Single Mode Input Shaping

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When a system has just one identical vibration as response to step command, need to reduce it by Single Mode Shaping. An early form of input shaper was the use of Posicast control. This control splits the reference signal into two parts. The size of the steps and the delay before introducing the second step are derived from the system dynamics. The modern technology that we call impulse shaping is a few generations removed from posicast control idea.
Input Shaping is a command generation technique which attempts to impart zero energy into a system at the frequencies at which it will vibrate. In order to ensure zero energy at the vibration frequencies, the commands given to the system must be modified, thus the term Input Shaping. Once the correct command for the system is found, the result will be a system that has no energy at frequencies for which it will vibrate, thus no vibration. The process relates back to the use of a series of impulses, which will cause zero vibration in the system; this series of impulses or shaper when convolved with the original command to the system yields a response that also causes zero vibration.
Let's assume we have a second order system defined as follows:

$$G(s) = \frac{1}{(\frac{s}{\omega_0})^2 + 2\zeta(\frac{s}{\omega_0}) + 1} \tag{0.1}$$

where ω_0 is the resonance frequency and ζ is damping coefficient. Time response of the system is depicted in Fig. 0.1. In this figure, overshoot has defined by $K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$ and damped period by $T_d = \frac{\pi}{\omega_d}$.
Input shaper is pre-filter defined as two or more impulses. As a first step in understanding how to generate commands that move systems without vibration, it is helpful to start with the simplest such command, an impulse. Fig. 0.2 shows that the first impulse (with amplitude A_1 at time t_1) causes a flexible system to vibrate, but a second properly timed (t_2) and sized impulse A_2 will cancel the vibration induced by the first impulse.

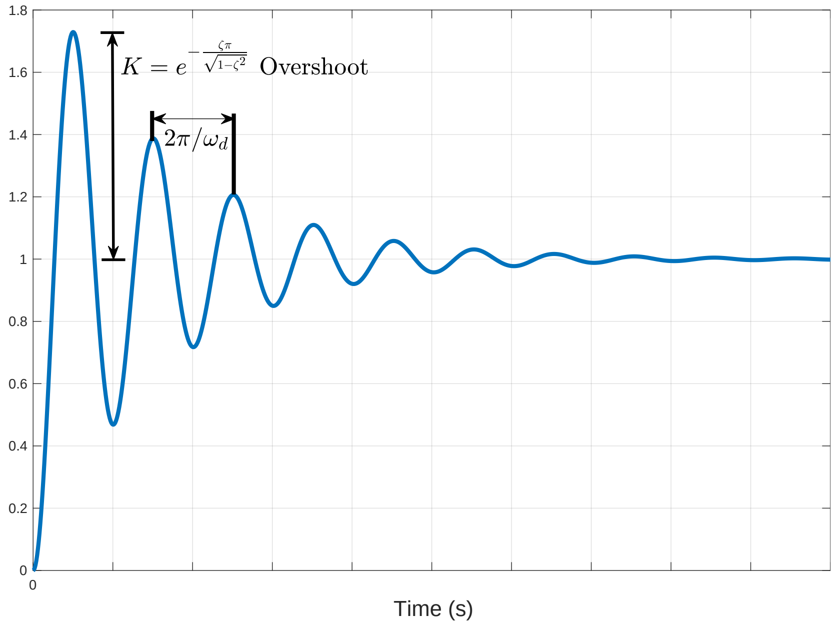


Figure 0.1: Time Response of Second Order System

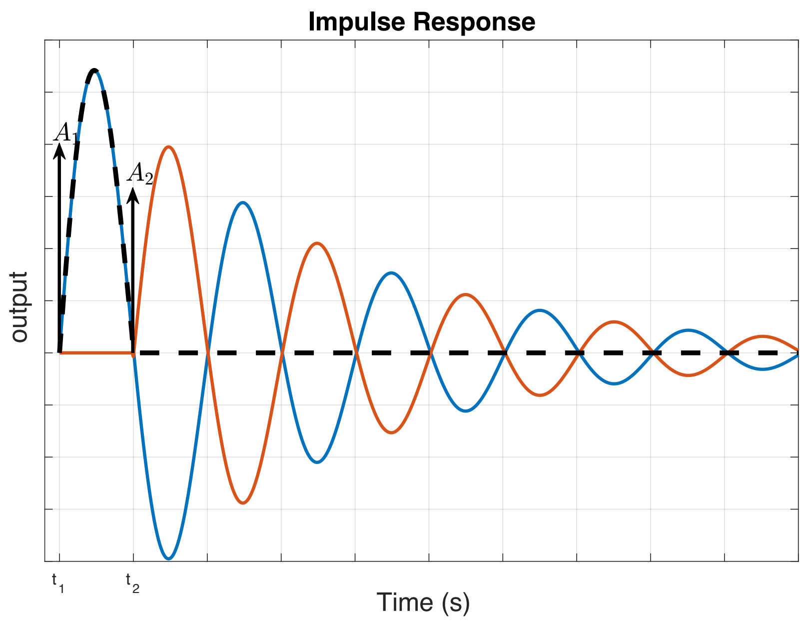


Figure 0.2: Impulse response

A Zero Vibration, ZV, input shaper is the simplest input shaper. The only constraints are minimal time and zero vibration at the modeling frequency. If these constraints are satisfied, the ZV shaper has the form of impulse amplitudes A_i and times t_i. The ZV shaper is useful in situations where the parameters of the system are known with a high level of accuracy. Also, if little faith is held in the input shaping approach, the application will never increase vibration beyond the level before shaping.
A Zero Vibration and Derivative, ZVD, shaper is a command generation scheme designed to make the input-shaping process more robust to modeling error. If another constraint is added to the formulation of the shaper by setting the derivative of the vibration with respect to frequency equal to zero. The application of ZVD shapers is for systems where rise time is still important, but either the system will change with time or the model is not accurate. If the model?s inaccuracy cannot be controlled with a ZVD shaper then other shaping techniques are available, such as those described in the subsequent sections.
It is possible to generate a more robust shaper by forming the second derivative of the residual vibration equation and setting it equal to zero. The shaper that results from satisfying this additional constraint is called a ZVDD shaper. This additional constraint increases the robustness, but also increases the shaper duration by one half period of the vibration. ZVDD shaper consists of four evenly spaced impulses lasting 1.5 periods of vibration. ZVDD is three ZV shapers convolved together. The advantage to doing this is that the input shaper parameters have less freedom, thereby simplifying the solution routine. However, by restricting the choice of input shaper parameters, the solution space is also restricted, meaning that there is the potential for optimal solutions to be missed. The matrix form of the three main Zero Vibration Shapers is shown in Fig. 0.3. Figure 0.4 shows the convolution of ZV, ZVD and ZVDD filters to a step input when a damping is zero i.e. $\zeta = 0$ thus, $K = 1$.

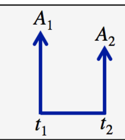
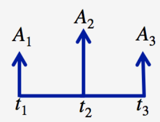
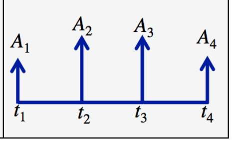
ZV	$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+K} & \frac{K}{1+K} \\ 0 & \frac{\pi}{\omega_d} \end{bmatrix}$	
ZVD	$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+K)^2} & \frac{2K}{(1+K)^2} & \frac{K^2}{(1+K)^2} \\ 0 & \frac{\pi}{\omega_d} & \frac{2\pi}{\omega_d} \end{bmatrix}$	
ZVDD	$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+K)^3} & \frac{3K}{(1+K)^3} & \frac{3K^2}{(1+K)^3} & \frac{K^3}{(1+K)^3} \\ 0 & \frac{\pi}{\omega_d} & \frac{2\pi}{\omega_d} & \frac{3\pi}{\omega_d} \end{bmatrix}$	

Figure 0.3: ZV, ZVD and ZVDD Input shapers

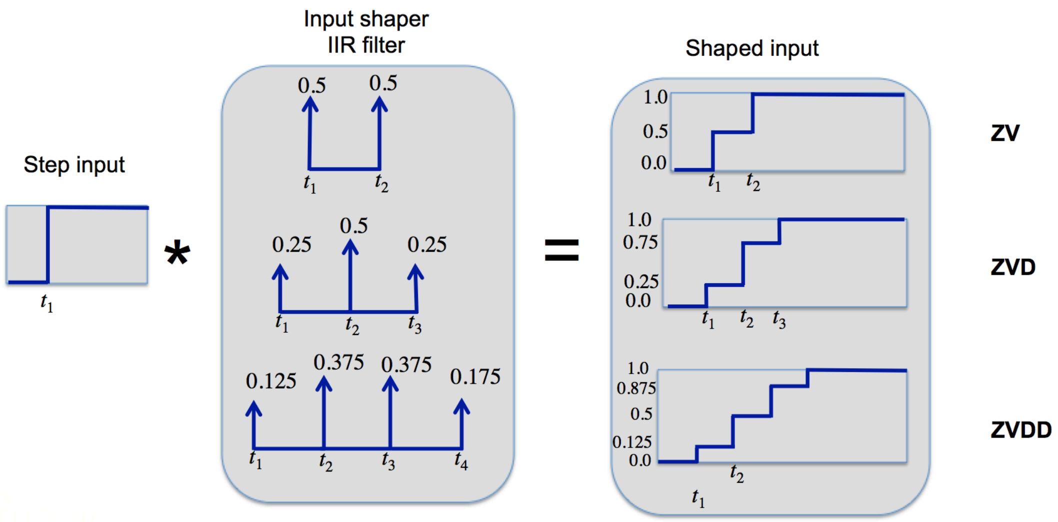


Figure 0.4: Shaped Input when $\zeta = 0$

Example: Suppose we have an undamped second order system as defined in equation (0.1) where the resonance frequency and damping coefficients are $\omega_0 = 1000$ and $\zeta = 0.01$, respectively. Find overshoot K and damped period T_d values to design a proper ZV, ZVD, ZVDD pre-filters.
Answer:

$$K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \approx 0.97 \tag{0.2}$$

$$T_d = \frac{\pi}{\omega_d} \approx \omega_0 = 2000\pi \tag{0.3}$$

Figure 0.5 shows the time response of the system while applying step and shaped input using ZV, ZVD, ZVDD filters. Figure 0.6 shows the performance of the designed filter against the uncertainty of the system a) $\zeta = 0.1$ and b) $\omega_0 = 1100$. It is obvious that the ZVDD is more robust against the variation of the parameters in the system.

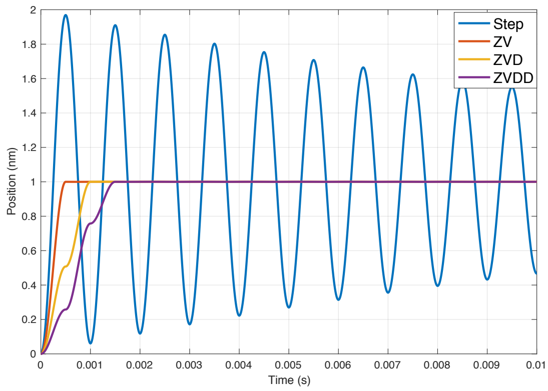


Figure 0.5: Time response of the system applying step and shaped input using ZV, ZVD, ZVDD filters

(*a*) (*b*)

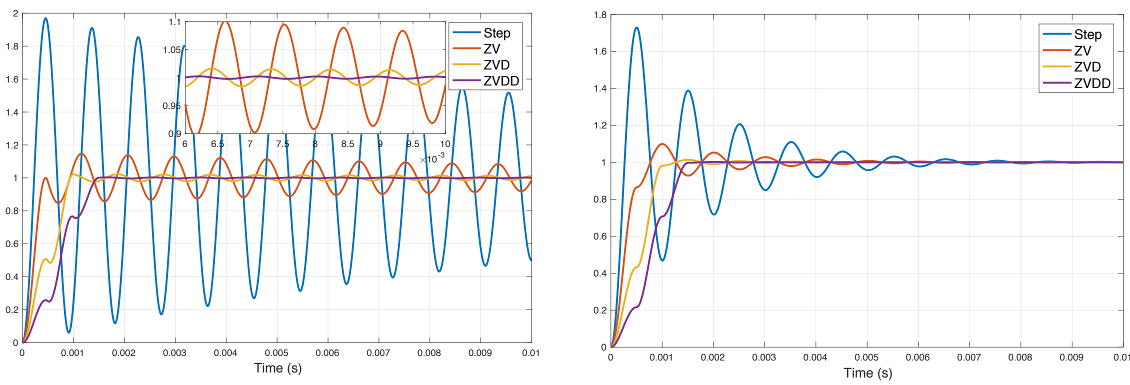


Figure 0.6: Robustness performance of the filters against uncertainty in the system a) $\zeta = 0.1$ b) $omega_0 = 1100$