UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL 2016 EXAMINATIONS
CSC418H1S: Computer Graphics

Duration: 3 hours

No aids allowed

There are 16 pages total (including this page)

- This exam has 2 parts and a total of 20 questions.
- There are a total of 116 marks.
- This is a closed book exam.
- Show your work and write legibly.
- Write your name and student number on the next page.
- In order to pass this course, your final grade on this exam must be at least 35%.
- The inverse of a 2x2 matrix A is the following:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Given name(s):

____

Family Name:

____

Student number:

____

Question	Marks
1	/2
2	/3
3	/1
4	/2
5	/1
6	/4
7	/2
8	/1
9	/3
10	/6
11	/9
12	/5
13	/12
14	/5
15	/7
16	/10
17	/15
18	/12
19	/8
20	/8
Total	/116

Shorter Answer Questions

1) (2 marks) Let P be an invertible homogeneous matrix representing a perspective projection that preserves the z value of the $z = f_0$ near plane and $z = f_1$ far plane, i.e., Where do points behind the viewer go to after perspective projection?

2) (3 marks) Give a homogeneous representation of the point $\bar{p} = (2,5,7)^T$. Give a homogeneous representation of the vector $\vec{v} = (2,5,7)^T$. Give a non-homogeneous representation of the point expressed in homogeneous coordinates as $(2,5,7,7)^T$.

3) (1 marks) Describe the main difference between an affine transformation and a homography.

4) (2 marks) Describe the main difference between an affine transformation and a linear transformation? Why do we use affine transformations in Computer Graphics?

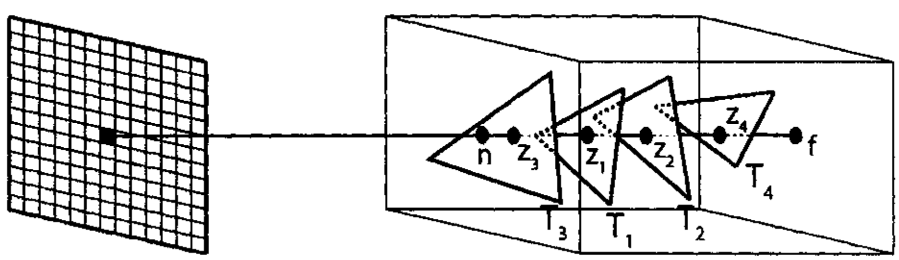
5) (1 mark) How does bump mapping differ from displacement mapping?

6) (4 marks) Give two advantages of key-frame animation over physics-based animation **and** two advantages of physics-based animation over key-frame animation.

7) (2 marks) At which point in the graphics pipeline is frustum culling done? Why?

8) (1 mark) Describe the difference between C_1 and G_1 continuity.

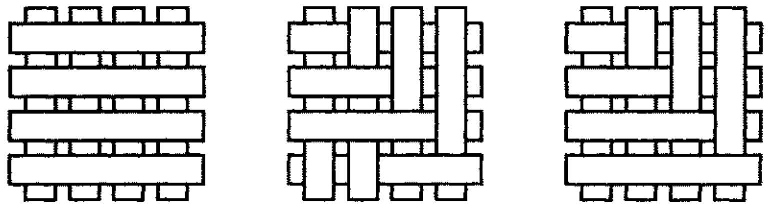
9) (3 marks) Assume the following simplified case (on the figure, next page) that illustrates four points from four triangles in 3D that are all mapped to the same pixel position on the 2D screen. What are the values in the z-Buffer at the indicated position if we draw the triangles in the order T_1, T_2, T_3, and T_4?



After initialization, the value in the z-Buffer is _______, after drawing T_1 it is _______, after drawing T_2 it is _______, after drawing T_3 it is _______, and after drawing T_4 it is _______. The z-value that is stored after projection for each triangle is called the __________.

Longer Answer Questions

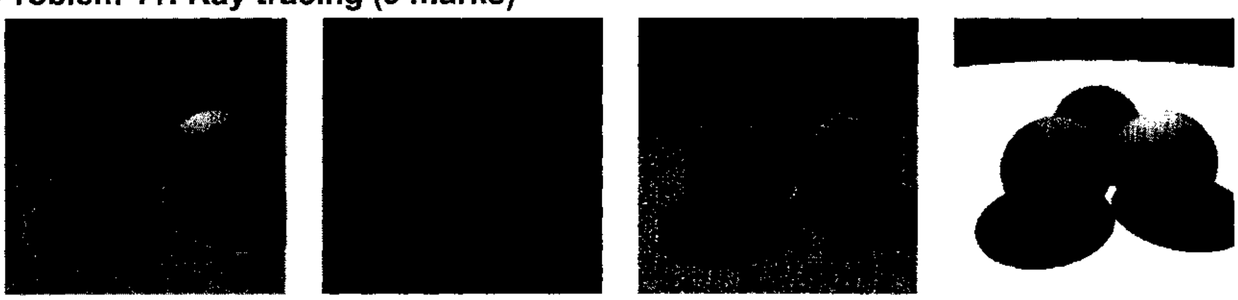
Problem 10: Visibility (6 marks) Which of the following scenes would cause problems for the Painter's Algorithm? Explain why or why not. A short verbal description is sufficient.



(a) (b) (c)

(These drawings are in image space; each rectangle is a single primitive)

Problem 11: Ray tracing (9 marks)



correct 1 2 3

Figure 1: Four images from a ray tracer. The left image is correct, and the other three were produced by introducing single-statement bugs into the program.

Look at each of the three images in Figure 1 that were produced by a Ray I ray tracer with various bugs. For **each** of the images, answer **each** of the following (in other words, three questions for each image):

a) Could it have been caused by a problem with ray generation?
b) Could it have been caused by a problem with ray intersection?
c) Could it have been caused by a problem with shading computations?

For each "yes" answer, back it up with an example of an error that would cause the observed symptoms. There is no right or wrong explanation; only plausible and implausible ones. But when there is a clearly plausible cause, very far-fetched explanations (which are roughly equivalent to a "no" answer) won't make full credit. Shadow computations count as part of shading. Computing surface normals counts as part of ray intersection.

Problem 12: Instancing (5 marks)

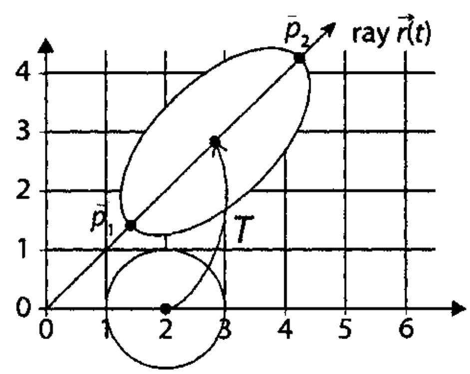


Figure 2: Circle transformed into ellipse via a linear transformation *T*. (Hint: looking at the image can save a lot of calculations)

a) **(3 marks)** The image in Figure 2 illustrates a rotated ellipse that was created by multiplying the depicted circle with the transformation matrix $\boldsymbol{T}$. $\boldsymbol{T}$ realizes a uniform scaling by the factor of 2 in x−direction followed by a counterclockwise rotation of 45° about the origin. Calculate the intersection points $\overline{p_1}$ and $\overline{p_2}$ of the ray $\vec{r}(t)$ going through the ellipse. The transformation matrix $\boldsymbol{T}$ is:

$$\boldsymbol{T} = \begin{pmatrix} \sqrt{2} & -\frac{\sqrt{2}}{2} \\ \sqrt{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

b) **(2 marks)** Suppose a normal vector to the circle at the point $(3, 0)$ is $\vec{n} = (1,0)^T$. Provide the transformed normal vector on the ellipse after transformation by $\boldsymbol{T}$.

Problem 13: Transformations (12 marks)

Consider the following classes of elementary 3D transformations:

- scale(s_x, s_y, s_z)
- rotate-x(θ) — rotate about x axis counterclockwise by θ
- rotate-y(θ)
- rotate-z(θ)
- translate(t_x, t_y, t_z)

Each of the following sequences of transformations happens to reduce to a single transformation from one of these classes. Find the equivalent elementary transformation for each sequence.

1. scale(2, 1, 1), then scale(1, 3, 4)
2. scale(2, 1, 1), then rotate-y(90°), then scale(3, 1, 1), then rotate-y(-90°)
3. rotate-x(90°), then rotate-y(90°), then rotate-z(90°)
4. rotate-z(90°), then translate(1, 0, 0), then rotate-z(-90°)

Problem 14: Affine Transformations (5 marks)

Show that affine transformations preserve parallel lines. (Hint: Recall the explicit parameterization of a line.)

Problem 15: Curves and Surfaces of Revolution (7 marks)

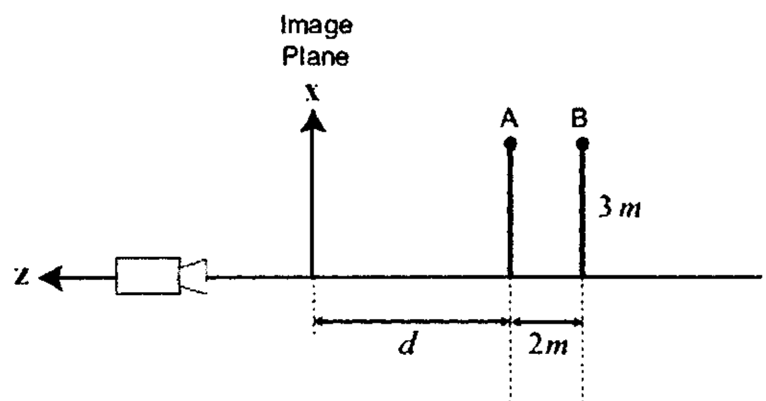
Give a parametric representation $f(u, v)$ of a sphere centered at $(1,2,3)$ that has the following properties:

$$f(0,0) = (2, 2, 3), \qquad f(1,1) = (0, 2, 3), \qquad f(0.5,0) = (1, 2, 4)$$

As usual, u and v both range from 0 to 1.

Problem 16: Projection (10 marks)

Consider a 2D scene consisting of two poles that are 3m high and 2m apart. Suppose that the scene is being viewed by a camera at a distance *d* away from the leftmost pole, as shown below:



a) **(5 marks)** Give the projected heights for each pole for a camera with focal length f.

b) **(5 marks)** How far should the camera be from Pole A so that the projected height of Pole A is within 10% of the projected height of Pole B?

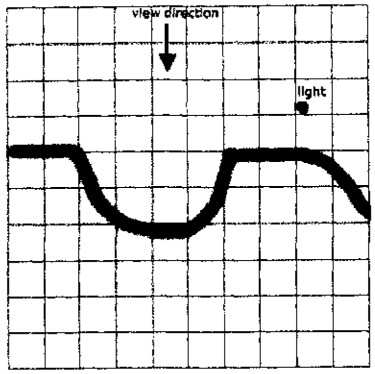
Problem 17: Shading (15 marks)

Given a triangle mesh, briefly explain the key differences in how triangle normals are used to evaluate (i) flat shading, (ii) Gouraud shading, and (iii) Phong shading. Use figures as needed.

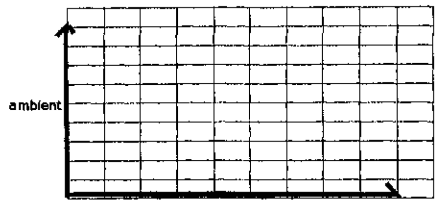
Problem 18: Phong Lighting (12 marks)

This question is about a Phong illumination model that includes shadows. For simplicity, assume a 1D surface in a 2D world, as shown below. The viewpoint is far above the scene looking straight down (as in orthographic projection) and the position of the light source is specified in the diagram. Sketch three graphs showing the ambient, diffuse, and specular components of the Phong model as applied to the scene, and properly account for shadows. In a fourth graph, show the total intensity that would be computed (the graphs should show the intensity as a function of x). For your convenience, two sets of graphs are provided. Use the first for a draft and the second for your final answer, drawn as accurately as you can. The parameters for this scene are:

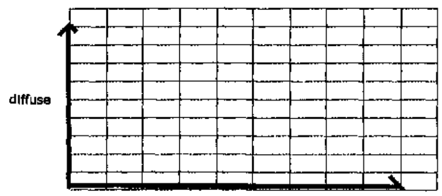
$$I = r_a + r_d(\max(0, \vec{n} \cdot \vec{s})) + r_s(\max(0, \vec{v} \cdot \vec{r}))^n \quad \text{(when not in shadow)}$$
$$r_a = 0.2, \quad r_d = 0.7, \quad r_s = 0.3, \quad n = 200$$



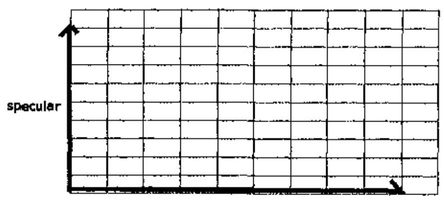
view direction
light



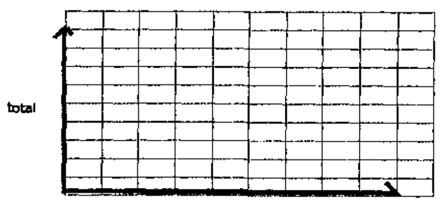
ambient



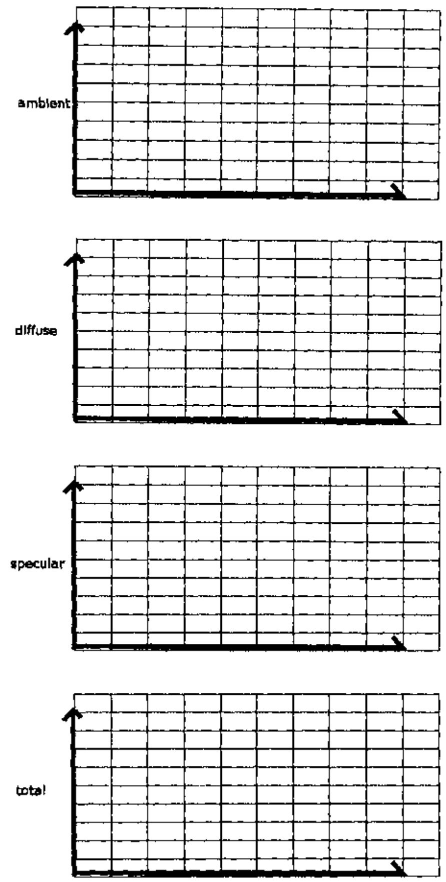
diffuse



specular



total



Problem 19: Bézier curves (8 marks)

Let $\overline{p_0} = (0{,}0)^T, \overline{p_1} = (1{,}1)^T, \overline{p_2} = (-1{,}1)^T$ be the control points for a quadratic Bézier curve $c(t) = \sum_{i=0}^{2} B_{i,2}(t)p_i$

(a) **(3 marks)** Evaluate the Bernstein basis functions at $t = 0.5$, and compute the position of the point $c(0.5)$.

(b) **(3 marks)** What is the derivative of the curve at $t = 0.5$? Show your work.

(c) **(2 marks)** Make a sketch of the control polygon and apply Decaslejau's algorithm to draw the position of $c(0.5)$. Does your picture agree with the answer to the first two parts of this question?

Problem 20: Cubic curves (8 marks)

Derive a cubic polynomial $x(t)$ that satisfies the following constraints:

- $x(0) = 1$
- $x(1) = 5$
- $x'(0) = 1$
- $x''(0) = 2$

END OF EXAM.

TOTAL PAGES = 16

TOTAL MARKS = 155