Advanced Motion Control

Part IX: Optimal and Robust Control Continued

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Previous lecture

- general control configuration
- norm-based control
- used a parametric model

This lecture

- how to obtain the parametric model
- what if model errors are made: robust control
- design for motion systems

Modeling for control

Robustness

Weighting filters for $\mathcal{H}_\infty$ motion control

Multivariable weighting filters: Scaling aspects

Design example

Summary

Modeling for control

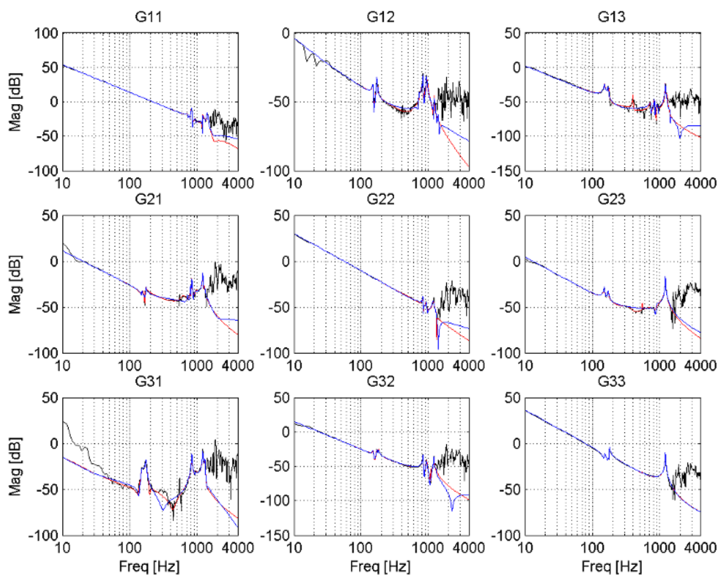
Modeling for $\mathcal{H}_\infty$-optimal control

- typical synthesis algorithms use a parametric model
- modeling approaches
  - white box modeling: physical laws, finite element models, ...
  - black box modeling: experimental modeling, system identification
    - time domain identification
    - frequency domain identification

- For motion systems:
  - identification is inexpensive, fast, and accurate
  - parametric models based on nonparametric FRFs for model validation(Oomen et al. 2014)

Modeling for control

Example from van de Wal et al. (2002)



Identification

1. FRF identification(Evers et al. 2020)
2. Fitting FRFs
   - element-by-element and stack(van de Wal et al. 2002)
   - directly MIMO(Oomen et al. 2014)

- possible model reduction

Clearly model errors introduced

Robustness

The need for robustness

- any mathematical model is an approximation of reality
  - model structure: e.g., LTI (no real system is fully linear)
  - model complexity: e.g., model order(Hughes 1987)
  - model parameters: e.g., parameter uncertainty(Hjalmarsson 2009)

Representing uncertainty

- general idea: given a model $\hat{G}(s)$, consider the set

$$\hat{G}(s) + w(s)\Delta(s), \text{ with } \|\Delta(s)\|_\infty \leq 1 \text{ and } w(s) \in \mathcal{RH}_\infty \text{ a weighting filter}$$

- this generates circles in the complex plane around $\hat{G}(j\omega)$ of radius $w(j\omega)$
- idea is to embed the true system P_o

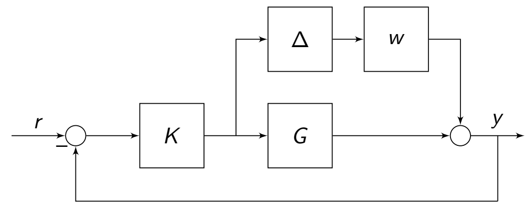
Robustness

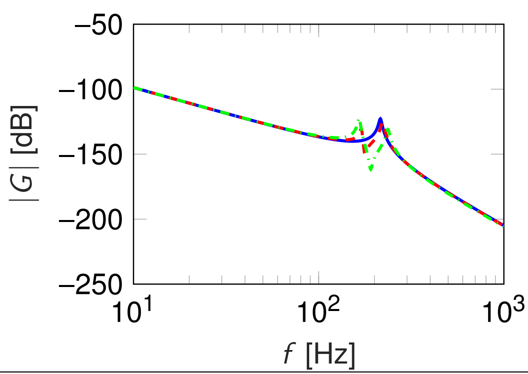
Robust control vs $\mathcal{H}_\infty$

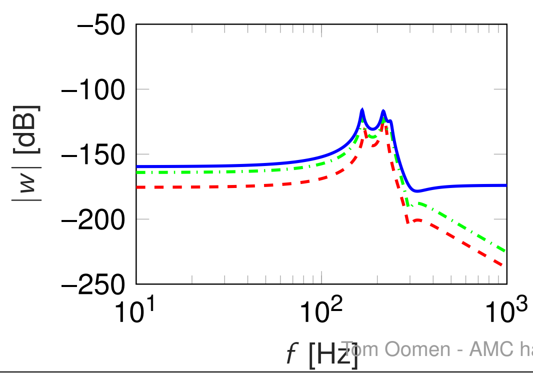
- robust control: achieving stability or performance in the presence of a certain description of uncertainty
- one systematic way: using $\mathcal{H}_\infty$-norm bounded perturbations
- must be explicitly modeled
  - controllers resulting from $\mathcal{H}_\infty$ optimization are not automatically robust!
  - either weigh certain closed-loop transfer functions(McFarlane & Glover 1990)
  - refinement: μ-synthesis: explicit knowledge of structure and size of uncertainty

Example uncertainty model

- earlier H-drive example (see also exercises)
- idea: $G(s) + w(s)\Delta(s)$, with $\|\Delta(s)\|_\infty \leq 1$:
  - take blue model as $\hat{G}$
  - compute model error $G - \hat{G}$ and $G - \hat{G}$
  - overbound the model error by weight $w(s)$







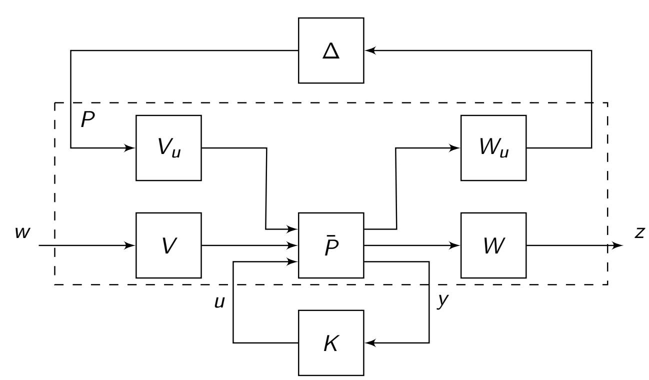
Design for Robust Performance

- pull out uncertainty Δ
- embed uncertainty weights V_u, W_u in P
- key point:
  - $\mathcal{H}_\infty$ bound on $w \mapsto z$ and Δ
  - to be kept separate!
- μ-synthesis!

$$\min_{K \text{ stabilizing}} \|\mathcal{F}_l(P, K)\|_\mu$$

- practical algorithm: $D-K$-iteration

$$\min_{K \text{ stabilizing}} \inf_{D} \left\| D\mathcal{F}_l(P, K)D^{-1} \right\|_\infty$$



Modeling for control

Robustness

Weighting filters for $\mathcal{H}_\infty$ motion control

Multivariable weighting filters: Scaling aspects

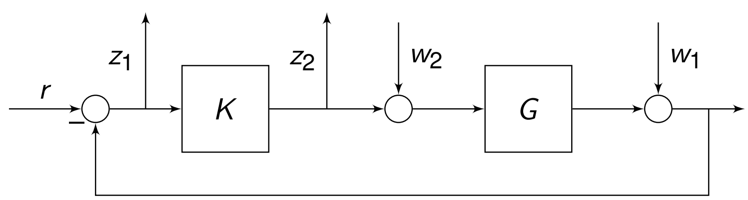
Design example

Summary

Weighting filters for $\mathcal{H}_\infty$ motion control

Four-block design

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = -\begin{bmatrix} S & SG \\ KS & KSG \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$



- guarantees internal stability
- facilitates low-order weighting functions

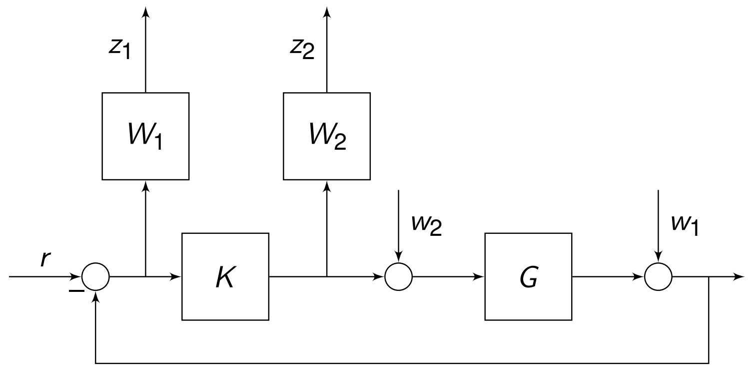
Basic assumptions

- assumption: G square and rigid-body decoupled
- facilitates design of filters
- allows interpretation in terms of loops
- result: K may become fully MIMO at high frequencies to compensate for interaction

Weighting filters for $\mathcal{H}_\infty$ motion control

Weighting filters

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = - \begin{bmatrix} W_1 S & W_1 SG \\ W_2 KS & W_2 KSG \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$



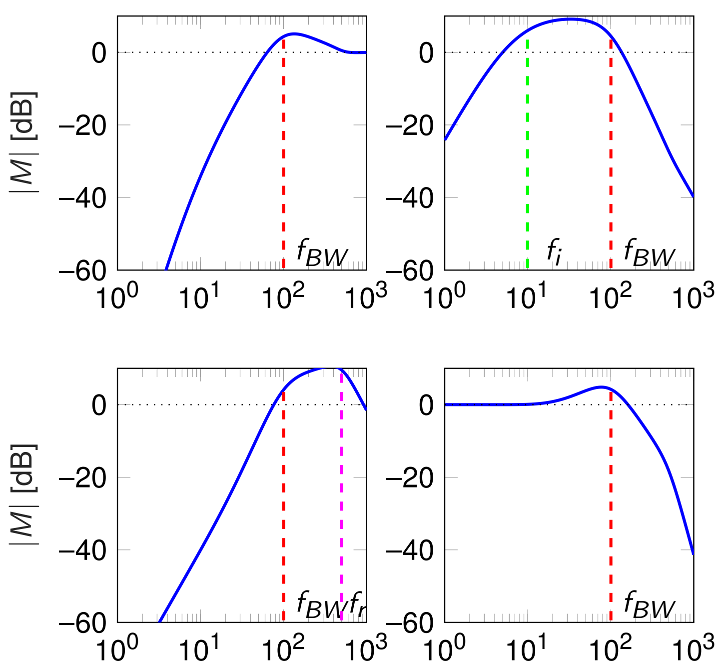
Design parameters

- f_{BW}: target bandwidth
- f_r: controller roll-off
- f_i: integrator frequency

Interpretation

- relates to manual loopshaping
- notches are included automatically
  - e.g., if the model includes resonance modes

Weighting filters for $\mathcal{H}_\infty$ motion control



Weighting filters

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = - \begin{bmatrix} W_1 S & W_1 SG \\ W_2 KS & W_2 KSG \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$
$$= - \begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Idea

- above f_{BW}: $|M_1|$ largest
  $\Rightarrow$ enforce roll-off
- below f_{BW}: $|M_2|$ largest
  $\Rightarrow$ enforce integral action

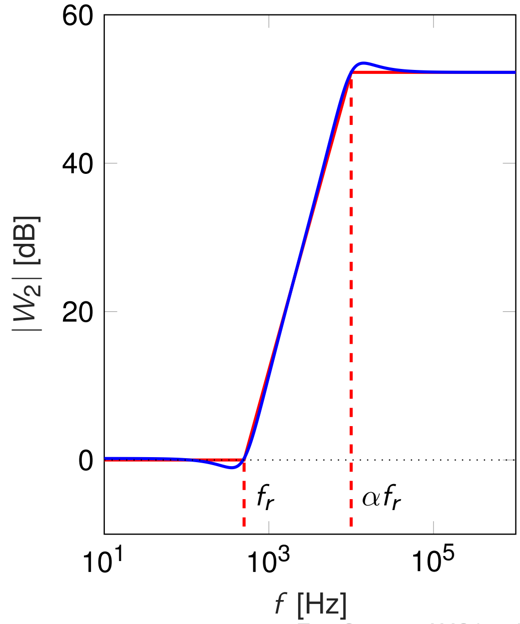
Weighting filters for $\mathcal{H}_\infty$ motion control

Designing W_2 for roll-off

- $\begin{bmatrix} M_1 & M_2 \end{bmatrix} = \begin{bmatrix} W_1 S & W_1 SG \\ W_2 KS & W_2 KSG \end{bmatrix}$
- elements of M_1 largest above f_{BW}
- $KS = \frac{K}{1+GK} \approx K$ for $|GK| \ll 1$
- W_2 slope $+1 \Rightarrow KS$ slope -1 $\Rightarrow K$ slope -1
- also: W_2 slope $+2 \Rightarrow KS$ slope -2 $\Rightarrow K$ slope -2
- select

$$W_2 = \frac{\frac{1}{2\pi f_r} s + 1}{\frac{1}{2\pi \alpha f_r} s + 1}$$

- zero at αf_r to ensure properness



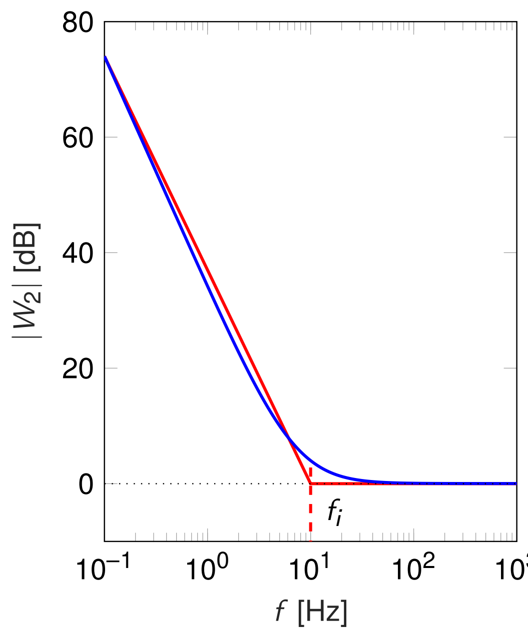
Weighting filters for $\mathcal{H}_\infty$ motion control

Designing W_1 for integral action

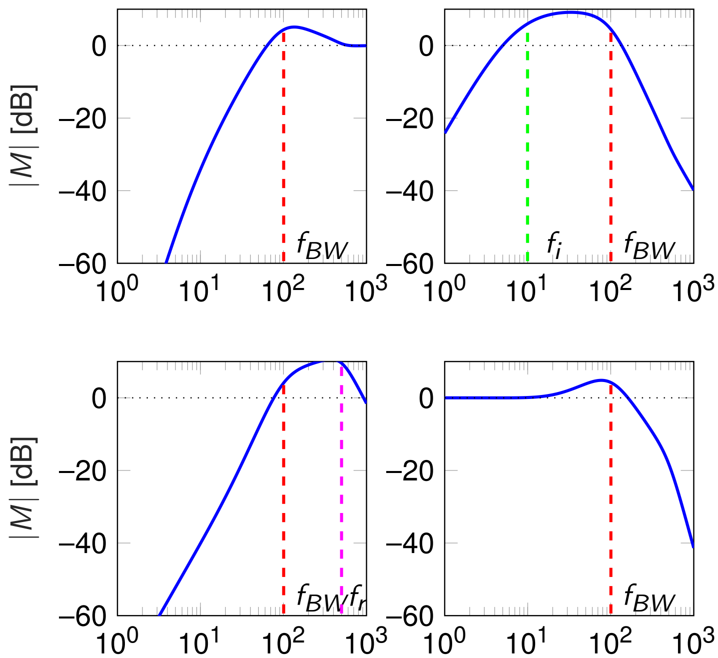
- $\begin{bmatrix} M_1 & M_2 \end{bmatrix} = \begin{bmatrix} W_1 S & W_1 SG \\ W_2 KS & W_2 KSG \end{bmatrix}$
- elements of M_2 largest above f_{BW}
- $SG = \frac{G}{1+GK} \approx \frac{1}{K}$ for $|GK| \gg 1$
- W_1 slope $-1 \Rightarrow GS$ slope +1 $\Rightarrow K$ slope -1
- also: W_1 slope $-2 \Rightarrow GS$ slope +2 $\Rightarrow K$ slope -2
- select

$$W_1 = \frac{s + 2\pi f_i}{s}$$

- pole at $s = 0$ may lead to violation of standard plant assumptions: see exercises



Weighting filters for $\mathcal{H}_\infty$ motion control



Design example

- $G = \frac{1}{s^2}$
- target bandwidth $f_{BW} = 100$
- controller roll-off $f_r = 500$
- integrator frequency $f_i = 10$

Multivariable weighting filters: Scaling aspects

- Elements of $M = \begin{bmatrix} W_1 S & W_1 SG \\ W_2 KS & W_2 KSG \end{bmatrix}$ become MIMO for MIMO systems
- Scaling is important!
- Example: the element S for a 2×2 system becomes

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

  - Q: size of diagonal elements $|S_{11}|$ and $|S_{22}|$?
  - Q: size of off-diagonal elements $|S_{12}|$ and $|S_{21}|$?
- Key point: $\bar{\sigma}(M) \geq \bar{\sigma}(S) \geq \max(|S_{11}|, |S_{12}|, |S_{21}|, |S_{22}|)$
  $\Rightarrow$ a large element of S leads to a large $\mathcal{H}_\infty$ norm
- Scaling
  - units of signals
  - relative importance (specs)

  $\Rightarrow$ embed these scalings into earlier weights

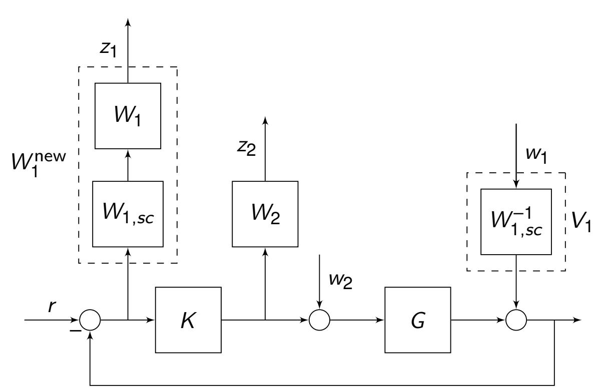
Multivariable weighting filters: Scaling aspects

Key idea

- scale error w.r.t. specification:
  $W_1^{new} = W_1 W_{1,sc}$
- pick

$$W_{1,sc} = \begin{bmatrix} z_1^{spec} & 0 & \dots & 0 \\ 0 & z_2^{spec} & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots \\ 0 & 0 & \dots & z_n^{spec} \end{bmatrix}^{-1}$$

- with z_i^{spec}, $i = 1, 2, \dots n$ the specification for exogenous output i
- $V_1 = W_{1,sc}^{-1}$: output disturbance w_1 assumed of same order of magnitude as error specs
- scaled sensitivity $S_{sc} = W_{1,sc} S W_{1,sc}^{-1}$



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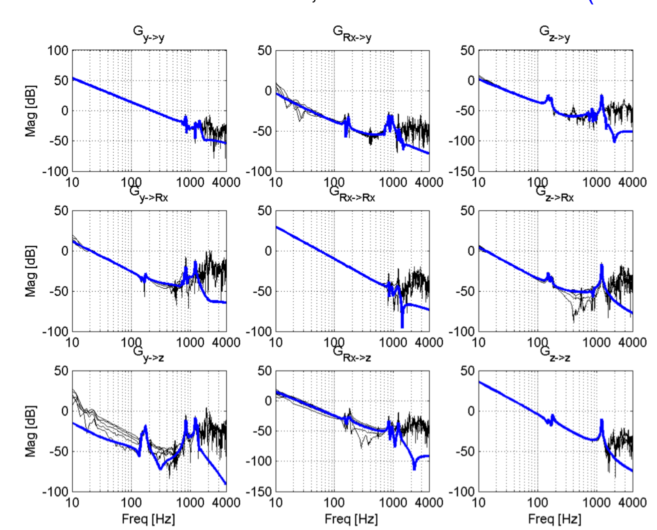
Design example

Example from van de Wal et al. (2002)

FRF at several locations, 53^{th} order model $\hat{G}$ (slide 4)



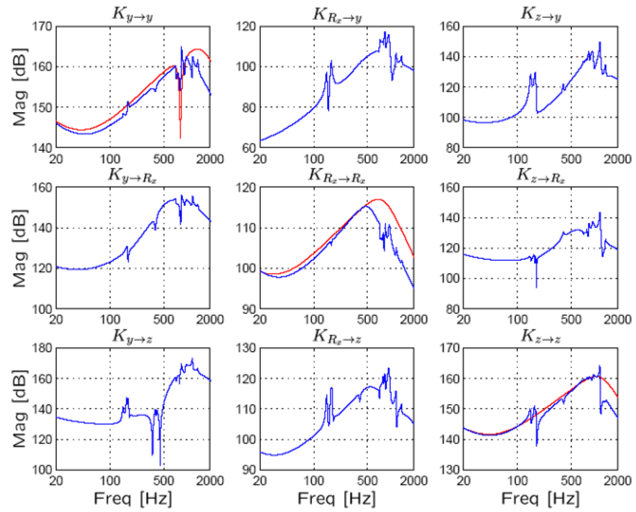
Starting point

- parametric model $\hat{G}$
- uncertainty model that covers all FRFs (slide 7)

Design example

Example from van de Wal et al. (2002)

Resulting controllers: 36$^{\text{th}}$ order optimal, 14$^{\text{th}}$ order manually tuned



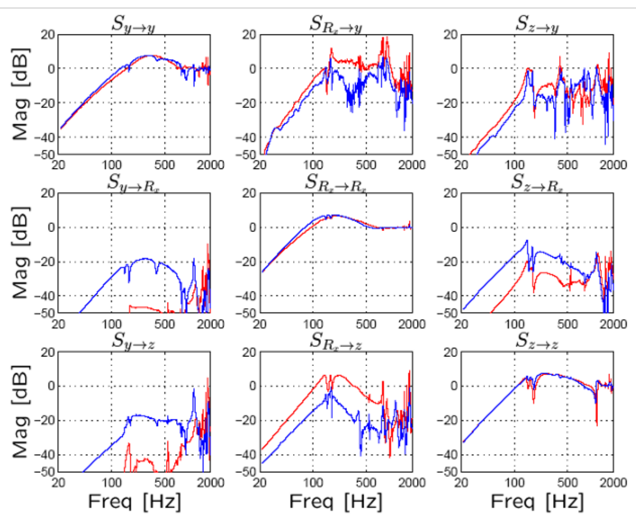
Controller synthesis

- optimal controller from μ-synthesis (slide 8)
- high-order due to $D-K$-iteration
- closed-loop model reduction
  (Wortelboer 1994)

Design example

Example from van de Wal et al. (2002)

Scaled sensitivity $S_{sc} = W_{1,sc} S W_{1,sc}^{-1}$: 36$^{\text{th}}$ order optimal, 14$^{\text{th}}$ order manually tuned



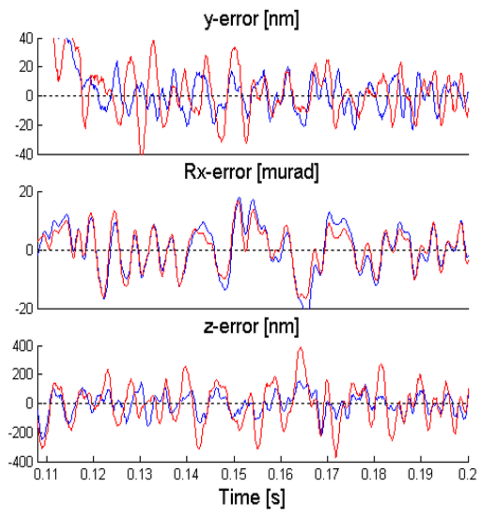
Varying results

- Optimal better for
  - $R_x \mapsto y$
  - $R_x \mapsto z$
  - $z \mapsto y$
- Manual tuning better for
  - $y \mapsto R_x$
  - $z \mapsto R_x$
  - $y \mapsto z$
- at least all elements of S_{sc} have magnitude < 1

Design example

Example from van de Wal et al. (2002)

Measured servo errors: 36^{th} order optimal, 14^{th} order manually tuned



Results

- Optimal better for
  - y
  - z
- Manual tuning equal for
  - R_x
- Optimal controller has improved performance
  - slightly different tuning
  - larger design space: centralized controller that can possibly deal more effectively with interaction

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Summary

Take-home messages

- model-based control provides a systematic framework for designing controllers
- larger design space: centralized controllers

Do-it-yourself

- first synthesis often does not lead to the desired result
- advice: pinpoint what causes the undesired result
  - theory: problem setting is conflicting with theoretical requirements, e.g., assumptions on the standard plant
  - numerical: computations are performed using finite precision and problem data may result in ill-conditioning. The theoretical result is invariant under the specific state-space realization. Try changing the state-space realization to investigate this: theoretically the same controller should result. Balanced coordinates often lead to better numerical properties compared to model coordinates.
  - design: problem may not be posed sensibly, e.g., the example on unweighted sensitivity minimization.

References I

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