习题 2-2.

1. 解：(1) $y' = 8x^3 + \frac{3\times 2x}{(x^2)^2} = 8x^3 + \frac{6}{x^3}$

(2) $y' = 2e^{2x} + 2^x\ln 2 + \frac{1}{x\ln 2}$

(3) $y' = 2x\sin x + x^2\cos x$

(4) $y' = 3x^2\ln x + \frac{x^3}{x} + \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = x^2(1+3\ln x) + \frac{1-\ln x}{x^2}$

(5) $y' = \frac{e^x(x^2+2x+1) - (2x+2)e^x}{(x^2+2x+1)^2} = \frac{(x-1)e^x}{(x+1)^3}$

(6) $y' = (2xe^x + x^2e^x)\cos x - x^2e^x\sin x = xe^x[(2+x)\cos x - x\sin x]$

(7) $y' = \frac{(\frac{2}{x}+3x^2)(3\ln x + x^2) - (2\ln x + x^3)(\frac{3}{x}+2x)}{(3\ln x + x^2)^2} = \frac{x(9x-4)\ln x + x^4 - 3x^2 + 2x}{(3\ln x + x^2)^2}$

(8) $y' = \frac{\sqrt{4-x^2} - \frac{x}{2\sqrt{4-x^2}}}{4-x^2} = 4(4-x^2)^{-\frac{3}{2}}$

(9) $y' = e^{\sqrt[3]{x}}\cdot(\frac{1}{3}x^{-\frac{2}{3}})$

(10) $y' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \times \left[1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)\right]$

(11) $y' = -\sin\frac{1-\sqrt{x}}{1+\sqrt{x}} \times \frac{-\frac{1}{2\sqrt{x}}\times(1+\sqrt{x}) - \frac{1}{2\sqrt{x}}\times(1-\sqrt{x})}{(1+\sqrt{x})^2} = \frac{1}{\sqrt{x}(1+\sqrt{x})}\cdot\sin\frac{1-\sqrt{x}}{1+\sqrt{x}}$

(12) $y' = \cos x\, e^{\cos x} + \sin x\, e^{\cos x}(-\sin x) = e^{\cos x}(\cos x - \sin^2 x)$

(13) $y' = \frac{1}{\sqrt{1+(\sqrt{x})^2}}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x-x^2}}$

(14) $y' = \frac{-\frac{1}{\sqrt{1-x^2}}(\sqrt{1-x^2}) - \frac{-2x}{2\sqrt{1-x^2}}\cdot\arccos x}{(\sqrt{1-x^2})^2} = \frac{x\arccos x - \sqrt{1-x^2}}{(1-x^2)^{\frac{3}{2}}}$

(15) $y' = \frac{1}{\arccos 2x}\times\frac{-1}{\sqrt{1-x^2}}\times 2 = \frac{-2}{\arccos 2x\cdot\sqrt{1-4x^2}}$

(16) $y' = \frac{\sqrt{x^2+a^2} - \frac{2x}{2\sqrt{x^2+a^2}}\cdot x}{(\sqrt{x^2+a^2})^2} = a^2(x^2+a^2)^{-\frac{3}{2}}$

(17) $y' = \frac{1}{\sqrt{\frac{1+\sin x}{1-\sin x}}}\cdot\frac{1}{2\sqrt{\frac{1+\sin x}{1-\sin x}}}\cdot\frac{\cos x(1-\sin x) - (-\cos x)(1+\sin x)}{(1-\sin x)^2} = \frac{1}{\cos x}$

(18) $y' = 2\sin(\cos 3x)\cdot\cos(\cos 3x)\cdot(-\sin 3x)\cdot 3 = -3\sin 3x\cdot\sin(2\cos 3x)$

(19). $y' = \dfrac{1}{\frac{x+\sqrt{1-x^2}}{x}} \cdot \dfrac{\left(1+2\frac{-2x}{\sqrt{1-x^2}}\right)\cdot x - (x+\sqrt{1-x^2})}{x^2} = \dfrac{-x+\sqrt{1-x^2}}{x(2x^2-1)\sqrt{1-x^2}}$

(20) $y' = \dfrac{1}{|\tan 2x|} \cdot \dfrac{1}{\cos^2 x} \cdot 2x = \dfrac{4}{\sin 4x}$

(21). $y' = 2^{\frac{x}{\ln x}} \ln 2 \cdot \dfrac{\ln x - \frac{1}{x}\cdot x}{(\ln x)^2} = 2^{\frac{x}{\ln x}} \cdot \dfrac{\ln x - 1}{\ln^2 x} \ln 2$

(22). $y' = 2\arccos\frac{1}{x} \cdot \dfrac{-1}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right) = 2\arccos\frac{1}{x} \cdot \dfrac{|x|}{x^2\sqrt{x^2-1}}$

(23) $y' = \frac{1}{3}\left(\frac{1+x}{1-x}\right)^{-\frac{2}{3}} \cdot \dfrac{1-x-(-1)(1+x)}{(1-x)^2} = \dfrac{-2}{3(x^2-1)}\sqrt[3]{\dfrac{x+1}{1-x}}$.

(24). $y' = \dfrac{1}{1+(1-2x)^4} \cdot (2(1-2x)) \cdot (-2) = \dfrac{-4(1-2x)}{1+(1-2x)^4}$

(25). $y' = \dfrac{1}{\csc x - \cot x}\left(\dfrac{\sin x}{(\cos x)^2} + \dfrac{1}{\sin^2 x}\right) = \csc x$.

(26). $y' = \dfrac{-1}{1+x^2-1} \cdot \dfrac{2x}{2\sqrt{x^2-1}} = \dfrac{-1}{x\sqrt{x^2-1}}$

(27). $y' = \dfrac{1}{\sqrt{x\sin x\sqrt{1-e^x}}} \times \dfrac{1}{2\sqrt{x\sin x\sqrt{1-e^x}}} \times \left[(\sin x + x\cos x)\sqrt{1-e^x} + \dfrac{-e^x}{2\sqrt{1-e^x}} x\sin x\right]$

$= \frac{1}{2}\left(\frac{1}{x} + \cot x + \dfrac{e^x}{2(e^x-1)}\right)$.

(28). $y' = \frac{1}{3}x^{-\frac{2}{3}} \cdot e^{\sin\frac{1}{x}} + \sqrt[3]{x}\, e^{\sin\frac{1}{x}} \cdot \cos\frac{1}{x} \cdot \left(\frac{1}{-x^2}\right) = e^{\sin\frac{1}{x}}\left(\frac{1}{3}x^{\frac{-2}{3}} - x^{\frac{-5}{3}}\cos\frac{1}{x}\right)$

(29). $y' = \dfrac{1}{2\sqrt{1+2\ln^2 x}} \cdot 4\ln x \cdot \frac{1}{x} = \dfrac{2\ln x}{x\sqrt{1+2\ln^2 x}}$

(30). $y' = n\sin^{n-1}x\cos x \cdot \cos nx + \sin^n x(-\sin nx)n = n\sin^{n-1}(x)\cos(n+1)x$.

(31) $y' = \operatorname{ch}x \cdot e^{\operatorname{ch}x} + \operatorname{sh}x\, e^{\operatorname{ch}x} \cdot \operatorname{sh}x = e^{\operatorname{ch}x}(\operatorname{ch}x + \operatorname{sh}^2 x)$

(32) $y' = \dfrac{(e^x+e^{x})(e^x+e^{-x}) - (e^x-e^{x})(e^x-e^{-x})}{(e^x+e^{-x})^2} = \dfrac{1}{\operatorname{ch}^2 x}$.

(33) $y' = \dfrac{1}{\sqrt{\frac{1-x}{1+x^2}}} \cdot \dfrac{1}{2\sqrt{\frac{1-x}{1+x^2}}} \cdot \dfrac{-1(1+x^2)-2x(1-x)}{(1+x^2)^2} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{2x}{1+x^2}\right)$

2. 解：(1) $y'=\frac{3}{(5-x)^2}+\frac{2}{5}x$. $y'|_{x=0}=\frac{3}{25}$，$y'|_{x=2}=\frac{17}{15}$

(2) $y'=\frac{1}{\sin(x-\frac{1}{x})}\cos(x-\frac{1}{x})(1+\frac{1}{x^2})$. $y'|_{x=2}=\frac{5}{4}\cot\frac{3}{2}$

(3) $y'=e^{3(\sin 2x)^2}\cdot 6\cdot\sin 2x\cdot\cos 2x\cdot 2$，$y'|_{x=\frac{\pi}{6}}=3\sqrt{3}e^{\frac{9}{4}}$.

3. 解：(1) $y'=f'(\sin^2x)\sin 2x+\sin 2f(x)\cdot f'(x)$

(2) $y'=e^x f'(e^x)e^{f(x)}+f(e^x)e^{f(x)}f'(x)$.

(3). $y'=\frac{3}{x}f^2(\ln x)f'(\ln x)+e^{f(\frac{1}{x})}f'(\frac{1}{x})\frac{-1}{x^2}$

4. 解：$y'=4ax^3+3bx^2+2cx$.

因曲线与 $y=11x-5$ 在点 $(1,6)$ 相切.

则 $y'|_{x=1}=11$. ① 且 $a+b+c+d=6$ ②

又经过 $(-1,8)$. 则 $a-b+c+d=8$ ③

在 $(0,3)$ 有水平切线. 则 过 $(0,3)$.

则 $d=3$ ④

由①～④得：$a=3$ $b=-1$. $c=1$. $d=3$.

5. 解：直线 $y'=1$. 对数曲线 $y'=\frac{1}{x\ln a}$

当 $1=\frac{1}{x\ln a}$ ①

$x=\log_a x$ ②

得 $a=e^{\frac{1}{e}}$. $x=e$.

则在 (e,e) 处相切.

6. 解：$y'=nx^{n-1}$. 则切线为 $y-1=n(x-1)$，令 $y=0$ 得 $\xi=x=\frac{n-1}{n}$.

则 $\lim\limits_{n\to\infty}y(\xi)=\lim\limits_{n\to\infty}\left(\frac{n-1}{n}\right)^n=\lim\limits_{n\to\infty}\left[(1-\frac{1}{n})^{-n}\right]^{-1}=\frac{1}{e}$

7. 解：曲线 $y'=\frac{1}{x}$，则 $\begin{cases}\frac{1}{x}=a\\ \ln x=ax\end{cases}\Rightarrow\begin{cases}x=e\\ a=\frac{1}{e}\end{cases}$

即 a 值为 $\frac{1}{e}$

8. 解: $\begin{cases} y=\frac{1}{2}(x^2+1) \\ y=1+\ln x \end{cases} \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases}$，即切点为 $(1,1)$

对 $y=1+\ln x$，$y'=\frac{1}{x}$. 令 $x=1$，则 $y'=1$.

则切线方程为 $y-1=1(x-1)$

即: $y=x$

9. 解: 设直线方程为 $y=kx$.

对于 $y=\frac{x+9}{x+5}$，有 $y'=\frac{(x+5)-(x+9)}{(x+5)^2}=\frac{-4}{(x+5)^2}$.

则 $\begin{cases} k\cancel{x}=\frac{-4}{(x+5)^2} \\ y=kx \\ y=\frac{-4}{(x+5)^2} \end{cases}$

$\Rightarrow k=-1$ 或 $k=-\frac{1}{25}$

则直线方程为 $y=-x$ 或 $y=-\frac{1}{25}x$.

10. 解: 由于 $x=0$. 所以只需考虑 $f(x)$ 展开式的一次项

显然 $f(x)$ 的一次项为: $100!\,x$.

则 $f'(0)=100!$

11. 解: (1). 当 $x<0$ 时. $f'(x)=-e^x$

当 $x=0$ 时. 由于 $f'_-(0)=-1 \neq f'_+(0)=0$. 则 $f'(0)$ 不存在

当 $x>0$ 时. $f'(x)=2x$.

(2). 当 $x<\sqrt{e}$ 时. $f'(x)=\frac{x}{e}$

当 $x=\sqrt{e}$ 时. 因 $f(\sqrt{e}+0)=0 \neq f(\sqrt{e}-0)=\frac{1}{2}$. 所以 $f'(\sqrt{e})$ 不存在

当 $x>\sqrt{e}$ 时. $f'(x)=\frac{1}{x}$.

12. (1) 因为 $\lim\limits_{x\to0^+} f(x) = \lim\limits_{x\to0^-} f(x) = f(0) = 0$

所以 $f(x)$ 在 $x=0$ 处连续.

且 $f'_-(0) = \lim\limits_{x\to0^-} \dfrac{x^2\sin\frac{1}{x} - 0}{x-0} = 0 = f'_+(0)$.

则 $x=0$ 时, $f'(0) = 0$.

且 $f'(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x} & x\neq0 \\ 0 & x=0 \end{cases}$ 成立.

又 $\lim\limits_{x\to0^+} f'(x) = -\infty \neq \lim\limits_{x\to0^-} f'(x) = +\infty$

则 $f'(x)$ 在 $x=0$ 不连续

(2) 因 $\lim\limits_{x\to0^+} f(x) = \lim\limits_{x\to0^-} f(x) = f(0) = 0$

所以 $f(x)$ 在 $x=0$ 处连续.

且 $f'_-(0) = \lim\limits_{x\to0^-} \dfrac{x\arctan\frac{1}{x^2} - 0}{x-0} = \dfrac{\pi}{2} = f'_+(0)$

则 $x=0$ 时. $f'(0) = \dfrac{\pi}{2}$. (只考虑 $[-\frac{\pi}{2}, \frac{\pi}{2}]$

则 $f'(x) = \begin{cases} \arctan\frac{1}{x^2} - \frac{2x^2}{x^4+1} & , x\neq0 \\ \frac{\pi}{2} & , x=0 \end{cases}$

又 $\lim\limits_{x\to0^+} f'(x) = \dfrac{\pi}{2} = \lim\limits_{x\to0^-} f'(x) = \dfrac{\pi}{2} = f'(0)$.

则 $f'(x)$ 在 $x=0$ 处连续