

$D_3(3)$

①

1) $\text{rank} = 2$

x_1, x_2 - баз. ; x_3, x_4 - свобод.

2) $x_1 = -3x_3 - x_4$
 $x_2 = x_3 + x_4$

Ответ:

$$\begin{cases} x_1 = -3x_3 - x_4 \\ x_2 = x_3 + x_4 \\ x_3, x_4 \in \mathbb{R} \end{cases}$$

②

Т.е. $\left. \begin{matrix} \text{rank}(A|b) = 3 \\ \text{rank}(A) = 2 \end{matrix} \right\} \Rightarrow$ сист. несовместна.

③

$b = 1 \cdot A_1 + 1 \cdot A_2 + 1 \cdot A_3$

④ $\text{rref}(A|b) = \left(\begin{array}{cccc|c} 1 & 0 & -8 & 7 & 0 \\ 0 & 1 & 6 & -5 & 0 \end{array} \right)$

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -8 & 6 \\ 7 & -5 \end{pmatrix}$$

	x_1	x_2	x_3	x_4
E_1	1	0	-8	7
E_2	0	1	6	-5

$$x_3 = -8x_1 + 6x_2$$

$$x_4 = 7x_1 - 5x_2$$

Отв: $\begin{cases} x_3 = -8x_1 + 6x_2 \\ x_4 = 7x_1 - 5x_2 \\ x_1, x_2 \in \mathbb{R} \end{cases}$

$$\textcircled{5} E_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, E_2 = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 2 \end{pmatrix}$$

x_1, x_2 - εβόδοι.

x_3, x_4 - δοζυε.

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 2 \end{array}$$

$$x_3 = 2x_1 + 3x_2$$

$$x_4 = x_1 + 2x_2$$

$$-2x_1 - 3x_2 + x_3 = 0$$

$$-x_1 - 2x_2 + 0 + x_4 = 0$$

$$\begin{array}{cccc} -2 & -3 & 1 & 0 \end{array}$$

$$\begin{array}{cccc} -1 & -2 & 0 & 1 \end{array}$$

$$\begin{array}{cccc} x_3 & x_4 & x_1 & x_2 \\ 1 & 0 & -2 & -3 \\ 0 & 1 & -1 & -2 \end{array}$$

Ολυστ:

$$\begin{cases} -2x_1 - 3x_2 + x_3 = 0 \\ -x_1 - 2x_2 + x_4 = 0 \end{cases}$$

$\textcircled{6}$

$$\begin{cases} a \cdot (-3)^4 + b \cdot (-3)^3 + c \cdot (-3)^2 + d \cdot (-3) + e = -17 \\ a \cdot (-2)^4 + b \cdot (-2)^3 + c \cdot (-2)^2 + d \cdot (-2) + e = -13 \\ a \cdot (-1)^4 + b \cdot (-1)^3 + c \cdot (-1)^2 + d \cdot (-1) + e = 1 \\ a \cdot 1 + b + c + d = -1 \\ a \cdot 16 + b \cdot 8 + c \cdot 4 + 2d + e = -17 \end{cases}$$

$$\text{rref}(A|b) = \left(\begin{array}{ccccc|c} 1 & 6 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 6 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right)$$

$$a = -1$$

$$b = 0$$

$$c = 0$$

$$d = -1$$

$$e = 1$$

$$f(x) = -x^4 - x + 1$$

$$\begin{cases} x_1 - 2x_2 + x_5 = 1 \\ x_3 + 4x_5 = 3 \\ x_4 = 0 \end{cases}$$

$$x_1 = 1 + 2x_2 - x_5 \quad x_2, x_5 \text{ - free variables}$$

$$x_3 = 3 - 4x_5$$

$$x_4 = 0$$

⑥

$$E_1 = \begin{pmatrix} 1 \\ 4 \\ -2 \\ 2 \\ -1 \end{pmatrix} \quad E_2 = \begin{pmatrix} 3 \\ 13 \\ -1 \\ -8 \end{pmatrix} \quad E_3 = \begin{pmatrix} 2 \\ 7 \\ -8 \\ 4 \\ -5 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 4 & -2 & 2 & -1 \\ 3 & 13 & -1 & 2 & 1 \\ 2 & 7 & -8 & 4 & -5 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 70 \\ -16 \\ 4 \\ 1 \\ 0 \end{pmatrix} \quad a_2 = \begin{pmatrix} -5 \\ 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$x_1 = 70x_4 - 5x_5$$

$$x_2 = -16x_4 + x_5$$

$$x_3 = 4x_4 - x_5$$

$$A = \begin{pmatrix} 70 & -16 & 4 & 1 & 0 \\ -5 & 1 & -1 & 0 & 1 \end{pmatrix}$$

$$\begin{cases} 70x_1 - 16x_2 + 4x_3 + x_4 = 0 \\ -5x_1 + x_2 - x_3 + x_5 = 0 \end{cases}$$

