CNM/Cold Neutral Medium (neutral atomic H), 21CM line

1

The splitting $C^+ : {}^2P^0_{1/2} \; ; \; {}^2P^0_{3/2}$

Low dense $\left\{ \begin{array}{l} \text{excitation : collision with } e^- \\ \text{de-exc : emission} \end{array} \right.$

$A_{10} = 2,4 \times 10^{-6} \, s^{-1}$

$\lambda_{10} = 157,74 \, \mu m$

$k_{10} = 4,53 \times 10^{-8} T_4^{-1/2} \, cm^3/s$

→ C^+ main e^- source ; $N_1(n_e, T_{gas})$

In the equilibrium we have that

(excit) (dexcit)

$n_0 \cdot n_e \cdot k_{01} = n_1 A_{10}$

Where $n_c = n_0 + n_1$

As C^+ is main source of e^- $n_c = n_e$

So then $n_e(n_e - n_1) k_{01} = n_1 A_{10}$

$$n_1 = \frac{n_e^2 k_{01}}{A_{10} + n_e k_{01}}$$

We have that $\left\{ \begin{array}{l} k_{01} = \dfrac{g_1}{g_0} k_{10} \, e^{-\Delta E/kT} \\ k_{10} = 4,53 \times 10^{-6} / \sqrt{T} \end{array} \right.$ where $\left\{ \begin{array}{l} g_1 = 2J+1 = 4 \\ g_0 = 2J+1 = 2 \end{array} \right.$

So then we have $\left\{ \begin{array}{l} k_{01} = \dfrac{4}{2} \cdot 4,53 \times 10^{-6} / \sqrt{T} \cdot e^{-91,21/T} \\ \Delta E = \dfrac{hc}{\lambda} = 7,8 \times 10^{-3} \, eV \to \Delta E \cdot k = 91,21 \, K \end{array} \right.$

Then replace into eq. for n_1 → $n_1 = \dfrac{n_e^2}{n_e + 0,265 \cdot \sqrt{T} \cdot e^{91,21/T}}$

The energy radiated per $(cm^{-3} s^{-1})$ in the 157,74 μm line is:

$$\Lambda = n_1 \cdot h\nu \cdot A_{10} = 2,32 \times 10^{-26} \, erg/cm^3/s$$

(2)

H hyperfine excitation

$\sigma = 5\times10^{-16}\ cm^{-2}$

$n_H \sim 100 cm^{-3}$
$T \sim 100K$

We know that $K_{ji} = \langle u\sigma_{ji} \rangle$ → collisional exc. coef. ($cm^3 s^{-1}$) units?
↳ coll. cross section (cm^2)

For 2 levels upper and lower
(u) (l)

$$k_{lu} = \frac{g_u}{g_l} k_{ul}\, e^{-\Delta E/k_B T}$$

We know that $\langle u \rangle = \left(\frac{8 k_B T}{\pi m_r}\right)^{1/2}$ → velocity

↳ $m_H/2$, red. mass

$m_r = \frac{m_c \cdot m_T}{m_c + m_T}$

Then we have that $k_{lu} \sim 1{,}025\times10^{-11}\ cm^3/s$

From Boltzmann eq. we have that → $k_{ul} = \frac{g_l}{g_u} \cdot k_{lu} \cdot e^{\Delta E/k_B T}$

$\left(\begin{array}{l} g_u = 3 \\ g_l = 1 \end{array} \quad \Delta E = \frac{hc}{\lambda} = 5{,}9\times10^{-6}\,eV \right)$ $\sim 3{,}419\times10^{-11}\ cm^3/s$

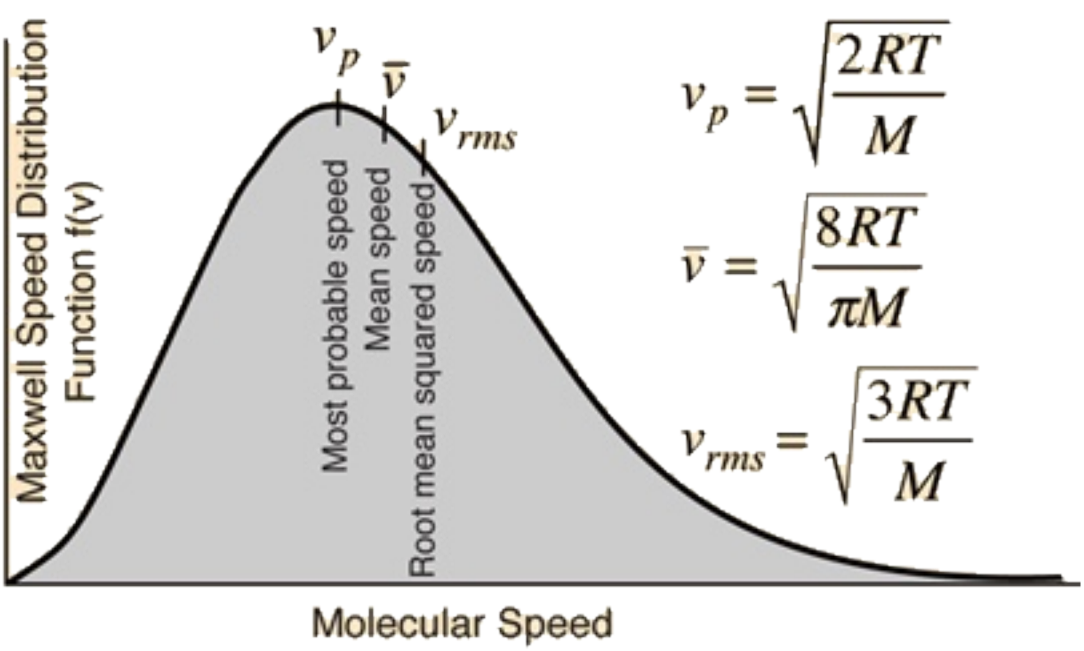
So then → $\frac{n_u}{n_l} \sim \frac{g_u}{g_l} \cdot e^{-\Delta E/k_B T} \sim 2{,}997 \sim 3$

$n(H) = n_u + n_l = 4 \cdot n_l$ $\left\{ \begin{array}{l} n_u = 75\ cm^{-3} \\ n_l = 25\ cm^{-3} \end{array} \right.$

The rates are $\left\{ \begin{array}{ll} \text{exc} \to & k_{lu} \cdot \underbrace{n_l}_{25\,cm^{-3}} \sim 2{,}56\times10^{-9}\ s^{-1} \\ \text{de-exc} \to & k_{ul} \cdot \underbrace{n_u}_{75\,cm^{-3}} \sim 2{,}56\times10^{-9}\ s^{-1} \end{array} \right.$

<u> is the mean molecular speed of the Maxwellian velocity distribution for the particles (target+collider), see next page or (https://farside.ph.utexas.edu/teaching/sm1/Thermalhtml/node87.html)

$$f(v) = 4\pi \left[\frac{M}{2\pi RT}\right]^{\frac{3}{2}} v^2 \exp\left[\frac{-Mv^2}{2RT}\right]$$



$T = 8000K$ $\quad$ $\beta(CaI = 3{,}8 \times 10 \; s^{-}$ / $\alpha_R(CaI) = 8{,}0 \times 10 \; cm^3/s$

③ $\quad$ $n_e = 10^3 cm^{-3}$ $\quad$ $\beta(CaII) = 4{,}0 \times 10^{-12} \; s^{-1}$ / $\alpha_R(CaII) = 4{,}28 \times 10^{-12} \; cm^3/s$

Eq. conditions: $n(X^r)\beta_{1,ph} = n(X^{r+1}) n_e \sum_j \alpha_j$

$\hookrightarrow$ photoioniz. $\qquad \hookrightarrow$ recomb.

$$① \quad n(CaI)\beta(CaI) = n(CaII) n_e \alpha_R(CaI)$$

$$② \quad n(CaII)\beta(CaII) = n(CaIII) n_e \alpha_R(CaII)$$

$$③ \quad n(CaI) + n(CaII) + n(CaIII) = n(Ca)$$

We can rewrite $\dfrac{n(CaI)}{n(Ca)} + \dfrac{n(CaII)}{n(Ca)} + \dfrac{n(CaIII)}{n(Ca)} = 1$

$$\varepsilon_I = 1 - \varepsilon_{II} - \varepsilon_{III} \quad ④$$

From ① $\quad \dfrac{n(CaI)}{n(Ca)} = \dfrac{n(CaII)}{n(Ca)} \cdot \dfrac{n_e \alpha_R(CaI)}{\beta(CaI)}$

$$\varepsilon_I = \varepsilon_{II} \frac{n_e \alpha_R(CaI)}{\beta(CaI)} \quad ⑤$$

$$\varepsilon_{II} = \varepsilon_{III} \frac{n_e \alpha_R(CaII)}{\beta(CaII)} \quad ⑥$$

Replacing ④ in ⑤ $\quad 1 - \varepsilon_{III} = \varepsilon_{II}\left(1 + \dfrac{n_e \alpha_R(CaI)}{\beta(CaI)}\right)$

$$\varepsilon_{II} = (1 - \varepsilon_{III})\left(1 + \frac{n_e \alpha_R(CaI)}{\beta(CaI)}\right)^{-1} \quad ⑦$$

And ⑦ into ⑥

$$(1 - \varepsilon_{III})\left(1 + \frac{n_e \alpha_R(CaI)}{\beta(CaI)}\right)^{-1} = \varepsilon_{III} \frac{n_e \alpha_R(CaII)}{\beta(CaII)}$$

Then $\quad (1 - \varepsilon_{III}) \cdot 0{,}299 = \varepsilon_{III} \cdot 1070 \rightarrow \varepsilon_{III} = 2{,}794 \times 10^{-4}$

Ans (6) $\varepsilon_{II} = 0{,}299$ /

(4) $\varepsilon_{I} = 0{,}7$ /

LTE applies inside the stars, NOT in the dilute ISM
Local thermodynamic equilibrium (LTE) requires all species, including ions, electrons neutrals, and photons collide with each other sufficiently frequently.)

④

The absorption of a line with optical depth τ is $I_\nu = I_\nu^0 \cdot e^{-\tau_\nu}$

Then we have that $\tau = T_{spin}^{-1} \cdot 1{,}38\,K$ (τ is inv. prop to T_{spin} so $\uparrow T_{spin}$ $\downarrow \tau$, more absorption)

We know that $\dfrac{I_\nu - I_{\nu_0}}{I_{\nu_0}} < 0{,}01$

$\downarrow$

$e^{-\tau_\nu} - 1 < 0{,}01 \rightarrow e^{-\tau_\nu} < 1{,}01$

From that we have $-\tau_\nu < \ln(1{,}01)$

$-1{,}38/T < \ln(1{,}01)$

$T > -1{,}38/\ln(1{,}01)$ $\boxed{T > 138{,}7\,K}$