习题 4-3

1. (1) $\int \frac{10x^3+3}{x^4}dx = \int (10x^{-1}+3x^{-4})dx = 10\ln|x| - x^{-3} + C$

(2) $\int \frac{(1-x)^2}{x\sqrt{x}}dx = \int \frac{x^2-2x+1}{x^{\frac{3}{2}}}dx = \int (x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + x^{-\frac{3}{2}})dx = \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + C$

(3) $\int \frac{x^2+7x+12}{x+4}dx = \int (x+3)dx = \frac{1}{2}x^2 + 3x + C$

(4) $\int (\frac{1}{2\sqrt{x}} - 3x\frac{1}{\sqrt{1-x^2}} + 2e^x)dx = \sqrt{x} - 3\arcsin x + 2e^x + C$

(5) $\int (2^x+3^x)^2 dx = \int (2^{2x} + 3^{2x} + 2\times 6^x)dx = \frac{4^x}{\ln 4} + \frac{9^x}{\ln 9} + \frac{2\times 6^x}{\ln 6} + C$

(6) $\int \frac{2\times 3^x - 5\times 2^x}{3^x}dx = \int (2 - 5\times(\frac{2}{3})^x)dx = 2x + \frac{5\cdot(\frac{2}{3})^x}{\ln\frac{3}{2}} + C$

(7) $\int \frac{\cos 2x}{\sin^2 x}dx = \int \frac{1-2\sin^2 x}{\sin^2 x}dx = \int (\frac{1}{\sin^2 x} - 2)dx = -\cot x - 2x + C$

(8) $\int \frac{x^4}{1+x^2}dx = \int (x^2 - 1 + \frac{1}{1+x^2})dx = \frac{1}{3}x^3 - x + \arctan x + C$

(9) $\int e^x(1 - \frac{e^{-x}}{\sqrt{x}})dx = \int (e^x - \frac{1}{\sqrt{x}})dx = e^x - 2\sqrt{x} + C$

(10) $\int \frac{dx}{1+\cos 2x} = \int \frac{dx}{2\cos^2 x} = \frac{1}{2}\tan x + C$

(11) $\int \frac{\cos 2x}{\cos x - \sin x}dx = \int \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}dx = \int (\cos x + \sin x)dx = \sin x - \cos x + C$

(12) $\int (1 - \frac{1}{x^2})\sqrt{x}\sqrt{x}\,dx = \int (x^{\frac{3}{4}} - x^{-\frac{5}{4}})dx = \frac{4}{7}x^{\frac{7}{4}} + 4x^{-\frac{1}{4}} + C$

2. 由 $y' = \frac{1}{x}$ 得 $y = \int y'dx = \int \frac{1}{x}dx = \ln|x| + C$

又因过 $(e^2, 3)$

则 $3 = \ln|e^2| + C \Rightarrow C = 1$

则 $y = \ln|x| + 1$

3. 由 $\frac{d^2s}{dt^2} = 5 - 2t$，得 $\frac{ds}{dt} = \int (5-2t)dt = 5t - t^2 + C_1$ 因 $t=0$ 时，$\frac{ds}{dt} = 2$，$\Rightarrow C_1 = 2$

则 $s = \int (5t - t^2 + 2)dt = \frac{5}{2}t^2 - \frac{1}{3}t^3 + 2t + C_2$ 因 $t=0$ 时，$s=0$ $\Rightarrow C_2 = 0$

则 $s = \frac{5}{2}t^2 - \frac{1}{3}t^3 + 2t$.