习题4-5

(1) $\int_0^{\frac{\pi}{2}} \cos^5 x \cdot \sin^2 x \, dx = \int_0^{\frac{\pi}{2}} \cos^4 x \sin^2 x \, d\sin x$

$= \int_0^{\frac{\pi}{2}} (1-\sin^2 x)^2 (\sin^2 x) \, d\sin x$

$= \int_0^{\frac{\pi}{2}} [\sin^6 x + 2\sin^4 x + \sin^2 x] \, d\sin x$

$= \left(\frac{1}{3}\sin^3 x + \frac{-2}{5}\sin^5 x + \frac{1}{7}\sin^7 x\right)\Big|_0^{\frac{\pi}{2}}$

$= \frac{1}{3} - \frac{2}{5} + \frac{1}{4} = \frac{8}{105}$

(2) $\int_1^{e^2} \frac{dx}{x\sqrt{1+\ln x}}$，令$t=\sqrt{1+\ln x}$，$t^2 = 1+\ln x$，$x = e^{t^2-1}$，$dx = e^{t^2-1}2t\,dt$

则原式 $= \int_1^{\sqrt{3}} \frac{e^{t^2-1}2t\,dt}{e^{t^2-1}\cdot t} = \int_1^{\sqrt{3}} 2\,dt = 2t\Big|_1^{\sqrt{3}} = 2\sqrt{3}-2$.

(3) 令$t = e^x - 1$，$t \in [1,3]$. $x = \ln(t+1)$，$dx = \frac{1}{t+1}dt$

则原式 $= \int_1^3 \left(\frac{1}{t}\cdot\frac{1}{t+1}\right)dt = \int_1^3 \left(\frac{1}{t} - \frac{1}{t+1}\right)dt = (\ln t - \ln(t+1))\Big|_1^3 = \ln\frac{3}{2}$

(4) 原式 $= \int_3^8 \frac{x+1-1}{\sqrt{x+1}}dx = \int_3^8 \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right)dx = \frac{2}{3}(x+1)^{\frac{3}{2}}\Big|_3^8 - 2\sqrt{1+x}\Big|_3^8$

$= 12 - \frac{4}{3} = \frac{32}{3}$

(5) 设$x = \sec t$，则$\cos t = \frac{1}{x}$，$dx = \sec t\cdot\tan t\,dt$，当$x=1$时，$t=0$，当$x=2$时，$t=\frac{\pi}{3}$

$\therefore$ 原式 $= \int_0^{\frac{\pi}{3}} \frac{\tan t}{\sec t}\cdot\sec t\cdot\tan t\,dt$

$= \int_0^{\frac{\pi}{3}} \tan^2 t\,dt = \int_0^{\frac{\pi}{3}} (\sec^2 t - 1)dt = \int_0^{\frac{\pi}{3}} \sec^2 t\,dt - \int_0^{\frac{\pi}{3}} dt$

$= \int_0^{\frac{\pi}{3}} d\tan t - \int_0^{\frac{\pi}{3}} dt = (\tan t - t)\Big|_0^{\frac{\pi}{3}}$

$= \sqrt{3} - \frac{\pi}{3}$

(6) 令 $x=\sin\theta$, $dx=\cos\theta d\theta$, $x\in[0,1]\to\theta\in[0,\frac{\pi}{2}]$

$$原式=\int_0^{\frac{\pi}{2}}\cos^3\theta\cdot\cos\theta d\theta=\int_0^{\frac{\pi}{2}}(\cos^2\theta)^2d\theta=\int_0^{\frac{\pi}{2}}\frac{1}{2}(1+\cos2\theta)^2d\theta$$

$$=\frac{1}{4}\int_0^{\frac{\pi}{2}}(1+2\cos2\theta+\cos^2 2\theta)d\theta=\frac{1}{4}\int_0^{\frac{\pi}{2}}(1+2\cos2\theta)d\theta+\frac{1}{8}\int_0^{\frac{\pi}{2}}(1+\cos4\theta)d\theta$$

$$=\frac{1}{4}(\theta+\sin2\theta)\Big|_0^{\frac{\pi}{2}}+\frac{1}{8}(\theta+\frac{1}{4}\sin4\theta)\Big|_0^{\frac{\pi}{2}}$$

$$=\frac{1}{4}\cdot\frac{\pi}{2}+\frac{1}{8}\cdot\frac{\pi}{2}=\frac{3\pi}{16}$$

(7) 令 $t=\frac{1}{x}$. $dx=\frac{-1}{t^2}dt$, $x\in[1,3]\to t\in[1,\frac{1}{3}]$

$$原式=\int_1^{\frac{1}{3}}\frac{t}{\sqrt{\frac{1}{t^2}+\frac{5}{t}+1}}\cdot\frac{-1}{t^2}dt=\int_{\frac{1}{3}}^{1}\frac{dt}{\sqrt{t^2+5t+1}}=\int_{\frac{1}{3}}^{1}\frac{dt}{\sqrt{(t+\frac{5}{2})^2-\frac{21}{4}}}$$

用公式: $\int\frac{dx}{\sqrt{x^2-a^2}}=\ln\left|x+\sqrt{x^2-a^2}\right|+C$

$$则原式=\int_{\frac{1}{3}}^{1}\frac{d(t+\frac{5}{2})}{\sqrt{(t+\frac{5}{2})^2-(\frac{\sqrt{21}}{2})^2}}=\ln\left|t+\frac{5}{2}+\sqrt{(t+\frac{5}{2})^2-\frac{21}{4}}\right|\Big|_{\frac{1}{3}}^{1}$$

$$=\ln\left|1+\frac{5}{2}+\sqrt{(1+\frac{5}{2})^2-\frac{21}{4}}\right|-\ln\left|\frac{1}{3}+\frac{5}{2}+\sqrt{(\frac{1}{3}+\frac{5}{2})^2-\frac{21}{4}}\right|$$

$$=\ln\frac{7+2\sqrt{7}}{9}$$

(若不用公式, 可令 $t+\frac{5}{2}=\sqrt{\frac{21}{4}}\sec\theta$, $\theta\in[0,\frac{\pi}{2}]$).

(8) $$原式=\int_0^{\pi}\sqrt{\sin^3x(1-\sin^2x)}dx=\int_0^{\pi}\sin^{\frac{3}{2}}x\cdot|\cos x|dx$$

$$=\int_0^{\frac{\pi}{2}}\sin^{\frac{3}{2}}x\cos x dx+\int_{\frac{\pi}{2}}^{\pi}\sin^{\frac{3}{2}}x(-\cos x)dx$$

$$=\int_0^{\frac{\pi}{2}}\sin^{\frac{3}{2}}x\, d\sin x-\int_{\frac{\pi}{2}}^{\pi}\sin^{\frac{3}{2}}x\, d\sin x$$

$$=(\frac{2}{5}\sin^{\frac{5}{2}}x)\Big|_0^{\frac{\pi}{2}}-(\frac{2}{5}\sin^{\frac{5}{2}}x)\Big|_{\frac{\pi}{2}}^{\pi}$$

$$=(\frac{2}{5}\times1-0)-(\frac{2}{5}\times0-\frac{2}{5}\times1)$$

$$=\frac{4}{5}$$

(9) 令 $e^x=\sin y$. $e^x dx=\cos y\,dy$ $dx=\frac{\cos y}{\sin y}dy$ 当 $x=0$, $y=\frac{\pi}{2}$ 当 $x=-\ln 2$, $y=\frac{\pi}{6}$

原式 $=\int_0^{-\ln 2}\sqrt{1-e^{2x}}\,dx=\int_{\frac{\pi}{2}}^{\frac{\pi}{6}}\frac{\cos^2 y}{\sin y}dy=\int_{\frac{\pi}{2}}^{\frac{\pi}{6}}\frac{1-\sin^2 y}{\sin y}dy$.

$=\int_{\frac{\pi}{2}}^{\frac{\pi}{6}}(\csc y-\sin y)dy=\left(\ln|\csc y-\cot y|+\cos y\right)\Big|_{\frac{\pi}{2}}^{\frac{\pi}{6}}$

$=\ln(2-\sqrt{3})+\frac{\sqrt{3}}{2}$

(10) 令 $x=\sin u$, $dx=\cos u\,du$, $x=\frac{1}{\sqrt{2}}$ 时 $u=\frac{\pi}{4}$, $x=1$ 时, $u=\frac{\pi}{2}$

原式 $=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos u\sqrt{1-\sin^2 u}}{\sin^2 u}du=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos^2 u}{\sin^2 u}du$

$=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^2 u\,du=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\csc^2 u-1)du$

$=(-\cot u-u)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}=-\frac{\pi}{2}+1+\frac{\pi}{4}$

$=1-\frac{\pi}{4}$

(11) 令 $x=\tan t$, 则 $dx=\sec^2 t\,dt$. $x\in[1,\sqrt{3}]\to t\in[\frac{\pi}{4},\frac{\pi}{3}]$

则原式 $=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sec^2 t\,dt}{\tan^2 t\sqrt{1+\tan^2 t}}$

$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sec^2 t\,dt}{\tan^2 t\cdot\sec t}$

$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sec t\,dt}{\tan^2 t}$

$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos^2 t}{\sin^2 t}\cdot\frac{1}{\cos t}dt$

$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos t}{\sin^2 t}dt=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{d\sin t}{\sin^2 t}$

$=-\frac{1}{\sin t}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$=\sqrt{2}-\frac{2\sqrt{3}}{3}$

(12) 原式$=\int_0^{\pi}\sqrt{2}\,|\cos x|\,dx=\int_0^{\frac{\pi}{2}}\sqrt{2}\cos x\,dx+\int_{\frac{\pi}{2}}^{\pi}\sqrt{2}(-\cos x)\,dx$

$=\sqrt{2}\sin x\Big|_0^{\frac{\pi}{2}}-\sqrt{2}\sin x\Big|_{\frac{\pi}{2}}^{\pi}=2\sqrt{2}$

2. (1) 因为x^4是偶函数，$\sin x$是奇函数. 则$x^4\sin x$是奇函数

则原式$=0$

(2) 因为$4\cos^4\theta$是偶函数,

则原式$=2\int_0^{\frac{\pi}{2}}4\cos^4\theta\,d\theta=2\times4\int_0^{\frac{\pi}{2}}\cos^4\theta\,d\theta$

运用例13结论：

原式$=2\times4\times\frac{3!!}{4!!}\times\frac{\pi}{2}=\frac{3}{2}\pi$

(3) 因为$\frac{(\arcsin x)^2}{\sqrt{1-x^2}}$为偶函数

则原式$=2\int_0^{\frac{1}{2}}\frac{(\arcsin x)^2}{\sqrt{1-x^2}}dx=2\int_0^{\frac{1}{2}}(\arcsin x)^2 d\arcsin x$

$=2\times\frac{1}{3}(\arcsin x)^3\Big|_0^{\frac{1}{2}}=\frac{2}{3}\left(\frac{\pi}{6}-0\right)^3=\frac{\pi^3}{324}$

(4) 显然 x^3为奇函数，$\frac{\sin^2 x}{x^4+2x^2+1}$为偶函数，则被积函数为奇函数

则原式$=0$

(5) 通过$y=\cos^5 x$的图像，便能得出原积分为0.

(6) 令$t=x-\pi$，则$x\in[0,2\pi]\to t\in[-\pi,\pi]$，$dx=dt$

原积分$=\int_{-\pi}^{\pi}\sin^3(t+\pi)\cos^{10}(t+\pi)\,dt$

$=-\int_{-\pi}^{\pi}\sin^3 t\cos^{10}t\,dt$

被积函数$\sin^3 t\cos^{10}t$为奇函数.

则原积分$=0$.

3. 令 $y=\frac{1}{t}$. 则 $t\in[x,1]\to y\in[\frac{1}{x},1]$, 且 $dy=-\frac{1}{t^2}dt$. $\Rightarrow dt=-\frac{1}{y^2}dy$.

则: $\int_x^1\frac{dt}{1+t^2}=\int_{\frac{1}{x}}^1\frac{1}{1+\frac{1}{y^2}}\cdot(-\frac{1}{y^2})dy=-\int_{\frac{1}{x}}^1\frac{1}{y^2+1}dy=\int_1^{\frac{1}{x}}\frac{1}{1+y^2}dy$.

因为改变y的符号积分值不变, 用t代替y.

则 $\int_x^1\frac{dt}{1+t^2}=\int_1^{\frac{1}{x}}\frac{1}{1+t^2}dt$

4. 因为 $\int_0^1 x^m(1-x)^n dx=-\int_1^0 x^m(1-x)^n dx=\int_1^0 x^m(1-x)^n d(-x)$, 设 $y=-x$. 则:

上式 $=\int_0^1(-y)^m(1+y)^n dy$, 再设 $y=x-1$, 则

上式 $=\int_0^1(1-x)^m x^n dx=\int_0^1 x^n(1-x)^m dx$

证毕.

则 $\int_0^1 x^2(1-x)^{20}dx=\int_0^1 x^{20}(1-x)^2 dx=\int_0^1 x^{20}(x^2+1-2x)dx$

$=\int_0^1(x^{22}-2x^{21}+x^{20})dx$

$=\frac{1}{23}x^{23}\Big|_0^1-\frac{1}{11}x^{22}\Big|_0^1+\frac{1}{21}x^{21}\Big|_0^1$

$=\frac{1}{23}-\frac{1}{11}+\frac{1}{21}=\frac{1}{5313}$

5. 令 $a+b-x=t$, 则 $dx=-dt$, $x\in[a,b]\to t\in[b,a]$

则 $\int_a^b f(a+b-x)dx=\int_b^a -f(t)dt=-\int_b^a f(t)dt=\int_a^b f(t)dt=\int_a^b f(x)dx$.

即: $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

证毕.

6. 因为 $f(x)$ 是连续函数, 则 $\int_0^a f(x)dx=\int_0^{\frac{a}{2}}f(x)dx+\int_{\frac{a}{2}}^a f(x)dx$

对 $\int_{\frac{a}{2}}^a f(x)dx$, 令 $t=a-x$. 则 $x\in[\frac{a}{2},a]\to t\in[\frac{a}{2},0]$. $dx=-dt$.

则 $\int_{\frac{a}{2}}^a f(x)dx=\int_{\frac{a}{2}}^0(-f(a-t))dt=\int_0^{\frac{a}{2}}f(a-t)dt=\int_0^{\frac{a}{2}}f(a-x)dx$

则 $\int_0^a f(x)dx=\int_0^{\frac{a}{2}}f(x)dx+\int_0^{\frac{a}{2}}f(a-x)dx=\int_0^{\frac{a}{2}}[f(x)+f(a-x)]dx$

证毕

6. 证：令 $t=x-\frac{\pi}{2}$，$x=t+\frac{\pi}{2}$. $x\in[0,\pi]\to t\in[-\frac{\pi}{2},\frac{\pi}{2}]$. $dx=dt$.

则 $\int_0^{\pi}\cos^n x\,dx=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos(t+\frac{\pi}{2}))^n dt=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^n t\,dt$

对于被积函数 $\sin^n t$，当 n 为奇数. 则 $\sin^n t$ 为奇函数

则原积分等于0，若 n 为偶数，则 $\sin^n t$ 为偶函数.

则原积分等于 $2\int_0^{\frac{\pi}{2}}\sin^n t\,dt=2\int_0^{\frac{\pi}{2}}\sin^n x\,dx$，由例13知，左式 $=2\int_0^{\frac{\pi}{2}}\cos^n x\,dx$.

综上 $\int_0^{\pi}\cos^n x\,dx=\begin{cases}0 & n\text{为奇数}.\\ 2\cdot\int_0^{\frac{\pi}{2}}\cos^n x\,dx & n\text{为偶数}.\end{cases}$

证毕.

7. 证：$\int_0^{\frac{\pi}{2}}\cos^m x\cdot\sin^m x\,dx=\int_0^{\frac{\pi}{2}}(\frac{1}{2}\sin 2x)^m dx=\frac{1}{2^{m+1}}\int_0^{\frac{\pi}{2}}\sin^m 2x\,d(2x)$

令 $x=2x\Rightarrow\frac{1}{2^{m+1}}\int_0^{\pi}\sin^m x\,dx$. 令 $x=\frac{\pi}{2}-x$

则原积分 $=\frac{1}{2^{m+1}}\cdot\int_{\frac{\pi}{2}}^{-\frac{\pi}{2}}\cdot\cos^m(\frac{\pi}{2}-x)(-dx)\cdot=\frac{1}{2^{m+1}}\cdot 2\int_0^{\frac{\pi}{2}}\cos^m x\,dx$

$=\frac{1}{2^m}\cdot\int_0^{\frac{\pi}{2}}\cos^m x\,dx$

证毕.

8. (1) 原式 $=\int_0^{\frac{1}{2}}x\arcsin x\,dx=x\arcsin x\Big|_0^{\frac{1}{2}}-\int_0^{\frac{1}{2}}\frac{x}{\sqrt{1-x^2}}dx=x\arcsin x\Big|_0^{\frac{1}{2}}+\sqrt{1-x^2}\Big|_0^{\frac{1}{2}}$

$=\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$

(2) 原式 $=\frac{1}{3}x^3\ln x\Big|_1^e-\frac{1}{3}\int_1^e x^3\cdot\frac{1}{x}dx$

$=\frac{1}{3}e^3-\frac{1}{3}\int_1^e x^2dx$

$=\frac{1}{3}e^3-\frac{1}{3}(\frac{1}{3}x^3)\Big|_1^e$

$=\frac{1}{3}e^3-\frac{1}{9}(e^3-1)$

$=\frac{2}{9}e^3+\frac{1}{9}$

(3) 原式 $= \frac{1}{2}x^2\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} \frac{1}{1+x^2}\cdot x^2 dx$

$= \frac{1}{2}x^2\arctan x\Big|_0^{\sqrt{3}} - \frac{1}{2}\int_0^{\sqrt{3}} (1-\frac{1}{1+x^2})dx$

$= \frac{1}{2}x^2\arctan x\Big|_0^{\sqrt{3}} - (\frac{x}{2}-\frac{1}{2}\arctan x)\Big|_0^{\sqrt{3}}$

$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

(4) 例用: $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(b\sin bx + a\cos bx) + C$

原式 $= \int_0^{\frac{\pi}{2}} e^{2x}\cos x dx = \frac{e^{2x}}{5}(\sin x + 2\cos x)\Big|_0^{\frac{\pi}{2}}$

$= \frac{e^{\pi}}{5}(1+0) - \frac{1}{5}(0+2)$

$= \frac{e^{\pi}}{5} - \frac{2}{5}$

(5) 先求不定积分:

$\int \arcsin\sqrt{\frac{x}{1+x}}dx = x\arcsin\sqrt{\frac{x}{1+x}} - \int \frac{x}{\sqrt{1-\frac{x}{x+1}}}\sqrt{\frac{x+1}{x}}\frac{1}{(x+1)^2}dx$

$= x\arcsin\sqrt{\frac{x}{1+x}} - \int \frac{\sqrt{x}}{2(x+1)}dx$

$= x\arcsin\sqrt{\frac{x}{1+x}} - \int (\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}\frac{1}{1+x})dx$

$= x\arcsin\sqrt{\frac{x}{1+x}} - \sqrt{x} - \arctan\sqrt{x} + C$

则原定积分 $= \frac{4}{3}\pi - \sqrt{3}$

(6) 直接运用公式 $\int_0^{\frac{\pi}{2}}\cos^7 x dx = \frac{6!!}{7!!} = \frac{6\times4\times2}{7\times5\times3} = \frac{16}{35}$

(7) 令 $t = \frac{x}{2}$, 则 $dx = 2dt$.

原式 $= \int_0^{\frac{\pi}{2}} \sin^8 t\, 2dt = 2\int_0^{\frac{\pi}{2}}\sin^8 t dt = 2\times\frac{7!!}{8!!}\times\frac{\pi}{2} = \frac{35\pi}{128}$.

(8) 因为 $x\cos x$ 为奇函数.

则原式 $= 0$

(9) 原式 $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\, d(-\cot x) = -x\cot x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x\, dx$

$= -x\cot x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} + \ln|\sin x| \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= -\frac{\sqrt{3}\pi}{9} + \ln\frac{\sqrt{3}}{2} + \frac{\pi}{4} - \ln\frac{\sqrt{2}}{2}$

$= (\frac{1}{4} - \frac{\sqrt{3}}{9})\pi + \frac{1}{2}\ln\frac{3}{2}$

(10) 分部积分：

$\int_1^e \sin\ln x\, dx = x\sin\ln x \Big|_1^e - \int_1^e \cos\ln x\, dx$

$= x\sin\ln x \Big|_1^e - x\cos\ln x \Big|_1^e - \int_1^e \sin\ln x\, dx$

$\Rightarrow 2\int_1^e \sin\ln x\, dx = x\sin\ln x \Big|_1^e - x\cos\ln x \Big|_1^e = e\sin 1 + 1 - e\cos 1$

$\Rightarrow \int_1^e \sin\ln x\, dx = \frac{1}{2}(e\sin 1 + 1 - e\cos 1)$

(11) 原式 $= \int_0^\pi x^2 \frac{1}{2}(1+\cos 2x)dx = \int_0^\pi \frac{1}{2} x^2 dx - \int_0^\pi \frac{1}{2} x^2\cos 2x\, dx$

$= \frac{1}{6}\pi^3 - \frac{1}{4}\int_0^\pi x^2 d\sin 2x = \frac{1}{6}\pi^3 + \frac{1}{2}\int_0^\pi x\sin 2x\, dx$

$= \frac{1}{6}\pi^3 - \frac{1}{4}\int_0^\pi x\, d\cos 2x = \frac{1}{6}\pi^3 - \frac{1}{4}\pi + \frac{1}{4}\int_0^\pi \cos 2x\, dx$

$= \frac{1}{6}\pi^3 - \frac{1}{4}\pi$.

(12) 原式 $= x\ln(x+\sqrt{x^2+a^2}) \Big|_0^\pi - \int_0^\pi x \cdot \frac{1+\frac{x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}} dx$

$= x\ln(x+\sqrt{x^2+a^2}) \Big|_0^\pi - \int_0^\pi \frac{x}{\sqrt{x^2+a^2}} dx$

$= x\ln(x+\sqrt{x^2+a^2}) \Big|_0^\pi - \sqrt{x^2+a^2} \Big|_0^\pi$

$= \pi\ln(\pi+\sqrt{\pi^2+a^2}) - \sqrt{\pi^2+a^2} + |a|$.

(13) 原式 $= \int_0^{\frac{\pi}{4}} \tan^2 x(\sec^2 x - 1)dx = \int_0^{\frac{\pi}{4}} \tan^2 x\, d\tan x - \int_0^{\frac{\pi}{4}} \tan^2 x\, dx$

$= \int_0^{\frac{\pi}{4}} \tan^2 x\, d\tan x - \int_0^{\frac{\pi}{4}} (\sec^2 x - 1)dx$

$= (\frac{1}{3}\tan^3\theta - \tan x + x) \Big|_0^{\frac{\pi}{4}}$

$= \frac{1}{3} - 1 + \frac{\pi}{4}$

$= \frac{\pi}{4} - \frac{2}{3}$

(14). 令 $x=\cos\theta$，$(1-x^2)^{\frac{9}{2}}=\sin^9\theta$　$x=1$时. $\theta=0$　$x=0$时 $\theta=\frac{\pi}{2}$，$dx=-\sin\theta d\theta$

$$原式=\int_{\frac{\pi}{2}}^{0}\sin^9\theta\cdot(-\sin\theta)d\theta$$

$$=\int_0^{\frac{\pi}{2}}\sin^{10}\theta\, d\theta$$

$$=\frac{9!!}{10!!}\cdot\frac{\pi}{2}$$

$$=\frac{63\pi}{512}$$

9. $\int_0^2 f(x)e^x dx=\int_0^1(1+x^2)e^x dx+\int_1^2(2-x)e^x dx$

对于 $\int_0^1(1+x^2)e^x dx=(1+x^2)e^x\Big|_0^1-\int_0^1 2xe^x dx$

$$=2e-1-2xe^x\Big|_0^1+2\int_0^1 e^x dx$$

$$=2e-1-2e+2(e-1)$$

$$=2e-3$$

对于 $\int_1^2(2-x)e^x dx=(2-x)e^x\Big|_1^2-\int_1^2(-1)e^x dx$

$$=-e+e^2-e$$

$$=e^2-2e$$

则 $\int_0^2 f(x)e^x dx=2e-3+e^2-2e=e^2-3$