习题4-2

1. (1) $f'(x)=\frac{1-x+x^2}{1+x+x^2}$, $f'(1)=\frac{1}{3}$

(2) $f'(x)=\frac{\ln e^x}{e^x}\cdot e^x=x$

(3) $f'(x)=-\frac{\sin\sqrt{x^2}}{x^2}\cdot 2x=\frac{2\sin x}{x}$

(4) $f'(x)=\ln(1+(\sqrt[3]{x})^6)(\sqrt[3]{x})'-\ln(1+(\sqrt{x})^6)(\sqrt{x})'=\frac{1}{3}\ln(1+x^2)x^{-\frac{2}{3}}-\frac{1}{2}\ln(1+x^3)x^{-\frac{1}{2}}$

2. $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{-t^2\ln t\cdot 1}{t\ln t\cdot 1}=-t$

3. 等式两边对x求导有:

$e^y y'+3\cos x=0$

$\Rightarrow y'=\frac{-3\cos x}{e^y}$

4. 对$y'=xe^{-x^2}$, 令$y'>0$, 得$x>0$, $y'=0$ 得$x=0$ $y'<0$ 得$x<0$.

则y在$x=0$处取极小值

5. (1) $\lim\limits_{x\to 0}\frac{\int_0^x\cos t^2dt}{x}=\lim\limits_{x\to 0}\frac{\cos x^2}{1}=1$ (洛比达法则)

(2) $\lim\limits_{x\to 0}\frac{\int_0^{\sin x}\sqrt{\tan t}\,dt}{\int_0^{\tan x}\sqrt{\sin t}\,dt}=\lim\limits_{x\to 0}\frac{\sqrt{\tan\sin x}\cos x}{\sqrt{\sin\tan x}\cdot\frac{1}{\cos^2 x}}=\lim\limits_{x\to 0}\frac{\sqrt{\tan\sin x}}{\sqrt{\sin\tan x}}$

$=\lim\limits_{x\to 0}\frac{\sqrt{\sin x}}{\sqrt{\tan x}}=\lim\limits_{x\to 0}\frac{\sqrt{x}}{\sqrt{x}}=1$

(3) $\lim\limits_{x\to 0}\frac{(\int_0^x e^{t^2}dt)^2}{\int_0^x te^{2t^2}dt}=\lim\limits_{x\to 0}\frac{2\int_0^x e^{t^2}dt\cdot e^{x^2}}{xe^{2x^2}}=\lim\limits_{x\to 0}\frac{2\int_0^x e^{t^2}dt}{x}$.

$=\lim\limits_{x\to 0}\frac{2e^{x^2}}{1}=2$

(4) $\lim\limits_{x\to 1}\frac{\int_1^x e^{t^2}dt}{\ln x}=\lim\limits_{x\to 1}\frac{e^{x^2}}{\frac{1}{x}}=\lim\limits_{x\to 1}xe^{x^2}=e$

(5) $\lim\limits_{x\to 0^+}\dfrac{\int_0^{x^2} t^{\frac{3}{2}}dt}{\int_0^x t(t-\sin t)dt} = \lim\limits_{x\to 0^+}\dfrac{(x^2)^{\frac{3}{2}}\cdot 2x}{x(\cancel{x}-\sin x)} = \lim\limits_{x\to 0^+}\dfrac{2x^3}{x-\sin x} = \lim\limits_{x\to 0^+}\dfrac{6x^2}{1-\cos x}$

$= \lim\limits_{x\to 0^+}\dfrac{12x}{\sin x} = 12$

6. 当 $0\le x<1$ 时.

$\int_0^x f(t)dt = \int_0^x t^2 dt = \dfrac{x^3}{3}$

当 $1\le x\le 2$ 时

$\int_0^x f(t)dt = \int_0^1 t^2dt + \int_1^x (1+t)dt = \dfrac{1}{3} + x + \dfrac{1}{2}x^2 - \dfrac{3}{2} = \dfrac{1}{2}x^2 + x - \dfrac{7}{6}$

综上 $\int_0^x f(t)dt = \begin{cases} \dfrac{x^3}{3} & 0\le x\le 1 \\ x+\dfrac{x^2}{2}-\dfrac{7}{6} & 1\le x\le 2 \end{cases}$

7. $f'(x) = \dfrac{1}{\sqrt{1+g(x)^3}}\cdot g'(x) = \dfrac{1}{\sqrt{1+g(x)^3}}\cdot(1+\sin(\cos^2 x))(-\sin x)$

则 $g(\frac{\pi}{2})=0$. 则 $f'(\frac{\pi}{2}) = 1\times(1+\sin 0)(-1) = -1$

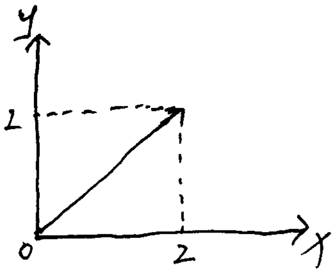
8. $F'(x) = \dfrac{f(x)}{x-a} - \dfrac{\int_a^x f(t)dt}{(x-a)^2} = \dfrac{(x-a)f(x) - \int_a^x f(t)dt}{(x-a)^2}$,

在 (a,b) 上 $f'(x)\le 0$. 则 $f(x)$ 单调递减, 则 $f(x)\le f(t)$ 对 $\forall t\in(a,x)$ 成立.

则 $\int_a^x f(t)dt \ge (x-a)f(x)$. 即 $(x-a)f(x) - \int_a^x f(t)dt \le 0$

又 $(x-a)^2>0$, 故 $F'(x) = \dfrac{(x-a)f(x)-\int_a^x f(t)dt}{(x-a)^2} \le 0$

9. (1)



(2)