UNIVERSITY OF TORONTO

Faculty of Arts and Science

Dec. 2016 EXAMINATIONS

CSC418H1F/ CSC2504H1F

Duration - 3 hours

No aids allowed

There are 8 pages, including this one. The examination is out of 60 marks and the value of each question is provided; please use this information to manage your time effectively.

Q.1:	/10 + 6
Q.2:	/5
Q.3:	/5
Q.4:	/6
Q.5:	/12
Q.6:	/8
Q.7:	/14 + 4
Total:	/60 + 10

1) [10 marks] Geometry

a) [2.5 marks] What is the normal vector **N** of a triangle in 3D, in terms of its vertices $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$?

b) [2.5 marks] What is the implicit equation **f** for the plane of a 3D triangle with normal vector **N** and vertices $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$ so that $\mathbf{f(p) > 0}$ if **p** is *above the plane*, $\mathbf{f(p) = 0}$ if **p** is *on the plane*, and $\mathbf{f(p) < 0}$ if **p** is *below the plane?*

c) [2.5 marks] Describe how to compute a bounding box (with box sides parallel to the global *x*, *y* and *z* axes) for a 3D triangle with vertices $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$? What are two diagonally opposite points that can be used to define this bounding box?

d) [2.5 marks] Describe how to compute a bounding sphere for a 3D triangle with vertices $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$? What is the center point and radius of this sphere?

e) **EXTRA CREDIT** [6 marks] Let $\mathbf{g(x,y,z) = 3xy + \sin(z) - 2y^2z = 0}$ describe the surface of some 3D shape, how can you determine the *unit normal* vector **n** at some point **(a,b,c)** on the surface? How can you determine *two* unit vectors spanning the tangent plane at **(a,b,c)?**

2) [5 marks] Animation

a) [2 marks] A parabolic function is used to interpolate a variable at two keyframes from $\mathbf{v_0}$ to $\mathbf{v_1}$, as time **t** goes from **0** to **1**, i.e. $\mathbf{v(t)=v_0 + (v_1-v_0)*t^2}$. Is the resulting motion of the variable **ease-in**? Is it **ease-out**?

d) [3 marks] Given a projectile's position and *velocity* at t=0 as **p**(0) and **v**(0) respectively, give an update rule to determine the position and velocity after a small discrete time step Δt. (Assume the only external force acting on the projectile is gravity: $\mathbf{g} = [0\ -9.8\ 0]^T$ m/s^2)

3) [5 marks] Real-time Graphics Pipeline

a) [3 marks] Enumerate the different stages in a graphics pipeline that a point on an object typically goes through to result in a pixel on the screen.

b) [2 marks] Describe a visual phenomenon that is difficult to produce using *direct local illumination* but easily achievable with *global illumination.*

4) [6 marks] Projection, visibility (True or False with reason, 2 marks each, **NO marks** without the correct reason).

a) Given three flat polygons in 3D, we can always find a depth ordering such that their visibility can be resolved using the Painter's algorithm without splitting the polygons. (accompany your answer with an illustration).

b) An equilateral triangle in 3D will only produce an equilateral triangle in 2D after projection if and only if its surface is parallel to the view-plane.

c) Removing the back-faces of a single non-convex object in a scene completely resolves scene visibility, i.e. all the remaining faces are visible.

5) [12 marks] Illumination (True or False with reason, 2 marks each, **NO marks** without the correct reason).

a) *Caustic* light patterns are examples of light transport paths of type E-S-S-D-D-S-S-S-L, where E is the eye, L a light and S and D, specular and diffuse objects respectively.

b) Chalk is a good example of a very specular object.

c) Bright sunlight is well approximated using an ambient light source.

d) Given a point light source that coincides with a view-point, we are guaranteed a specular highlight on a completely visible specular sphere, no matter where it is placed in the 3D scene.

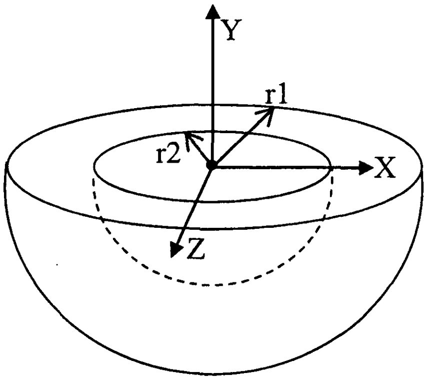
e) When refracting from air into some material like glass, the angle a light ray *bends* depends on the viewing direction.

f) Given a surface defined by the solution to a quadratic equation (e.g., $\mathbf{p}^T \mathbf{A} \mathbf{p} + \mathbf{p}^T\mathbf{b} + \mathbf{c} = \mathbf{0}$), it is always possible to analytically determine the number and location of intersections with a ray in 3D.

6) [8 marks] Ray-object intersections

Consider the intersections of a ray (starting from point **S** with direction **D**) and a hollow hemispherical cup centered at the origin with its hemisphere below the **XZ** plane (i.e. with **Y** negative), and outer and inner radii **r1** and **r2** (assume **r1**>**r2**>0).

[2 marks] What is the maximum number of intersections between the cup and a ray?



[6 marks] Your task is to implement:

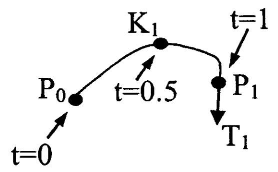
int **cupIntersect**(point **S**, vector **D**, float **r1**, float **r2**, float &t[]) ;

returning the number of intersections and the corresponding parameter values in the array **t** in increasing order. You may use *without implementing* the following functions:

int **sphereAtOriginIntersect(**point S, vector **D**, float **r,** float &t[]) ;
int **XZplaneIntersect(**point **S**, vector **D**, float &t[]) ;

7) [14 + 4 marks] Curves

a) [4 marks] Consider a *cubic* curve **C(t)** with $\mathbf{0 \le t \le 1}$,defined by 4 geometric constraints, such that:



a. **C(0)** $= \mathbf{P_0}$
b. **C(1)** $= \mathbf{P_1}$
c. **C'(1)** $= \mathbf{T_1}$
d. **C''(0.5)** $= \mathbf{K_1}$

Write an expression for the basis matrix of the cubic curve when the constraints are written as

$[\,\mathbf{P_0}\ \mathbf{P_1}\ \mathbf{T_1}\ \mathbf{K_1}\,]^T$

b) [3 marks] A curve **D(t)** is defined using two pieces:
$\mathbf{D(t) = (t,t^2)}$ for $\mathbf{t<0}$ and $\mathbf{D(t) = (t^3+t^2-t,\ 1-\cos(t))}$ for $\mathbf{t>=0}$.
What is the level of geometric $G^?$, and parametric $C^?$, continuity of the overall curve?

c) [3 marks] Give three reasons why cubic curves are popular in computer graphics.

d) [4 marks] A curve $\mathbf{C}(t)$ over $\mathbf{0 <= t <= 1}$ is defined using a set of $\mathbf{n}$ basis functions $\mathbf{B}_i(t)$ corresponding to points $\mathbf{P}_i$, such that $\mathbf{C}(t) = \sum_i (\mathbf{B}_i(t) * \mathbf{P}_i)$. We say that the basis for the curve is *affine invariant* if the curve produced by applying any affine transform $\mathbf{A}$ to the control points is the same as applying the affine transform to the curve: $\mathbf{A\ C(t)} = \sum_i (\mathbf{B}_i(t) * \mathbf{A\ P}_i)$. Show that a basis is *affine invariant* if and only if the basis functions sum to one for any value of t.

e) **EXTRA CREDIT** [4 marks] Extend the notion of G^1/C^1 continuity to surfaces. If two surfaces meet up along a shared curve, what must be true about both surfaces along this curve for strict C^1 continuity? For geometry G^1 continuity?

End of Examination.