Kyle Bishop Quiz 1 - Assignment 4 002-24-8328

1 $\begin{bmatrix} x_{cam} \\ y_{cam} \\ w \end{bmatrix} = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \left[\begin{array}{c|c} R & t \end{array}\right] \cdot \begin{bmatrix} x_{world} \\ y_{world} \\ z_{world} \\ 1 \end{bmatrix}$

Camera points — Intrinsic Matrix — Extrinsic Matrix — World points

2 a) $R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ $R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ $R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $R_x \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}$ R_y, $\theta = 60°$ $\begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{bmatrix}$ =

$(1,1) = 1 \cdot \frac{1}{2} + 0 \cdot 0 + 0 \cdot (-\frac{\sqrt{3}}{2}) = \frac{1}{2}$
$(1,2) = 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 = 0$
$(1,3) = 1 \cdot \frac{\sqrt{3}}{2} + 0 \cdot 0 + 0 \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$
$(2,1) = 0 \cdot \frac{1}{2} + \frac{1}{2} \cdot 0 + (-\frac{\sqrt{3}}{2}) \cdot (\frac{\sqrt{3}}{2}) = \frac{3}{4}$
$(2,2) = \frac{1}{2}$
$(2,3) = -\frac{\sqrt{3}}{4}$
$(3,1) = -\frac{\sqrt{3}}{4}$
$(3,2) = \frac{\sqrt{3}}{2}$
$(3,3) = \frac{1}{4}$

$R_x R_y$ $\begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ \frac{3}{4} & \frac{1}{2} & -\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} & \frac{1}{4} \end{bmatrix}$ R_z $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ =

$(1,1) = \frac{1}{4}$
$(1,2) = -\frac{\sqrt{3}}{4}$
$(1,3) = \frac{\sqrt{3}}{2}$
$(2,1) = \frac{3}{8} + \frac{\sqrt{3}}{4} = \frac{3+2\sqrt{3}}{8}$
$(2,2) = -\frac{3\sqrt{3}}{8} + \frac{1}{4} = \frac{2-3\sqrt{3}}{8}$
$(2,3) = -\frac{\sqrt{3}}{4}$
$(3,1) = -\frac{\sqrt{3}}{8} + \frac{3}{4} = \frac{6-\sqrt{3}}{8}$
$(3,2) = \frac{3}{8} + \frac{\sqrt{3}}{4} = \frac{3+2\sqrt{3}}{8}$
$(3,3) = \frac{1}{4}$

$R_x R_y R_z$ $\begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} \\ \frac{3+2\sqrt{3}}{8} & \frac{2-3\sqrt{3}}{8} & -\frac{\sqrt{3}}{4} \\ \frac{6-\sqrt{3}}{8} & \frac{3+2\sqrt{3}}{8} & \frac{1}{4} \end{bmatrix}$ R_y $\begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{bmatrix}$ =

$(1,1) = \frac{1}{4} \cdot \frac{1}{2} + (-\frac{\sqrt{3}}{4}) \cdot 0 + \frac{\sqrt{3}}{2} \cdot (\frac{\sqrt{3}}{2}) = \frac{1}{8} + \frac{3}{4} = \frac{5}{8}$
$(1,2) = \frac{1}{4} \cdot 0 + (-\frac{\sqrt{3}}{4}) \cdot 1 + \frac{\sqrt{3}}{2} \cdot 0 = -\frac{\sqrt{3}}{4}$
$(1,3) = \frac{1}{4} \cdot \frac{\sqrt{3}}{2} + 0 + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{8}$
$(2,1) = \frac{3+2\sqrt{3}}{8} \cdot \frac{1}{2} + \frac{2-3\sqrt{3}}{8} \cdot 0 - \frac{\sqrt{3}}{4} \cdot (\frac{\sqrt{3}}{2}) = \frac{3+2\sqrt{3}}{16} + \frac{3}{8} = \frac{9+2\sqrt{3}}{16}$
$(2,2) = 0 + \frac{2-3\sqrt{3}}{8} \cdot 1 + 0 = \frac{2-3\sqrt{3}}{8}$
$(2,3) = \frac{3+2\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} + \frac{2-3\sqrt{3}}{8} \cdot 0 + (-\frac{\sqrt{3}}{4}) \cdot \frac{1}{2} = \frac{2+3\sqrt{3}}{16} - \frac{\sqrt{3}}{8} = \frac{2+\sqrt{3}}{16}$
$(3,1) = \frac{6-\sqrt{3}}{8} \cdot \frac{1}{2} + 0 + \frac{1}{4} \cdot (-\frac{\sqrt{3}}{2}) = \frac{6-\sqrt{3}}{16} - \frac{\sqrt{3}}{8} = \frac{6-3\sqrt{3}}{16}$
$(3,2) = 0 + \frac{3+2\sqrt{3}}{8} \cdot 1 + 0 = \frac{3+2\sqrt{3}}{8}$
$(3,3) = \frac{6-\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} + 0 + \frac{1}{4} \cdot \frac{1}{2} = \frac{6\sqrt{3}-3}{16} + \frac{1}{8} = \frac{6\sqrt{3}-1}{16}$

$R_x R_y R_z R_y$ $\begin{bmatrix} -\frac{5}{8} & -\frac{\sqrt{3}}{4} & \frac{3\sqrt{3}}{8} \\ \frac{9+2\sqrt{3}}{16} & \frac{2-3\sqrt{3}}{8} & \frac{2+3\sqrt{3}}{16} \\ \frac{6-3\sqrt{3}}{16} & \frac{3+2\sqrt{3}}{8} & \frac{6\sqrt{3}-1}{16} \end{bmatrix}$

$x = \theta = \cos^{-1}\left(-\frac{5}{8}\right), y = \cos^{-1}\left(\frac{2-3\sqrt{3}}{8}\right), z = \cos^{-1}\left(\frac{6\sqrt{3}-1}{16}\right)$

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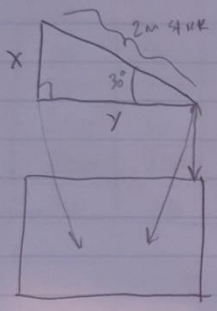
3 a) ${}^{c}p = {}^{c}R_{w}\,{}^{w}p + {}^{c}t_{\rightarrow w}$

$${}^{c}p = \begin{bmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{bmatrix} {}^{w}p + \begin{bmatrix} 9 \\ 0 \\ -4 \end{bmatrix}$$

b)



$\frac{x}{2m} = \sin(30^\circ)$ $\frac{y}{2m} = \cos(30^\circ)$

$x = 2\sin(30^\circ)$ $y = 2\cos(30^\circ)$

$x' = \frac{fx}{z}$ $y' = \frac{fy}{z}$

$x' = \frac{(1000)(2\sin(30))}{4}$

$y' = \frac{(1000)(2\cos(30))}{4}$

Size of stick = x' x y' pixels

4 a) principal point = the origin of the camera in reference to the real world point (O_x, O_y)

b) $M = M_{\text{intrinsic}} \cdot M_{\text{extrinsic}}$

$$\begin{bmatrix} -f/s_x & 0 & O_x \\ 0 & f/s_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \left[\begin{array}{c:c} R & t \end{array}\right]$$

intrinsic extrinsic

f = focal length
(s_x, s_y) = size of pixels
(O_x, O_y) = principle point
R = rotation matrix
t = translation vector

Kyle Bishop Quiz 2 - Assignment 4 CSCI-24-8328

1. a)

Fig 1

250	50	250	50	250
50	250	50	250	50
250	50	250	50	250
50	250	50	250	50
250	50	250	50	250

Fig 2

250	255	250	240	250
101	98	102	120	240
250	110	250	99	250
250	250	101	100	250
250	255	250	255	250

Fig 3

50	50	50	50	50
50	250	250	250	50
50	250	10	250	50
50	250	250	250	50
50	50	50	50	50

AVG

183.33	116.33	183.33	113.33	183.33
67	199.33	134	206.66	113.33
183.33	136.66	170	133	183.33
116.66	250	133.66	200	116.66
183.33	116.33	183.33	116.33	183.33

3x3 Gaussian Kernel to $\sigma=1$

0.0585	0.0965	0.0585
0.0965	0.159	0.0965
0.0585	0.0965	0.0585

(row, column)

Convolve around AVG

(1,1) = 183.33 · 0.159 + 0.0965 · 116.33 + 67 · 0.0965 + 199.33 · 0.0585 =
= 29.15 + 11.42 + 6.47 + 11.66 = 58.7

(1,2) = 183.33 · 0.0965 + 0.159 · 116.33 + 0.0965 · 183.33 + 67 · 0.0585 + 199.33 · 0.0965 + 134 · 0.0585 =
= 17.69 + 11.42 + 17.69 + 3.92 + 19.24 + 7.84 = 77.8

(1,3) = 116.33 · 0.0965 + 183.33 · 0.159 + 113.33 · 0.0965 + 199.33 · 0.0585 + 134 · 0.0965 + 206.66 · 0.0585 =
= 11.42 + 29.15 + 10.94 + 11.66 + 12.93 + 12.09 = 88.19

(1,4) = 183.33 · 0.0965 + 113.33 · 0.159 + 183.33 · 0.0965 + 134 · 0.0585 + 206.66 · 0.0965 + 113.33 · 0.0585 =
= 17.69 + 18.02 + 17.69 + 7.84 + 19.94 + 6.63 = 88.03

(1,5) = 113.33 · 0.0965 + 183.33 · 0.159 + 206.66 · 0.0585 + 113.33 · 0.0965 =
= 10.94 + 29.15 + 12.09 + 10.94 = 63.12

(2,1) = 183.33 · 0.0965 + 116.33 · 0.0585 + 67 · 0.159 + 199.33 · 0.0965 + 183.33 · 0.0965 + 136.66 · 0.0585 =
= 17.69 + 6.92 + 10.65 + 19.24 + 17.69 + 7.99 = 80.18

(2,2) = 183.33 · 0.0585 + 116.33 · 0.0965 + 183.33 · 0.0585 + 67 · 0.0965 + 199.33 · 0.159 + 134 · 0.0965
+ 183.33 · 0.0585 + 136.66 · 0.0965 + 170 · 0.0585 = 10.72 + 11.42 + 10.72 + 6.47
+ 31.69 + 12.93 + 10.72 + 13.19 + 9.95 = 117.81

(2,3) = 116.33 · 0.0585 + 183.33 · 0.0965 + 113.33 · 0.0585 + 199.33 · 0.0965 + 134 · 0.159 + 206.66 · 0.0965
+ 136.66 · 0.0585 + 170 · 0.0965 + 133 · 0.0585 = 6.92 + 17.69 + 6.63 + 19.24 + 21.31 + 19.94
+ 7.99 + 16.41 + 7.78 = 123.91

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(2,4) = 183.33 · 0.0585 + 113.33 · 0.0965 + 183.33 · 0.0585 + 134 · 0.0965 + 206.66 · 0.159
10.72 + 10.94 + 10.72 + 12.93 + 32.86
+ 113.33 · 0.0965 + 170 · 0.0585 + 133 · 0.0965 + 183.33 · 0.0585 =
+ 10.94 + 9.95 + 12.83 + 10.72 = 122.66

(2,5) = 113.33 · 0.0585 + 183.33 · 0.0965 + 206.66 · 0.0965 + 113.33 · 0.159 + 133 · 0.0585 + 183.33 · 0.0965
= 6.63 + 17.69 + 19.94 + 18.02 + 7.78 + 17.69 = 87.75

(3,1) = 67 · 0.0965 + 199.33 · 0.0585 + 183.33 · 0.159 + 136.66 · 0.0965 + 116.66 · 0.0965 + 136.66 · 0.0585 =
6.47 + 11.66 + 29.15 + 13.19 + 11.26 + 7.99 = 79.72

(3,2) = 67 · 0.0585 + 199.33 · 0.0965 + 134 · 0.0585 + 183.33 · 0.0965 + 136.66 · 0.159 + 170 · 0.0965 + 116.66 · 0.0585
3.92 + 19.24 + 7.84 + 17.69 + 21.73 + 16.41 + 6.82
+ 250 · 0.0965 + 136.66 · 0.0585 = 155.77
24.13 + 7.99

(3,3) = 199.33 · 0.0585 + 134 · 0.0965 + 206.66 · 0.0585 + 136.66 · 0.0965 + 170 · 0.159 + 133 · 0.0965
11.66 + 12.93 + 12.09 + 13.19 + 27.03 + 12.83
+ 250 · 0.0585 + 133.66 · 0.0965 + 200 · 0.0585 = 128.9
14.63 12.9 11.7

(3,4) = 134 · 0.0585 + 206.66 · 0.0965 + 113.33 · 0.0585 + 170 · 0.0965 + 133 · 0.159 + 183.33 · 0.0965
7.84 + 19.94 + 6.63 + 16.41 + 21.15 + 17.69
+ 133.66 · 0.0585 + 200 · 0.0965 + 116.66 · 0.0585 = 123.6
7.82 + 19.3 + 6.82

(3,5) = 206.66 · 0.0585 + 113.33 · 0.0965 + 133 · 0.0965 + 183.33 · 0.159 + 200 · 0.0585 + 116.66 · 0.0965
12.09 + 10.94 + 12.83 + 29.15 + 11.7 + 11.26 = 87.97

(4,1) = 183.33 · 0.0965 + 136.66 · 0.0585 + 116.66 · 0.159 + 250 · 0.0965 + 183.33 · 0.0965 + 183.33 · 0.0585 =
17.69 + 7.82 + 18.55 + 24.13 + 17.89 + 6.92 = 92.8

(4,2) = 183.33 · 0.0585 + 136.66 · 0.0965 + 170 · 0.0585 + 116.66 · 0.0965 + 250 · 0.159 + 133.66 · 0.0965
10.72 + 13.19 + 9.95 + 11.26 + 39.75 + 12.9
+ 183.33 · 0.0585 + 113.33 · 0.0965 + 183.33 · 0.0585 = 130.63
10.72 11.42 10.72

(4,3) = 136.66 · 0.0585 + 170 · 0.0965 + 133 · 0.0585 + 250 · 0.0965 + 133.66 · 0.159 + 200 · 0.0965
7.82 + 16.41 + 7.78 + 24.13 + 21.25 + 19.3
+ 113.33 · 0.0585 + 183.33 · 0.0965 + 113.33 · 0.0585 = 128.22
6.92 17.69 6.92

(4,4) = 170 · 0.0585 + 133 · 0.0965 + 183.33 · 0.0585 + 133.66 · 0.0965 + 200 · 0.159 + 116.66 · 0.0965
9.95 + 12.83 + 10.72 + 13.19 + 31.8 + 11.26
+ 183.33 · 0.0585 + 113.33 · 0.0965 + 183.33 · 0.0585 = 122.61
10.72 11.42 10.72

(4,5) = 133 · 0.0585 + 183.33 · 0.0965 + 200 · 0.0965 + 116.66 · 0.159 + 113.33 · 0.0585 + 183.33 · 0.0965
= 7.78 + 17.69 + 19.3 + 18.55 + 6.92 + 17.69 = 87.93

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$(5,1) = 116.66 \cdot 0.0965 + 250 \cdot 0.0585 + 183.33 \cdot 0.159 + 116.33 \cdot 0.965 =$
$11.26 + 14.625 + 29.15 + 11.42 = 66.46$

$(5,2) = 116.66 \cdot 0.0585 + 250 \cdot 0.0965 + 133.66 \cdot 0.0585 + 183.33 \cdot 0.0965 + 116.33 \cdot 0.159 + 183.33 \cdot 0.0965 =$
$6.82 + 24.13 + 7.82 + 17.69 + 16.81 + 17.69 = 92.96$

$(5,3) = 250 \cdot 0.0585 + 133.66 \cdot 0.0965 + 200 \cdot 0.0585 + 116.33 \cdot 0.0965 + 183.33 \cdot 0.159 + 116.33 \cdot 0.0965 =$
$14.63 + 12.9 + 11.7 + 11.42 + 29.15 + 11.42 = 91.22$

$(5,4) = 133.66 \cdot 0.0585 + 200 \cdot 0.0965 + 116.66 \cdot 0.0585 + 183.33 \cdot 0.0965 + 116.33 \cdot 0.159 + 183.33 \cdot 0.0965 =$
$7.82 + 19.3 + 6.82 + 17.69 + 16.81 + 17.69 = 88.13$

$(5,5) = 200 \cdot 0.0585 + 116.66 \cdot 0.0965 + 116.33 \cdot 0.0965 + 183.33 \cdot 0.159 =$
$11.7 + 11.26 + 11.42 + 29.15 = 63.53$

Avg w/ kernel applied (3x3; $\sigma = 1$)

58.7	77.8	88.19	88.08	63.12
80.18	117.81	123.91	122.66	87.75
79.72	155.77	128.9	123.6	87.97
92.8	130.63	128.22	122.81	87.93
66.46	92.96	91.22	88.13	63.53

b) The Algorithm will search through the image patch to find 3 corners, And if those corners connect to form a triangle (Edges)

1) corner Detection (Harris corner Detection):

$$x(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Where w is the window patch being tested,
u and v are the coordinates of the patch,
and x and y are the coordinates of the image

simplifies to: $M = \sum w(x,y)\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \Rightarrow E(u,v) = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$

($\downarrow \lambda_1$, $\downarrow \lambda_2$)

$\det M = \lambda_1 \lambda_2$

trace $M = \lambda_1 + \lambda_2$ then;

R (recognition) $\Rightarrow R = \det M - K(\text{trace } M)^2$

K is weighted value from image traversal

This will lead us to a value of R that is either positive (has corners) or negative (Not a corner)

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R = +ve (has corner)
R = -ve (has edges)

If the algorithm never finds a R value that is positive (R=+ve) then there is NO triangle in the image. However if it is found, then we will shift to follow an edge value (R=-ve) until we reach another corner, followed by a 3rd corner, and finally back to the original. If the 3rd edge does not follow back to the original corner, then there is still no triangle.

In order to find an appropriate following corner, there is an additional step needed to calculate an Edge.

2) Canny Edge Detection:

1) the first step is to Apply a Gaussian filter to remove Noise

2) calculate the Angle of the edge heading away from the corner found: $\theta = \tan^{-1}\left(\frac{\frac{dI}{dy}}{\frac{dI}{dx}}\right)$

As well As the gradient: Magnitude $G = \sqrt{\left(\frac{dI}{dx}\right)^2 + \left(\frac{dI}{dy}\right)^2}$

3) Non-max suppression: Check the gradients of neighbouring points to ensure the direction of the Edge.
For point P, P_A & P_B (Neighbours). Array N for storing the values. If $G(P_A) < G(P) < G(P_B)$, then $N(P) = G(P)$
Else, $N(P) = 0$

4) Trace Edge With Hysteresis:
T_{min} and T_{max} (T is time). $N(P)$ must be $\geq T_{max}$
Plot all pixels where $N(P) \geq T_{min}$ on the resulting image to trace the Edge, And repeat when brought to Another corner.

Programming environment:

Kyle Bishop Quiz 3 - Assignment 4 CSCI-24-6328

1. Image filtering → RANGE
   EX: Gaussian Blur
   Image warping → Domain
   EX: Scaling by factor x = factor · u y = factor · v
   or Holistic tracking of an image patch

2. $I_x u + I_y v + I_t = 0$ $u(x,y) = u$ $V(x,y) = x - y$

$I_x u + I_y(x - y) + I_t = 0$

$I_x u + I_y x - I_y y + I_t = 0$

$u = \dfrac{-I_y x + I_y y - I_t}{I_x}$

(u: unknown)

3. $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1600 & 0 & 960 \\ 0 & -1600 & 540 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

M

$M^{-1} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$