Advanced Motion Control

Part VI: Decentralized Control

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Introduction

Motivation

- we now have notions for multivariable
  - poles
  - zeros
  - stability:
    - open-loop & closed-loop
    - frequency-response-based tests: generalized Nyquist stability theorem

This part

- multivariable control design based on frequency response functions

Design: factorized Nyquist techniques

Design: sequential loop closing

Summary and reading

Tom Oomen - AMC handout

Design: factorized Nyquist techniques

Yet another Nyquist test: factorized version

- multivariable Nyquist tests enable analysis of multivariable control systems for a given K
- if unstable
  - how to redesign K?
  - if K is decentralized, which element of K should be retuned?
- answer:
  - many approaches possible
  - first: robustness for interaction (next)

Tom Oomen - AMC handout

Design: factorized Nyquist techniques

Yet another Nyquist test: factorized version

- off-diagonal terms lead to interaction
- let $G \in \mathcal{R}^{m \times m}$ be a square, stable system with elements g_{ij}
- diagonal terms: $\tilde{G} = \mathrm{diag}(g_{ii})$
- off-diagonal terms: $G - \tilde{G}$
- normalized off-diagonal terms: $E = (G - \tilde{G})\tilde{G}^{-1}$
- decentralized controller: $K = \mathrm{diag}(k_{ii})$
- also, $\tilde{T} = \mathrm{diag}\left(\frac{g_{ii}k_{ii}}{1 + g_{ii}k_{ii}}\right)$ and $\tilde{S} = \mathrm{diag}\left(\frac{1}{1 + g_{ii}k_{ii}}\right)$
- then,

$$(I + GK) = (I + E\tilde{T})(I + \tilde{G}K)$$

or

$$\underbrace{(I + GK)^{-1}}_{=S} = \underbrace{(I + \tilde{G}K)^{-1}}_{=\tilde{S}} \underbrace{(I + E\tilde{T})^{-1}}_{\text{interaction}}$$

Design: factorized Nyquist techniques

Yet another Nyquist test: factorized version

$$\underbrace{(I + GK)^{-1}}_{=S} = \underbrace{(I + \tilde{G}K)^{-1}}_{=\tilde{S}} \underbrace{(I + E\tilde{T})^{-1}}_{\text{interaction}}$$

Theorem (10.3 Factorized Nyquist)

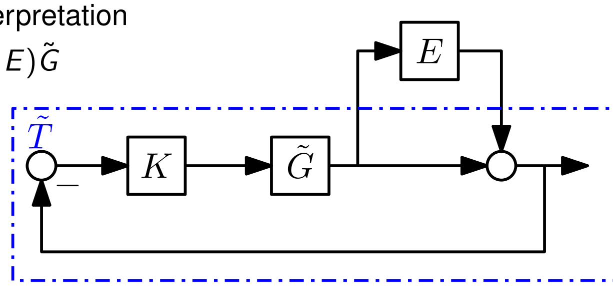
Under the assumptions of slide 4, S is stable if

- *$\tilde{S}$ is stable $\Rightarrow$ iff $\det(I + \tilde{G}K)$ does not encircle the origin as s traverses Nyquist D-contour (independent SISO designs!)*
- *$(I + E\tilde{T})^{-1}$ is stable $\Rightarrow$ various tests:*
  a. *iff $\det(I + E\tilde{T})$ does not encircle the origin as s traverses Nyquist D-contour*
  b. *and a. is satisfied if $\rho(E(j\omega)\tilde{T}(j\omega)) < 1, \forall\omega$*
  c. *and b. is satisfied if $\bar{\sigma}(\tilde{T}(j\omega)) < \mu_{\tilde{\mathbf{T}}}^{-1}(E(j\omega)), \forall\omega$. Here, SSV $\mu(E)$ is computed w.r.t. diagonal structure of $\tilde{T}$ (see (Skogestad & Postlethwaite 2005, Section 8.8))*

Design: factorized Nyquist techniques

Yet another Nyquist test: factorized version

- a robust control interpretation
- $G = G + \tilde{G} = (I + E)\tilde{G}$

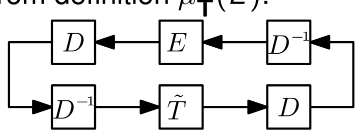


- typical robust control: RS if $\max_{E, \bar{\sigma}(E) \leq 1} \rho(E\tilde{T}) < 1, \forall \omega$
- same as $\mu_{\boldsymbol{\Delta}}(\tilde{T}) < 1, \forall \omega$, where $\boldsymbol{\Delta}$ has structure of E and $\bar{\sigma}(E) \leq 1$
- here: we will vary K and hence $\tilde{T}$ (E is not a function of K)
- so, take $\boldsymbol{\Delta}$ to have the structure $\tilde{\mathbf{T}}$ of $\tilde{T}$
- next, $\rho(E\tilde{T}) \leq \mu_{\tilde{\mathbf{T}}}(E\tilde{T}) \leq \bar{\sigma}(\tilde{T})\mu_{\tilde{\mathbf{T}}}(E) \Rightarrow \bar{\sigma}(\tilde{T}) < \mu_{\tilde{\mathbf{T}}}^{-1}(E)$ for stability (see Skogestad & Postlethwaite (2005, (8.92)))

Design: factorized Nyquist techniques

Intermezzo: computing μ

- so we need $\bar{\sigma}(\tilde{T}) < \mu_{\tilde{\mathbf{T}}}^{-1}(E)$
- here: $\mu_{\tilde{\mathbf{T}}}^{-1}(E) = \min_{k_m} \left\{ k_m \middle| \det(I - k_m E\tilde{T}) = 0, \tilde{T} \in \tilde{\mathbf{T}}, \bar{\sigma}(\tilde{T}) < 1 \right\}$
- this does not really help to compute it. Upper bound: $\mu_{\tilde{\mathbf{T}}}(E) \leq \bar{\sigma}(E)$
- can be tightened! From definition $\mu_{\tilde{\mathbf{T}}}(E)$:



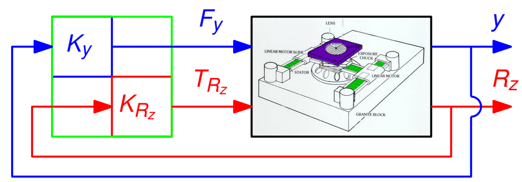
- if D commutes with $\tilde{T}$, i.e., $D\tilde{T}D^{-1} = \tilde{T}$, then $\mu_{\tilde{\mathbf{T}}}(DED^{-1}) = \mu_{\tilde{\mathbf{T}}}(E)$
- 2×2: let $D = \left[\begin{smallmatrix} 1 & 0 \\ 0 & d \end{smallmatrix}\right]$, optimize $\mu_{\tilde{\mathbf{T}}}(E) \leq \min_{d>0} \bar{\sigma}(DED^{-1})$
- see also (Skogestad & Postlethwaite 2005, Sec. 8.8) (Packard & Doyle 1993) (Scherer 2015)

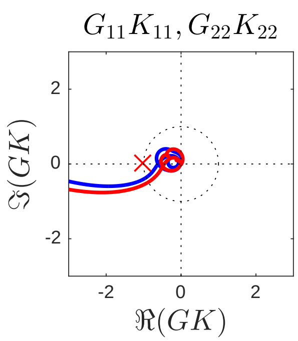
Design: factorized Nyquist techniques

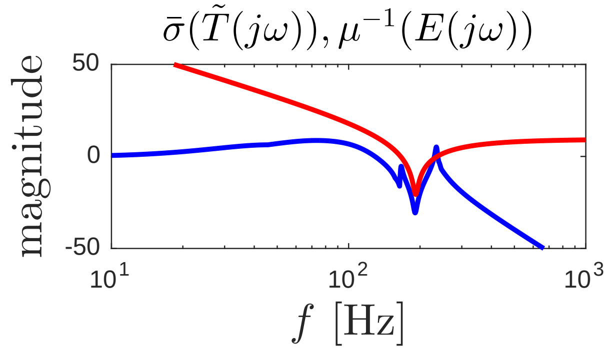
Example revisited

Factorized nyquist:

- $\det(I + \tilde{G}K)$: stable
- $\bar{\sigma}(\tilde{T}(j\omega)) < \mu^{-1}_{\tilde{\mathbf{T}}}(E(j\omega)),\ \forall\omega$:
  not satisfied
  $\Rightarrow$ stability cannot be guaranteed (sufficient condition)





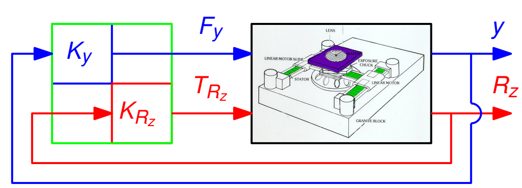


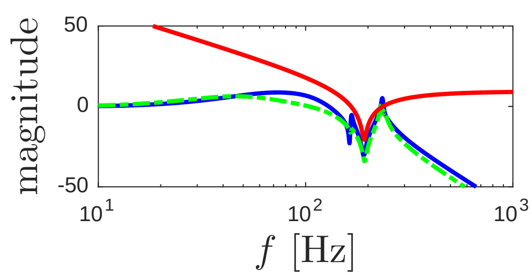
Design: factorized Nyquist techniques

Example revisited - continued

Factorized nyquist:

- use for design!
- note that
  $\sigma(\tilde{T}(j\omega)) = \{|\tilde{T}_{11}(j\omega)|, |\tilde{T}_{22}(j\omega)|\}$
- example:
  - retune K_1 such that $|\tilde{T}_{11}(j\omega)| < \mu^{-1}_{\tilde{\mathbf{T}}}(E(j\omega)),\ \forall\omega$
  - retune K_2 such that $|\tilde{T}_{22}(j\omega)| < \mu^{-1}_{\tilde{\mathbf{T}}}(E(j\omega)),\ \forall\omega$





Design: factorized Nyquist techniques

Yet another Nyquist test: summary & extensions

- test useful for decentralized SISO controller designs
  - only requires computation of structured singular value once for varying K (it is independent of K)
  - provides robustness for interaction terms
  - can be used for decentralized design
- $\mu_{\tilde{\mathbf{T}}}(E(j\omega))$ is also an interaction measure!

Definition (10.1)

A matrix G is generalized diagonally dominant iff $\mu(E(j\omega)) < 1$

Interaction measures for 2×2 systems

- RGA: $\lambda(G) = \frac{1}{1-\phi(G)}$, goal $\lambda(G) \approx 1$
- Rijnsdorp interaction measure: $\phi(G) = \frac{g_{12}g_{21}}{g_{11}g_{22}}$, goal $\phi(G) < 1$
- generalized dominance $\mu(E(j\omega)) = \sqrt{|\phi(G)|}$, goal $\mu(E(j\omega)) < 1$
  $\Rightarrow$ essentially identical for 2×2 systems

Design: factorized Nyquist techniques

Design: sequential loop closing

Summary and reading

Design: sequential loop closing

Analysis

So far we investigated:

- analysis: stability for a given controller
- design: only robustness for interaction (factorized Nyquist)

Synthesis

- sometimes robustness for interaction very conservative, e.g., using overactuation/oversensing, see (van Herpen et al. 2014)
- How to synthesize/design while explicitly addressing interaction?

Sequential Loop Closing

- close one loop at the time
- address interaction in each loop design
- assume open-loop stable (can be extended to unstable systems)

Design: sequential loop closing

Intuitive idea

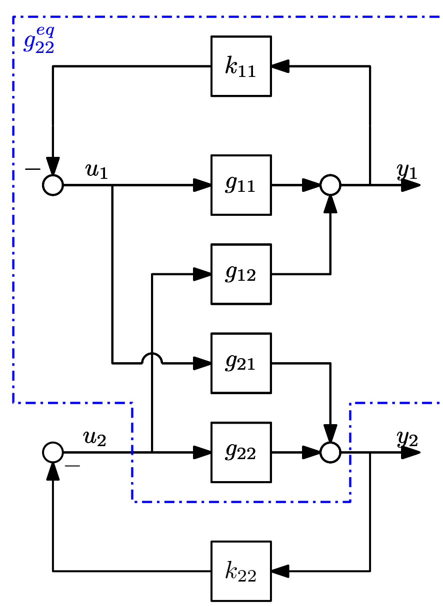
- consider a 2×2 system G
- design controller k_{11} for loop 1
- define 'equivalent plant'

$$g_{22}^{eq} = g_{22} - \frac{g_{21} k_{11} g_{12}}{1 + k_{11} g_{11}}$$

- design k_{22} for g_{22}^{eq}

General situation

- naturally extends to more loops



Does it work?

- recall that $\det(I+GK) = c\frac{\phi_{cl}}{\phi_{ol}}$
- let $g_{22}^{eq} = g_{22} - \frac{g_{21}k_{11}g_{12}}{1+k_{11}g_{11}}$, $l_{11} = g_{11}k_{11}$, and $l_{22}^{eq} = g_{22}^{eq}k_{22}$
- next,

$$\begin{aligned}
\det(I+GK) &= \begin{vmatrix} 1+g_{11}k_{11} & g_{12}k_{22} \\ g_{21}k_{11} & 1+g_{22}k_{22} \end{vmatrix} \\
&= (1+g_{11}k_{11})(1+g_{22}k_{22}) - g_{21}k_{11}g_{12}k_{22} \\
&= (1+g_{22}k_{22} - \frac{g_{21}k_{11}g_{12}k_{22}}{1+k_{11}g_{11}})(1+g_{11}k_{11}) \\
&= \left(1+\underbrace{\left(g_{22} - \frac{g_{21}k_{11}g_{12}}{1+k_{11}g_{11}}\right)}_{=g_{22}^{eq}}k_{22}\right)(1+g_{11}k_{11}) \\
&= (1+l_{22}^{eq})(1+l_{11})
\end{aligned}$$

Does it work? - continued

- let $g_{22}^{eq} = g_{22} - \frac{g_{21}k_{11}g_{12}}{1+k_{11}g_{11}}$, $l_{11} = g_{11}k_{11}$, and $l_{22}^{eq} = g_{22}^{eq}k_{22}$
- thus $\det(I+GK) = (1+l_{22}^{eq})(1+l_{11})$

Analysis

- k_{11} designed such that zeros of $1+l_{11}$ in OLHP
- k_{22} designed such that zeros of $1+l_{22}^{eq}$ in OLHP
  $\Rightarrow$ closed-loop stability guaranteed

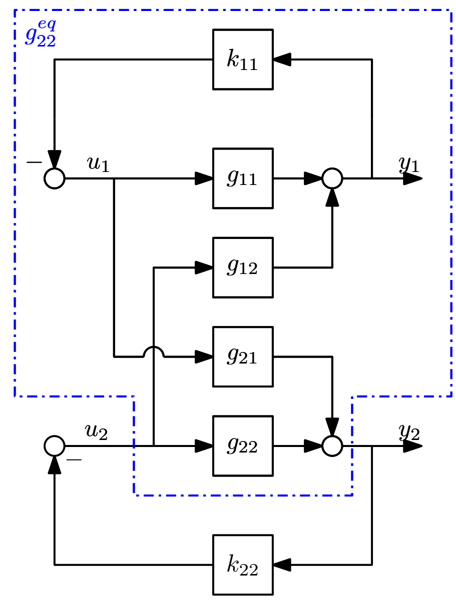
Remark: using same argument

- let $g_{11}^{eq} = g_{11} - \frac{g_{12}k_{22}g_{21}}{1+k_{22}g_{22}}$, $l_{22} = g_{22}k_{22}$, and $l_{11}^{eq} = g_{11}^{eq}k_{11}$
- thus $\det(I+GK) = (1+l_{11}^{eq})(1+l_{22})$
- stability invariant under change of ordering loop-closing

Design: sequential loop closing

Example

- suppose k_{11} design to have phase margin 30° for g_{11}
- suppose k_{22} design to have phase margin 30° for g_{22}^{eq}



Q:

- stable?
- phase margin loop 2?
- phase margin loop 1?
- now set $k_{11} = 0$. what happens?
  k_{22} designed for g_{22}^{eq}, but now implemented on g_{22}...

Design: sequential loop closing

Sequential loop closing - summary

- sequential close loops to take interaction into account
- nominal stability guaranteed
- margins (and more generally robustness) not guaranteed
  - if not sufficient, try different sequence
  - don't open previous loops in a different sequence

Design: factorized Nyquist techniques

Design: sequential loop closing

Summary and reading

Summary and reading

Take-home messages

- factorized Nyquist-based design: robustness for interaction
  - always works as long as other loops are stabilized
- sequential loop closing: design for interaction
  - always closed-loop stability, but robustness strongly depend on other loops

Multivariable design procedure

1. interaction analysis
2. decouple
3. try independent SISO design + MIMO stability analysis
4. try using robustness for interaction: factorized Nyquist-based
5. performance insufficient? try designing for interaction using sequential loop closing

Next

- exploiting physical system properties (modal): enables systematic sequential loop closing

Summary and reading I

Reading

- general decentralized control: (Skogestad & Postlethwaite 2005, Section 10.6)

Additional reading material

- Nyquist stability theory: Maciejowski (1989, Section 2.9)
- characteristic loci: Skogestad & Postlethwaite (2005, Section 4.9.3)
- factorized Nyquist test: Skogestad & Postlethwaite (2005, Section 10.6.3-10.6.4), see Skogestad & Postlethwaite (2005, Section 8.8.3) for μ
- alternative decentralized design techniques, e.g., Nyquist array techniques Maciejowski (1989, Section 4.6), Characteristic locus methods Maciejowski (1989, Section 4.3)
- connections interaction measures: Grosdidier & Morari (1986)
- details on μ: Skogestad & Postlethwaite (2005, Section 8.8), Packard & Doyle (1993), (Scherer 2015, Part 6)

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