

2006 级概率与数理统计期末试题 A 卷答案

一、填空题（共 20 分 每题 2 分）

1、 $\frac{1}{3}$;

2、 $b(5, 0.5)$;

3、 $\frac{5}{9}$, $\frac{8}{9}$

4、 1, $3/2$;

5、 σ^2 ;

6、 $\frac{1}{2}$;

7、 不是;

8、 $(-0.98, 0.98)$.

二、(共 12 分)

解：设 A 表示“目标被击落”， B_1, B_2, B_3 依次表示“甲、乙、丙击中目标”，

C_i 表示“有 i 个人击中目标”， $i=1, 2, 3$ 。

则有题设有： $P(B_1)=0.4$, $P(B_2)=0.5$, $P(B_3)=0.7$

$$C_1 = B_1 \bar{B}_2 \bar{B}_3 \cup \bar{B}_1 B_2 \bar{B}_3 \cup \bar{B}_1 \bar{B}_2 B_3$$

$$P(C_1) = P(B_1 \bar{B}_2 \bar{B}_3) + P(\bar{B}_1 B_2 \bar{B}_3) + P(\bar{B}_1 \bar{B}_2 B_3)$$

$$= P(B_1)P(\bar{B}_2)P(\bar{B}_3) + P(\bar{B}_1)P(B_2)P(\bar{B}_3) + P(\bar{B}_1)P(\bar{B}_2)P(B_3)$$

$$= 0.4 \times 0.5 \times 0.3 + 0.6 \times 0.5 \times 0.3 + 0.6 \times 0.5 \times 0.7 = 0.36$$

$$C_2 = B_1 B_2 \bar{B}_3 \cup B_1 \bar{B}_2 B_3 \cup \bar{B}_1 B_2 B_3 \quad \text{同理} \quad P(C_2) = 0.41$$

$$C_3 = B_1 B_2 B_3 \quad P(C_3) = 0.14$$

由全概率公式得：

$$P(A) = \sum_{i=0}^3 P(C_i)P(A|C_i) \quad \dots\dots\dots+2$$

$$= 0.36 \times 0.2 + 0.41 \times 0.6 + 0.14 \times 1 = 0.458 \quad \dots\dots\dots+10$$

三、(共 14 分)

解: (1) $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

(2) 设 X 和 Y 的分布函数分别为 $F_X(x), F_Y(y)$.

$$F_Y(y) = P\{Y \leq y\} = P\{X^2 \leq y\}$$

当 $y < 0$ 时, $F_Y(y) = 0$.

$$\text{当 } y \geq 0 \text{ 时, } F_Y(y) = P\{-\sqrt{y} \leq X \leq \sqrt{y}\} = F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

$$f_Y(y) = F'_Y(y)$$

$$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} [f_X(\sqrt{y}) + f_X(-\sqrt{y})], & y \geq 0 \\ 0, & \text{其它} \end{cases} = \begin{cases} \frac{1}{\sqrt{2\pi}\sqrt{y}} e^{-\frac{y}{2}}, & y \geq 0 \\ 0, & \text{其它} \end{cases}$$

(3) $\Phi(1) = 0.8413, \Phi(2) = 0.9972$

$$P\{Z = 1\} = P\{-1 \leq X \leq 1\} = \Phi(1) - \Phi(-1)$$

$$= 2\Phi(1) - 1 = 2 \times 0.8413 - 1 = 0.6826$$

$$P\{Z = 2\} = P\{-2 \leq X < -1\} + P\{1 < X \leq 2\} = 2[\Phi(2) - \Phi(1)]$$

$$= 2 \times (0.9972 - 0.8413) = 0.3118$$

$$P\{Z = 3\} = 1 - P\{Z = 1\} - P\{Z = 2\} = 1 - 0.6826 - 0.3118 = 0.0056$$

$Z \mid$	1	2	3
$P \mid$	0.6826	0.3118	0.0056

四、(共 12 分)

解: (1) D 的面积 $m(D) = 2$, 所以, (X, Y) 的联合密度

$$f(x, y) = \begin{cases} \frac{1}{2} & (x, y) \in D, \\ 0 & \text{其它.} \end{cases} \dots\dots\dots+4$$

(2) 设 X 与 Y 的边际密度函数分别为 $f_X(x)$ 和 $f_Y(y)$,

$$f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \int_1^x \frac{1}{2} dy = \frac{1}{2}(x-1), \quad (1 \leq x \leq 3).$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \int_y^3 \frac{1}{2} dx = \frac{1}{2}(3-y), \quad (1 \leq y \leq 3).$$

因为 $f(x, y) \neq f_X(x)f_Y(y)$, 所以 X 与 Y 不独立。 $\dots\dots\dots+4$

$$(3) \quad f_Z(z) = \int_{-\infty}^{+\infty} f_X(x, z-x) dx$$

$$\text{非零区域} \begin{cases} 1 \leq x \leq 3 \\ 1 \leq z-x \leq x \end{cases} \Rightarrow \begin{cases} 1 \leq x \leq 3 \\ 1+x \leq z \leq x \end{cases}$$

$$\text{当 } 2 \leq z < 4 \text{ 时, } f_Z(z) = \int_{\frac{z}{2}}^{z-1} \frac{1}{2} dx = \frac{z}{4} - \frac{1}{2}$$

$$\text{当 } 4 \leq z \leq 6 \text{ 时, } f_Z(z) = \int_{\frac{z}{2}}^3 \frac{1}{2} dx = -\frac{z}{4} + \frac{3}{2}$$

$$\text{其它, } f_Z(z) = 0$$

$$\therefore f_Z(z) = \begin{cases} \frac{z}{4} - \frac{1}{2}, & 2 \leq z < 4 \\ -\frac{z}{4} + \frac{3}{2} & , \quad 4 \leq z \leq 6 \\ 0 & , \quad \text{其它} \end{cases} \dots\dots\dots+4$$

五、(共 10 分)

解: (1) $\because X_1 \sim B(1, 0.8), \quad X_2 \sim B(1, 0.1)$

$$\therefore EX_1 = 0.8, \quad DX_1 = 0.8 \times 0.2 = 0.16$$

$$EX_2 = 0.1, \quad DX_2 = 0.1 \times 0.9 = 0.09$$

则 (X_1, X_2) 的联合分布律为

		X_2	
		0	1
X_1	0	0.1	0.1
	1	0.8	0

.....+4

如 $P(X_1 = 1, X_2 = 0) = \frac{80}{100} = 0.8$

(2) $X_1 X_2$ 的分布律为

$X_1 X_2$	0	1
P	1	0

$$\therefore E(X_1 X_2) = 0 \quad \text{.....+2}$$

$$\begin{aligned} \text{cov}(X_1, X_2) &= EX_1 X_2 - EX_1 EX_2 \\ &= 0 - 0.08 \times 0.1 = -0.08 \quad \text{.....+2} \end{aligned}$$

$$\rho_{X_1 X_2} = \frac{\text{cov}(X_1, X_2)}{\sqrt{DX_1} \sqrt{DX_2}} = \frac{-0.08}{\sqrt{0.16} \sqrt{0.09}} = -\frac{2}{3} \quad \text{.....+2}$$

六、(8 分)

解: $X_{n+1} \sim N(\mu, \sigma^2)$, $\bar{X}_n \sim N(\mu, \frac{\sigma^2}{n})$ $X_{n+1} - \bar{X}_n \sim N(0, \frac{n+1}{n}\sigma^2)$ 。

故
$$U = \frac{X_{n+1} - \bar{X}_n}{\sqrt{\frac{n+1}{n}\sigma^2}} = \frac{X_{n+1} - \bar{X}_n}{\sigma} \sqrt{\frac{n}{n+1}} \sim N(0,1) \dots\dots\dots+3$$

$$W = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \dots\dots\dots+2$$

且 X_{n+1} 与 \bar{X}_n 及 S^2 都独立, 所以 U 与 W 也独立, $\dots\dots\dots+1$

于是有

$$\frac{U}{\sqrt{W/n-1}} = \frac{X_{n+1} - \bar{X}_n}{\sigma} \sqrt{\frac{n}{n+1}} / \sqrt{\frac{(n-1)S^2/\sigma^2}{n-1}} = \frac{X_{n+1} - \bar{X}_n}{S} \sqrt{\frac{n}{n+1}} \sim t(n-1) \dots\dots\dots+2$$

七、(12 分)

解: (1) $E(X) = \int_{-\infty}^{\infty} xf(x; \theta)dx = \int_0^{\infty} x \frac{2}{\sqrt{2\pi\theta}} e^{-\frac{x^2}{2\theta}} dx = \frac{2\theta}{\sqrt{2\pi\theta}} (-e^{-\frac{x^2}{2\theta}}) \Big|_0^{\infty} = \frac{\sqrt{2\theta}}{\sqrt{\pi}} \dots\dots\dots+3$

$$\text{令 } \frac{\sqrt{2\theta}}{\sqrt{\pi}} = \bar{X} \dots\dots\dots+2$$

$$\text{得 } \theta \text{ 的矩估计 } \hat{\theta} = \frac{\pi}{2} \bar{X}^2 \dots\dots\dots+1$$

$$(2) \quad L(\theta) = \prod_{i=1}^n f(x_i; \theta) = \prod_{i=1}^n \frac{2}{\sqrt{2\pi\theta}} e^{-\frac{x_i^2}{2\theta}} = (0.5\pi\theta)^{-\frac{n}{2}} e^{-\frac{\sum_{i=1}^n x_i^2}{2\theta}} \dots\dots\dots+2$$

$$\ln L(\theta) = -\frac{n}{2} \ln(0.5\pi\theta) - \frac{\sum_{i=1}^n x_i^2}{2\theta} \dots\dots\dots+1$$

$$\frac{d(\ln L(\theta))}{d\theta} = -\frac{n}{2\theta} + \frac{\sum_{i=1}^n x_i^2}{2\theta^2} = 0 \dots\dots\dots+2$$

$$\text{得 } \theta \text{ 的极大似然估计 } \hat{\theta} = \frac{\sum_{i=1}^n x_i^2}{n} \dots\dots\dots+1$$

八、（共 12 分）

解： 作统计假设

$$H_0: \sigma^2 = 0.108^2, \quad H_1: \sigma^2 \neq 0.108^2; \quad \dots\dots\dots+2$$

选取统计量

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1); \quad \dots\dots\dots+2$$

$$\text{拒绝域} \quad \frac{(n-1)S^2}{\sigma_0^2} > \chi_{\frac{\alpha}{2}}^2(n-1) \text{ 或者 } \frac{(n-1)S^2}{\sigma_0^2} < \chi_{1-\frac{\alpha}{2}}^2(n-1) \quad \dots\dots\dots+3$$

查表得

$$\chi_{0.025}^2(4) = 11.143, \quad \chi_{0.975}^2(4) = 0.484,$$

计算得：

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{4 \times 0.052}{0.108^2} = 17.83 > 11.43, \quad \dots\dots\dots+4$$

这表明拒绝 H_0 ，即不可能认为此公司股价波动仍为 0.108^2 . $\dots\dots\dots+1$