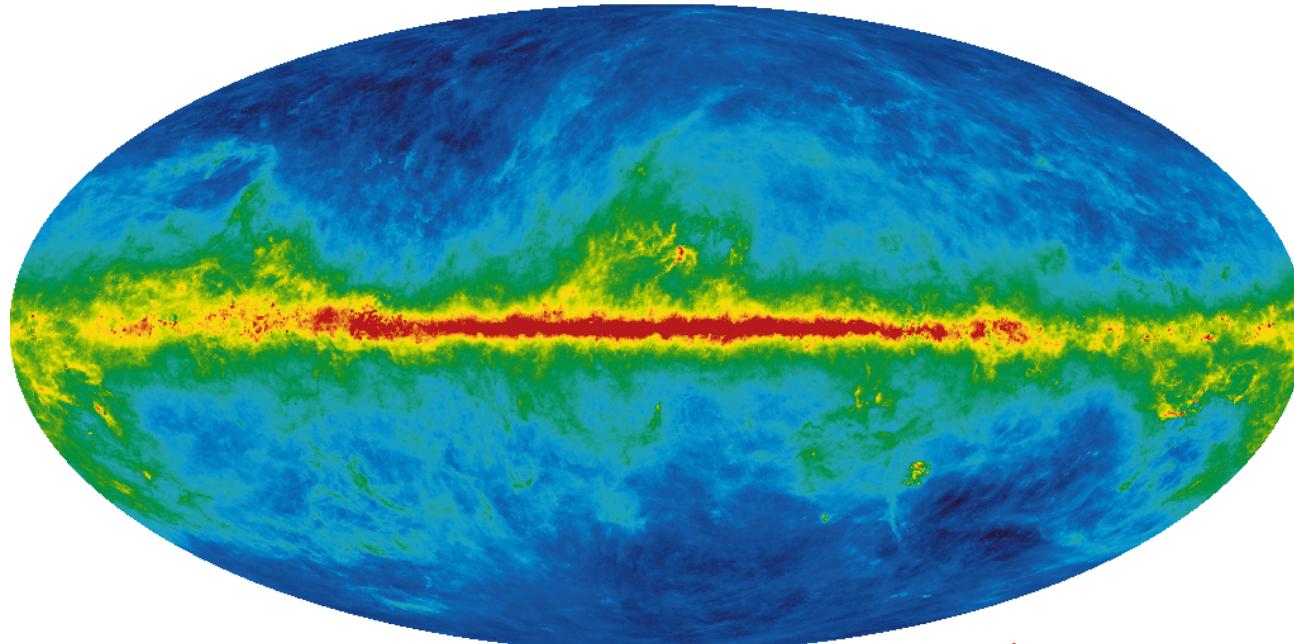
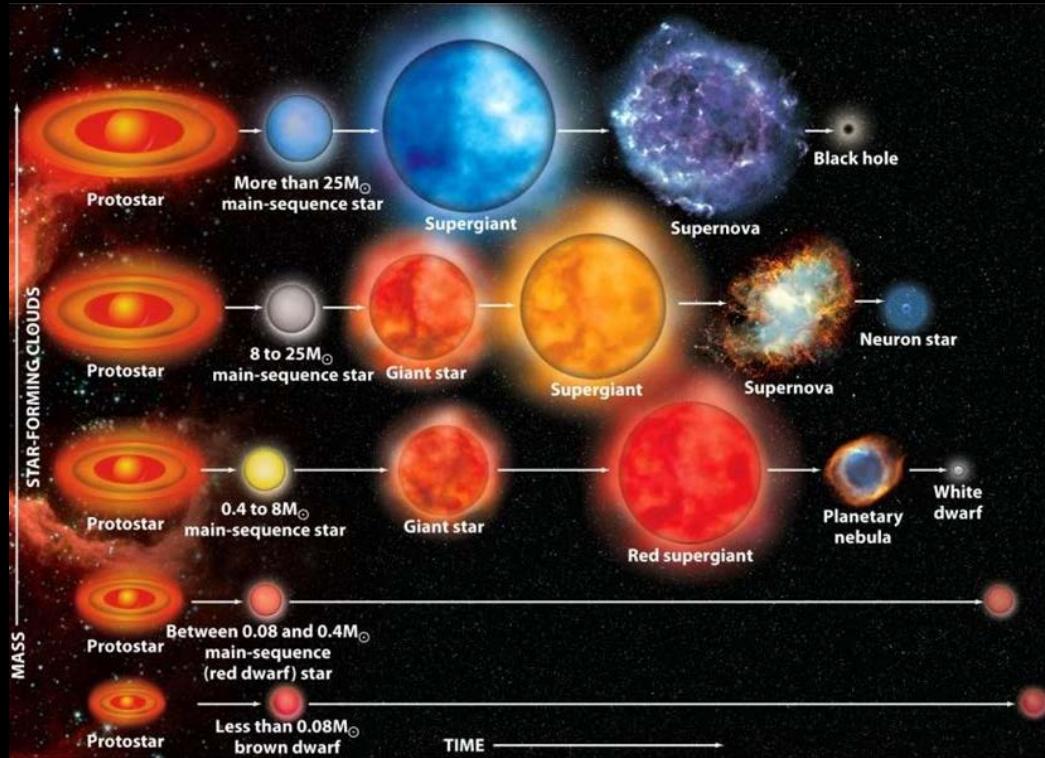


# Dark Matter and Cosmology



Astroparticle Physics  
Part 5 – WS 2019  
Federico Ambrogi

# Stellar Evolution (i.e. Where BHs and NSs come from)



Star evolution mostly depends on the mass of the original gas cloud

Metallicity i.e. chemical composition impacts fusion processes

For initial mass  $M_i > 0.08 M_{\odot} \rightarrow$  white dwarfs

For  $M_i > 8 M_{\odot} \rightarrow$  Supernova (SN) explosion and NS remnant

For  $M_i > 8 M_{\odot} \rightarrow$  SN explosion and formation of BHs (fallback or direct)

*Stellar evolution end with the creation of White Dwarfs, SNe, Neutron Satrs, Black Holes which are sources of gravitational waves!*

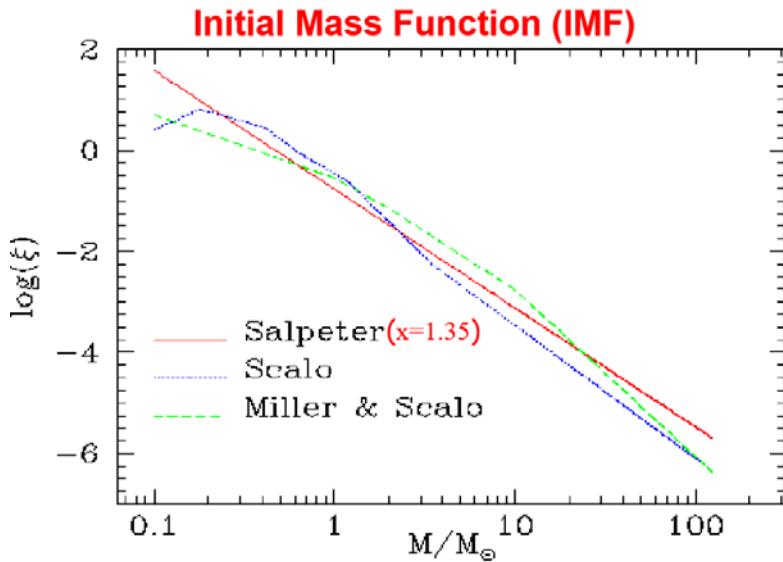
[ Typical numbers of neutron star:

Mass  $\sim 1.4 M_{\odot}$

radius  $\sim 10$  Km

magnetic field  $10^{12}$  Gauss (LHC: 4 T = 40k Gauss, average in Milky way  $\sim$  microGauss, Earth surface  $\sim$  0.2 Gauss) ]

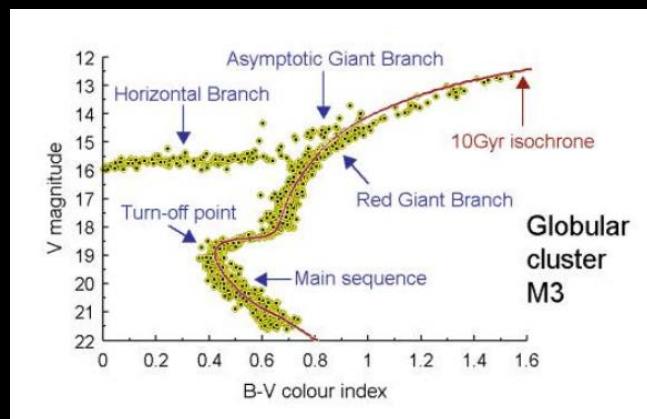
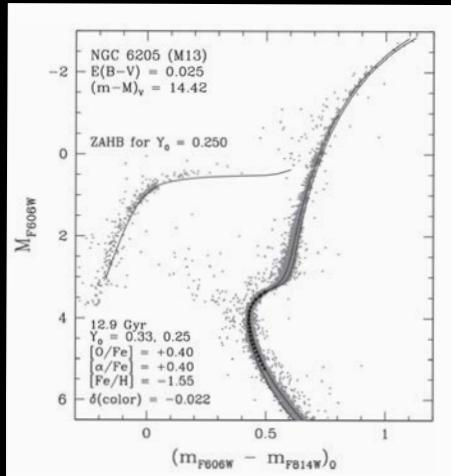
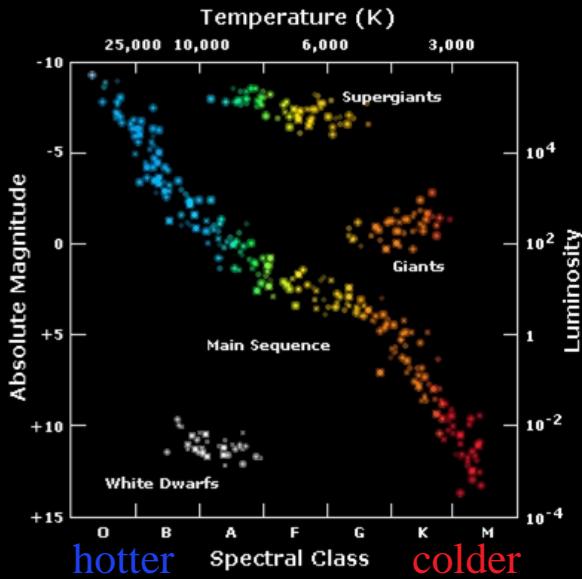
## Probability of a star forming with a given mass



## Stellar populations in our Galaxy

- **Population I**
  - $Z = 0.01 - 0.04$  ( $Z_{\text{Sun}} \approx 0.02$ ) (from low to high metallicity)
  - circular orbits within plane of Galaxy
  - young: Myr to 10 Gyr (e.g. **open clusters**)
- **Population II**
  - stellar halo:  $Z < 0.002$  (very low metallicity)
  - bulge  $Z \sim 0.02$
  - eccentric orbits (high-velocity stars)
  - old: 12 – 13.5 Gyr (e.g. **globular clusters**)
- **Relics of Population III objects ?**
  - Theoretical idea = the first stars formed  $> 13$  Gyr ago
  - Relics in the halo with extremely low metallicities ?

## Stellar Evolution (i.e. Where BHs and NSs come from)



Isochrone: curve on the HR diagram representing a population of stars of ~ same age (like e.g. globular clusters)

By fitting the stellar population with a model isochrone, it is possible to estimate the population age

Each feature (e.g. turning points, branches,...) of the isochrone can be explained by models of stellar evolution (type/size/region of nuclear fusion processes)

# Pulsars

<http://www.jb.man.ac.uk/distance/frontiers/pulsars/section1.html>

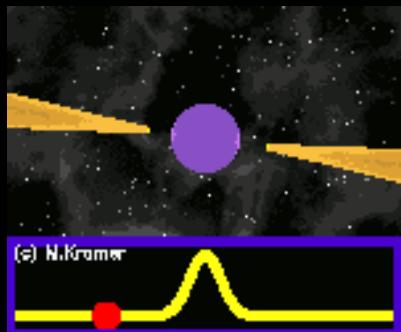
Pulsars (**Pulsating Radio Source**) are rapidly spinning neutron stars

Their rotation period span from the order of milliseconds to the order of ~8 seconds

Pulse: collection of radio waves of different frequencies

Rotation period extremely stable, although the rotation speed slows down with time

„Lighthouse“ effect



Optical image of the Crab nebula (NOAO).  
The arrow marks the position of the pulsar.

Composite optical/X-ray image of the Crab Nebula, showing synchrotron emission in the surrounding pulsar wind nebula, powered by injection of magnetic fields and particles from the central pulsar.

- In this lecture we will see the role of Dark Matter in the evolution of the universe and in the formation of cosmic structure
- Cosmology is a discipline that studies the origin and evolution of the universe
- It is a very fascinating theory, yet very complicated
- It is based on Einstein's theory of general relativity (GR)
- The most favoured and largely accepted model of Cosmology is called  $\Lambda$ CDM

*How, when did the universe form?*

*How did it evolve?*

*What is the geometry of the universe? And what is the energy content?*

*How did structures (from the first stars to galaxies) form?*

*What is the CMB? Why is it so homogeneous*

*In temperature?*

*How will the universe „end“ ?*

- The first and most important feature of the universe aka the "Cosmological Principle" (CP) : at sufficiently large scales, is homogeneous and isotropic
- It was introduced to make Einstein's fields equation of General Relativity solvable in particular simple situations, e.g. systems with spherical symmetry. It was finally observed that the principle holds effectively on scales larger than about 300 Mpc, well beyond typical the scale of formed structures in the universe (galaxy superclusters, the largest structures in the present universe, reach dimensions of a few tens of Mpc)
- At smaller and smaller scales the universe is very inhomogeneous, with large portions of void between formed structures (planets, stars, galaxies, clusters ...)
- The best evidence for isotropy is the Cosmic Microwave Background Radiation (CMB), the relic of the initial phase of the universe (the so called "Big Bang") originated at the decoupling of matter and radiation. It is found at exactly the same average temperature of  $\sim 2.73$  K in any direction we observe it, with fluctuations  $< 10^{-5}$  K.

## Cosmology: building blocks

Thanks to the CP, we can apply Einstein's theory of gravitation in a much simpler form

Ricci Tensor	$R_{ij} - \frac{1}{2}g_{ij}R - \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$	Energy Tensor
Metric tensor		
	Cosmological Constant	
		Pressure, mass energy, 4-velocity tensors

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - dx^2 - dy^2 - dz^2 \quad \text{the invariant interval between two events at coordinates } (t,x,y,z) \text{ and } (t+dt,x+dx,y+dy,z+dz)$$

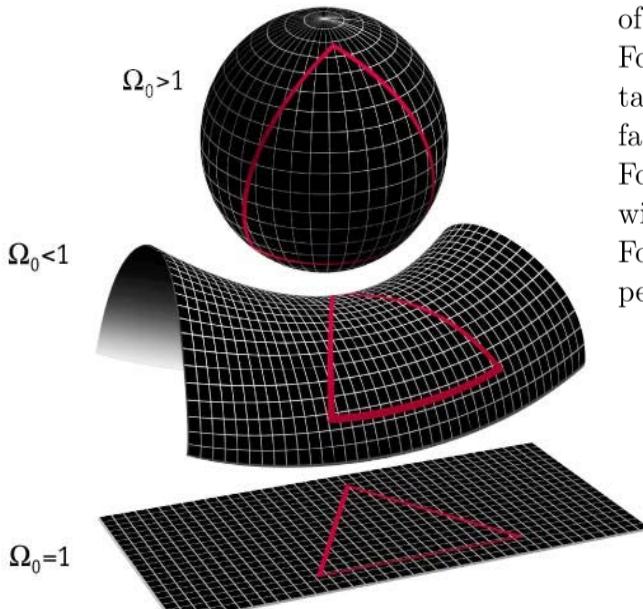
In particular we can look at the universe as a continuous, homogeneous and isotropic fluid. For such system, ***the most general metric*** of the space-time, called Robertson-Friedmann-Walker, for an expanding universe takes the form

$$ds^2 = (c dt)^2 - a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

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- $r, \theta, \phi$  are the comoving dimensionless coordinates;
- $a(t)$  is called cosmic expansion or scale parameter and has the dimension of a length. It is the variable that controls the rate expansion or contraction of the universe giving its dimension at a particular time, and the evolution varies according to the laws of the different models of universe. Geometrically it is a function of the time coordinate  $t$ ;
- $t$  is the proper cosmic time, as measured by a clock moving through two distinct events at  $r, \theta, \phi$  fixed;
- $K$  is the curvature parameter, and after proper renormalization it can be parametrized by the set of values (-1, 0, 1). It is related to the curvature of space:  $R_{curv} \equiv a(t)/\sqrt{|K|}$ .

# Geometry of the Universe



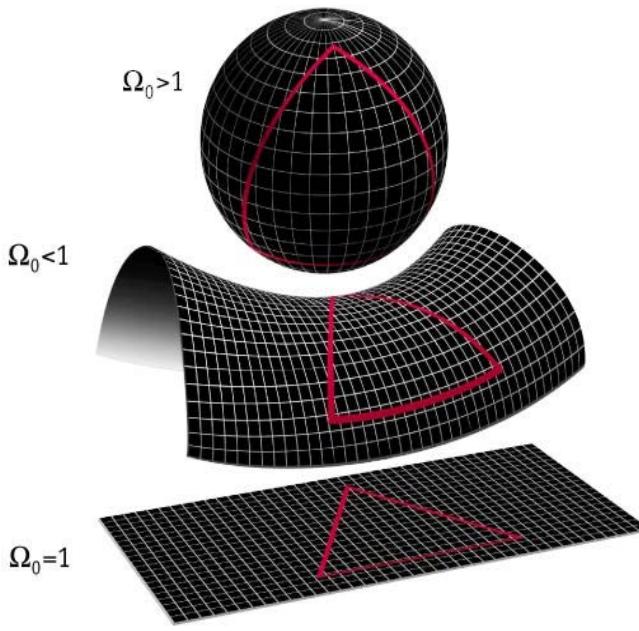
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The different values of the curvature parameter reflect three different possible geometries of the universe.

For  $\mathbf{K} = \mathbf{0}$  the spatial part of the metric represents an Euclidean flat space, with radial distance given by  $a(t)r$ ; the curvature of the space-time instead is granted by the cosmic scale factor  $a(t)$ .

For  $\mathbf{K} = \mathbf{1}$  the metric shows a singularity at  $r = 1/\sqrt{K}$ ; the geometry is that of a hypersphere with *positive* curvature and the universe has finite dimensions.

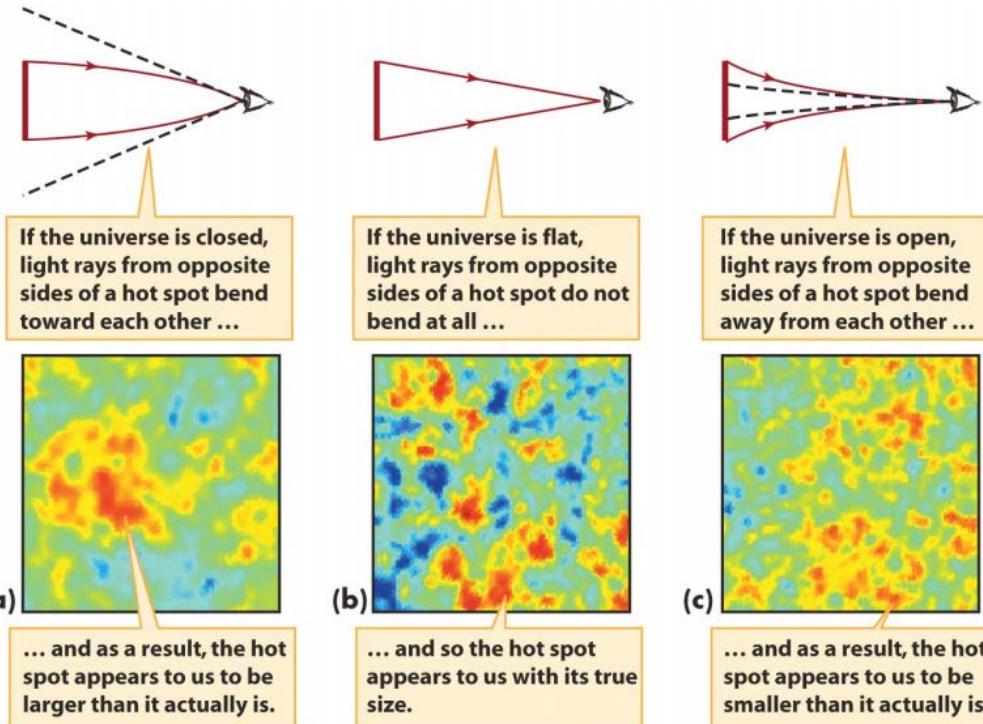
For  $\mathbf{K} = \mathbf{-1}$  the universe doesn't have a finite dimension and the geometry is that of a hyperbolic open space, with *negative* curvature.



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Depending on the geometry, the dimension of distant object changes according to the angle formed by the light

## Curvature of the Universe



$$\dot{a}^2 + Kc^2 = \frac{8}{3}\pi G\rho a^2$$

The two solutions are not independent, but can be retrieved by considering the adiabatic condition:

$$\ddot{a} = -\frac{4\pi}{3}G\left(\rho + 3\frac{p}{c^2}\right)a$$

$$\begin{aligned} d(\rho c^2 a^3) &= -p da^3 \\ dU &= -pdV \end{aligned}$$

Such kind of Universe cannot be static, unless the pressure assumes a negative value:

$$\rho = -3\frac{p}{c^2}$$

This lead Einstein to introduce the Cosmological Constant  $\Lambda$

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G}{c^4}T_{ij} \quad \longrightarrow \quad R_{ij} - \frac{1}{2}g_{ij}R - \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$$

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The two solutions are not independent, but can be retrieved by considering the adiabatic condition:

$$\begin{aligned} d(\rho c^2 a^3) &= -p da^3 \\ dU &= -pdV \end{aligned}$$

“ With an appropriate choice of  $\Lambda$ , one can obtain a static cosmological model. Equation ... represents the most general possible modification of the Einstein equations that still satisfies the condition that  $T_{ij}$  is equal to a tensor constructed from the metric  $g_{ij}$  and its first and second derivatives, and is linear in the second derivative. The strongest constraint one can place on  $\Lambda$  from observations is that it should be sufficiently small so as not to change the laws of planetary motion (from Cosmology by Coles-Lucchin)“

$$R_{ij} - \frac{1}{2}g_{ij}R = \frac{8\pi G}{c^4}T_{ij} \quad \longrightarrow \quad R_{ij} - \frac{1}{2}g_{ij}R - \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$$

## Some Useful Formulas

### Equation of state

Each component of the cosmic fluid is treated separately, with a proper equation of state ,  
Which relates the pressure of the component to its density, e.g.

$$P = P(\rho) \quad P(\rho) = w\rho c^2$$

Example: Ideal gas of nonrelativistic particles of mass  $m_p$ , temperature T, density  $\rho$  and adiabatic index  $\gamma$

$$p = nk_B T = \frac{k_B T}{m_p c^2} \rho m_p c^2 = \frac{k_B T}{m_p c^2} \frac{\rho c^2}{1 + (k_B T / ((\gamma - 1)m_p c^2))} = w(T)\rho c^2$$

It is also possible to calculate the adiabatic sound speed of the fluid:

$$v_s = \left( \frac{\partial p}{\partial \rho} \right)_S^{1/2} = c\sqrt{w}$$



Since it cannot exceed the speed of light, w must be necessarily  $< 1$

## Some Useful Formulas

$$z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e}$$

Redshift  $z$  : given a source emitting light, the redshift is calculated as the shift between the observed and emitted wave length (at source), divided by the wavelength at source

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3} \rho(t) - \frac{K}{a_0(t)^2 a(t)^2}$$

$$H^2 = H_0^2 \left(\frac{a_0}{a}\right)^2 \left[ 1 - \sum_w \Omega_{0,w} + \sum_w \Omega_{0,w} \left(\frac{a_0}{a}\right)^{1+3w} \right]$$

$$H^2(z) = H_0^2 (1+z)^2 \left[ 1 - \sum_w \Omega_{0,w} + \sum_w \Omega_{0,w} (1+z)^{1+3w} \right]$$

## Evoution of the Components of the Universe

$$\Omega_w(t) = \frac{\rho_w(t)}{\rho_c(t)}$$

Density of each component of the Universe

$$\rho_{cr} = \frac{3H_0^2}{8\pi G}$$

Critical density ( esitmaed:  $1-3 \times 10^{-29}$  g/m<sup>3</sup>)

$\rho < p_{cr}$	$\Leftrightarrow$	$\Omega < 1$	$\Leftrightarrow$	$K = -1$
$\rho = p_{cr}$	$\Leftrightarrow$	$\Omega = 1$	$\Leftrightarrow$	$K = 0$
$\rho > p_{cr}$	$\Leftrightarrow$	$\Omega > 1$	$\Leftrightarrow$	$K = +1$

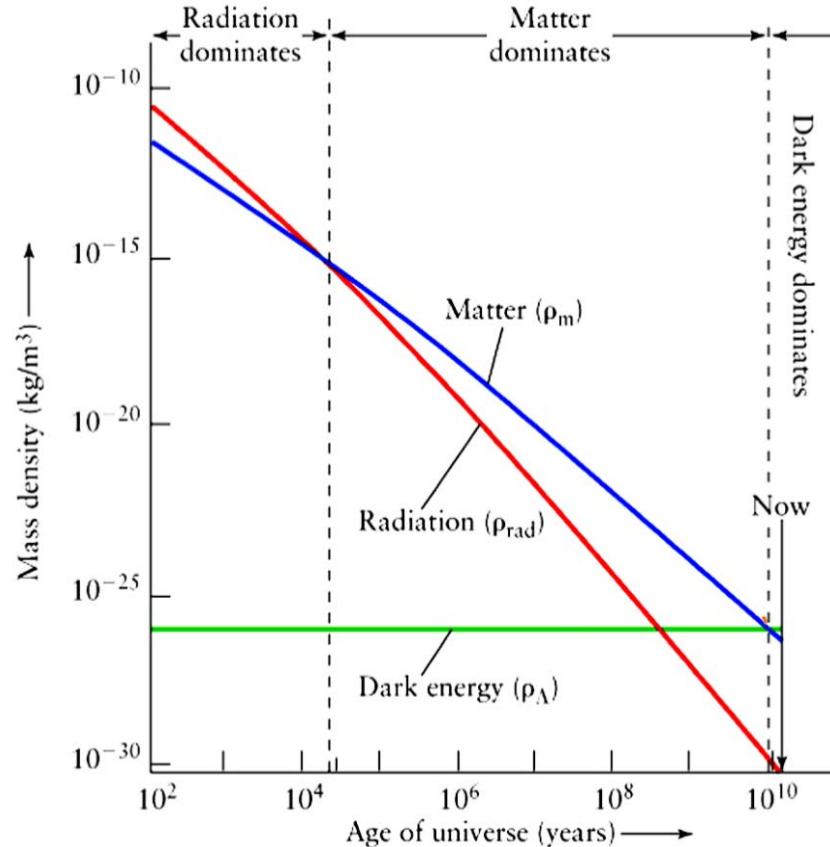
Currently, the most favoured geometry of the Universe is the „flat“ case  
i.e. the total energy density of all components equals the critical density

# Evolution of the Components of the Universe

Combining the adiabaticity condition and the equation of state, for specific values of  $w$ :

$$d(\rho c^2 a^3) = -p da^3 \quad \downarrow \quad p = w(T) \rho c^2$$

Component	$w$	Eq. of State	Density Evolution
Matter(dust)	0	$p = \frac{1}{3} \rho_r c^2$	$\rho_r = \rho_{0,r}(1+z)^4$
Radiation	$\frac{1}{3}$	$p = 0$	$\rho_m = \rho_{0,m}(1+z)^3$
Cosm. Constant $\Lambda$	-1	$p = -\rho_\Lambda c^2$	const.



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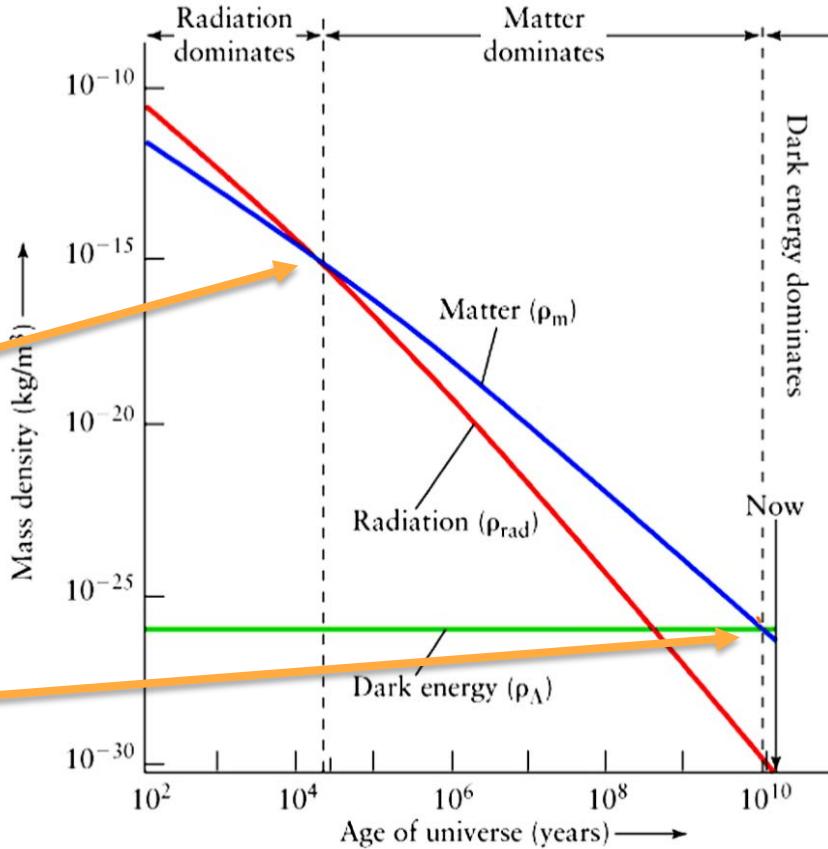
Matter-radiation equivalence

$$\rho_m(z_{eq}) \equiv \rho_r(z_{eq}) \rightarrow \rho_{0m}(1+z_{eq})^3 \equiv \rho_{0r}(1+z_{eq})^4$$

$$1+z_{eq} = \left( \frac{\rho_{0r}}{\rho_{0m}} \right)^{-1} = \left( \frac{\Omega_{0r}}{\Omega_{0m}} \right)^{-1} \simeq 4 \cdot 10^4 (\Omega_{0m} h^2)$$

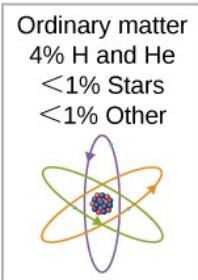
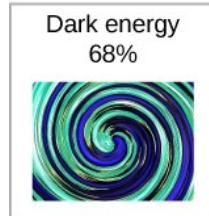
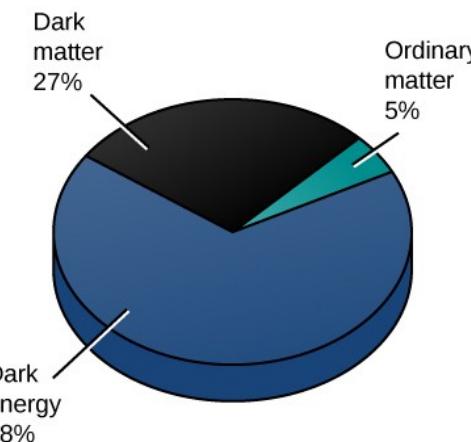
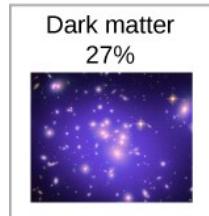
Matter-Lambda equivalence

$$1+z_\Lambda = \left( \frac{\rho_{0\Lambda}}{\rho_{0m}} \right)^{1/3} \simeq \left( \frac{0.75}{0.25} \right)^{1/3} \simeq 3^{1/3}$$

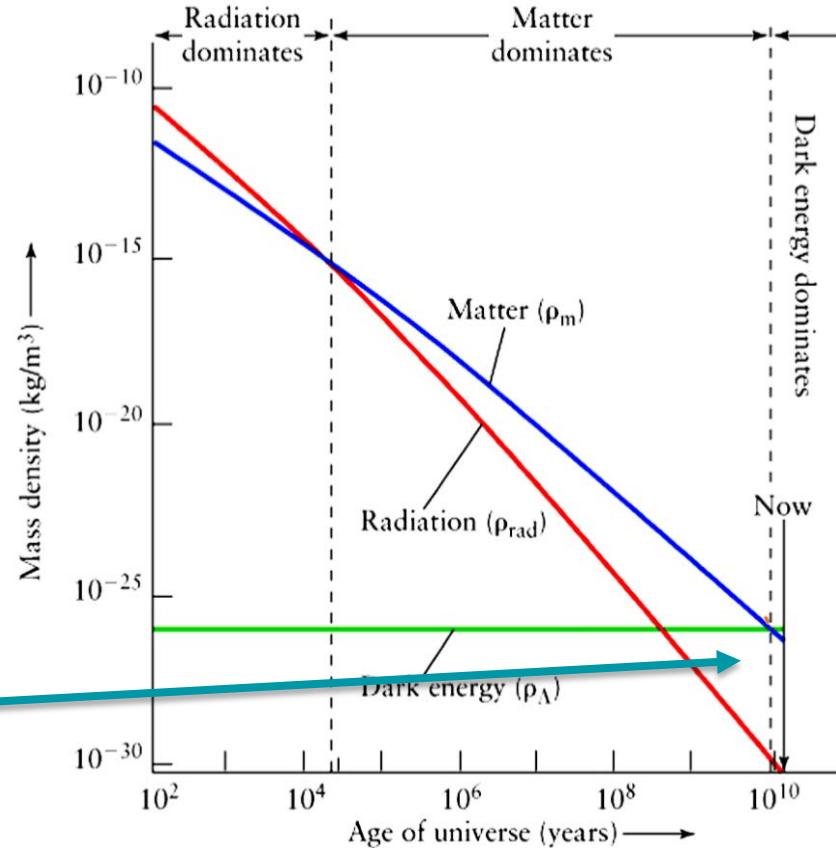


# Evoution of the Components of the Universe

## Composition of the Universe



Radiation (photons and relativistic particles such as neutrinos) does not contribute significantly **anymore** to the energy budget of the universe



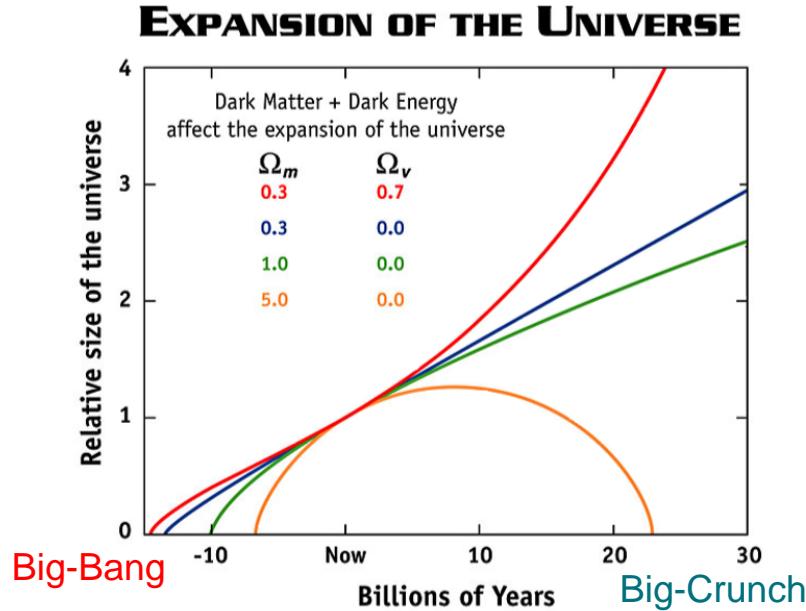
## Evoution of the Components of the Universe

Example: equations for a flat universe ( $k=1$ ) composed of a single fluid (matter or radiation)

Equation	Matter( $w = 0$ )	Radiation( $w = \frac{1}{3}$ )
$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$	$a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/3}$	$a(t) = a_0 \left(\frac{t}{t_0}\right)^{1/2}$
$t = t_0(1+z)^{-\frac{3(1+w)}{2}}$	$t = t_0(1+z)^{-3/2}$	$t = t_0(1+z)^{-2}$
$H(t) = \frac{2}{3(1+w)t}$	$H(t) = \frac{2}{3t}$	$H(t) = \frac{1}{2t}$
$q = \frac{1+3w}{2}$	$q = \frac{1}{2}$	$q = 1$
$t_0 = \frac{2}{3(1+w)} \frac{1}{H_0}$	$t_0 = \frac{2}{3} \frac{1}{H_0}$	$t_0 = \frac{1}{2H_0}$
$\rho(t) = \frac{1}{6\pi G(1+w)^2} \frac{1}{t^2}$	$\rho(t) = \frac{1}{8\pi G t^2}$	$\rho(t) = \frac{3}{32\pi G t^2}$

Table 1.3: Summary of properties of matter and radiation single-fluid flat universe.

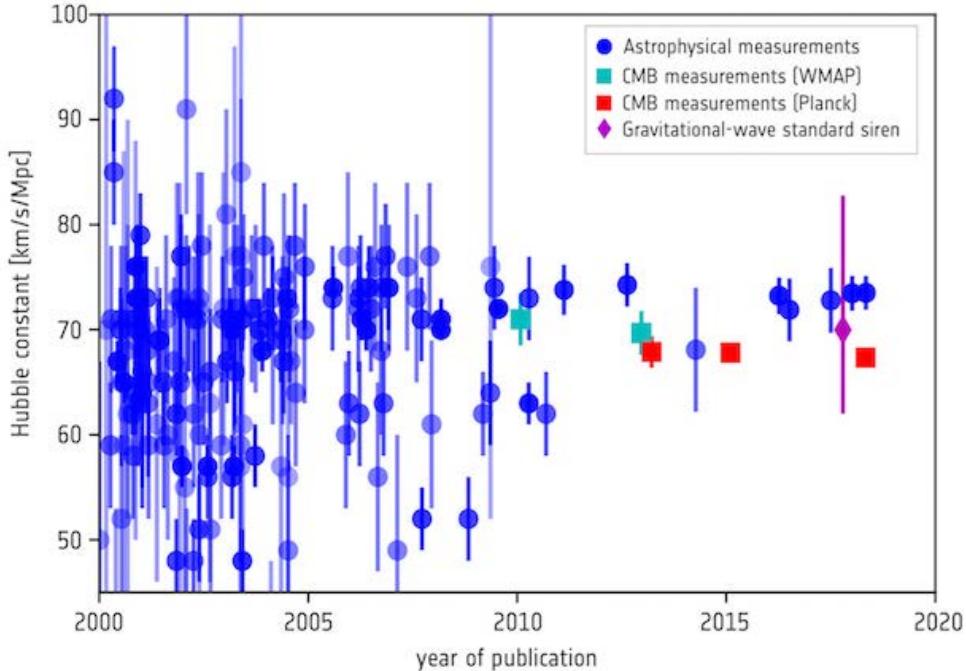
# Fate of the Universe ?



- Depending on the geometry and on the energy content of the universe (divided into its different components), the fate of the universe will be different
- We are in a situation “now” which is compatible with most models

Figure 1.3: Time evolution of the scale parameter, or size of the universe, for different values of the matter and dark energy densities. Times run to the right of the x-axes or analogously, redshift increases to the left of x-axis. The point where the curves cross the time axis (on the left) is known as Big Bang; the time where the closed-model curve intersects the time axis (on the right) is known as Big Crunch. Different models have different ages of the universe, hence different times for the Big Bang. Picture taken from [www.nasa.gov](http://www.nasa.gov).

# The Hubble Parameter



«Alternatively, the Hubble Constant can also be estimated from the cosmological model that fits observations of the cosmic microwave background, which represents the very young Universe, and calculate a prediction for what the Hubble Constant should be today. Measurements based on this method using data from NASA's WMAP satellite are shown in green, and those obtained using data from ESA's Planck mission are shown in red. On the one hand, it is extraordinary that two such radically different ways of deriving the Hubble constant – one using the local, mature Universe, and one based on the distant, infant Universe – are so close to each other. On the other hand, in principle these two figures should agree to within their respective uncertainties, causing what cosmologists call a 'tension' – an oddity that still needs explaining.»

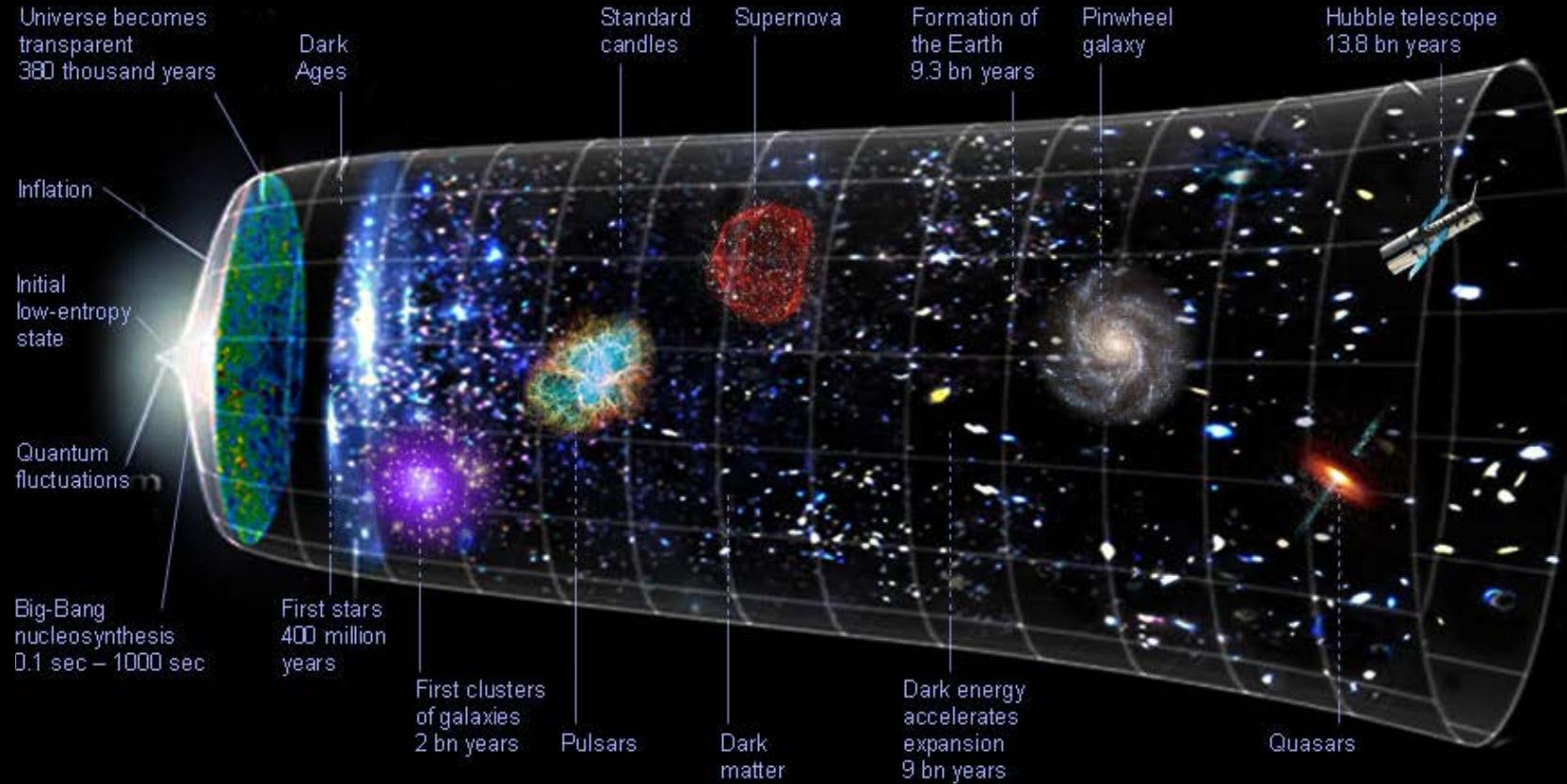
<https://sci.esa.int/web/planck/-/60504-measurements-of-the-hubble-constant>

# The Hubble Parameter

Measurement of the Hubble constant

Date published	Hubble constant $\pm$ (km/s)/Mpc	Observer	Citation	Remarks / methodology
2019-10-14	74.2 $^{+2.7}_{-3.0}$	STRIDES	[76]	Modelling the mass distribution & time delay of the lensed quasar DES J0408-5354.
2019-09-12	76.8 $\pm 2.6$	SHARP/H0LiCOW	[77]	Modelling three galactically lensed objects and their lenses using ground-based adaptive optics and the Hubble Space Telescope
2019-08-15	73.5 $\pm 1.4$	M. J. Reid, D. W. Pesce, A. G. Riess	[78]	Measuring the distance to <a href="#">Messier 106</a> using its supermassive black hole, combined with measurements of eclipsing binaries in the Large Magellanic Cloud.
2019-07-16	69.8 $\pm 1.9$	Hubble Space Telescope	[56][57][58]	Distances to <a href="#">red giant stars</a> are calculated using the <a href="#">tip of the red-giant branch</a> (TRGB) distance indicator.
2019-07-10	73.3 $^{+1.7}_{-1.8}$	H0LiCOW collaboration	[79]	Updated observations of multiply imaged <a href="#">quasars</a> , now using six quasars, independent of the cosmic distance ladder and independent of the cosmic microwave background measurements.
2019-07-08	70.3 $^{+5.3}_{-5.0}$	LIGO and Virgo detectors	[55]	Uses radio counterpart of <a href="#">GW170817</a> , combined with earlier gravitational wave (GW) and electromagnetic (EM) data.
2019-03-28	68.0 $^{+4.2}_{-4.1}$	Fermi-LAT	[80]	Gamma ray attenuation due to extragalactic light. Independent of the cosmic distance ladder and the cosmic microwave background.
2019-03-18	74.03 $\pm 1.42$	Hubble Space Telescope	[61]	Precision HST <a href="#">photometry</a> of <a href="#">Cepheids</a> in the <a href="#">Large Magellanic Cloud (LMC)</a> reduce the uncertainty in the distance to the LMC from 2.5% to 1.3%. The revision increases the tension with <a href="#">CMB</a> measurements to the $4.4\sigma$ level ( $P=99.999\%$ for Gaussian errors), raising the discrepancy beyond a plausible level of chance. Continuation of a collaboration known as Supernovae, $H_0$ , for the Equation of State of Dark Energy (SHoES).
2019-02-08	67.78 $^{+0.91}_{-0.87}$	Joseph Ryan et al.	[81]	<a href="#">Quasar</a> angular size and <a href="#">baryon acoustic oscillations</a> , assuming a flat LambdaCDM model. Alternative models result in different (generally lower) values for the Hubble constant.
2018-11-06	67.77 $\pm 1.30$	Dark Energy Survey	[82]	Supernova measurements using the <a href="#">inverse distance ladder</a> method based on baryon acoustic oscillations.
2018-09-05	72.5 $^{+2.1}_{-2.3}$	H0LiCOW collaboration	[83]	Observations of multiply imaged <a href="#">quasars</a> , independent of the cosmic distance ladder and independent of the cosmic microwave background measurements.
2018-07-18	67.66 $\pm 0.42$	Planck Mission	[84]	Final Planck 2018 results.
2018-04-27	73.52 $\pm 1.62$	Hubble Space Telescope and <a href="#">Gaia</a>	[85][86]	Additional HST <a href="#">photometry</a> of galactic <a href="#">Cepheids</a> with early <a href="#">Gaia</a> <a href="#">parallax</a> measurements. The revised value increases tension with <a href="#">CMB</a> measurements at the $3.8\sigma$ level. Continuation of the SHoES collaboration.
2018-02-22	73.45 $\pm 1.66$	Hubble Space Telescope	[87][88]	Parallax measurements of galactic Cepheids for enhanced calibration of the <a href="#">distance ladder</a> ; the value suggests a discrepancy with CMB measurements at the $3.7\sigma$ level. The uncertainty is expected to be reduced to below 1% with the final release of the <a href="#">Gaia</a> catalog. SHoES collaboration.

# History of the Universe



# History of the Universe

*Summary of fundamental steps in the evolution of the Universe:*

- *Big bang*
- *Quantum fluctuations and local DM overdensities*
- *Inflation*
- *Baryogenesis and Nucleosynthesis*
- *Release of the CMB from the last scatter surface*
- *Recombination*
- *Dark ages*
- *First stars (Population III)*
- *Reionization*
- *First galaxies, first quasars*
- (...)

***Very complex yet fascinating***

*Need to take into account particle physics (which particle are present in each epoch? Are they relativistic? Are they in thermal equilibrium? Were there more particles than today (i.e. „X“ boson that unifies the 4 forces – electromagnetic, weak, strong gravitational ? Quantum gravity?*

*What causes inflation? How and when it ended?*

*Why the CMB is so isotropic and uniform? Causally-connected regions outside the horizon?*

*Cosmic neutrino background?*

*Why baryogenesis?*

*What is DM? How does it drive structure formation?*

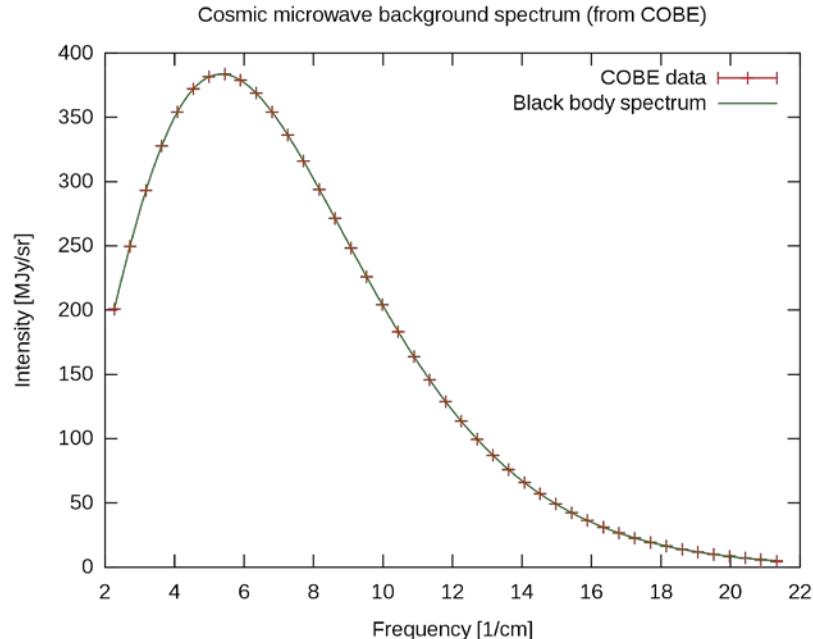
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**How to investigate all of these questions?**

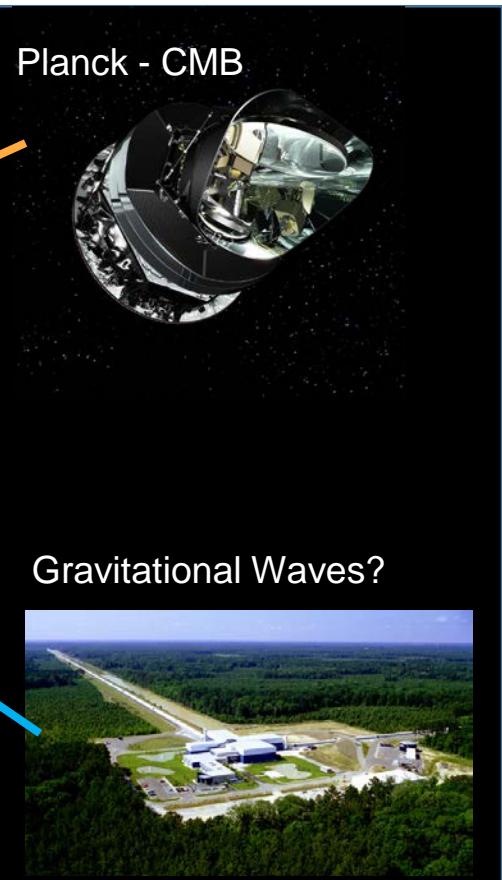
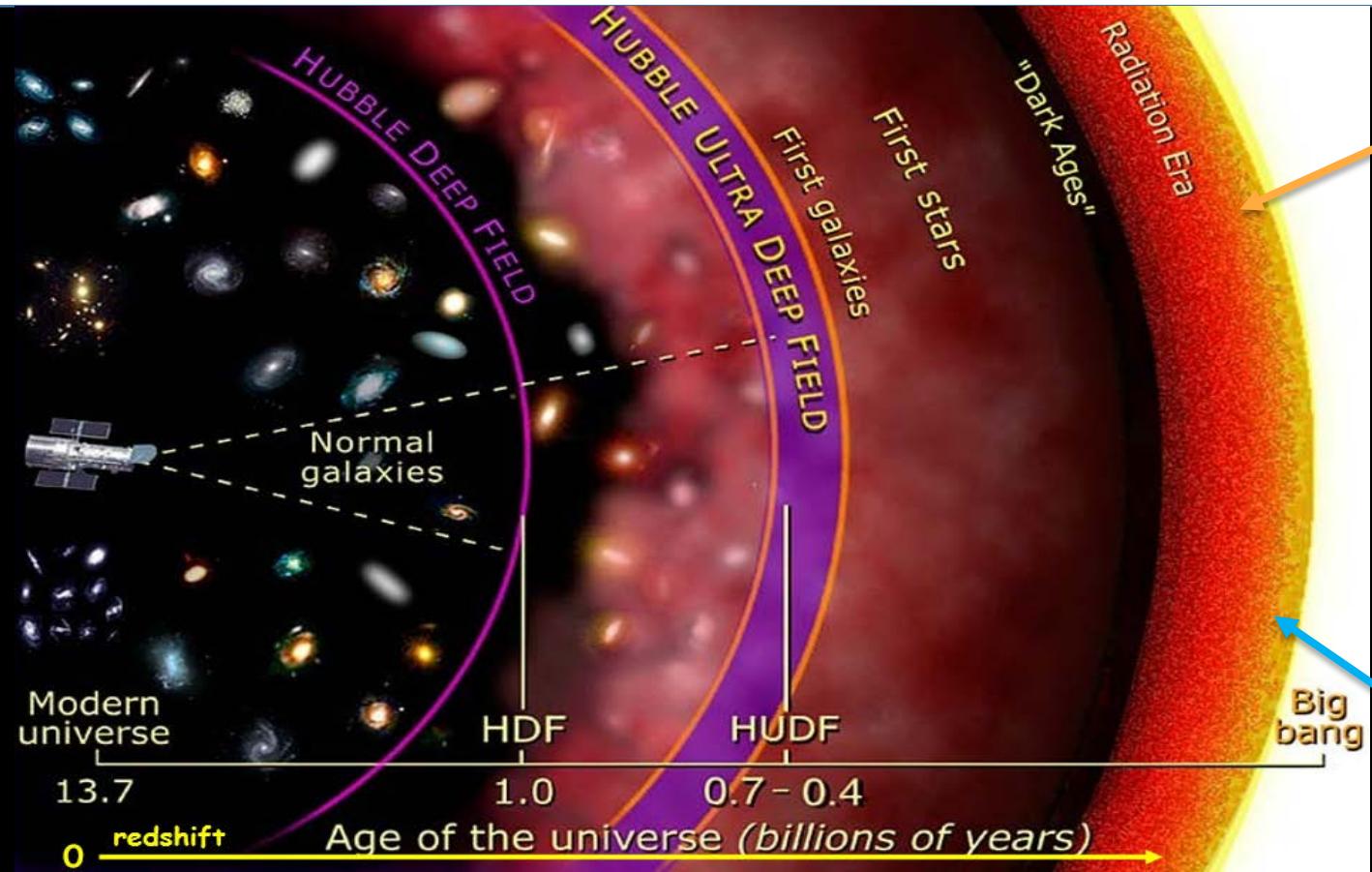
# The Cosmic Microwave Background

- The cosmic microwave background radiation is an emission of uniform, black body spectrum coming from all parts of the sky
- It is a consequence of **recombination**, i.e. the moment in the history of the Universe when charged electrons and protons form electrically neutral hydrogen atoms (at a redshift of  $z = 1100$ )
- Photons and charged particles were electrically coupled, and the Thomson scattering would prevent photons to free stream
- Today Energy density:  $\sim 0.25 \text{ eV/cm}^3$ ,  $400\text{--}500 \text{ photons/cm}^3$
- Temperature evolution with redshift:

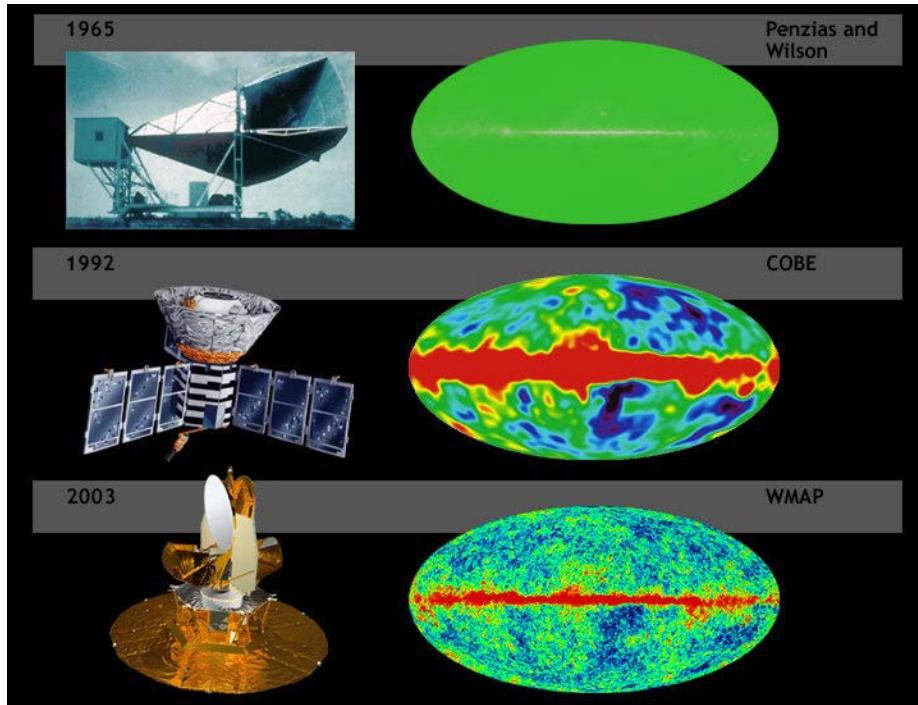
$$T_r = 2.725 \cdot (1 + z)$$



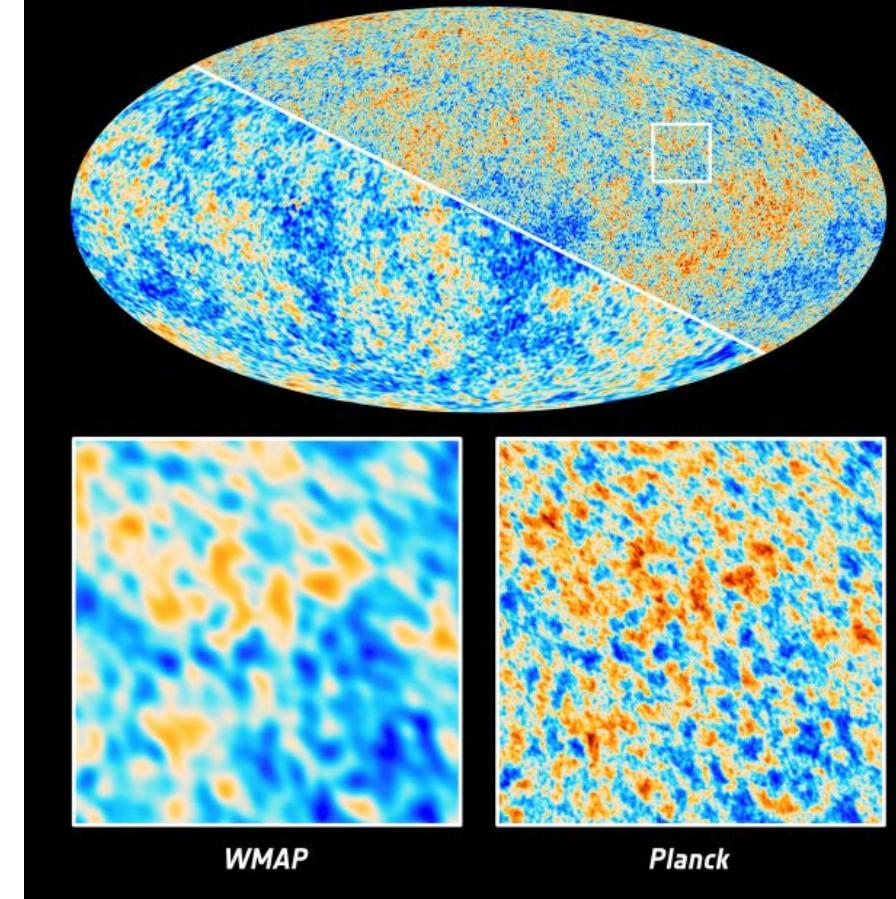
Graph of cosmic microwave background spectrum measured by the FIRAS instrument on the COBE, the most precisely measured black body spectrum in nature. The error bars are too small to be seen even in an enlarged image, and it is impossible to distinguish the observed data from the theoretical curve.



# The Cosmic Microwave Background

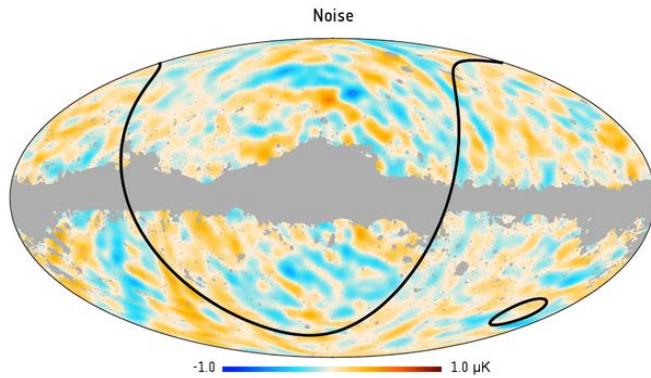
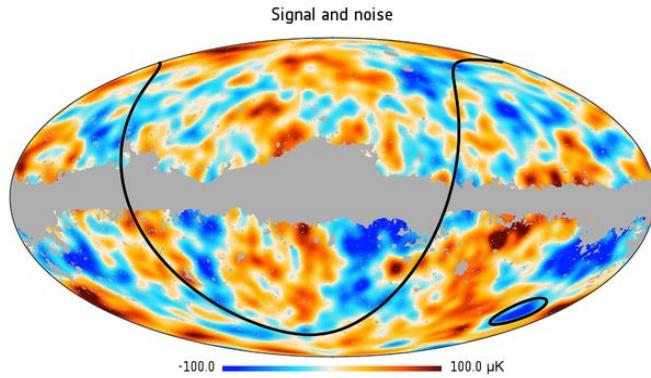


*The Cosmic Microwave Background as seen by Planck and WMAP*



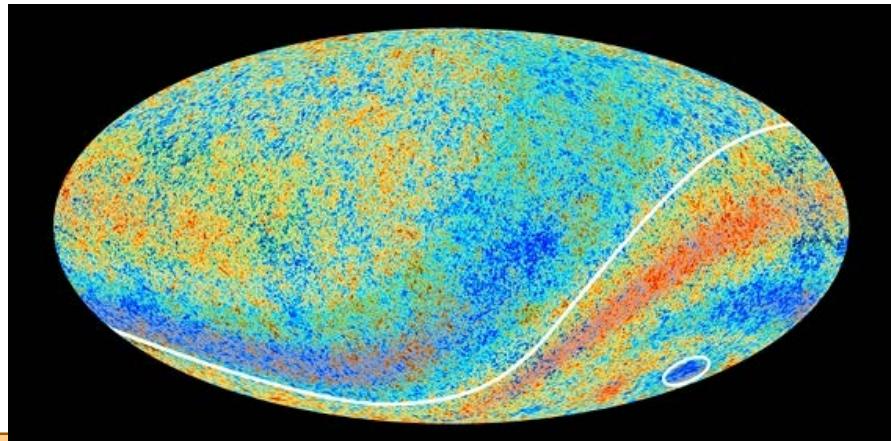
# The Cosmic Microwave Background

Cosmic microwave background temperature fluctuations  
Filtered to show scales around  $5^\circ$  and larger



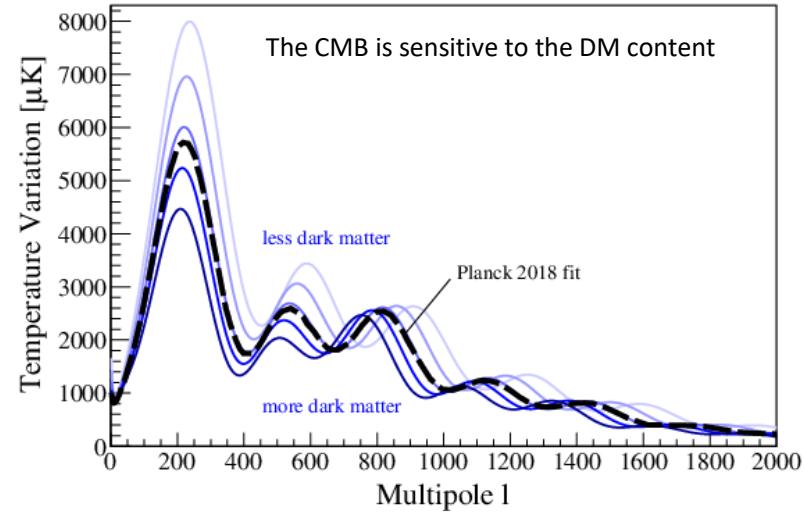
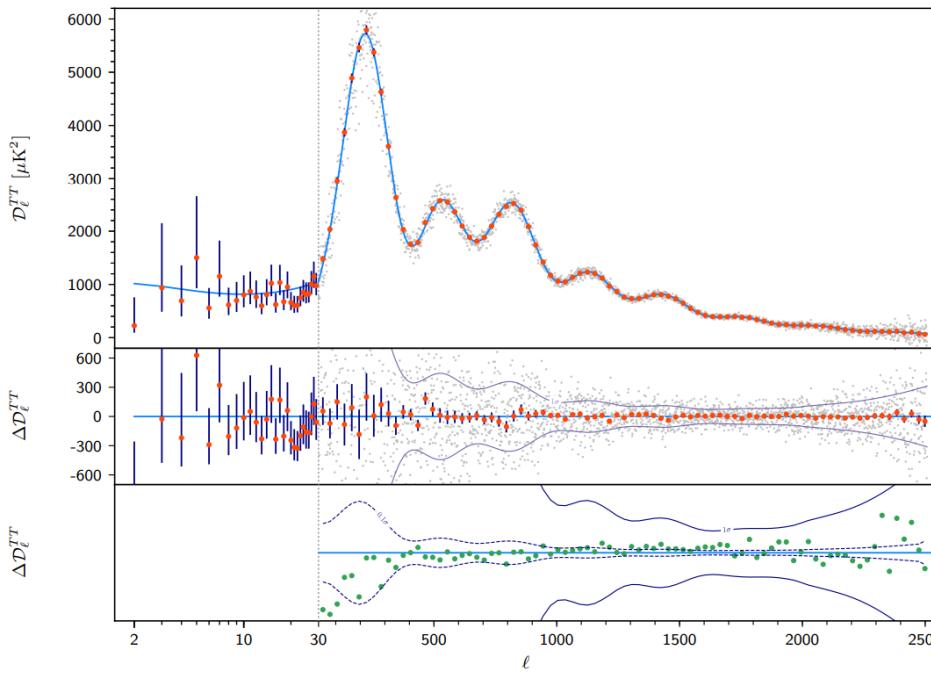
The most serious anomaly is a deficit in the signal observed on scales around 5 degrees, which is about ten per cent weaker than predicted. Other anomalous traits are a significant discrepancy of the signal as observed in the two opposite hemispheres of the sky (the two hemispheres are outlined by the large, roughly u-shaped curve in the image, the northern one being at the centre) and a so-called 'cold spot' – a large, low-temperature spot with an unusually steep temperature profile (also outlined in the lower right).

Planck's anomalous sky: the hemispheric asymmetry and the cold spot. Credit: ESA and the Planck Collaboration



# The Cosmic Microwave Background

Planck 2018: <https://arxiv.org/pdf/1907.12875.pdf>



arXiv:1903.03026

Residual vs CDM model

Residual vs 2015 data

## Big-Bang Nucleosynthesis

- Nucleosynthesis is the fundamental process through which elements were created right after the Big-bang
- It explains the very homogeneous distribution of ~light elements (H, He, Li) found in the universe
- “Metals” in Astronomy : elements other than H and He
- The element which is typically tracked is Iron (Fe), since it is assumed that the relative abundance of the other elements (e.g. C, N, Ca...) is proportional
- Metallicity =  $[Fe/H]$  : how many Fe atoms are present per H atoms

$$[Fe/H] = \log \frac{(Fe/H)}{(Fe/H)_{Sun}} = \log(Fe/H) - \log(Fe/H)_{Sun}$$

Examples:

$[Fe/H] = -1 \rightarrow$  10% of solar metallicity

$[Fe/H] = 0 \rightarrow$  solar metallicity

**Range in our Galaxy:  $-5 < [Fe/H] < +1$**

Convenient to describe the chemical content of a star by **fractional mass** in different elements w.r.t. total mass:

X = H

Y = He

**X+Y+Z = 1**

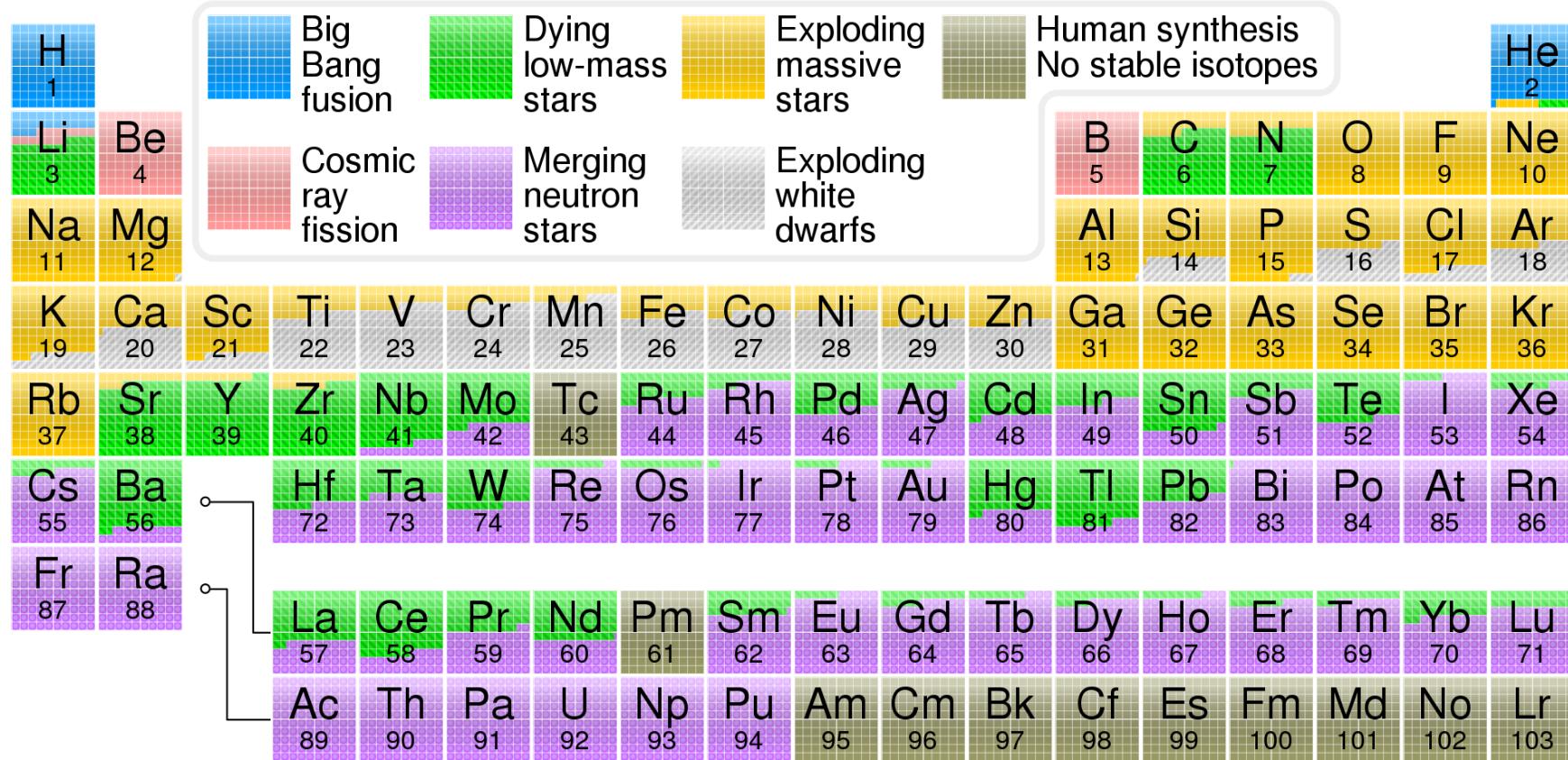
**Z = all the rest**

$X_{\odot} \sim 0.7$

$Y_{\odot} \sim 0.28$  Solar values

$Z_{\odot} \sim 0.02$

# Big-Bang Nucleosynthesis



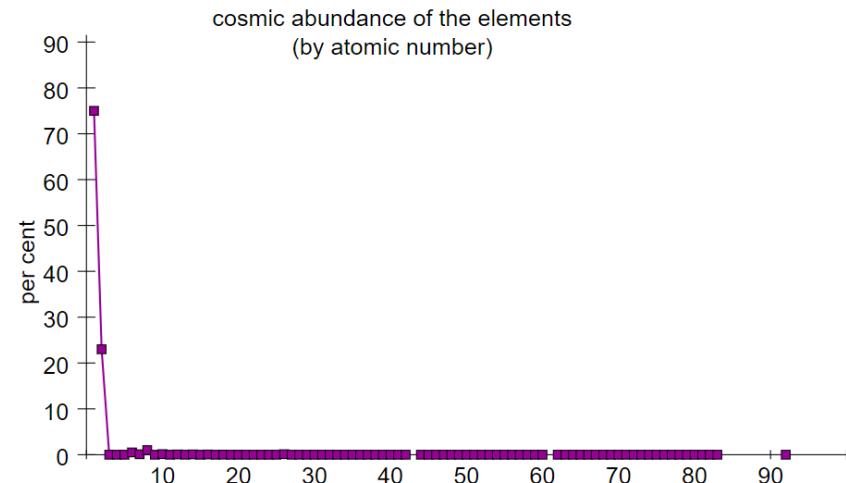
# Big-Bang Nucleosynthesis

"By the first millisecond, the universe had cooled to a few trillion kelvins ( $10^{12}$  K) and quarks finally had the opportunity to bind together into free protons and neutrons. Free neutrons are unstable with a half-life of about ten minutes (614.8 s) and formed in much smaller numbers. The abundance ratio was about seven protons for every neutron. Before one neutron half-life passed nearly every neutron had paired up with a proton, and nearly every one of these pairs had paired up to form helium. By this time the universe had cooled to a few billion kelvins ( $10^9$  K) and the rate of nucleosynthesis had slowed down significantly.

By the time the universe was three minutes old the process had basically stopped and the relative abundances of the elements was fixed at ratios that didn't change for a very long time: **75% hydrogen**, **25% helium**, with trace amounts of deuterium (hydrogen-2), helium-3, and lithium-7.

Big Bang nucleosynthesis produced no elements heavier than lithium. To do that you need stars, which means waiting around for at least 200 billion years"

<https://physics.info/nucleosynthesis/>



# Big-Bang Nucleosynthesis

The hypotheses usually made to explain the cosmological origin of the light elements are as follows.

1. The Universe has passed through a hot phase with  $T \geq 10^{12}$  K, during which its components were in thermal equilibrium.
2. General Relativity and known laws of particle physics apply at this time.
3. The Universe is homogeneous and isotropic at the time of nucleosynthesis.
4. The number of neutrino types is not high (in fact we shall assume  $N_\nu \approx 3$ ).
5. The neutrinos have a negligible degeneracy parameter.
6. The Universe is not composed in such a way that some regions contain matter and others antimatter.
7. There is no appreciable magnetic field at the epoch of nucleosynthesis.
8. The density of any exotic particles (photinos, gravitinos, etc.) at  $T_e$  is negligible compared with the density of the photons.

Weak interaction: maintain the thermal equilibrium between neutrons and protons (Boltzman distribution)



$$\frac{n_n}{n_p} \approx \exp\left(-\frac{Q}{k_B T}\right) = \exp\left(-\frac{1.5 \times 10^{10} \text{ K}}{T}\right)$$

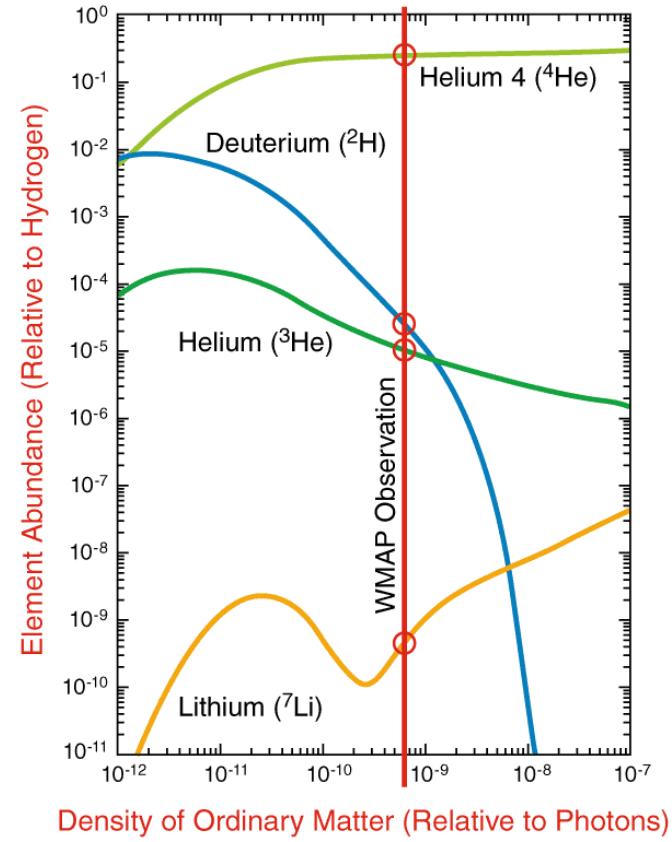
$$Q = (m_n - m_p)c^2 \approx 1.3 \text{ MeV}$$

is the difference in rest-mass energy between 'n' and 'p', corresponding to a temperature  $T_{pn} \equiv Q/k_B \approx 1.5 \times 10^{10}$  K. For  $T \gg T_{pn}$ , the number of protons is virtually identical to the number of neutrons.

(...calculations...)

- It can be shown that the predicted fraction of Helium produced is  $Y \sim 0.25$
- All the other isotopes and elements are below %

# Big-Bang Nucleosynthesis



Solar values

$$\begin{aligned} X_{\odot} &\sim 0.7 \\ Y_{\odot} &\sim 0.28 \\ Z_{\odot} &\sim 0.02 \end{aligned}$$

## Brief Chronology of Dark Matter

- ❑ 1933 : discovery of galaxy clusters  $\sigma_{\text{vel}} \approx 1000 \text{ km/s}$  (Zwicky)
- ❑ 1970s : discovery of galaxy flat rotation curves
- ❑ 1980 : astronomers convince that DM binds galaxies and clusters

- ❑ 1980-83 : short life of HDM theory
- ❑ 1982-84 : CDM theory proposed



**CDM** = Cold Dark Matter  
**HDM** = Hot Dark Matter

- ❑ 1992 : COBE discovers CMB fluctuations as predicted by  $\Lambda$ CDM
- ❑ 1998 : SNe Ia evidence of Dark Energy
- ❑ 2000 :  $\Lambda$ CDM becomes the standard cosmological model
- ❑ 2003-2010: WMAP + galaxy LSS confirm  $\Lambda$ CDM

### Properties of particle DM in the

- DM particles interact gravitationally in the  $\Lambda$ CDM
- (however DM particles might interact via weak interaction or other unknown interactions – “dark sector” )
- This means that they cannot dissipate energy via electromagnetic processes i.e. DM is dissipationless
- DM can only convert energy: kinetic  $\leftrightarrow$  potential

# Structure Formation: the Cold DM paradigm

## Jeans Mass

$\rho$  = constant = mass density

$\overline{m}$  = mean mass particle

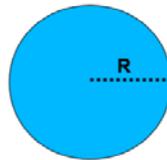
$$|E_g| = GM^2/R \quad \text{gravitational energy}$$

$$|dE_g| = \frac{GM^2}{R^2} dR \quad \text{radial compression, gravitational energy becomes more negative}$$

$$dV = 4\pi R^2 dR \quad \text{corresponding decrease of the volume}$$

$$dE_{th} = PdV = nkT 4\pi R^2 dR = \frac{M}{\frac{4}{3}\pi R^3 m} kT 4\pi R^2 dR = \frac{3M}{m} kT \frac{dR}{R} \quad \text{Increase of gas thermal energy}$$

$|dE_g| > dE_{th}$  condition for the cloud to become unstable  
(remind that an increase of  $E_{th}$  implies an increase of pressure)



## Jeans Mass

$$|dE_g| > dE_{th} \Rightarrow \frac{GM^2}{R^2} dR > \frac{3M}{m} kT \frac{dR}{R}$$

$$\Rightarrow M_J = \frac{3kT}{Gm} R$$

a cloud will collapse only if its mass is larger than the Jeans mass  $M_J$

$$R_J = \frac{\bar{Gm}M}{3kT} \quad \text{a cloud will collapse only if its radius is smaller than the Jeans radius } R_J$$

$$\rho_J = \frac{M_J}{\frac{4}{3}\pi R_J^3} \quad \text{a cloud will collapse only if its density is larger than the Jeans density } \rho_J$$

$$R_J = \frac{\bar{Gm}M}{3kT} \approx \frac{\bar{Gm}\rho R_J^3}{3kT} = R_J \approx \left( \frac{3kT}{\bar{Gm}\rho} \right)^{1/2} \quad \text{Expression of Jeans length as a function of temperature and density}$$

*Take home message:*

*There is a parameter (or there are parameters) which regulate the gravitational collapse of DM (or matter in general)*

# Structure Formation: the Cold DM paradigm

HDM - hot DM, if dark matter particles are still relativistic at time of decoupling

CDM - cold DM, if dark matter particles are no more relativistic at time of their decoupling

This distinction is critical for the formation of cosmic structure, in particular it influences the value of the Jeans mass, i.e. the minimum mass for which structures become gravitationally bound.

In general, the Jeans scale for a certain species depends on the speed of sound of that particular species, and on the density of the dominant species  $\rho_{dom}$  of the universe in that moment:

$$\lambda_{J,\chi} \propto \frac{v_\chi}{\sqrt{G\rho_{dom}}} .$$

For a relativistic species, the speed is of order of the speed of light,  $v^2 \approx \frac{c^2}{3}$ . For ordinary matter, it depends on pressure that in turn depends on the dominant component, but in general is the sum of the radiation pressure plus the gas pressure.

- for hot dark matter (and example candidates would be massive neutrinos) the maximum Jeans mass reached at  $a = a_{eq}$  is of order:

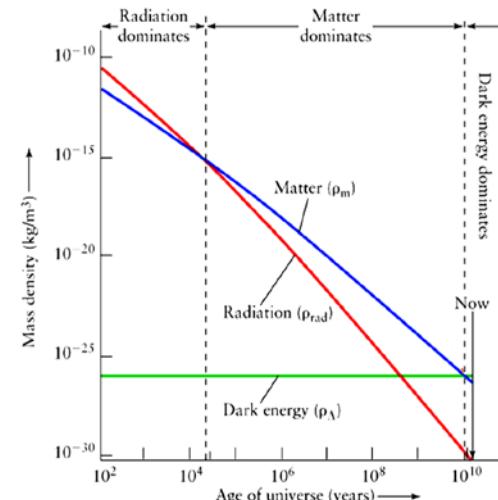
$$M_{J,HDM} \simeq 3.5 \times 10^{15} (\Omega_{HDM} h^2) M_\odot ;$$

- for cold dark matter, this maximum mass is much smaller:

$$M_{J,CDM} \simeq 10^{5-6} M_\odot ;$$

- for baryons, Jeans Mass at decoupling is:

$$M_{J,bar} \simeq 3.1 \times 10^{16} M_\odot \left( \frac{\Omega_{0B}}{\Omega_0} \right) (\Omega_0 h^2)^{-1/2}$$



For neutrinos, the free streaming mass depends on the mass of this particle. If  $> 0.2$  eV, it exceeds the mass of the largest supercluster ever observed.

$$M_{Fs} \approx 10^{17} M_\odot \left( \frac{eV}{m_\nu} \right)^2$$

So: what is the typical size of galaxies that we observe as a function of the redshift, or what is the typical size of DM halos that we get from simulations, for different types of cosmological models?

# Structure Formation: the Cold DM paradigm

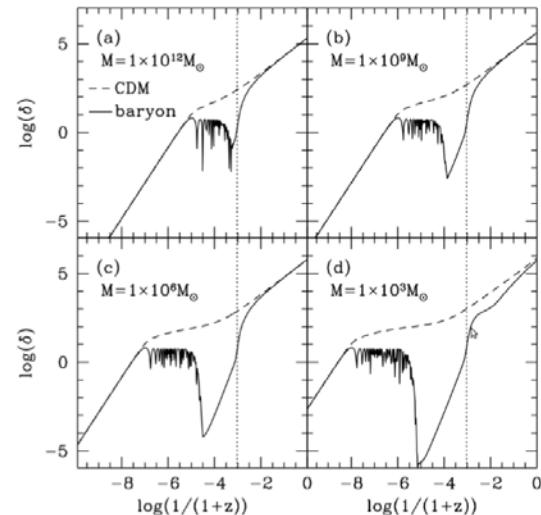


Figure 2.1: Evolution of baryons and CDM fluctuations for different baryons mass scales; cosmological parameters are set as:  $h = 0.5$ ,  $\Omega_0 = 1$  and  $\Omega_b = 0.1$ . The plots show the power-law growing phase of baryon fluctuations after the diffusion damping and the moment of equivalence, indicated by the vertical dotted line. The curve right after decoupling is the baryon catch-up. Image taken from reference [56].

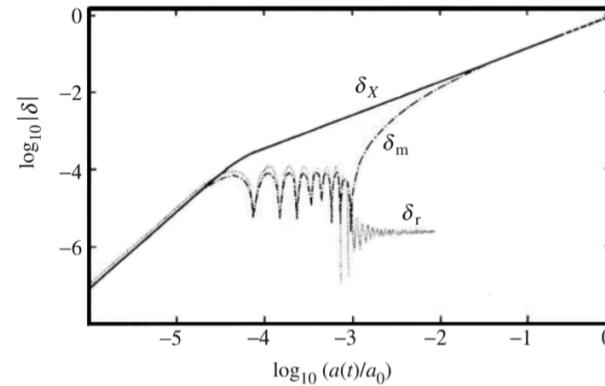


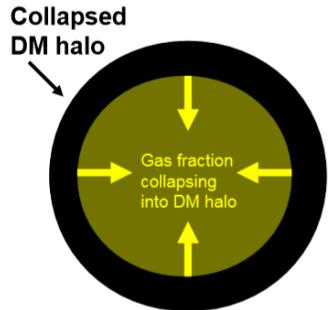
Figure 2.2: Evolution of the densities of different components of the universe on a scale of  $M \sim 10^{15} M_\odot$  as a function of the scale factor (or time).  $X$   $m$  and  $r$  denote dark matter, baryonic matter and radiation respectively. Image taken from reference [26].

# Structure Formation: the Cold DM paradigm

## Paradigm:

The collapses of baryonic matter (that constitute the galaxies and clusters) follow the gravitational collapse of the DM halos

The physics of baryonic matter is much more complicated than the physics of DM (collisionless, dissipationless)



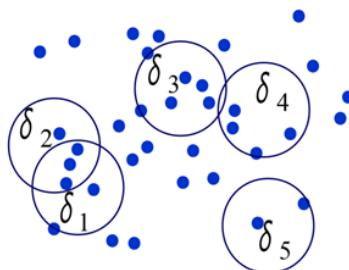
Perturbations in baryonic matter **follow** those of DM after decoupling

After recombination, baryons fall into DM potential wells assuming the same contrast density of DM halos. Entering in DM halos at supersonic speeds, the baryonic gas is heated by the shock waves and compressions to the virial temperature of the halo

This gas component is usually called **hot** and it is assumed that it is heated to the virial temperature of the halo (hydrostatic equilibrium)

The virial radius of the baryonic content of a structure can become much smaller than that of the DM halo because baryons can cool by radiative processes and contract further

# Distributions of Dark Matter Halos



Press-Schechter theory (1974)

Method to estimate the *number of DM halos, with a given mass, at a given redshift*

Assumptions:

- The DM over-densities follow a random Gaussian profile
- The over-densities grow linearly with time
- The DM particles collapse spherically (spherical Halos)

$$n(M; t)dM = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M^2} \frac{\delta_c(t)}{\sigma(M)} \left| \frac{d \ln \sigma}{d \ln M} \right| \times \exp \left[ -\frac{\delta_c^2(t)}{2\sigma^2(M)} \right] dM,$$

$n(M, t)dM$  = number of DM halos at time  $t$  with mass between  $M$  and  $M+dM$

$\rho_0$  = average mass density of the DM halo ( $M \approx \rho_0 R^3$ )

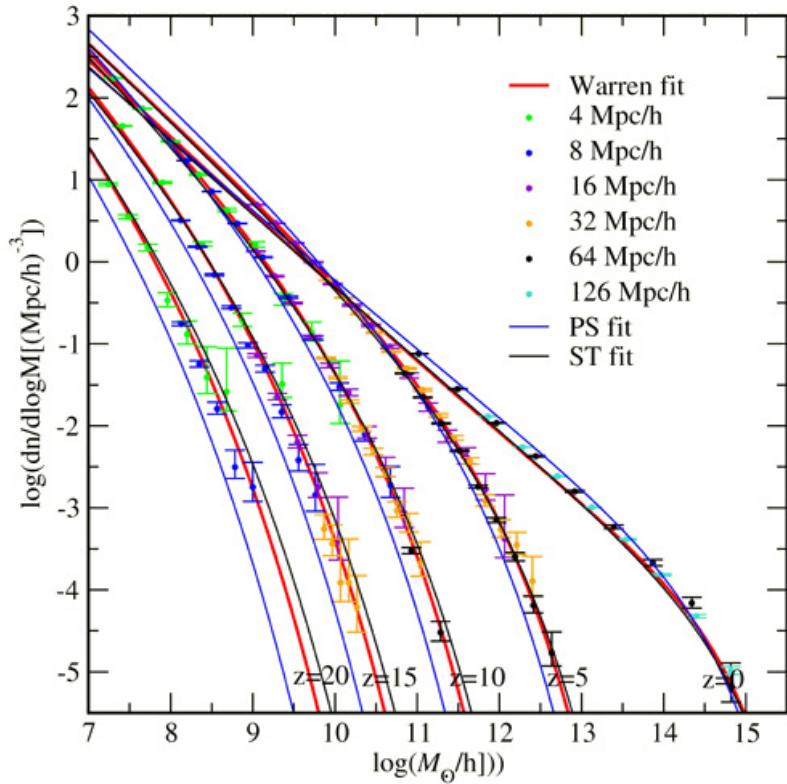
$\delta_c(t)$  = threshold in the density perturbation  $\delta = (\delta\rho/\rho) = (\rho - \langle \rho \rangle)/\langle \rho \rangle$

Fluctuations above the threshold  $\delta_c$  correspond to collapsed regions)

$\delta_c(t) = \delta_c/D(t) = 1.69/D(t)$ ,  $D(t)$  = linear growth rate normalized to 1 at  $z=0$

$\sigma(M)$  = dispersion of the mass  $M$  (gaussian random field of masses)

# Distributions of Dark Matter Halos



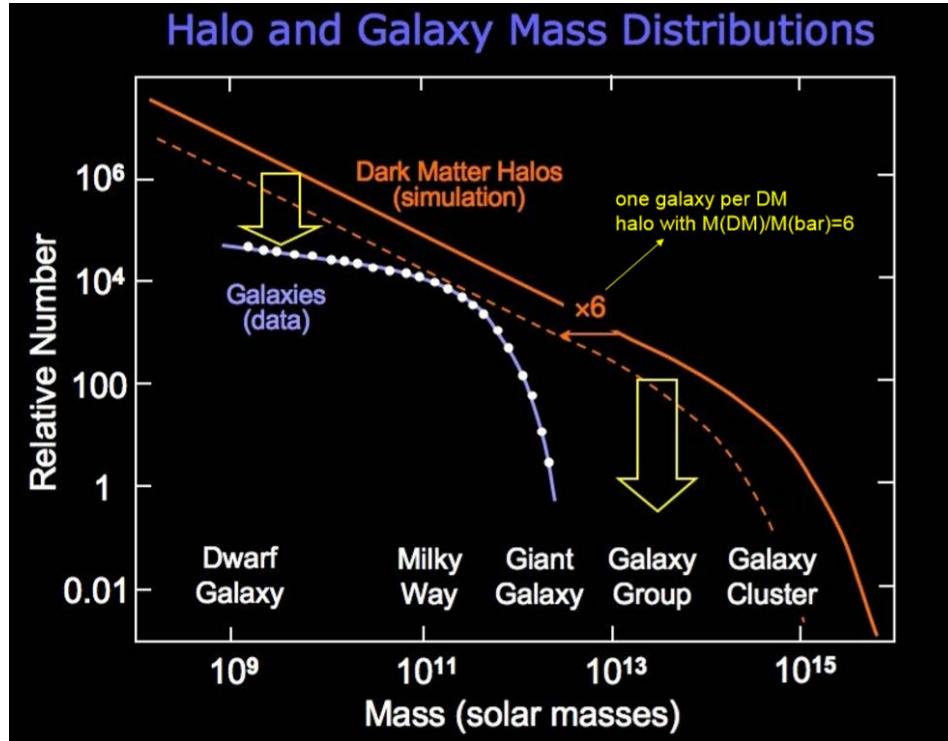
[http://zarija.com/s\\_structureFormation.html](http://zarija.com/s_structureFormation.html)

PS = Press-Schechter

ST = Sheth-Tormen

For now, in our simulations we consider only the most dominant matter component, responsible for the overall dynamics - the dark matter. We examined "concordance" cosmology ( $n=1$ ,  $h=0.7$ , 25.2% dark matter, 70% dark energy, baryons 4.8%), and we run 62 simulations with different box sizes (from 4 Mpc on a side to 256 Mpc), and with numerous number of realizations for each box size. The fact we have simulations spanning from small regions of the universe ( $4^3 \text{ Mpc}^3$ ) up to a reasonably big ones ( $256^3 \text{ Mpc}^3$ ) enables us to probe very large range of masses and redshifts, and since we have a lot of statistical samples for each box size, we have excellent statistics. We have compared mass function starting at very high redshift ( $z=20$ ) with analytical predictions and numerical fits, and our finding is that Warren et al. (2005) fit works the best over the whole redshift and mass range.

# Distributions of Dark Matter Halos



DM Halo mass function



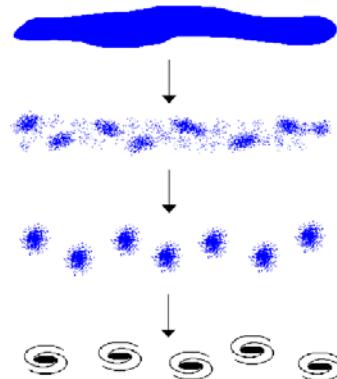
Galactic mass function

# Galaxy Formation: top-down vs bottom-up

Depending on the size of the initial DM halos (and on the Jean's mass), the cosmic structures might have two possible evolution scenarios

## Top-Down Structure Formation

In a top-down scenario, very large „pancakes“ of matter form. Eventually they will fragment and divide into smaller galaxies and structures

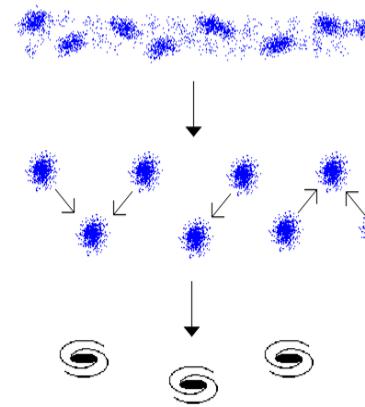


## HDM

First scales to condense are clusters/superclusters ( $\sim 10^{15-16}$  Msun)  
Smaller structures form later by fragmentation  
→ “Top-down” scenario, fragmentation of “pancakes”  
Inconsistent with: galaxy clustering and CMB fluctuations

## Bottom-Up Structure Formation

In a bottom-up scenario, galaxies of the size of dwarf galaxies (5 Mo) form. Larger scale structure will then form through events of merging

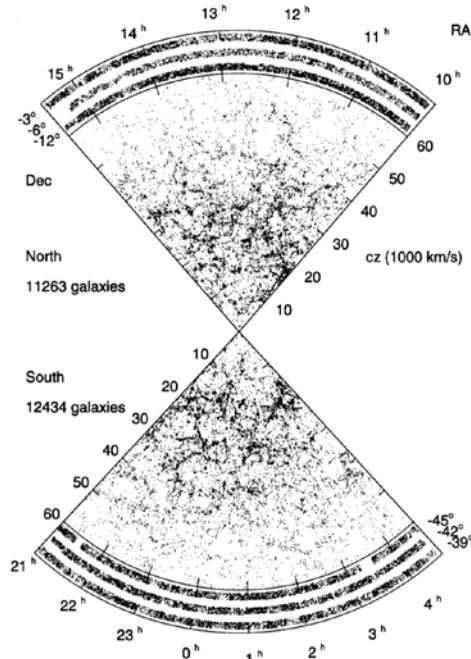


## CDM

Objects of  $\sim 10^6$  Msun form first  
Galaxies and clusters form after in hierarchical structure and mergers  
→ “Bottom-up” scenario, hierarchical merging

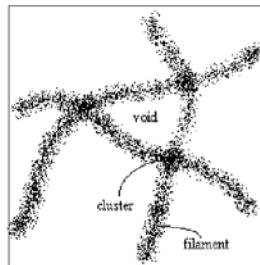
# Matching Observations and Simulations

## Las Campanas Redshift survey

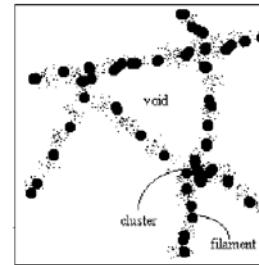


### Large Scale Structure

HDM and the top-down scenario predict smooth, weak features in the large scale distribution of galaxies

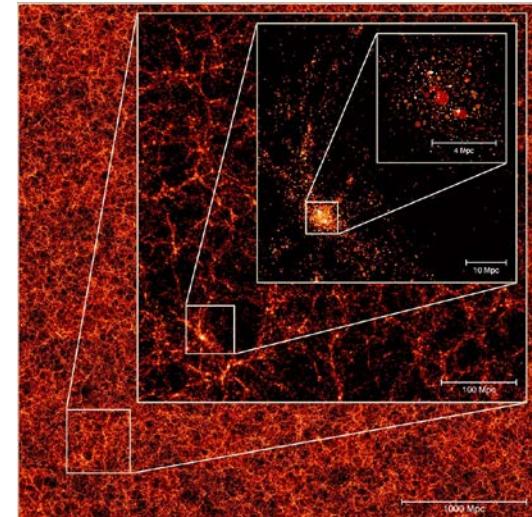


HDM



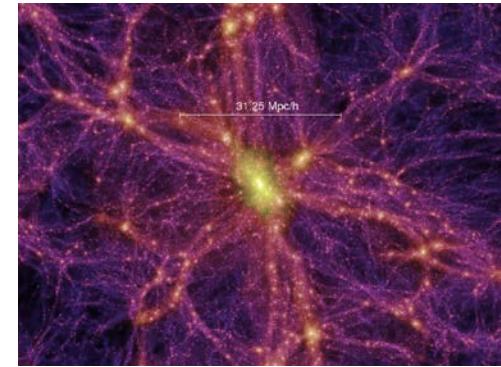
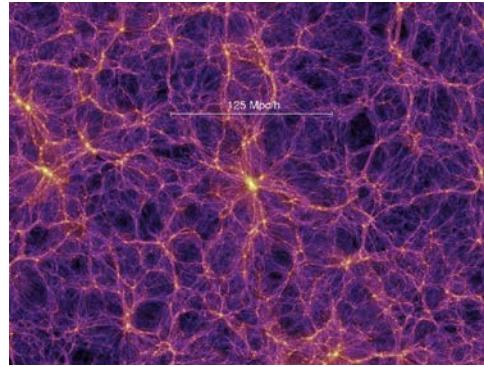
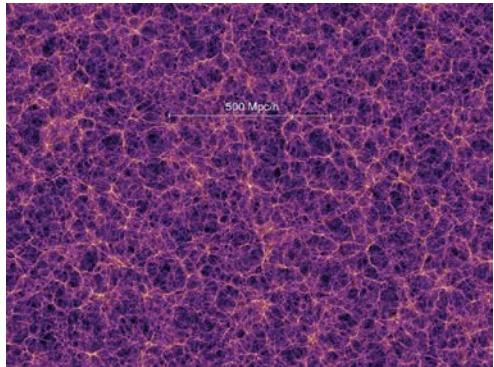
CDM

CDM and the bottom-up scenario predict sharp features with weak connecting filaments

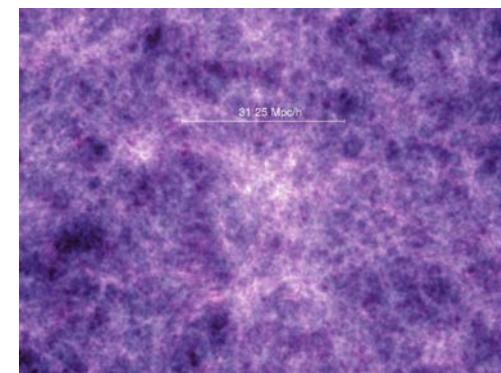
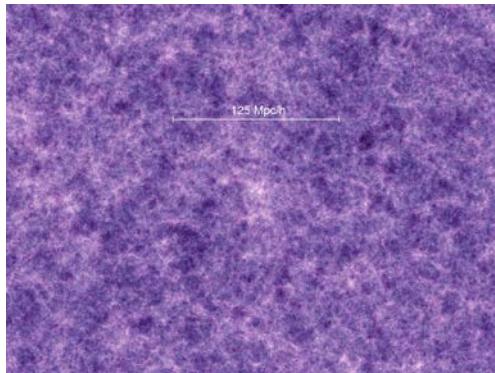
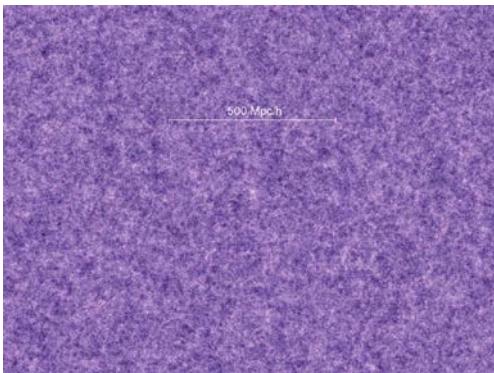


# Matching Observations and Simulations

Redshift  $z=0$  ( $t = 13.6$  Gyr)

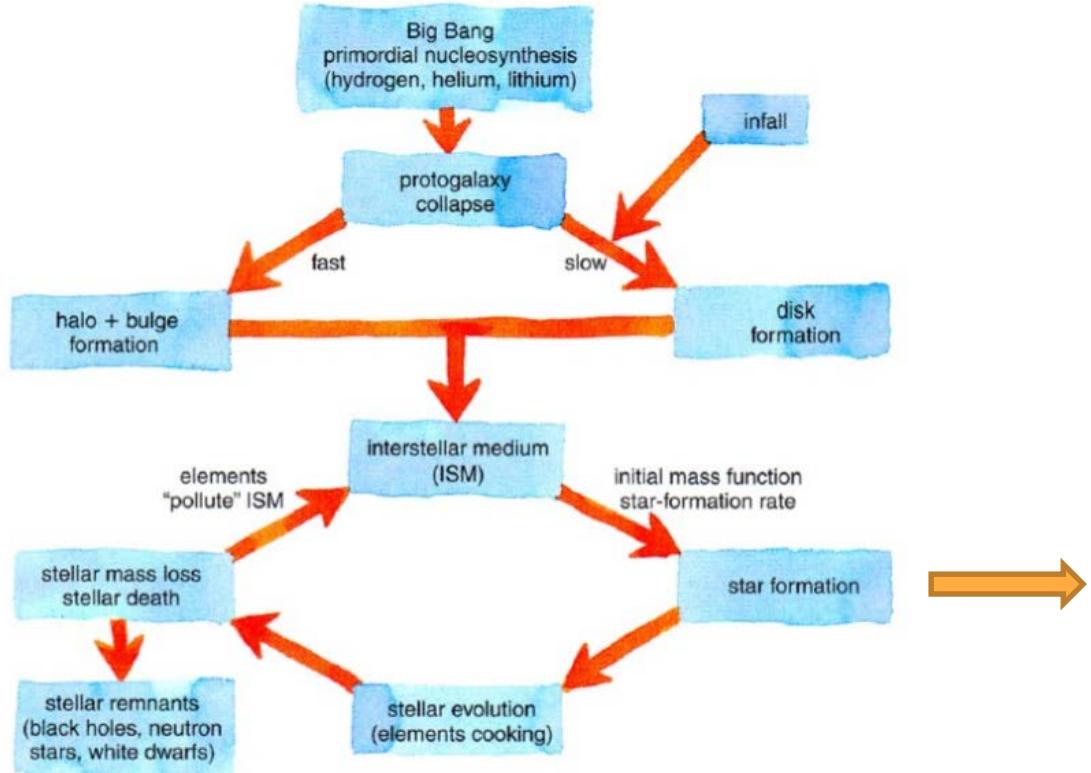


Redshift  $z=18.3$  ( $t = 0.21$  Gyr)



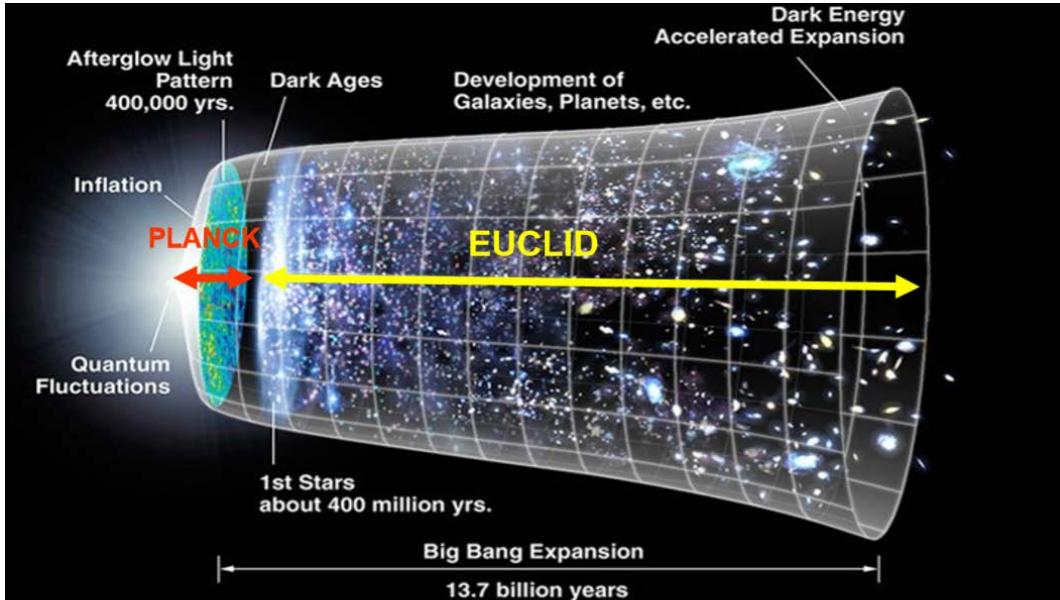
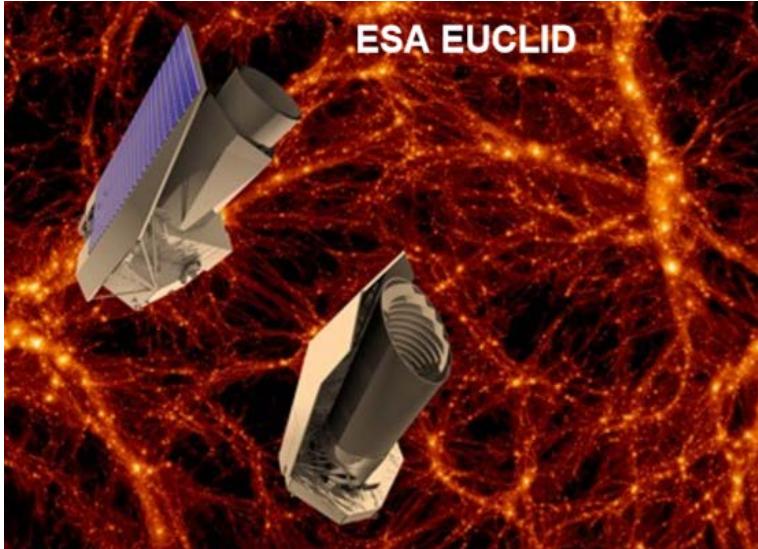
<https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/>

# Galaxy Formation



Very complex due to the feedback produced by stars (supernovae, black holes, eventually AGNs...)

## The Future: ESA EUCLID



***Euclid*:** visible to near-infrared space telescope  
The launch date is planned for June 2022

- ❑ “The” high precision Dark Energy & Cosmology mission
- ❑ Essential and unbeatable synergy of imaging + spectroscopy
- ❑ Euclid will impact the whole astrophysics and cosmology for decades to come

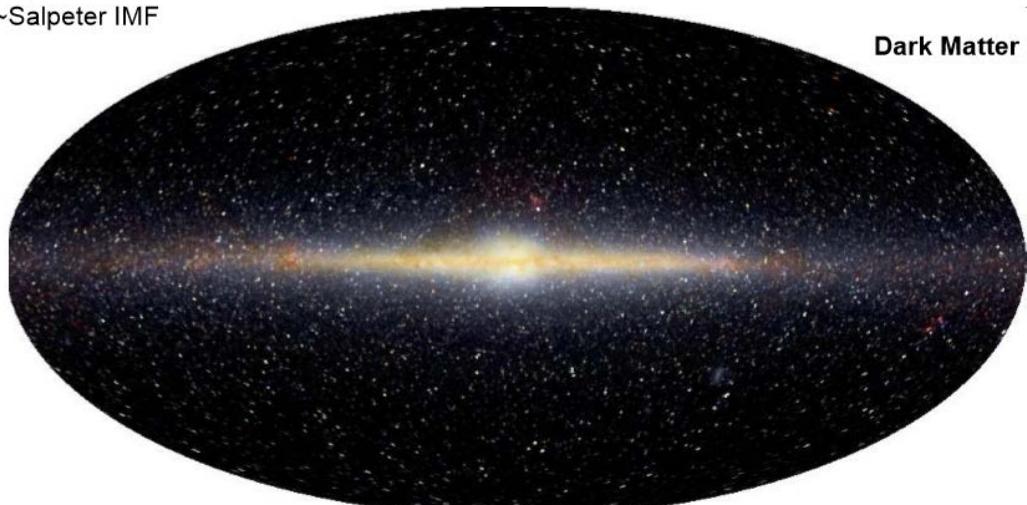
# Extras

# Our Galaxy: summary

Sb type  
Halo+bulge+disk  
Pop I & Pop II stars  
Pop III relics ?  
~Salpeter IMF

Central supermassive  
black hole ( $3 \times 10^6$  Msun)

Mass(total)  $\sim 10^{12}$  Msun  
Mass(stars)  $\sim 5 \times 10^{10}$  Msun  
Mass(gas)  $\sim 5 \times 10^9$  Msun  
Mass(dust)  $\sim 3 \times 10^7$  Msun



SFR  $\sim 1$  Msun yr $^{-1}$

Heterogeneous metallicity  
Age–metallicity relation with large scatter  
Metallicity gradient

Not representative  
of majority of spirals ?

## The Origin of the Density Fluctuations

- Likely that **quantum-gravity effects** in the inflation era provide origin
- **Power spectrum** of fluctuations also unknown
- Postulate a power-law spectrum (gravity has no characteristic scale)

$$\frac{\delta \rho}{\rho} = \frac{\delta M}{M} = A M^{-\alpha}$$

- Cannot have too significant **small-scale** perturbations: would lead to abundant primordial black holes
- Cannot have too significant **large-scale** perturbations: would give an inhomogeneous universe
- Expect  $\alpha$  close to 0 to 1
- **Inflationary models** predict :  $\alpha = 2/3$  (Harrison-Zel'dovich spectrum)  
 $A = \text{amplitude} = 10^{-4} - 10^{-5}$

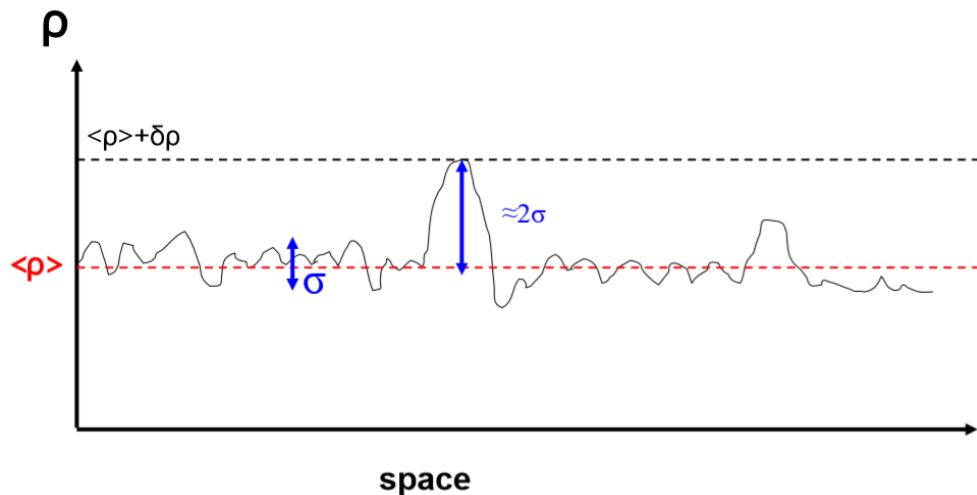
# Perturbations

The idea is to understand what happens if a fluid is perturbed :

$$\rho = \langle \rho \rangle + \delta \rho$$

$$\delta \equiv \frac{\delta \rho}{\rho} = \frac{\rho - \langle \rho \rangle}{\langle \rho \rangle}$$

Density field



## Two Possible Forms of the Spectrum of Fluctuations at Recombination depending on the nature and speed of the particles

### CDM

Objects of  $\sim 10^6$  Msun form first

Galaxies and clusters form after in hierarchical structure and mergers

→ “Bottom-up” scenario, hierarchical merging

### HDM

First scales to condense are clusters/superclusters ( $\sim 10^{15-16}$  Msun )

Smaller structures form later by fragmentation

→ “Top-down” scenario, fragmentation of “pancakes”

Inconsistent with: galaxy clustering and CMB fluctuations