

## Lab 6, Week 8

### Normalisation (Solutions)

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The purpose of this lab is to help you understand the normal forms 3NF and BCNF. In particular, you need to understand:

- What is BCNF? What is 3NF?
- What are the differences between 3NF and BCNF?

## 1 Normalisation - Inspection Example

Consider the following relation INSPECTION held at the MyHome real estate agency, in which {PropertyNo, Date} is the primary key:

PropertyNo	Address	Date	Time	StaffNo	StaffName	CameraID
PR4	6 Masson St	18-Oct-11	10:00	S137	Mike Jenk	C211
PR16	8 Berry St	22-Apr-12	09:00	S114	Sue Wang	C323
PR4	6 Masson St	01-Oct-13	12:00	S114	Sue Wang	C323
PR16	8 Berry St	21-Apr-12	13:00	S114	Sue Wang	C323

A set  $\Sigma$  of FDs for representing the business rules of INSPECTION is as follows:

- {PropertyNo}  $\rightarrow$  {Address};
- {StaffNo}  $\rightarrow$  {StaffName};
- {PropertyNo, Date}  $\rightarrow$  {StaffNo, Time};
- {StaffNo, Date}  $\rightarrow$  {CameraID};

- $\{\text{StaffNo}, \text{Date}, \text{Time}\} \rightarrow \{\text{PropertyNo}\};$
- $\{\text{Date}, \text{Time}, \text{CameraID}\} \rightarrow \{\text{PropertyNo}\}.$

(1) *Find all the keys and prime attributes w.r.t.  $\Sigma$ .*

Solution: The keys are:

- $\{\text{PropertyNo}, \text{Date}\};$
- $\{\text{Date}, \text{Time}, \text{CameraID}\};$
- $\{\text{StaffNo}, \text{Date}, \text{Time}\}.$

This is because the closure of  $\{\text{Date}, \text{PropertyNo}\}$ ,  $\{\text{Date}, \text{Time}, \text{CameraID}\}$  or  $\{\text{StaffNo}, \text{Date}, \text{Time}\}$  is the set of all attributes of INSPECTION with respect to  $\Sigma$  and they are minimal.

The prime attributes are: Date, Time, PropertyNo, StaffNo and CameraID. The non-prime attributes are: StaffName and Address.

(2) *Is the given set of FDs minimal? If not, give a minimal cover.*

Solution: It's not minimal. One possible solution is as follows:

- $\{\text{StaffNo}\} \rightarrow \{\text{StaffName}\};$
- $\{\text{PropertyNo}\} \rightarrow \{\text{Address}\};$
- $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{StaffNo}\};$
- $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{Time}\};$
- $\{\text{StaffNo}, \text{Date}\} \rightarrow \{\text{CameraID}\};$
- $\{\text{Date}, \text{Time}, \text{CameraID}\} \rightarrow \{\text{PropertyNo}\}.$

The steps are as follows:

Starting with the given set  $\Sigma$  of FDs,

- by **Step 2** of the algorithm, we replace  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{StaffNo}, \text{Time}\}$  with  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{StaffNo}\}$  and  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{Time}\}$ . Then we have:  $\{\{\text{StaffNo}\} \rightarrow \{\text{StaffName}\}, \{\text{PropertyNo}\} \rightarrow \{\text{Address}\}, \{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{StaffNo}\}, \{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{Time}\}, \{\text{StaffNo}, \text{Date}\} \rightarrow \{\text{CameraID}\}, \{\text{StaffNo}, \text{Date}, \text{Time}\} \rightarrow \{\text{PropertyNo}\}, \{\text{Date}, \text{Time}, \text{CameraID}\} \rightarrow \{\text{PropertyNo}\}\};$

- by **Step 3** of the algorithm, we still have the same set of FDs as in the previous step;
- by **Step 4** of the algorithm, we calculate the closure of the determinant of a FD in terms of other FDs, if the closure contains the dependent of the FD, then the FD is redundant and can be removed. In doing so, we can only remove  $\{\text{StaffNo}, \text{Date}, \text{Time}\} \rightarrow \{\text{PropertyNo}\}$  because the closure of  $\{\text{StaffNo}, \text{Date}, \text{Time}\}$  in terms of the other FDs contains PropertyNo.

After the above three steps, we can obtain the minimal cover.

**(3)** *Is INSPECTION in 3NF w.r.t.  $\Sigma$ ? If not, determine a lossless and dependency preserving 3NF decomposition. Are the relation schemas you have obtained in the decomposition in BCNF? Justify your answers.*

Solution:

INSPECTION is not in 3NF w.r.t.  $\Sigma$ . This can be verified by testing each FD:  $X \rightarrow A$  defined on INSPECTION: either  $X$  is a superkey or  $A$  is a prime attribute. In accordance with the results in Exercise (2), it is clear that  $\{\text{PropertyNo}\} \rightarrow \{\text{Address}\}$  and  $\{\text{StaffNo}\} \rightarrow \{\text{StaffName}\}$  are problematic. Using the minimal cover in Exercise (3) and the corresponding 3NF decomposition algorithm, we may decompose INSPECTION into the following relation schemas.

- STAFF={StaffNo, StaffName} with the FD:  $\{\text{StaffNo}\} \rightarrow \{\text{StaffName}\}$ ;
- PROPERTY={PropertyNo, Address} with the FD:  $\{\text{PropertyNo}\} \rightarrow \{\text{Address}\}$ ;
- INSPECTION1={PropertyNo, Date, staffNo, Time} with the FD:  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{StaffNo}, \text{Time}\}$ ;
- INSPECTION2={StaffNo, Date, CameraID} with the FD:  $\{\text{StaffNo}, \text{Date}\} \rightarrow \{\text{CameraID}\}$ ;
- INSPECTION3={PropertyNo, Date, Time, CameraID} with the FDs:  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{Time}\}$ , and  $\{\text{Date}, \text{Time}, \text{CameraID}\} \rightarrow \{\text{PropertyNo}\}$ .

The above decomposition into 3NF is lossless and dependency preserving.

Now let's discuss why the above 3NF decomposition preserves all FDs in the original set  $\Sigma$ . Assume the above minimal cover derived from  $\Sigma$  is  $\Sigma_{min}$ . Note that  $\Sigma$  and  $\Sigma_{min}$  must be equivalent according to the definition of the minimal cover. It is obvious that the 3NF decomposition preserves all FDs in  $\Sigma_{min}$  and therefore can imply any FD in the  $\Sigma$ .

The relation schemas STAFF, PROPERTY, INSPECTION1 and INSPECTION2 are in BCNF. INSPECTION3 seems not in BCNF due to  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{Time}\}$  and  $\{\text{PropertyNo}, \text{Date}\}$  seems not a PK of INSPECTION3 based on two surviving FDs in INSPECTION3. However, we should consider all surviving FDs in this decomposition and thus  $\{\text{PropertyNo}, \text{Date}\}$  is actually a PK of INSPECTION3 because  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{StaffNo}, \text{Time}\}$  and  $\{\text{StaffNo}, \text{Date}\} \rightarrow \{\text{CameraID}\}$  together imply  $\{\text{PropertyNo}, \text{Date}\} \rightarrow \{\text{CameraID}, \text{Time}\}$  and thus INSPECTION3 is also in BCNF.

## 2 Normalisation - Meeting Example

Consider the following relation:

MEETING = {CRN, Name, Date, Time, Officer, Cabin}

with the following set  $\Sigma$  of FDs:

fd1:  $\{\text{CRN}, \text{Date}, \text{Time}\} \rightarrow \{\text{Officer}\};$

fd2:  $\{\text{Date}, \text{Time}, \text{Cabin}\} \rightarrow \{\text{CRN}\};$

fd3:  $\{\text{Officer}, \text{Date}, \text{Time}\} \rightarrow \{\text{CRN}\};$

fd4:  $\{\text{Date}, \text{Officer}\} \rightarrow \{\text{Cabin}\};$

fd5:  $\{\text{CRN}\} \rightarrow \{\text{Name}\}.$

(4) *List all the keys of MEETING w.r.t.  $\Sigma$ .*

Solution: Let us find out all the keys for MEETING. Note the  $\{\text{Date}, \text{Time}\}$  must be part of all keys as they never appear in the dependent of any FD,

1.  $\{\text{CRN}, \text{Date}, \text{Time}\}$  is a key (minimal superkey)
2.  $\{\text{Cabin}, \text{Date}, \text{Time}\}$  is a key (minimal superkey)
3.  $\{\text{Officer}, \text{Date}, \text{Time}\}$  is a key (minimal superkey)

(5) *Find all the prime attributes of MEETING w.r.t.  $\Sigma$ .*

Solution:

$\{\text{CRN}, \text{Date}, \text{Time}, \text{Officer}, \text{Cabin}\}$  is the set of all prime attributes of MEETING with respect to  $\Sigma$ .

(6) Does MEETING satisfy 3NF w.r.t.  $\Sigma$ ? If not, determine a minimal cover of  $\Sigma$ , and a lossless and dependency preserving 3NF decomposition. Justify your answers.

Solution:

- MEETING doesn't satisfy 3NF because, in fd5:  $\{\text{CRN}\} \rightarrow \{\text{Name}\}$ , neither CRN is a superkey nor Name is a prime attribute.
- $\Sigma' = \{\{\text{CRN}, \text{Date}, \text{Time}\} \rightarrow \{\text{Officer}\}, \{\text{Date}, \text{Time}, \text{Cabin}\} \rightarrow \{\text{CRN}\}, \{\text{Date}, \text{Officer}\} \rightarrow \{\text{Cabin}\}, \{\text{CRN}\} \rightarrow \{\text{Name}\}\}$  is a minimal cover.
- By applying the corresponding algorithm, we can achieve a lossless and dependency preserving 3NF decomposition for MEETING as follows:
- From  $\Sigma'$ , we add  $R_1 = \{\text{CRN}, \text{Date}, \text{Time}, \text{Officer}\}$ ;  $R_2 = \{\text{Date}, \text{Time}, \text{Cabin}, \text{CRN}\}$ ;  $R_3 = \{\text{Officer}, \text{Date}, \text{Cabin}\}$  and  $R_4 = \{\text{CRN}, \text{Name}\}$  to S.
- Since  $R_1$  is a superkey, we don't need to add a key. Thus  $S := \{R_1, R_2, R_3, R_4\}$
- Therefore MEETING is decomposed into the following relations in 3NF:
  - $R_1 = \{\text{CRN}, \text{Date}, \text{Time}, \text{Office}\}$  with  $\Sigma_1 = \{\text{CRN}, \text{Date}, \text{Time}\} \rightarrow \{\text{Officer}\}$ ;
  - $R_2 = \{\text{Date}, \text{Time}, \text{Cabin}, \text{CRN}\}$  with  $\Sigma_2 = \{\text{Date}, \text{Time}, \text{Cabin}\} \rightarrow \{\text{CRN}\}$ ;
  - $R_3 = \{\text{Officer}, \text{Date}, \text{Cabin}\}$  with  $\Sigma_3 = \{\text{Date}, \text{Officer}\} \rightarrow \{\text{Cabin}\}$ ;
  - $R_4 = \{\text{CRN}, \text{Name}\}$  with  $\Sigma_4 = \{\text{CRN}\} \rightarrow \{\text{Name}\}$ .

Why the above 3NF decomposition preserves all fds in  $\Sigma$ ?

The surviving fds ( $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Sigma_4$ ) is equivalent to  $\Sigma$ .

(7) Does MEETING satisfy BCNF w.r.t.  $\Sigma$ ? If not, determine a lossless decomposition for MEETING into BCNF. Does your decomposition preserve all dependencies of MEETING?

Solution:

- Since the determinants of the FDs:  $\{\text{Date}, \text{Officer}\} \rightarrow \{\text{Cabin}\}$  and  $\{\text{CRN}\} \rightarrow \{\text{Name}\}$  are not superkeys, MEETING doesn't satisfy BCNF.
- By applying the corresponding algorithm,

- Let  $S := \{\text{MEETING}\}$ .
- Since MEETING is not in BCNF, we pick the FD:  $\{\text{Date}, \text{Officer}\} \rightarrow \{\text{Cabin}\}$  that violates BCNF, and replace MEETING in  $S$  by two relation schemas  $R_1 = \{\text{CRN}, \text{Date}, \text{Time}, \text{Officer}, \text{Name}\}$  with  $\Sigma_1 = \{\{\text{CRN}, \text{Date}, \text{Time}\} \rightarrow \{\text{Officer}\}, \{\text{Officer}, \text{Date}, \text{Time}\} \rightarrow \{\text{CRN}\}, \{\text{CRN}\} \rightarrow \{\text{Name}\}\}$  and  $R_2 = \{\text{Date}, \text{Officer}, \text{Cabin}\}$  with  $\Sigma_2 = \{\{\text{Date}, \text{Officer}\} \rightarrow \{\text{Cabin}\}\}$ . So we have  $S := \{R_1, R_2\}$ .
- Now we easily see that  $R_1$  is still not in BCNF because the determinant of the FD  $\{\text{CRN}\} \rightarrow \{\text{Name}\}$  and  $\{\text{CRN}\}$  is not a superkey with respect to  $\Sigma_1$ . We pick this problematic fd and further decompose  $R_1$  into two relations  $R_{11} = \{\text{CRN}, \text{Date}, \text{Time}, \text{Officer}\}$  with  $\Sigma_{11} = \{\{\text{CRN}, \text{Date}, \text{Time}\} \rightarrow \{\text{Officer}\}, \{\text{Officer}, \text{Date}, \text{Time}\} \rightarrow \{\text{CRN}\}\}$  and  $R_{12} = \{\text{CRN}, \text{Name}\}$  with  $\Sigma_{12} = \{\{\text{CRN}\} \rightarrow \{\text{Name}\}\}$ .
- Now we have  $S := \{R_{11}, R_{12}, R_2\}$ .
- This decomposition is lossless, which is ensured by the algorithm. However the FD:  $\text{Date}, \text{Time}, \text{Cabin} \rightarrow \text{CRN}$  isn't preserved in the decomposition of MEETING into  $R_{11}$ ,  $R_{12}$  and  $R_2$  because it cannot be inferred from  $\Sigma_{11}$ ,  $\Sigma_{12}$  or  $\Sigma_2$ .