习题2-5

1. 解：当$\Delta x=0.1$时，$\Delta y=(2+0.1)^3-2.1-(2^3-2)=1.161$

$dy=f'(x_0)\Delta x=(12-1)\times 0.1=1.1$

$\Delta y-dy=0.061$

当$\Delta x=0.01$时. $\Delta y=(2+0.01)^3-2.01-(2^3-2)=0.110601$

$dy=f(x)\cdot\Delta x=(12-1)\times 0.01=0.11$

$\Delta y-dy=0.000601$

2. 解：(1) $y'=\dfrac{\sqrt{x^2+1}-\frac{2x}{2\sqrt{x^2+1}}\cdot x}{x^2+1}=\dfrac{1}{(x^2+1)^{\frac{3}{2}}}$

$dy=y'dx=\dfrac{dx}{(x^2+1)^{\frac{3}{2}}}$

(2) $y'=2\ln(1-x)\cdot\dfrac{-1}{1-x}=\dfrac{2\ln(1-x)}{x-1}$

$dy=y'dx=\dfrac{2\ln(1-x)}{x-1}dx$

(3) $y'=-e^{-x}\cos(3-x)+e^{-x}(-\sin(3-x))(-1)=e^{-x}[\sin(3-x)-\cos(3-x)]$

$dy=y'dx=e^{-x}[\sin(3-x)-\cos(3-x)]dx$

(4) $y'=\dfrac{1}{1+\left(\frac{1-x^2}{1+x^2}\right)^2}\cdot\dfrac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}=\dfrac{-2x}{1+x^4}$

$dy=y'dx=\dfrac{-2x}{1+x^4}dx$

(5) $y'=2\tan(1+2x^2)\sec^2(1+2x^2)\cdot 4x=8x\tan(1+2x^2)\sec^2(1+2x^2)$

$dy=y'dx=8x\tan(1+2x^2)\sec^2(1+2x^2)\cdot dx$

(6) $y'=\dfrac{1}{3}\left(\dfrac{1-x}{1+x}\right)^{-\frac{2}{3}}\cdot\dfrac{-(1+x)-(1-x)}{(1+x)^2}=\dfrac{-2}{3(1+x)^{\frac{4}{3}}(1-x)^{\frac{2}{3}}}$

$dy=\dfrac{-2}{3(1+x)^{\frac{4}{3}}(1-x)^{\frac{2}{3}}}dx$

(7) $y'=\dfrac{-2\sin 2x-\cos x\cos 2x}{(1+\sin x)^2}=\dfrac{-\cos x(2\sin^2x+4\sin x+1)}{(1+\sin x)^2}$

$dy=y'dx=\dfrac{-\cos x(2\sin^2x+4\sin x+1)}{(1+\sin x)^2}dx$

(8) $y' = \frac{-1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$

$dy = y'dx = \frac{-dx}{x\sqrt{1-\ln^2 x}}$

(9) $y' = f'(e^{f(x)}) \cdot e^{f(x)} f'(x)$

$dy = y'dx = f'(e^{f(x)}) e^{f(x)} f'(x) dx$

3. (1) 原式两边求导:

$\frac{1}{2} \cdot \frac{1}{x^2+y^2} \cdot (2x+2yy') = \frac{1}{1+(\frac{y}{x})^2} \cdot \frac{y'x-y}{x^2}$

$\Rightarrow y' = \frac{x+y}{x-y}$

$\therefore dy = y'dx = \frac{x+y}{x-y} dx$.

(2) 原式两边求导

$2(x+y)(1+y')(2x-y)^3 + (x+y)^2 3(2x-y)^2 \cdot (2-y') = 0$

又 $(x+y)^2(2x-y)^3 = 5$

则 $y' = \frac{2x+8y}{7x+y}$

$dy = y'dx = \frac{2x+8y}{7x+y} dx$

(3) 原式两边求导

$y' = e^{-\frac{x}{y}} \cdot \frac{-y+y'x}{y^2} = \frac{-y+y'x}{y}$

$\Rightarrow y' = \frac{y}{x-y}$.

则 $dy = y'dx = \frac{y}{x-y} dx$

(4) 原式两边求导.

$e^{x+y}(1+y') - y - xy' = 0$

$\Rightarrow y' = \frac{y-xy}{xy-x}$

则 $dy = y'dx = \frac{y-xy}{xy-x} dx$

(5) $3x^2+3y^2y'-3\cos 3x+6y'=0$

$$\Rightarrow y'=\frac{\cos 3x-x^2}{y^2+2}$$

则 $dy=y'dx=\dfrac{\cos 3x-x^2}{y^2+2}dx$.

$$dy|_{x=0}=\frac{1}{2}dx$$

4. 解：$m=8.9\times\frac{4}{3}\pi r^3$,

则 $\Delta m\approx dm=\dfrac{dm}{dr}\Big|_{r=1}\cdot\Delta r=8.9\times4\pi1^2\cdot0.01=1.11784$

即大约用1.11784g铜

5. 解：(1) $S=\dfrac{\alpha}{360^\circ}\pi R^2$. $S'_\alpha=\dfrac{\pi R^2}{360^\circ}$

$$\Delta S=\frac{\pi(\alpha+\Delta\alpha)}{180\cdot2\pi}\pi R^2-\frac{\pi\alpha}{180\cdot2\pi}\pi R^2\approx ds=f'(\alpha_0)\cdot\Delta\alpha$$

$=\dfrac{\pi R^2}{360^\circ}\cdot0.5^\circ=\dfrac{10000\pi}{720}\approx43.63cm$，即减少 $43.63cm^2$

(2) $S'_R=\dfrac{\alpha\pi}{180^\circ}\cdot R$

$$ds=\frac{\alpha\pi}{180^\circ}R\cdot\Delta R=\frac{\pi}{3}\times100\approx104.72cm^2$$

即增加 $104.72cm^2$

6. 解：$dT=f'(l)\Delta l$

$$\Rightarrow 0.05=\pi\left(\frac{l}{g}\right)^{-\frac{1}{2}}\cdot\frac{l}{g}\cdot\Delta l$$

$$\Rightarrow 0.05=\frac{7\pi}{980}\Delta l$$

$$\Delta l=\frac{7}{\pi}\approx2.23cm$$

摆长约需增加长 2.23cm

7. 解：(1) 设 $f(x)=\cos x$. 则 $f'(x)=-\sin x$. 则有：

$\cos x\approx\cos x_0-\sin x_0(x-x_0)$

现在 $x=29^\circ=\dfrac{29\pi}{180}$，取 $x_0=\dfrac{30\pi}{180}=\dfrac{\pi}{6}$，于是

$\cos29^\circ\approx\cos\dfrac{\pi}{6}-\sin\dfrac{\pi}{6}\left(\dfrac{29\pi}{180}-\dfrac{\pi}{6}\right)=\dfrac{\sqrt3}{2}-\dfrac{1}{2}\left(\dfrac{-\pi}{180}\right)\approx0.8747$

(2) 设$f(x)=x^{\frac{1}{3}}$，则$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$，有

$$x^{\frac{1}{3}}\approx x_0^{\frac{1}{3}}+\frac{1}{3}x_0^{-\frac{2}{3}}(x-x_0)$$

现在$x=1.02$ 取$x_0=$ ~~0.02~~ 1. 于是

$$\sqrt[3]{1.02}\approx 1^{\frac{1}{3}}+\frac{1}{3}(0.02)=1.0067$$

(3) 设$f(x)=x^{\frac{1}{2}}$. 则$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$，有

$$x^{\frac{1}{2}}\approx x_0^{\frac{1}{2}}+\frac{1}{2}x_0^{-\frac{1}{2}}(x-x_0)$$

现在$x=25.4$，取$x_0=25$，于是

$$\sqrt{25.04}\approx 25^{\frac{1}{2}}+\frac{1}{2}\cancel{25.4}\times 25^{-\frac{1}{2}}(25.4-25)=5.04.$$

(4) 设$f(x)=\ln x$. 则$f'(x)=\frac{1}{x}$. 有

$$\ln x\approx \cancel{x_0}\ln x_0+\frac{1}{x_0}(x-x_0)$$

现$x=1.01$，$x_0=1$. 于是

$$\ln 1.01\approx \ln 1+\frac{1}{1}(0.01)=0.01$$

(5). 设$f(x)=\arctan x$. 则$f'(x)=\frac{1}{1+x^2}$，有.

$$\arctan x\approx \arctan x_0+\frac{1}{1+x_0^2}(x-x_0)$$

现$x=1.02$. $x_0=1$. 于是

$$\arctan 1.02\approx \arctan 1+\frac{1}{1+1}\times(0.02)=45^\circ 34'$$

(6). 设$f(x)=\frac{x}{\sqrt{x^2-9}}$，$f'(x)=\dfrac{\sqrt{x^2-9}-\frac{x^2}{\sqrt{x^2-9}}}{x^2-9}=\dfrac{-9}{(x^2-9)^{\frac{3}{2}}}$

则原式$=f(5)+f'(5)\times 0.03$

$=1.25-0.0041=1.246$

8. 解：$l=\cancel{4}2\times 200\sin\frac{\alpha}{2}=400\sin\frac{\alpha}{2}$.

则$l'_\alpha=200\cos\frac{\alpha}{2}$

则$\varepsilon(\alpha)=\dfrac{\varepsilon(l)}{l'(55^\circ)}=\dfrac{0.1}{200\cos 27.5^\circ}\approx 0.00056\ rad$.

(9). 解：$V = \pi R^2 h = 25\pi R^2 \quad V' = 50\pi R$, $V_0 = 25\pi(20)^2 = 10000\pi$.

$$\varepsilon(V_0) = V' \varepsilon(R_0) = 50\pi R \cdot \times 0.05 = 50\pi \times 20 \times 0.05 = 50\pi$$

$$\therefore \varepsilon_r(V_0) = \frac{\varepsilon(V_0)}{|V_0|} = \frac{50\pi}{10000\pi} = 0.5\%$$

$S_{\text{侧}} = 2\pi R h = 50\pi R$. $S'_{\text{侧}} = 50\pi$. $S_{\text{侧}(0)} = 1000\pi$.

$$\varepsilon(S_{\text{侧}0}) = S'_{\text{侧}} \varepsilon(R_0) = 50\pi \times 0.05 = 2.5\pi.$$

$$\therefore \varepsilon_r(S_{\text{侧}0}) = \frac{\varepsilon(S_{\text{侧}0})}{|S_{\text{侧}0}|} = \frac{2.5\pi}{10000\pi} = 0.25\%$$