Mechanism Design Basics

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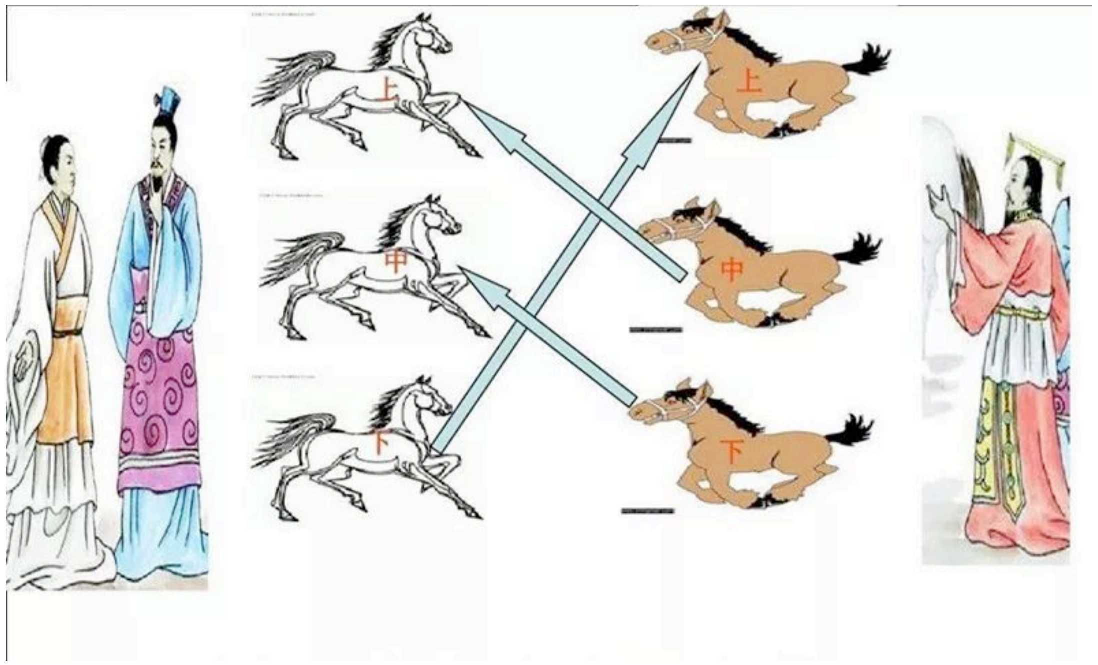
What is MD?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Sotheby's
Sotheby's
LOT 1
HK$ 100,000,000
USD 12,900,000
EUR 11,790,000
JPY 1,541,330,000
IDR 167,241,040,000
Premium Lot


5.9
26
9.8

Designing “rules of the game” to achieve desired outcomes

https://www.cs.jhu.edu/~mdinitz/classes/AGT/Spring2022/
http://timroughgarden.org/f13/f13.html

We need “MONEY”

- MD w/ money
  - Helps us to quantify the utility:)
- MD w/o money
  - Preference comparison instead of money
  - Typically hard…

Single-Item Auctions

- The simplest setting to think about
- n (rational) bidders, an auctioneer with only one item
- What are the behaviors in auctions? We need a model/assumption…
  - Value: private $v_i \geq 0$, she wants to take the item only if the price is at most v_i
  - Utility: quasilinear model, i.e., $u_i = v_i - p$ when she gets the item where p is the price of the item, $u_i = 0$ otherwise.

Sealed Bid Auction

1. Each bidder $i \in [n]$ has a private value v_i to the item.
2. Each bidder sends a private bid b_i to the auctioneer.
3. The auctioneer looks all the bids, and decides the followings:
   - (Allocation) Who takes the item
   - (Payment) How much to charge them for it

What's our goal?

What if …

give it to the bidder with the highest value?

We don't care about the revenue!

1. Give it with price 0, without any payment?

2. First-price auction: set $p = \max_{i \in [n]} b_i$ and give it to $\mathrm{argmax}_{i \in [n]} b_i$

    - It could be better if $b_i < v_i$; and the winner only needs to bid higher than the second-highest bidder.

3. Second-price (Vickrey) auction: set $p = \max_{i \neq i^*} b_i$, where

    $i^* = \mathrm{argmax}_{i \in [n]} b_i$

Vickery is good!

Theorem 1

For any $i \in [n]$, bidding $b_i = v_i$ is a dominant strategy for bidder i. That is, no matter what others do, one will be better off by using such an action.

Fix i, and let $B = \max_{j \neq i} b_j$

- If $B > v_i$, bidder i cannot get the item, and her utility is 0.
  - If $b_i > B$, her utility $v_i - B < 0$; otherwise, her utility is always zero.
- If $B \leq v_i$, bidder i gets the item, and her utility is $v_i - B \geq 0$.
  - If $b_i \geq B$, her utility $v_i - B$; otherwise, her utility is zero.

Theorem 2

In a Vickery auction, every truthtelling bidder is guaranteed non-negative utility.

"Ideal" Auctions

Definition 1(DSIC)

A mechanism is Dominant Strategy Incentive Compatible (DSIC) if:

1. [IC] Truthful bidding is a dominant strategy for anyone;
2. [IR] No player ever gets negative utility.

Vickery auction is an ideal auction since:

1. DSIC
2. Truthful bidding ⇒ social welfare maximization! $\sum_{i \in [n]} v_i x_i$

   where $x_i \in \{0,1\}$ is the allocation of the bidder i.
3. In P, actually linear time.

Why important?

- DSIC: easy for bidders to play and easy for the auctioneer to analyze and predict.
- SWM: our ultimate goal
- In P: we are studying AGT…

In the next few lectures, we try our best to build auctions with these properties, of course, for more complicated settings.

For AGT, we care about the third point… In contrast, the economics community doesn't.

Sponsored Search Auctions

- k slots where ads can be placed.

- The CTR (click-through rate) of slot j is α_j, wlog, we assume that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$.

- For some keyword (e.g., car, phone), n bidders compete for the slots.

- Bidder i gets v_i per click, that is, the utility when her ads placed on slot j is $\alpha_j v_i$.

- More complicated, but similarly, each bidder has her own single parameter v_i.

Our Approach

- Separate the allocations and the payments.
- We first construct the best allocation and then set the price.
  - We also need to show that the MD is DSIC.
  - And the whole algorithm is efficient.
- Assume truthful bidding, what's the allocation (an injective function) $A : [k] \to [n]$?
  - To max the social welfare $\sum_{j \in [k]} \alpha_j v_{A(j)}$
- Given the allocation, what's the payment? (Next lecture…)

Q&A?

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