UNIVERSITY OF TORONTO
Department of Computer Science

APRIL 2017 EXAMINATIONS
CSC418H1S and CSC2504HS
-- Computer Graphics

Duration: 3 hours

No aids allowed

There are 12 pages total (including this page)

- This exam has 3 parts and a total of 22 questions.
- There are a total of 80 marks.
- This is a closed book exam.
- Show your work and write *legibly*. Not like this.
- Write your name and student number on the next page.

Some notes:

- The inverse of a 2x2 matrix A is the following:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- $c \in \mathbb{R}^N$ is a vector in the form $(c_1, \ldots, c_N)$. c^T is the vector represented as a column matrix. i.e. $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$. The dot product of two vectors c, d can be written as $c \cdot d$ or $c^T d$. The matrix product needs to be well-define (i.e. the dimensions match up)

Given name(s): ____________________

Family Name: ____________________

Student number: ____________________

Question	**Marks**
Multiple Choice	---
1	/1
2	/1
3 a) b) c)	/1 /1 /1 = /3
4	/1
5	/1
6	/1
7	/1
Short Answer	---
1	/2
2	/2
3 a) b) c) d)	/1 /2 /1 /1 = /5
4	/2
5 a) b) c)	/5
6	/1
7	/1
8	/2
9	/1 /2 /2 = /5
10	/5
Long Answer	---
1	/5
2	/5
3 a) b)	/3 /3 = /6
4	/15
5	/10
Total	**/80**

Multiple Choice Questions

Circle the correct answer. No explanation required. One correct answer per question. Each one mark.

1. In the following, $\cdot$ denotes the dot product of two vectors and $\times$ denotes their cross product. If $\vec{v}, \vec{w}$ are two 3D vectors and s, t are two scalar values, then $(s\vec{v} \times t\vec{w}) + (s\vec{v} \cdot t\vec{w})$ is...
   a. undefined
   b. a scalar value
   c. a 2D vector
   d. a 3D vector
   e. a unit vector

2. In the following, $\boldsymbol{A}$ is 3x3 matrix and $\vec{t}$ is a vector 3x1 vector. The following is a transformation on $\bar{p}$: $F(\bar{p}) = A\bar{p} + \vec{t}$. This is ...
   a. an affine transformation in homogeneous coordinates
   b. an affine transformation in non-homogeneous coordinates
   c. a homography in homogeneous coordinates
   d. a homography in non-homogeneous coordinates

3. The following properties concern themselves with a *piecewise linear curve* which represents the position of a point as a function of time $(x(t), y(t))$.

   a. What is the continuity of this curve?
      i. C^0
      ii. C^1
      iii. L_2
      iv. NP

   b. What is the acceleration of the point over each line segment?
      i. Positive or negative infinity
      ii. undefined
      iii. 0

   c. What is the acceleration of the point when it moves from one segment to the next (i.e at a vertex of the curve)?
      i. Positive or negative infinity
      ii. undefined
      iii. 0

4. Given points $\bar{p}_1$ and $\bar{p}_2$ in homogeneous coordinates, their sum is a...
   a. new point $\bar{p}_3$
   b. direction vector $\vec{v}$
   c. undefined
   d. the vector of parameters defining the line through $\bar{p}_1$ and $\bar{p}_2$

5. Homogenous equality of two points means that
   a. Two points have the same x,y and z components
   b. Both points lie on the same line passing through the origin
   c. The dot product of one point with the other is equal to zero

6. Given a point $P(x, y, z)$ in non-homogeneous coordinates, when converted to homogeneous form and after an affine transformation, its homogeneous coordinate...
   a. remains unchanged
   b. is scaled up
   c. is scaled down
   d. changes to the homogenous coordinate of a vector (i.e. a point at infinity)

7. Curvature is a...
   a. global property
   b. local property
   c. invariant property
   d. none of these

Shorter Answer Questions

Note that for questions in this section, a short answer, (e.g., one or two sentences) can be sufficient to get full credit. Put simply: keep it short and to the point.

1. [2 marks] Give one example for when it would make more sense to use a physically-based simulation and one example for when it would make more sense to use an animation approach. Also, for each, give an advantage over the other. (*You need to write four things here: One example for each and one advantage for each*)

2. [2 marks] How does scaling a homography matrix (i.e. multiplying the homography $\boldsymbol{H}$ by some scalar $s \in \mathbb{R}$) affect the transformation? Prove your claim (*hint*: this "proof" is barely a line).

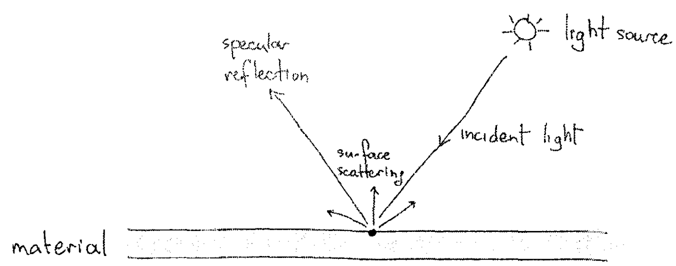
3.
   a. [1 mark] The view frustum is transformed into the orthographic view volume. Why?

b. [2 marks] What happens to the depth after transformation?

c. [1 marks] What *type* of relationship does the original depth have with the transformed depth?

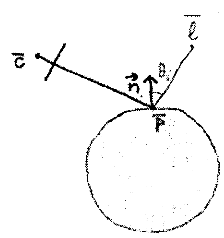
d. [1 marks] What do we call this new "depth"?

4. [2 marks] In class, we simplified the "common modes of light transport" for the Phong Reflectance Model to what is in the image below. We omitted two modes of light transport to simplify the model. What were they?

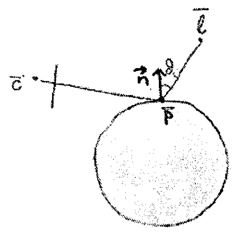


5. [5 marks] For a diffuse object pictured below, what is the intensity at point $\bar{p}$ with normal $\vec{n}$ as seen by the camera center $\bar{c}$ lit by a point source light with position $\bar{l}$ where θ_i is the angle between the normal and the vector $\overrightarrow{\bar{l} - \bar{p}}$? Fill in the blanks only for the diffuse component.

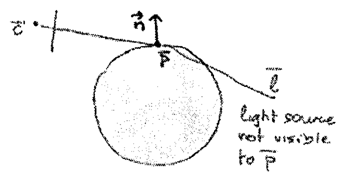
a.



b. (*Camera position $\bar{c}'$ slightly below where $\bar{c}$ was in part a.*)



c. *(Camera position $\bar{c}'$ in same position as in b.)*



6. [1 mark] Explain the difference between object space and image space algorithms.

7. [1 mark] Shading with Gouraud shading can have a triangular appearance. Why is that?

8. [2 marks] Which shading algorithms seen in class do not have any visible seems between mesh triangles? What makes this so?

9. [5 marks] Consider the following perspective projection matrix:

$$\boldsymbol{P} = \begin{bmatrix} f_0 & 0 & 0 & 0 \\ 0 & f_0 & 0 & 0 \\ 0 & 0 & f_0 + f_1 & -f_0 f_1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

a. [1 mark] $\boldsymbol{P}$ is an invertible homogeneous matrix. It preserves the z value of the $z = f_0$ near plane and $z = f_1$ plane. Where do points behind the viewer go to after perspective projection?

b. [2 marks] Which points in $\mathbb{R}^3$ get mapped to points at infinity?

c. [2 marks] Which points at infinity get mapped to points in $\mathbb{R}^3$?

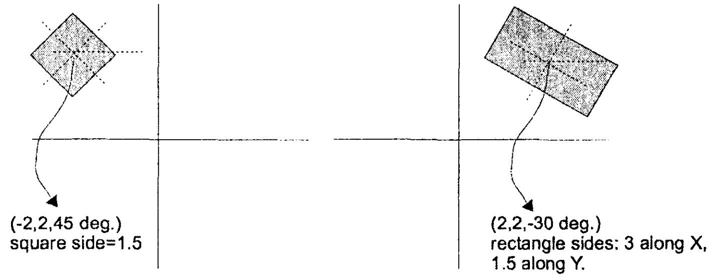
10. [5 marks] Explain each part of the *specular* component of the Phong Illumination Model as seen in class (i.e. explain it but keep it to one or two sentences max for each part).

Longer Answer Questions

Though these are the longer answer questions, avoid superfluous detail. Keep it to the point. The questions are designed to make you think. Avoid the brute-force approach, if possible.

1. [5 marks] You are in the middle of an exam and you need to find out a parametric equation for a sphere centered at some point. You tried memorizing it and you can't remember it. Stress does that and memory work takes you nowhere. However, you know about surfaces of revolution. How can you figure out some parameterization using this knowledge and save – or partially save – the day? (*To get full marks for this question, you will need to show a surface parametrization for a sphere along with bounds for its parameters. You will have to show the steps on how to derive the surface mathematically)*

2. [5 marks] Give the sequence of 2D affine transformations that maps the object in the left figure to the object in the right figure. You can use R, T, S, Sh, Re, to express rotation, translation, scaling, shear, and reflection operations respectively.



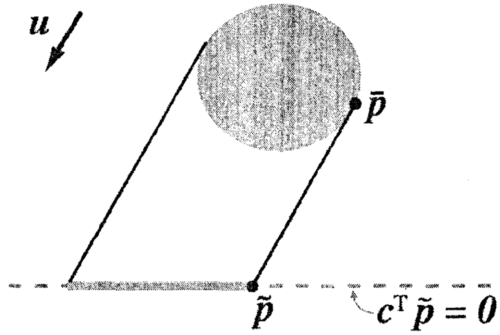
3. [6 marks total] Let $\bar{p} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)^T$ be a point in non-homogeneous coordinates on a sphere with center at the origin and with radius equal to 1. Let $\vec{n} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)^T$ be a normal vector in non-homogeneous coordinates of the sphere at point $\bar{p}$. Given transformations $\boldsymbol{T}, \boldsymbol{S}, \boldsymbol{R}$:

$$\boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \boldsymbol{R} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. [3 marks] What is the position of the point $\bar{p}$ on the sphere after the sphere is transformed by the product $\boldsymbol{TSR}$? Show your work by applying each transformation separately rather than computing the product of the three matrices.

b. [3 marks] What is the normal of the point $\bar{p}$ on the sphere after the sphere is transformed by the product $\boldsymbol{TSR}$? Show your work by applying each transformation separately.

4. [13 marks] Consider a directional light source (remember, this type of light source emits parallel rays of light in the direction of the unit vector $\vec{u}$), and the planar shadow that is created by projecting each mesh vertex position $\bar{p}$ to its shadow point $\tilde{p}$ onto the 3D plane specified (in homogeneous coordinates) by $c^T\tilde{p} = 0$ where $c \in \mathbb{R}^4$. Derive a *formula* for the 4x4 projection matrix $\boldsymbol{A}$ that maps a homogeneous object point, $\bar{p} = (x, y, z, 1)^T$, to its shadow point $\tilde{p} = \boldsymbol{A}\bar{p}$. (*Hint*: Consider the ray $\bar{p} + t\vec{u}$. Your goal is to find a formula for mapping $\bar{p}$ to $\tilde{p}$ and infer $\boldsymbol{A}$)



5. [12 marks] Suppose you would like to fit a polynomial curve $\boldsymbol{p}(t) = (x(t), y(t), z(t))$ to two given 3D points $\boldsymbol{p}(0)$ and $\boldsymbol{p}(1)$ and suppose $\boldsymbol{p}'(1)$ and $\boldsymbol{p}''(1)$ are also given. That is, you are given the positions at $t = 0{,}1$ and you are given the first and second derivatives at $t = 1$. What type curve should you fit in order to satisfy these constraints and why? How do you find its coefficients?

END OF EXAM

TOTAL PAGES = 12

TOTAL MARKS = 80