Problem Set 4 - Interstellar Medium 2023-2024 IIa

February 22, 2024

1. Consider the lowest energy level of C^+ and its fine structure splitting $^2P^{\rm o}_{1/2}$ and $^2P^{\rm o}_{3/2}$. In a low density environment like the CNM, the excitation will be driven by collisions, while de-excitation due to spontaneous emission ($A_{10} = 2.4 \times 10^{-6}$ s^{-1}, $\lambda_{10} = 157.74\,\mu m$) will dominate over collisional de-excitation. Assume excitation is mainly due to electrons and that the de-excitation rate coefficient at low temperature ($T < 100$ K) is $k_{10} = 4.53 \times 10^{-8} T_4^{-0.5}$ cm^{3}s^{-1}.

   a) Derive an approximation for the upper level population in terms of the electron density and gas temperature if carbon is the main electron source in the gas.

   b) Suppose the gas has a temperature of 50 K and electron density of 0.003 cm^{-3}. What is the amount of energy radiated away by C^+ per cm^3 per second in the $157.74\,\mu m$ fine-structure line?

2. Hydrogen hyperfine level excitation is a process where the electron spin is reversed by collisions with H atoms. Suppose the hyperfine excitation cross section for Hydrogen is constant at low temperatures ($T \sim 100$ K) and given as $\sigma = 5 \times 10^{-16}$ cm^2[1]. Assume $n_{\rm H} \sim 100\,{\rm cm}^{-3}$. Calculate the rate for hyperfine de-excitation and compare it to the excitation rate assuming detailed balance.

3. Consider a situation where ionization originates exclusively from photon absorption (photoionization) and recombination is driven by radiative processes (i.e. a free electron is captured by an ion to form an atom in a certain excited state j). In general, only the ground state is populated in the interstellar medium and photoionizations are made essentially from that level. However, recombinations can be done for every level j of the atom, followed by decay until the ground level is reached. When photoionization balances recombination, the equilibrium conditions can be written

$$n({\rm X}^r)\beta_{1,{\rm ph}} = n({\rm X}^{r+1})\ n_e \sum_j \alpha_j. \tag{1}$$

   a) Estimate the fraction of ions Ca I, Ca II, and Ca III in an interstellar cloud with $T = 8000$ K and $n_e = 10^3$ cm^{-3}, assuming only photoionization and radiative recombination. Given: β(CaI)$= 3.8 \times 10^{-10}$ s^{-1}, β(CaII)$= 4.0 \times 10^{-12}$ s^{-1}, α_R(CaI)$= 8.90 \times 10^{-13}$ cm^3 s^{-1}, and α_R(CaII)$= 4.28 \times 10^{-12}$ cm^3 s^{-1}. In all cases, assume that α_R, is the total recombination coefficient, $\alpha_R = \sum_j \alpha_j$ and that β is the photoionization rate from the ground level.

   b) Estimate the ratio of Ca II to Ca I using the Saha equation ($g_{1,1} = 1$, $g_{2,1} = 2$) for the same conditions. Compare your answer to the above and explain your finding.

4. Let $dN({\rm H\,I})/dv \times \Delta v$ be the column density of H I in the radial velocity interval Δv. The optical depth in the 21-cm line can be written as

$$\tau = \frac{3}{32\pi} A_{ul} \frac{hc\lambda^2}{kT_{\rm spin}} \frac{dN({\rm H\,I})}{dv}$$

$$= 0.552\Big(\frac{100}{T_{\rm spin}}\Big) \frac{dN({\rm H\,I})/dv}{10^{20}\,{\rm cm}^{-2}/({\rm km\,s}^{-1})}$$

   Now, suppose we observe a background radio continuum point source through a layer of "foreground" H I with $dN({\rm H\,I})/dv = 5 \times 10^{19}\,{\rm cm}^{-2}/(20\,{\rm km\,s}^{-1})$, where v is the radial velocity. If the

[1] The collisional excitation cross section relates to the collisional excitation coefficient via $k_{\rm ji} = \langle v\sigma_{\rm ji}\rangle$ where v is the relative velocity of the colliding particles

measured flux density of the background continuum source changes by less than 1% on-line to off-line[2], what can be said about the spin temperature of the H I? Assume the beamsize is very small.

[2]The on-line to off-line change in the flux density has to be interpreted as the change in the flux value when it is measured at the center of the absorption line and at the continuum next to the line, respectively.