小测验参考答案

1. (d); 2. (b); 3. $B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}$;

4. $(I-A)^{-1} = I + A + A^2 + \cdots + A^{k-1}$;

5. 充分性：若A可逆，则对任何的b,在$AX = b$
   两边同时左乘A^{-1},有$X = A^{-1}b$,即$AX = b$对任何
   b的取值都有解.充分性得证.

   必要性：若$AX = b$对任何b都有解,证A可逆.

特别地，当取b分别为$\varepsilon_1 = (1,0,0,\cdots,0), \varepsilon_2 = (0,1,0,\cdots,0),$
$\cdots, \varepsilon_n = (0,0,0,\cdots,1)$时，方程$AX = b$均有解，记它们的
解依次为$\alpha_1, \alpha_2, \cdots, \alpha_n$,则有：$A\alpha_i = \varepsilon_i (i = 1,2,\cdots,n)$

即 $A[\alpha_1,\alpha_2,\cdots,\alpha_n] = [A\alpha_1, A\alpha_2,\cdots,A\alpha_n]$

$$= [\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n] = I_n$$

记 $B = [\alpha_1,\alpha_2,\cdots,\alpha_n]$,则有$AB = I$.从而$A$可逆.

6. $\begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & B^{-1} \\ A^{-1} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 2 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$.

7. $A + B = \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ -\frac{1}{15} & -\frac{1}{15} & \frac{1}{5} & 0 \\ -\frac{1}{15} & -\frac{1}{15} & 0 & \frac{1}{5} \end{bmatrix}$;

8. $\lambda \neq -6$时有唯一解；$\lambda = -6$时有无穷多解.

9. 当$5a - 3b - c = 0$且$a - 3b + 2d = 0$时，有无穷多解.

   通解为：$\begin{cases} x_1 = -\dfrac{5}{2} - \dfrac{5}{2}x_3 \\ x_2 = \dfrac{3}{2} + \dfrac{3}{2}x_3 \end{cases}$, $(x_3$任意$)$.