4CM00: Control Engineering

FeedForward control

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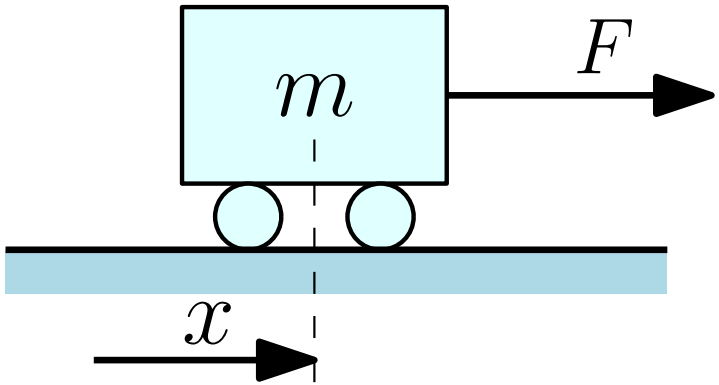
Where innovation starts

Influence of the setpoint

Consider a simple motion system

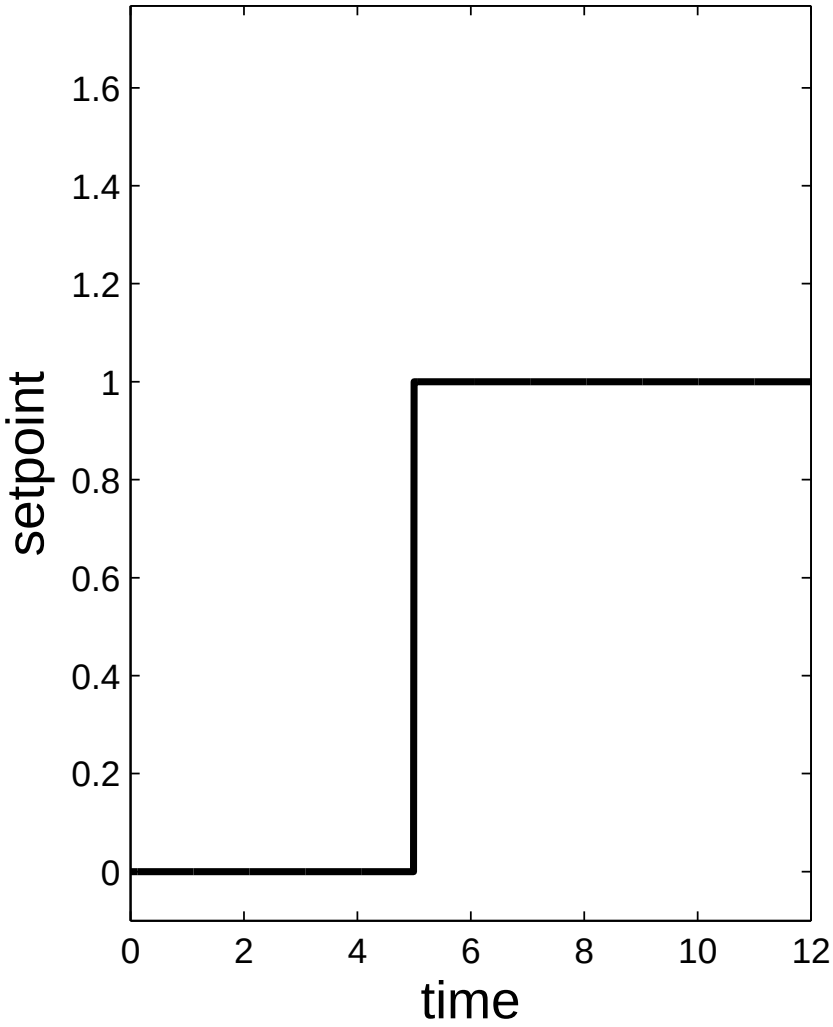


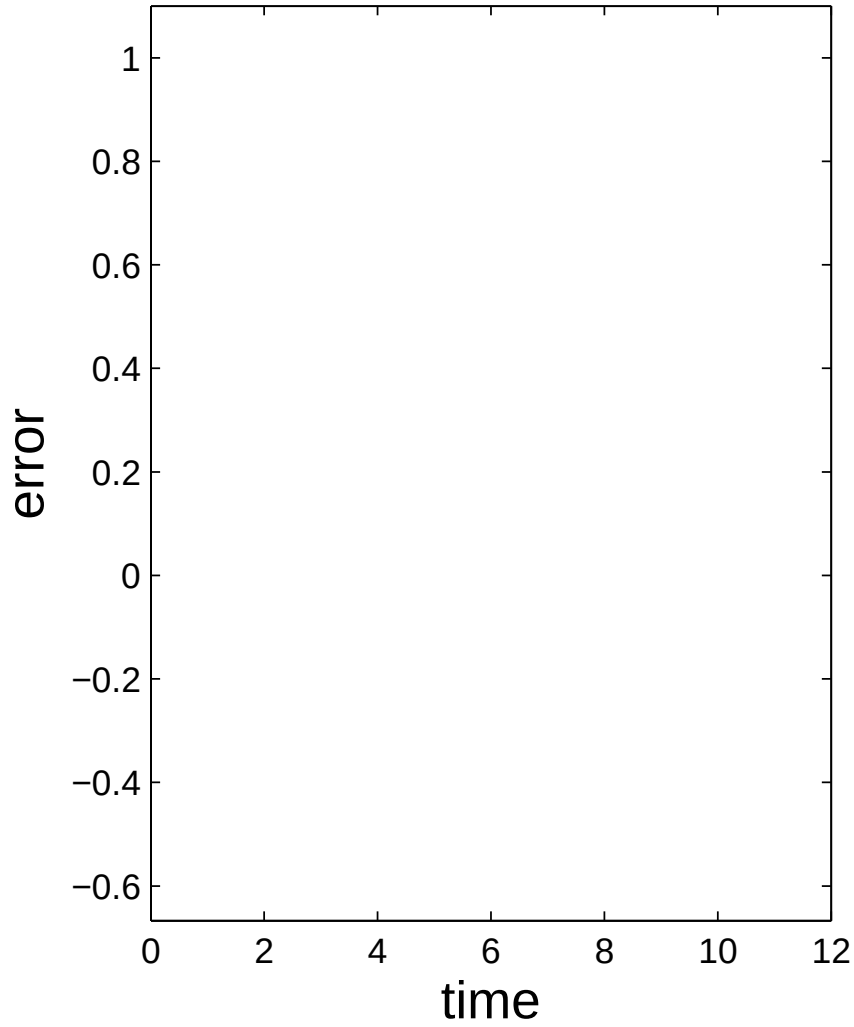
Objective: Move the mass move from position 0 to 1

Question 1: How does the error depend on the setpoint trajectory x_s? I.e. how do we get from 0 to 1?

Question 2: Can we obtain (nearly) perfect tracking using just a simple PD-controller?

Closed loop response: 0th order setpoint





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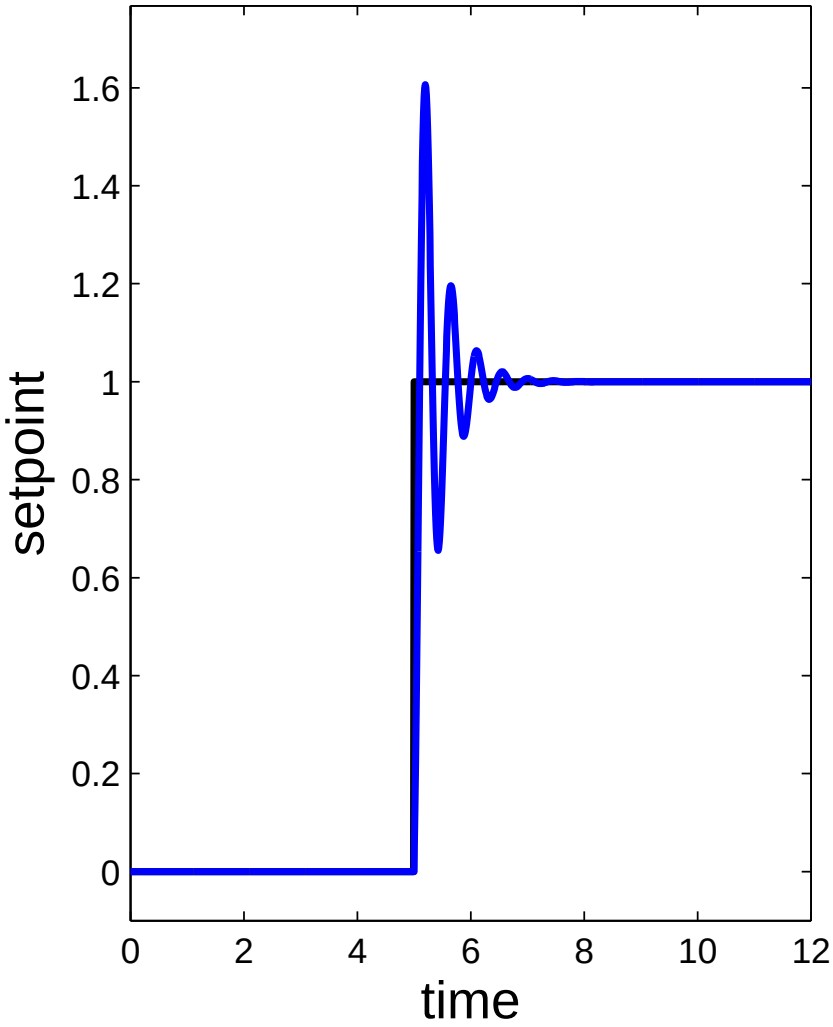
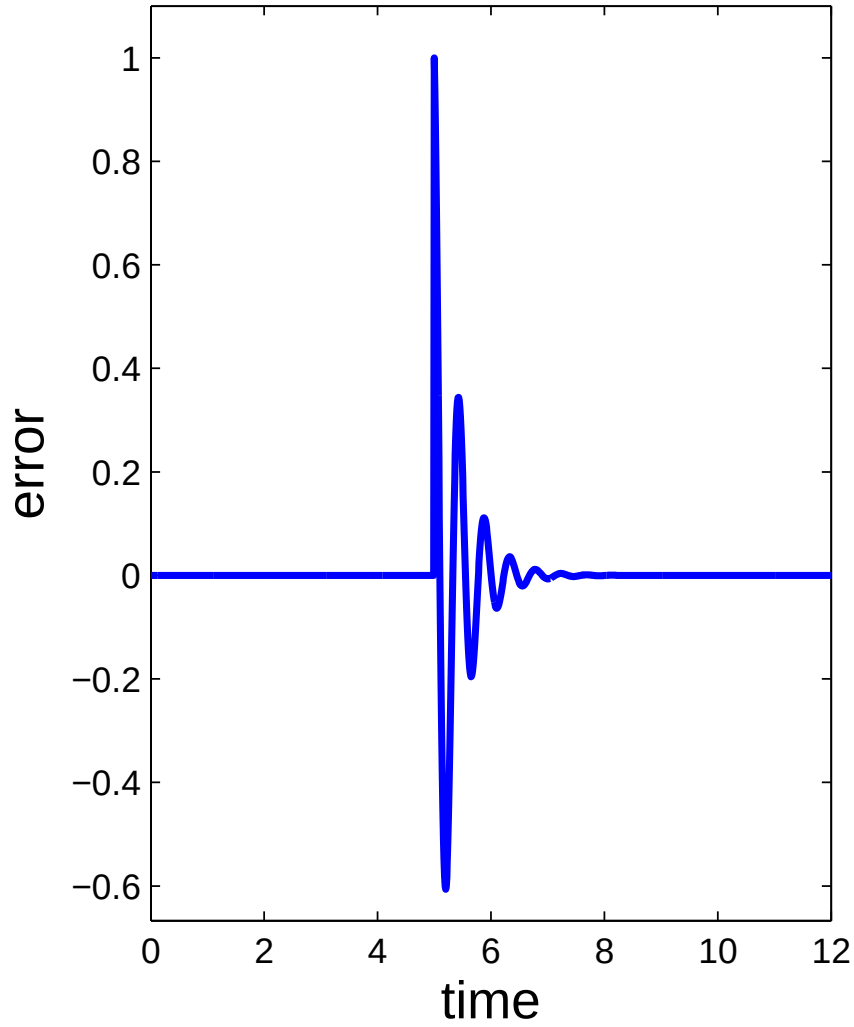
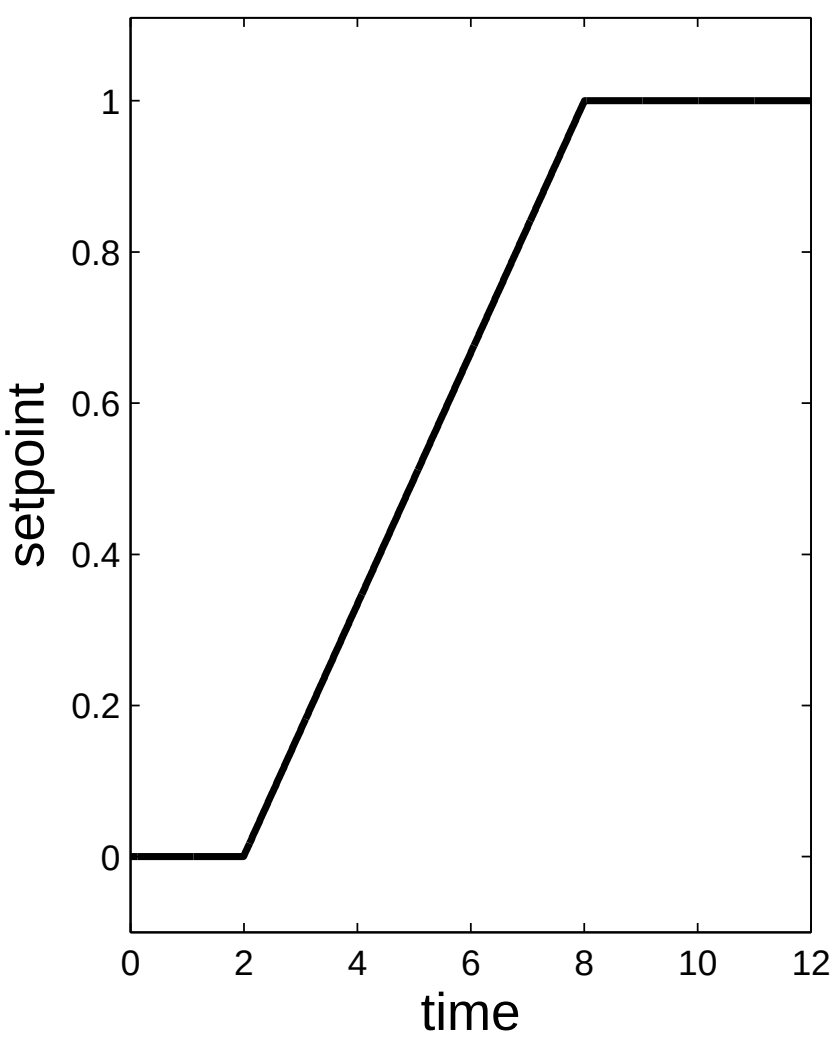
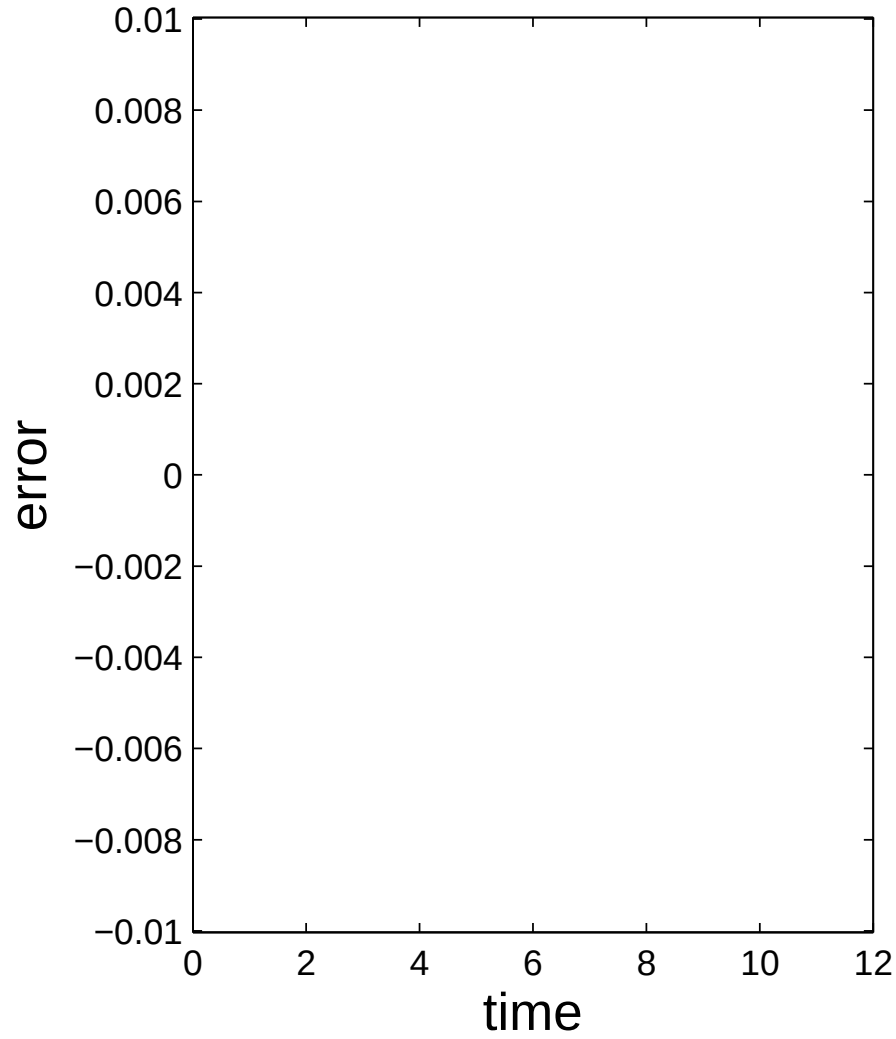




Maximal error / overshoot: 0.6

Closed loop response: 1st order setpoint





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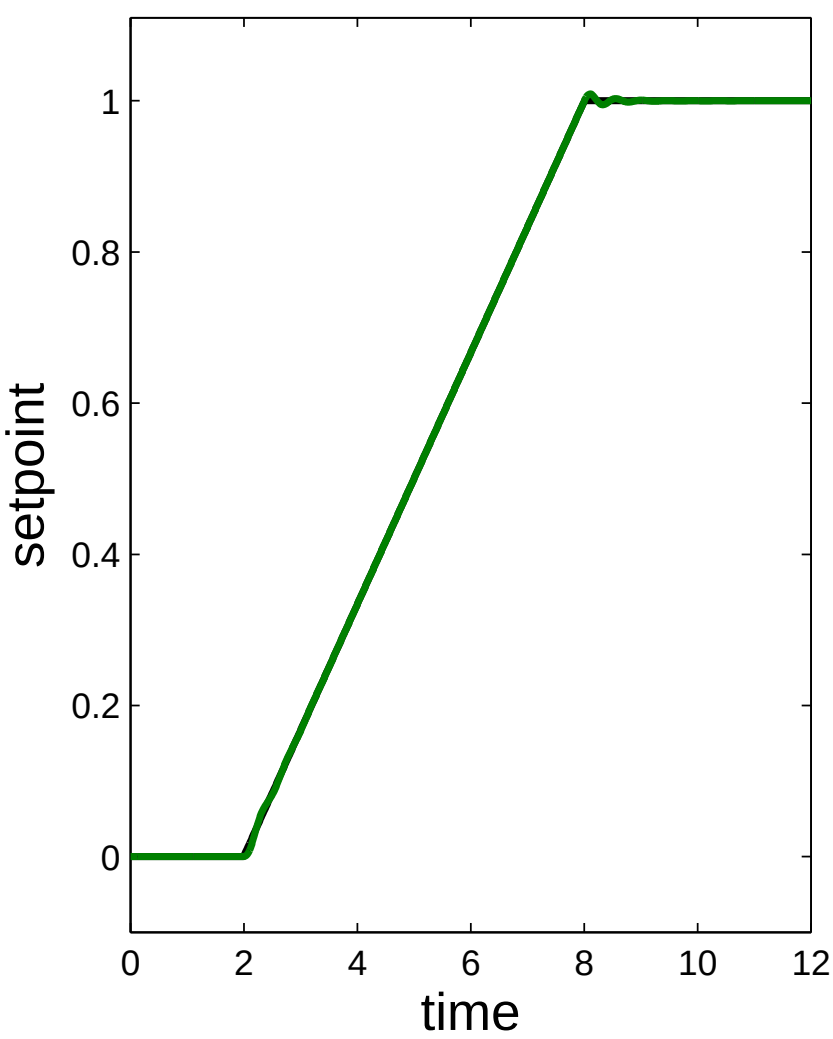
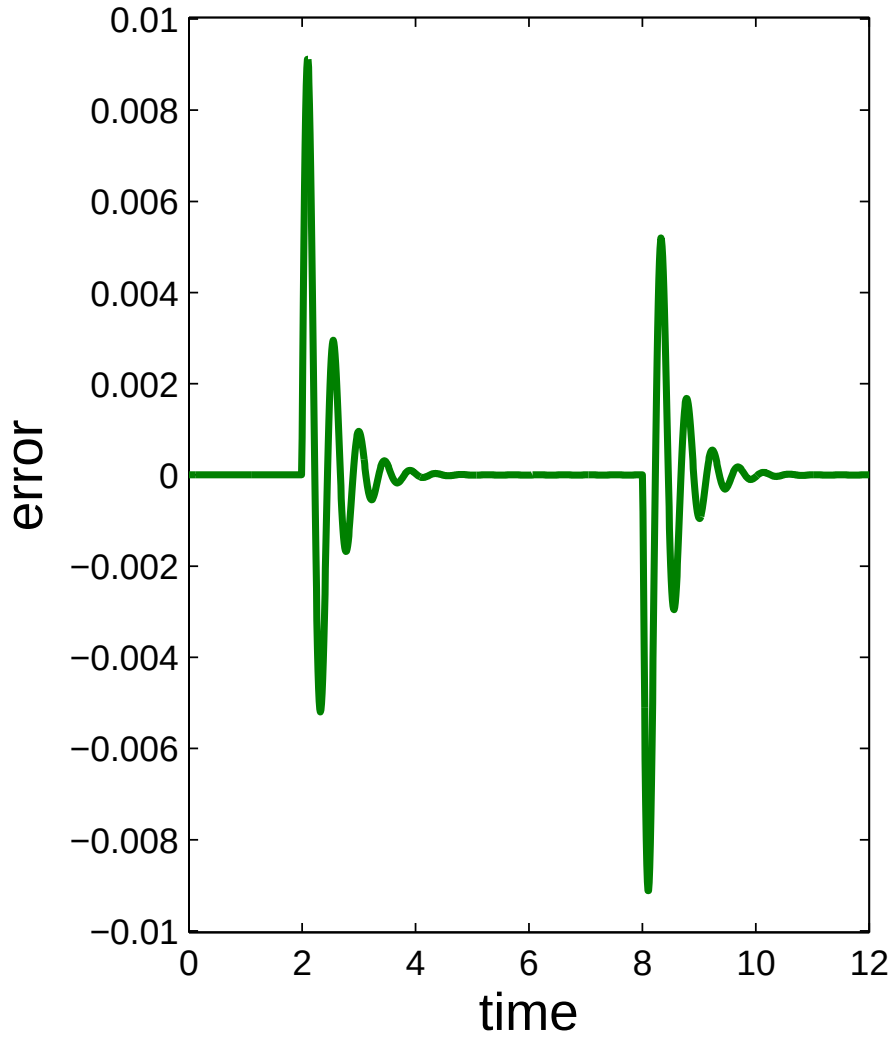
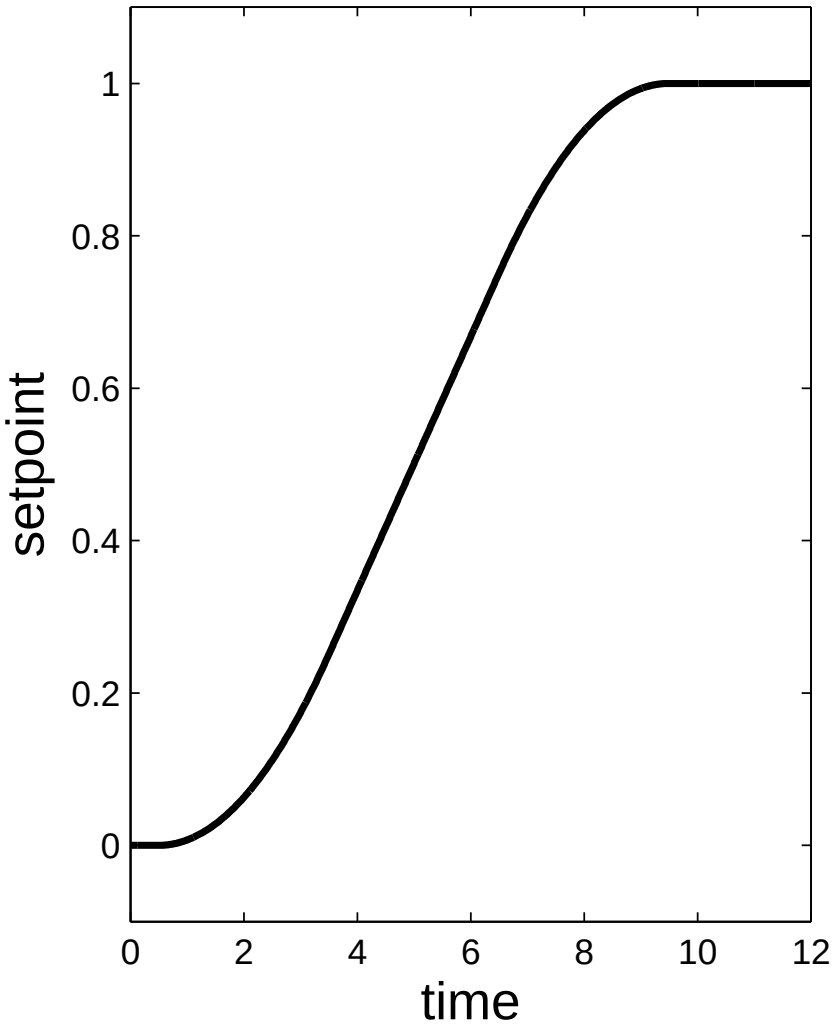
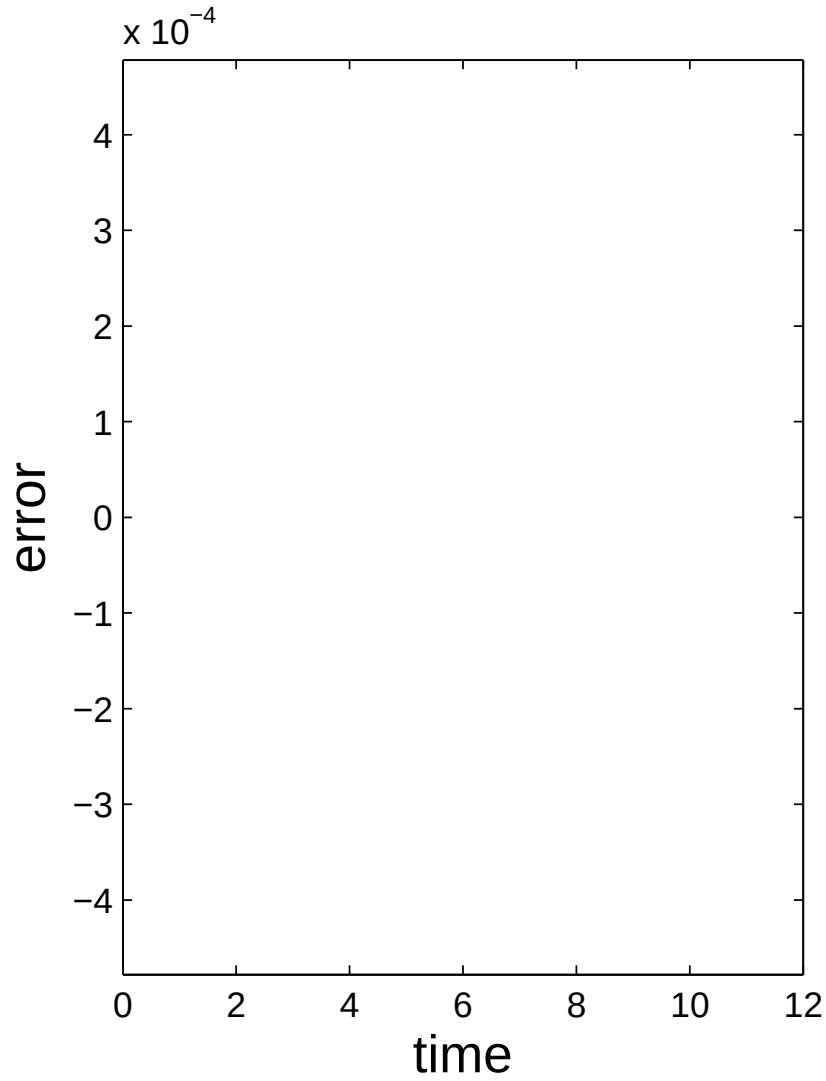




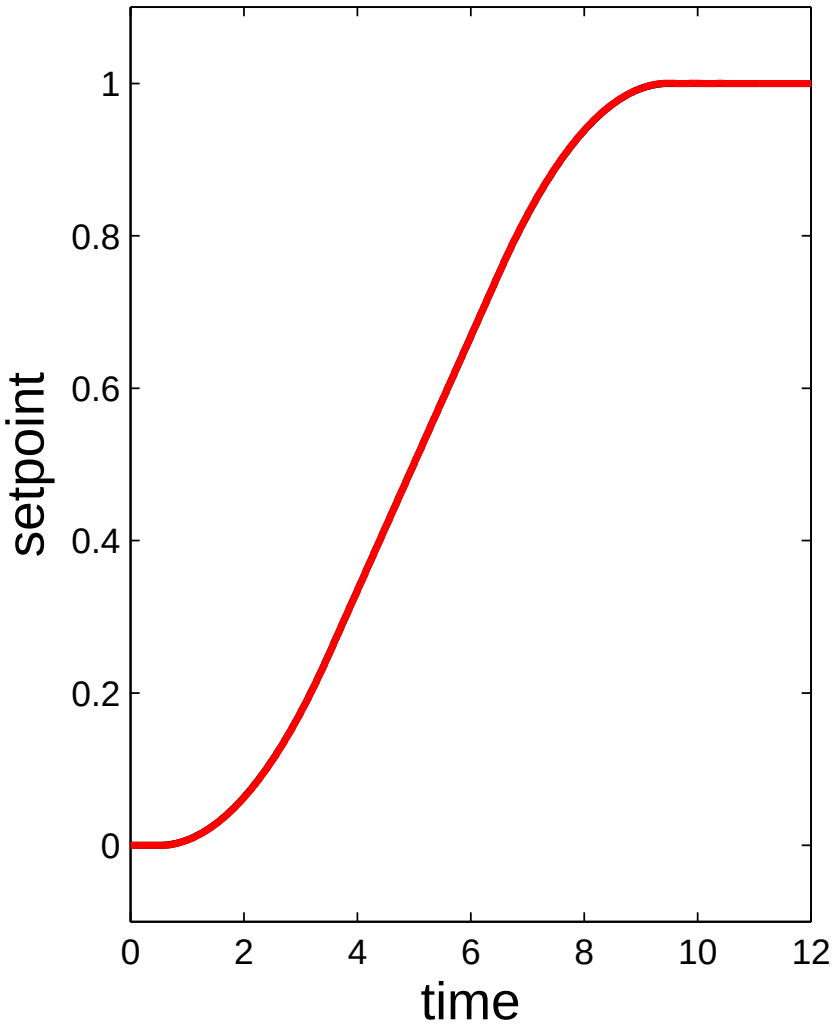
Maximal error / overshoot: 0.009

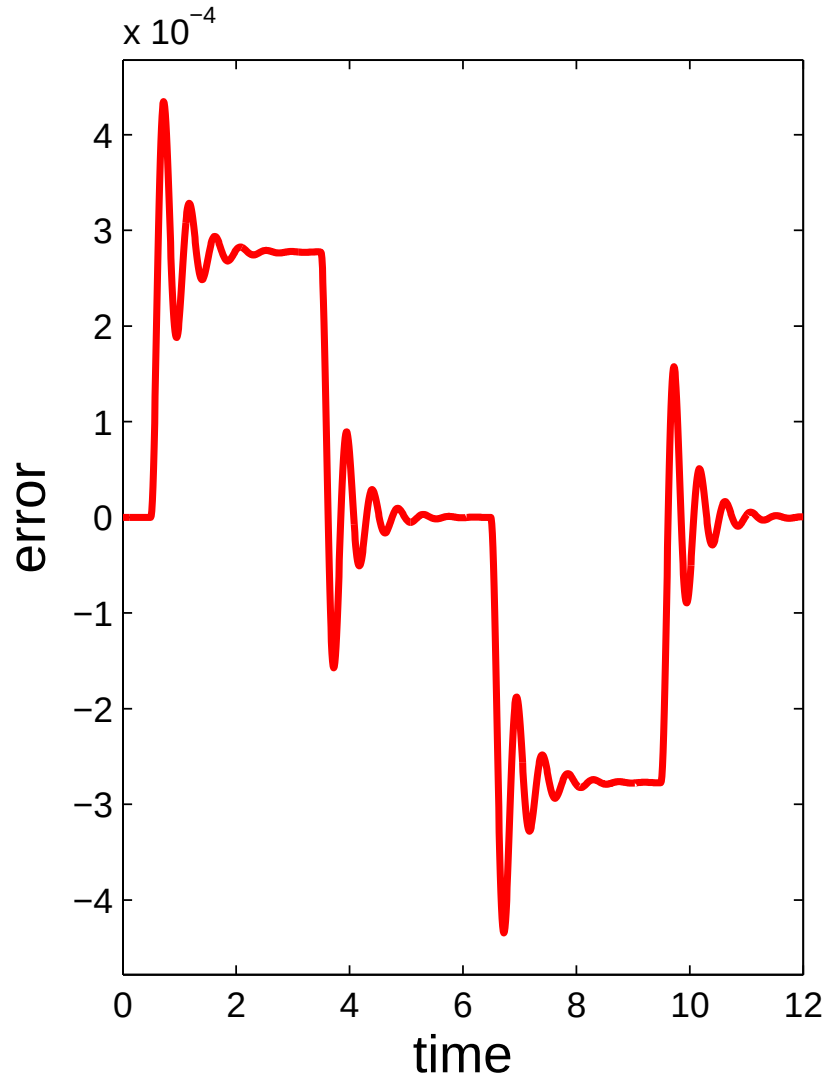
Closed loop response: 2nd order setpoint





Closed loop response: 2nd order setpoint





Maximal error / overshoot: 0.0004

Trajectory planning

Very good tracking results can be obtained

- using a simple (low bandwidth) controller
- using a 'smart' trajectory for x_s
- basic idea: shape trajectory $\rightarrow$ restrict $\dot{x}_s$, $\ddot{x}_s$, etc

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- less oscillations
- possibly faster settling times
- smaller control signals (less saturation issues)

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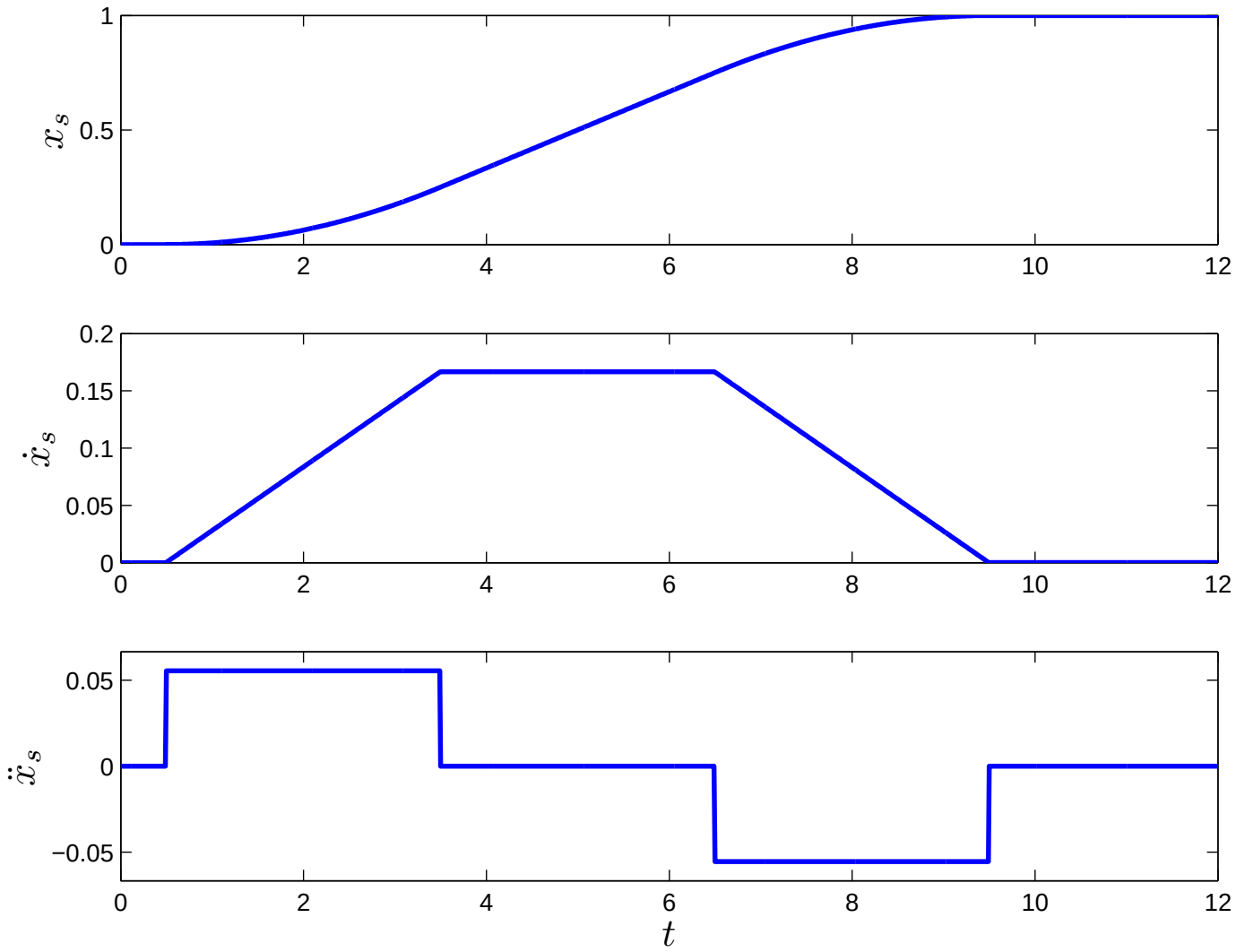
One can wonder...

- Can we quantify the tracking error?
- Is there a way to further reduce this tracking error?

Understanding the servo error

2nd order trajectory

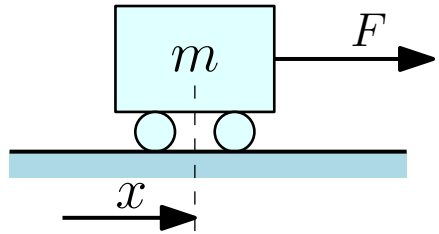
For now, assume only x_s, $\dot{x}_s$ and $\ddot{x}_s$ are restricted



Tracking example 1: error dynamics

Equation of motion:

$$m\ddot{x} = F$$



PD-controller:

$$\begin{aligned} F &= K_p e + K_v \dot{e} \\ &= K_p (x_s - x) + K_v (\dot{x}_s - \dot{x}) \end{aligned}$$

Hence

$$\begin{aligned} m\ddot{x} + K_v (\dot{x} - \dot{x}_s) + K_p (x - x_s) &= 0 \\ m (\ddot{x}_s - \ddot{x}) + K_v (\dot{x}_s - \dot{x}) + K_p (x_s - x) &= m\ddot{x}_s \end{aligned}$$

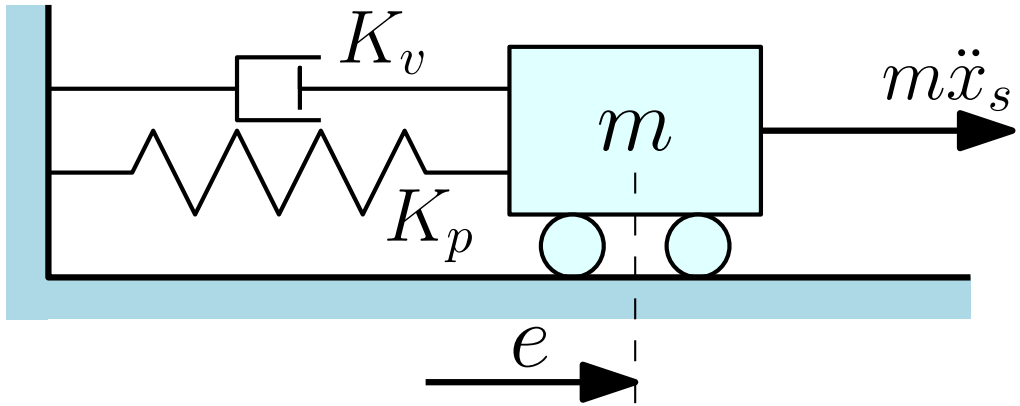
Using $e = x_s - x$, the tracking error dynamics becomes

$$m\ddot{e} + K_v \dot{e} + K_p e = m\ddot{x}_s$$

Tracking example 1: error dynamics

$$\underbrace{m\ddot{e} + K_v\dot{e} + K_p e}_{\text{mass - spring - damper}} = \underbrace{m\ddot{x}_s}_{\text{force on mass}}$$

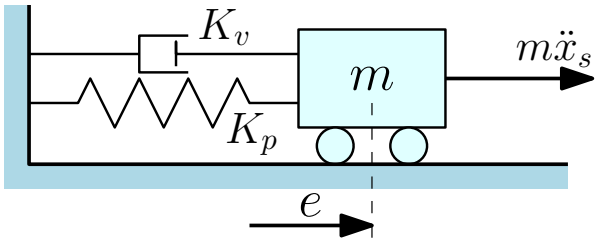
Does e go to zero?



- Iff $\ddot{x}_s = 0$ then $e \to 0$
- Tracking error whenever $\ddot{x}_s \neq 0$

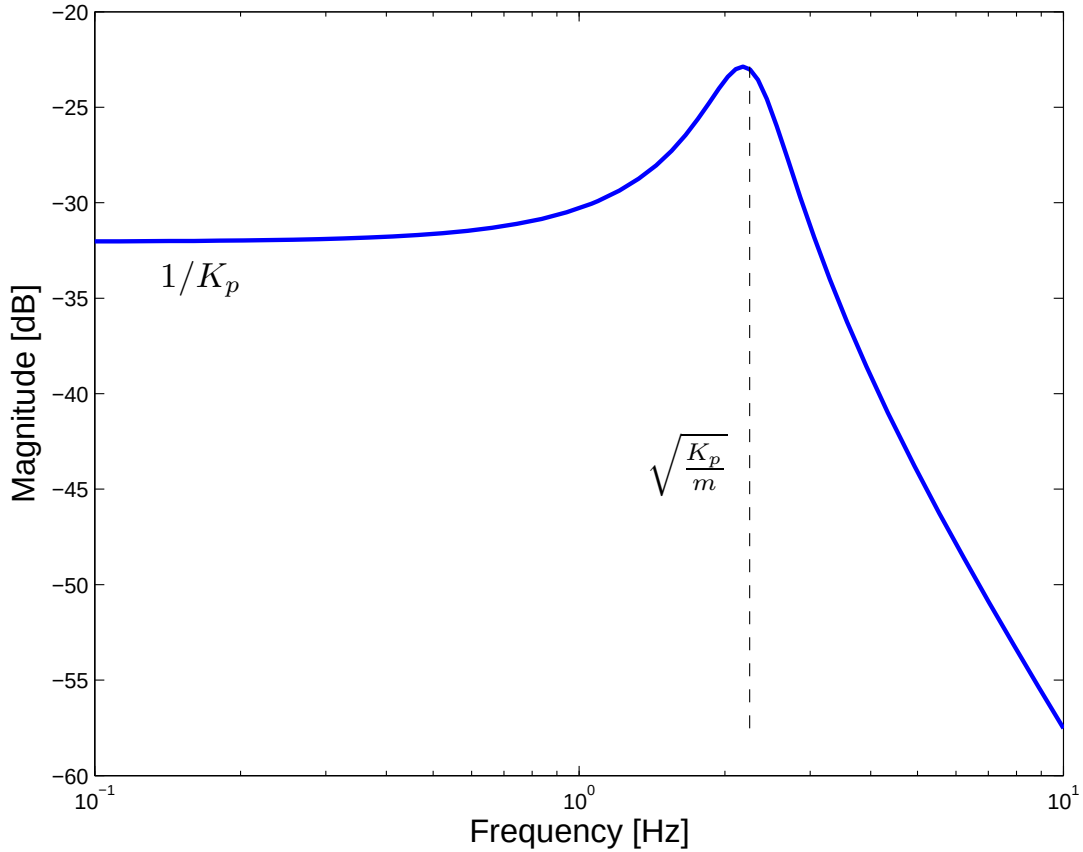
Whenever the mass needs to be accelerated, there is a tracking error!

Tracking example 1: error dynamics



$$m\ddot{e} + K_v\dot{e} + K_p e = u$$

- DC-gain: $\frac{1}{K_p}$
- Eigenfrequency: $\sqrt{\frac{K_p}{m}}$
- $u = m\ddot{x}_s$



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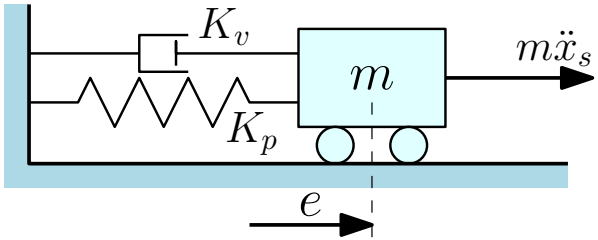

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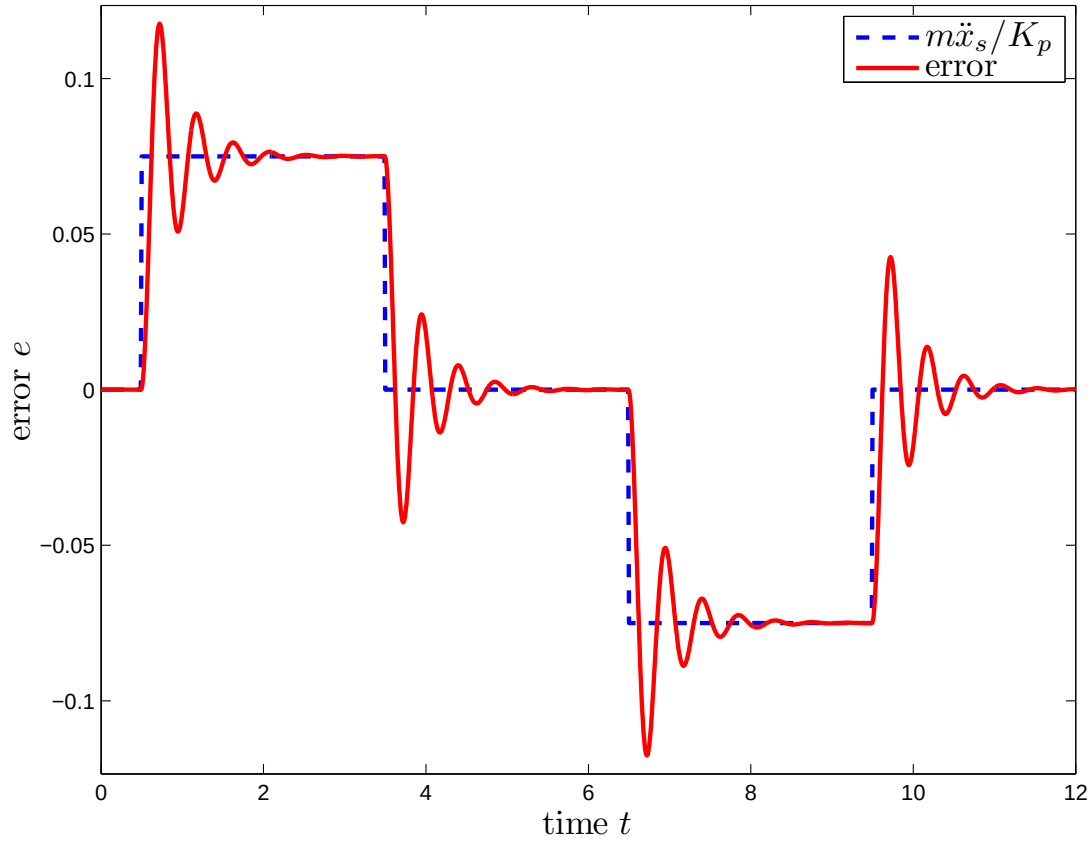
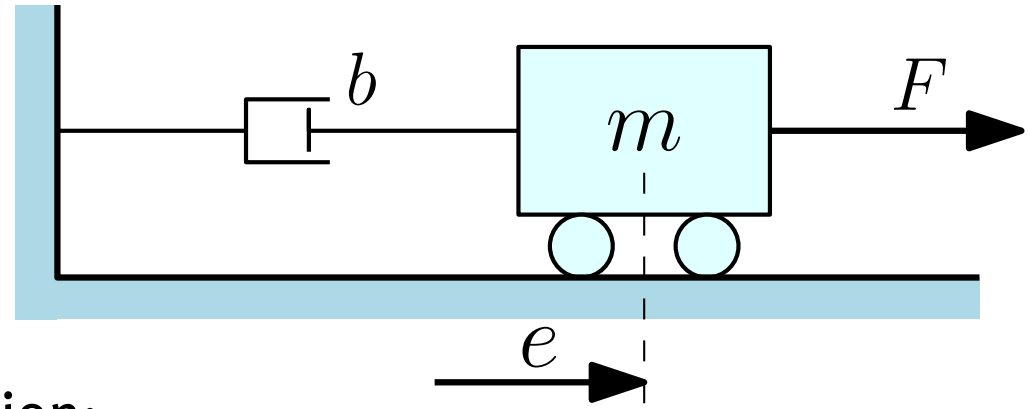


Tracking example 2: error dynamics

Suppose the mass encounters friction:



Equation of motion:

$$m\ddot{x} + b\dot{x} = \underbrace{K_p\left(x_s - x\right) + K_v\left(\dot{x}_s - \dot{x}\right)}_{\text{PD-controller}}$$

Hence, the error dynamics becomes

$$\begin{aligned} m\left(\ddot{x}_s - \ddot{x}\right) + \left(K_v + b\right)\left(\dot{x}_s - \dot{x}\right) + K_p\left(x_s - x\right) &= m\ddot{x}_s + b\dot{x}_s \\ m\ddot{e} + \left(K_v + b\right)\dot{e} + K_p e &= m\ddot{x}_s + b\dot{x}_s \end{aligned}$$

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$$\underbrace{m\ddot{e} + (K_v + b)\,\dot{e} + K_p\,e}_{\text{mass - spring - damper}} \quad = \quad \underbrace{m\ddot{x}_s + b\dot{x}_s}_{\text{force on mass}}$$

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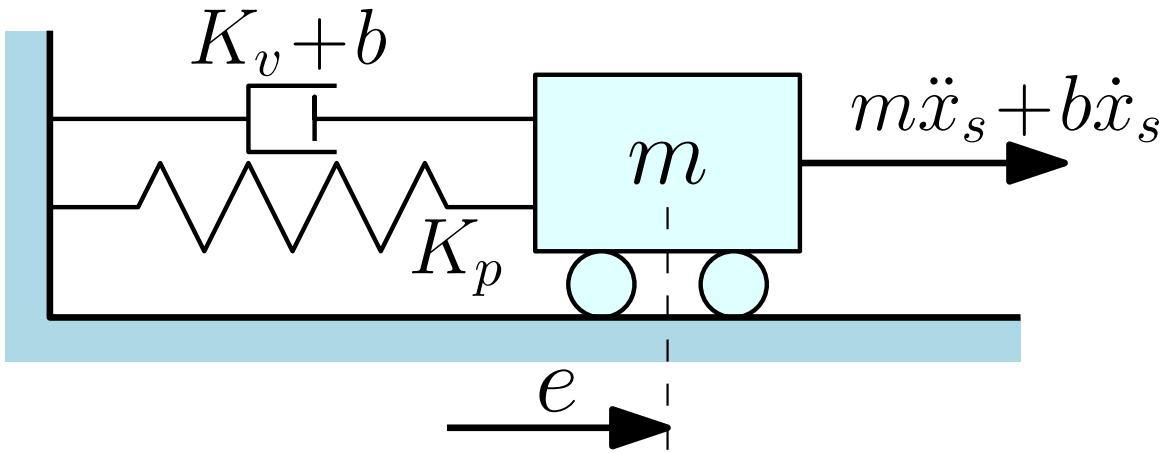

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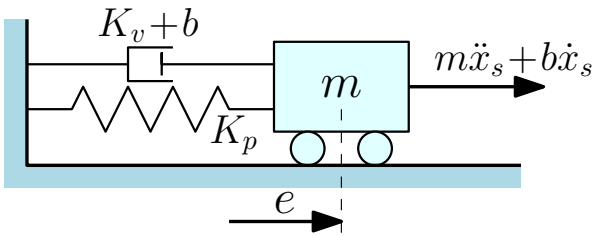

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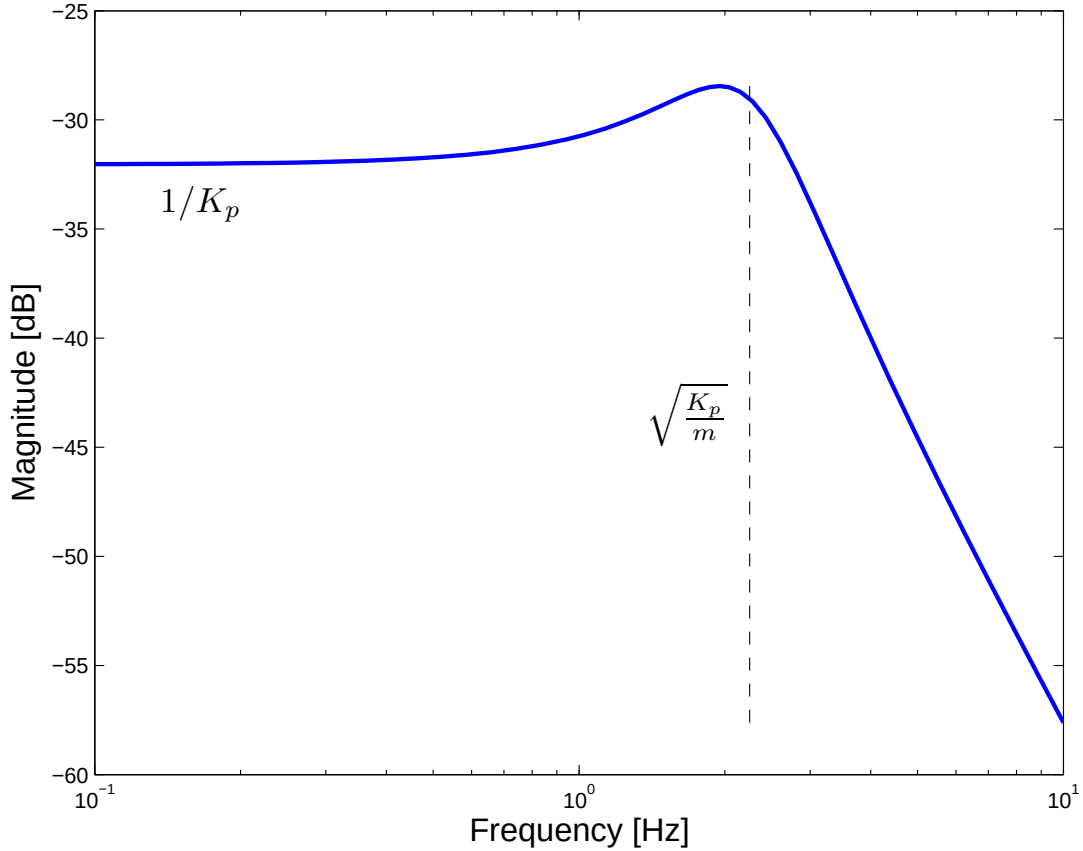

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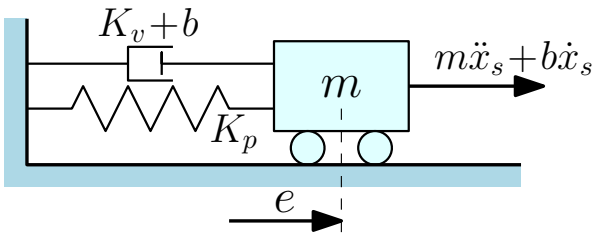

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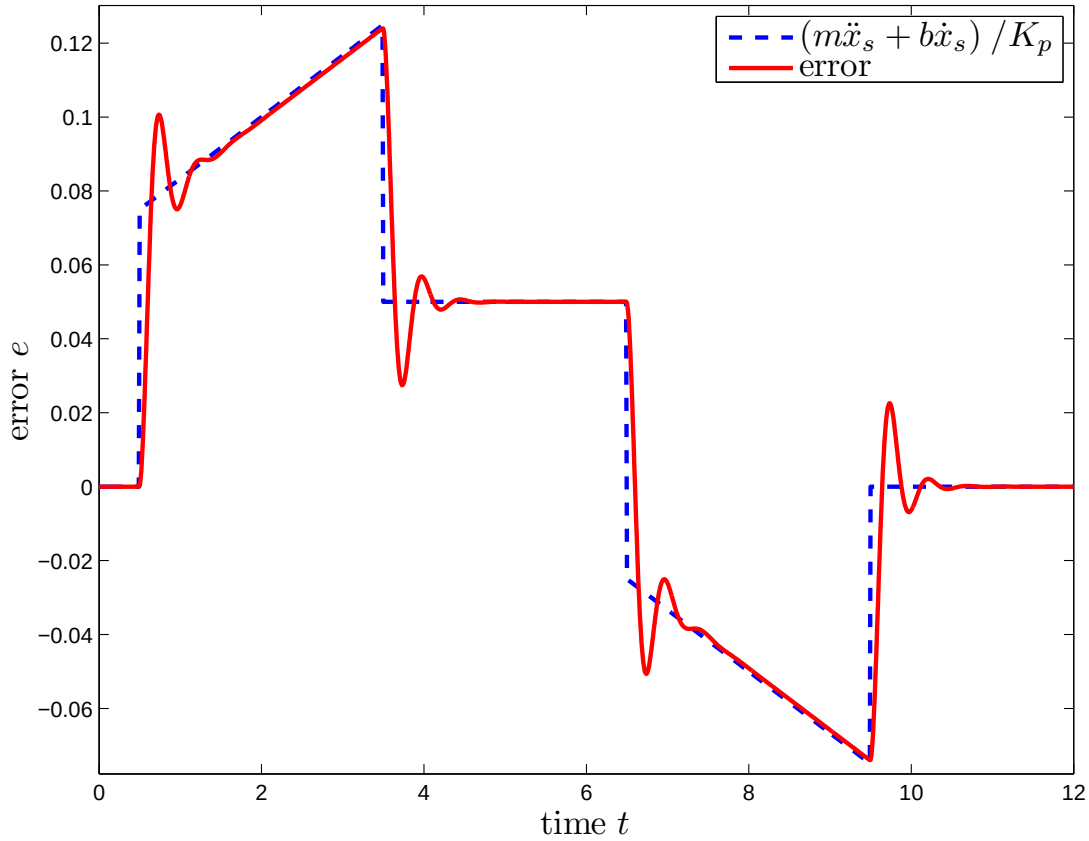

Error dynamics

How can we decrease this error?

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- Using feedback
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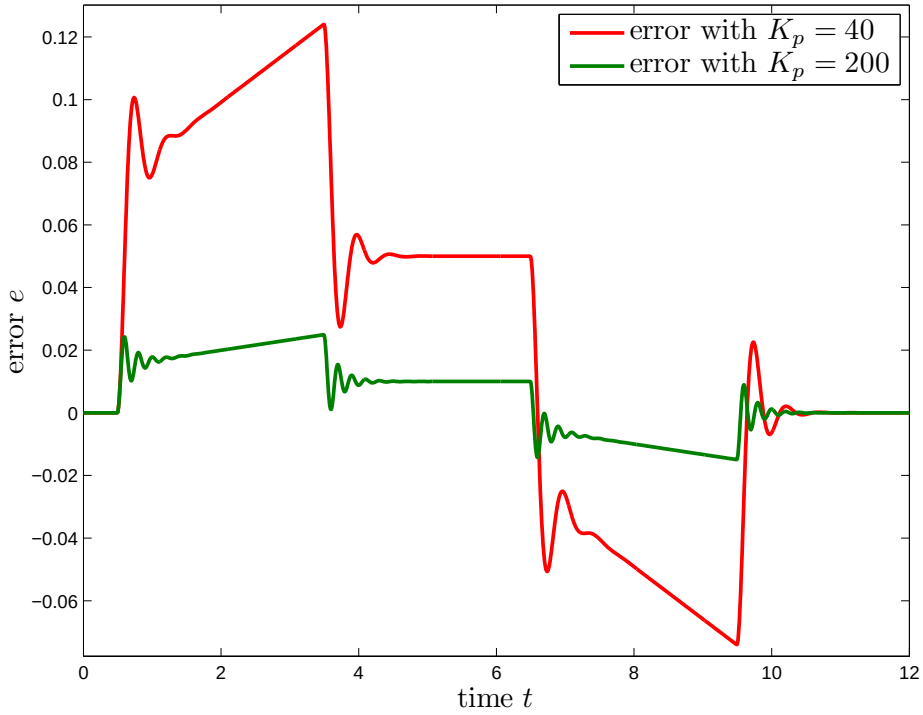

There is still a (steady-state) error...

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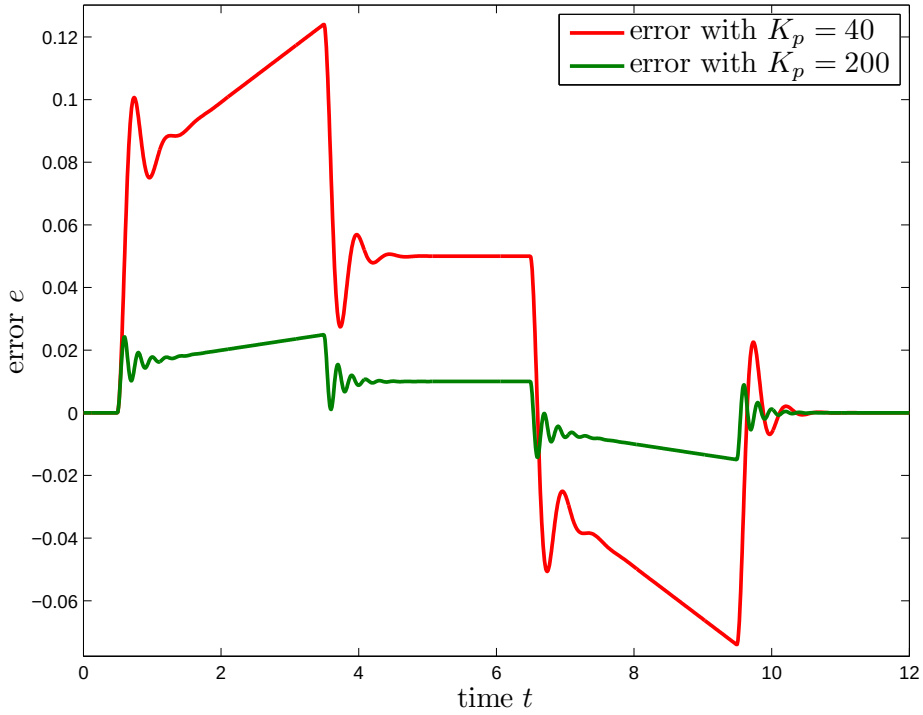

There is still a (steady-state) error...

- Using feedforward

$$m\ddot{e} + K_v\dot{e} + K_p e = \underbrace{m\ddot{x}_s}_{\text{is known!}}$$

- Given x_s, can we compensate the error *beforehand*?

Feedback vs feedforward

Feedback:

- Controller responds to an error
- Error thus already exists
- Feedback = looking back
  - you’re always lagging behind

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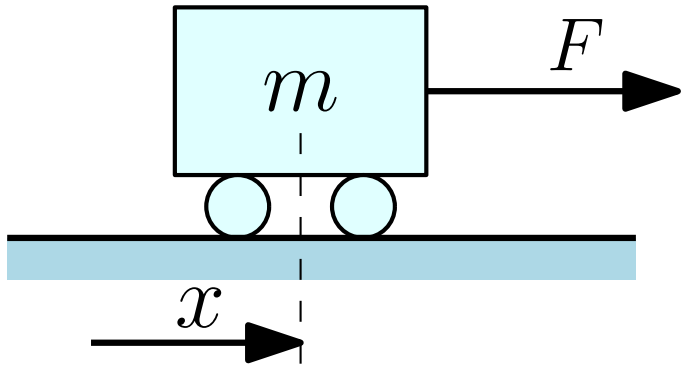
Feedforward:

- Use knowledge about the input
- Correct *before* the error can occur
- Feedforward = looking ahead
  - anticipate what happens or what is needed

Feedforward for motion systems

The inverse problem (1)

How does feedforward work?



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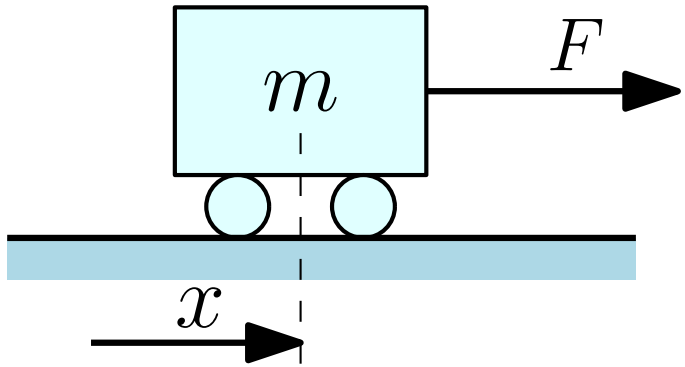

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$$F = \underbrace{m\ddot{x}_s}_{\text{feedforward}} \quad \underbrace{-K_v\dot{e} - K_p e}_{\text{PD-controller}}$$

Feedforward in the loop (1)

Note that x_s is known and

$$\mathcal{L}\{m\ddot{x}_s\} = ms^2\mathcal{L}\{x_s\}$$

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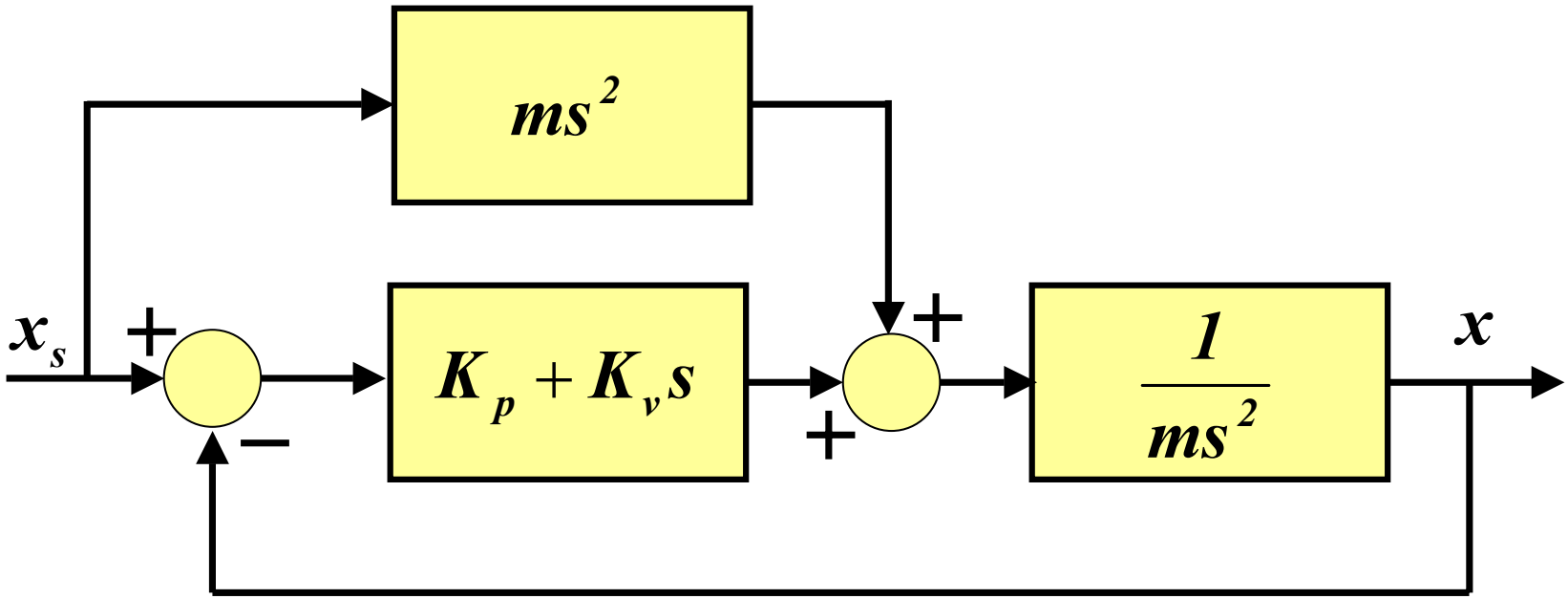

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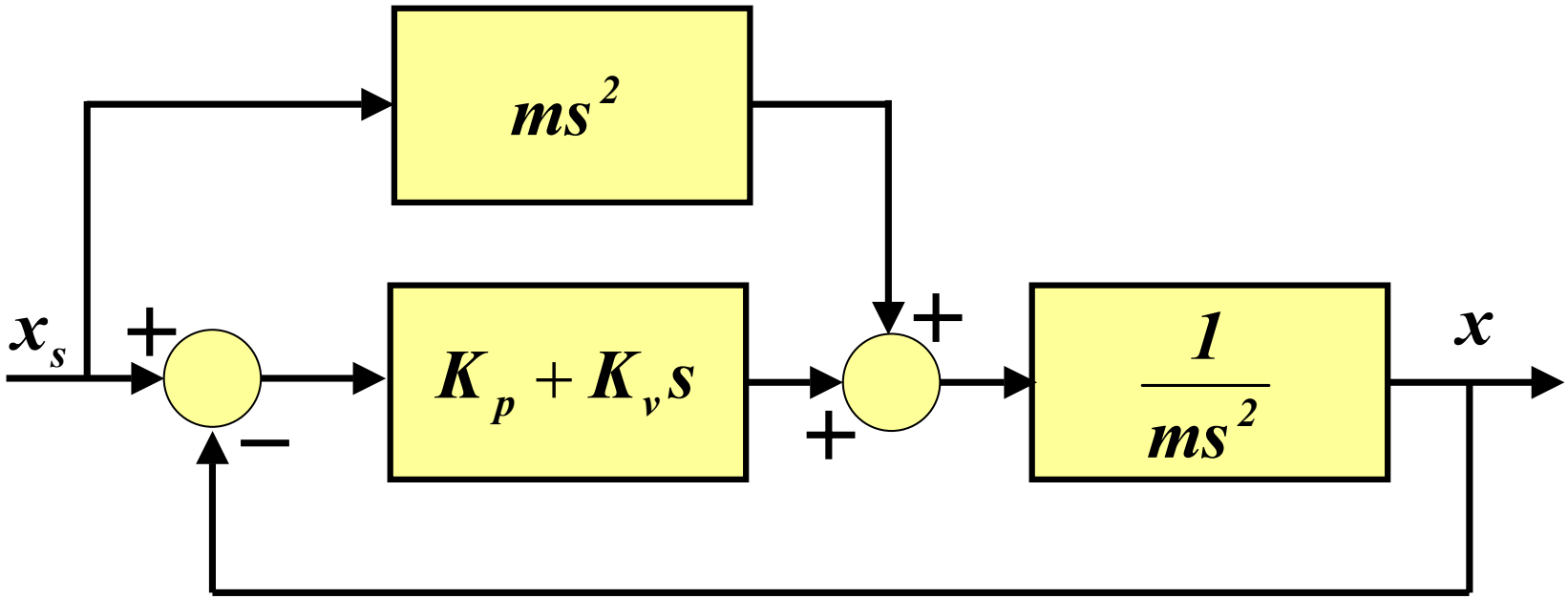
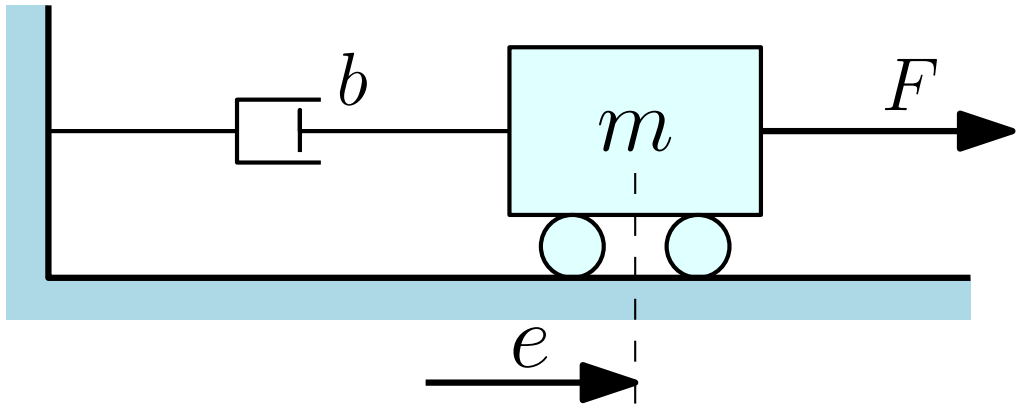


Forward feed / prescribe in advance . . .

- . . . the force necessary to accelerate the rigid mass.

The inverse problem (2)



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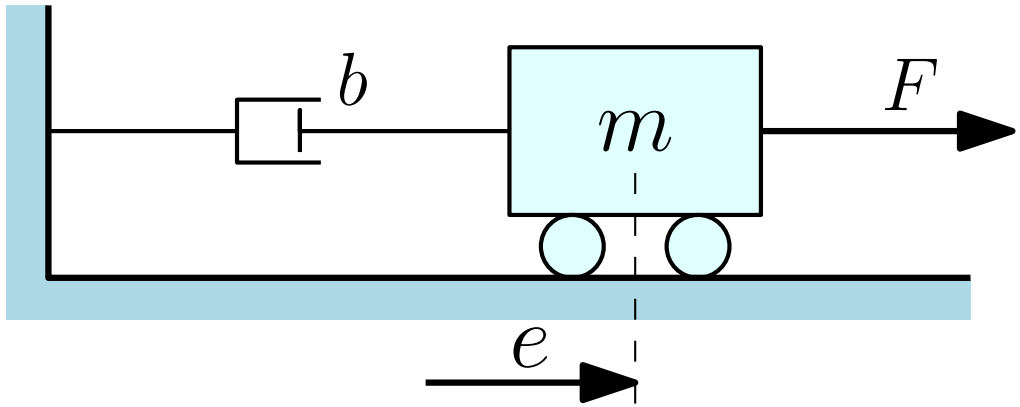

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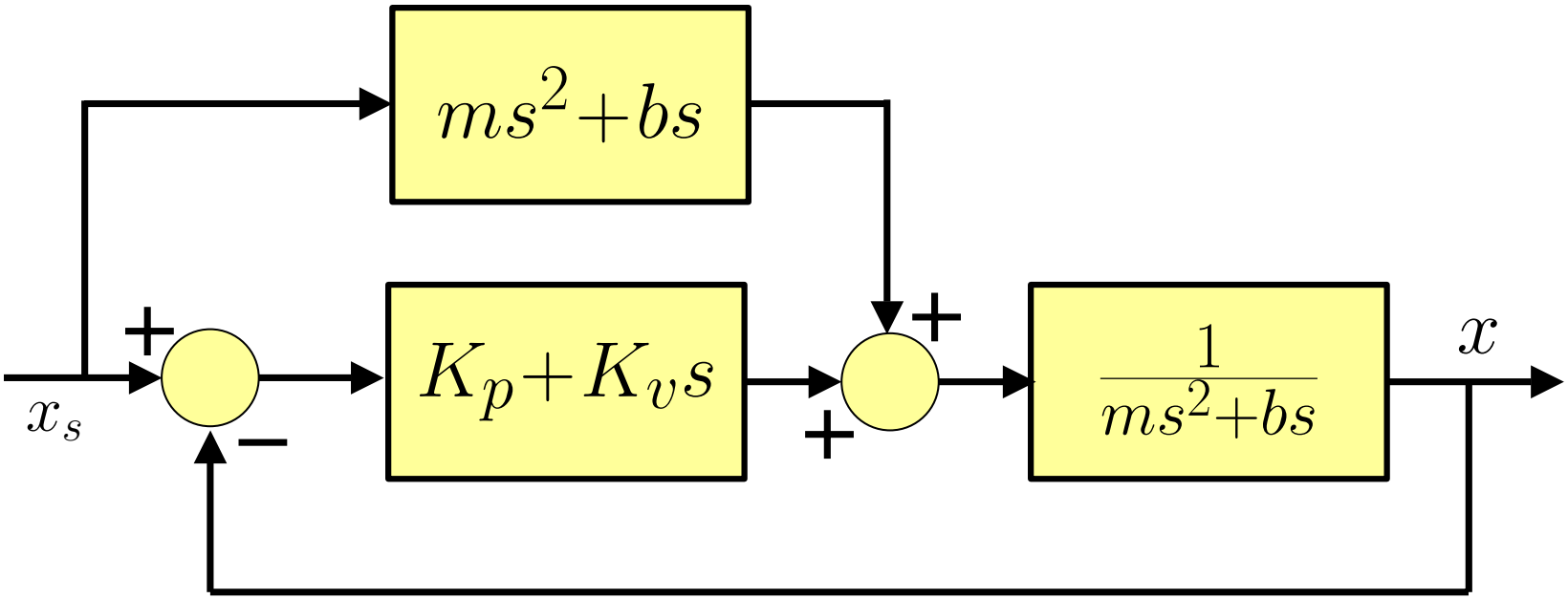

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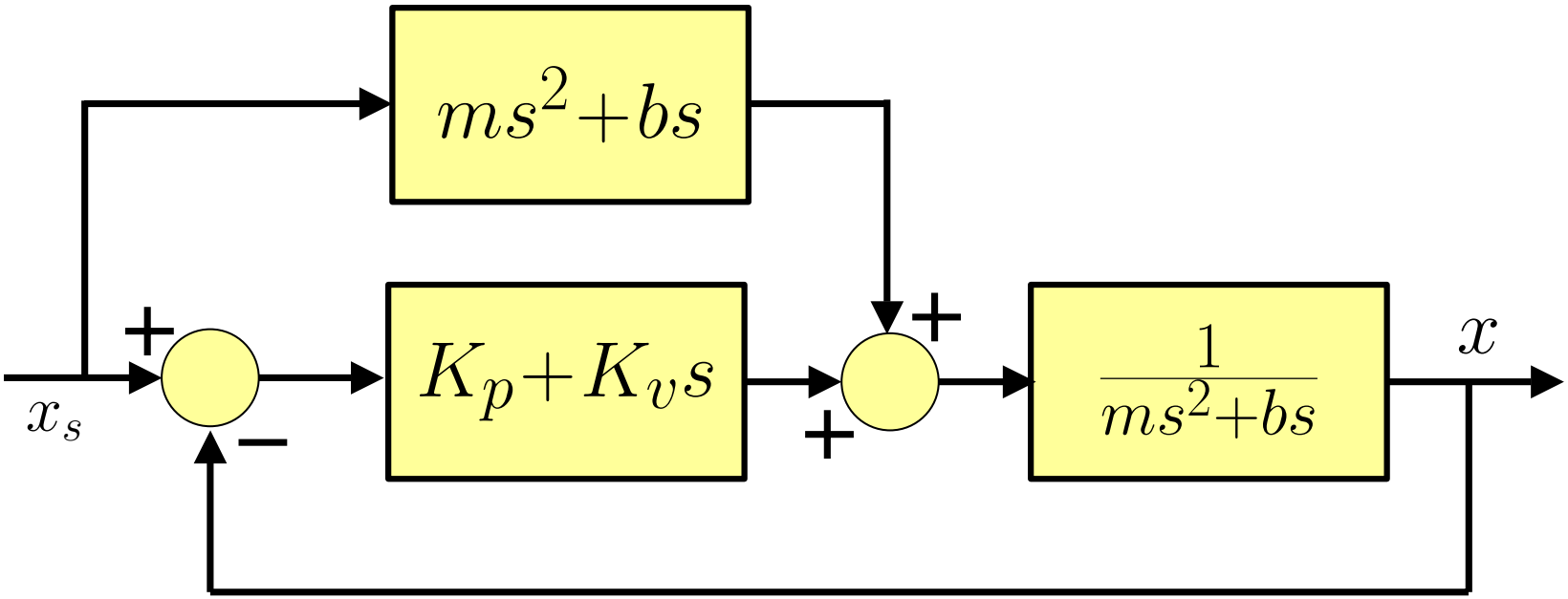
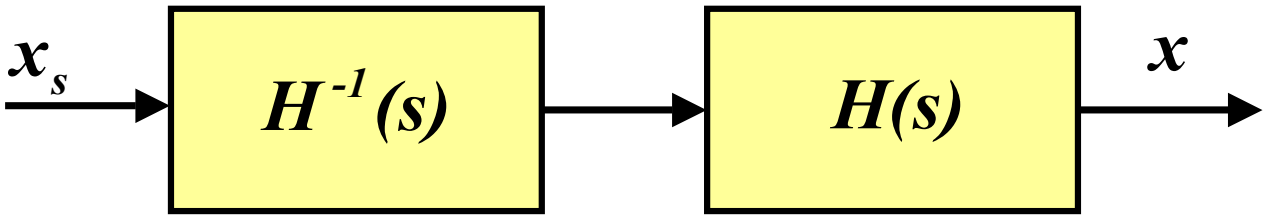


Forward feed / prescribe in advance . . .

- . . . the force necessary to accelerate the rigid mass,
- . . . the force required to overcome the friction.

Pure feedforward

In theory, a feedback controller is not necessary:

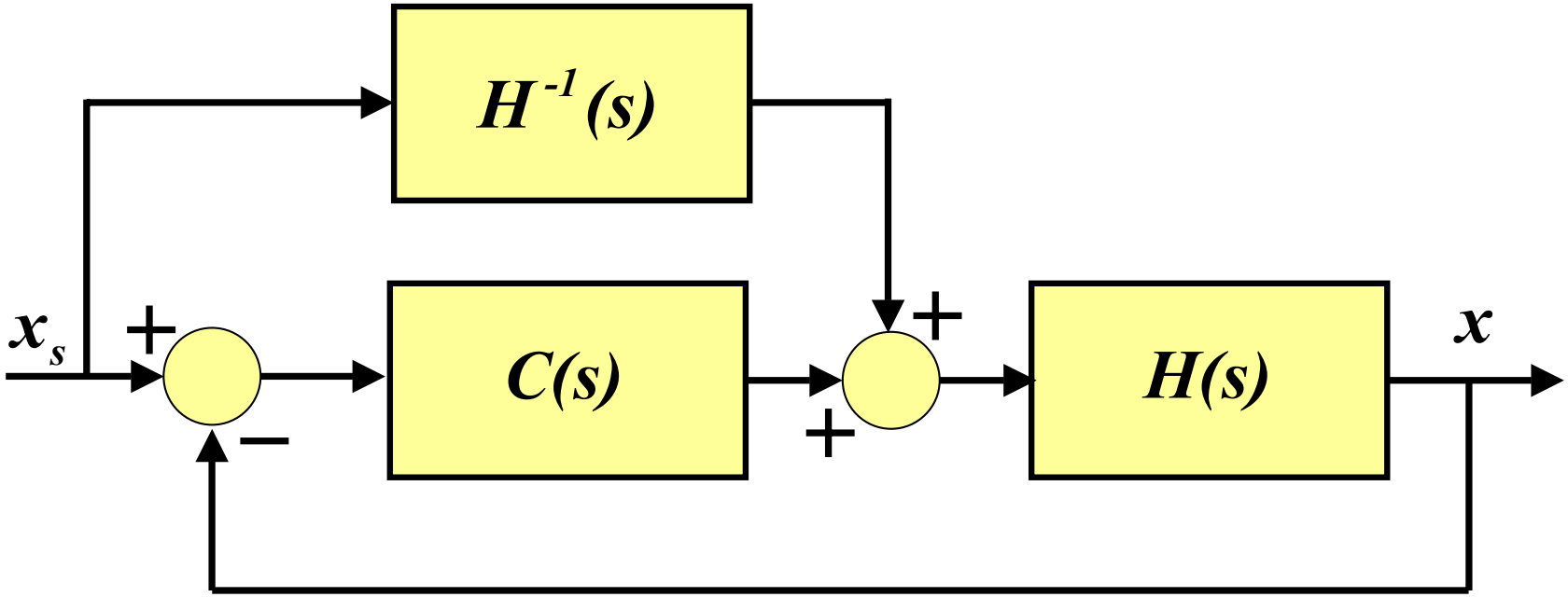


Ideal feedforward:

- $C_{ff}(s) = H^{-1}(s)$
- $x = H(s)H^{-1}(s)x_s = x_s$
- use the plant inverse to compute the plant input *in advance*

Combination feedback and feedforward

In practice, feedback is needed to compensate disturbances around x_s:



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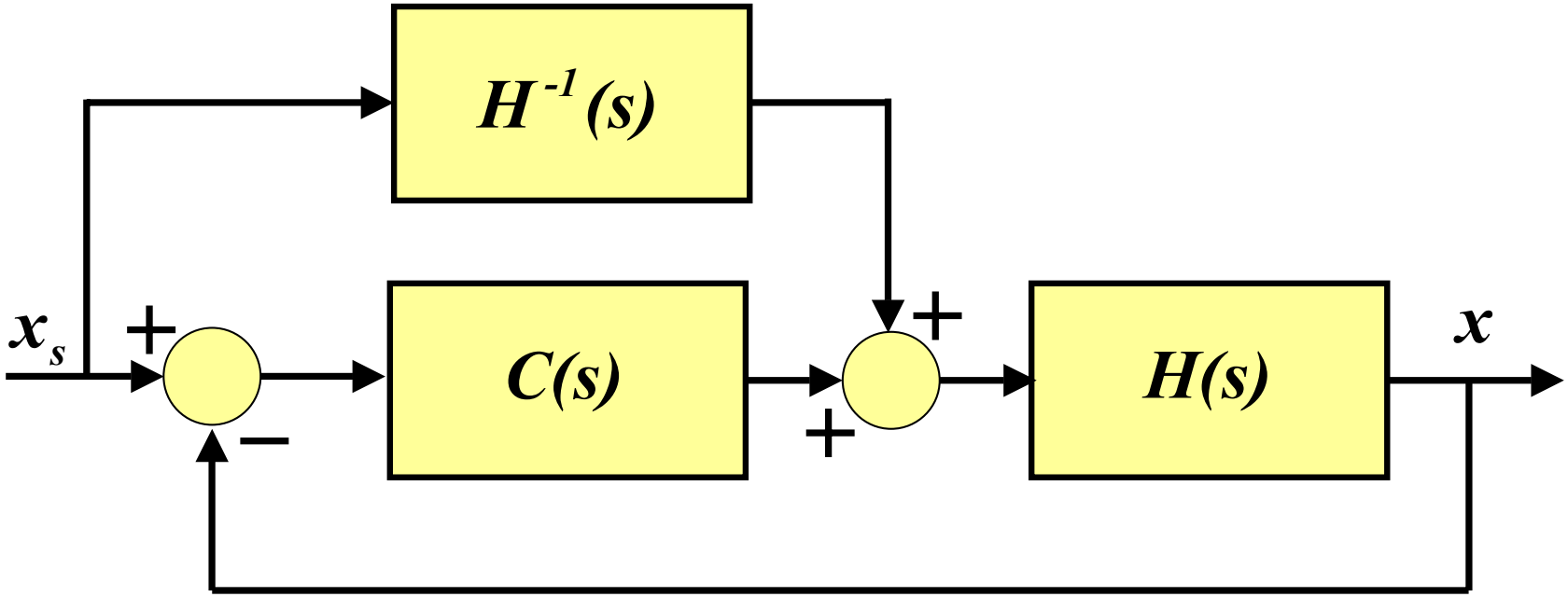

However,

- what if $H^{-1}(s)$ is unstable?
- what if $H(s)$ is unknown?

Then we try to approximate $H^{-1}(s)$ in time domain.

General feedforward (1)

Forward feeding the force required to...

- ...accelerate the rigid body mode of the plant: $F_a = K_{fa}\ddot{x}_s$
- ...overcome the viscous friction with the fixed world: $F_v = K_{fv}\dot{x}_s$

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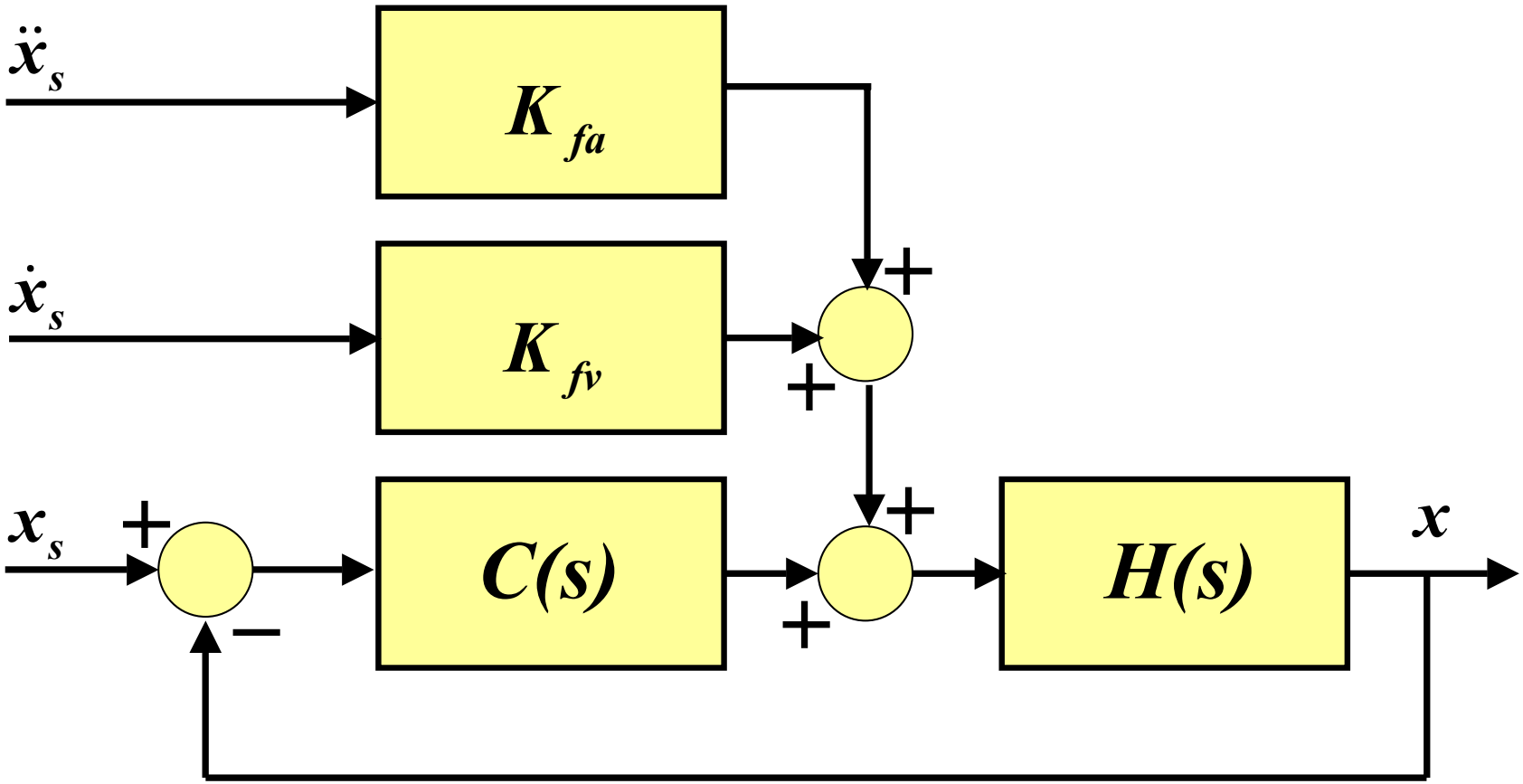
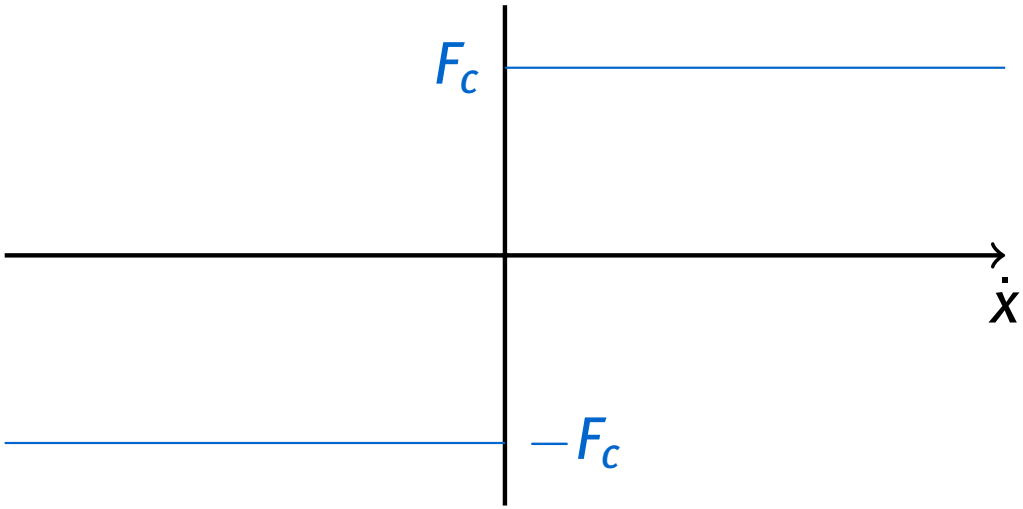


Coulomb friction

What if Coulomb friction (dry friction) is present?

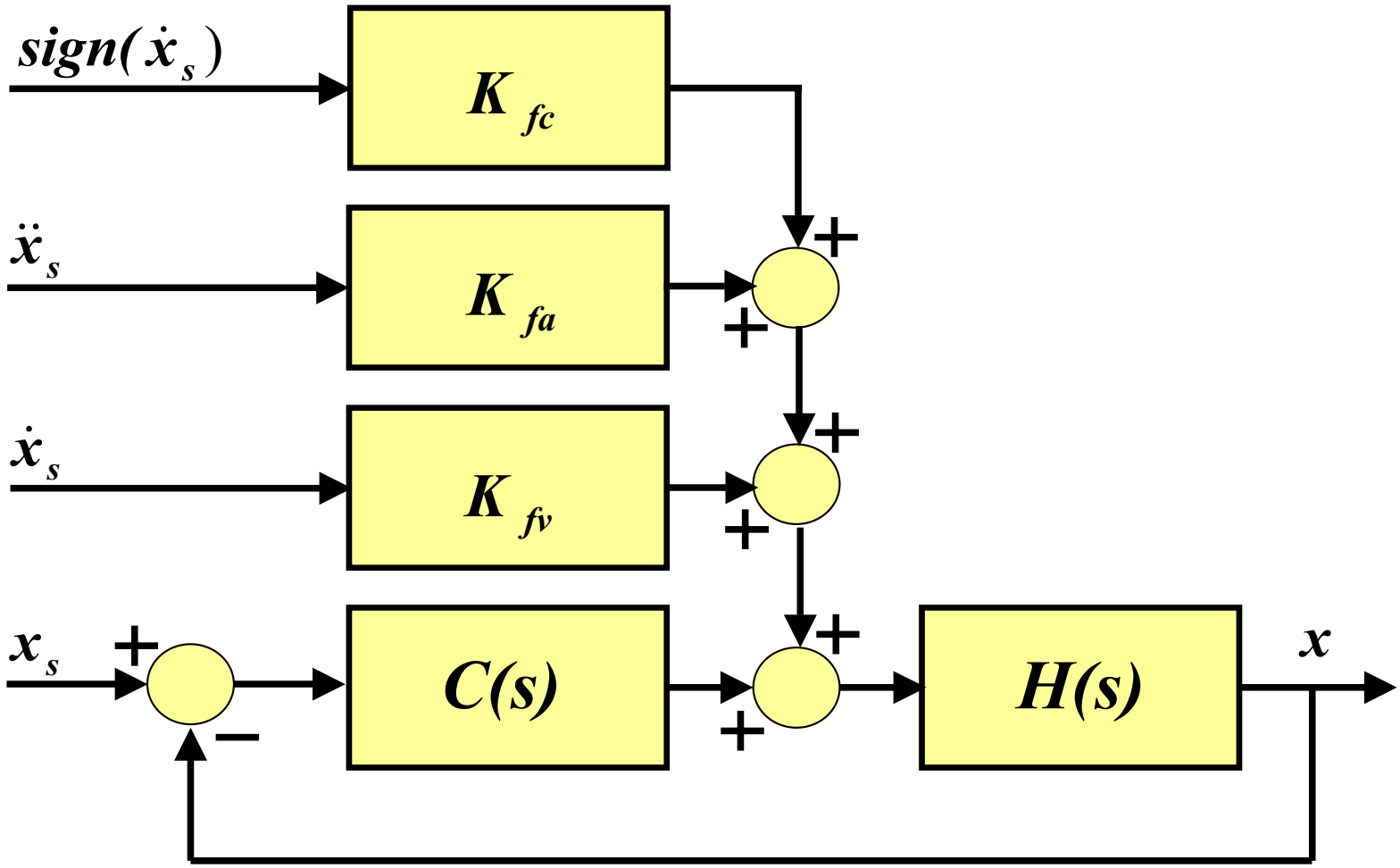
- constant force, opposing direction of motion
- direction dependent



Hence, to overcome this force in advance, we need a feedforward

$$F_c = K_{fc} \cdot \text{sign}\left(\dot{x}_s\right)$$

General feedforward (2)



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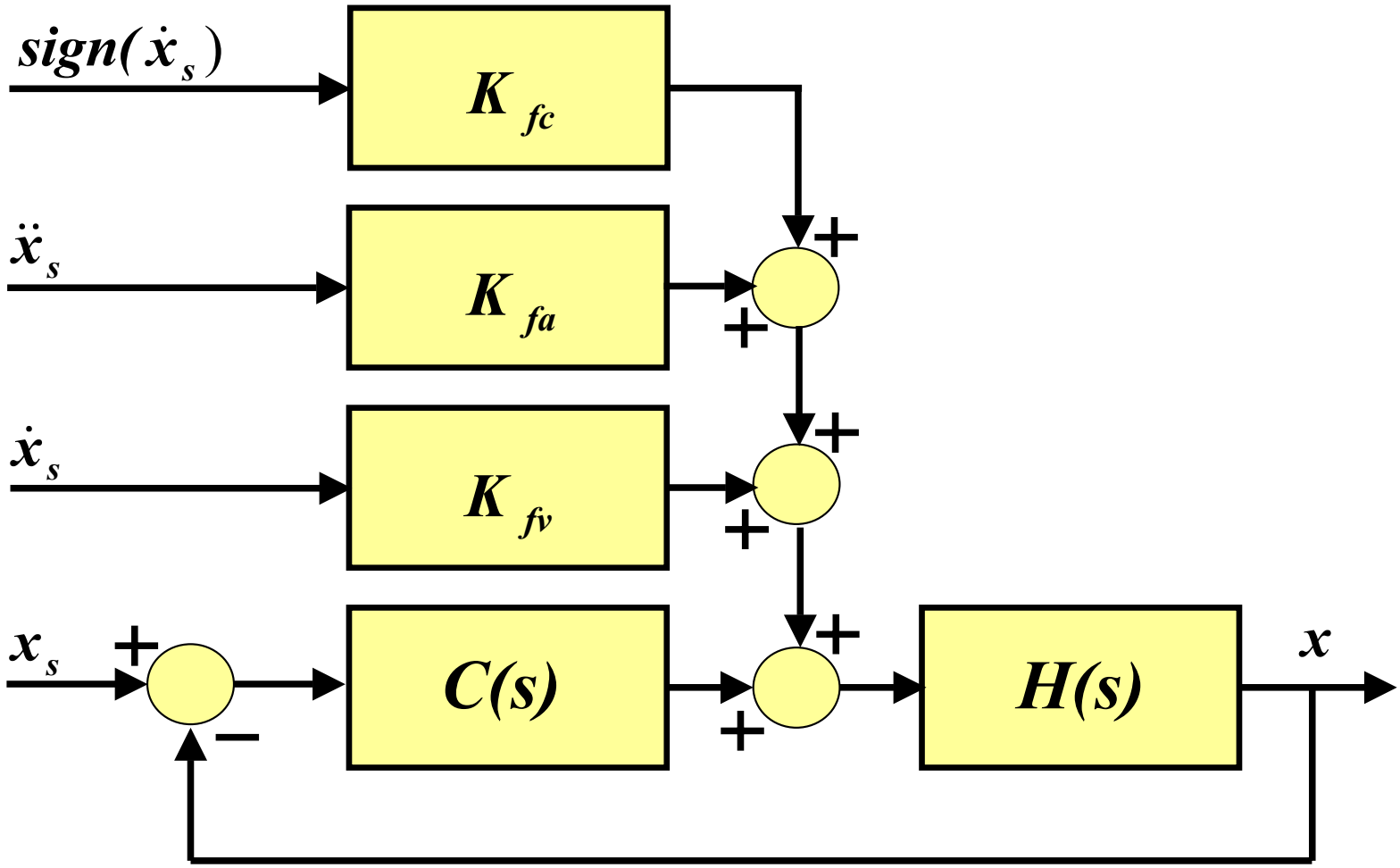
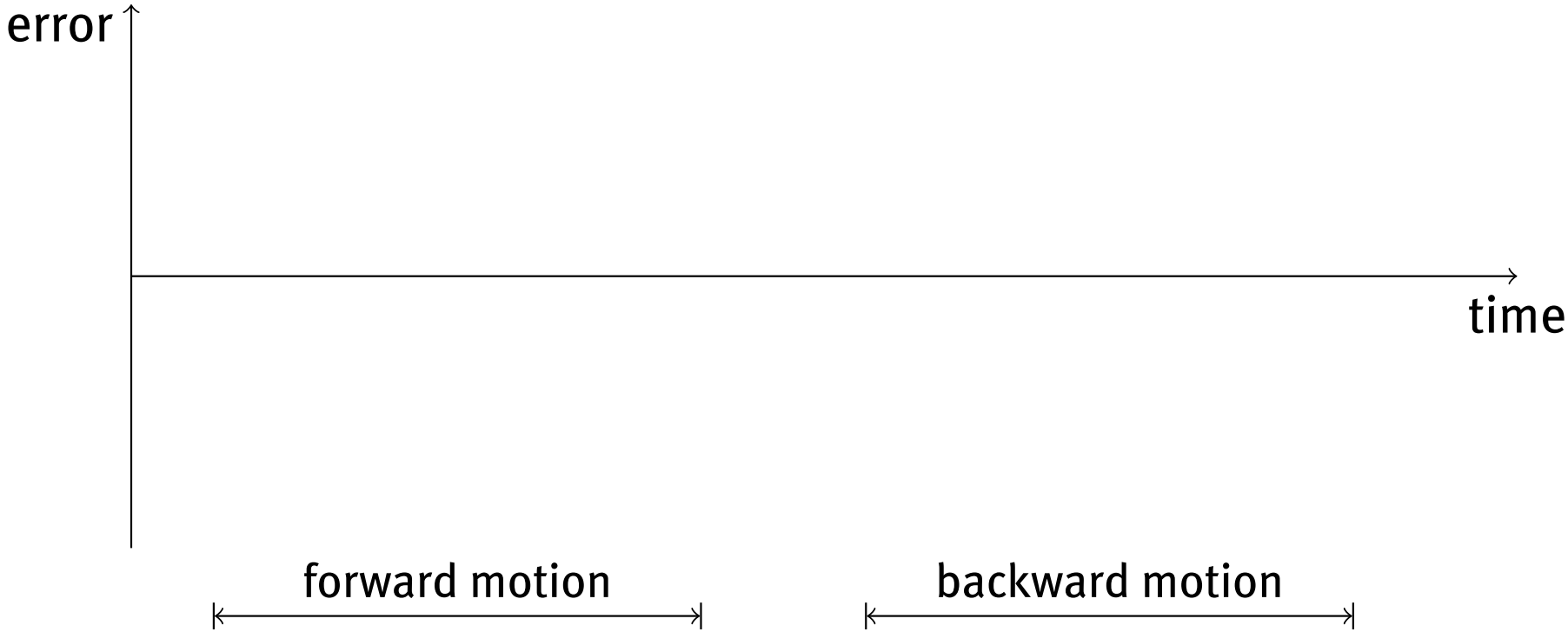


$$F_{ff} = K_{fa}\ddot{x}_s + K_{fv}\dot{x}_s + K_{fc}\,\mathrm{sign}\,(\dot{x}_s)$$

Tuning feedforward parameters

Error profile

Feedforward tuning is based on error profile in time domain



There is an error due to:

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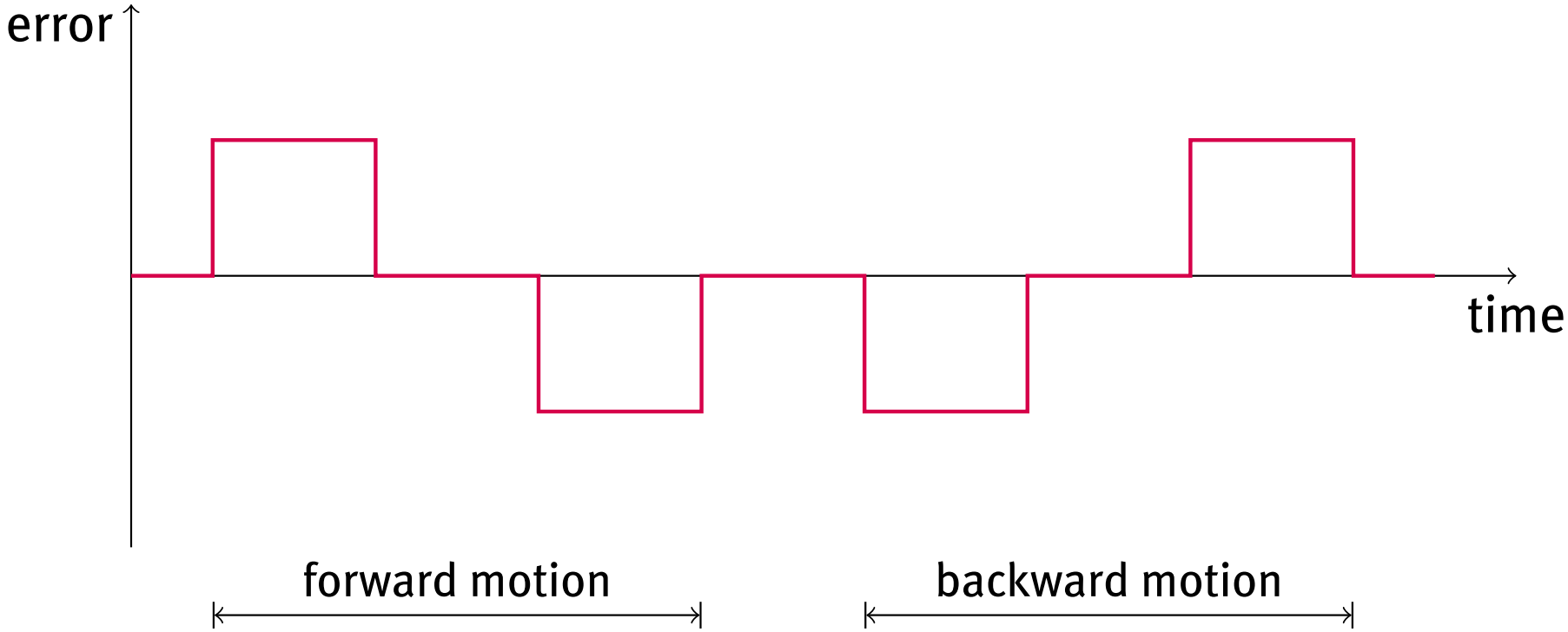

There is an error due to:

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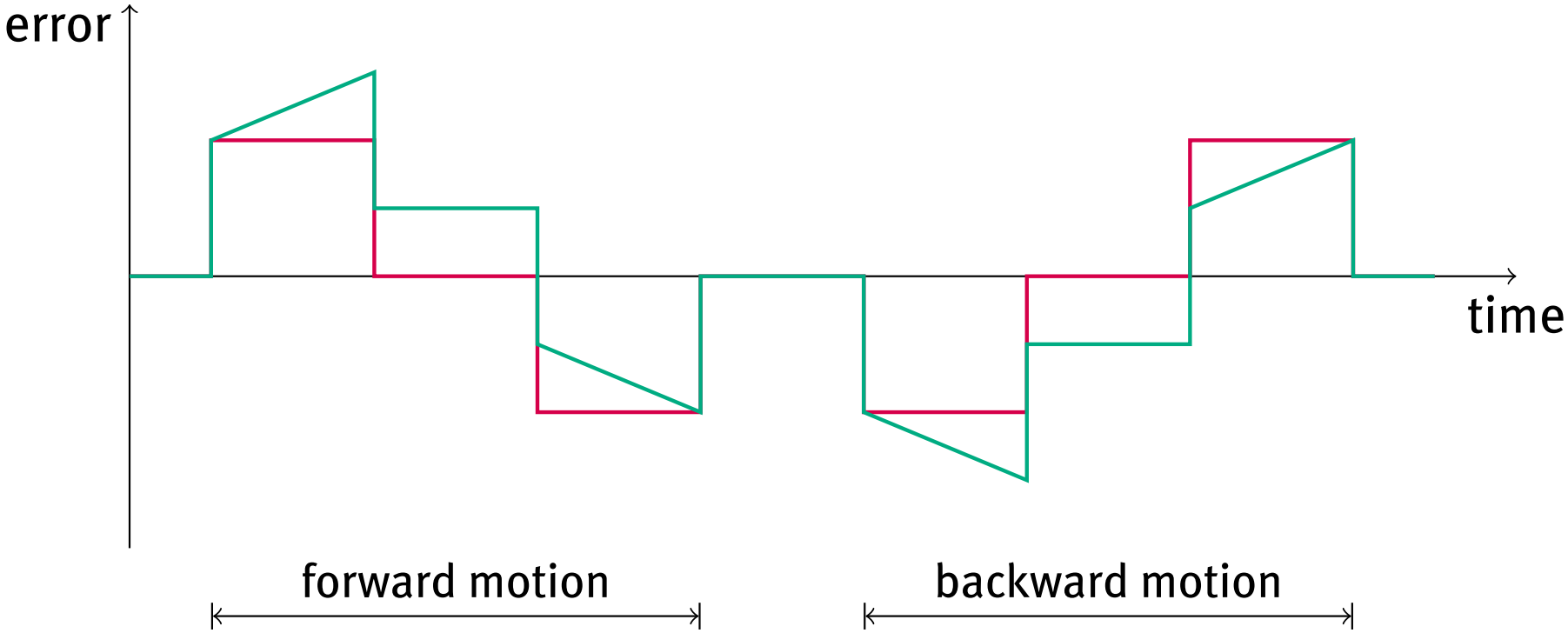

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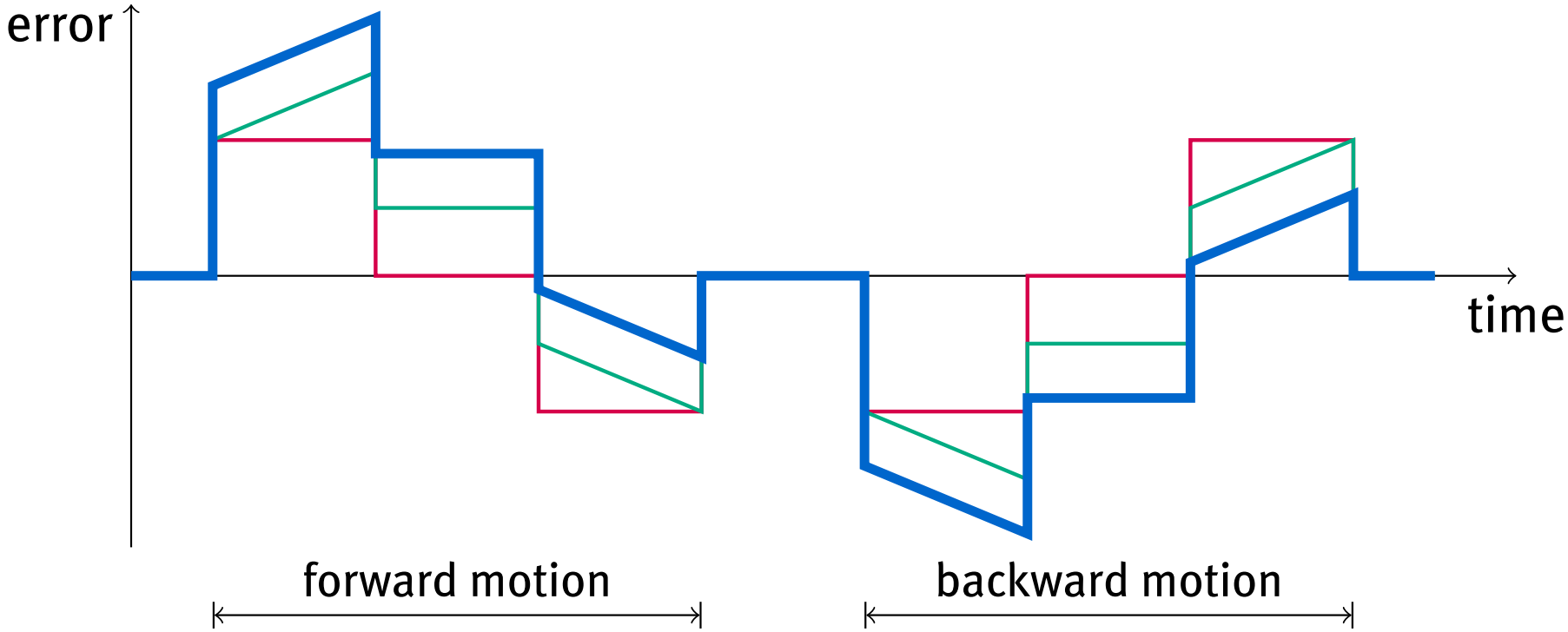

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- lack of force to accelerate (K_{fa})
- lack of force to compensate viscous friction (K_{fv})
- lack of force to compensate Coulomb friction (K_{fc})

Feedforward tuning procedure

- Define a suitable (2nd or higher order) setpoint trajectory
  - both forward and backward movement (with a small pause)
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  - approx 1/3 acceleration, 1/3 constant velocity, 1/3 deceleration

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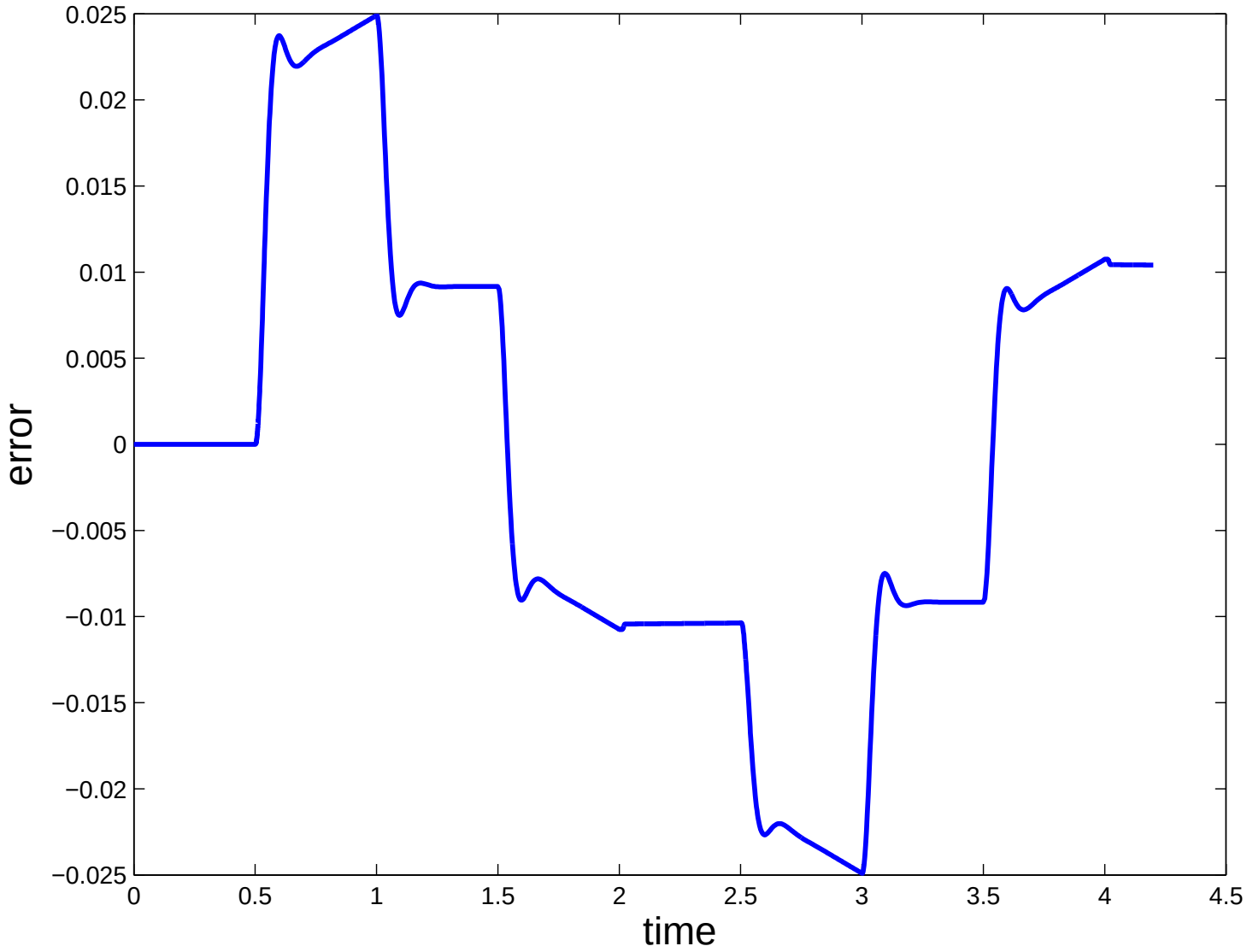
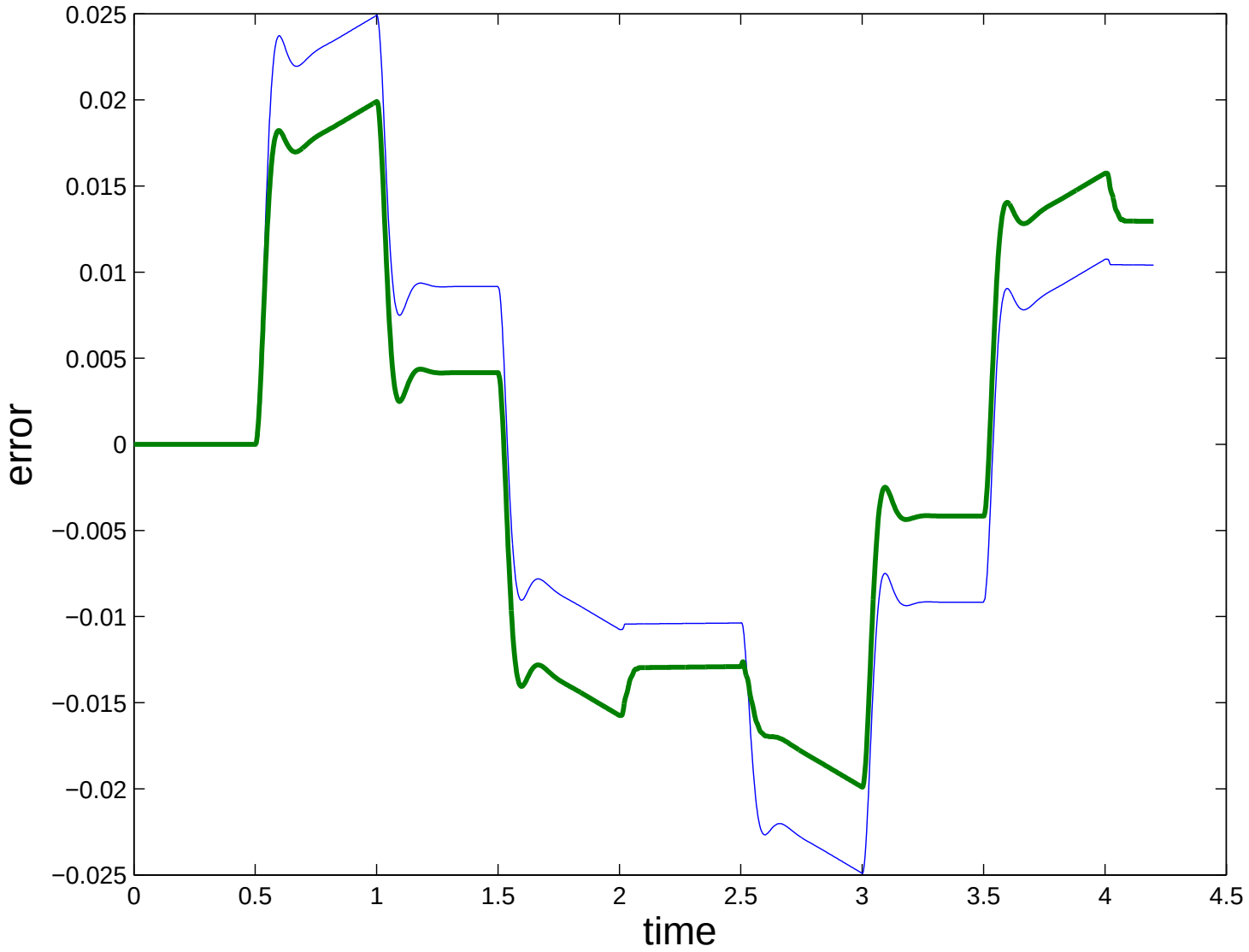
- If desired, tune up the feedback controller

Feedforward tuning



- No FF

Feedforward tuning



- No FF
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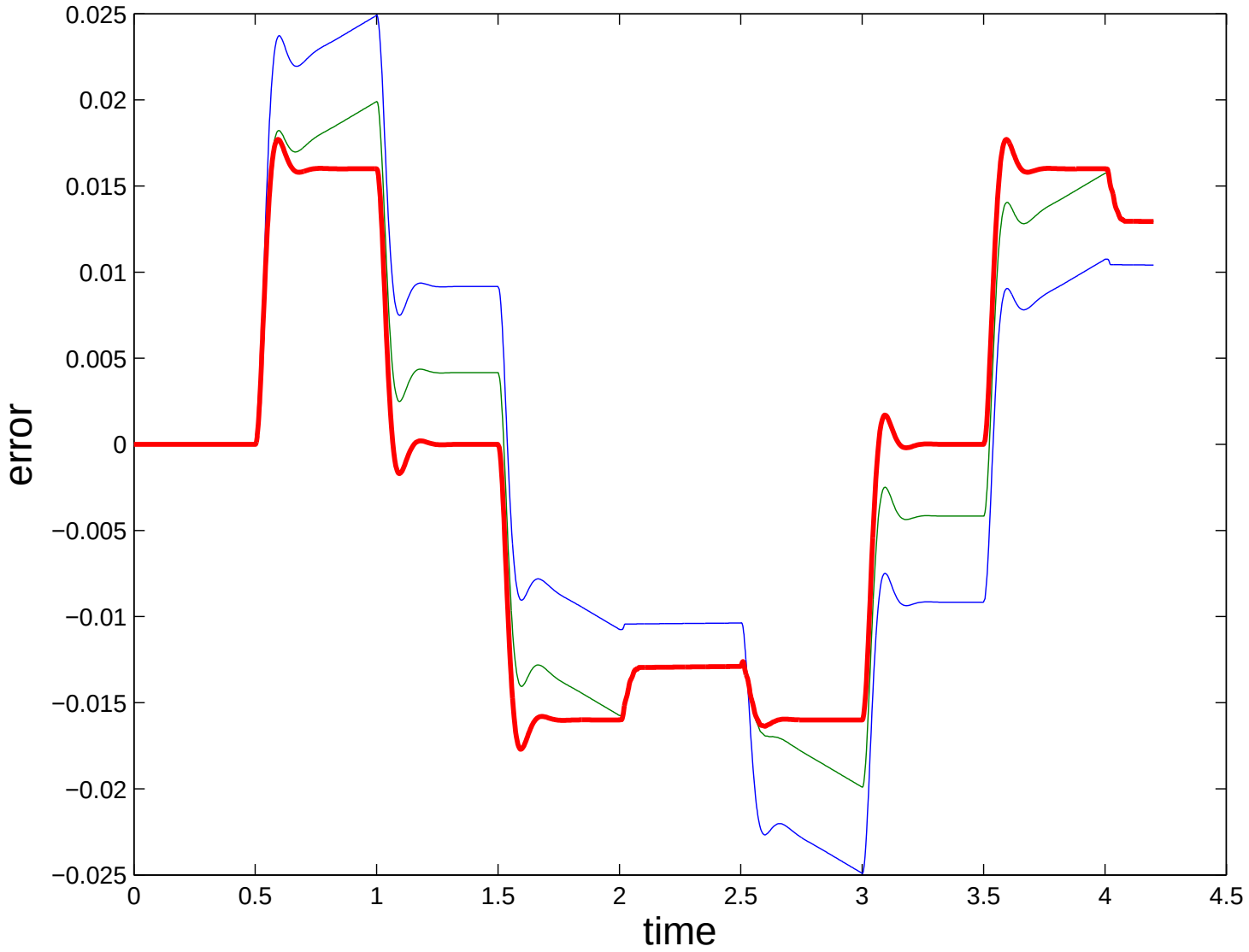
Feedforward tuning



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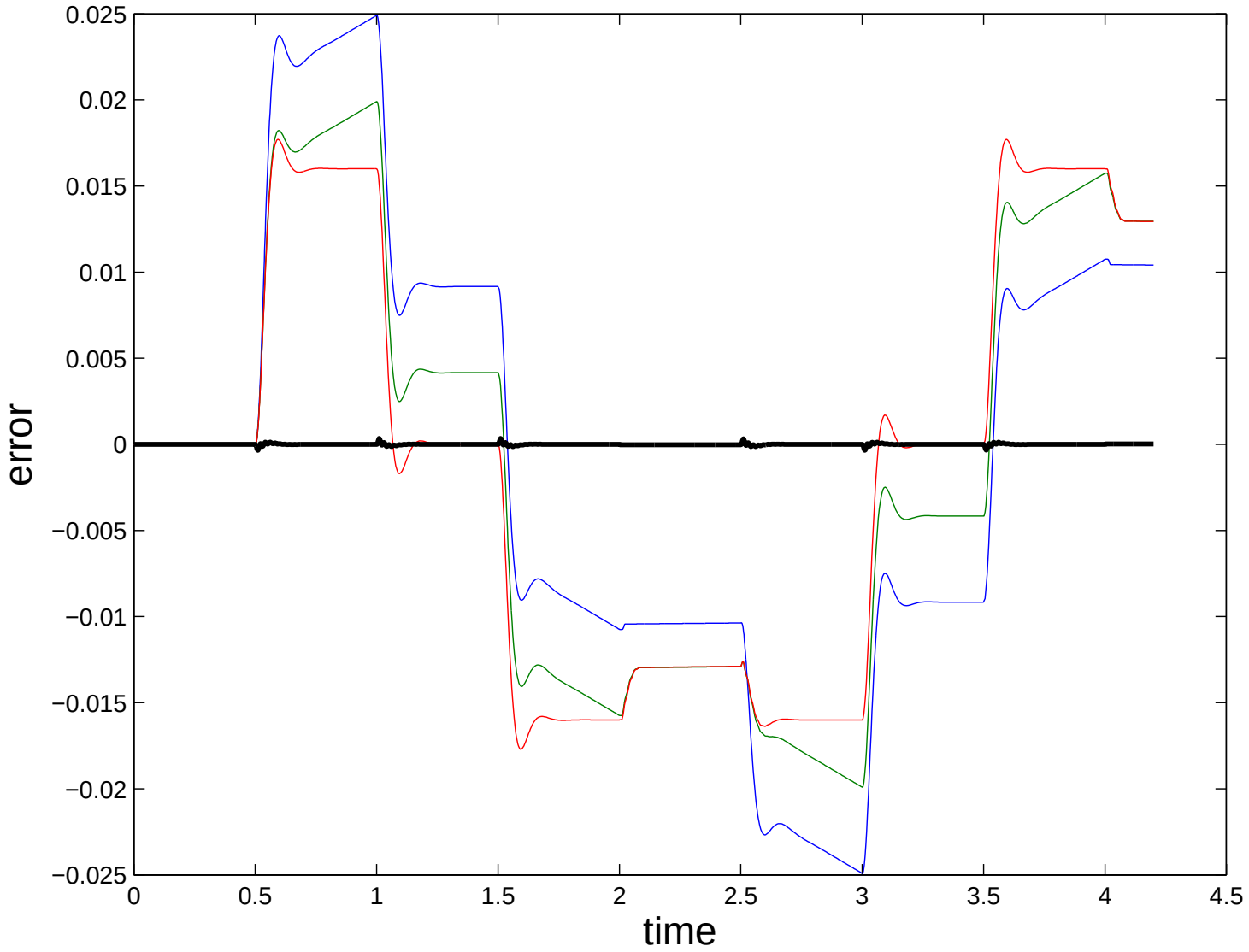

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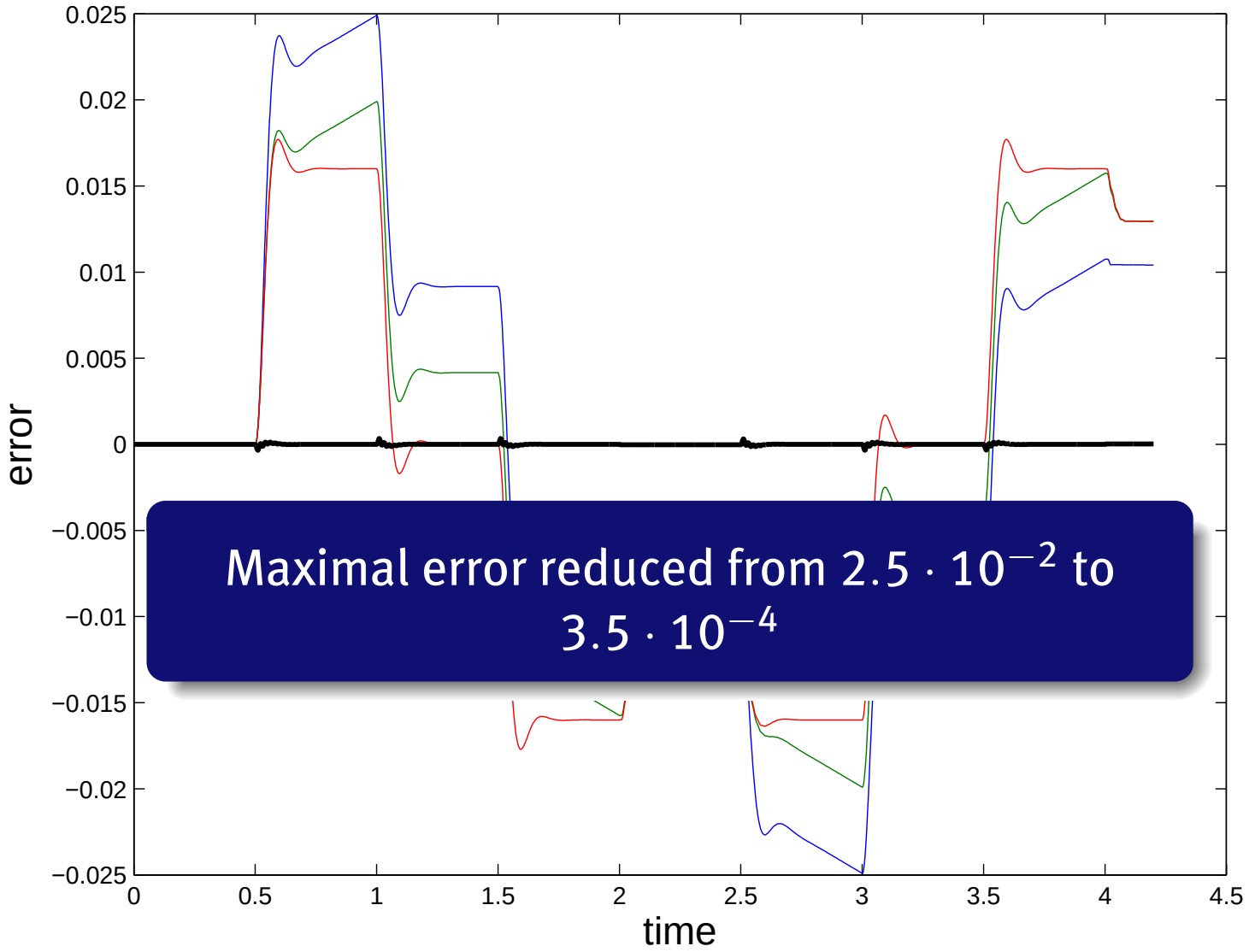

Maximal error reduced from $2.5 \cdot 10^{-2}$ to $3.5 \cdot 10^{-4}$

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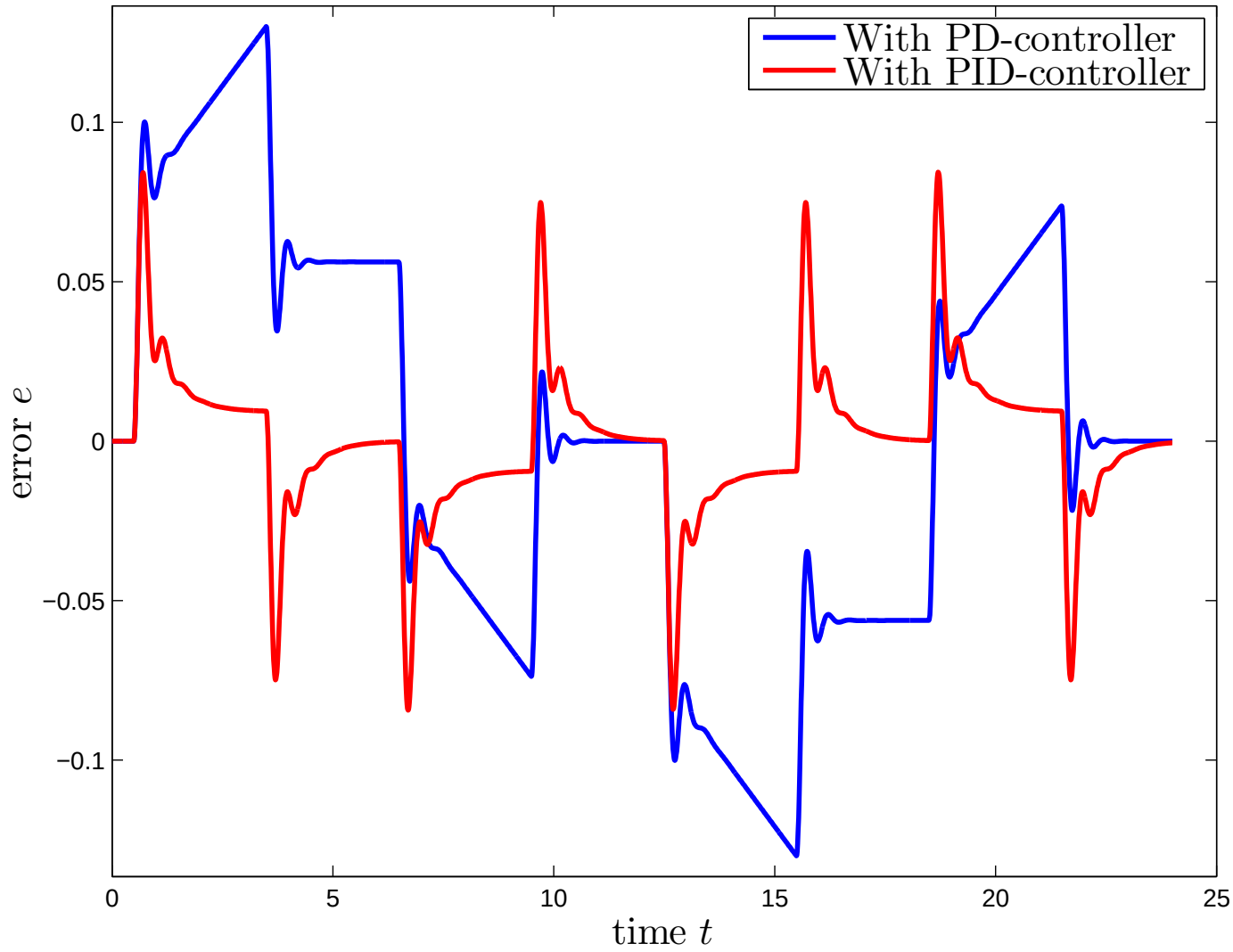
Integral controller action

To tune feedforward, don't use a high performance feedback controller!



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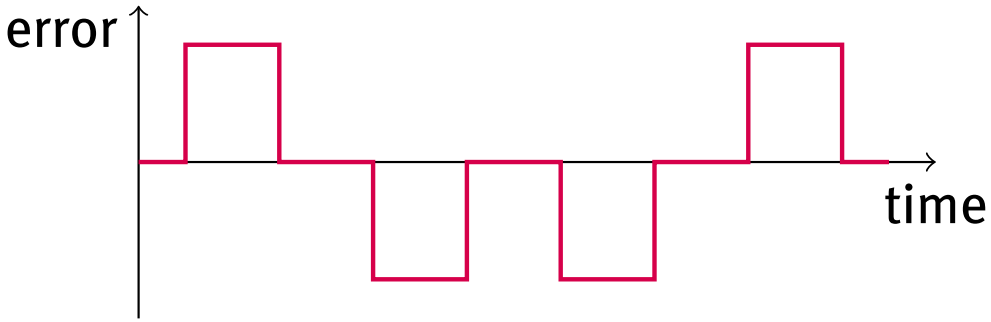

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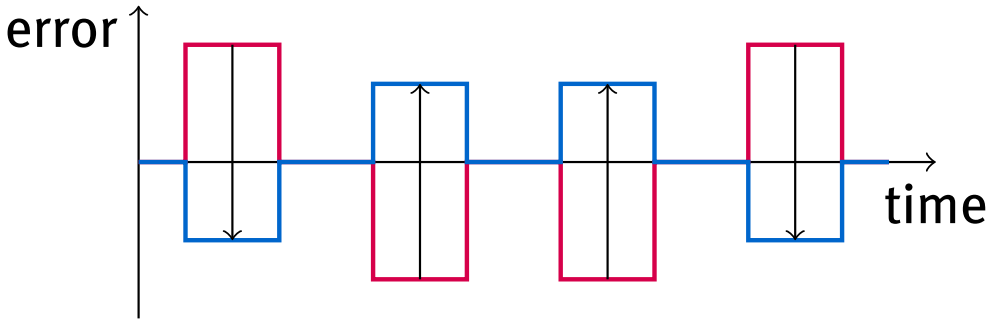

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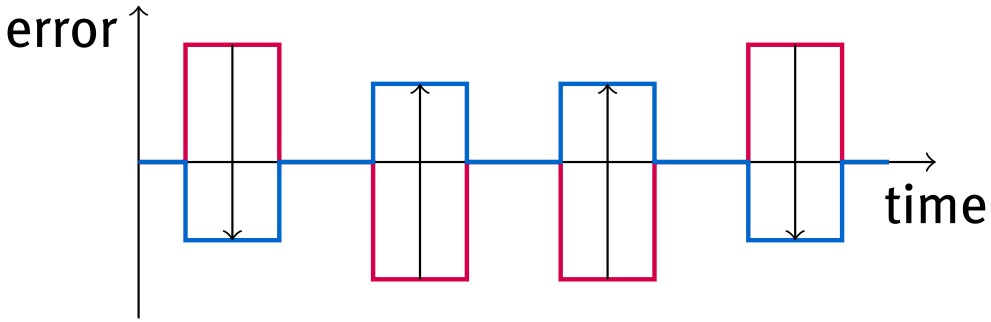

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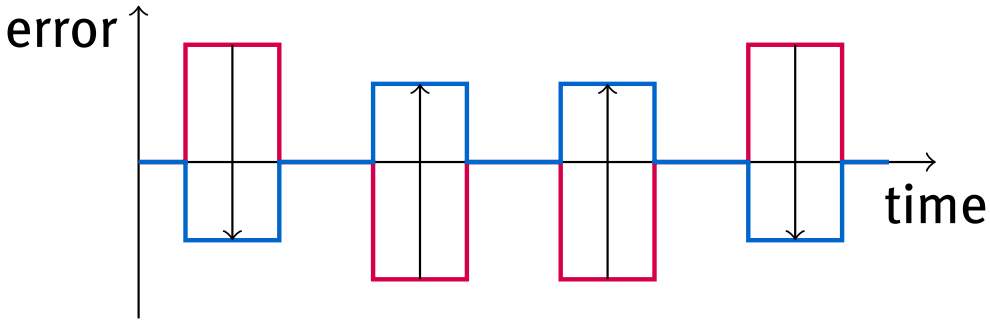

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  $\Rightarrow$ Terms represent system parameters, which do not change

Smoothing feedforward

Second order trajectory may not be sufficient

- jump in $\ddot{x}_s \Rightarrow$ jump in feedforward force
- can excite parasitic (higher order) dynamics
- can introduce oscillations
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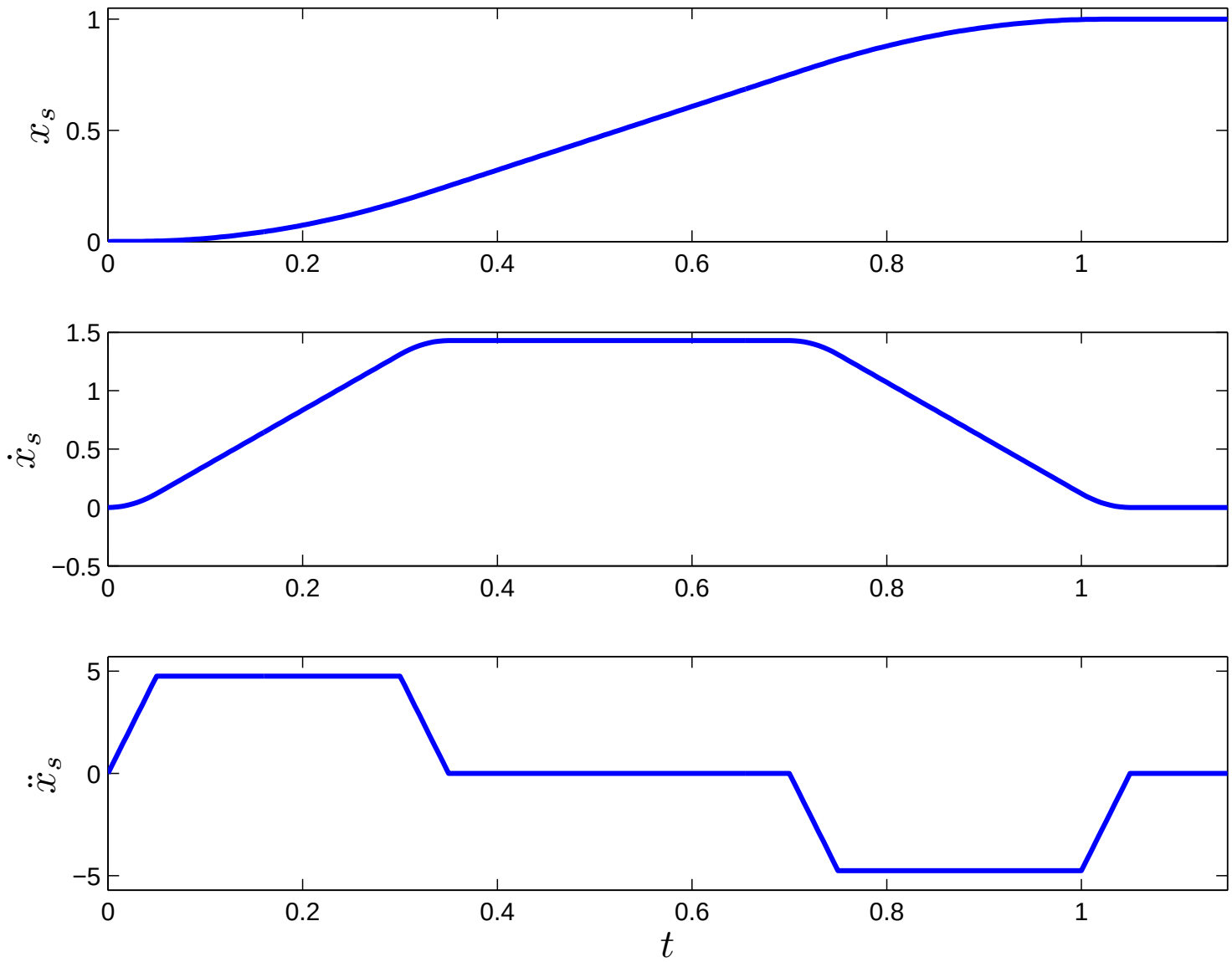
How to solve this?

- Higher order setpoints
  - gradual increase of feedforward forces (smoothing)
  - by limiting higher derivatives (e.g. jerk)
- Here: using 3rd order setpoints

3rd order setpoint



Higher order setpoints

$$\begin{aligned} \textit{Momentum} &= \textit{mass} \times \textit{velocity} \\ \textit{Force} &= \textit{mass} \times \textit{acceleration} \\ \textit{Yank} &= \textit{mass} \times \textit{jerk} \\ \textit{Tug} &= \textit{mass} \times \textit{snap} \\ \textit{Snatch} &= \textit{mass} \times \textit{crackle} \\ \textit{Shake} &= \textit{mass} \times \textit{pop} \end{aligned}$$

More advanced methods

Other more advanced techniques:

- Model-based input shaping
- Learning input shaping
  - adapt the trajectory after each trial
- Iterative Learning Control (ILC)
  - adapt the feedforward signal after each trial
  - frequency domain, lifted domain, or using basis functions

See other courses (4SC070: Learning Control) and current research.