Advanced Motion Control

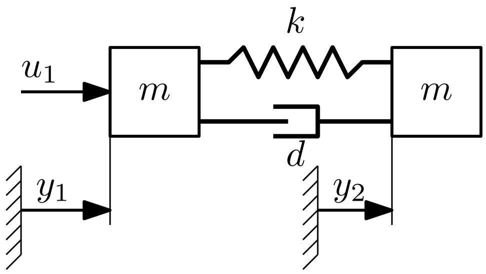
Part VII: Modal Control

Tom Oomen

Control Systems Technology Group
Department of Mechanical Engineering
Eindhoven University of Technology

2022-2023

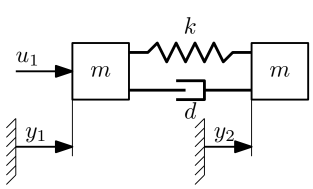
A benchmark system

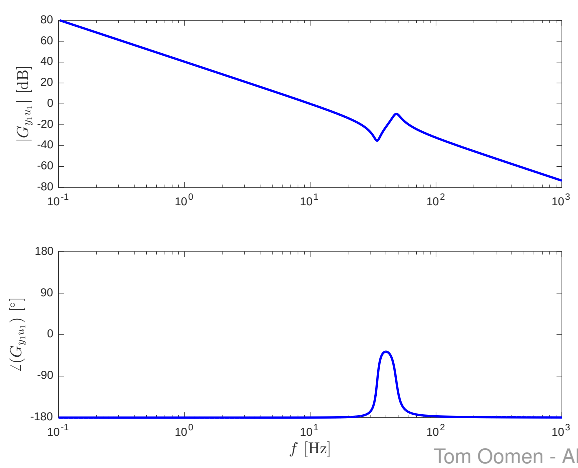


- what limits the control performance?
  - e.g., in terms of achievable bandwidth
  - on the real-life setup
  - performance limitations?

A benchmark system

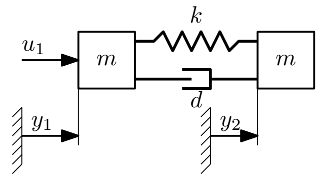
- input u_1
- output y_1
- collocated
- performance limitations?
  - not too many in practice
  - implementation
    - computational delay
    - sensitivity integral (Sung & Hara 1988)
  - higher-order modes
    - encoder

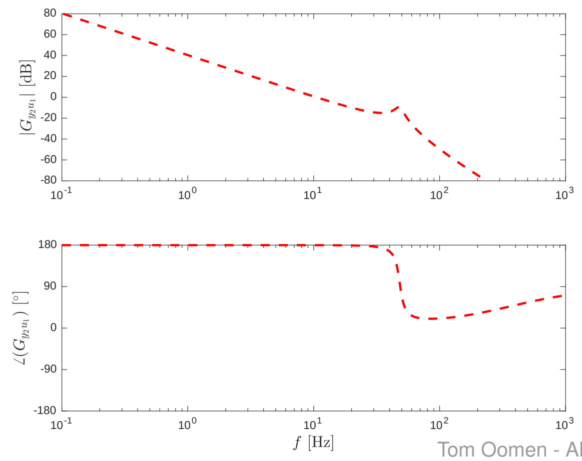




A benchmark system

- input u_1
- output y_2
- non-collocated
- performance limitations?
- in practice bandwidth seems limited by resonance frequency
- why?
  - algebraic constraints?
  - analytic constraints?
  - NMP zeros?
  - …?

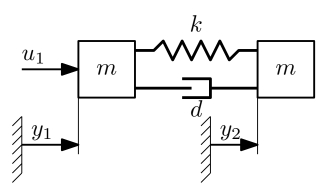


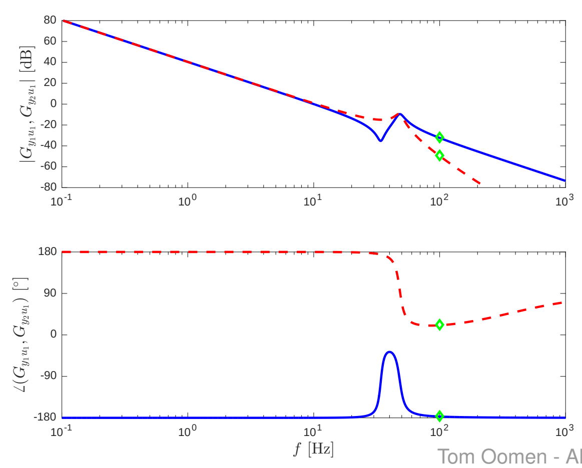


Tom Oomen - AMC handout

A benchmark system

- example: aim for bandwidth of 100 Hz
- manual loopshaping
- performance limitations in typical mechanical systems?



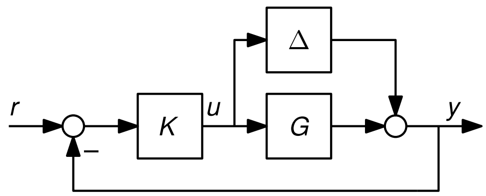


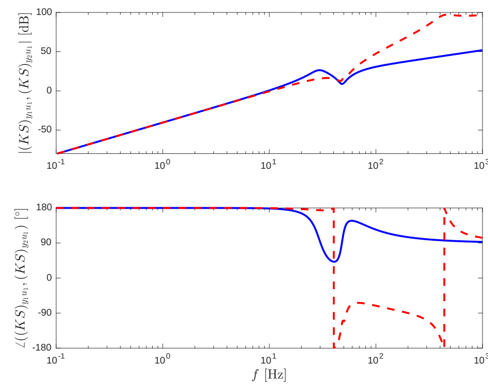
Tom Oomen - AMC handout

Introduction

A benchmark system

- example: aim for bandwidth of 100 Hz
- performance-robustness tradeoff
- RS test: $\|M\Delta\|_\infty < 1$
- Additive uncertainty: $M = -K(I + GK)^{-1}$
- non-collocated: admissible uncertainty 100 times smaller!
- non-collocated: typically bandwidth below $f_{\text{res}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$
- this part: beyond f_{res}?





Introduction

Modal models

Modal control

Modal control - experimental case studies

Summary and reading

Modal models

Nodal coordinates

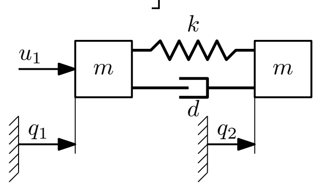
- often used in analysis of structural dynamics
  - finite element models
  - lumped parameter models

$$M\ddot{q} + D\dot{q} + Kq = B_o u$$
$$y = C_{oq}q + C_{ov}\dot{q}$$

- q: nodal displacements
- relation state-space $\dot{x} = Ax + Bu$, then, e.g.,

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad \Rightarrow \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$$

- example:



Modal models

Modal coordinates

- different system representation may facilitate analysis & control
- consider the undamped ($D = 0$), free ($u = 0$) vibration of the equations of motion

$$M\ddot{q} + Kq = 0$$

- postulate solution of the form $q = \phi e^{j\omega t}$, hence $\ddot{q} = -\omega^2 \phi e^{j\omega t}$

$$\left(K - \omega^2 M\right) \phi e^{j\omega t} = 0$$

- generalized eigenvalue problem: eigenvalues ω_i, eigenvectors ϕ_i:

$$\Omega = \operatorname{diag}\left(\omega_1, \omega_2, \ldots, \omega_n\right) \quad \Phi = \begin{bmatrix} \phi_1 & \phi_2 & \ldots & \phi_n \end{bmatrix}$$

Modal models

Modal coordinates

- Let $q = \Phi q_m$
- substution into

$$M\ddot{q} + D\dot{q} + Kq = B_o u$$
$$y = C_{oq} q + C_{ov}\dot{q}$$

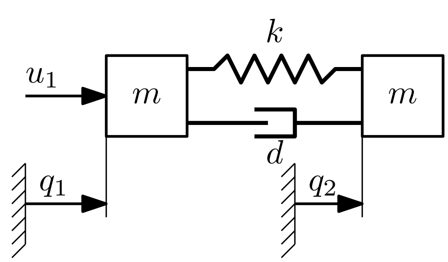
and pre-multiplying by Φ^T yields

$$\underbrace{\Phi^T M \Phi}_{M_m} \ddot{q}_m + \underbrace{\Phi^T D \Phi}_{D_m} \dot{q}_m + \underbrace{\Phi^T K \Phi}_{K_m} q_m = \Phi^T B_o u$$
$$y = C_{oq}\Phi q_m + C_{ov}\Phi \dot{q}_m$$

- assume proportial damping, e.g., through $D = \alpha_1 K + \alpha_2 M$, then M_m, K_m, D_m diagonal
- Q: Interpretation?
  A: response is sum of modal responses y_i

Modal models

Modal coordinates: example



- derivation of equations of motion

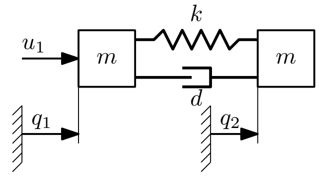
$$m\ddot{q}_1 = -k(q_1 - q_2) - d(\dot{q}_1 - \dot{q}_2) + u_1$$
$$m\ddot{q}_2 = -k(q_1 - q_2) - d(\dot{q}_1 - \dot{q}_2)$$

- which is equal to

$$\underbrace{\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}}_{M} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} d & -d \\ -d & d \end{bmatrix}}_{D} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{K} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} u_1 \\ 0 \end{bmatrix}}_{u}$$

Modal models

Modal coordinates: example



- equations of motion:
  undamped ($D = 0$), free ($u = 0$)

$$\underbrace{\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}}_{M} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{K} = 0$$

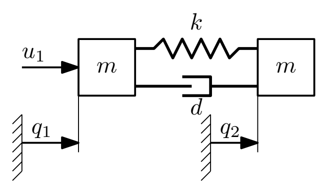
- so that $\det(K - \omega^2 M) = 0$ leads to characteristic equation

$$\begin{aligned} \begin{vmatrix} k - m\omega^2 & -k \\ -k & k - m\omega^2 \end{vmatrix} &= (k - m\omega^2)^2 - k^2 \\ &= \cancel{k^2} - 2km\omega^2 + m^2\omega^4 - \cancel{k^2} \\ &= m\omega^2(m\omega^2 - 2k) = 0 \end{aligned}$$

- hence $\omega^2 = 0$ or $\omega^2 = \frac{2k}{m}$

Modal models

Modal coordinates: example



- eigenvectors for $\omega_1 = 0$: $\phi_1 \neq 0$ such that

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \phi_1 = 0 \quad \Rightarrow \phi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- eigenvectors for $\omega_2 = \pm\sqrt{\frac{2k}{m}}$: $\phi_2 \neq 0$ such that

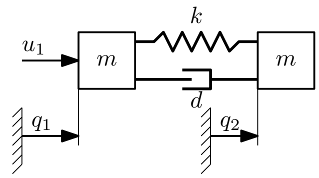
$$\begin{bmatrix} k - \frac{m \cdot 2k}{m} & -k \\ -k & k - \frac{m \cdot 2k}{m} \end{bmatrix} \phi_2 = 0 \quad \Rightarrow \phi_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

- Thus

$$\Phi = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Modal models

Modal coordinates: example



- recall

$$\underbrace{\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}}_{M} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} d & -d \\ -d & d \end{bmatrix}}_{D} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{K} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} u_1 \\ 0 \end{bmatrix}}_{u}$$

- with $\Phi = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $\underbrace{\Phi^T M \Phi}_{M_m} \ddot{q}_m + \underbrace{\Phi^T D \Phi}_{D_m} \dot{q}_m + \underbrace{\Phi^T K \Phi}_{K_m} q_m = \Phi^T B_o u$ yields

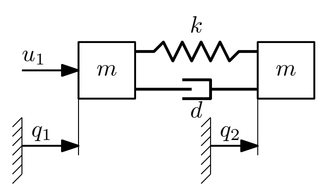
$$\underbrace{\begin{bmatrix} 2m & 0 \\ 0 & 2m \end{bmatrix}}_{M_m} \ddot{q}_m + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 4d \end{bmatrix}}_{D_m} \dot{q}_m + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 4k \end{bmatrix}}_{K_m} q_m = \begin{bmatrix} u_1 \\ u_1 \end{bmatrix}$$

- and

$$q_m = \Phi^{-1} q = \begin{bmatrix} \frac{1}{2} q_1 + \frac{1}{2} q_2 \\ \frac{1}{2} q_1 - \frac{1}{2} q_2 \end{bmatrix}$$

Modal models

Modal coordinates: example



- diagonal matrices:
  - $2m\ddot{q}_{m1} = u_1 \Rightarrow \frac{q_{m1}(s)}{u_1(s)} = \frac{1}{2ms^2}$
  - $2m\ddot{q}_{m2} + 4d\dot{q}_{m2} + 4kq_{m2} = u_1 \Rightarrow \frac{q_{m2}(s)}{u_1(s)} = \frac{1}{2ms^2+4ds+4k}$
- hence $G(s)$ can be written as the sum of modal transfer functions

$$\begin{aligned} y &= C_{oq} q \\ &= C_{oq} \Phi q_m \end{aligned}$$

  - collocated case: $y = q_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} q_m = q_{m1} + q_{m2}$

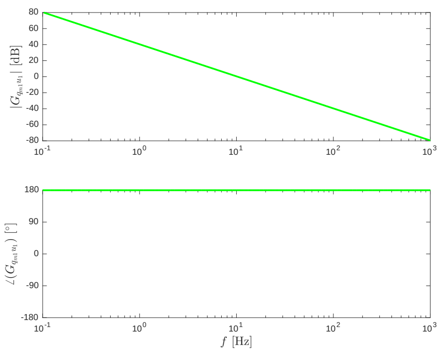
    hence $\frac{y(s)}{u(s)} = \frac{1}{2ms^2} + \frac{1}{2ms^2+4ds+4k}$
  - non-collocated case: $y = q_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} q_m = q_{m1} - q_{m2}$

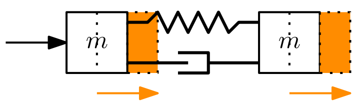
    hence $\frac{y(s)}{u(s)} = \frac{1}{2ms^2} - \frac{1}{2ms^2+4ds+4k}$

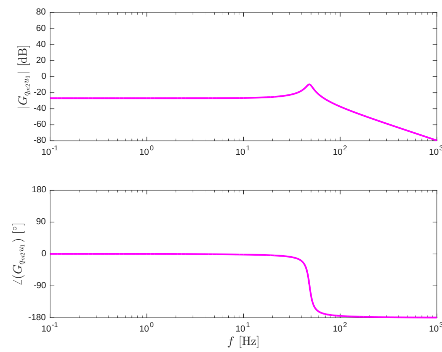
Modal coordinates: example

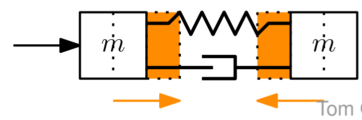
$$\frac{q_{m1}(s)}{u_1(s)} = \frac{1}{2ms^2}, \quad \phi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{q_{m2}(s)}{u_1(s)} = \frac{1}{2ms^2+4ds+4k}, \quad \phi_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



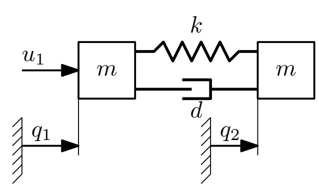




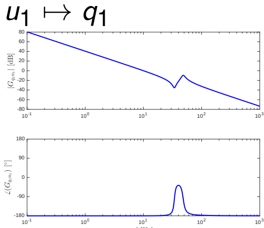


Modal coordinates: example

- collocated case $u_1 \mapsto q_1$

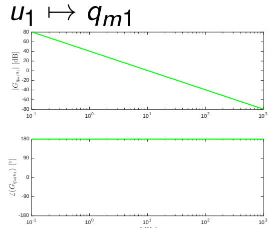


$u_1 \mapsto q_1$



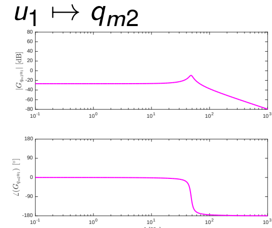
$=$

$u_1 \mapsto q_{m1}$



$+$

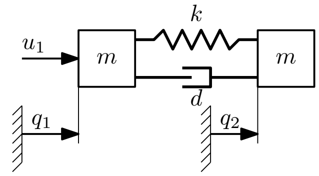
$u_1 \mapsto q_{m2}$



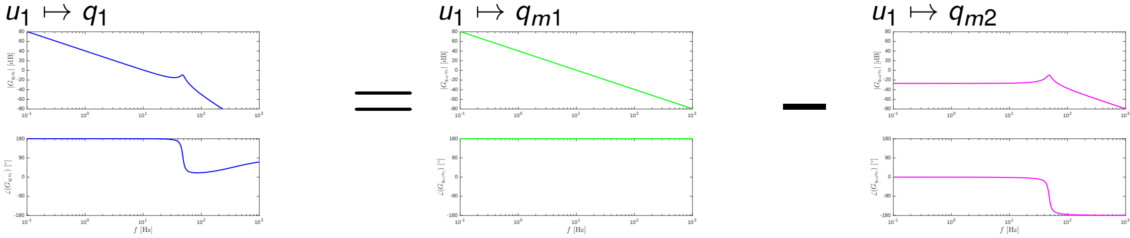
Modal models

Modal coordinates: example

- non-collocated case $u_1 \mapsto q_2$



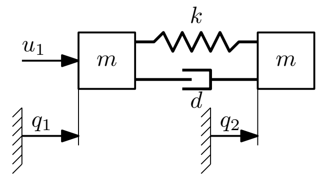
$u_1 \mapsto q_1$ $=$ $u_1 \mapsto q_{m1}$ $-$ $u_1 \mapsto q_{m2}$

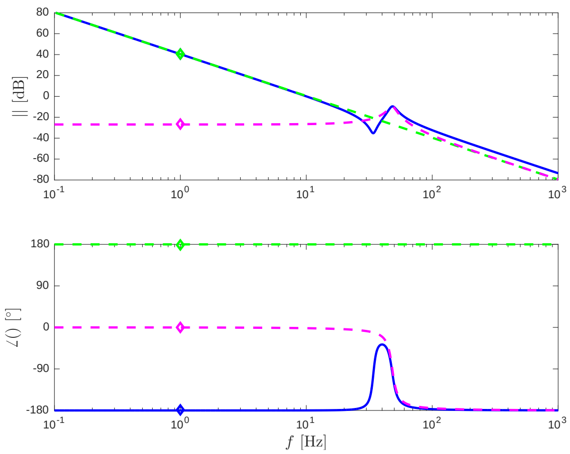


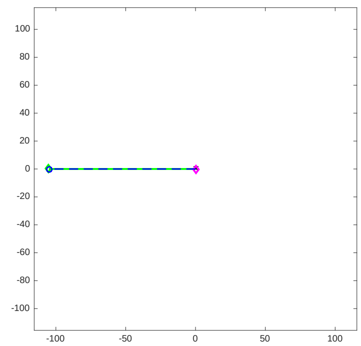
Modal models

Modal coordinates: example

- collocated case $u_1 \mapsto q_1$
- evaluation at low frequencies $f = 1$ Hz

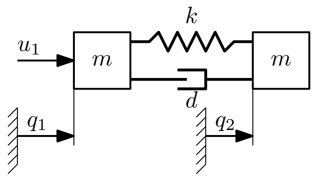


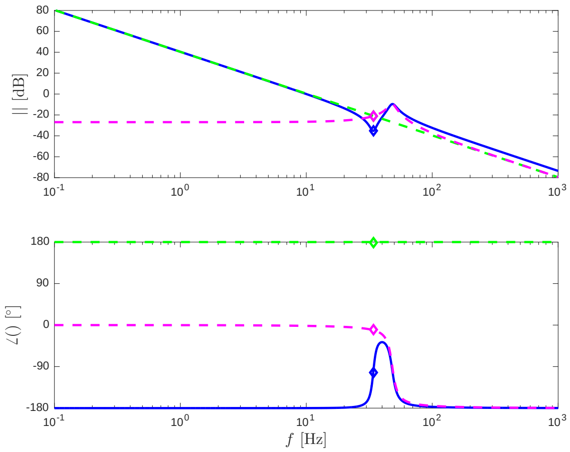


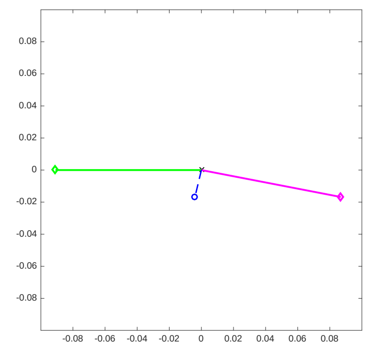


Modal coordinates: example

- collocated case $u_1 \mapsto q_1$
- evaluation at mid frequencies $f = 34$ Hz

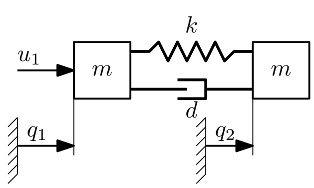


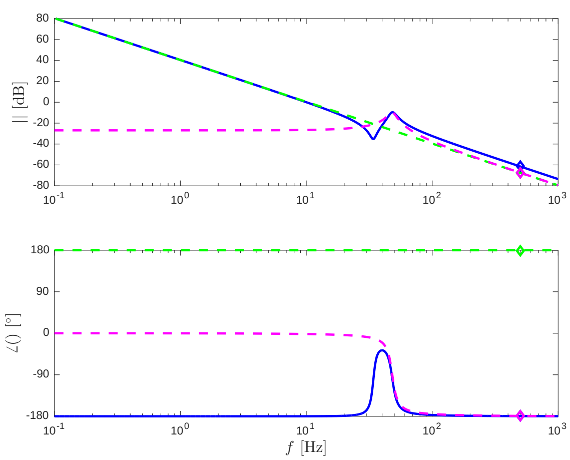


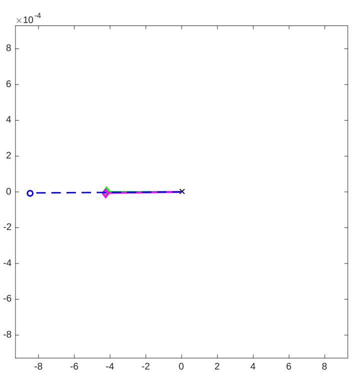


Modal coordinates: example

- collocated case $u_1 \mapsto q_1$
- evaluation at high frequencies $f = 500$ Hz

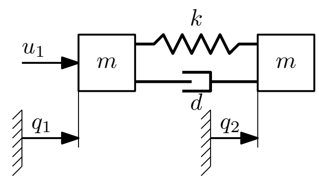


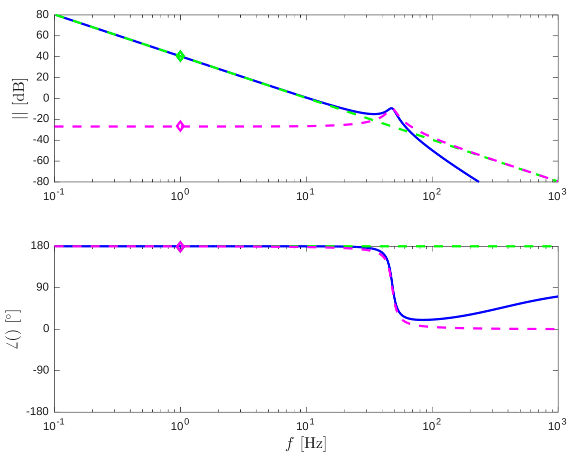


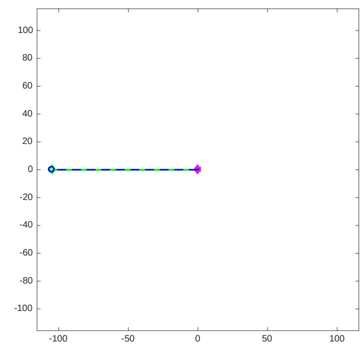


Modal coordinates: example

- non-collocated case $u_1 \mapsto q_2$
- evaluation at low frequencies $f = 1$ Hz

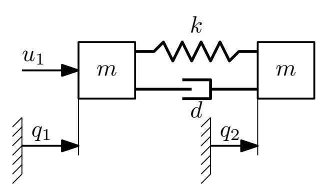


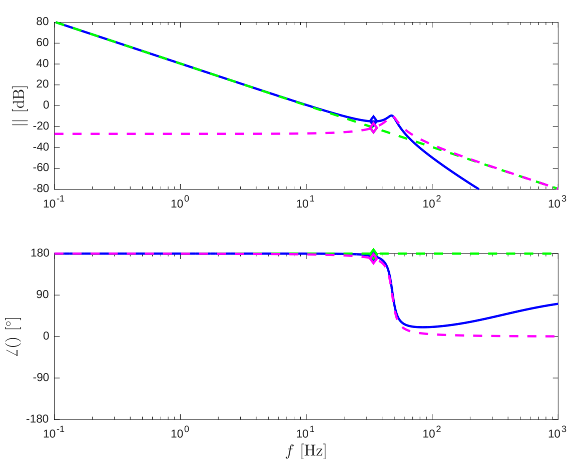


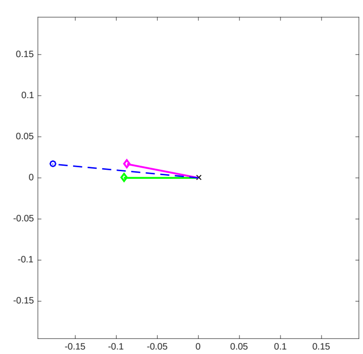


Modal coordinates: example

- non-collocated case $u_1 \mapsto q_2$
- evaluation at low frequencies $f = 34$ Hz



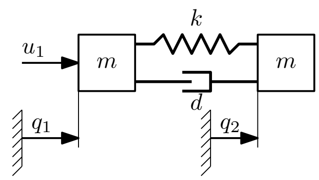


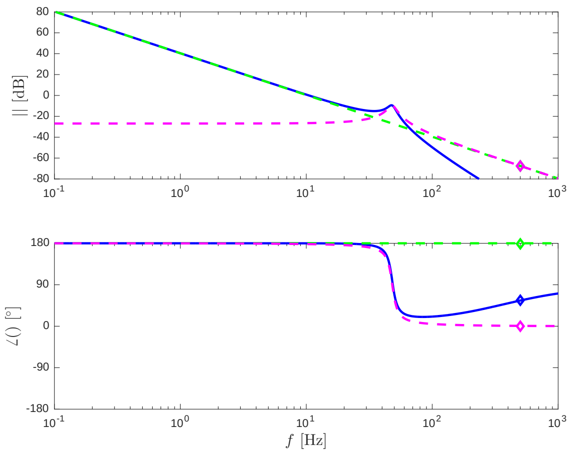


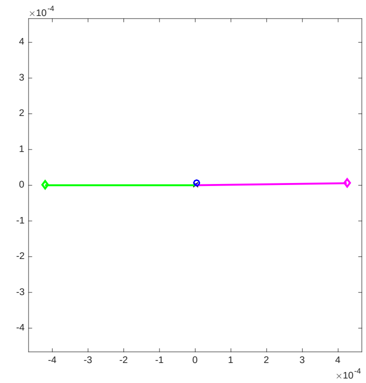
Modal models

Modal coordinates: example

- non-collocated case $u_1 \mapsto q_2$
- evaluation at low frequencies $f = 500$ Hz







Modal models

Modal coordinates: MIMO systems

- the results directly extend to MIMO systems:

$$G_m = \underbrace{\sum_{i=1}^{n_{RB}} \frac{c_i b_i^T}{s^2}}_{\text{rigid–body modes}} + \underbrace{\sum_{i=N_{rb}+1}^{n_s} \frac{c_i b_i^T}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}}_{\text{flexible modes}},$$

- decoupling at least aims to decouple the rigid-body modes

$$G = T_y G_m T_u = \frac{1}{s^2} I_{n_{RB}} + G_{\text{flex}},$$

Modal control

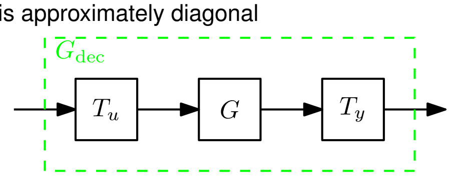
Modal decoupling

- System description in modal coordinates (assuming $C_{ov} = 0$)

$$\underbrace{\Phi^T M \Phi}_{M_m} \ddot{q}_m + \underbrace{\Phi^T D \Phi}_{D_m} \dot{q}_m + \underbrace{\Phi^T K \Phi}_{K_m} q_m = \Phi^T B_o u$$

$$y = C_{oq} \Phi q$$

- Recall decoupling:
  - determine $T_y \in \mathbb{R}^{l \times l}$ and $T_u \in \mathbb{R}^{m \times m}$
  - such that $T_y G T_u$ is approximately diagonal



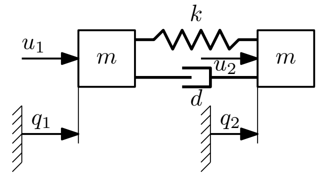
- idea: if B_o and C_{oq} square, then a suitable choice is given by

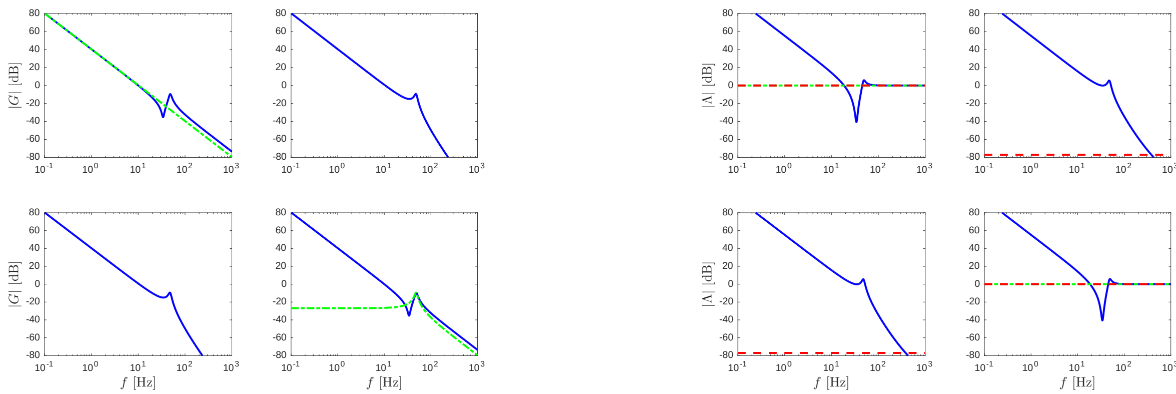
$$T_u = B_o^{-1} (\Phi^{-1})^T, \quad T_y = \Phi^{-1} C_{oq}^{-1}$$

Modal control

Modal decoupling: example

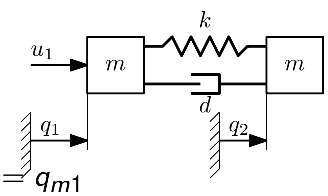
- earlier example revisited
- original plant G
- decoupled plant $T_y G T_u$



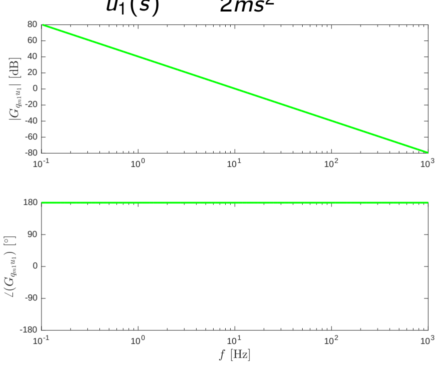


Modal control

Modal control: another example



- suppose you have the system $u_1 \mapsto \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$
- suppose you decouple it using modal transformation matrices: $y_{new} = q_{m1}$
- hence $\frac{y_{new}(s)}{u_1(s)} = \frac{1}{2ms^2}$



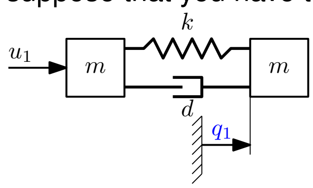
Control

- using y_{new}
- implications?
  - bandwidth limitation?
  - underlying control 'objective'?
  - desirable in practice?

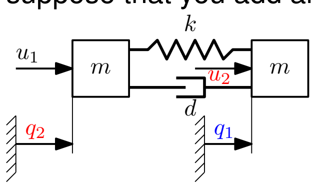
Modal control

Modal control: systematically enhancing performance

- suppose that you have the non-collocated system



- and performance is desired at q_1
- slide 4: performance often bounded by resonance frequency
- suppose that you add an actuator and sensor

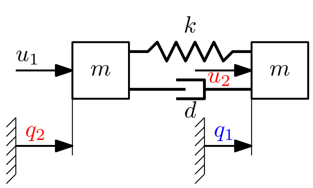


- what can be done using modal representations?

Modal control

Modal control: systematically enhancing performance

- system with swapped outputs
- cast in modal form
- compute



$$\tilde{T}_y = \left[\begin{array}{c} \tilde{T}_{y1} \\ \hline \tilde{T}_{y2} \end{array}\right] = \Phi^{-1} C_{oq}^{-1} \quad \tilde{T}_u = \left[\begin{array}{c|c} T_{u1} & \tilde{T}_{u2} \end{array}\right] = B_o^{-1} (\Phi^{-1})^T,$$

- implement

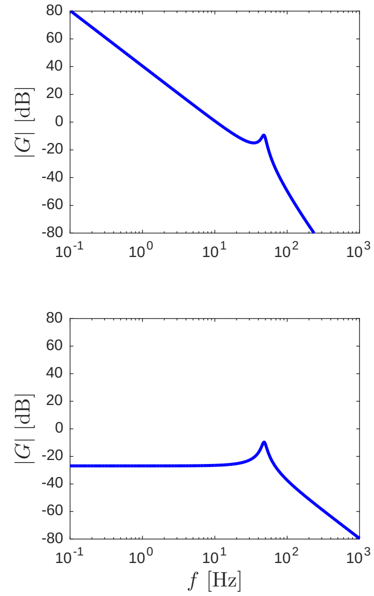
$$T_y = \left[\begin{array}{cc} 1 & 0 \\ \hline \multicolumn{2}{c}{\tilde{T}_{y2}} \end{array}\right] \quad T_u = \left[\begin{array}{c|c} \begin{array}{c} 1 \\ 0 \end{array} & \tilde{T}_{u2} \end{array}\right]$$

- Q: $T_y G T_u$ decoupled?

Modal control

Resulting system

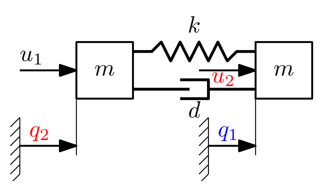
- $T_y G T_u$

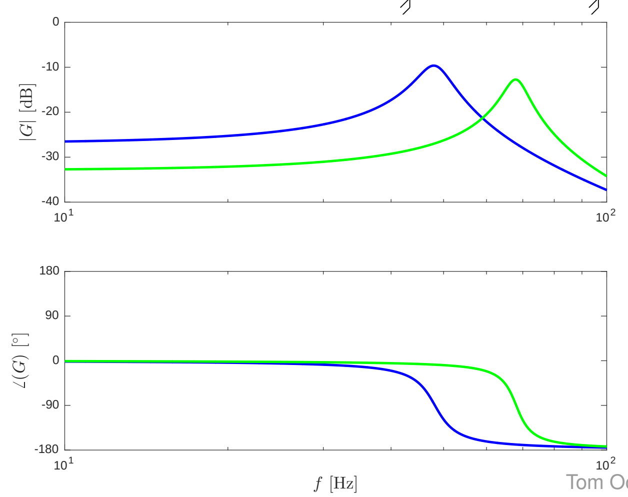


Modal control

Sequential loop closing

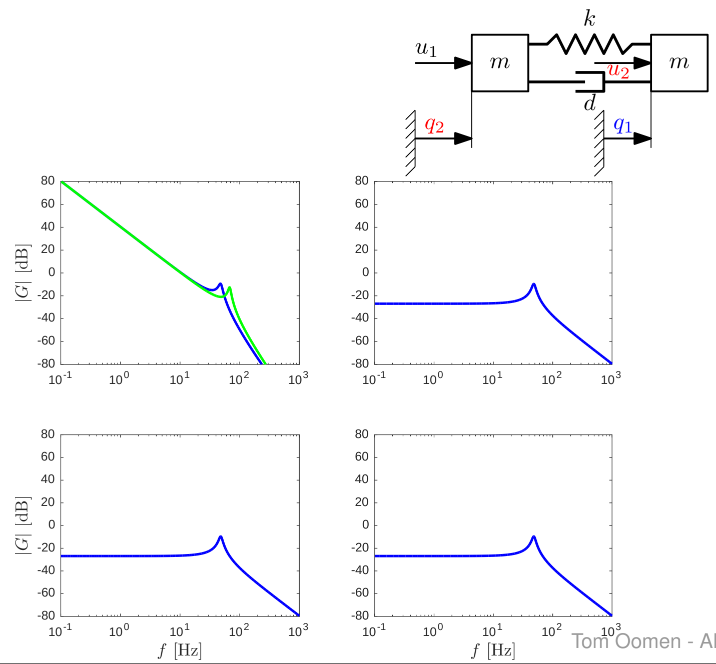
- open-loop g_{22}
- close loop 2:
  k_{22}: PD
- $\frac{g_{22}}{1+g_{22}k_{22}}$





Sequential loop closing

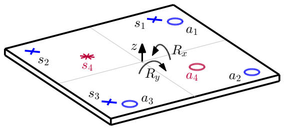
- open-loop G
- $g_{11}^{eq} = g_{11} - \frac{g_{12}k_{22}g_{21}}{1+k_{22}g_{22}}$
- f_{res} increased
- potential performance increase $u_1 \mapsto q_1$
- fully actuated example (simpler solutions possible)
- idea useful in practice: exploiting additional actuators and sensors

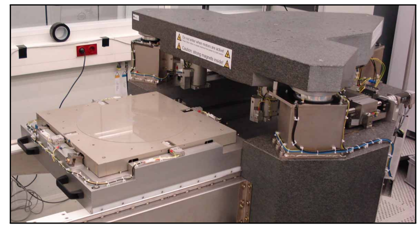


Modal control - experimental case studies

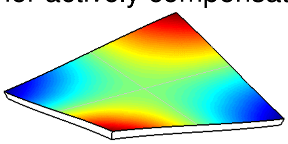
Active control ⇒ more inputs u & outputs y (van Herpen et al. 2014)

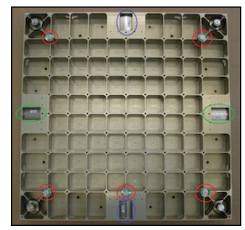
- prototype wafer stage
- many actuators and sensors





- for actively compensating torsion bending mode

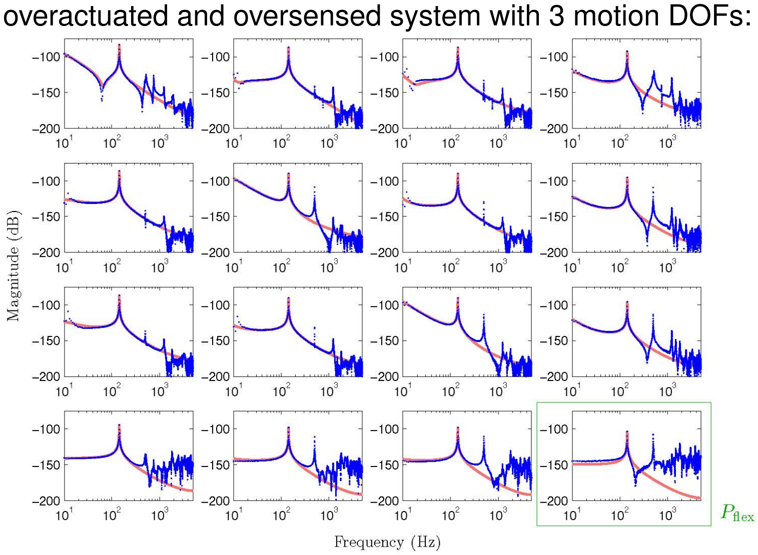




Modal control - experimental case studies

Active control ⇒ more inputs u & outputs y (van Herpen et al. 2014)

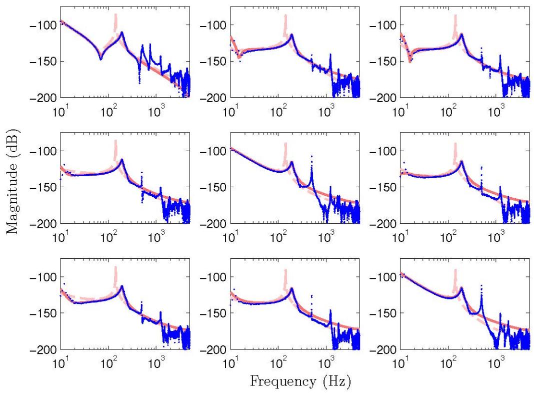
- overactuated and oversensed system with 3 motion DOFs:

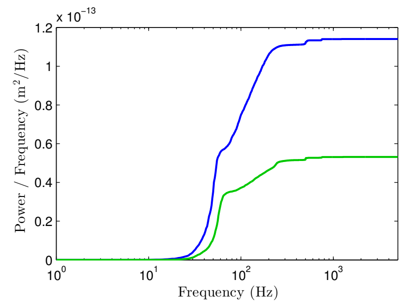


Modal control - experimental case studies

Active control $\Rightarrow$ more inputs u & outputs y (van Herpen et al. 2014)

- active control increases damping and stiffness of torsion mode:





- in turn enables 35% higher bandwidth and smaller error
- achieved in (van Herpen et al. 2014) (Tacx & Oomen 2022) via robust control (next lectures)

Modal control - experimental case studies

Global spatial optimal feedforward (de Rozario et al. 2017)

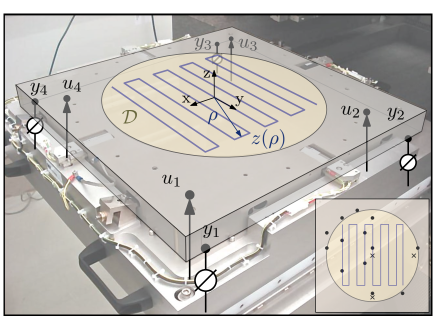


Figure: OAT setup

Minimize error during scan at $z(\rho)$.
Approach: minimize total spatial vibrations using local LTI feedforward control using u_i.

1. Estimate spatial modal model $G(\rho)$
2. Use spatial norm

$$\|s\|_{\mathcal{D}} \triangleq \sqrt{\sum_{k=-\infty}^{\infty} \int_{\mathcal{D}} s^\top(\rho, k) s(\rho, k) d\rho},$$

3. Feedforward control problem: minimize spatial norm over domain $\mathcal{D}$ for a single LTI feedforward controller

Modal control - experimental case studies

Global spatial optimal feedforward (de Rozario et al. 2017)

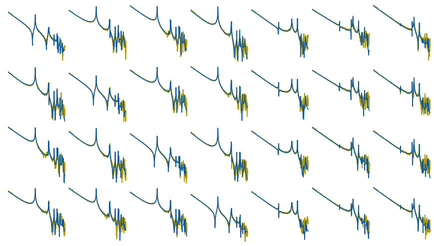


Figure: Estimate parametric model from FRF-data

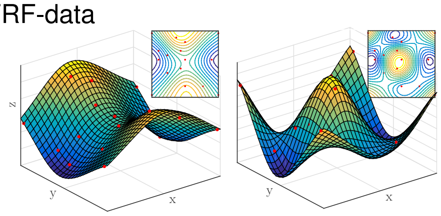


Figure: Interpolate modes

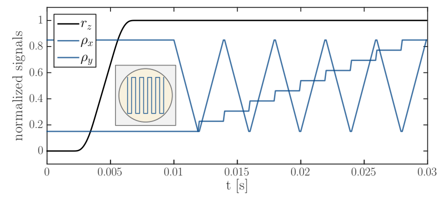


Figure: Local reference and scanning trajectory $z(\rho)$

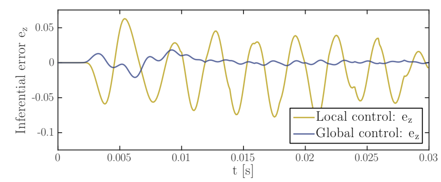


Figure: Global error improved w.r.t. local control approach

Introduction

Modal models

Modal control

Modal control - experimental case studies

Summary and reading

Summary and reading

Take-home messages

- performance limitations in motion systems: often performance/robustness tradeoff due to resonances
- exploiting more actuators & sensors to alleviate performance limitations
- modal control facilitates
  - decoupling
  - sequential loop closing
  - ...

Next

- centralized, optimal, and robust control design

Summary and reading

Additional reading

- modal models & control: Gawronski (2004) Preumont (2004) Inman (2006)
- application to an ‘overactuated’/‘oversensed’ motion system: van Herpen et al. (2014), Tacx & Oomen (2022)
- application to spatial performance: de Rozario et al. (2017) Moheimani et al. (2003)

References I

Gawronski, W. K. (2004), *Advanced Structural Dynamics and Active Control of Structures*, Springer, New York, New York, United States.

van Herpen, R., Oomen, T., Kikken, E., van de Wal, M., Aangenent, W. & Steinbuch, M. (2014), 'Exploiting additional actuators and sensors for nano-positioning robust motion control', *Mechatronics* **24**(6), 619–631.

Inman, D. J. (2006), *Vibration with Control*, John Wiley & Sons.

Moheimani, S., Halim, D. & Fleming, A. J. (2003), *Spatial Control of Vibration: Theory and Experiments*, World Scientific, Singapore.

Preumont, A. (2004), *Vibration Control of Active Structures: An Introduction*, Vol. 96 of *Solid Mechanics and Its Applications*, second edn, Kluwer Academic Publishers, New York, New York, United States.

de Rozario, R., Voorhoeve, R., Aangenent, W. & Oomen, T. (2017), Spatio-temporal identification of mechanical systems: With application to global feedforward control of an industrial wafer stage, *in* 'IFAC 2017 Triennial World Congress', Toulouse, France, pp. 15140–15145.

Sung, H.-K. & Hara, S. (1988), 'Properties of sensitivity and complementary sensitivity functions in single-input single-output digital control systems', *International Journal of Control* **48**(6), 2429–2439.

Tacx, P. & Oomen, T. (2022), A one-step approach for centralized overactuated motion control of a prototype reticle stage, *in* 'nsy2022 Modeling, Estimation and Control Conference (MECC)'.