第2章 信息的表示与处理

100076202： 计算机系统导论

比特，字节和整数

Bits, Bytes, and Integers

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议题: 比特、字节和整数
Bits, Bytes, and Integers

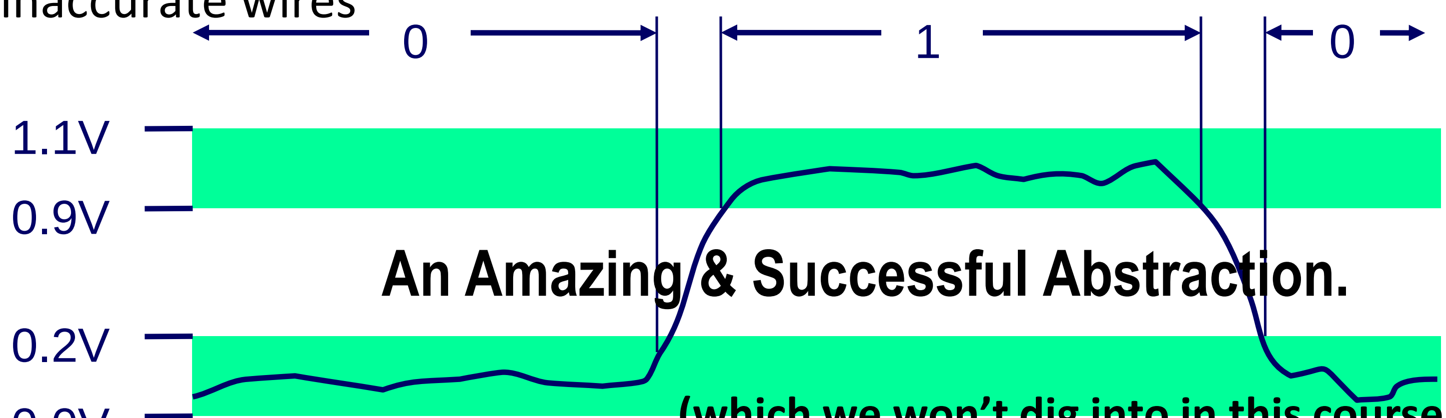
- **用比特表示信息 Representing information as bits**
- **比特级操作 Bit-level manipulations**
- **整数 Integer**
  - 无符号数和有符号数表示 Representation: unsigned and signed
  - 转换和强制类型转换 Conversion, casting
  - 扩展和截断 Expanding, truncating
  - 加、补码非、乘和移位 Addition, negation, multiplication, shifting
  - 小结 Summary
- **内存中的表示、指针和字符串 Representations in memory, pointers, strings**

十进制表示 The Decimal Representation

- 基数为10 Base-10
- 已经使用了超过1000年 Has been in use for over 1000 years
- 起源于印度 Developed in India
- 12世纪被阿拉伯数学家改进 Improved by Arab mathematicians in the 12th century
- 13世纪被带到西方 Brought to the West in the 13th century by
  - 意大利数学家 the Italian mathematician Leonardo Pisano,
    - 更为大家熟悉的名字是斐波那契 better known as Fibonacci.

一切均是比特位 Everything is bits

- **每个比特是0或1 Each bit is 0 or 1**
- **以各种方式编码/解释比特位集合 By encoding / interpreting sets of bits in various ways**
  - 计算机确定要做什么（指令）Computers determine what to do (instructions)
  - 。。。以及表示和操作数值、集合、字符串等。。。 ... and represent and manipulate numbers, sets, strings, etc...
- **为何是比特？电信号实现Why bits? Electronic Implementation**
  - 易于用稳态元件存储 Easy to store with bistable elements
  - 在有噪声和不精确的电缆中可靠传输 Reliably transmitted on noisy and inaccurate wires



An Amazing & Successful Abstraction.

(which we won't dig into in this course)

举例：用二进制计数
For example, can count in binary

- **基数为2的数值表示 Base 2 Number Representation**
  - 0, 1, 10, 11, 100, 101, …
  - 十进制整数表示为二进制整数 Represent 15213_{10} as 11101101101101_2
  - 十进制小数表示为二进制小数 Represent 1.20_{10} as $1.0011001100110011[0011]\ldots_2$
  - 十进制科学计数法表示为二进制科学计数法 Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$
- **负数表示为。。。？ Represent negative numbers as …?**

二进制数的性质
Binary Number Property

声明/断言 Claim

$$1 + 1 + 2 + 4 + 8 + \ldots + 2^{w-1} = 2^w$$

$$1 + \sum_{i=0}^{w-1} 2^i = 2^w$$

- **w = 0:**
  - $1 = 2^0$
- **假设对于w-1为真 Assume true for w-1:**
  - $\underbrace{1 + 1 + 2 + 4 + 8 + \ldots + 2^{w-1}}_{=\ 2^w} + 2^w = 2^w + 2^w = 2^{w+1}$

多个位组成组 Group Bits

- 孤立地看，单个位不是很有用 **In isolation, a single bit is not very useful**
- 在英语中，它的字母表中有26（或52）个字符。它们单独使用也没有用 **In English, there are 26(or 52) characters in its alphabet. They are not useful either in isolation**
- 然而，它的词汇表中有很多单词，这是如何实现的? **However, there are plenty of words in its vocabulary. How is this achieved?**
- 同样，我们可以使用多个位（而不是单个位）来表示任何有限集的元素 **Similarly, we are able to represent the elements of any finite set by using <u>bits</u> (instead of bit)**

多个位组成组 Group Bits

维纳 · 布赫霍尔兹

- **为此，我们 To do this, we**
  - 首先把多个比特组合在一起 first group bits together
  - 然后应用某种解释给不同的可能位模式 then apply some *interpretation* to the different possible bit patterns
    - 给每个模式一个意义 that gives meaning to each patterns
- **8位数据块组织成一个字节 8-bit chunks are organized as a byte**
  - 1956年7月维纳博士提出 Dr. Werner Buchholz in July 1956
  - IBM草稿计算机早期设计阶段 during the early design phase for the IBM Stretch computer

多个位的值 Value of Bits

```
位序列 Bits          01010
值 Value            0*2^4+1*2^3+0*2^2+1*2^1+0*2^0 = 10

值 Value            102(1100110)
位序列 Bits     102 = 51*2 + 0 (0)
                51  = 25*2 + 1 (1)
                25  = 12*2 + 1 (1)
                12  =  6*2 + 0 (0)
                 6  =  3*2 + 0 (0)
                 3  =  1*2 + 1 (1)
                 1  =  0*2 + 1 (1)
```

编码字节值 Encoding Byte Values

- **一个字节包含8比特位 Byte = 8 bits**
  - 十进制取值范围 Decimal: 0_{10} to 255_{10}
    - $255=2^8 - 1$
  - 二进制取值范围 Binary 00000000_2 to 11111111_2
  - 十六进制取值范围 Hexadecimal 00_{16} to FF_{16}
    - 基数为16的数值表示Base 16 number representation
    - 字符0-9和A-F Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - C语言中写作前导‘0x’，以下情况之一 Write in C with leading ‘0x’, either case
      - $0101\ 1010_2$ = `0x5a` = `0x5A` = `0X5a`

十六进制 Hex	十进制 Decimal	二进制 Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$15213:\ \underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$$

十六进制对二进制 Hexadecimal vs. Binary

```
0x173A4C
Hexadecimal      1    7    3    A    4    C
Binary        0001 0111 0011 1010 0100 1100

1111001010110110110011
Binary              11 1100 1010 1101 1011 0011
Hexadecimal          3    C    A    D    B    3
0x3CADB3
```

十六进制对十进制 Hexadecimal vs. Decimal

```
Hexadecimal     0xA7
Decimal         10*16+7 = 167

Decimal         314156 = 19634*16 + 12 (C)
                 19634 =  1227*16 +  2 (2)
                  1227 =    76*16 + 11 (B)
                    76 =     4*16 + 12 (C)
                     4 =     0*16 +  4 (4)
Hexadecimal     0x4CB2C
```

十六进制对二进制 Hexadecimal vs. Binary

- 1100100101111011 -> C97B
- 1001101110011110110101 -> 2 6 E 7 B 5

十进制、十六进制和二进制
Decimal, Hexadecimal, Binary

Decimal	Binary	Hexadecimal
62	00111110	0x3E
3*16+7=55	0011 0111	0x37
5*16+2=82	01010010	0x52

十六进制 Hexadecimal

- 0x503c + 0x8 = 0x5044
- 0x503c – 0x40 = 0x4ffc
- 0x503c + 64 = 0x507c
- 0x50ea -0x503c = 0xae

组合字节以创建标量数据类型
Combine bytes to make *scalar data types*

	大小(字节数)Size(# of bytes)	
C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8
	“ILP32”	“LP64”

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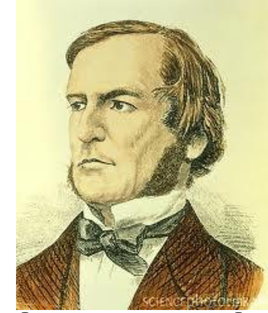
布尔代数 Boolean Algebra



- **19世纪由布尔开发 Developed by George Boole in 19th Century**
  - 逻辑的代数表示 Algebraic representation of logic
    - 逻辑值真和假编码为1和0 Encode “True” as 1 and “False” as 0

与 And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

或 Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

非 Not

- **~A = 1 when A=0**

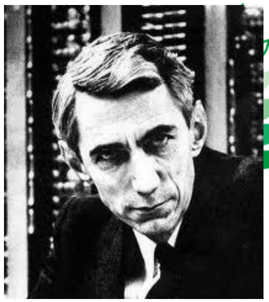
~	
0	1
1	0

异或 Exclusive-Or (Xor)

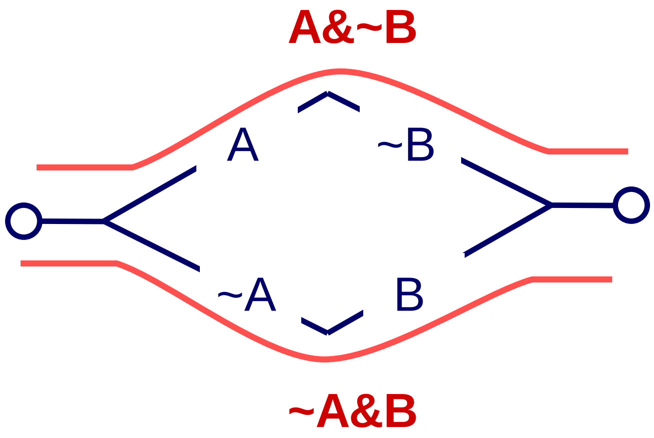
- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

布尔代数的应用
Application of Boolean Algebra



- **由香农应用到数字系统中，信息论的奠基人 Applied to Digital Systems by Claude Shannon， founded the information theory**
  - 1937年MIT硕士论文 1937 MIT Master’s Thesis
  - 对继电器开关网络进行推理 Reason about networks of relay switches
    - 开关闭合编码为1，开关打开为0 Encode closed switch as 1, open switch as 0



当满足下面条件是连通
Connection when

A&~B | ~A&B

= A^B

一般布尔代数
General Boolean Algebras

- **对比特位向量进行运算 Operate on Bit Vectors**
  - 将运算运用到每个比特位 Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001     01111101     00111100     10101010
```

- **布尔代数的所有性质都可运用 All of the Properties of Boolean Algebra Apply**

举例：小整数集合
Example: Sets of Small Integers

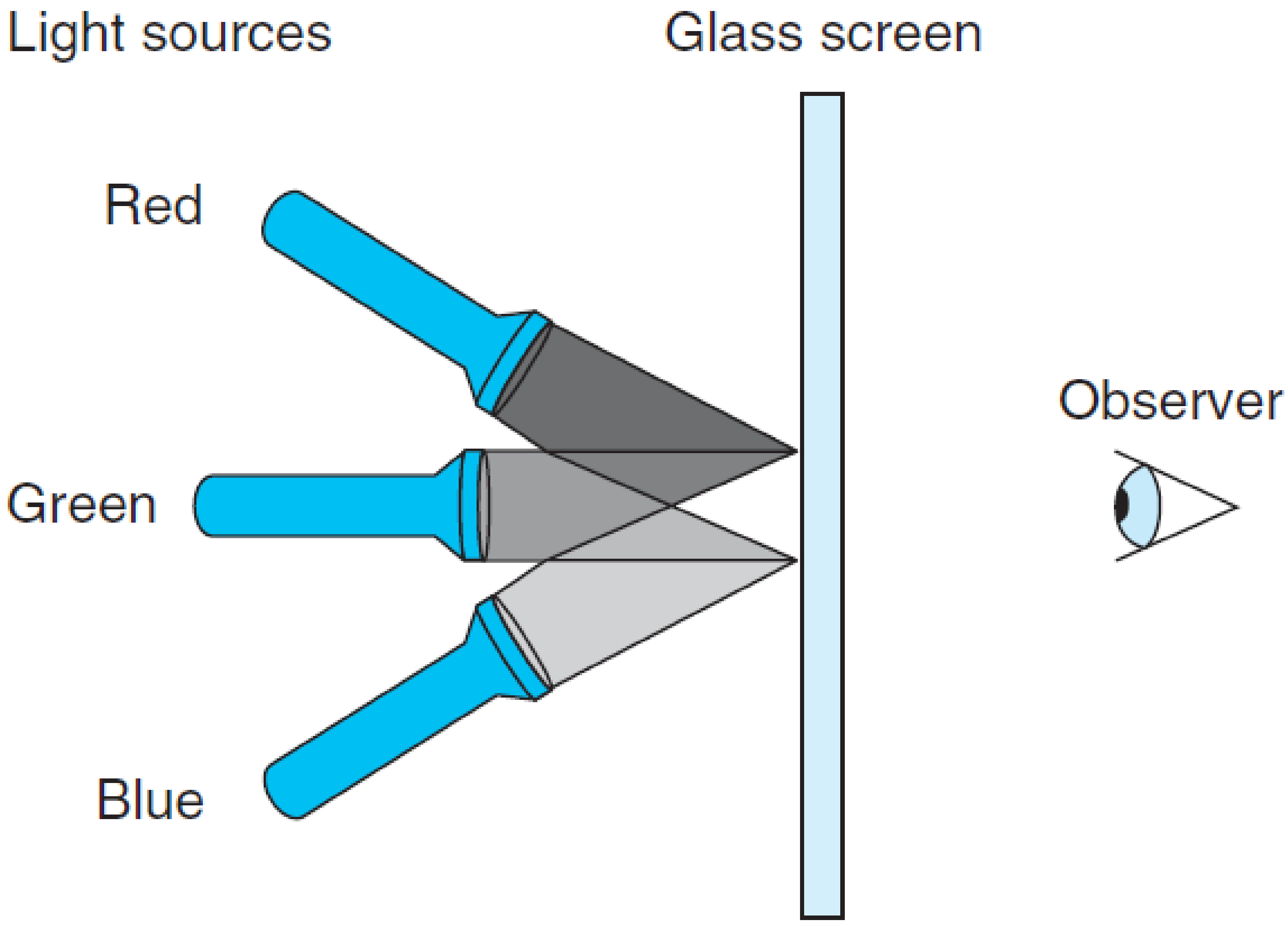
- **表示 Representation**
  - 宽度为w的比特位向量表示{0, …, w–1}的子集 Width w bit vector represents subsets of {0, …, w–1}
  - 当j属于A集合时，a_j为1　$a_j = 1$ if $j \in A$

    - 01101001　　{ 0, 3, 5, 6 }
    - *76543210*

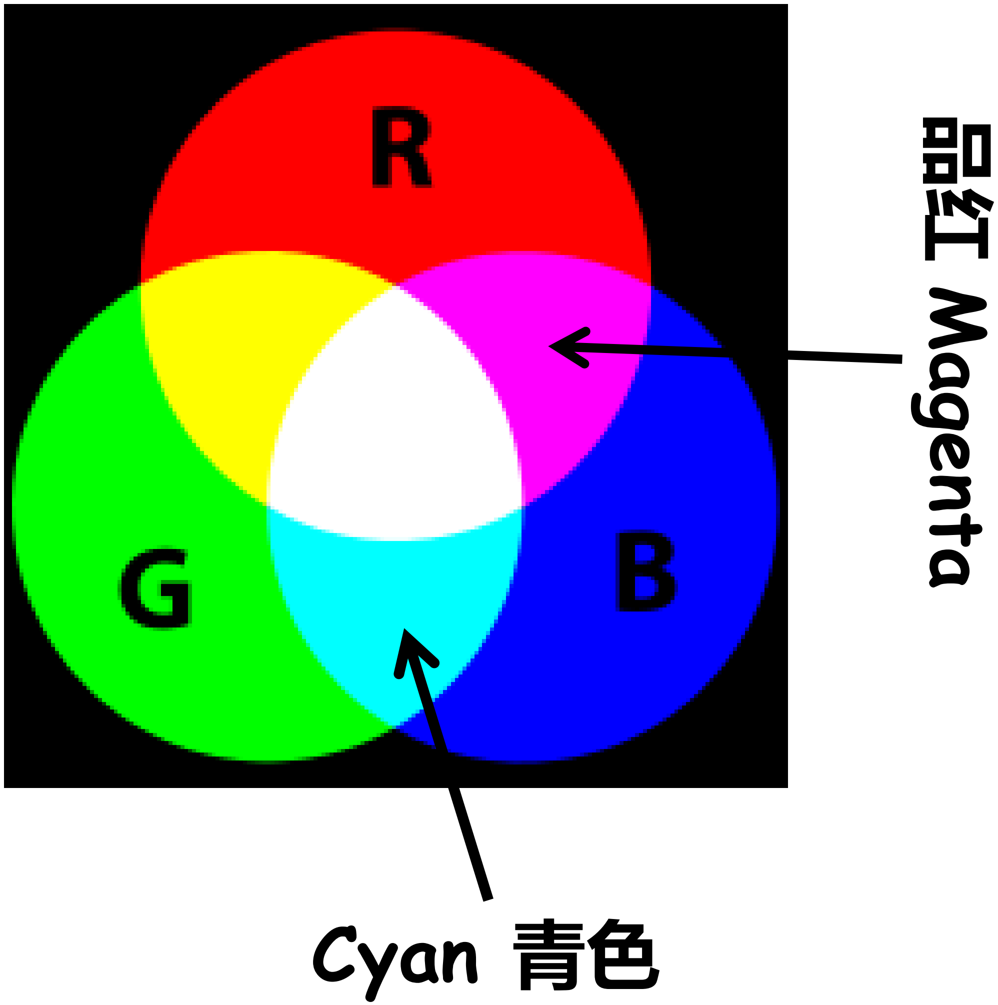
    - 01010101　　{ 0, 2, 4, 6 }
    - *76543210*

- **运算 Operations**
  - &　交 Intersection　01000001　{ 0, 6 }
  - |　并 Union　01111101　{ 0, 2, 3, 4, 5, 6 }
  - ^　对称差 Symmetric difference　00111100　{ 2, 3, 4, 5 } 异或
  - ~　补Complement　10101010　{ 1, 3, 5, 7 }

三基色模型 RGB Color Model



三基色模型 RGB Color Model



R	*G*	*B*	Color
0	0	0	Black
0	0	1	Blue
0	1	0	Green
0	1	1	Cyan
1	0	0	Red
1	0	1	Magenta
1	1	0	Yellow
1	1	1	White

C语言中的比特级运算
Bit-Level Operations in C

- **C中可用运算 Operations &, |, ~, ^ Available in C**
  - 运用到任何整数类数据类型 Apply to any “integral” data type
    - long, int, short, char, unsigned
  - 参数视为比特位向量 View arguments as bit vectors
  - 参数运用到每个比特位 Arguments applied bit-wise
- **举例（字符数据类型） Examples (Char data type)**
  - ~0x41 →
  - ~0x00 →
  - 0x69 & 0x55 →
  - 0x69 | 0x55 →

C语言中的比特级运算
Bit-Level Operations in C

- **C中可用运算 Operations &, |, ~, ^ Available in C**
  - 运用到任何整数类数据类型 Apply to any “integral” data type
    - long, int, short, char, unsigned
  - 参数视为比特位向量 View arguments as bit vectors
  - 参数运用到每个比特位 Arguments applied bit-wise
- **举例（字符数据类型） Examples (Char data type)**
  - ~0x41 → 0xBE
    - $\sim 0100\ 0001_2 \rightarrow 1011\ 1110_2$
  - ~0x00 → 0xFF
    - $\sim 0000\ 0000_2 \rightarrow 1111\ 1111_2$
  - 0x69 & 0x55 → 0x41
    - $0110\ 1001_2\ \&\ 0101\ 0101_2 \rightarrow 0100\ 0001_2$
  - 0x69 | 0x55 → 0x7D
    - $0110\ 1001_2 \mid 0101\ 0101_2 \rightarrow 0111\ 1101_2$

用异或进行很酷的操作
Cool Stuff with Xor

- **比特位级异或是一种加法 Bitwise Xor is form of addition**
- 具有额外的性质，即每个值都是其自身的加法逆元 **With extra property that every value is its own additive inverse**
  - A ^ A = 0

用异或进行很酷的操作
Cool Stuff with Xor

```
int  inplace_swap(int *x, int *y)
{
   *x = *x ^ *y;    /* #1 */
   *y = *x ^ *y;    /* #2 */
   *x = *x ^ *y;    /* #3 */
}
```

Step	`*x`	`*y`
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A^(B^B) = A^0 = A
3	(A^B)^A = (B^A)^A = B^(A^A) = B^0 = B	A
End	B	A

用异或进行很酷的操作
Cool Stuff with Xor

```
void reverse_array(int a[], int cnt) {
    int first, last;
    for (first = 0, last = cnt-1;
         first <= last;
         first++,last--)
        inplace_swap(&a[first], &a[last]);
}
```

掩码操作 Mask Operations

- **位模式 Bit pattern**
  - 0xFF
    - 最低8个有效位为1 Having 1s for the least significant eight bits
    - 指明一个字的最低字节 Indicates the lower-order byte of a word
- **掩码操作 Mask Operation**
  - X = 0x89ABCDEF
  - X & 0xFF =?
- **位模式~0 Bit Pattern ~0**
  - 为何不是0xFFFFFFFF? Why not 0xFFFFFFFF?
  - 无论字长是多少 No matter how word size

掩码操作 Mask Operations

- 写C表达式使其对任何字长w大于等于8的数都能适用 Write C expressions that work for any word size $w \geq 8$
- For $x = 0x87654321$, with $w = 32$
- x的最低有效字节，其它位都设为0 The least significant byte of x, with all other bits set to 0
  - [0x00000021]

x & 0xFF

掩码操作 Mask Operations

- **x的最低有效字节不变，所有其它位变反（取补） All but the least significant byte of complemented, with the least significant byte left unchanged**
  - [0x789ABC21] x ^ ~0xFF
- **x的最低有效字节设置为全1，所有其它字节保持不变 The least significant byte set to all 1s, and all other bytes of x left unchanged.**
  - [0x876543FF]. x | 0xFF

位设置和位清除 Bis & Bic

- **设置结果z为x并进行修改 Set result z to x and modify it**
  - X data， m mask
- **z = bis (int x, int m) (位设置 bit set) = x | m**
  - 在m为1的每个位置上，将结果z的对应位设置为1 Set result z to 1 at each bit position where m is 1
- **z = bic(int x, int m) (位清除 bit clear) = x & ~m**
  - 在m为1的每个位置上，将结果z的对应位设置为0 set result z to 0 at each bit position where m is 1
- **DEC公司的VAX机 The Digital Equipment VAX**
- **使用位设置和位清除来实现 Use bis and bic to implement**
  - Or(int x, int y)   bis(x, y)
  - Xor(int x, int y)   bis(bic(x, y), bic(y,x))
    - (x&~y) | (y&~x)

对比：C语言中的逻辑运算
Contrast: Logic Operations in C

- **对比比特位级的操作符 Contrast to Bit-Level Operators**
  - **逻辑运算 Logic Operations: &&, ||, !**
    - 视0为假 View 0 as "False"
    - 任何非零视为真 Anything nonzero a
    - 总是返回0或1 Always retur
    - 提前终止 Early termination
- **举例（字符数据类型） Exa**
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41→ 0x01

  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)避免访问空指针

注意&& vs. &(以及|| vs. |)...
超级常见的C编程错误!
Watch out for && vs. & (and || vs. |)...
Super common C programming pitfall!

逻辑运算的快捷方式
Short Cut in Logical Operations

- **a && 5/a**
  - 如果a为零，不会计算5/a If a is zero, the evaluation of 5/a is stopped
  - 避免了被零除 avoid division by zero
- **p && *p**
  - 不会导致间接引用空指针 Never cause the dereferencing of a null pointer
- **仅使用位级和逻辑操作 Using only bit-level and logical operations**
  - 实现x==y Implement x == y
  - x和y相等时返回1，否则返回0 it returns 1 when x and y are equal, and 0 otherwise
  - !(x^y)

移位运算 Shift Operations

- **左移 Left Shift: `x << y`**
  - 位向量x左移y位 Shift bit-vector **x** left **y** positions
    - 丢弃左边多余比特 Throw away extra bits on left
    - 右边填0 Fill with 0's on right
- **右移 Right Shift: `x >> y`**
  - 位向量x右移y位 Shift bit-vector **x** right **y** positions
    - 丢弃右边多余的比特 Throw away extra bits on right
  - 逻辑移位 Logical shift
    - 左边填0 Fill with 0's on left
  - 算术移位 Arithmetic shift
    - 左边复制最高有效位 Replicate most significant bit on left
- **未定义行为 Undefined Behavior**
  - 移位量小于零或大于等于字长 Shift amount < 0 or ≥ word size

Argument **x**	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument **x**	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

C语言中的移位操作
Shift Operations in C

- **会发生什么情况? What happens ?**
  - int lval = 0xFEDCBA98 << 32;
  - int aval = 0xFEDCBA98 >> 36;
  - unsigned uval = 0xFEDCBA98u >> 40;
- **可能的情况 It may be   k mod w**
  - lval 0xFEDCBA98 (0)
  - aval 0xFFEDCBA9 (4)
  - uval 0x00FEDCBA (8)
- **要小心 Be careful about**
  - 1<<2 + 3<<4 means 1<<(2 + 3)<<4
  - 512
  - Not 52

位计数 bitCount

- 返回字中1的个数 Returns number of 1's a in word
- 例如： Examples: bitCount(5) = 2, bitCount(7) = 3
- 合法的运算符： Legal ops: ! ~ & ^ | + << >>
- 最大运算符：40   Max ops: 40

求和4位一组共8组
Sum 8 groups of 4 bits each

```
int bitCount(int x) {
    int m1 = 0x11 | (0x11 << 8);
    int mask = m1 | (m1 << 16);
    int s = x & mask;
    s += x>>1 & mask;
    s += x>>2 & mask;
    s += x>>3 & mask;
```

将和组合在一起 Combine the sums

```
/* Now combine high and low order sums */
 s = s + (s >> 16);

 /* Low order 16 bits now consists of 4 sums.
    Split into two groups and sum */
 mask = 0xF | (0xF << 8);
 s = (s & mask) + ((s >> 4) & mask);
 return (s + (s>>8)) & 0x3F;
}
```

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编码整数 Encoding Integers

无符号 Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

补码 Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

符号位 Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- C语言并不强制使用二进制补码 C does not mandate using two's complement
  - 然而大部分机器一般会用二进制补码进行运算, 我们也如此假设 But, most machines do, and we will assume so
- C语言中short为2字节长 C `short` 2 bytes long

	十进制 Decimal	十六进制 Hex	二进制 Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

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符号位
Sign
Bit

```
short int x =  15213;
short int y = -15213;
```

- **符号位 Sign Bit**
  - 对于补码最高有效位是符号位 For 2's complement, most significant bit indicates sign
    - 0表示非负 0 for nonnegative
    - 1表示负 1 for negative

补码：简单示例
Two-complement: Simple Example

		-16	8	4	2	1	
10	=	0	1	0	1	0	8+2 = 10

		-16	8	4	2	1	
-10	=	1	0	1	1	0	-16+4+2 = -10

补码编码举例

Two-complement Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

无符号数表示 Unsigned Representation

- **二进制（物理上） Binary (physical)**
  - 位向量 Bit vector [x_{w-1},x_{w-2},x_{w-3},...x_0]
- **二进制到无符号数（逻辑上） Binary to Unsigned (logical)**

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

补码 Two's Complement

- **二进制（物理上） Binary (physical)**
  - 位向量 Bit vector $[x_{w-1}, x_{w-2}, x_{w-3}, \ldots x_0]$
- **二进制到有符号数（逻辑上） Binary to Signed (logical)**

$$B2T(X) \quad = \quad -\underbrace{x_{w-1}}_{\text{符号位 Sign Bit}} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

- **补码 2's complement**

从补码到二进制
From Two's Complement to Binary

- 如果是非负数 If nonnegative
  - 不需要变化 Nothing changes
- 如果是负数，其二进制表示为 If negative, its binary representation is
  - $1x_{w-2}\ldots x_1x_0$
  - 其值为x Its value is x
  - 假设其绝对值为y=-x Assume its absolute value is y=-x
- y的二进制表示为 The binary representation of y is
  - $0y_{w-2}\ldots y_1y_0$

从补码到二进制(x是负数)
From Two's Complement to Binary

$$x = -2^{w-1} + \sum_{i=0}^{w-2} x_i 2^i = -y = -\sum_{i=0}^{w-2} y_i 2^i$$

$$\sum_{i=0}^{w-2} x_i 2^i = 2^{w-1} - \sum_{i=0}^{w-2} y_i 2^i = \sum_{i=0}^{w-2} (1 - y_i) 2^i + 1$$

$$2^{w-1} = \sum_{i=0}^{w-2} 2^i + 1 \qquad x_{w-1} = 1 \qquad y_{w-1} = 0$$

补码 Two's Complement

$$\sum_{i=0}^{w-1} x_i 2^i = \sum_{i=0}^{w-1} (1 - y_i) 2^i + 1$$

- **这个公式意味着什么呢？ What does it mean?**
  - 计算x的绝对值成w位的二进制数 Computing the negation of x into binary with w-bits
  - 对结果变反（求补） Complementing the result
  - 加一 Adding 1

从十进制数变换成补码
From a Number to Two's Complement

- **-5**
  - 5
  - 0101 (5的二进制表示 binary for 5)
  - 1010 (变反 after complement)
  - 1011 (加一 add 1)

补码编码示例
Two's Complement Encoding Examples

二进制/十六进制表示 Binary/Hexadecimal Representation for -12345

二进制 Binary: 0011 0000 0011 1001 (12345)

十六进制 Hex: 3 0 3 9

二进制 Binary: 1100 1111 1100 0110 (变反后 after complement)

十六进制 Hex: C F C 6

二进制 Binary: 1100 1111 1100 0111 (加一 add 1)

十六进制 Hex: C F C 7

数值范围 Numeric Ranges

- 无符号值 Unsigned Values
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1

- 补码值 Two's Complement Values
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1

- 其它值 Other Values
  - 负一（-1）Minus 1
    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

不同字长的取值范围
Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- **观察 Observations**
  - $|TMin| = TMax + 1$
    - 非对称范围 Asymmetric range
  - $UMax = 2 * TMax + 1$
  - Question: abs(TMin)?

- **C语言编程 C Programming**
  - #include <limits.h>
  - 声明常量 Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - 值随平台而定 Values platform specific

无符号和有符号数的数值
Unsigned & Signed Numeric Values

X	B2U(X)	B2T(X)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **等同的 Equivalence**
  - 非负值同样的编码 Same encodings for nonnegative values
- **惟一性 Uniqueness**
  - 每个比特模式表示惟一的整数值 Every bit pattern represents unique integer value
  - 每个可表示的整数有惟一的比特位编码 Each representable integer has unique bit encoding
- **⇒ 能够逆映射Can Invert Mappings**
  - U2B(x) = B2U^{-1}(x)
    - 无符号整数比特模式Bit pattern for unsigned integer
  - T2B(x) = B2T^{-1}(x)
    - 补码整数比特模式 Bit pattern for

其它有符号数表示

Alternative representations of signed numbers

- **反码：与补码相同，除了最高位权值为$-(2^{w-1}-1)$而不是-2^{w-1}**
  Ones' complement：The same as two's complement, except that the most significant bit has weight $-(2^{w-1}-1)$ rather than -2^{w-1}

$$B2O_w(\vec{x}) = -x_{w-1}\left(2^{w-1}-1\right) + \sum_{i=0}^{w-2} x_i 2^i$$

  - 过去有使用反码的机器，现在都用补码 use in the past，Now use two's complement
  - +0：00000000　　　　-0：11111111
- **原码：最高有效位是符号位，决定剩余位是负权还是正权**
  Sign magnitude：The most significant bit is a sign bit that determines whether the remaining bits should be given negative or positive weight

$$B2S_w(\vec{x}) = (-1)^{x_{w-1}} \cdot \left(\sum_{i=0}^{w-2} x_i 2^i\right)$$

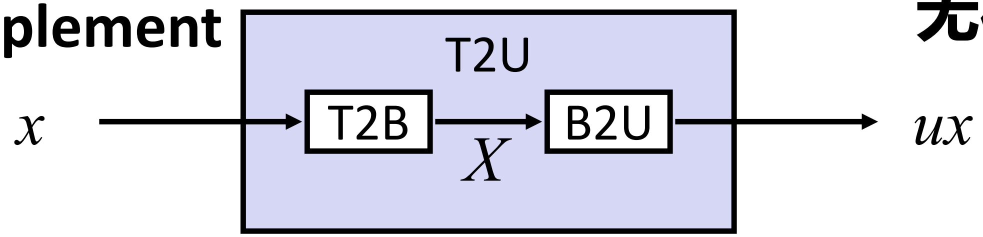
  - 在浮点数表示中还在使用 Used with floating-point numbers
  - +0：00000000　　　　-0：10000000

议题: 比特、字节和整数
Bits, Bytes, and Integers

- 用比特表示信息 Representing information as bits
- 比特级操作 Bit-level manipulations
- **整数 Integers**
  - 无符号数和有符号数表示 Representation: unsigned and signed
  - **转换和强制类型转换 Conversion, casting**
  - 扩展和截断 Expanding, truncating
  - 加、补码非、乘和移位 Addition, negation, multiplication, shifting
  - 小结 Summary
- 内存中的表示、指针和字符串 Representations in memory, pointers, strings

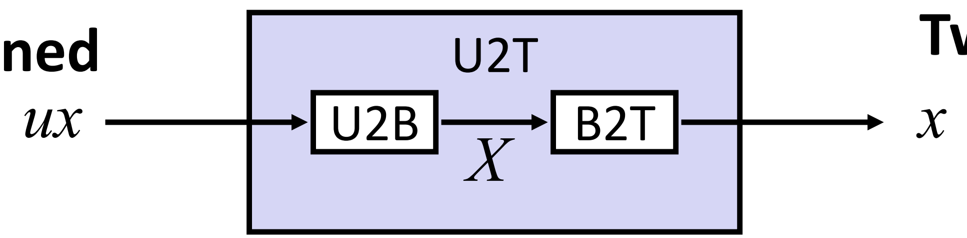
有符号数和无符号数之间进行映射
Mapping Between Signed & Unsigned

补码Two’s Complement **无符号数Unsigned**



保持相同的比特位模式 Maintain Same Bit Pattern

Unsigned **Two’s Complement**

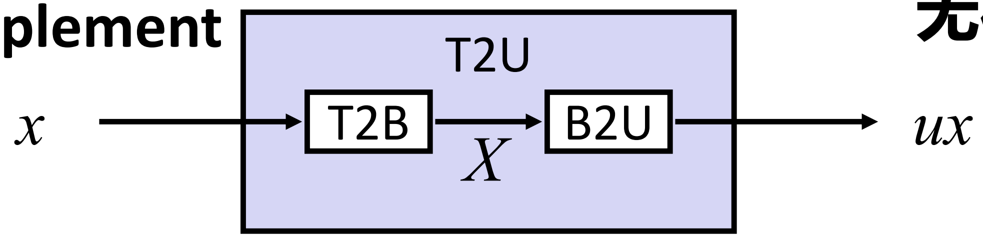


保持相同的比特位模式 Maintain Same Bit Pattern

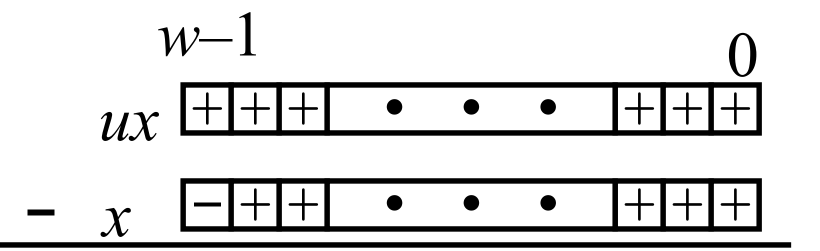
- **无符号数和补码之间进行映射Mappings between unsigned and two’s complement numbers:**
  保持位表示并重新解释 **Keep bit representations and reinterpret**

有符号数和无符号数之间进行映射
Mapping Between Signed & Unsigned

补码Two's Complement **无符号数Unsigned**



保持相同的比特位模式 Maintain Same Bit Pattern



$$+2^{w-1} - -2^{w-1} = 2*2^{w-1} = 2^w$$

$$ux = \begin{cases} x & x \ge 0 \\ x + 2^w & x < 0 \end{cases}$$

有符号和无符号数之间映射

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

→ T2U →

← U2T ←

有符号和无符号数之间映射

Mapping Signed ↔ Unsigned

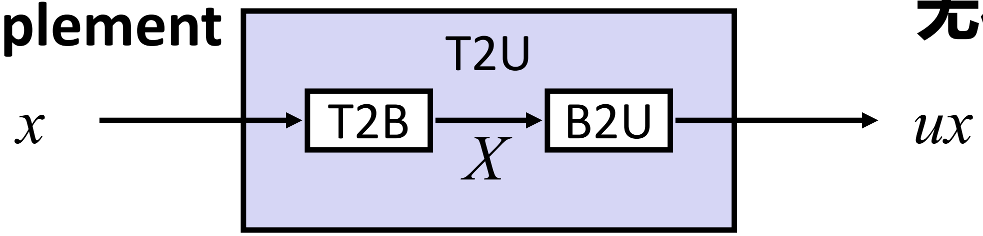
Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

=

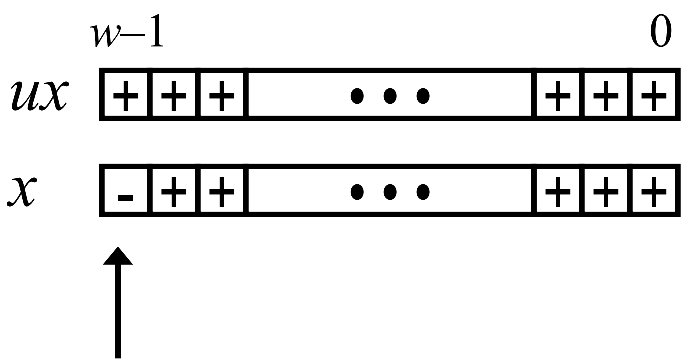
+/- 16

有符号数和无符号数之间的关系
Relation between Signed & Unsigned

补码Two's Complement **无符号Unsigned**



保持相同的比特位模式 Maintain Same Bit Pattern



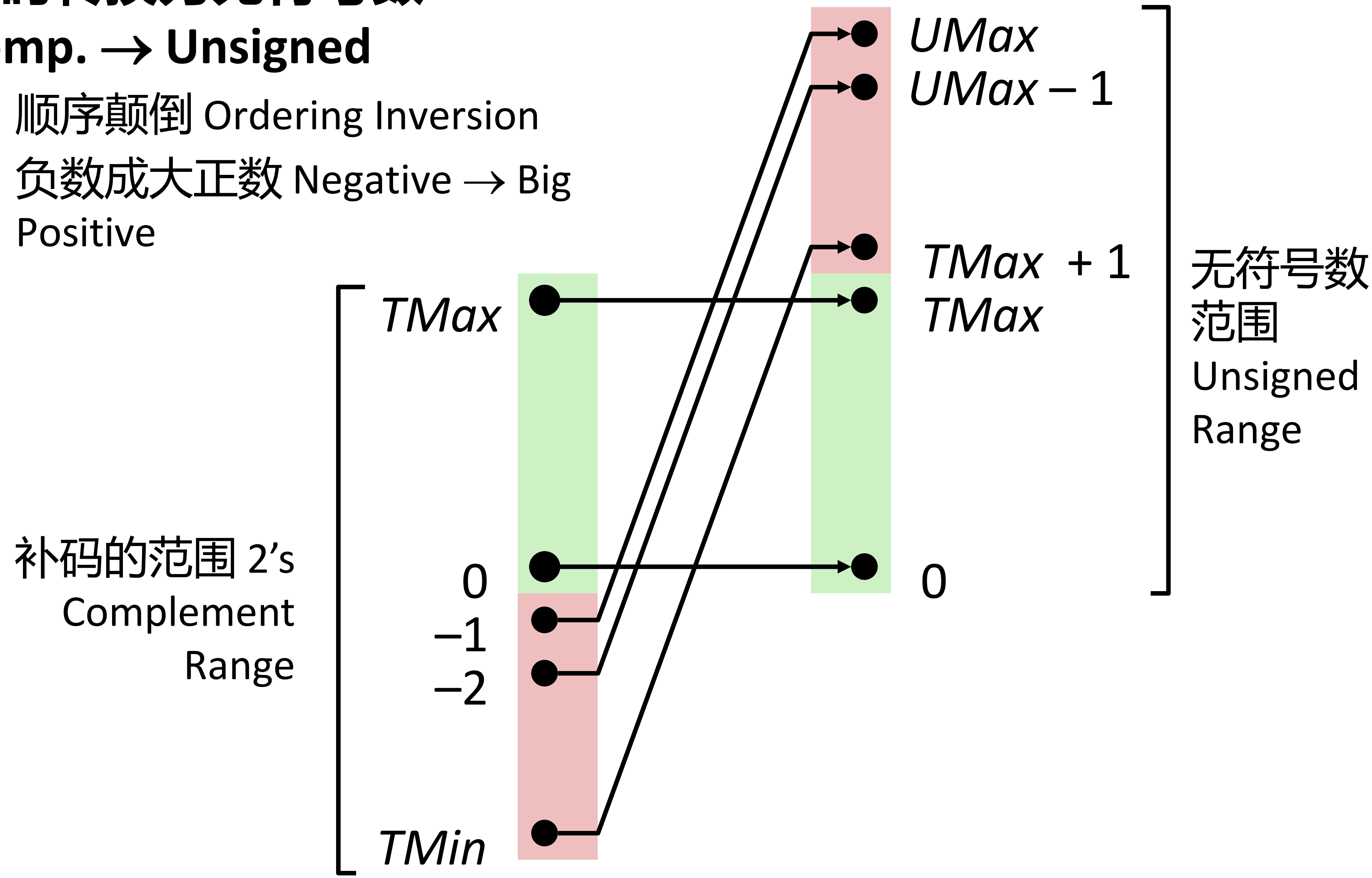
大的负权Large negative weight

变成becomes

大的正权Large positive weight

转换可视化 Conversion Visualized

- **补码转换为无符号数 2's Comp. → Unsigned**
  - 顺序颠倒 Ordering Inversion
  - 负数成大正数 Negative → Big Positive



C语言中的整数数据类型
Integral data type in C

- **有符号类型 Signed type (整型数 for integer numbers)**
  - char, short [int], int, long [int]
- **无符号类型 Unsigned type (非负数 for nonnegative numbers)**
  - unsigned char, unsigned short [int], unsigned [int], unsigned long [int]
- **Java没有无符号数据类型 Java has no unsigned data type**
  - 使用字节代替字符 Using byte to replace the char

C语言整数数据类型典型取值范围-32位
Typical Ranges for C integral data types 32

C语言声明 C declaration	典型32位 Typical 32-bit	
	最小 minimum	最大 maximum
char unsigned char	-128 0	127 255
short [int] unsigned short	-32,768 0	32,767 65,535
int unsigned [int]	-2,147,483,648 0	2,147,483,647 4,294,967,295
long [int] unsigned long	-2,147,483,648 0	2,147,483,647 4,294,967,295
int32_t uint32_t	-2,147,483,648 0	2,147,483,647 4,294,967,295
int64_t uint64_t	-9,223,372,036,854,775,800 0	9,223,372,036,854,775,800 18,446,744,073,709,551,615

C语言整数数据类型典型取值范围-64位
Typical Ranges for C integral data types 64

C语言声明 C declaration	典型64位 Typical 64-bit	
	最小 minimum	最大 maximum
char unsigned char	-128 0	127 255
short [int] unsigned short	-32,768 0	32,767 65,535
int unsigned [int]	-2,147,483,648 0	2,147,483,647 4,294,967,295
long [int] unsigned long	-9,223,372,036,854,775,800 0	9,223,372,036,854,775,800 18,446,744,073,709,551,615
int32_t uint32_t	-2,147,483,648 0	2,147,483,647 4,294,967,295
int64_t uint64_t	-9,223,372,036,854,775,800 0	9,223,372,036,854,775,800 18,446,744,073,709,551,615

C语言整数数据类型确保取值范围
Guaranteed Ranges for C integral data types

C语言声明 C declaration	典型32位 Typical 32-bit	
	最小 minimum	最大 maximum
char unsigned char	-127 0	127 255
short [int] unsigned short	-32,767 0	32,767 65,535
int unsigned [int]	-32,767 0	32,767 65,535
long [int] unsigned long	-2,147,483,648 0	2,147,483,647 4,294,967,295
int32_t uint32_t	-2,147,483,648 0	2,147,483,647 4,294,967,295
int64_t uint64_t	-9,223,372,036,854,775,800 0	9,223,372,036,854,775,800 18,446,744,073,709,551,615

C语言有/无符号数之间强制类型转换
Casting among Signed and Unsigned in C

- **C语言允许一种类型的变量解释为另一种数据类型**
  C Allows a variable of one type to be interpreted as other data type
  - 类型转换（隐式） Type conversion (implicitly)
  - 强制类型转换（显式） Type casting (explicitly)

C语言中的有符号数和无符号数
Signed vs. Unsigned in C

- **常量 Constants**
  - 默认为有符号整数 By default are considered to be signed integers
  - 有U做后缀表示无符号数 Unsigned if have “U” as suffix

    ```
    0U, 4294967259U
    ```
- **强制类型转换 Casting**
  - 显示强制类型转换有/无符号数等同于U2T/T2U Explicit casting between signed & unsigned same as U2T and T2U

    ```
    int tx, ty;
    unsigned ux, uy;
    tx = (int) ux;
    uy = (unsigned) ty;
    ```
  - 隐式强制类型转换通过赋值和过程调用也会发生 Implicit casting also occurs via assignments and procedure calls

    ```
    tx = ux;
    uy = ty;
    ```

强制从有符号数转换成无符号数
Casting from Signed to Unsigned

```
short int          x  =  12345;
unsigned short int ux = (unsigned short) x;
short int          y  = -12345;
unsigned short int uy = (unsigned short) y;
```

- 结果值 **Resulting Value**
  - 位表示没有变化 No change in bit representation
  - 非负数值没有变化 Nonnegative values unchanged
    - *ux* = 12345
  - 负值变成大的正值 Negative values change into (large) positive values
    - *uy* = 53191

Weight	12,345		−12,345		53,191	
	Bit	Value	Bit	Value	Bit	Value
1	1	1	1	1	1	1
2	0	0	1	2	1	2
4	0	0	1	4	1	4
8	1	8	0	0	0	0
16	1	16	0	0	0	0
32	1	32	0	0	0	0
64	0	0	1	64	1	64
128	0	0	1	128	1	128
256	0	0	1	256	1	256
512	0	0	1	512	1	512
1,024	0	0	1	1,024	1	1,024
2,048	0	0	1	2,048	1	2,048
4,096	1	4096	0	0	0	0
8,192	1	8192	0	0	0	0
16,384	0	0	1	16,384	1	16,384
±32, 768	0	0	1	−32,768	1	32,768
Total		12,345		−12,345		53,191

强制类型转换奇怪之处 Casting Surprises

- **表达式求值 Expression Evaluation**
  - 如果单一表达式中混合了无符号数和有符号数 If there is a mix of unsigned and signed in single expression,
    有符号值隐含强制转换成无符号数***signed values implicitly cast to unsigned***
  - 包括比较运算 Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 ,** **TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	**==**	**unsigned**
-1	0	**<**	**signed**
-1	0U	**>**	**unsigned**
2147483647	-2147483647-1	**>**	**signed**
2147483647U	-2147483647-1	**<**	**unsigned**
-1	-2	**>**	**signed**
(unsigned)-1	-2	**>**	**unsigned**
2147483647	2147483648U	**<**	**unsigned**
2147483647	(int) 2147483648U	**>**	**signed**

小结 Summary
有/无符号数强制转换：基本规则
Casting Signed ↔ Unsigned: Basic Rules

- **比特位模式保持不变 Bit pattern is maintained**
- **但是需要重新解释 But reinterpreted**
- **可能有不期望的效果：加或减2^w　Can have unexpected effects: adding or subtracting 2^w**

- **表达式包含带符号和无符号整数时 Expression containing signed and unsigned int**
  - 有符号数强制类型转换为无符号数 `int` is cast to `unsigned`!!

议题: 比特、字节和整数
Bits, Bytes, and Integers

- 用比特表示信息 Representing information as bits
- 比特级操作 Bit-level manipulations
- **整数 Integers**
  - 无符号数和有符号数表示 Representation: unsigned and signed
  - 转换和强制类型转换 Conversion, casting
  - **扩展和截断 Expanding, truncating**
  - 加、补码非、乘和移位 Addition, negation, multiplication, shifting
  - 小结 Summary
- 内存中的表示、指针和字符串 Representations in memory, pointers, strings

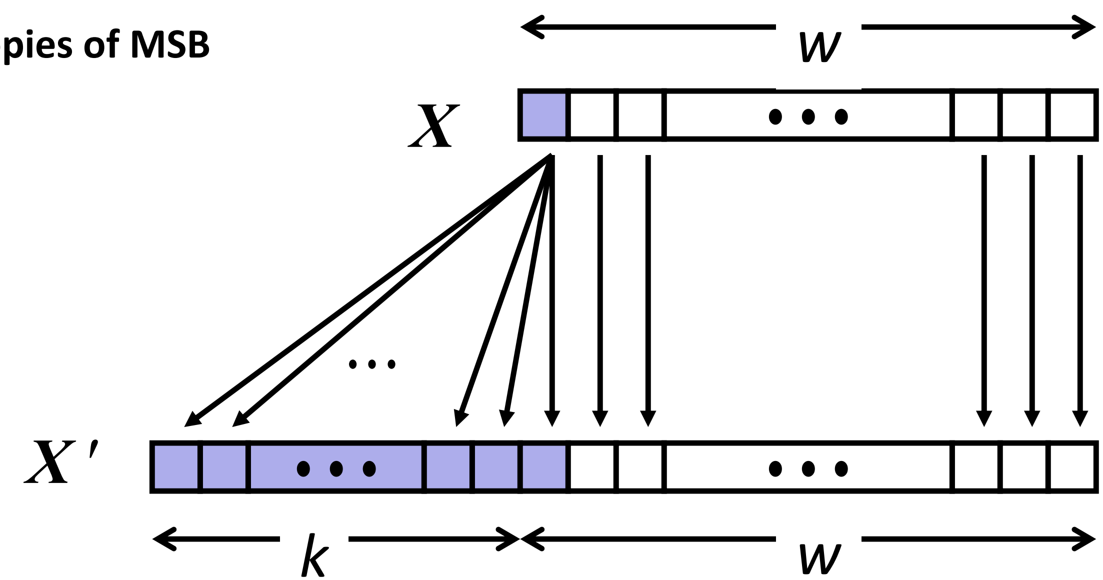
从短到长的转换 From short to long

```
short int x =  12345;
int ix = (int) x;
short int y = -12345;
int iy = (int) y;
```

- 我们需要扩展数据长度 We need to expand the data size
- 无符号类型之间强制类型转换正常 Casting among unsigned types is normal
- 有符号类型之间强制类型转换需要技巧 Casting among signed types is trick

符号扩展 Sign Extension

- **任务 Task:**
  - 给定w位的带符号整数x Given *w*-bit signed integer *x*
  - 转换成数值相同的w+k位整数 Convert it to *w*+*k*-bit integer with same value
- **规则 Rule: MSB-最高有效位**
  - 把符号位复制k位 Make *k* copies of sign bit:
  - $X' = \underbrace{x_{w-1},\ldots, x_{w-1}}_{k\ \textbf{copies of MSB}}, x_{w-1}, x_{w-2},\ldots, x_0$



符号位扩展：简单示例
Sign Extension: Simple Example

正数 Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

负数 Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

更大型符号位扩展举例
Larger Sign Extension Example

```
short int x =  15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- 从小整数数据类型转换到大整数数据类型 Converting from smaller to larger integer data type
- C语言自动执行符号扩展 C automatically performs sign extension

从短到长的扩展 From short to long

```
short int sx = 12345; : 0x0309
int x = (int) sx;       : 0x00000309
short int sy =-12345;: 0xcfc7
int y = (int) sy;       : 0xffffcfc7
usinged ux = sy;        : 0xffffcfc7
```

从短到长的扩展 From short to long

```
int fun1(unsigned word) {
        return (int) ((word << 24) >> 24);
}
int fun2(unsigned word) {
        return ((int) word << 24) >> 24;
}
```

w	fun1(w)	fun2(w)
0x00000076	**00000076**	**00000076**
0x87654321	**00000021**	**00000021**
0x000000C9	**000000C9**	**FFFFFFC9**
0xEDCBA987	**00000087**	**FFFFFF87**

描述每个函数执行时字中的有用计算 Describe in words the useful computation each of these functions performs.

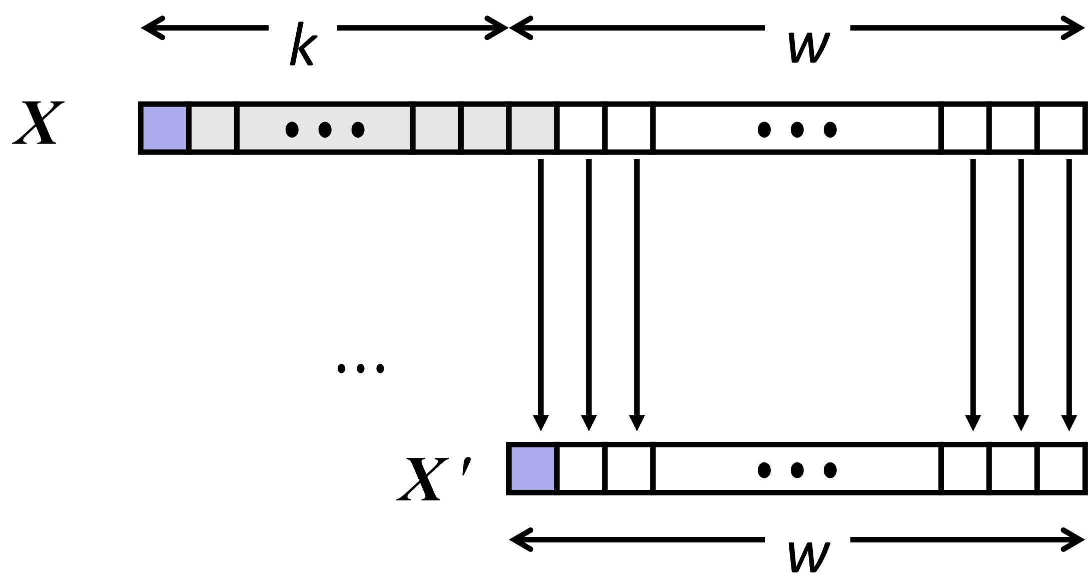
从长到短的转换 From long to short

```
int          x  = 53191;
short int sx = x;
int          y  = -12345;
Short int sy = y;
```

- 我们需要截断数据大小 We need to truncate the data size
- 强制类型转换从长到短需要技巧 Casting from long to short is trick

截断 Truncation

- **任务 Task:**
  - 给定k+w位有符号或无符号整数 X Given k+*w*-bit signed or unsigned integer *X*
  - 转换成w位整数X’，对于“足够小的”X具有同样的值 Convert it to *w*-bit integer X’ with same value for “small enough” X
- **规则 Rule:**
  - 丢弃头k位 Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



截断：简单示例 Truncation: Simple Example

符号位没变 No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

2 mod 16 = 2

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

-6 mod 16 = 26U mod 16 = 10U = -6

符号位变化 Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

10 mod 16 = 10U mod 16 = 10U = -6

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

-10 mod 16 = 22U mod 16 = 6U = 6

截断数值 Truncating Numbers

- Unsigned Truncating

$$B2U_w([x_w, x_{w\text{-}1}, \cdots, x_0]) mod 2^k$$
$$= B2U_k([x_k, x_{k-1}, \cdots, x_0])$$

- Signed Truncating

$$B2\mathrm{T}_k([x_k, x_{k-1}, \cdots, x_0])$$
$$= B2\mathrm{T}_k(B2U_w([x_w, x_{w-1}, \cdots, x_0]) mod 2^k)$$

小结 Summary:
扩展和截断：基本规则
Expanding, Truncating: Basic Rules

- **扩展（例如short扩展成int）Expanding (e.g., short int to int)**
  - 无符号数：添加零 Unsigned: zeros added
  - 有符号数：符号位扩展 Signed: sign extension
  - 都还会产生期望的结果 Both yield expected result

- **截断（例如无符号int截断成无符号short）Truncating (e.g., unsigned to unsigned short)**
  - 无/有符号数：比特位截断 Unsigned/signed: bits are truncated
  - 结果重新解释 Result reinterpreted
  - 无符号数：模取余运算 Unsigned: mod operation
  - 有符号数：类似模取余 Signed: similar to mod
  - 对于小的数值还会产生期望的行为 For small numbers yields expected behavior