CSC418/2504S LEC0101/2001 Midterm Q1

a) Modelling
Animation
Rendering

b) $f(t) = [f_1(t), f_2(t), \cdots f_n(t)]$, $t \in [a, b]$

c) True, Third derivative is constant $\Rightarrow$ continuous

d) True, usually but not always

e) Point: $[x, y, z, 1]^T$
Vector: $[x, y, z, 0]^T$

f) Parallelism

g) False, lines not perpendicular with the view direction converge.

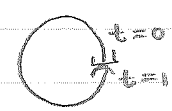
CSC 418/2504S Karan's Section Midterm Q1

a) - Keyframe interpolation
- Physical simulation

b) No, quadratics are defined by 3 points, which defines a plane.

c) No $p(t) = (\cos(2\pi t^2), \sin(2\pi t^2))$ $t \in [0,1)$ is a circle

$p'(t) = (-4\pi t\cos(2\pi t^2), 4\pi \sin(2\pi t^2))$



$\lim_{t \to 0} p'(t) = (0, 0)$

$\lim_{t \to 1} p'(t) = (-4\pi, 0) \neq \lim_{t \to 0} p'(t)$

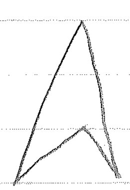
So it's not even C^1 continuous
It is however G^2 continuous.

d) Yes, uniform scaling preserves angles

e) $\begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$

f) The image will be blurred

g)



Karan's midterm

Q2)

i) $[6,4,2] \times [0,5,1] =$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 2 \\ 0 & 5 & 1 \end{vmatrix} = (4-10)\hat{i} - 6\hat{j} + 30\hat{k}$$

$$= [-6 \quad -6 \quad 30]$$

or

$$[-1 \quad -1 \quad 5]$$

or

$$[-3 \quad -3 \quad 15]$$

or $[-2 \quad -2 \quad 10]$

ii) $x^3 + 2xy - y^2 + 3 = 0$

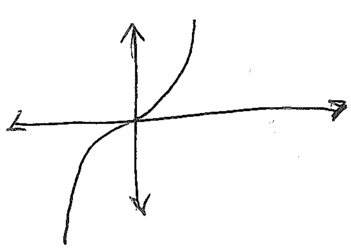
$\frac{\partial}{\partial x} = 3x^2 + 2y$

$\frac{\partial}{\partial y} = 2x - 2y$

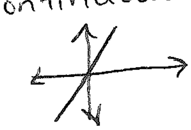
tangent $= \left[\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y}\right]$

normal $= \left[\frac{\partial}{\partial y} \quad -\frac{\partial}{\partial x}\right]$

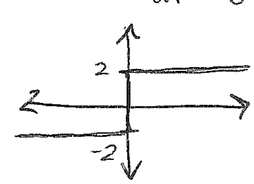
iii) $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$



$\frac{\partial f}{\partial x} = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$ → both are continuous



$\frac{\partial^2 f}{\partial x^2} = \begin{cases} 2 & \text{if } x \geq 0 \\ -2 & \text{if } x < 0 \end{cases}$ → discontinuous at 0



C^1 continuous

Dave's midterm

Q3)

a) quadratic polynomial:

$x(t) = a_0 + a_1 t + a_2 t^2$

in terms of t

we need to solve for the coefficients a_0, a_1, a_2

we know:

$x(0) = 0$

$x(5) = 1$

$x(10) = 2$

we can construct the following matrix and equation:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 25 \\ 1 & 10 & 100 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

to solve, we can rearrange:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 25 \\ 1 & 10 & 100 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

and similarly for $\theta(t)$:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 5 & 25 \\ 1 & 10 & 100 \end{bmatrix}^{-1} \begin{bmatrix} 0^\circ \\ 90^\circ \\ 180^\circ \end{bmatrix}$$

b) $T(t) = T_{(x(t))} R_{(-\theta(t))}$

$T_{(x(t))}$ and $R_{(-\theta(t))}$

come from the previous part

$$T_{(x(t))} = \begin{bmatrix} 1 & 0 & x(t) \\ 0 & 1 & y(t) \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{(-\theta(t))} = \begin{bmatrix} \cos(\theta(t)) & \sin(\theta(t)) & 0 \\ -\sin(\theta(t)) & \cos(\theta(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

CSC418 LEC0101/2001 Midterm Q2.

a) i) $P_1 = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$ $P_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ $\quad \ell = P_1 \times P_2 = [2, -2, -6]$

or equivalently $[1, -1, -3]$

ii) $r(s) = p_1 + s(p_2 - p_1) \quad s \in \mathbb{R}$

$= \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$

iii) let our point of intersection be q. $\ell_1 = [2, -2, -6]$ $\ell_2 = [1, -2, 7]$

$q = \ell_1 \times \ell_2 = [-26, -20, -2]$

which in cartesian coordinates is $\left(\frac{-26}{-2}, \frac{-20}{-2}\right)$

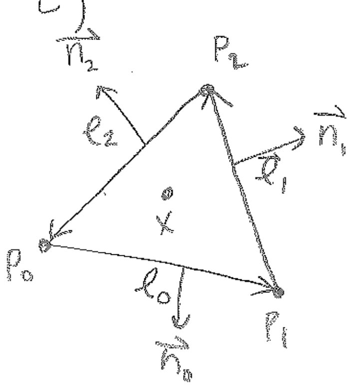
$= (13, 10)$

b)

$$S(t, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ 0 \end{bmatrix}$$

$$= (x(t), y(t)\cos(\theta), y(t)\sin(\theta))$$

c)



let $\left.\begin{aligned} \vec{\ell}_0 &= P_1 - P_0 \\ \vec{\ell}_1 &= P_2 - P_1 \\ \vec{\ell}_2 &= P_0 - P_2 \end{aligned}\right\}$ Edge vectors

then obtain normal vectors $\vec{n}_i$ by rotating $\vec{\ell}_i$ (counterclockwise or clockwise, as long as it's consistent for all i) by $90°$

Then x is inside the triangle if, and only if each $\vec{n}_i \cdot (x - p_i)$ are all positive, or all are negative.

i. Let the polynomial describing the x-coordinate w.r.t time be

$$x(t) = a_0 + a_1\, t + a_2\, t^2 + a_3 t^3.$$

Differentiating $x(t)$, we get the velocity polynomial $x'(t) = a_1 + 2a_2 t + 3a_3 t^2$.

We have four constraints given for finding the coefficients

$$x(0) = 0, x(5) = 1, x(10) = 2, x'(0) = 0.$$

In matrix form, the constraints can be expressed as

$$MA = [0\ 1\ 2\ 0]^T, \text{ where}$$

$$A = [a_0\ a_1\ a_2\ a_3]^T, M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 5 & 25 & 125 \\ 1 & 10 & 100 & 1000 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

And can be solved as $A = M^{-1}[0\ 1\ 2\ 0]^T$.

Similarly, one can find the coefficients for $y(t)$ (optional), and for $\theta(t)$ (required).

Alternatively, you can directly solve for the polynomial coefficients as well, without constructing a matrix. In that solution, the actual values of the coefficients is expected.

ii. The matrix $T(t)$ should be a change of basis matrix which moves the point $\boldsymbol{p}$ from local coordinates to global coordinates. Therefore,

$$T(t) = T_{[x(t)\ y(t)]} R_{-\theta(t)},$$

Where $T_{[x(t)\ y(t)]}$ is the 2D translation matrix $\begin{bmatrix} 1 & 0 & x(t) \\ 0 & 1 & y(t) \\ 0 & 0 & 1 \end{bmatrix}$,

and $R_{-\theta(t)}$ is the 2D rotation matrix $\begin{bmatrix} cos\theta(t) & sin\theta(t) & 0 \\ -sin\theta(t) & cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$. We use $-\theta(t)$ since $\theta(t)$ constructs an angle in the clockwise direction.

Q4

a) Basis is u, v, w

$u = -g$

$u = \frac{u}{\|u\|}$

$v = u \times up$

$w = v \times u$

b, c) $M_{camera} = M_{uvw} \cdot M_e = \begin{bmatrix} \begin{bmatrix} M_{uvw} \end{bmatrix} & \begin{bmatrix} -M_{uvw} \cdot e \end{bmatrix} \\ 0 \; 0 \; 0 & 1 \end{bmatrix}$

← vector $-e$

$$M_{uvw} = \begin{bmatrix} [- & u & -] & 0 \\ [- & v & -] & 0 \\ [- & w & -] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_e = \begin{bmatrix} 1 & 0 & 0 & \begin{bmatrix} 1 \\ -e \\ 1 \end{bmatrix} \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection question was ambiguous and not very well defined. So I was giving points as long as you defined matricies for orthographic / perspective projections.

(Pavel's section)

$$M_{xy\text{-plane}} = M_{ortho} \cdot M_{camera} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot M_{camera} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(Karan's section)

$$M_{xy\text{-plane}} = M_{persp} \cdot M_{camera} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

any M_{persp} given in lecture were correct.