Второй тип задания

МСР N8

① $$\lim_{n\to\infty}\left(\frac{n^3+2n-4}{n^3+n-2}\right)^{5n^2}=\left[\frac{\infty}{\infty}^{\infty}\right]=$$

$$\lim_{n\to\infty}\left(\frac{1+\frac{2}{n^2}-\frac{4}{n^3}}{1+\frac{1}{n^2}-\frac{2}{n^3}}\right)^{5n^2}=\lim_{n\to\infty}\left(1+\frac{1-1+\frac{2}{n^2}-\frac{1}{n^2}-\frac{4}{n^3}+\frac{2}{n^3}}{1+\frac{1}{n^2}-\frac{2}{n^3}}\right)^{5n^2}=$$

$$=\lim_{n\to\infty}\left(1+\frac{\frac{1}{n^2}-\frac{2}{n^3}}{1+\frac{1}{n^2}-\frac{2}{n^3}}\right)^{5n^2}=e^{\lim\limits_{n\to\infty}\frac{\left(\frac{1}{n^2}-\frac{2}{n^3}\right)5n^2}{1+\frac{1}{n^2}-\frac{2}{n^3}}}=$$

$$=e^{\lim\limits_{n\to\infty}\frac{5-\cancel{\frac{10}{n}}\to 0}{1+\cancel{\frac{1}{n^2}}-\cancel{\frac{2}{n^3}}\to 0}}=e^5$$

② $$\lim_{n\to\infty}\frac{(8n^2-3\cos 2n)(15n^4-2n^2+4)}{(4n^2-n+1)^3}=$$

$:n^6$

$$=\lim_{n\to\infty}\frac{120n^6-45\cos 2n\, n^4+\ldots}{64n^6+\ldots}=\frac{15}{8}$$

③ $$\lim_{x\to1}\left(\frac{3\operatorname{tg}(x-1)}{-7x^2+4x+3}\right)=\begin{cases}(x-1)\to 0\\ \operatorname{tg}(x-1)=(x-1)\end{cases}=\lim_{x\to1}\left(\frac{3(x-1)}{-7x^2+4x+3}\right)=$$

$$=\lim_{x\to1}\frac{3x-1}{-7x^2+4x+3}\overset{\text{ЛОП}}{=}\lim_{x\to1}\frac{3}{-14x+4}=-0,3$$

④ $$\lim_{x\to\infty}\left(x^2\ln\left(\cos\frac{3\pi}{x}\right)\right)=\begin{cases}\frac{3\pi}{x}\to 0\\ \cos\frac{3\pi}{x}\approx 1-\frac{\left(\frac{3\pi}{x}\right)^2}{2}\end{cases}=$$

$$=\lim_{x\to\infty}\left(x^2\cdot\left(\ln\left(1-\frac{9\pi^2}{2x^2}\right)\right)\right)=\lim_{x\to\infty}\left(x^2\cdot-\frac{9\pi^2}{2x^2}\right)=-4,5\pi^2$$

(5)

$$\lim_{\substack{x\to 0\\ y\to 2}} \frac{\operatorname{arctg}(3xy)}{x^2y - xy} = \lim_{\substack{x\to 0\\ y\to 2}} \frac{3\cancel{xy}}{\cancel{xy}(x-1)} = -3$$

формулы б.м экв для замены:

при $x \to 0$

$\sin x \sim x$

$\operatorname{tg} x \sim x$

$\arcsin x \sim x$

$\operatorname{arctg} x \sim x$

$\cos x \sim 1 - \frac{x^2}{2}$

$(1+x)^a \sim 1 + ax$

$\log_a(1+x) \sim \frac{x}{\ln a}$

$\ln(1+x) \sim x$

$a^x \sim 1 + x \cdot \ln a$

$e^x \sim 1 + x$

Первый замечательный предел:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Второй замечательный предел:

$$\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^x = e^{-1}$$

Рон из МСР:

① $$\lim_{n\to\infty} \frac{(3n^2-4\cos n)(9n^4+4)}{(3n^3-4n+3)^2} =$$

$:n^6$

$$= \lim_{n\to\infty} \frac{27n^6 - 4\cos n \cdot 9n^4 + 12n^2 - 16\cos n}{9n^6 - 24n^4 + 18n^3 + 16n^2 - 24n + 9} = 3$$