ECE 470 HW1

2-1

$$V_1 \cdot V_2 = V_1^T V_2$$
$$= (R_0^1 V_0)^T V_2$$
$$= V_0^T R_0^{1T} V_2$$
$$= (R_1^0 V_1)^T R_0^{1T} V_2$$
$$= V_1^T R_1^{0T} R_0^{1T} V_2$$
$$= V_1 (R_0^1)(R_0^{1T}) V_2$$
$$= V_1 \cdot I \cdot V_2$$
$$= V_1 \cdot V_2$$

2-2

$$V = \begin{bmatrix} ax \\ by \end{bmatrix} \qquad \|V\| = \left((ax)^2 + (by)^2\right)^{\frac{1}{2}}$$

$$RV = \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} ax \\ by \end{bmatrix} = \begin{bmatrix} axC_\theta - S_\theta by \\ axS_\theta + C_\theta by \end{bmatrix}$$

$$\|RV\| = \left((axC_\theta - S_\theta by)^2 + (axS_\theta + C_\theta by)^2\right)^{\frac{1}{2}}$$
$$= \left((ax)^2 C_\theta^2 + S_\theta^2 (by)^2 - \underline{2axbyC_\theta S_\theta} + (ax)^2 S_\theta^2 + (by)^2 C_\theta^2 + \underline{2axbyC_\theta S_\theta}\right)^{\frac{1}{2}}$$
$$= \left((ax)^2(C_\theta^2 + S_\theta^2) + (by)^2(S_\theta^2 + C_\theta^2)\right)^{\frac{1}{2}}$$
$$= \left((ax)^2 + (by)^2\right)^{\frac{1}{2}}$$
$$= \|V\|$$

2-10 $R_{y,\psi} \cdot R_{x,\phi} \cdot R_{z,\theta}$

$$\begin{bmatrix} C\psi & 0 & S\psi \\ 0 & 1 & 0 \\ -S\psi & 0 & C\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2-11 $R_{z,\theta}\, R_{x,\phi}\, R_{x,\psi}$

$$\begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix}$$

2-12 $R_{z,\alpha} R_{x,\phi}\, R_{z,\theta}\, R_{x,\psi}$

$$\begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix}$$

2-13 $R_{z,\alpha}\, R_{z,\theta}\, R_{x,\phi}\, R_{x,\psi}$

$$\begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & -S\phi \\ 0 & S\phi & C\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix}$$

2-15 $R_3^1 = R_2^1 R_3^2$

$$R_3^2 = R_3^1 (R_2^1)^{-1} = \underline{R_3^1 (R_2^1)^T} = R_3^1 R_1^2$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{bmatrix}$$

2-23

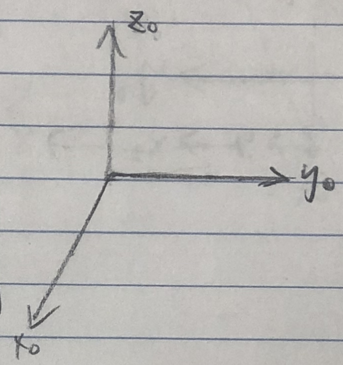
$$R_{k,\theta} = R_{y,90} R_{z,45}$$

$$= \begin{bmatrix} \cos 90 & 0 & \sin 90 \\ 0 & 1 & 0 \\ -\sin 90 & 0 & \cos 90 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

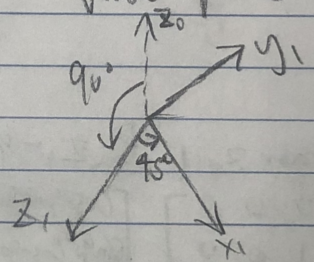
$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

initial frame



final frame



$$\theta = \cos^{-1}\left(\frac{Tr(R)-1}{2}\right)$$

$$= \cos^{-1}\left(\frac{0+\frac{\sqrt{2}}{2}+0-1}{2}\right)$$

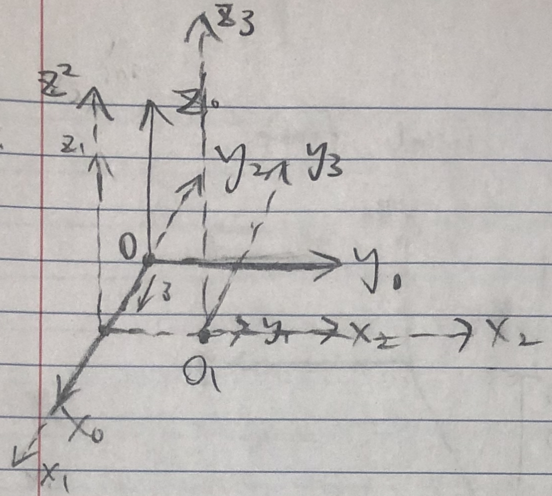
$$= 98.42°$$

$$k = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

$$= \frac{1}{2\sin 98.42°}\begin{bmatrix} \frac{\sqrt{2}}{2} - 0 \\ 1 + \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} - 0 \end{bmatrix}$$

$$= \frac{1}{2\sin 98.42°}\begin{bmatrix} \frac{\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

2-37:



$$H = Trans_{x,3}\ R_{z,\frac{\pi}{2}}\ Trans_{y,1}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2-38: H_1^0: $Trans_{z,1}\ R_{z,-90°}\ R_{y,90°}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos{-90} & -\sin{90°} & 0 \\ \sin 90° & \cos{-90} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90° & 0 & \sin 90° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90° & 0 & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H_2^0: $Tran_{y,1}\ R_{x,90°}\ R_{y,-90°}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90° & -\sin 90° \\ 0 & \sin 90° & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos{-90°} & 0 & \sin 90° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90° & 0 & \cos{-90°} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H_2^1: $Trans_{X,1}\ Trans_{z,-1}\ R_{x,90°}\ R_{z,90°}$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 0 \\ \sin 90° & \cos 90° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0\ H_2^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2-39

$$H_1^0 = \overset{\text{Trans } y,1}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \overset{\text{Trans } z,1}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

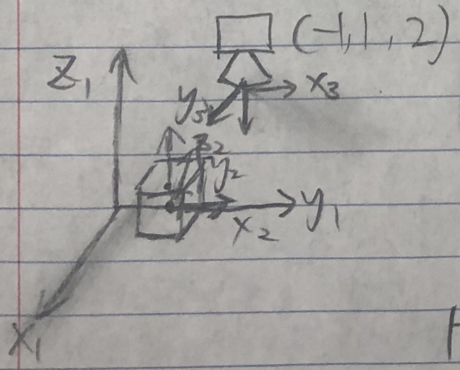
$$H_2^1 = \overset{\text{Trans } y_1, \frac{l}{2}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{l}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \overset{\text{Trans } x_1, -\frac{l}{2}}{\begin{bmatrix} 1 & 0 & 0 & -\frac{l}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & -\frac{l}{2} \\ 0 & 1 & 0 & \frac{l}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 H_2^1 H_3^2 = \begin{bmatrix} 1 & 0 & 0 & -\frac{l}{2} \\ 0 & 1 & 0 & 1+\frac{l}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & -\frac{l}{2} \\ 1 & 0 & 0 & 1+\frac{l}{2} \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2-41



$$H_{cam}^{block} \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_{base}^{block} \begin{bmatrix} 0 & -1 & 0 & -1.5 \\ 1 & 0 & 0 & -0.8 \\ 0 & 0 & 1 & -1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$