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An Advanced Signature Scheme:

Schnorr Algorithm and its Benefits to the Bitcoin Ecosystem

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School of Industrial and Information Engineering
Master of Science in Mathematical Engineering

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Introduction

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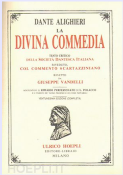
1. Efficiency (DER encoding, no batch validation, modular inversion);
2. Poor implementation of higher level constructions (low privacy and fungibility, scales badly);
3. Not provably secure (malleable).

Outline

Hash functions ($\simeq$ Random functions)

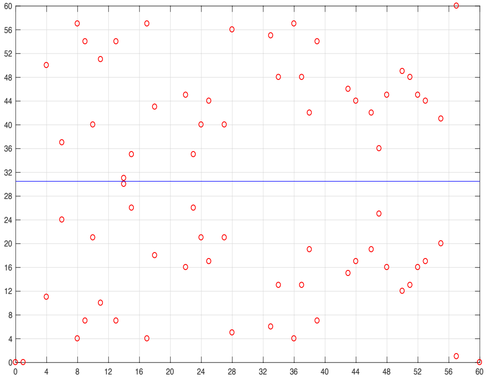
Input		Output	
"Politecnico di Milano"	$\xrightarrow{\text{SHA-256}}$ / $\not\leftarrow$	52c61456bd3d60c08599a8c61e113e7b c39118b75d06b2643c55957ece91269b	
	$\xrightarrow{\text{SHA-256}}$ / $\not\leftarrow$	1f12f8384d5941e8e273926635be646a f786405a8662e0ba6dd1fd0526ec0528	
	$\xrightarrow{\text{SHA-256}}$ / $\not\leftarrow$	721a14d89cb463b51cf20ecc707aa290 5b7c8f32d9114dc19fe353e611494bf1	

Elliptic curve cryptography

An elliptic curve over a finite field is defined by:

$$E(\mathbb{F}_p):\; y^2 = x^3 + ax + b \pmod p.$$

It is possible to define:



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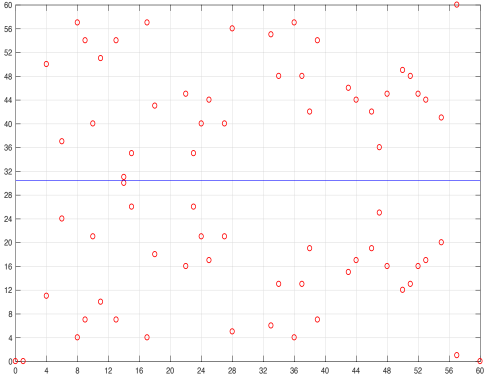

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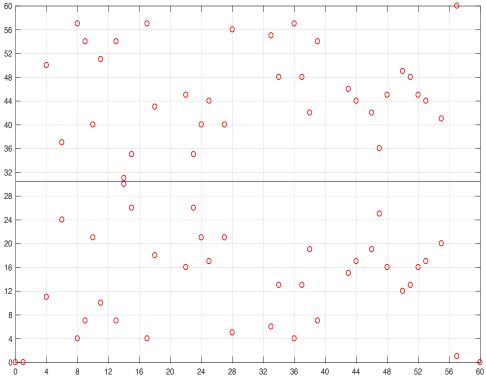

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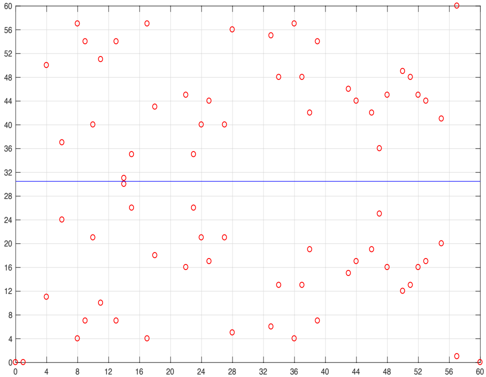
Multiplication's computational asymmetry is the core of ECC.



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Double and add algorithm: $q = 41$

$$41 = 1 + 8 + 32 \implies 41G = G + 8G + 32G.$$

5 point doubling and 2 additions vs. 40 additions.

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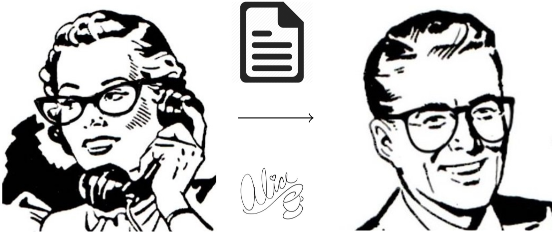
Digital signature



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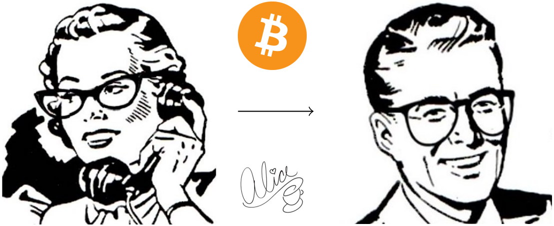

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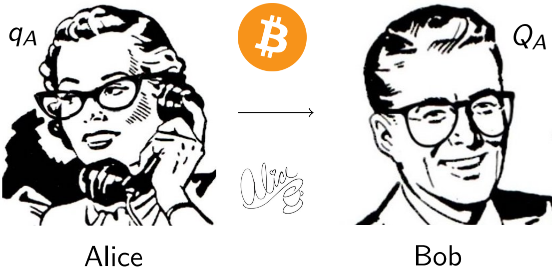

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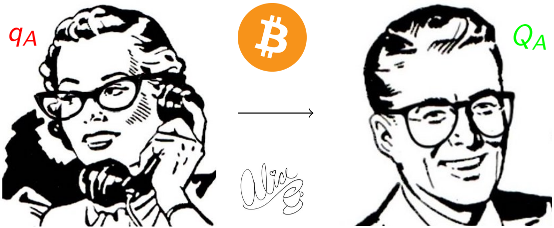

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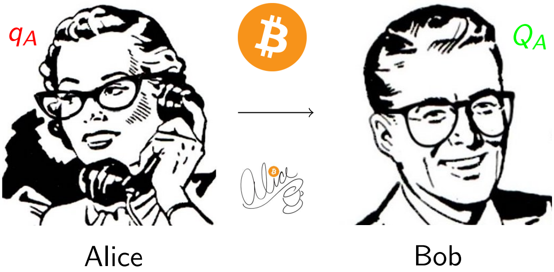

- Authentication: the recipient is confident that the message comes from the alleged sender;
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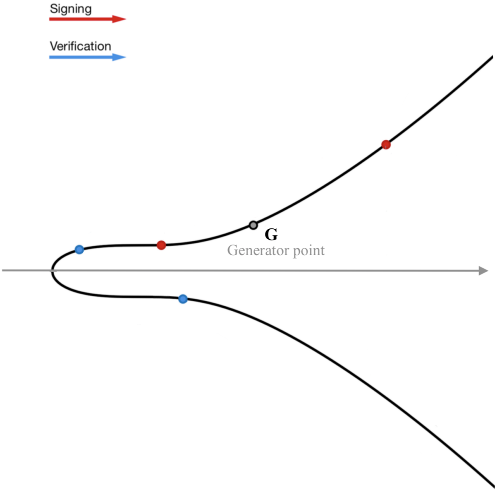
- Integrity: ensures that the message has not been altered during transmission.

Elliptic curve digital signature algorithm

ECDSA_SIG(m, q):



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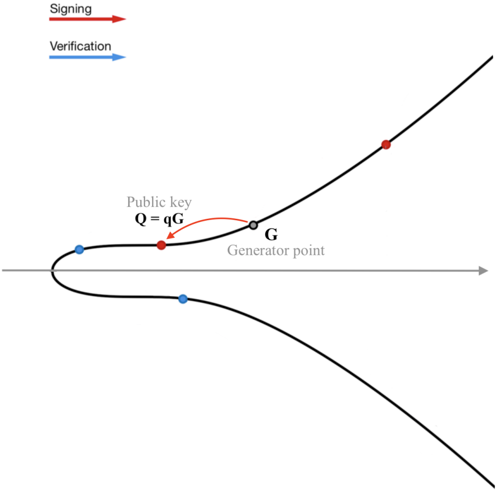

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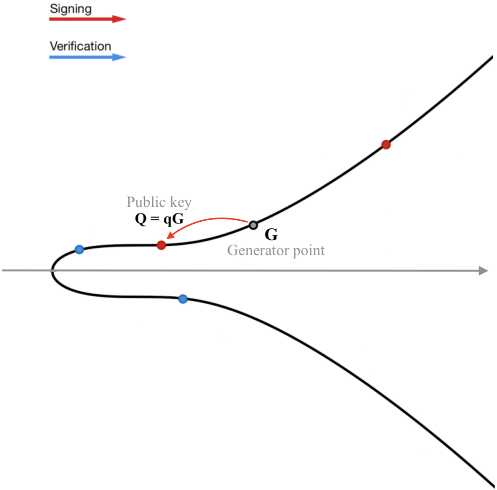

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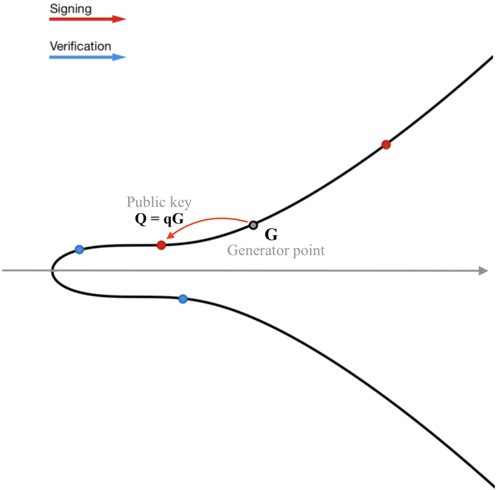

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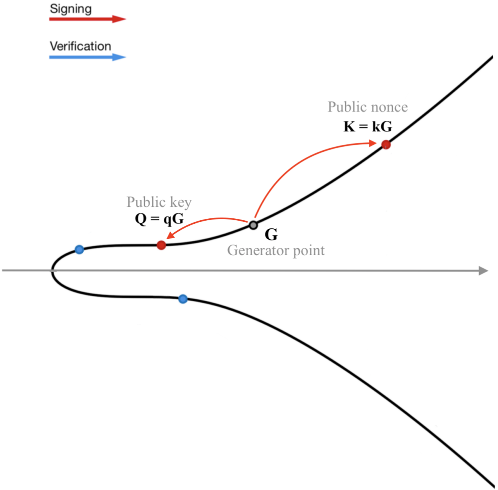

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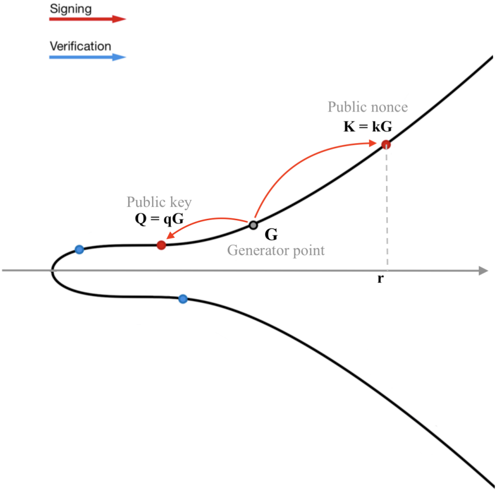

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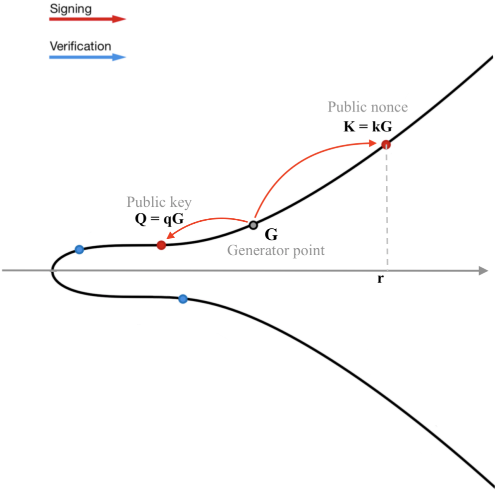

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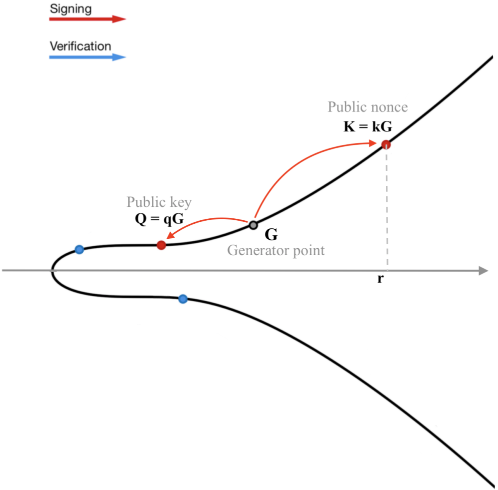

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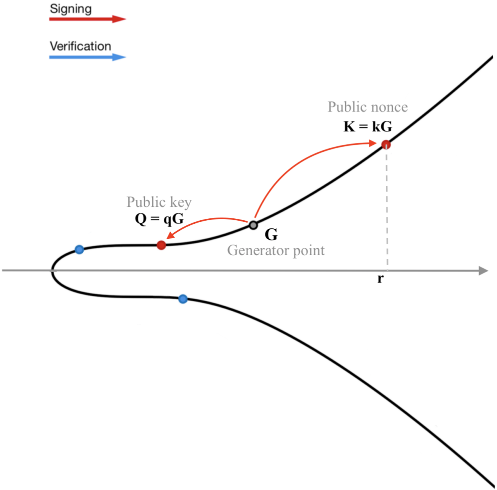

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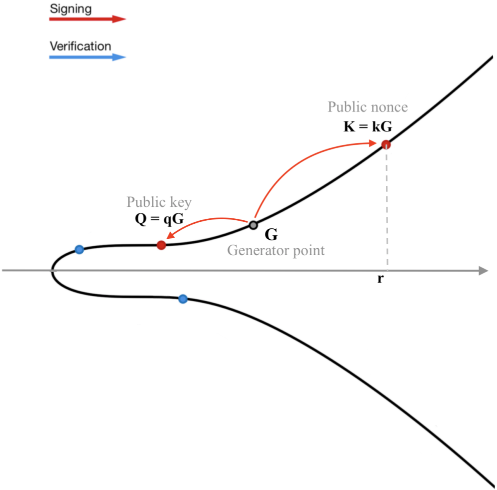

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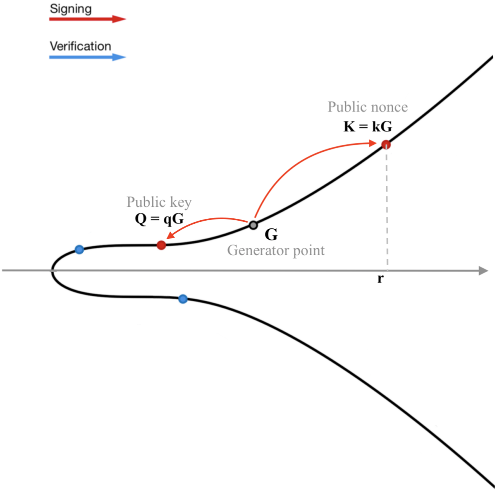

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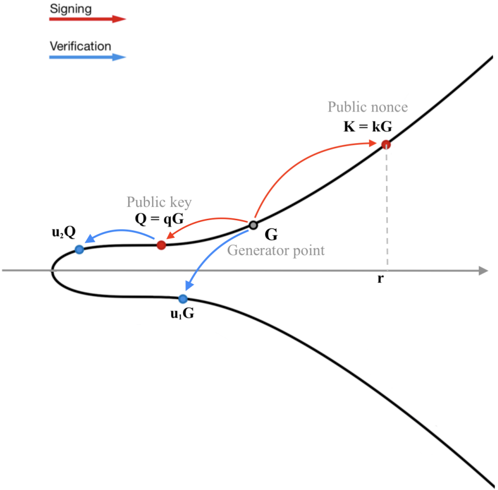

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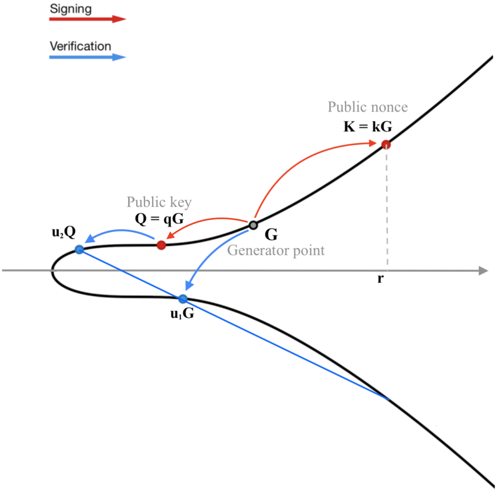

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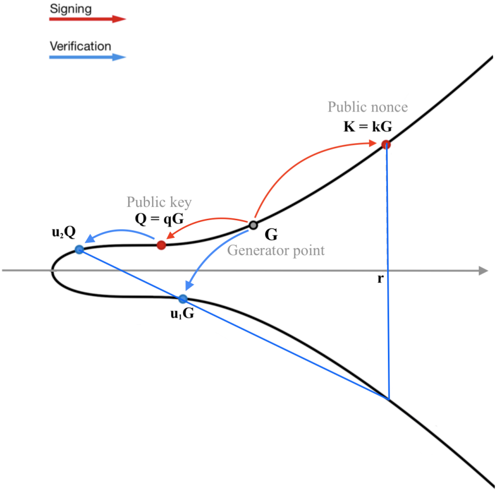

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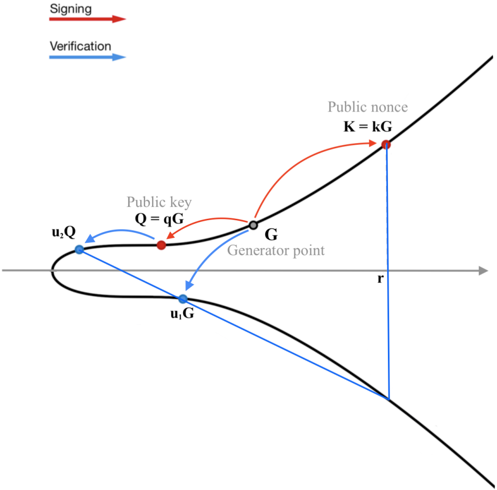

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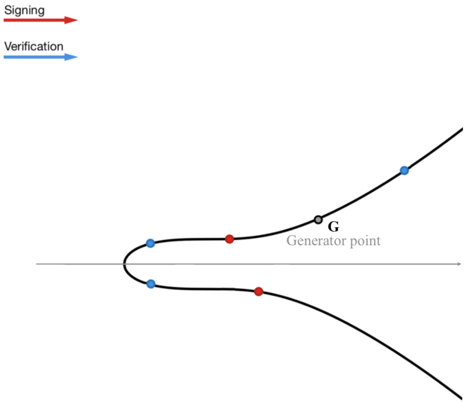

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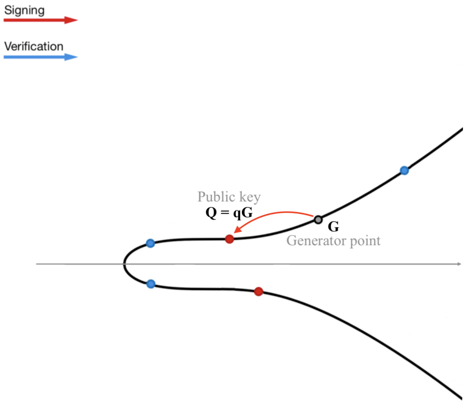

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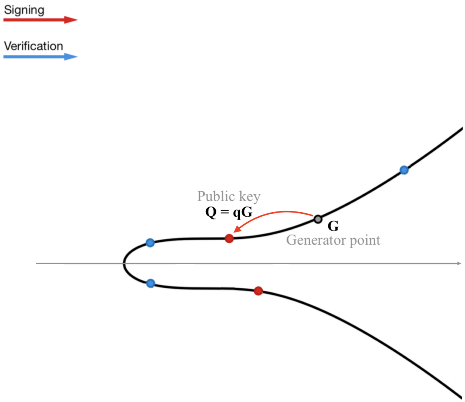

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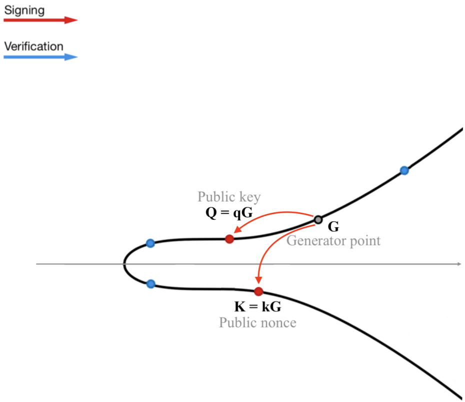

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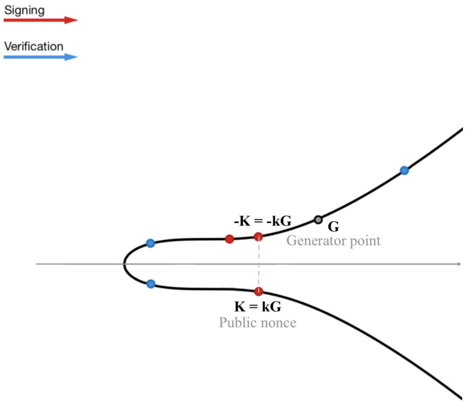

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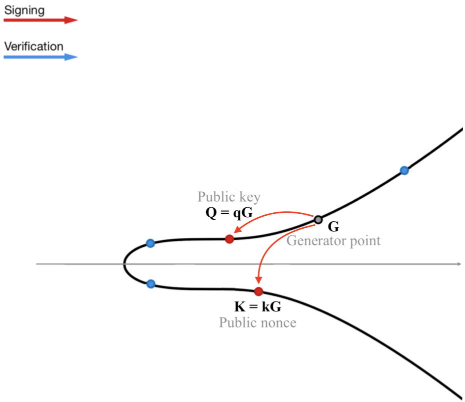

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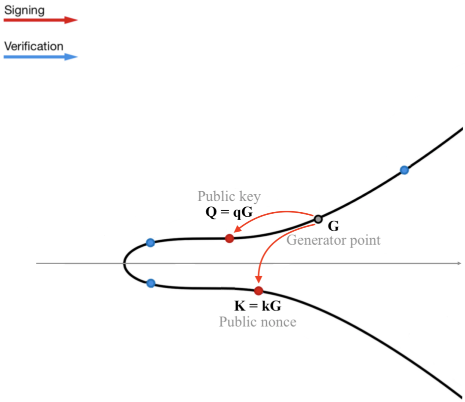

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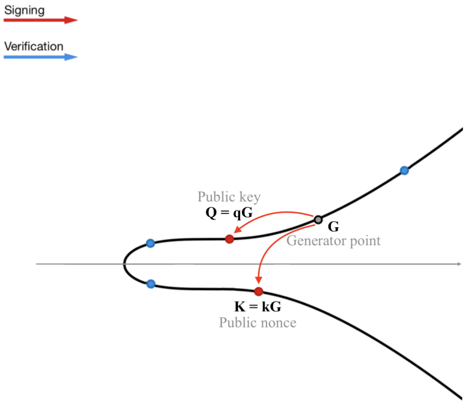

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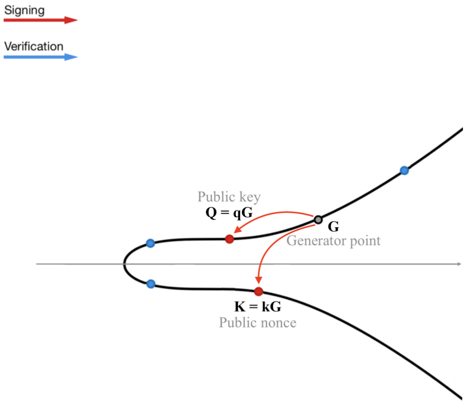

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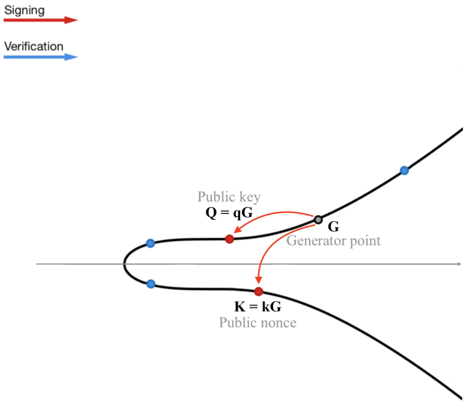

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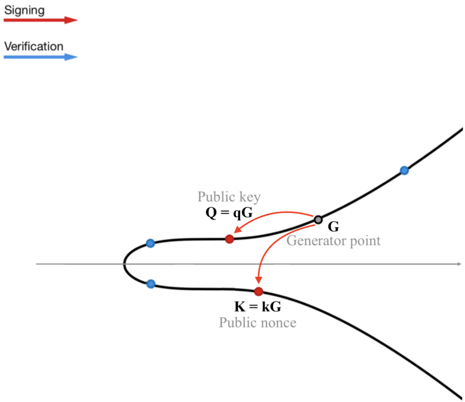

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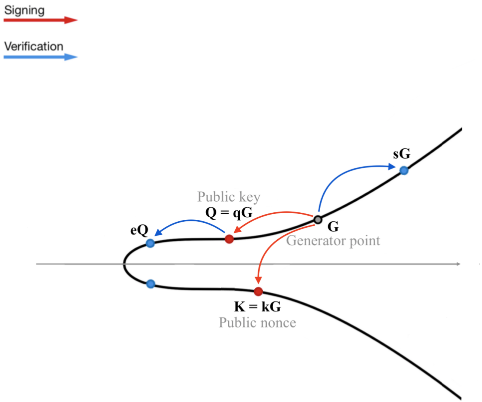

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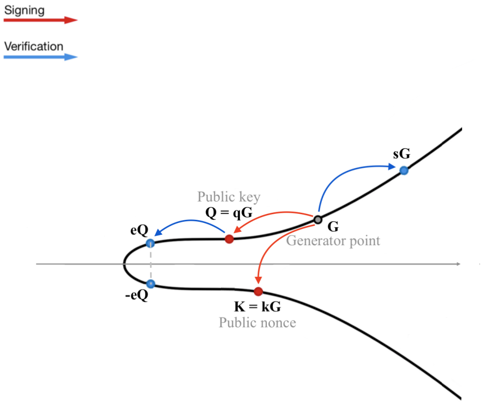

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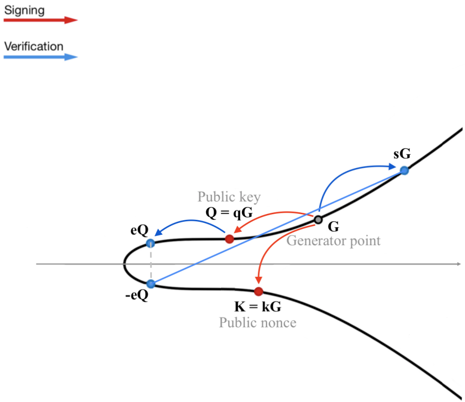

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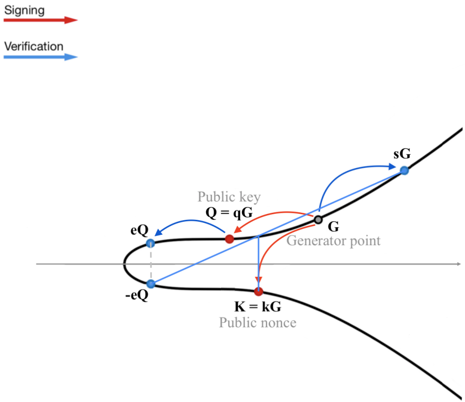

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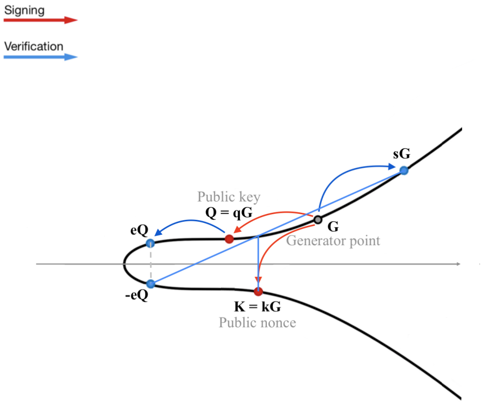

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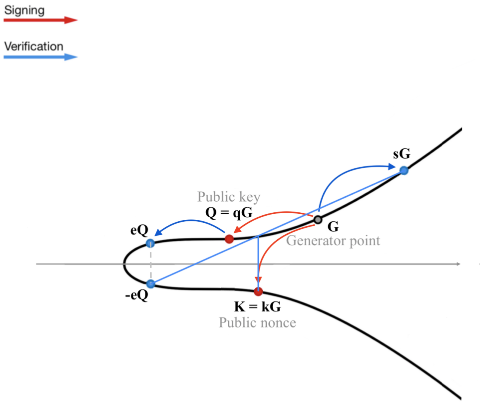

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Batch validation

A signature (K, s) is valid if $K = sG - \text{hash}(x_K \mid\mid Q \mid\mid m)Q$.
Thus, two valid signatures (K_0, s_0) and (K_1, s_1) satisfies:

$$K_0+K_1 = (s_0+s_1)G-\text{hash}(x_{K_0} \mid\mid Q_0 \mid\mid m_0)Q_0-\text{hash}(x_{K_1} \mid\mid Q_1 \mid\mid m_1)Q_1.$$

Insecure: introduction of random factors.

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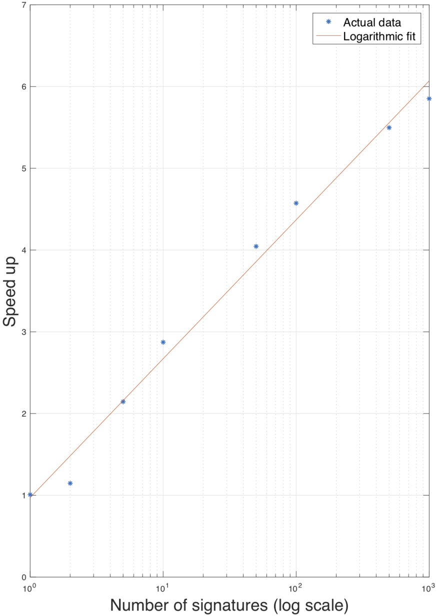

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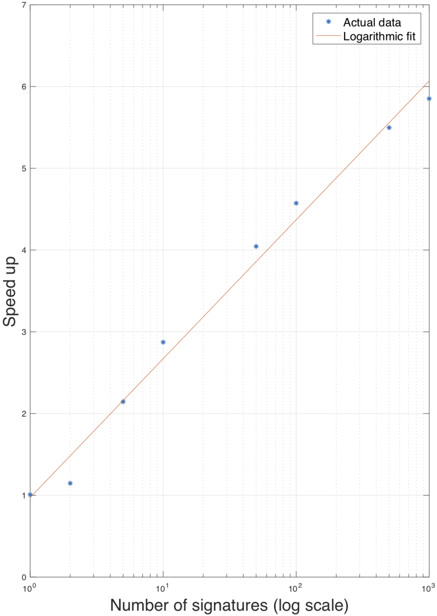

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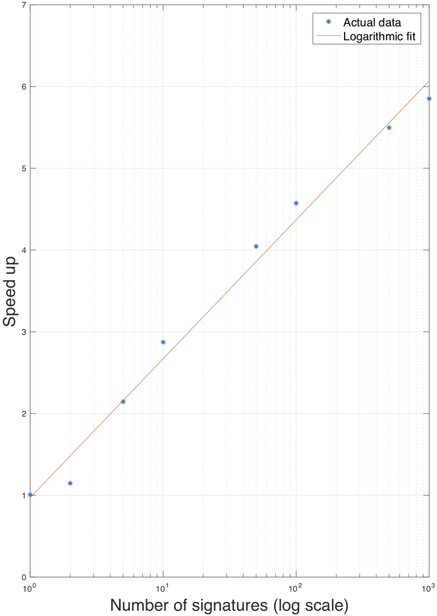

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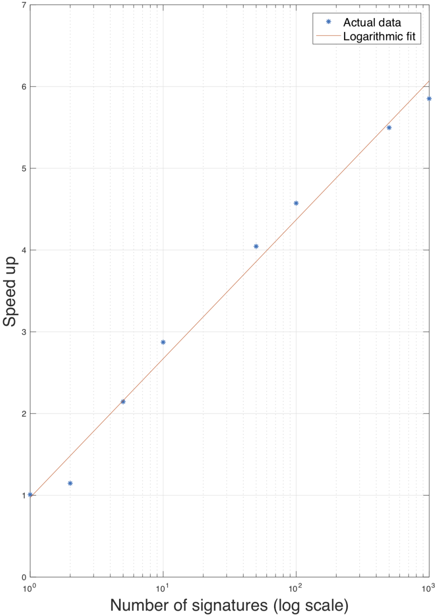

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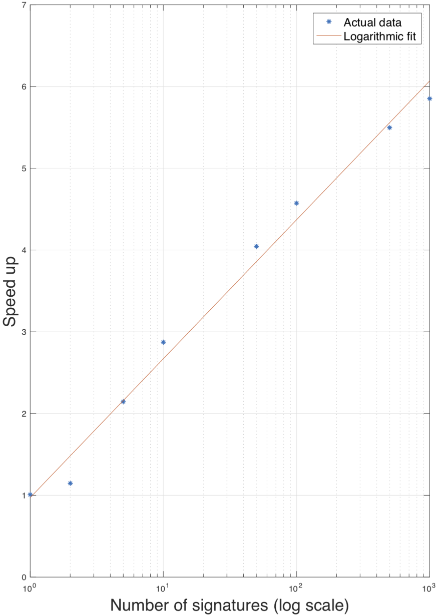

Outline

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INSECURE: rogue key attack!

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$\text{MuSig}(m, q_1, \langle L \rangle)$:

1: Alice

2: Bob

3: Charlotte

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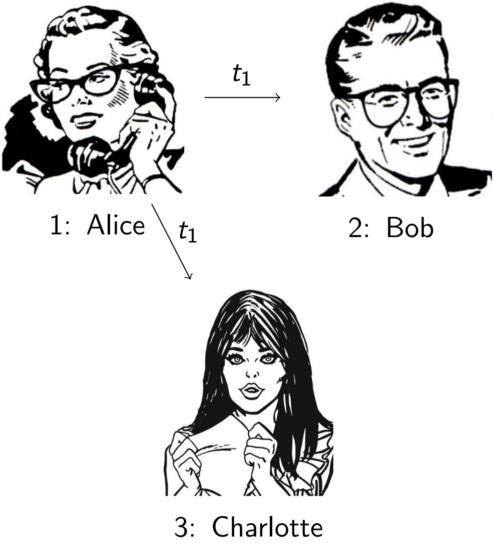

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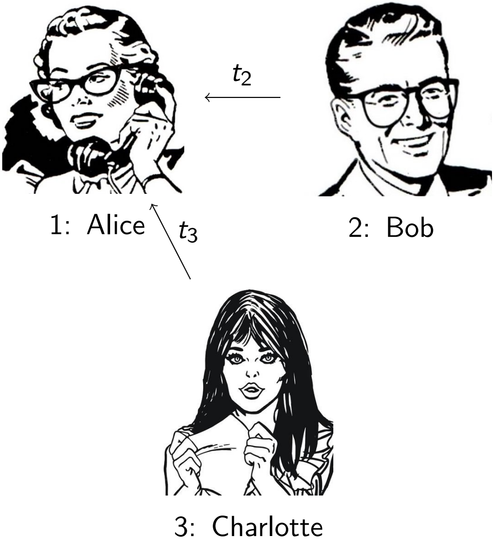

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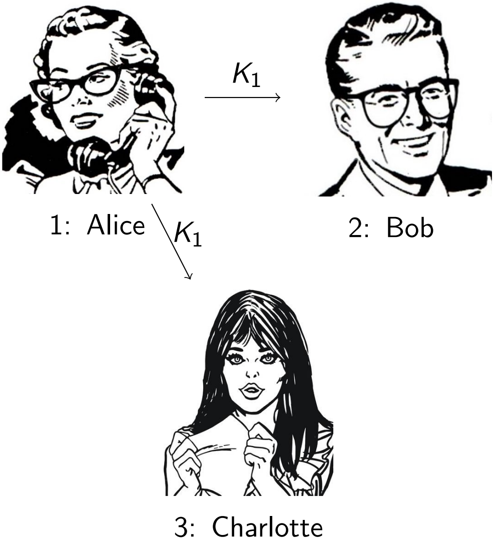

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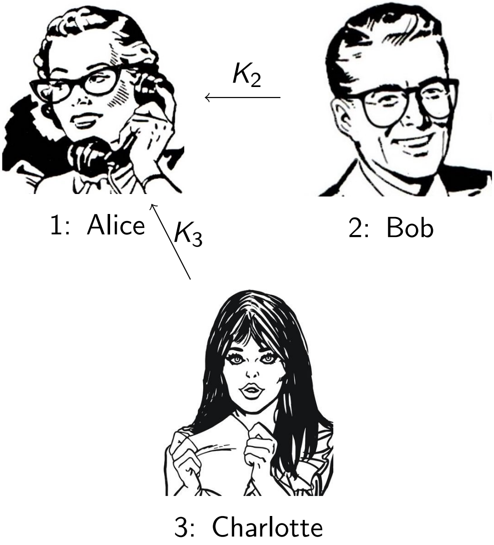

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1. **for** $i \leftarrow 1, m$ **do**:
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4. $K_1 \leftarrow k_1 G$, $t_1 \leftarrow \text{hash}(K_1)$;
5. **send** t_1, K_1;
6. $K \leftarrow \sum_{i=1}^{m} K_i$;
7. $c \leftarrow \text{hash}(x_K || Q || m)$;

c

1: Alice

2: Bob

3: Charlotte

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8. $s_1 \leftarrow k_1 + c a_1 q_1 \pmod{n}$;

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2: Bob

3: Charlotte

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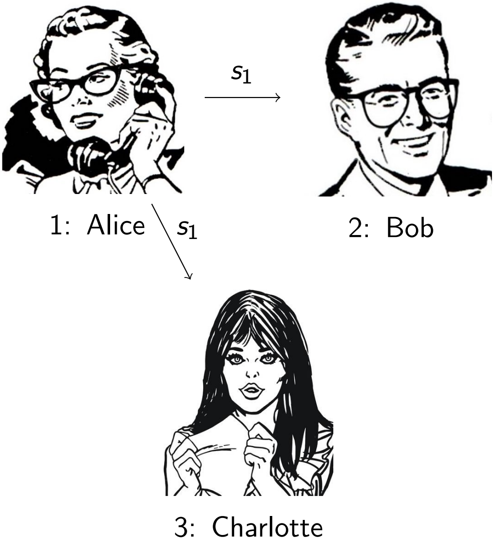

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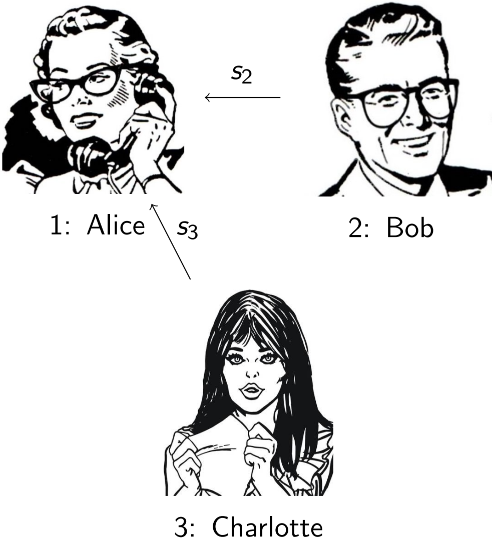

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9. **send** s_1;
10. $s \leftarrow \sum_{i=1}^{m} s_i \pmod{n}$;

1: Alice

2: Bob

3: Charlotte

MuSig: compact m-of-m signature scheme

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7. $c \leftarrow \text{hash}(x_K || Q || m)$;
8. $s_1 \leftarrow k_1 + c a_1 q_1 \pmod{n}$;
9. **send** s_1;
10. $s \leftarrow \sum_{i=1}^{m} s_i \pmod{n}$;
11. **return** (x_K, s).

1: Alice

(x_K, s)

2: Bob

3: Charlotte

MuSig: compact m-of-m signature scheme

MuSig ($\mu\Sigma$):

- Compact: same size as the single user case;

1: Alice

2: Bob

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- Secure in the plain public key model: cross input aggregation at transaction level;

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MuSig: compact m-of-m signature scheme

MuSig ($\mu\Sigma$):

- Compact: same size as the single user case;
- Secure in the plain public key model: cross input aggregation at transaction level;
- Key aggregation: signature indistinguishable from the single user case.

1: Alice

2: Bob

3: Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

Alice

Bob

Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

↓

Protocol for the generation of a random shared secret

Alice

Bob

Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

↓

Protocol for the generation of a random shared secret

↓

Threshold scheme

Alice

Bob

Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The dealer:

Dealer: Alice

1: Bob

2: Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The dealer:

- generates secret s and $s' \xleftarrow{\$} \{1, ..., n-1\}$;

Dealer: Alice

s, s'

1: Bob

2: Charlotte

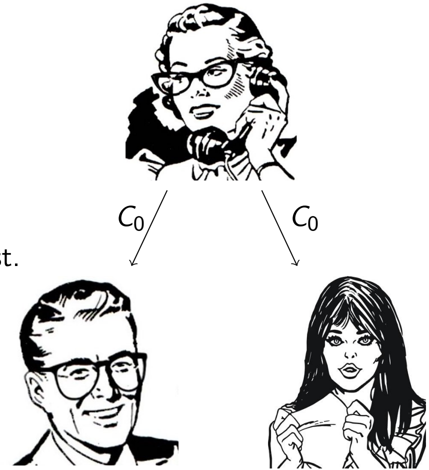
Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The dealer:

- generates secret s and $s' \xleftarrow{\$} \{1, \ldots, n-1\}$;
- commits to them through the Pedersen commitment $C_0 = sG + s'H$: C_0 is broadcast.

Dealer: Alice



1: Bob

2: Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The dealer:

- chooses random polynomials:
  $f(u) = s + f_1 u + ... + f_{t-1} u^{t-1}$,
  $f'(u) = s' + f'_1 u + ... + f'_{t-1} u^{t-1}$,
  $f_j,\ f'_j \xleftarrow{\$} \{1, ..., n-1\}$;

Dealer: Alice

f, f'

1: Bob

2: Charlotte

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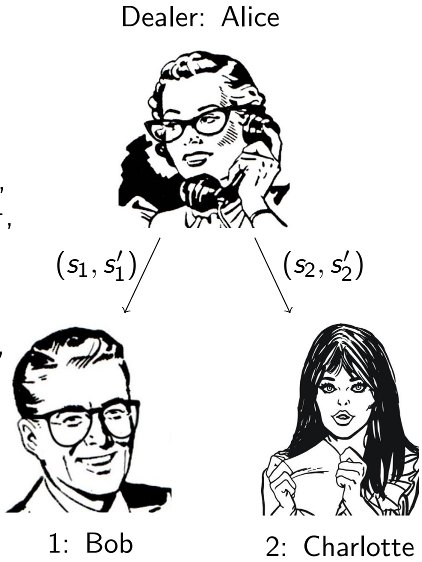
- computes $(s_i, s'_i) = (f(i) \pmod{n}, f'(i) \pmod{n})$, $i \in \{1, ..., m\}$ and sends them secretly to P_i;



Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

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- computes $(s_i, s'_i) = (f(i) \pmod{n}, f'(i) \pmod{n})$, $i \in \{1, ..., m\}$ and sends them secretly to P_i;
- broadcasts the commitment to the sharing polynomials:
  $C_j = f_j G + f'_j H$,
  $j \in \{1, ..., t-1\}$.

Dealer: Alice

1: Bob

2: Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The participants:

Dealer: Alice

1: Bob

2: Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The participants:

- verify the consistency of their shares of secret: $s_i G + s_i' H = \sum_{j=0}^{t-1} i^j C_j$;

Dealer: Alice

1: Bob

2: Charlotte

Threshold signature scheme (t-of-m)

Verifiable secret sharing scheme

The participants:

- verify the consistency of their shares of secret: $s_i G + s_i' H = \sum_{j=0}^{t-1} i^j C_j$;
- to reconstruct the secret they rely on Lagrange's interpolation formula:
  $f(u) = \sum_i f(i)\omega_i(u)$, where
  $\omega_i(u) = \prod_{j \neq i} \frac{u-j}{i-j} \pmod{n}$.
  $s = f(0) = \sum_i s_i \omega_i$, with
  $\omega_i = \omega_i(0) = \prod_{j \neq i} \frac{j}{j-i} \pmod{n}$.

Dealer: Alice

$\overset{s}{\longleftrightarrow}$

1: Bob

2: Charlotte

Threshold signature scheme (t-of-m)

Protocol for the generation of a random shared secret

Each participant:

1: Alice

2: Bob

3: Charlotte

Threshold signature scheme (t-of-m)

Protocol for the generation of a random shared secret

Each participant:

- acts as the dealer in the previous protocol ($f_i(u) = \sum_{j=0}^{t-1} a_{ij} u^j$, $a_{i0} = r_i$);

1: Alice

r_1, f_1

r_2, f_2

2: Bob

r_3, f_3

3: Charlotte

Threshold signature scheme (t-of-m)

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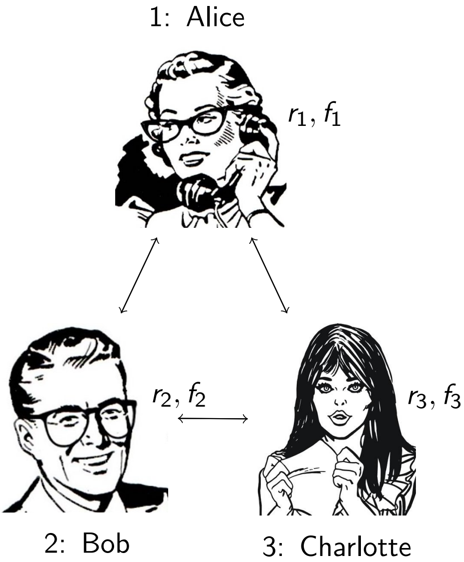

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- the shared secret is $r = \sum_{i=1}^{m} r_i \pmod{n}$ with shares $s_i = \sum_{j=1}^{m} f_j(i) \pmod{n}$;

1: Alice

s_1

s_2

2: Bob

s_3

3: Charlotte

Threshold signature scheme (t-of-m)

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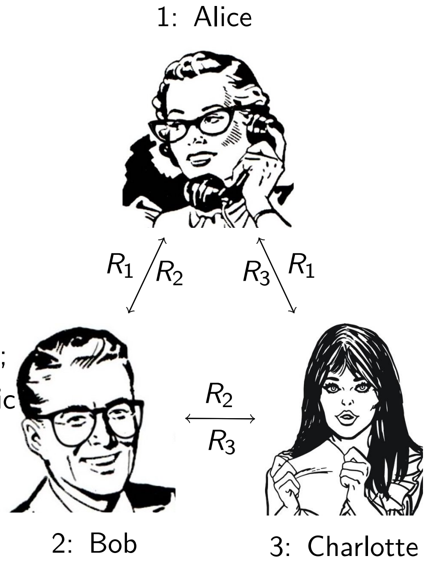
- the shared secret is $r = \sum_{i=1}^{m} r_i \pmod{n}$ with shares $s_i = \sum_{j=1}^{m} f_j(i) \pmod{n}$;
- broadcast his share of the public key $R_j = r_j G$ ($R = \sum_{j=1}^{m} R_j = \sum_{j=1}^{m} r_j G = rG$).



Threshold signature scheme (t-of-m)

Threshold scheme

After having established a distributed key pair $(\alpha_1, \ldots, \alpha_m) \xleftrightarrow{\text{(t, m)}} (q|Q)$ through the protocol for the generation of a random shared secret (that acts as key generation protocol) the signers:

1: Alice

α_1

$\updownarrow$

α_3

3: Charlotte

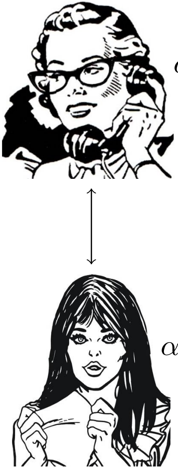
Threshold signature scheme (t-of-m)

Threshold scheme

After having established a distributed key pair $(\alpha_1, ..., \alpha_m) \xleftrightarrow{\text{(t, m)}} (q|Q)$ through the protocol for the generation of a random shared secret (that acts as key generation protocol) the signers:

- run again the same protocol to produce a nonces pair: $(\beta_1, ..., \beta_m) \xleftrightarrow{\text{(t, m)}} (k|K)$.

1: Alice



α_1, β_1

α_3, β_3

3: Charlotte

Threshold signature scheme (t-of-m)

Threshold scheme

Then each signer i:

- checks whether jacobi$(y_K) \neq 1$; if it is the case she sets $\beta_i = n - \beta_i$;

1: Alice

3: Charlotte

Threshold signature scheme (t-of-m)

Threshold scheme

Then each signer i:

- checks whether $\mathrm{jacobi}(y_K) \neq 1$;
  if it is the case she sets
  $\beta_i = n - \beta_i$;
- reveals her partial signature:
  $\gamma_i = \beta_i + e\alpha_i \pmod{n}$, with
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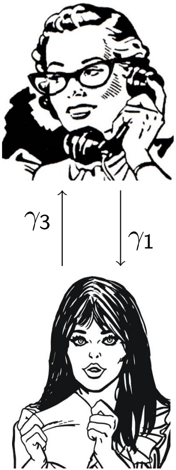
1: Alice



3: Charlotte

Threshold signature scheme (t-of-m)

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- computes $\sigma = \sum_{j=1}^{t} \gamma_j \omega_j \pmod{n}$, with $\omega_j = \prod_{h \neq j} \frac{h}{h-j} \pmod{n}$: σ is such that $\sigma = k + eq \pmod{n}$;

1: Alice

σ

σ

3: Charlotte

Threshold signature scheme (t-of-m)

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- the signature is (x_K, σ).

1: Alice

(x_K, σ)

(x_K, σ)

3: Charlotte

ECDSA vs. ECSSA (multi-signature)

ECDSA:

- Locking script: t <pubKey1> <pubKey2> ... <pubKeym> m OP_CHECKMULTISIG
- Unlocking script: 0 <sig1> <sig2> ... <sigt>

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ECSSA:

- Locking script: <jointPubKey> OP_SCHNORR
- Unlocking script: <jointSig>

Adaptor signatures

Building block for *scriptless script*: aim at encapsulating the flexibility of script semantics in fixed size signatures.

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How?

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How? The idea is to add to the public nonce K a random $T = tG$ but still consider k as private nonce: this results in an invalid signature, however learning t is equivalent to learning a valid signature.

Adaptor signatures

Building block for *scriptless script*: aim at encapsulating the flexibility of script semantics in fixed size signatures.

How? The idea is to add to the public nonce K a random $T = tG$ but still consider k as private nonce: this results in an invalid signature, however learning t is equivalent to learning a valid signature.

If t is some necessary data for the execution of a separate protocol, arbitrary steps of arbitrary protocols can be made equivalent to signature production.

Cross-chain atomic swaps

Exchange of different crypto-currencies among two distrustful users in an atomic and decentralized way.

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    OP_HASH160 <Bob address>
OP_ELSE
    <num> OP_CHECKSEQUENCEVERIFY OP_DROP
    OP_DUP OP_HASH160 <Alice addres>
OP_ENDIF
OP_EQUALVERIFY OP_CHECKSIG
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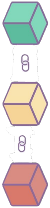
OP_ENDIF
OP_EQUALVERIFY OP_CHECKSIG
```

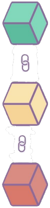
Easily identifiable (lack of privacy) and cumbersome (high fees).

Cross-chain atomic swaps via adaptor signatures

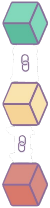


Alice

Bob



Cross-chain atomic swaps via adaptor signatures

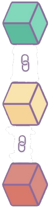


$Q_A^{฿}, Q_A^{\Xi}$

Alice

$Q_B^{฿}, Q_B^{\Xi}$

Bob



Cross-chain atomic swaps via adaptor signatures