Advanced Motion Control

Part VIII: Optimal and Robust Control

Tom Oomen

Control Systems Technology Group
Department of Mechanical Engineering
Eindhoven University of Technology

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Introduction

Advanced motion control design procedure (Oomen & Steinbuch 2020) (Oomen 2018)

1. interaction analysis: e.g., RGA. Decoupled?
   - yes: independent SISO designs. no: next step
2. static decouping: Decoupled?
   - yes: independent SISO designs. no: next step
3. decentralized MIMO design: loop closing procedures
   - robustness for interaction, e.g., using factorized Nyquist
   - design for interaction, e.g., sequential loop closing

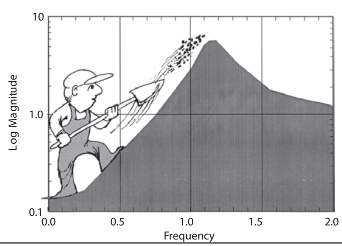
Key point

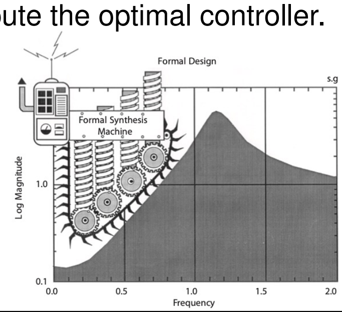
- All steps 1-3 can be done using non-parametric FRF models
- What if we are not successful?
  this lecture: model-based control

Introduction

What is model-based control?

1. A parametric model is used.
   Typically, state-space models are used, which is in sharp contrast to non-parametric FRF models used earlier.
2. The controller is optimal.
   The control engineer has to translate the control requirements in a suitable (scalar) criterion, typically involving a certain norm.
3. The controller is synthesized.
   An optimization algorithm is used to compute the optimal controller.





Introduction

Some differences

Traditional (James et al. 1947)	Model-based
manual tuning	computer algorithms
non-optimal	optimal
non-parametric models	parametric models
bandwidth, gain/phase margin	norms ($\mathcal{H}_2$, $\mathcal{H}_\infty$, μ, $\ldots$)
Bode/Nyquist	Riccati, Lyapunov, LMIs
PID, notches	state-space, transfer function matrices
simple problems	complex problems
SISO	MIMO

Similarities

both require skill and experience!

Introduction

Motivation for model-based control

1. To solve complex control design problems, including
   a. rigid-body decoupling is unsuccessful
   b. disturbance-based redesign
   c. additional actuators and/or sensors
   d. measured variables $\neq$ performance variables
   e. dealing with uncertainty
2. To automatically and quickly design a controller, e.g., through automated loopshaping
3. To determine NS, NP, RS, RP for a given controller

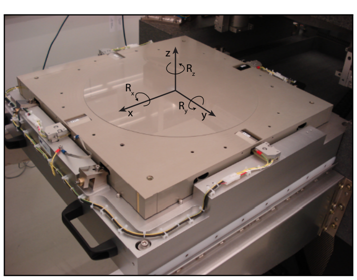
Introduction

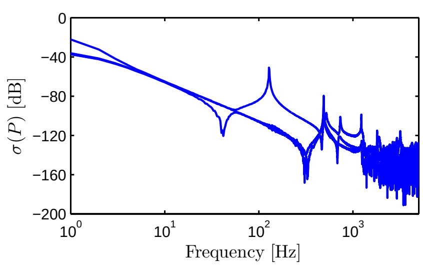
1a. Rigid-body decoupling unsuccessful

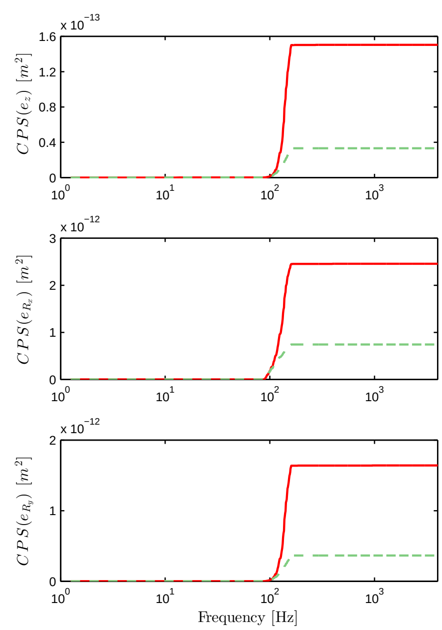
- model system

$$G_m = \underbrace{\sum_{i=1}^{N_{RB}} \frac{c_i b_i^T}{s^2}}_{\text{rigid–body modes}} + \underbrace{\sum_{i=N_{rb}+1}^{N_{nrb}} \frac{c_i b_i^T}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}}_{\text{flexible modes}}$$

- each mode has an associated direction $c_i b_i^T$
- requires a MIMO controller(Boeren et al. 2015)



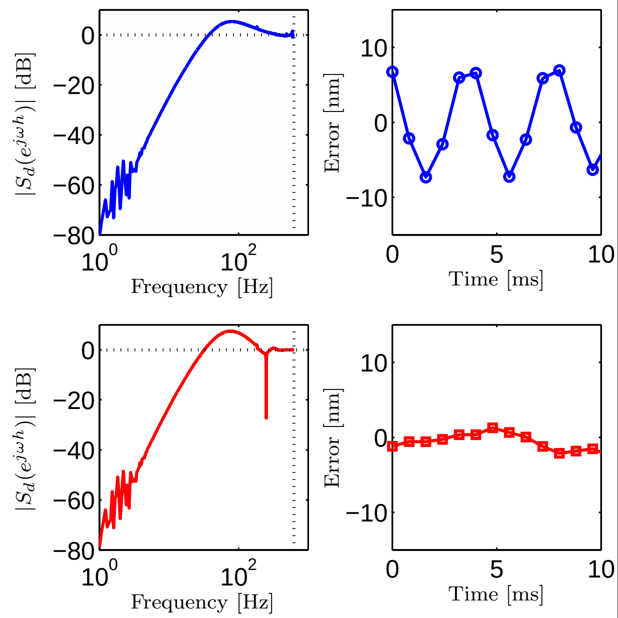




1b. Disturbance-based redesign (Oomen et al. 2007)

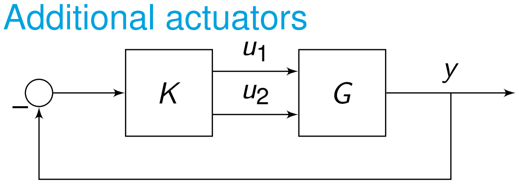
Sensitivity functions

- initial controller:
  dominant frequency component
- disturbance-based $\mathcal{H}_\infty$ controller:
  suppresses disturbance



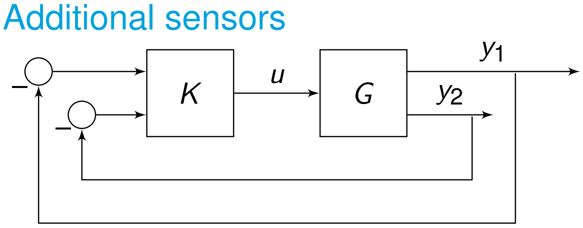
1c. Additional actuators and/or sensors

Additional actuators



- dual-stage wafer stages (Oomen et al. 2014)
- optical disc drives
- hard disk drives

Additional sensors

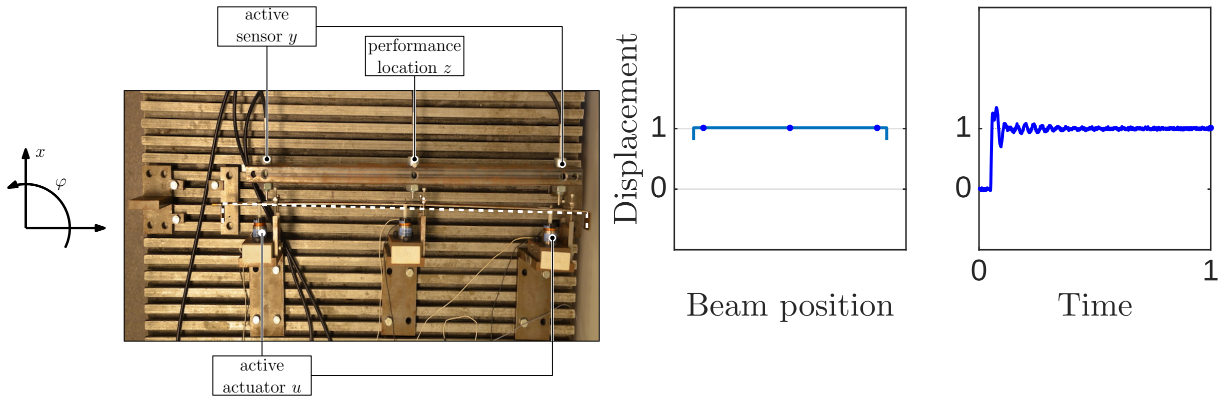


- velocity sensors
- accelerometers

- How to design K using manual design? See also (Skogestad & Postlethwaite 2005, Section 10.5.3)
- What are the benefits? See also (van Zundert et al. 2019)

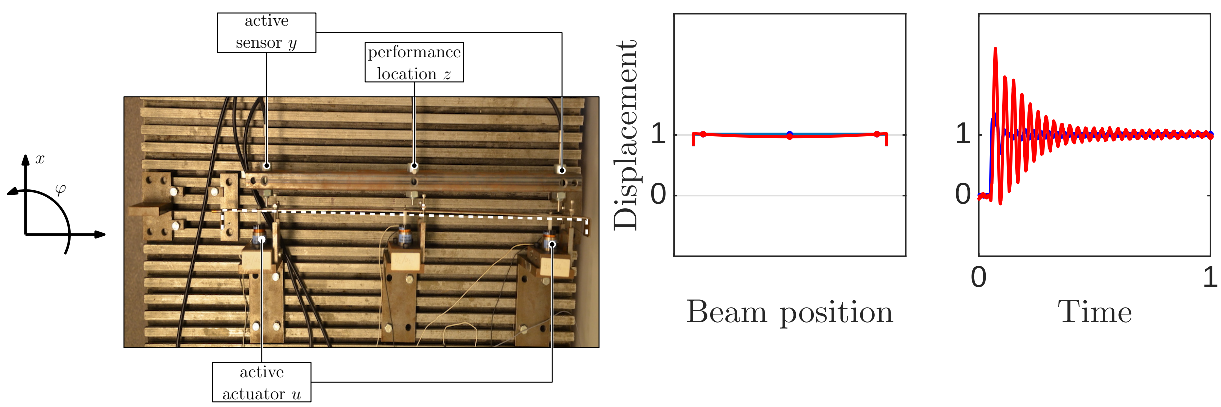
1d. Measured variables $\neq$ performance variables(Oomen et al. 2015)

- blue: what you think happens at z if you assume rigid-body dynamics

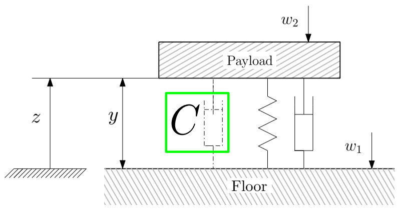


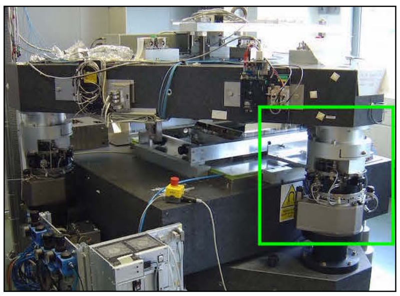
1d. Measured variables $\neq$ performance variables(Oomen et al. 2015)

- blue: what you think happens at z if you assume rigid-body dynamics
- red: what actually happens if you measure at z



1d. Measured variables $\neq$ performance variables(Voorhoeve et al. 2015)





Control goal

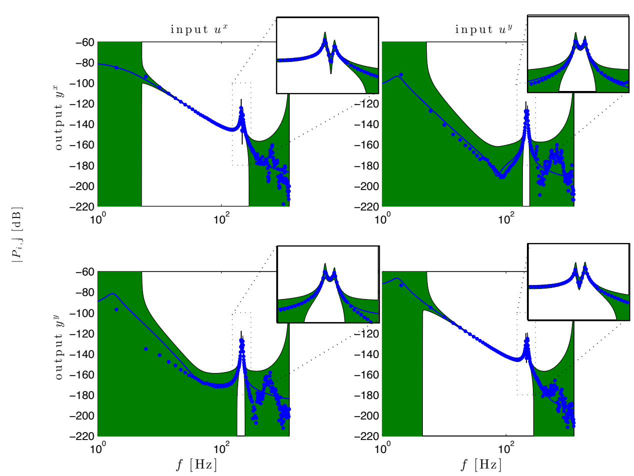
- performance variable $z = 0$
- in presence of disturbances w_1 and w_2
- given measurement y

High-gain or low-gain controller?

1e. Dealing with uncertainty(Oomen et al. 2014)

Models are always an approximation of reality

- explicit characterization of uncertainty
- leads to model set $G_p = G + E$
- E: unknown but bounded perturbation, e.g., $E = w\Delta$, $\Delta \in \mathcal{B}\boldsymbol{\Delta}$
- design a controller that works well for all candidate models G_p: design for RP



General control configuration

Norm-based control

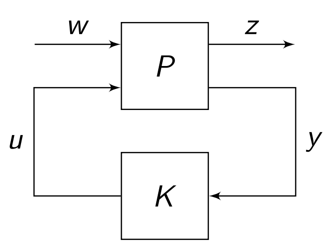
Weighting filter design

$\mathcal{H}_\infty$ Norm for Multivariable Systems

$\mathcal{H}_\infty$ Controller Synthesis

Summary and reading

General control configuration



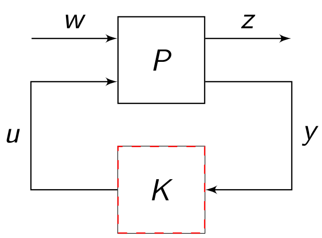
General control configuration

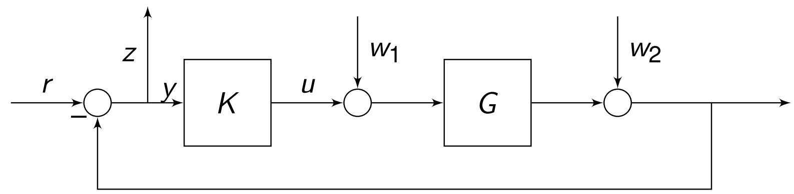
- w: (weighted) exogenous inputs
- z: (weighted) exogenous outputs
- y: measured variables
- u: manipulated variables

Control goal: design K such that the closed-loop norm from w to z is minimized
- z and y need not be equal
- dimensions of y and u need not be equal: K may be MIMO and nonsquare

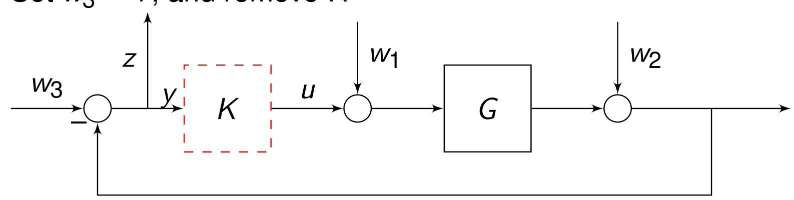
General control configuration

Putting any problem into the general control configuration



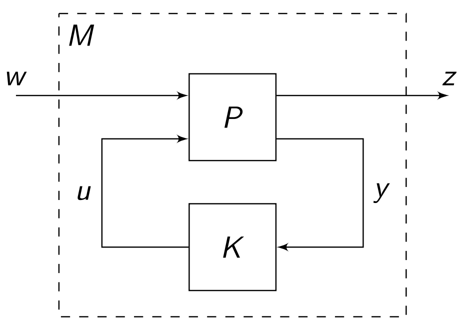


Set $w_3 = r$, and remove K



What is P?

General control configuration



Control goal

- design K such that a norm from w to z is minimized
- equivalent to minimizing a norm of M: norm-based control
- ideally: $\|M\| = 0$

Linear Fractional Transformation (LFT)

$$M = \mathcal{F}_l(P, K)$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

$$M = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

How to derive M?

General control configuration

Norm-based control

Weighting filter design

$\mathcal{H}_\infty$ Norm for Multivariable Systems

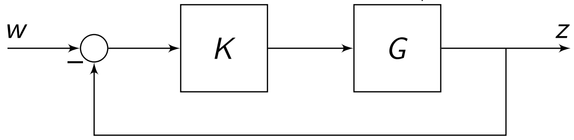
$\mathcal{H}_\infty$ Controller Synthesis

Summary and reading

Norm-based control

Norms to quantify control goals

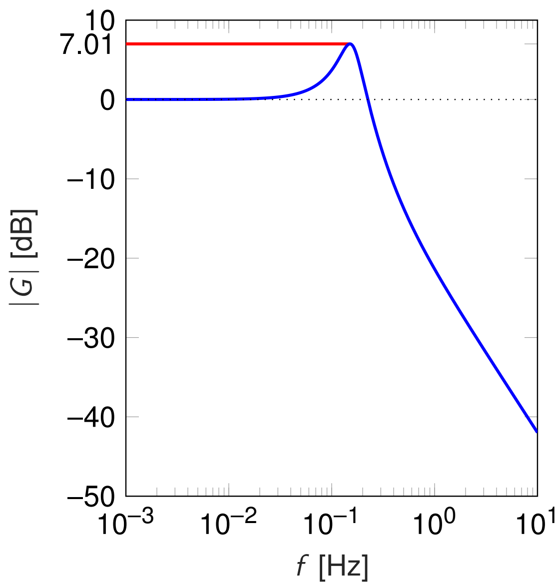
- consider scalar w, z, with $M = \frac{GK}{1+GK}$



- $\mathcal{H}_2$ norm for stable and strictly proper M: $\|M\|_2 = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} |M(j\omega)|^2 d\omega\right)^{\frac{1}{2}}$
- $\mathcal{H}_\infty$ norm for stable and proper M: $\|M\|_\infty = \sup_\omega |M(j\omega)|$
- $\mathcal{H}_\infty$ norm is limit case of $\mathcal{H}_p$ norm: $\sup_\omega |M(j\omega)| = \lim_{p\to\infty} \left(\int_{-\infty}^{\infty} |M(j\omega)|^p \, d\omega\right)^{\frac{1}{p}}$

Norm-based control

Norms to quantify control goals



Interpretation of norms

- $\|M\|_\infty = \sup_\omega |M(j\omega)| = 7.01$
  - peak magnitude
  - 'worst-case' frequency
- $\|M\|_2 = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} |M(j\omega)|^2 d\omega\right)^{\frac{1}{2}} = 1.11$
  - 'surface' under magnitude plot (square root of squared surface)
  - 'all' frequency

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Norm-based control

$\mathcal{H}_\infty$ is suitable for both performance and robustness control goals

- performance: enables loop-shaping
  - open-loop shaping (McFarlane & Glover 1990) (Vinnicombe 2001)
  - closed-loop shaping (van de Wal et al. 2002)
- robustness: enables quantifying model uncertainty in the frequency domain

$\mathcal{H}_2$

- performance: ideal for disturbance attenuation (Boeren et al. 2018)
- robustness: hard to enforce, LQG control is very similar to $\mathcal{H}_2$ (Skogestad & Postlethwaite 2005, Section 9.3.3)

Guaranteed Margins for LQG Regulators

JOHN C. DOYLE

Abstract—There are none.

INTRODUCTION

Considerable attention has been given lately to the issue of robustness of linear–quadratic (LQ) regulators. The recent work by Safonov and

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General control configuration

Norm-based control

Weighting filter design

$\mathcal{H}_\infty$ Norm for Multivariable Systems

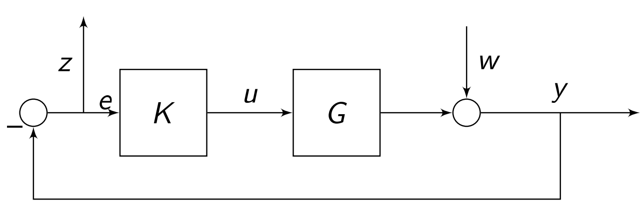
$\mathcal{H}_\infty$ Controller Synthesis

Summary and reading

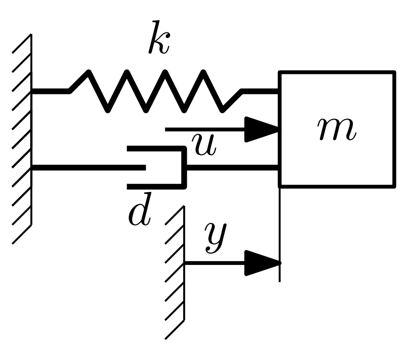
Weighting filter design

Example

- consider $G = \frac{1}{ms^2+ds+k}$
- interconnection structure



- thus $z = Mw$, with $M = -\frac{1}{1+GK}$
- question: compute $K^{\text{opt}} = \arg\min \|M\|_\infty$



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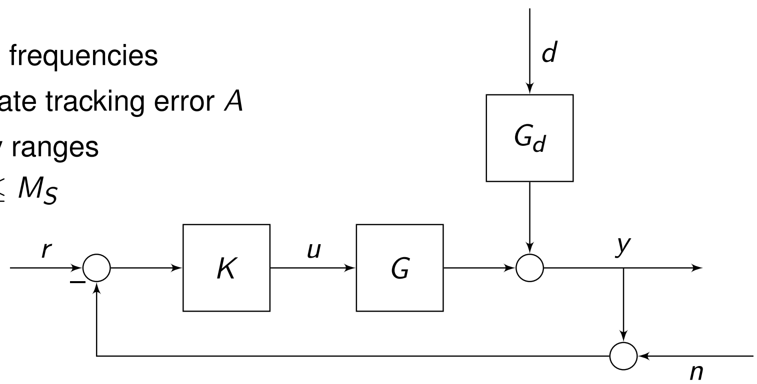
Weighting filter design

Loopshaping design

$$e = y - r = -\frac{1}{1+GK}r + \frac{1}{1+GK}G_d d - \frac{GK}{1+GK}n$$

Specifications for S (note the definition of e!)

1. minimum bandwidth f_{BW}
2. maximum tracking error at selected frequencies
3. system type or maximum steady-state tracking error A
4. shape of S over selected frequency ranges
5. maximum peak magnitude $\|S\|_\infty \leq M_S$
   - GM $\geq \frac{M_S}{M_S-1}$, PM $\geq \frac{1}{M_S} \cdot \frac{180}{\pi}$



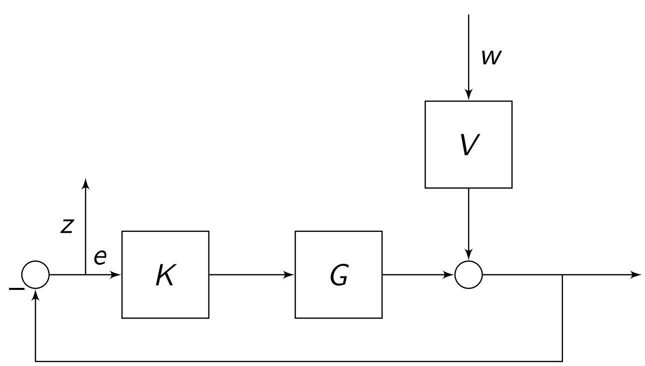
Ideally:

$$e = 0 \cdot r + 0 \cdot d - 0 \cdot n$$

Weighting filter design

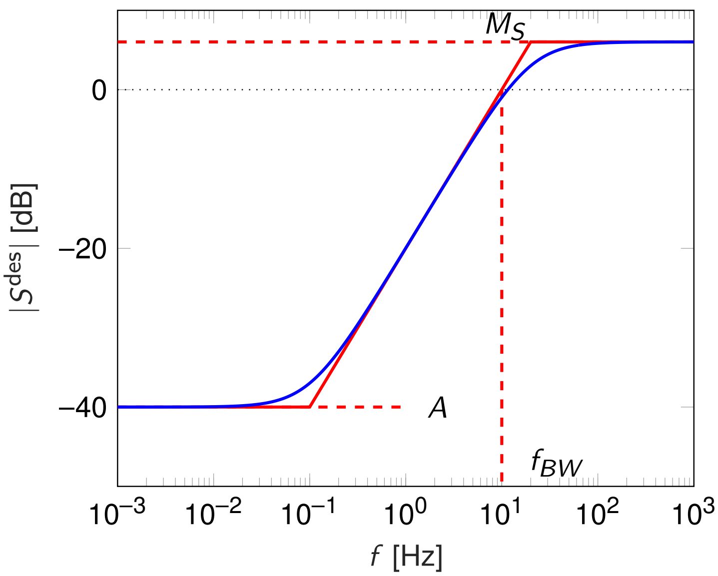
Loopshaping design

- idea: specify $|S(j\omega)|$ by $\left|S^{\text{des}}(j\omega)\right|$
- then $|S(j\omega)| < \left|S^{\text{des}}(j\omega)\right| \Leftrightarrow \left|\frac{S(j\omega)}{S^{\text{des}}(j\omega)}\right| < 1$
- from $z = -SVw$, set $V = \frac{1}{S^{\text{des}}}$
- so that $\left|\frac{S(j\omega)}{S^{\text{des}}(j\omega)}\right| < 1 \Leftrightarrow |S(j\omega)V(j\omega)| < 1$
- next, $|S(j\omega)V(j\omega)| < 1 \forall\omega \Leftrightarrow \|SV\|_\infty < 1$



Weighting filter design

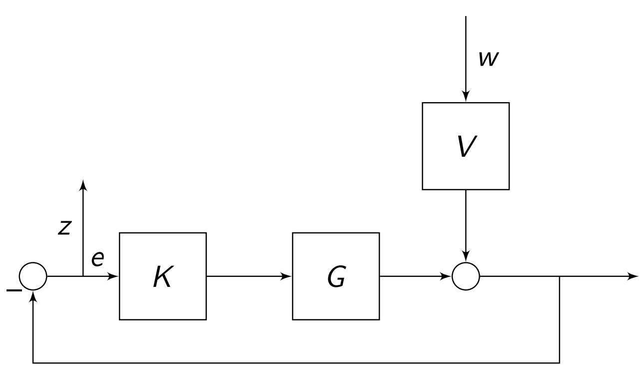
Loopshaping design



Example $V(s)$

$$S^{\text{des}} = V^{-1} = \frac{\frac{s}{M_S} + 2\pi f_{BW}}{s + 2\pi f_{BW} A}$$

Weighting filter design



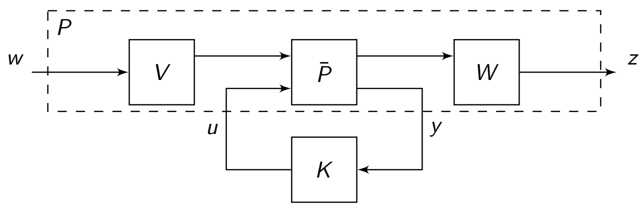
So $z = SVw$

Control goals

- $\mathcal{H}_\infty$-sub-optimal design: compute a stabilizing K such that $\|SV\|_\infty < 1$
- $\mathcal{H}_\infty$-optimal design: compute a stabilizing K such that $\|SV\|_\infty$ is minimized

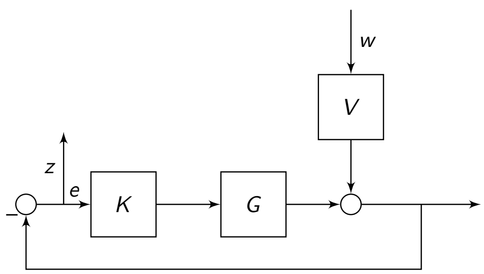
Weighting filter design

Weighting filters in the general control configuration



Approach

- absorb weights W, V into M
- goal: $K^{\text{opt}} = \arg\min_K \|\mathcal{F}_l(P, K)\|_\infty$



Sensitivity example

- $W = 1$, $V = \frac{1}{S^{\text{des}}}$
- $M = -SV$

General control configuration

Norm-based control

Weighting filter design

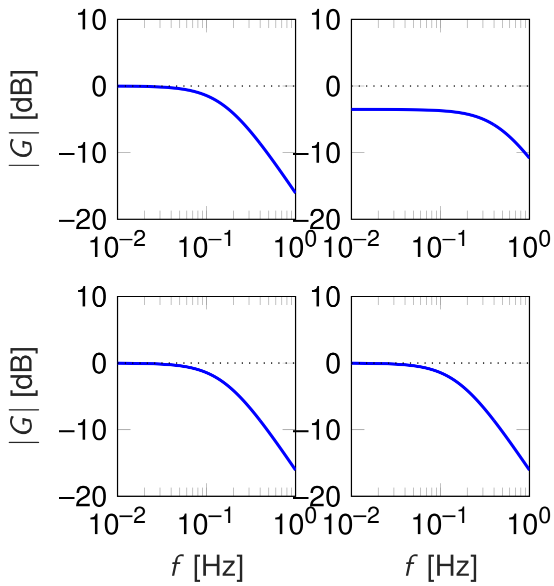
$\mathcal{H}_\infty$ Norm for Multivariable Systems

$\mathcal{H}_\infty$ Controller Synthesis

Summary and reading

$\mathcal{H}_\infty$ Norm for Multivariable Systems

Bode diagram



Multivariable systems

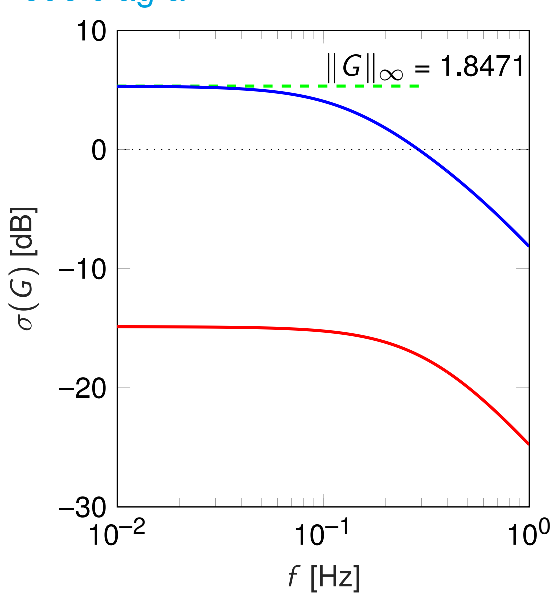
$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

System norms

- SISO case: $\|G\|_\infty = \sup_\omega |G(j\omega)|$
- how to generalise to MIMO?

$\mathcal{H}_\infty$ Norm for Multivariable Systems

Bode diagram



Multivariable systems

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+3} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

System norms

- SISO case: $\|G\|_\infty = \sup_\omega |G(j\omega)|$
- MIMO case: $\|G\|_\infty = \sup_\omega \bar{\sigma}(G(j\omega))$
- interpretation $\mathcal{H}_\infty$:
  - worst-case frequency, worst-case direction
- interpretation $\mathcal{H}_2$:
  - average frequency, average direction
  - see (Skogestad & Postlethwaite 2005, Sec. 4.10.1)
- scaling-dependent

$\mathcal{H}_\infty$ Controller Synthesis

$\mathcal{H}_\infty$ synthesis

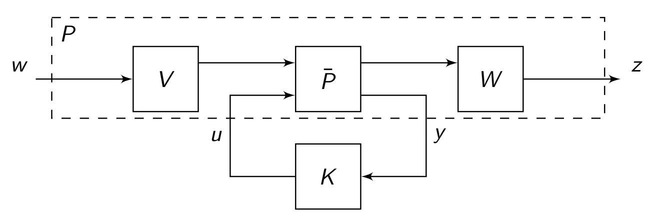
- $P(s)$ is input to software, with

$$\begin{bmatrix} z \\ y \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix}$$

$$P \stackrel{s}{=} \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

- available software
  - Matlab Robust Control Toolbox
  - Matlab μ-Analysis and Synthesis Toolbox (old)
  - Matlab LMI Control Toolbox (old)
  - LMI Parser & Solver (free versions available, e.g., YALMIP, CVX)

General control configuration



$\mathcal{H}_\infty$ Controller Synthesis

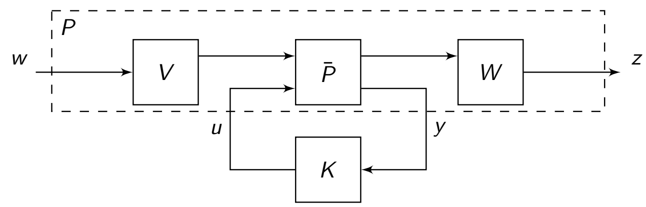
$\mathcal{H}_\infty$ synthesis

- Assumptions on

$$P \overset{s}{=} \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

- (A, B_2) stabilizable
  - V must be stable
- (C_2, A) detectable
  - W must be stable
- D_{12} must have full column rank
  - penalise control inputs to ensure proper and realizable controllers
- D_{21} must have full row rank
  - all measurements are corrupted by noise to ensure proper and realizable controllers
- several more: see (Skogestad & Postlethwaite 2005, Sec. 9.3.1)

General control configuration



$\mathcal{H}_\infty$ Controller Synthesis

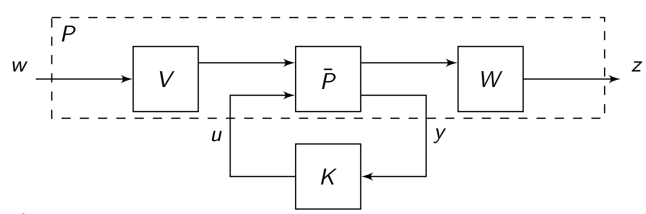
$\mathcal{H}_\infty$ synthesis

- $\mathcal{H}_\infty$ synthesis of the optimal controller:

$$\min_{K \text{ stabilizing}} \|\mathcal{F}_l(P, K)\|_\infty$$

- suboptimal solutions
  - in practice computationally and theoretically simpler
  - $\|\mathcal{F}_l(P, K)\|_\infty < \gamma$
- bisection:
  - given some γ, compute stabilizing K such that $\|\mathcal{F}_l(P, K)\|_\infty < \gamma$
  - if it exists, decrease γ
  - if it does not exists, increase γ
  - continue until tolerance on γ is below threshold

General control configuration



$\mathcal{H}_\infty$ Controller Synthesis

Reliable $\mathcal{H}_\infty$ synthesis via Two-Riccati approach (Doyle et al. 1989)

Let the assumptions of Slide 27 be satisfied. Then, a stabilizing controller such that $\|\mathcal{F}_l(P, K)\|_\infty < \gamma$ exists if and only if

1. $X_\infty \succeq 0$ is a solution to the algebraic Riccati equation

$$A^T X_\infty + X_\infty A + C_1^T C_1 + X_\infty(\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty = 0$$

   such that $\text{Re}\left(\lambda_i\left(A + (\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty\right)\right) < 0 \forall i$

2. $Y_\infty \succeq 0$ is a solution to the algebraic Riccati equation

$$A Y_\infty + Y_\infty A^T + B_1 B_1^T + Y_\infty(\gamma^{-2} C_1^T C_1 - C_2^T C_2) Y_\infty = 0$$

   such that $\text{Re}\left(\lambda_i\left(A + Y_\infty(\gamma^{-2} C_1^T C_1 + C_2^T C_2)\right)\right) < 0 \forall i$

3. $\rho(X_\infty Y_\infty) < \gamma^2$ Then, all controllers are parameterized as $K = \mathcal{F}_l(K_c, Q)$, for any Q that satisfies $\|Q\|_\infty < \gamma$, and K_c is the central controller.

$\mathcal{H}_\infty$ Controller Synthesis

Reliable $\mathcal{H}_\infty$ synthesis via Two-Riccati approach (Doyle et al. 1989)

- state dimension of K_c is identical to P
  - keep order G, V, and W small
  - or use controller order reduction afterwards
- the feedback interconnection of K_c and P is guaranteed to be stable
- stability of K_c cannot be guaranteed
- K_c is typically a centralized controller
- enforcing additional structure in K_c typically leads to a nonconvex synthesis
  - e.g., decentralized

General control configuration

Norm-based control

Weighting filter design

$\mathcal{H}_\infty$ Norm for Multivariable Systems

$\mathcal{H}_\infty$ Controller Synthesis

Summary and reading

Summary and reading

Take-home messages

- general control configuration: fits any control problem
- automated synthesis algorithms available
- control objective must be translated into a norm
- weighting filters are essential
- often uses parametric model of system

Next

- how to incorporate the model and model errors?
- how to specify weighting filters for motion systems?

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