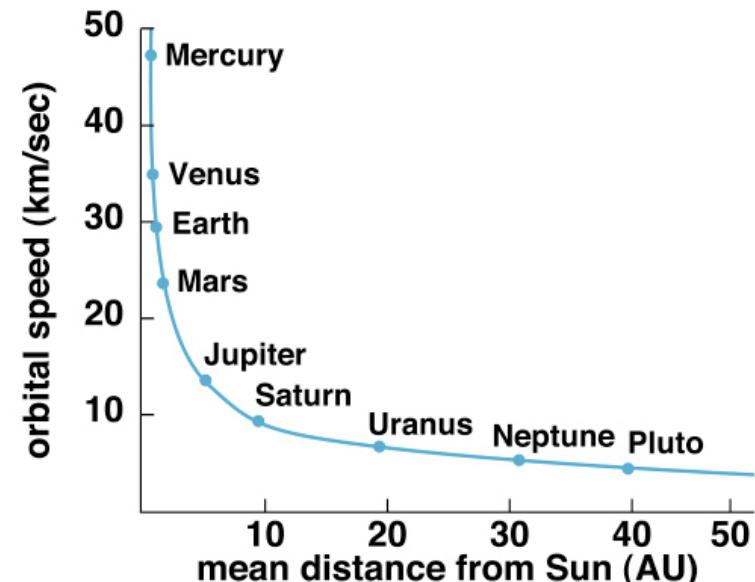
- Consider the mean orbital speed of planets around the Sun. The gravitational (attractive) force must be equal to the centripetal force:

$$F_G = \frac{G m M}{r^2} = \frac{m v^2}{r}$$

$v(r) = \sqrt{\frac{GM}{r}}$

➡ $v(r) \propto \sqrt{1/r}$

- This is observed for the planets in the solar system
- This should also hold for other dynamical systems e.g. the motion of stars or blobs of hydrogen in spiral galaxies orbiting around the galactic centre (hosting e.g. a supermassive black hole)



(b)

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Rotation Velocity and Galaxy Rotation Curve

Vera C. Rubin: Pioneering American astronomer (1928–2016)

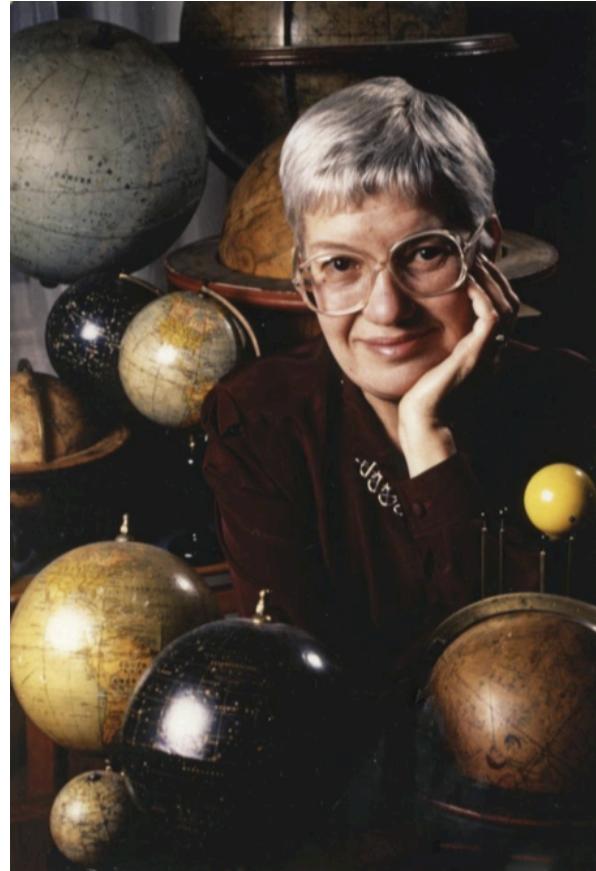
<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5338491/>

"Vera Cooper Rubin, an icon of astronomy whose work revolutionized our understanding of the universe by confirming the existence of dark matter"

"Her research showed that spiral galaxies rotate quickly enough that they should fly apart, if the gravity of their constituent stars was all that was holding them together; because they stay intact, a large amount of unseen mass must be holding them together, a conundrum that became known as the galaxy rotation problem.

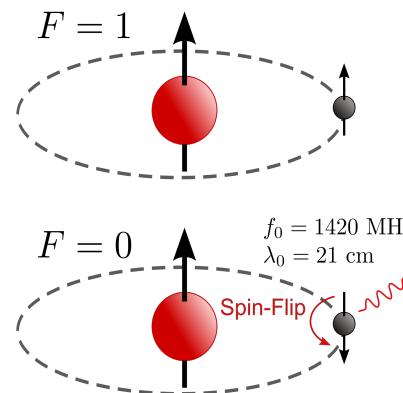
Rubin's calculations showed that galaxies must contain at least five to ten times as much dark matter as ordinary matter. Rubin's results were confirmed over subsequent decades, and became the first persuasive results supporting the theory of dark matter, initially proposed by Fritz Zwicky in the 1930s.

This data was confirmed by radio astronomers, the discovery of the cosmic microwave background, and images of gravitational lensing."



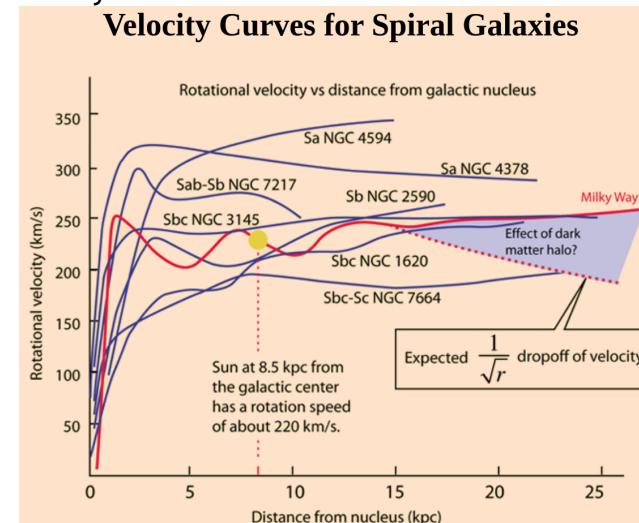
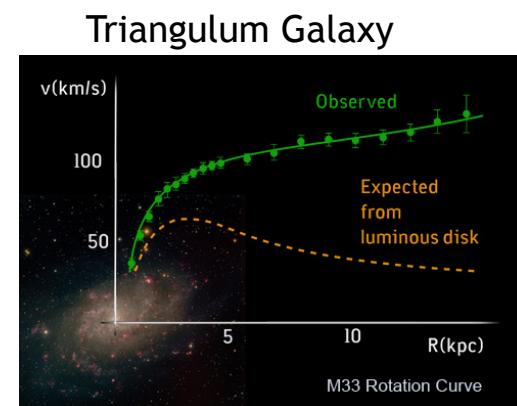
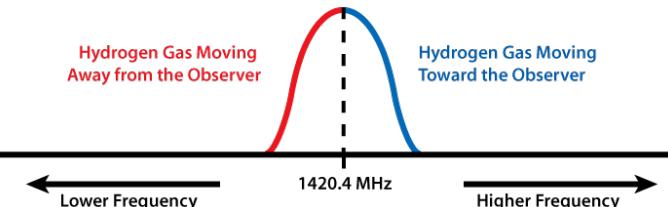
Rotation Velocity and Galaxy Rotation Curve

- However: this behaviour is not observed in the case of spiral galaxies; instead of the expected fall, the rotation curves all seem to flatten with increasing distance from the centre
- Idea: use the neutral hydrogen clouds 21cm radio emission to map the rotational velocity



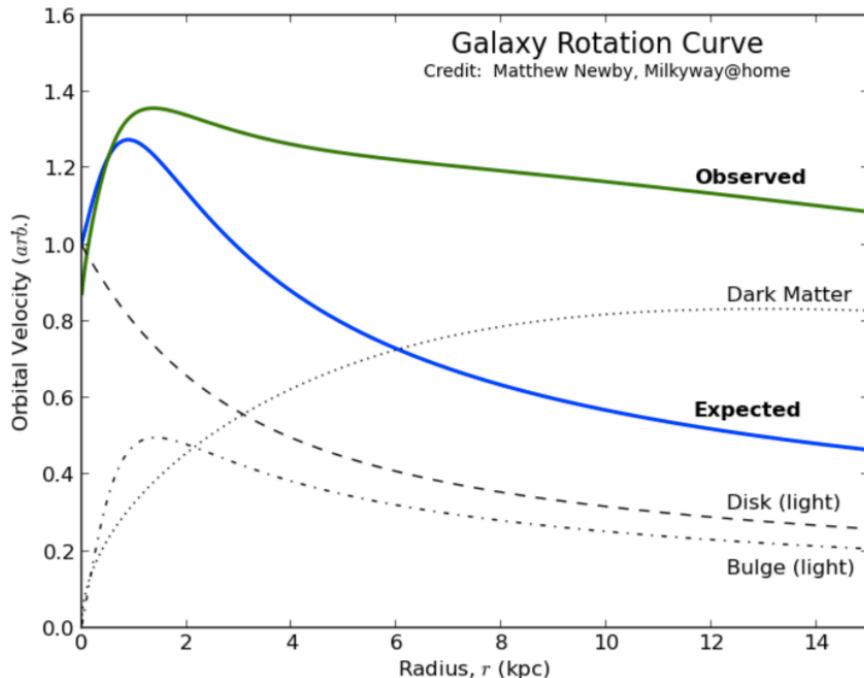
Hyperfine splitting or “spin flip”: transition to two different spin states of the electron. The motion wrt the observer will cause blue or red-shift (Doppler effect)

Spiral Galaxy Redshift Characteristics



<http://hyperphysics.phy-astr.gsu.edu/hbase/Astro/velcurv.html>

Rotation Velocity and Galaxy Rotation Curve



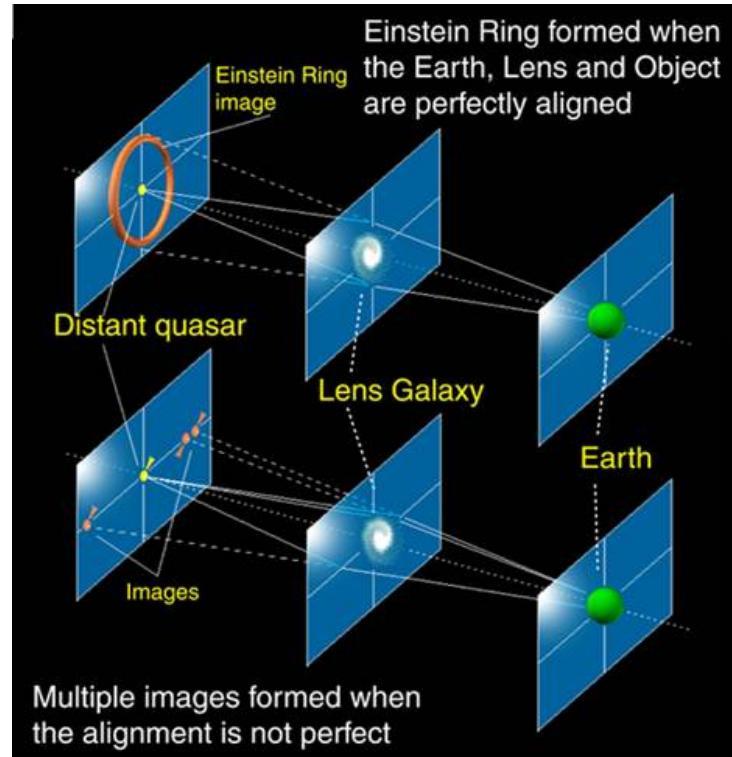
How to explain the observed flat curve:

- Observe the luminous matter distributions of the two components of the galaxy (i.e. the bulge and the disk)
- Calculate the gravitational potential, and resulting velocity distribution of stars as a function of the distance
- Add the expected contributions → they do not match the observed curve
- Add a hypothetical distribution of matter, that is not observed, to obtain velocities compatible with observations

- Calculations/simulations suggest a presence of a “halo” of non-visible matter, that surrounds the galaxy, and explain the flattening of the rotation curves
- The halo component dominates at large radii, while it is less important near the centre

Gravitational Lensing

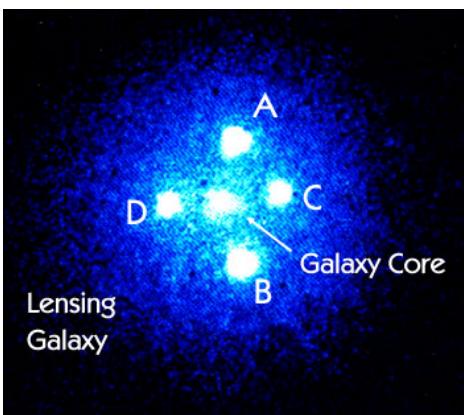
- Yet, another prediction from Einstein general relativity
- In the presence of a large mass, the space-time is distorted and particles e.g. photons follow accordingly different geodetics
- The image of a distant source (like a quasar) appears on the sky at a different position, or at multiple positions, or distorted due to the intense gravitational field caused by a massive astrophysics placed in between the source and the observer
- Depending on the alignment of the luminous source - lensing object - observer, multiple effects are possible



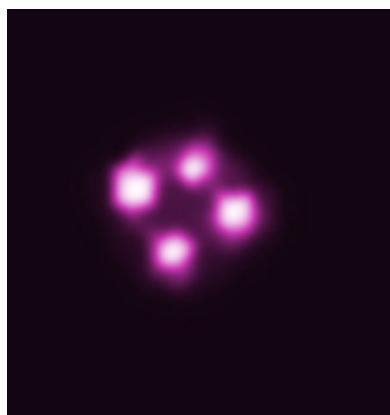
Gravitational Lensing

<http://www.astronomy.com/news/2015/04/alma-sees-einstein-ring-in-stunning-image-of-lensed->

- The Einstein Cross (Q2237+030 or QSO 2237+0305) is a gravitationally lensed quasar
- Due to strong gravitational lensing caused by the lensing galaxy (Huchra's Lens), the light coming from the quasar splits into four different images



The above long-exposure photograph shows the position of the lensed images of the quasar



Chandra satellite X-Ray observation

Constraining Quasar Relativistic Reflection Regions and Spins with Microlensing

“The X-rays detected by Chandra are produced when the accretion disk surrounding the black hole creates a multimillion-degree cloud, or corona above the disk near the black hole. X-rays from this corona reflect off the inner edge of the accretion disk, and the strong gravitational forces near the black hole distort the reflected X-ray spectrum, that is, the amount of X-rays seen at different energies. The large distortions seen in the X-ray spectra of the quasars studied here imply that the inner edge of the disk must be close to the black holes, giving further evidence that they must be spinning rapidly.”

<https://arxiv.org/pdf/1901.06007.pdf>

<https://chandra.harvard.edu/blog/node/731>

<https://phys.org/news/2019-03-einstein.html>

Einstein Cross (March 2019) found by The Hubble Space Telescope

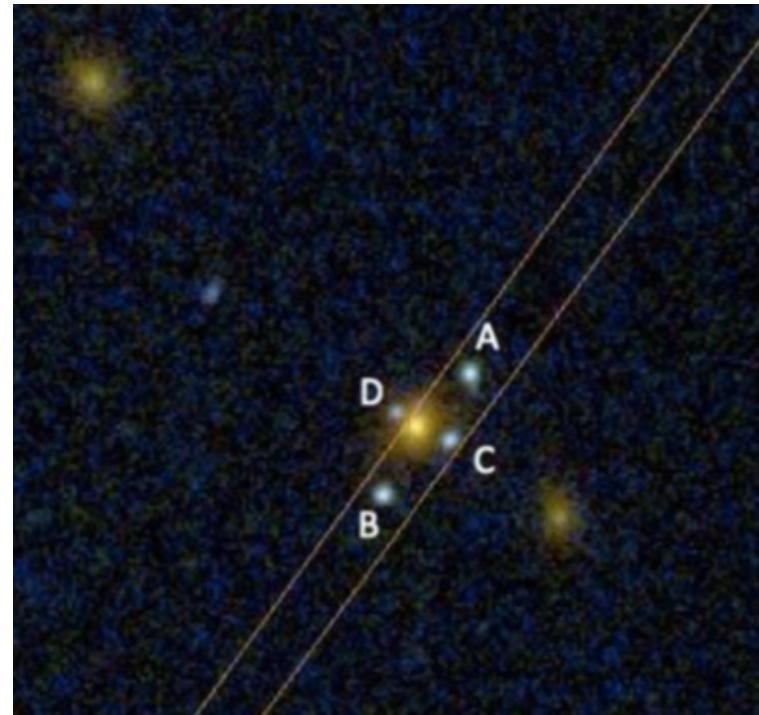
"The object acting as a lens turns out to be an elliptical galaxy located at a distance of approximately 7 billion light years ($z = 0.556$), while the source is at least 20 billion light years away ($z = 3.03$). "Normally the source is a quasar, it was with great surprise that we realized the source in this case was another galaxy, in fact a galaxy with very intense emission lines which indicates it is a young object still forming large amounts of stars"

[...] Gravitational lenses are important because they allow the study of the Universe in a unique way. Because the light of the different images, initially the same light, follows different paths in the Universe, thus any spectral differences must be due to the material that is between us and the source. Moreover, if the source is variable, we can see a time delay (one image illuminates before the others), which provides valuable information about the shape of the Universe.

Of course, the mass of the lens responsible for bending the light can be accurately derived, providing an important independent method to weight galaxies.

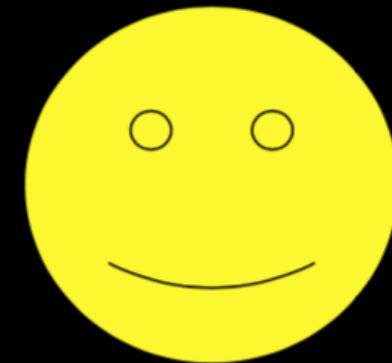
Finally, as with a normal glass lens, the gravitational lens concentrates toward us the light from the source, making it possible to see intrinsically unreachable objects. In this case it could be calculated that the source is 5 times brighter than it would be without the lens. "

<https://phys.org/news/2019-03-einstein.html>



“Hubble sees a smiling lens”

<https://www.nasa.gov/content/hubble-sees-a-smiling-lens>

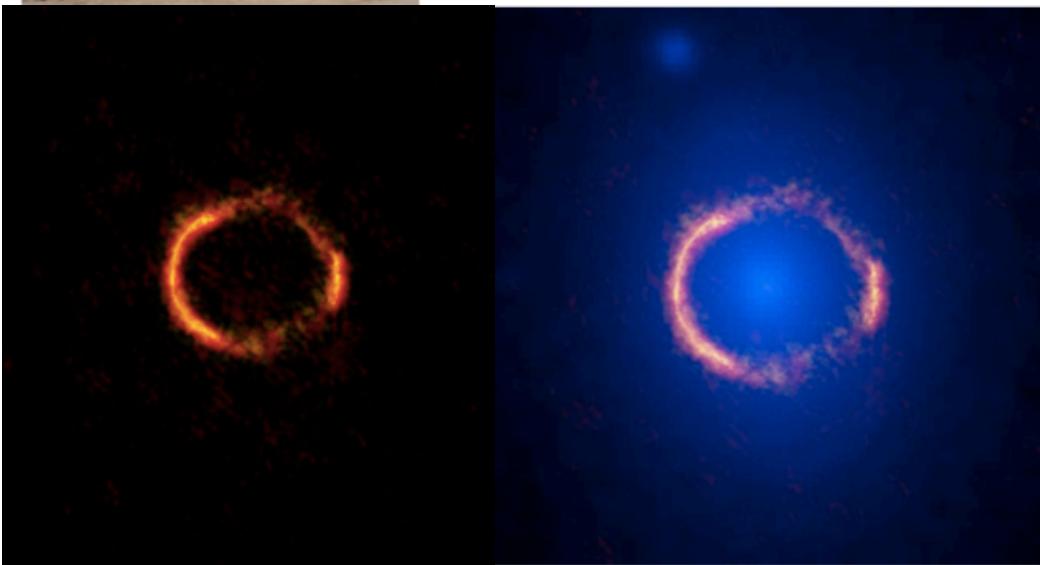


ALMA Ring

Atacama Large Millimiter Array (ALMA, Chile)



<https://public.nrao.edu/news/alma-ring-lens/#PRimage3>



ALMA/Hubble composite image of the gravitationally lensed galaxy SDP.81. The bright orange central region of the ring (ALMA's highest resolution observation ever) reveals the glowing dust in this distant galaxy.

The surrounding lower-resolution portions of the ring trace the millimeter wavelength light emitted by carbon monoxide.

The diffuse blue element at the center of the ring is from the intervening lensing galaxy, as seen with the Hubble Space Telescope.

Bullet Cluster (1E 0657–56)



- One “smoking gun” of the existence of Dark Matter
It is the smallest cluster of a two cluster systems, which is bullet-shaped
- The dynamics of visible components (stars and hot gas) is different from the dynamics of Dark Matter
- The study of gravitational lensing produced by the system can be explained only by the presence of two DM Halos
- The lensing is strongest in two separated regions, which do not overlap with the X-ray emission (from the hot gas)
i.e. the centre of mass of the baryonic matter

- One of the strongest evidence against alternative gravitation theories (like MOND, Modified Newtonian Dynamics)
- It is possible to derive a model independent DM annihilation / interaction cross section upper limit

Bullet Cluster (1E 0657–56)

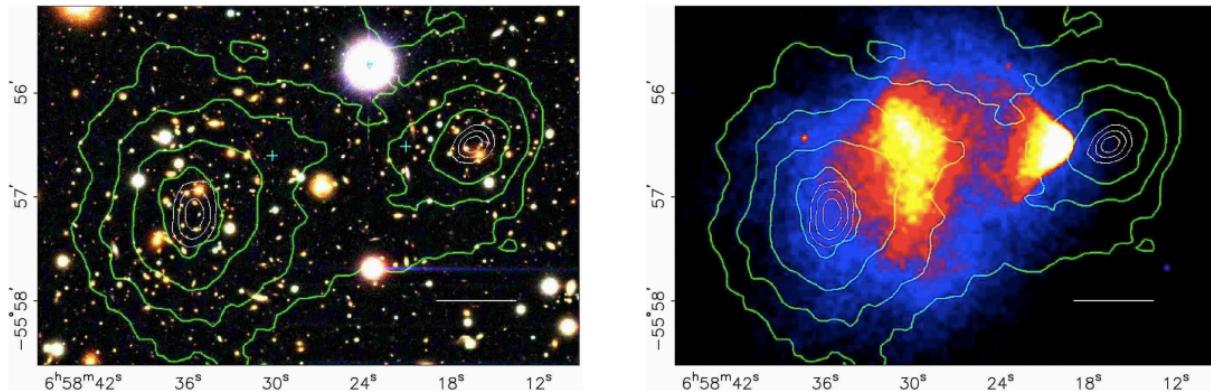


FIG. 1.— Shown above in the top panel is a color image from the Magellan images of the merging cluster 1E0657–558, with the white bar indicating 200 kpc at the distance of the cluster. In the bottom panel is a 500 ks Chandra image of the cluster. Shown in green contours in both panels are the weak lensing κ reconstruction with the outer contour level at $\kappa = 0.16$ and increasing in steps of 0.07. The white contours show the errors on the positions of the κ peaks and correspond to 68.3%, 95.5%, and 99.7% confidence levels. The blue +s show the location of the centers used to measure the masses of the plasma clouds in Table 2.

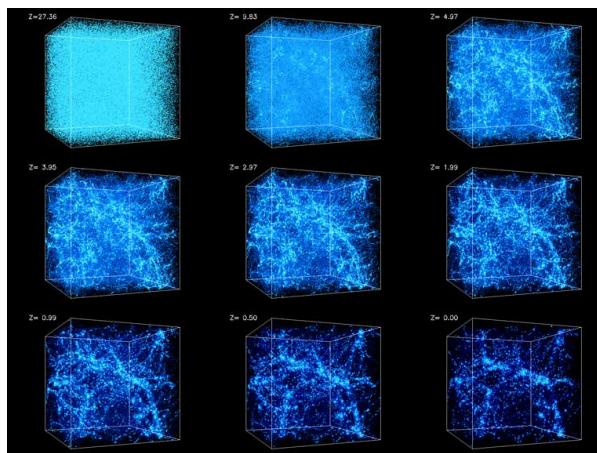
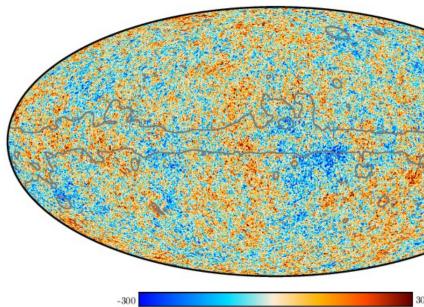
<https://arxiv.org/pdf/astro-ph/0608407.pdf>

A DIRECT EMPIRICAL PROOF OF THE EXISTENCE OF DARK MATTER

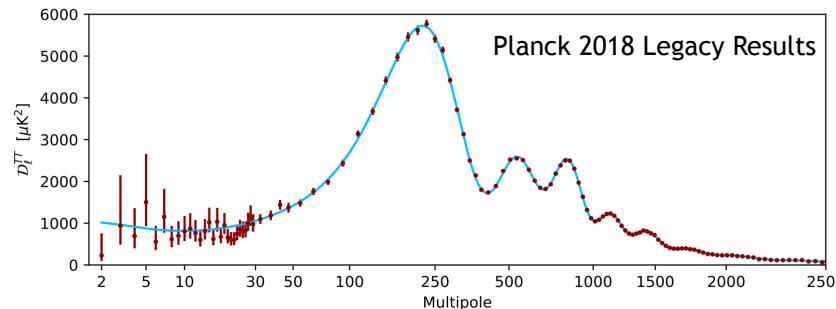
“We present new weak lensing observations of 1E0657–558 ($z=0.296$), a unique cluster merger, that enable a direct detection of dark matter, independent of assumptions regarding the nature of the gravitational force law. Due to the collision of two clusters, the dissipationless stellar component and the fluid-like X-ray emitting plasma are spatially segregated. By using both wide-field ground based images and HST/ACS images of the cluster cores, we create gravitational lensing maps which show that the gravitational potential does not trace the plasma distribution, the dominant baryonic mass component, but rather approximately traces the distribution of galaxies. An 8σ significance spatial offset of the center of the total mass from the center of the baryonic mass peaks cannot be explained with an alteration of the gravitational force law, and thus proves that the majority of the matter in the system is unseen.”

DM in the Cosmic Microwave Background and in Cosmology

CMB Spectrum



Large Scale Structure of the Universe and Structure Formation



Baryonic component



DM component



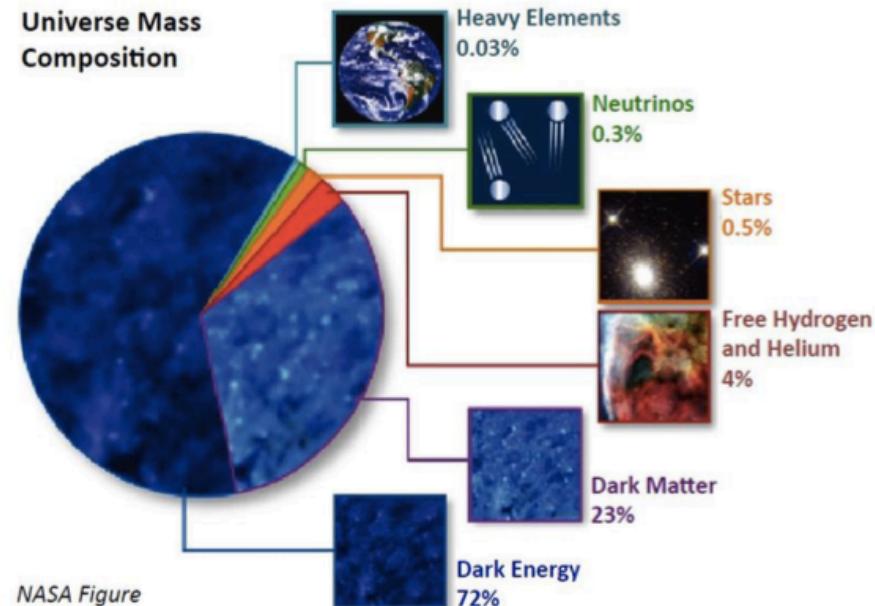
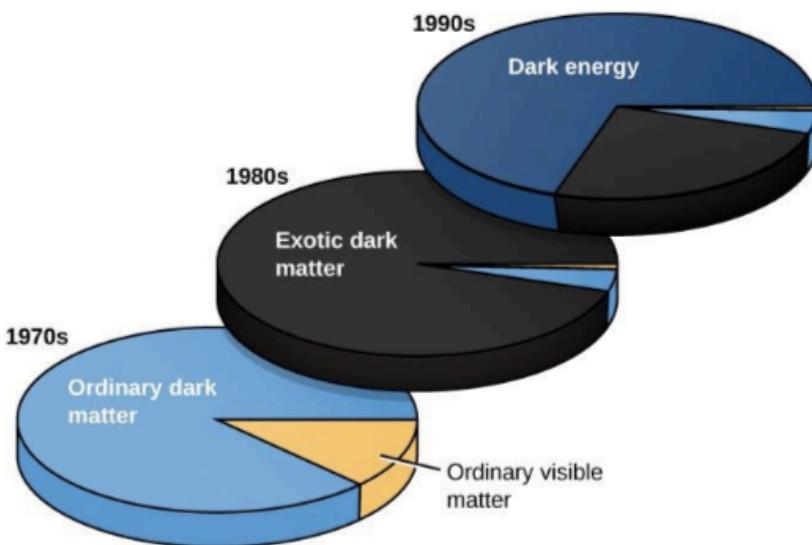
Dark Energy component



DM + Baryons



Parameter	TT+lowE 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022
$\Omega_c h^2$	0.1206 ± 0.0021
$100\theta_{MC}$	1.04077 ± 0.00047
τ	0.0522 ± 0.0080
$\ln(10^{10} A_s)$	3.040 ± 0.016
n_s	0.9626 ± 0.0057
$H_0 [\text{km s}^{-1} \text{ Mpc}^{-1}]$. . .	66.88 ± 0.92
Ω_Λ	0.679 ± 0.013
Ω_m	0.321 ± 0.013
Age [Gyr]	13.830 ± 0.037



(Numbers ~ change according to the experiment taken into account)

Part 2. A bit of Cosmology

Introduction: Cosmology

- Cosmology is a discipline that studies the origin and evolution of the universe
- It is a very fascinating theory, yet very complicated (because of the maths and physics required)
- It is based on Einstein's theory of general relativity (GR)
- The most favoured and largely accepted model of Cosmology is called Λ CDM, where Λ = Einstein's constant and CDM = Cold Dark Matter

How, when did the universe form?

How did it evolve?

What is the geometry of the universe? And what is the energy content?

How did structures (from the first stars to galaxies) form?

What is the CMB? Why is it so homogeneous?

How will the universe „end“ ?

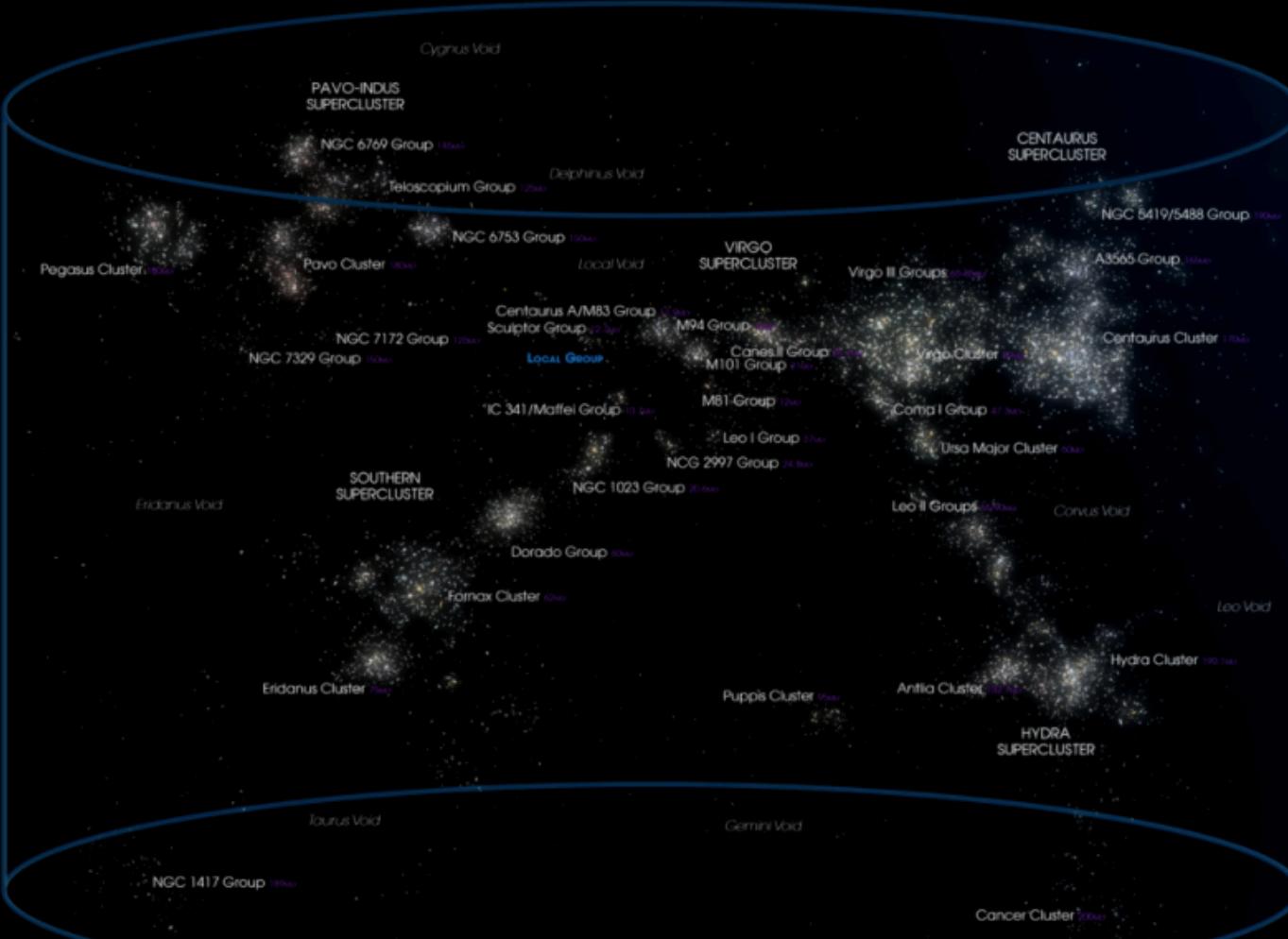


Introduction: Cosmology

- The first and most important feature of the universe is the "Cosmological Principle" (CP) : at sufficiently large scales, is homogeneous and isotropic
- It was introduced to make Einstein's fields equation of General Relativity solvable in particular simple situations, e.g. systems with spherical symmetry. It was finally observed that the principle holds effectively on scales larger than about 300 Mpc, well beyond typical the scale of formed structures in the universe (galaxy superclusters, the largest structures in the present universe, reach dimensions of up to ~100 Mpc)
- At smaller and smaller scales the universe is very inhomogeneous, with large portions of void between formed structures (planets, stars, galaxies, clusters ...)
- The best evidence for isotropy is the Cosmic Microwave Background Radiation (CMB), the relic of the initial phase of the universe (the so called "Big Bang") originated at the decoupling of matter and radiation. It is found at exactly the same average temperature of ~ 2.73 K in any direction we observe it, with fluctuations $< 10^{-5}$ K.

Local Supercluster: ~159 Mpc

LANIAKEA



Robertson-Friedmann-Walker Metric

Thanks to the CP, we can apply Einstein's theory of gravitation in a much simpler form

$$R_{ij} - \frac{1}{2}g_{ij}R - \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$$

Diagram illustrating the components of the Einstein field equation:

- Ricci Tensor (left)
- Metric tensor (center)
- Cosmological Constant (Λ) (center)
- Energy Tensor (right)

Equation for the Energy Tensor:

$$T_{ij} = (p + \rho c^2)u_i u_j - p g_{ij}$$

P:Pressure, rho:energy density, u:4-velocity tensors

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - dx^2 - dy^2 - dz^2$$

the invariant interval between two events at coordinates (t,x,y,z) and (t+dt,x+dx,y+dy,z+dz)

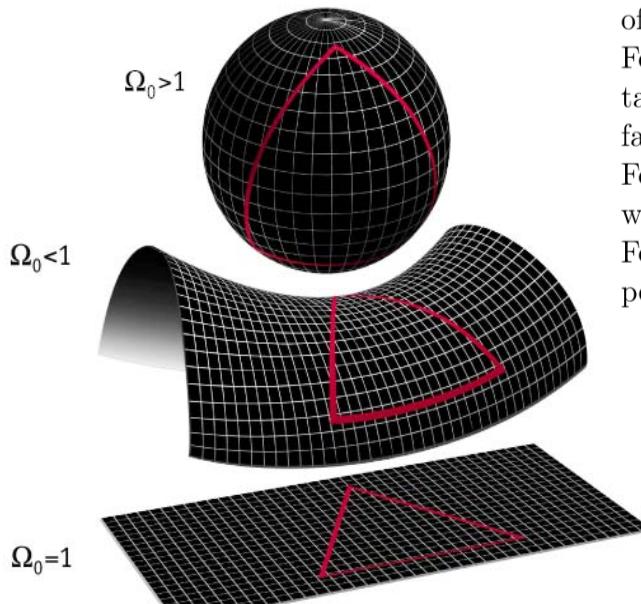
In particular we can look at the universe as a continuous, homogeneous and isotropic fluid. For such system, ***the most general metric*** of the space-time, called Robertson-Friedmann-Walker, for an expanding universe takes the form

$$ds^2 = (c dt)^2 - a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

$$ds^2 = (c dt)^2 - a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

- r, θ, ϕ are the comoving dimensionless coordinates;
- $a(t)$ is called cosmic expansion or scale parameter and has the dimension of a length. It is the variable that controls the rate expansion or contraction of the universe giving its dimension at a particular time, and the evolution varies according to the laws of the different models of universe. Geometrically it is a function of the time coordinate t ;
- t is the proper cosmic time, as measured by a clock moving through two distinct events at r, θ, ϕ fixed;
- K is the curvature parameter, and after proper renormalization it can be parametrized by the set of values (-1, 0, 1). It is related to the curvature of space: $R_{curv} \equiv a(t)/\sqrt{|K|}$.

Geometry of the Universe



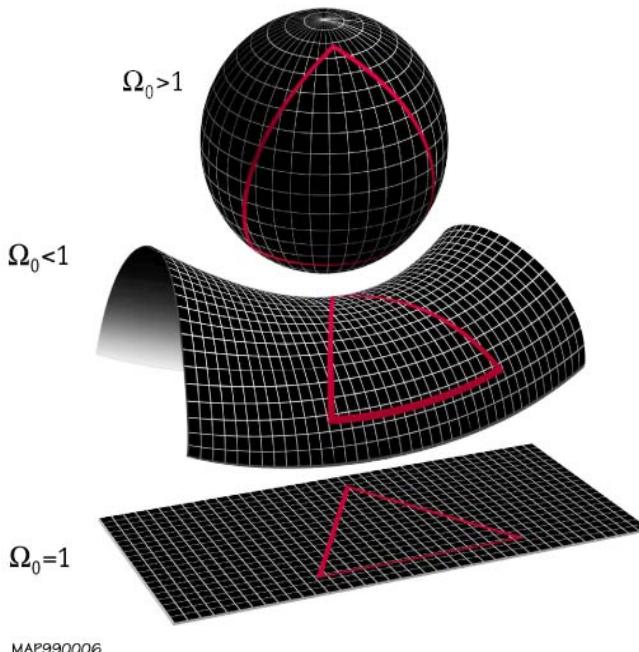
MAP990006

The different values of the curvature parameter reflect three different possible geometries of the universe.

For $\mathbf{K} = \mathbf{0}$ the spatial part of the metric represents an Euclidean flat space, with radial distance given by $a(t)r$; the curvature of the space-time instead is granted by the cosmic scale factor $a(t)$.

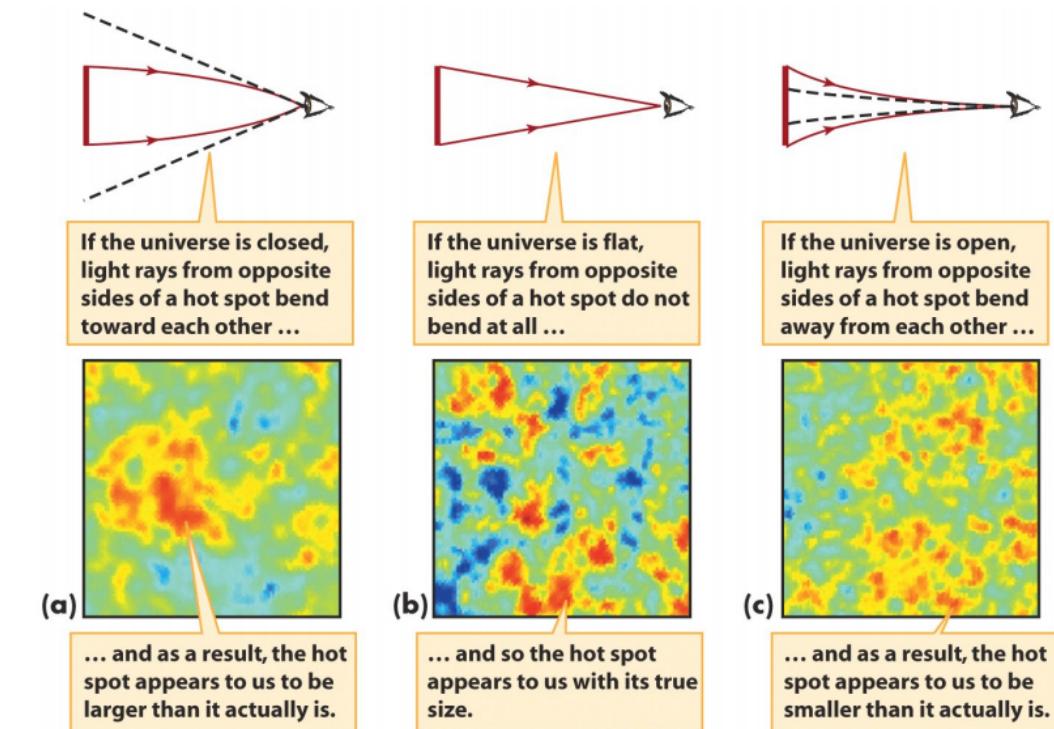
For $\mathbf{K} = \mathbf{1}$ the metric shows a singularity at $r = 1/\sqrt{K}$; the geometry is that of a hypersphere with *positive* curvature and the universe has finite dimensions.

For $\mathbf{K} = -\mathbf{1}$ the universe doesn't have a finite dimension and the geometry is that of a hyperbolic open space, with *negative* curvature.



Depending on the geometry, the dimension of distant object changes according to the angle formed by the light

Curvature of the Universe



$$\dot{a}^2 + Kc^2 = \frac{8}{3}\pi G\rho a^2$$

The two solutions are not independent, but can be retrieved by considering the adiabatic condition:

$$\ddot{a} = -\frac{4\pi}{3}G\left(\rho + 3\frac{p}{c^2}\right)a$$

$$\begin{aligned} d(\rho c^2 a^3) &= -p da^3 \\ dU &= -pdV \end{aligned}$$

Such kind of Universe cannot be static, unless the pressure assumes a negative value:

$$\rho = -3\frac{p}{c^2}$$

This lead Einstein to introduce the Cosmological Constant Λ

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G}{c^4}T_{ij} \quad \rightarrow \quad R_{ij} - \frac{1}{2}g_{ij}R - \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$$

$$\dot{a}^2 + Kc^2 = \frac{8}{3}\pi G\rho a^2$$

$$\ddot{a} = -\frac{4\pi}{3}G\left(\rho + 3\frac{p}{c^2}\right)a$$

The two solutions are not independent, but can be retrieved by considering the adiabatic condition:

$$\begin{aligned} d(\rho c^2 a^3) &= -p da^3 \\ dU &= -pdV \end{aligned}$$

“With an appropriate choice of Λ , one can obtain a static cosmological model. Equation ... represents the most general possible modification of the Einstein equations that still satisfies the condition that T_{ij} is equal to a tensor constructed from the metric g_{ij} and its first and second derivatives, and is linear in the second derivative. The strongest constraint one can place on Λ from observations is that it should be sufficiently small so as not to change the laws of planetary motion (from Cosmology by Coles-Lucchin)“

$$R_{ij} - \frac{1}{2}g_{ij}R - \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$$

The Λ term plays the role of a new component in the universe that is called “dark energy”, responsible for the accelerated expansion of the universe (1998)

Some Useful Formulas and Hubble's Law

Equation of state

Each component (regular matter or dust, radiation, dark matter and dark energy) of the cosmic fluid is treated separately, with a proper equation of state, which relates the pressure of the component to its density, e.g.

$$P = P(\rho) \quad P(\rho) = w\rho c^2$$

Example from standard thermodynamics:

→ Ideal gas of non-relativistic particles of mass m_p , temperature T, density ρ and adiabatic index γ

$$p = nk_B T = \frac{k_B T}{m_p c^2} \rho m_p c^2 = \frac{k_B T}{m_p c^2} \frac{\rho c^2}{1 + (k_B T / ((\gamma - 1)m_p c^2))} = w(T)\rho c^2$$

It is also possible to calculate the adiabatic speed of sound of the fluid:

$$v_s = \left(\frac{\partial p}{\partial \rho} \right)_S^{1/2} = c\sqrt{w}$$



Since it cannot exceed the speed of light, w must be necessarily < 1

Some Useful Formulas and Hubble's Law

$$z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e}$$

Redshift z : given a source emitting light, the redshift is calculated as the shift between the observed and emitted wave length (at source), divided by the wavelength at source

- Hubble (Lemaître) Law (1927)

$$v = H_0(t) D$$

v = recession velocity

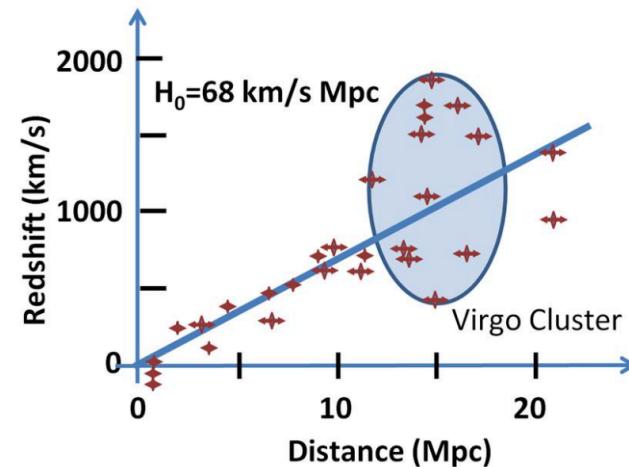
H = Hubble parameter at time of observation [km/s/Mpc]

D = proper distance galaxy-observer

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3} \rho(t) - \frac{K}{a_0(t)^2 a(t)^2}$$

$$H^2 = H_0^2 \left(\frac{a_0}{a}\right)^2 \left[1 - \sum_w \Omega_{0,w} + \sum_w \Omega_{0,w} \left(\frac{a_0}{a}\right)^{1+3w} \right]$$

$$H^2(z) = H_0^2 (1+z)^2 \left[1 - \sum_w \Omega_{0,w} + \sum_w \Omega_{0,w} (1+z)^{1+3w} \right]$$



Take-home message: depending on the content of the universe i.e. the parameters in the Λ CDM you obtain a different parameter H which gives a different expansion rate

—> What can you measure experimentally?

Evolution of the Components of the Universe

$$\Omega_w(t) = \frac{\rho_w(t)}{\rho_c(t)}$$

Density of each component of the Universe
(ordinary matter or dust, dark matter, radiation and dark energy)

$$\rho_{cr} = \frac{3H_0^2}{8\pi G}$$

Critical density (estimated: $1-3 \times 10^{-29} \text{ g/m}^3$)

$\rho < p_{cr}$	\Leftrightarrow	$\Omega < 1$	\Leftrightarrow	$K = -1$
$\rho = p_{cr}$	\Leftrightarrow	$\Omega = 1$	\Leftrightarrow	$K = 0$
$\rho > p_{cr}$	\Leftrightarrow	$\Omega > 1$	\Leftrightarrow	$K = +1$

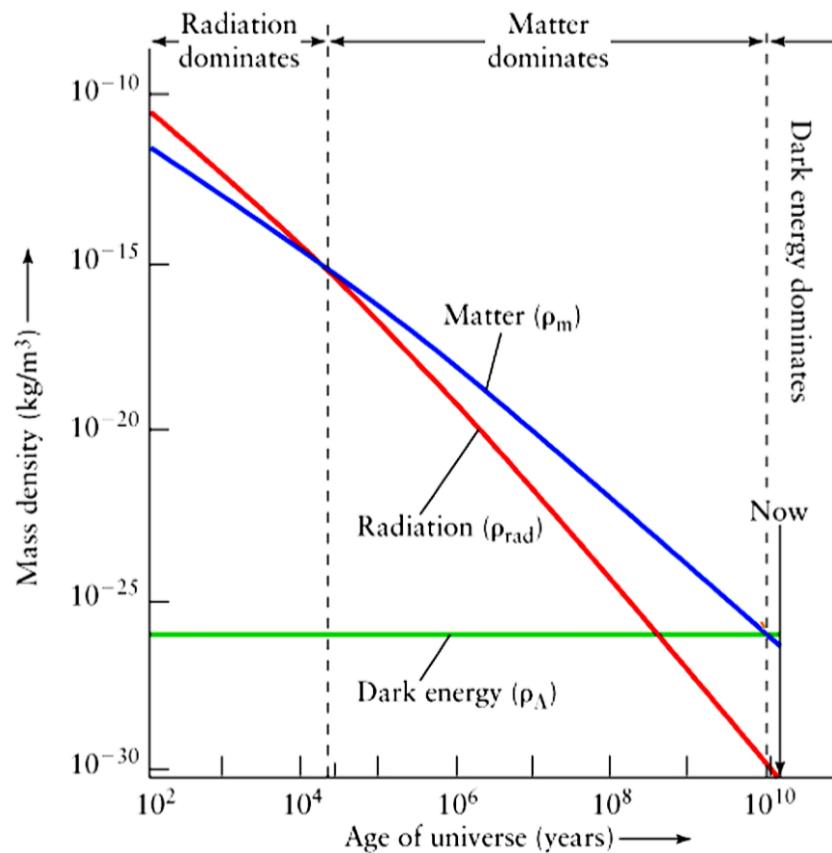
Currently, the most favoured geometry of the Universe is the „flat“ case
i.e. the total energy density of all components equals the critical density

Evolution of the Components of the Universe

Combining the adiabaticity condition and the equation of state, for specific values of w :

$$d(\rho c^2 a^3) = -p da^3 \quad \downarrow \quad p = w(T) \rho c^2$$

Component	w	Eq. of State	Density Evolution
Matter(dust)	0	$p = 0$	$\rho_m = \rho_{0,m}(1+z)^3$
Radiation	$\frac{1}{3}$	$p = \frac{1}{3}\rho_r c^2$	$\rho_r = \rho_{0,r}(1+z)^4$
Cosm. Constant Λ	-1	$p = -\rho_\Lambda c^2$	const.



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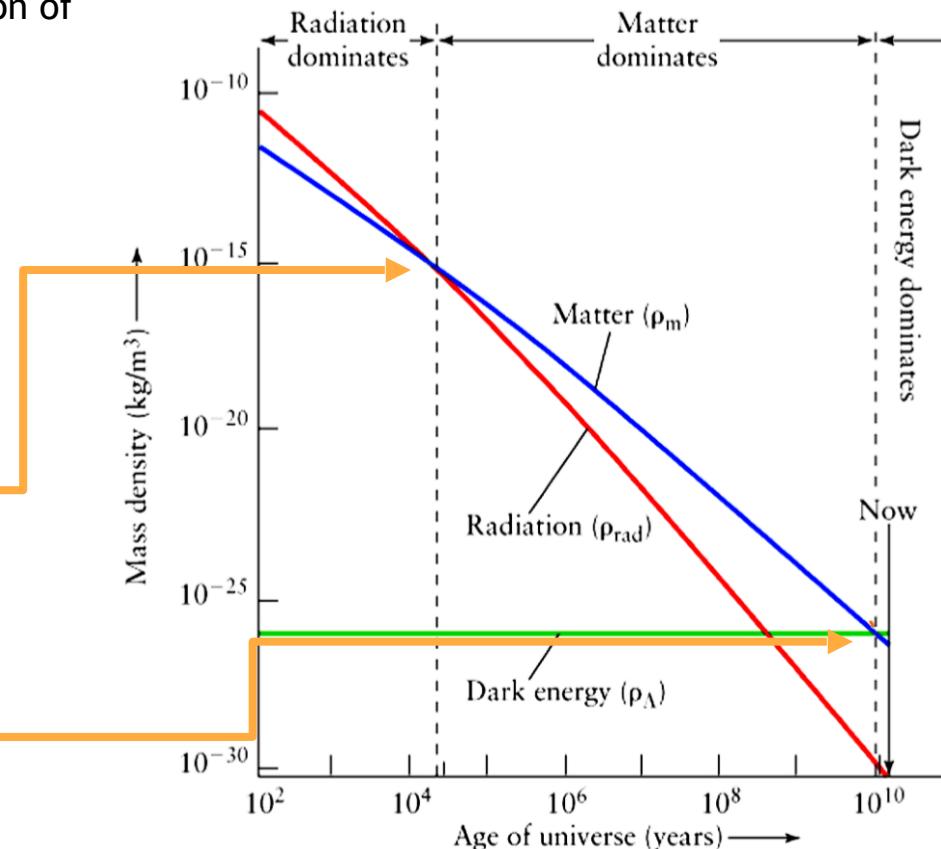
Matter-radiation equivalence

$$\rho_m(z_{eq}) \equiv \rho_r(z_{eq}) \rightarrow \rho_{0m}(1+z_{eq})^3 \equiv \rho_{0r}(1+z_{eq})^4$$

$$1+z_{eq} = \left(\frac{\rho_{0r}}{\rho_{0m}} \right)^{-1} = \left(\frac{\Omega_{0r}}{\Omega_{0m}} \right)^{-1} \simeq 4 \cdot 10^4 (\Omega_{0m} h^2)$$

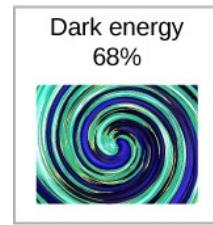
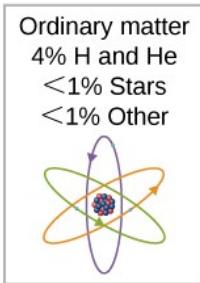
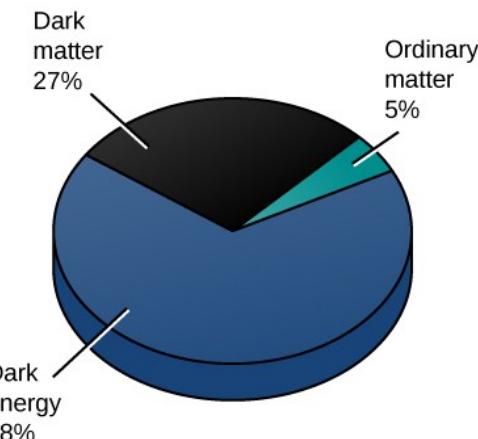
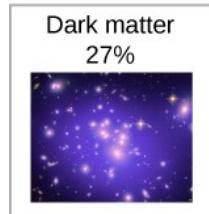
“Matter-Lambda” equivalence

$$1+z_\Lambda = \left(\frac{\rho_{0\Lambda}}{\rho_{0m}} \right)^{1/3} \simeq \left(\frac{0.75}{0.25} \right)^{1/3} \simeq 3^{1/3}$$

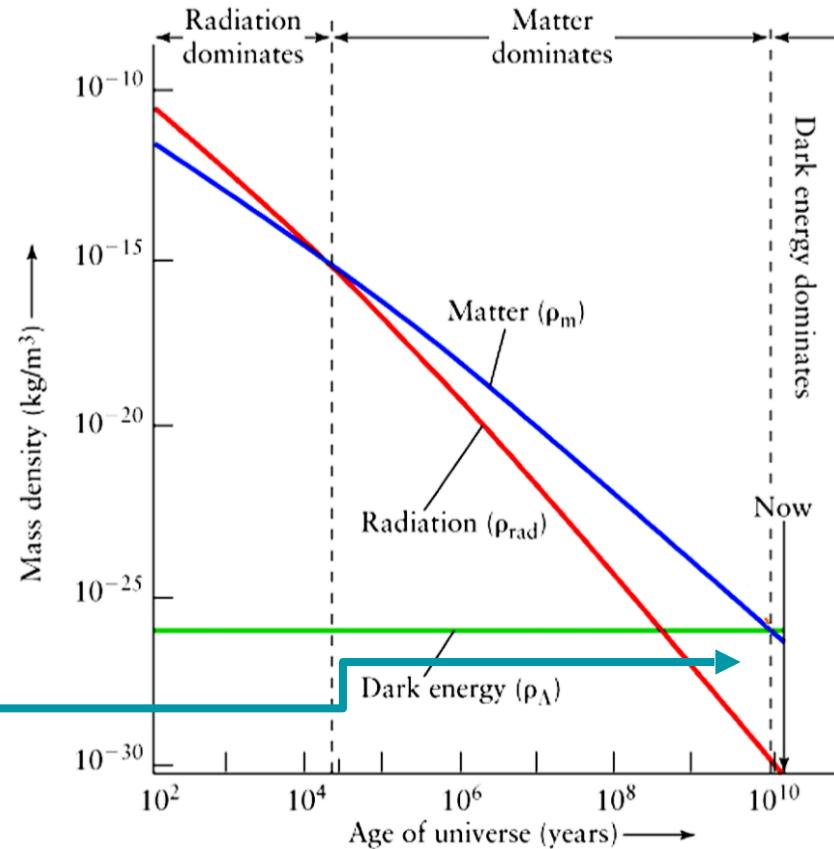


Evolution of the Components of the Universe

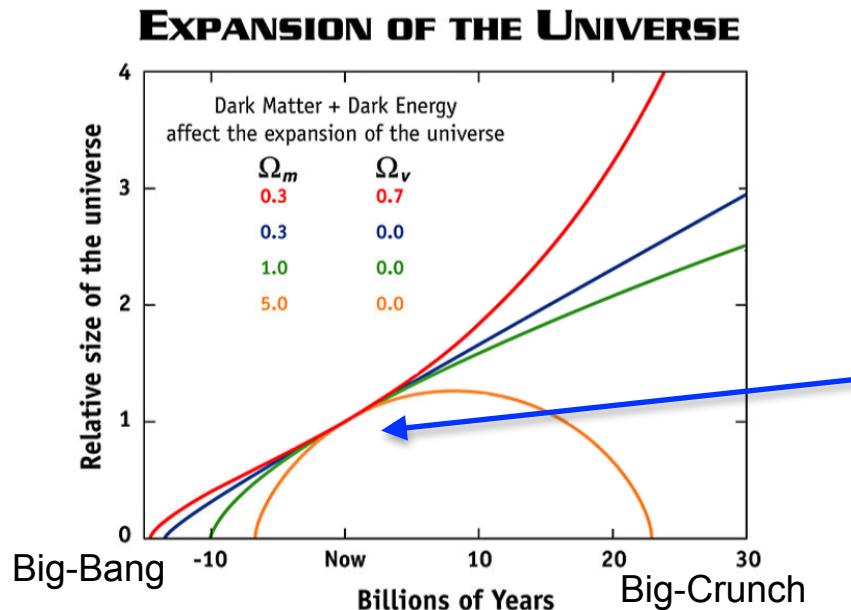
Composition of the Universe



Radiation (photons and relativistic particles such as neutrinos) does not contribute significantly anymore to the energy budget of the universe



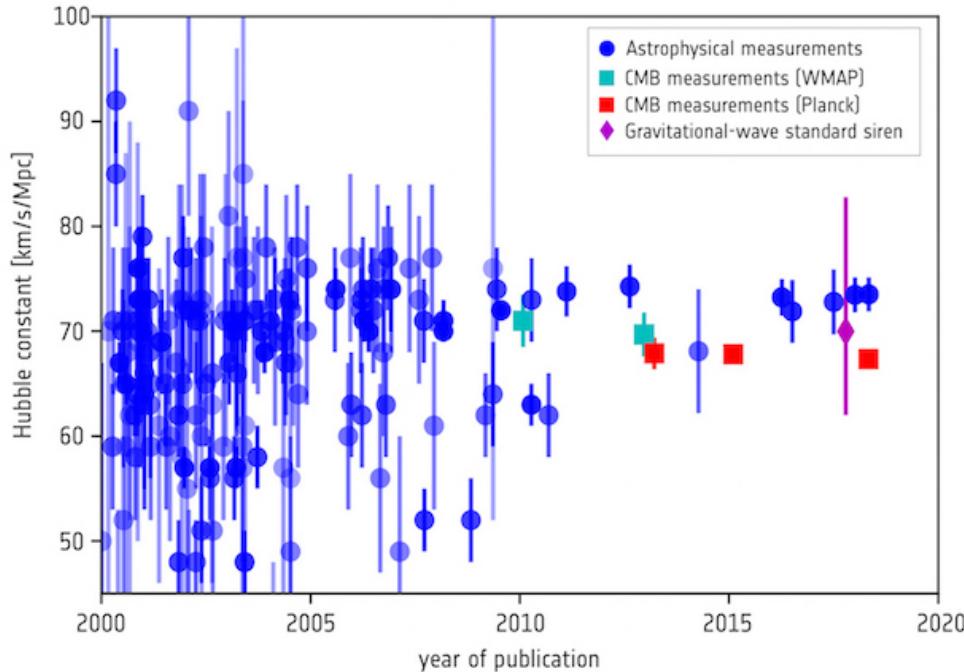
Fate of the Universe ?



- Depending on the geometry and on the energy content of the universe (divided into its different components), the fate of the universe will be different
- We are in a point i.e. “now” which is compatible with most models

Figure 1.3: Time evolution of the scale parameter, or size of the universe, for different values of the matter and dark energy densities. Times run to the right of the x-axes or analogously, redshift increases to the left of x-axis. The point where the curves cross the time axis (on the left) is known as Big Bang; the time where the closed-model curve intersects the time axis (on the right) is known as Big Crunch. Different models have different ages of the universe, hence different times for the Big Bang. Picture taken from www.nasa.gov.

The Hubble Parameter



[from 2018]

<https://sci.esa.int/web/planck/-/60504-measurements-of-the-hubble-constant>

«Alternatively, the Hubble Constant can also be estimated from the cosmological model that fits observations of the cosmic microwave background, which represents the very young Universe, and calculate a prediction for what the Hubble Constant should be today. Measurements based on this method using data from NASA's WMAP satellite are shown in green, and those obtained using data from ESA's Planck mission are shown in red. On the one hand, it is extraordinary that two such radically different ways of deriving the Hubble constant - one using the local, mature Universe, and one based on the distant, infant Universe - are so close to each other. On the other hand, in principle these two figures should agree to within their respective uncertainties, causing what cosmologists call a 'tension' - an oddity that still needs explaining.»

The Hubble Parameter

Measurement of the Hubble constant

Date published	Hubble constant (km/s)/Mpc	Observer	Citation	Remarks / methodology
2019-10-14	74.2 ^{+2.7} _{-3.0}	STRIDES	[76]	Modelling the mass distribution & time delay of the lensed quasar DES J0408-5354.
2019-09-12	76.8 \pm 2.6	SHARP/H0LiCOW	[77]	Modelling three galactically lensed objects and their lenses using ground-based adaptive optics and the Hubble Space Telescope
2019-08-15	73.5 \pm 1.4	M. J. Reid, D. W. Pesce, A. G. Riess	[78]	Measuring the distance to Messier 106 using its supermassive black hole, combined with measurements of eclipsing binaries in the Large Magellanic Cloud.
2019-07-16	69.8 \pm 1.9	Hubble Space Telescope	[56][57][58]	Distances to red giant stars are calculated using the tip of the red-giant branch (TRGB) distance indicator.
2019-07-10	73.3 ^{+1.7} _{-1.8}	H0LiCOW collaboration	[79]	Updated observations of multiply imaged quasars, now using six quasars, independent of the cosmic distance ladder and independent of the cosmic microwave background measurements.
2019-07-08	70.3 ^{+5.3} _{-5.0}	LIGO and Virgo detectors	[55]	Uses radio counterpart of GW170817, combined with earlier gravitational wave (GW) and electromagnetic (EM) data.
2019-03-28	68.0 ^{+4.2} _{-4.1}	Fermi-LAT	[80]	Gamma ray attenuation due to extragalactic light. Independent of the cosmic distance ladder and the cosmic microwave background.
2019-03-18	74.03 \pm 1.42	Hubble Space Telescope	[61]	Precision HST photometry of Cepheids in the Large Magellanic Cloud (LMC) reduce the uncertainty in the distance to the LMC from 2.5% to 1.3%. The revision increases the tension with CMB measurements to the 4.4σ level ($P=99.99\%$ for Gaussian errors), raising the discrepancy beyond a plausible level of chance. Continuation of a collaboration known as Supernovae, H_0 , for the Equation of State of Dark Energy (SHoES).
2019-02-08	67.78 ^{+0.91} _{-0.87}	Joseph Ryan et al.	[81]	Quasar angular size and baryon acoustic oscillations, assuming a flat LambdaCDM model. Alternative models result in different (generally lower) values for the Hubble constant.
2018-11-06	67.77 \pm 1.30	Dark Energy Survey	[82]	Supernova measurements using the inverse distance ladder method based on baryon acoustic oscillations.
2018-09-05	72.5 ^{+2.1} _{-2.3}	H0LiCOW collaboration	[83]	Observations of multiply imaged quasars, independent of the cosmic distance ladder and independent of the cosmic microwave background measurements.
2018-07-18	67.66 \pm 0.42	Planck Mission	[84]	Final Planck 2018 results.
2018-04-27	73.52 \pm 1.62	Hubble Space Telescope and Gaia	[85][86]	Additional HST photometry of galactic Cepheids with early Gaia parallax measurements. The revised value increases tension with CMB measurements at the 3.8σ level. Continuation of the SHoES collaboration.
2018-02-22	73.45 \pm 1.66	Hubble Space Telescope	[87][88]	Parallax measurements of galactic Cepheids for enhanced calibration of the distance ladder; the value suggests a discrepancy with CMB measurements at the 3.7σ level. The uncertainty is expected to be reduced to below 1% with the final release of the Gaia catalog. SHoES collaboration.