The Australian National University, School of Computing
COMP2400/6240 (Relational Databases)
Semester 2, 2022

Lab 5, Week 7

Functional Dependencies (Solutions)

The purpose of this lab is to deepen your understanding of the notion of functional dependencies (FDs). We will first do some exercises on identifying FDs using sample data or following data requirements, then will practice on finding implied FDs and keys. In addition to these, we will also look into minimal cover (e.g., how is a minimal cover computed).

1 Update anomalies

To motivate the study of FDs, we start with discussing several update anomalies that may occur when database design allows the existence of redundant data.

(1) *Consider the relation shown in Figure 1 which has the primary key {SSN, Pnumber} together with the following functional dependencies. What update anomalies may occur in the relation? Give an example if possible.*

$$\{SSN\} \rightarrow \{EmployeeName\} \text{ and } \{Pnumber\} \rightarrow \{ProjectName, Plocation\}$$

SSN	Pnumber	Hours	EmployeeName	ProjectName	Plocation
11	1	32.5	Smith	Newbenefit	Bellaire
11	2	7.5	Smith	Databases	Sugarland
22	3	40	Narayan	Softwares	Houston
33	1	20	English	Newbenefit	Bellaire
33	2	7.5	English	Databases	Sugarland

Figure 1: A relation for Exercise (1)

Solution:

The FDs $\{SSN\} \rightarrow \{EmployeeName\}$ and $\{Pnumber\} \rightarrow \{ProjectName, Plocation\}$ can cause the anomalies. For example,

- If a project temporarily has no employees working on it, its information (Pnumber,ProjectName, Plocation) will not be represented in the database when the last employee working on it is removed (deletion anomaly).
- A new project cannot be added unless at least one employee is assigned to work on it (insertion anomaly).
- Changing the name of a project or an employee requires us to update every tuple that records the name of this project or employee (modification anomaly).
- Inserting a new tuple concerning adding an existing employee to an existing project requires checking both partial dependencies. For example, if a different value is entered for Plocation than those values in other tuples with the same value for Pnumber, we get an update anomaly. Similar anomalies can occur on the employee information. An example is shown in Figure 2.

SSN	Pnumber	Hours	EmployeeName	ProjectName	Plocation
11	1	32.5	Smith	Newbenefit	Bellaire
11	2	7.5	Smith	Databases	Sugarland
22	3	40	Narayan	Softwares	Houston
33	1	20	English	Newbenefit	Bellaire
33	2	7.5	English	Databases	Sugarland
33	2	7.5	Problem3	Problem2	Sugarland

Figure 2: Relation with an update anomaly

The reason for the occurrence of these anomalies is that the given relation represents the relationship between employees and projects, and at the same time represents information concerning the entities *employee* and *project*.

2 Functional Dependencies and Implication

(2) *Consider a relation schema* $R = \{A, B, C, D, E, F\}$ *with the following set* Σ *of functional dependencies:*

$$AB \rightarrow C,\ CF \rightarrow B,\ BC \rightarrow AD \text{ and } D \rightarrow E.$$

(2.1) Does $AB \rightarrow D$ hold on any relation of R that satisfies Σ? If so, explain why; otherwise, give a counterexample.

Solution:

Yes, $AB \rightarrow D$ holds. There are two approaches to show this:

- compute the closure of AB, i.e.,

$$\begin{aligned} (AB)^+ &= AB && \text{by } AB \rightarrow AB \\ &= ABC && \text{by } AB \rightarrow C \\ &= ABCD && \text{by } BC \rightarrow AD \end{aligned}$$

- by formal proof:

$AB \rightarrow B$	by reflexivity
$AB \rightarrow BC$	by union and the given FD $AB \rightarrow C$
$AB \rightarrow AD$	by transitivity and the given FD $BC \rightarrow AD$
$AB \rightarrow D$	decomposition

Remark: Compare two approaches and observe that the second approach is usually simpler than the first approach.

(2.2) Does $B \rightarrow C$ hold on any relation of R that satisfies Σ? If so, explain why; otherwise, give a counterexample.

Solution:

No, $B \rightarrow C$ does not hold. This can be proven by computing the closure of B in terms of Σ. Since $(B)^+ = B$ and $C \notin (B)^+$, $B \rightarrow C$ does not hold.

A counterexample can be made by adding two tuples into the relation that satisfies all the given functional dependencies but does not satisfy $B \rightarrow C$.

A	B	C	D	E	F
1	1	1	2	1	1
2	1	2	2	1	2

3 Identifying Functional Dependencies

(3) *Consider the following relation schema* INTERVIEW={*Client_ID, Client_Name, Staff_No, Date, Room*} *based on the following constraints:*

- Each staff member is allocated to a specific room on any given day.
- Each client has a unique ID.
- Each client is only interviewed by one staff member on any given day.

Your task is to find FDs over INTERVIEW based on the above requirements.

Solution:

Let Σ be the set of FDs containing the following:

- $\{Date, Staff\_No\} \rightarrow \{Room\_No\}$.
- $\{Client\_ID\} \rightarrow \{Client\_Name\}$;
- $\{Date, Client\_ID\} \rightarrow \{Staff\_No\}$.

(4) *Consider the relation shown in Figure 3.*

X	Y	Z
a_1	b	c_1
a_1	b	c_2
a_2	b	c_1
a_2	b	c_3

Figure 3: A relation for Exercise (2)

(4.1) List all the FDs that this relation satisfies.

Solution:

The following functional dependencies hold on R:

$$Z \rightarrow Y, XZ \rightarrow Y \text{ and } X \rightarrow Y.$$

The set of all FDs that this relation satisfies is the set of all FDs implied by $\{Z \rightarrow Y, X \rightarrow Y\}$, for example, $XZ \rightarrow Y$.

(4.2) Assume that the value of attribute Z of the last record in the relation in Figure 3 is changed from c_3 to c_1. Now list all the functional dependencies that this relation instance satisfies.

Solution:

The set of following functional dependencies hold on R remain unchanged, i.e., same as the previous one.

4 Finding keys

(5) *Consider a relation $R = \{A, B, C, D\}$ with the following functional dependencies:*

$$C \to D,\ AB \to C \text{ and } D \to A$$

(5.1) List all the keys of R.

Solution:

There are three keys of R

- {A,B}
- {B,C}
- {B,D}

(5.2) Find all the prime attributes of R.

Solution:

All the attributes of R are also prime attributes.

Remark: Discuss the systematical steps for finding the keys (refer to the lecture slides or the textbook).

5 Equivalence of functional dependencies

(6) *Consider a relation $R = \{A, B, C, D\}$. For each two sets of FDs given below, identity whether or not these two sets of FDs are equivalent*

(6.1) $\Sigma_1 = \{A \to B, AB \to C\}$ and $\Sigma_2 = \{A \to B, A \to C\}$

Solution: they are equivalent because $\Sigma_1 \models \Sigma_2$ and $\Sigma_2 \models \Sigma_1$.

(6.2) $\Sigma_1 = \{A \to B, A \to C\}$ and $\Sigma_2 = \{B \to A, A \to C\}$

Solution: they are not equivalent because $\Sigma_1 \not\models B \to A$

6 Minimal cover

(7) *Consider a relation $R = \{A, B, C, D\}$. For each set of FDs given below, identity whether or not the set of FDs is a minimal cover. If not, find a minimal cover for the set of FDs.*

(7.1) $\{A \to B,\ AB \to C\}$.

Solution: it's not a minimal cover. A minimal cover should be $\{A \to B,\ A \to C\}$. Note that $\{A \to B,\ AB \to C\}$ and $\{A \to B,\ A \to C\}$ are equivalent as shown in Exercise (6.1).

(7.2) $\{A \to B,\ A \to CB,\ B \to C\}$.

Solution: it's not a minimal cover. A minimal cover should be $\{A \to B,\ B \to C\}$

(7.3) $\{A \to B,\ B \to A,\ B \to C\}$.

Solution: it's a minimal cover.

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