

3-2

ECE 470 HW 2

LINK	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	$d_1$	0	0	$\theta_1$
2	$d_2$	0	0	$\theta_2$
3	$d_3$	0	0	$-\theta_3$

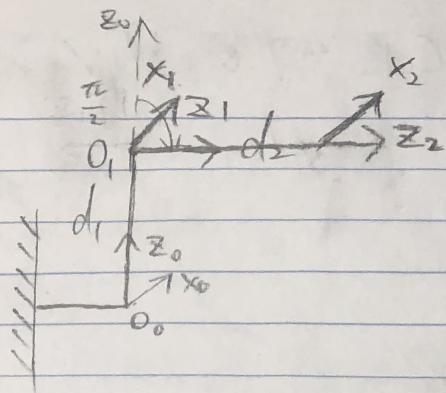
$$T_2^1 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & d_1 \cos \theta_1 \\ \sin \theta_1 & \cos \theta_1 & 0 & d_1 \sin \theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & d_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & d_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} \cos \theta_3 & \sin \theta_3 & 0 & d_3 \cos \theta_3 \\ -\sin \theta_3 & \cos \theta_3 & 0 & -d_3 \sin \theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^1 = T_2^1 T_3^2 T_4^3$$

3-3

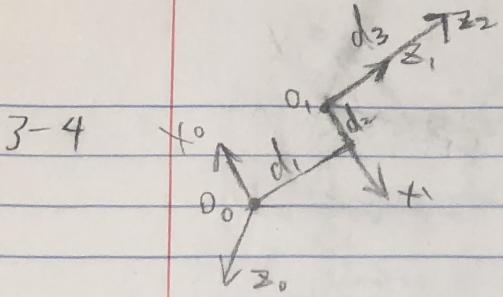


LINK	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	0	$\frac{\pi}{2}$	$d_1$	0
2	0	0	$d_2$	0

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = T_1^0 T_2^1$$



3-4

LINK  $a_i \alpha_i d_i \theta_i$

1  $d_2 \frac{\pi}{2} d_1 \pi$

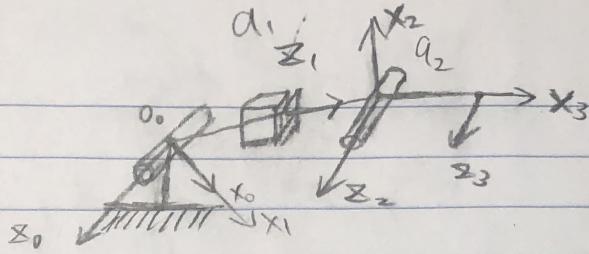
2 0 0  $d_3$  0

$$T_1^o = \begin{bmatrix} \cos \pi & -\sin \pi \cos \frac{\pi}{2} & \sin \pi \sin \frac{\pi}{2} & d_2 \cos \pi \\ \sin \pi & \cos \pi \cos \frac{\pi}{2} & -\cos \pi \sin \frac{\pi}{2} & d_2 \sin \pi \\ 0 & \sin \frac{\pi}{2} & \cos \frac{\pi}{2} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{T}_2^o = T_1^o T_2^l$$

3-5



LINK  $a_i \alpha_i d_i \theta_i$

$$1 \quad 0 \quad -\frac{\pi}{2} \quad 0 \quad 0$$

$$2 \quad 0 \quad -\frac{\pi}{2} \quad a_1 \quad \pi$$

$$3 \quad a_2 \quad 0 \quad 0 \quad -\frac{\pi}{2}$$

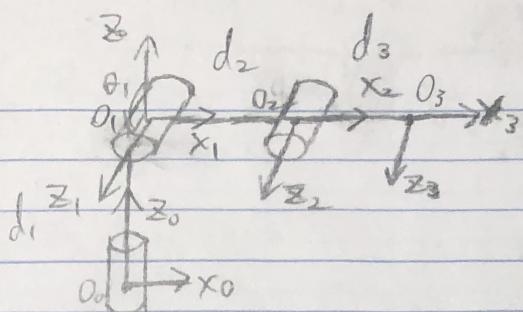
$$H_1^0 = \begin{bmatrix} C_0 & -S_0 C_{-\frac{\pi}{2}} & S_0 S_{-\frac{\pi}{2}} & 0 \\ S_0 & C_0 C_{-\frac{\pi}{2}} & -C_0 S_{-\frac{\pi}{2}} & 0 \\ 0 & S_{-\frac{\pi}{2}} & C_{-\frac{\pi}{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} C\pi & -S\pi C_{\frac{\pi}{2}} & S\pi S_{\frac{\pi}{2}} & 0 \\ S\pi & C\pi C_{\frac{\pi}{2}} & -C\pi S_{\frac{\pi}{2}} & 0 \\ 0 & S_{\frac{\pi}{2}} & C_{\frac{\pi}{2}} & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} C_{\frac{\pi}{2}} & -S_{\frac{\pi}{2}} C_0 & S_{\frac{\pi}{2}} S_0 \cdot a_2 C_{\frac{\pi}{2}} & 0 \\ S_{\frac{\pi}{2}} & C_{\frac{\pi}{2}} C_0 & -C_{\frac{\pi}{2}} S_0 \cdot a_2 S_{\frac{\pi}{2}} & 0 \\ 0 & S_0 & C_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

3-6



LINK  $a_1 \ d_1 \ d_2 \ \theta_1$

$$1 \quad 0 \quad \theta_1 \quad d_1 \quad 0$$

$$2 \quad d_2 \quad 0 \quad 0 \quad 0$$

$$3 \quad d_3 \quad 0 \quad 0 \quad 0$$

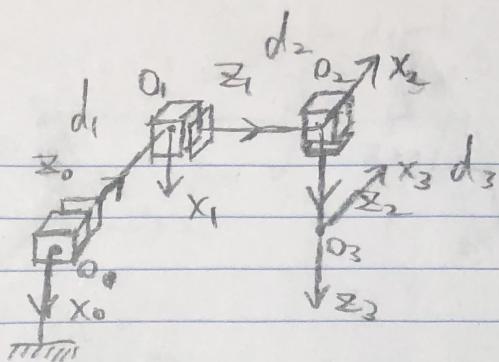
$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\theta_1} & -S_{\theta_1} & 0 \\ 0 & S_{\theta_1} & C_{\theta_1} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

3-7

LINK  $d_1$   $d_2$   $d_3$   $\theta_1$ 

1	0	$\frac{\pi}{2}$	$d_1$	0
2	0	$\frac{\pi}{2}$	$d_2$	$\frac{\pi}{2}$
3	0	0	$d_3$	0

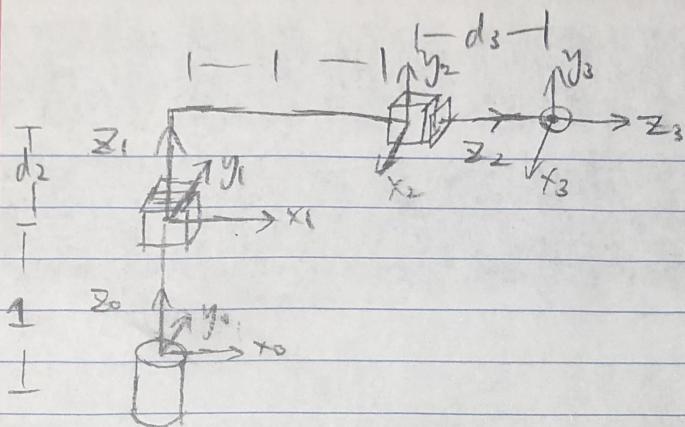
$$H_1^o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = H_3^o = H_1^o H_2^1 H_3^2$$

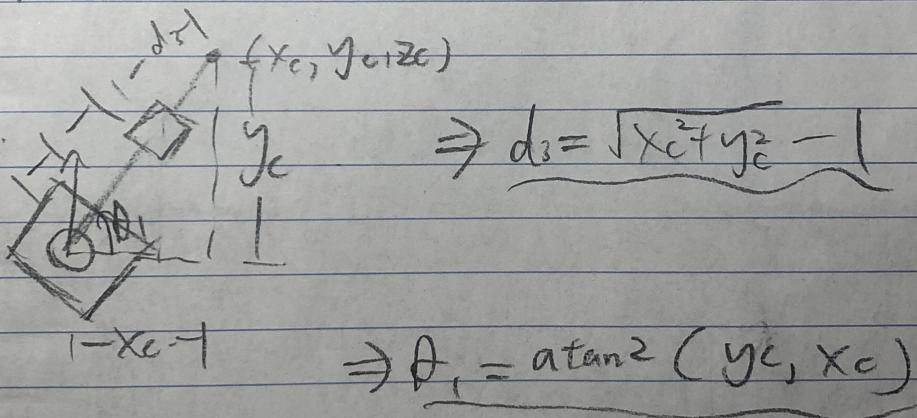
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Side view -  $\Rightarrow d_2 = z_c - 1$

Side view diagram showing a stepped block. The total height is  $d_3$ , the depth of the slot is  $d_1$ , and the horizontal distance from the center of the slot to the back face is  $z_c$ . The equation  $d_2 = z_c - 1$  is derived.

Top View



$$\Rightarrow d_3 = \sqrt{x_c^2 + y_c^2} - 1$$

$$\Rightarrow \theta_1 = \arctan^2(y_c, x_c)$$