$ln[+] := Eigenvectors [{{(n + 1/2) w + \Delta/2, \lambda Sqrt[n + 1]}, {\lambda Sqrt[n + 1], (n + 1/2) w - \Delta/2}}]$

Out[
$$\circ$$
]= $\left\{ \left\{ -\frac{-\Delta + \sqrt{\Delta^2 + 4 \lambda^2 + 4 n \lambda^2}}{2 \sqrt{1 + n} \lambda}, 1 \right\}, \left\{ -\frac{-\Delta - \sqrt{\Delta^2 + 4 \lambda^2 + 4 n \lambda^2}}{2 \sqrt{1 + n} \lambda}, 1 \right\} \right\}$