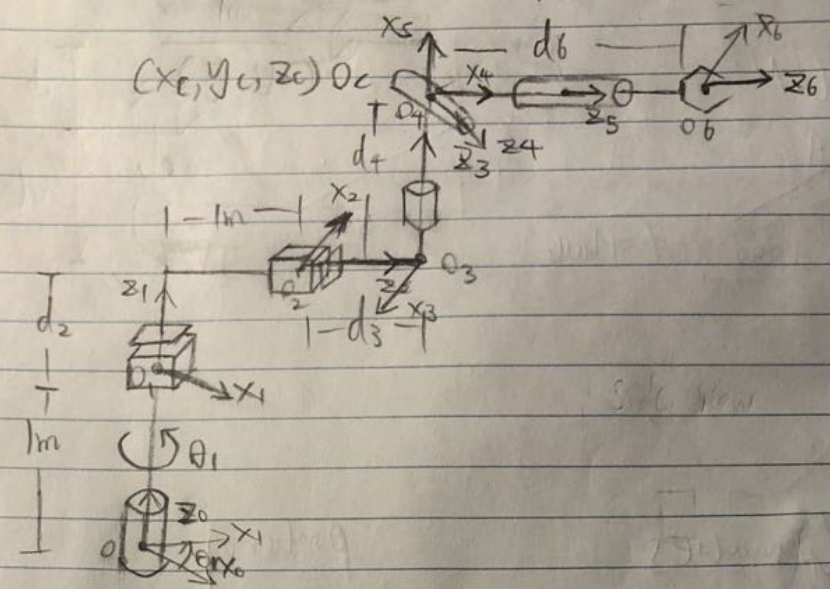
ece 470 HW 3

3-15



LINK	a	α	d	θ
1	0	0	1	θ_1
2	1	$\frac{\pi}{2}$	d_2	$\frac{\pi}{2}$
3	0	$\frac{\pi}{2}$	d_3	π
4	0	$-\frac{\pi}{2}$	d_4	θ_4
5	0	$\frac{\pi}{2}$	0	θ_5
6	0	0	d_6	θ_6

$H_d \rightarrow$ desired Matrix
$R_d \rightarrow$ desired Rotation Matrix
$O_d \rightarrow$ desired position Matrix

$$O_c = O_d - R_d \cdot \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$$

$$\theta_1 = \operatorname{atan2}(O_{cy}, O_{cx})$$

$$d_2 = z_c - 1$$

$$d_3 = \sqrt{x_c^2 + y_c^2} - 1$$

$$H_3^2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & 0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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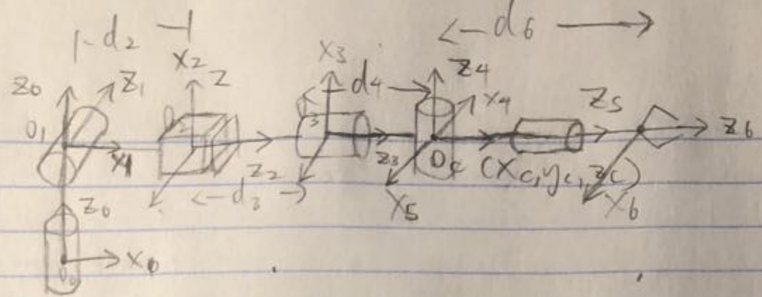


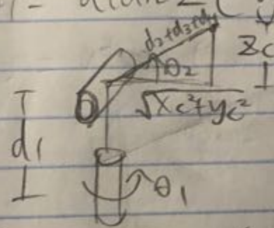
LINK	a	α	d	θ
1	0	$-\frac{\pi}{2}$	d_1	θ_1
2	0	$\frac{\pi}{2}$	d_2	θ_2
3	0	0	d_3	0
4	0	$-\frac{\pi}{2}$	d_4	θ_4
5	0	$\frac{\pi}{2}$	0	θ_5
6	0	0	d_6	θ_6

1. $R = R_6^0 = R_d \qquad o = o_d$

$$o_c = o_d - R_d \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix}$$

2. $\theta_1 = \text{atan2}(y_c, x_c)$ Not Unique!

4 Solutions



$$\theta_2 = \text{atan2}\left(z_c - d_1, \sqrt{x_c^2 + y_c^2}\right)$$

$$d_3 = \sqrt{x_c^2 + y_c^2 + (z_c - d_1)^2} - d_2 - d_4$$

3. $$H_1^0 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2^1 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$H_3^0 = H_1^0 H_2^1 H_3^2$

$R_3^0 = H_3^0(1:3, 1:3)$

$$R_6^3 = R_3^{0T} R_d = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$\theta_4 = \text{atan2}(a_{23}, a_{13})$

$\theta_5 = \text{atan2}\left(\sqrt{1 - a_{33}^2}, a_{33}\right)$

$\theta_6 = \text{atan2}(a_{32}, -a_{31})$

$$R = \begin{bmatrix} \cos\psi\cos\phi\cos\theta - \sin\phi\sin\psi, & -\sin\psi\cos\phi\cos\theta - \sin\phi\cos\psi, & \cos\phi\sin\theta \\ \cos\psi\sin\phi\cos\theta + \cos\phi\sin\psi, & -\sin\psi\sin\phi\cos\theta + \cos\phi\cos\psi, & \sin\phi\sin\theta \\ -\sin\theta\cos\psi & , \sin\theta\sin\psi & , \cos\theta \end{bmatrix}$$

4-13

$$\frac{d}{dt}R = \frac{dR}{d\phi}\cdot\frac{d\phi}{dt} + \frac{dR}{d\psi}\frac{d\psi}{dt} + \frac{dR}{d\theta}\frac{d\theta}{dt}$$

$$\frac{d}{dt}R_{11} = (-\cos\psi\sin\phi\cos\theta - \cos\phi\sin\psi)\dot\phi + (-\sin\psi\cos\phi\cos\theta - \sin\phi\cos\psi)\dot\psi$$
$$+ (-\cos\psi\cos\phi\sin\theta)\dot\theta$$

$$\frac{d}{dt}R_{12} = (\sin\psi\sin\phi\cos\theta - \cos\phi\cos\psi)\dot\phi + (-\cos\psi\cos\phi\cos\theta + \sin\phi\sin\psi)\dot\psi$$
$$+ (\sin\psi\cos\phi\sin\theta)\dot\theta$$

$$\frac{d}{dt}R_{13} = (-\sin\phi\sin\theta)\dot\phi + (\cos\phi\cos\theta)\dot\theta$$

$$\frac{d}{dt}R_{21} = (\cos\psi\cos\phi\cos\theta - \sin\phi\sin\psi)\dot\phi + (-\sin\psi\sin\phi\cos\theta + \cos\phi\cos\psi)\dot\psi$$
$$+ (-\cos\psi\sin\phi\sin\theta)\dot\theta$$

$$\frac{d}{dt}R_{22} = (-\sin\psi\cos\phi\cos\theta - \sin\phi\cos\psi)\dot\phi + (-\cos\psi\sin\phi\cos\theta - \cos\phi\sin\psi)\dot\psi$$
$$+ (\sin\psi\sin\phi\sin\theta)\dot\theta$$

$$\frac{d}{dt}R_{23} = (\cos\phi\sin\theta)\dot\phi + (\sin\phi\cos\theta)\dot\theta$$

$$\frac{d}{dt}R_{31} = (\sin\theta\sin\psi)\dot\psi - (\cos\theta\cos\psi)\dot\theta$$

$$\frac{d}{dt}R_{32} = (\sin\theta\cos\psi)\dot\psi + (\cos\theta\sin\psi)\dot\theta$$

$$\frac{d}{dt}R_{33} = -\sin\theta\,\dot\theta$$

$$w = (C\phi S_\theta\dot\psi - S\phi\dot\theta)\vec{i} + (S\phi S_\theta\dot\psi + C\phi\dot\theta)\vec{j} + (\dot\phi + C_\theta\dot\psi)\vec{k}$$

$-a_{12}$

$-a_{23}$ $\qquad$ a_{13}

$$S(w) = \begin{bmatrix} 0 & -\dot\phi - C_\theta\dot\psi & S\phi S_\theta\dot\psi + C\phi\dot\theta \\ \dot\phi + C_\theta\dot\psi & 0 & -C\phi S_\theta\dot\psi + S\phi\dot\theta \\ -S\phi S_\theta\dot\psi - C\phi\dot\theta & C\phi S_\theta\dot\psi - S\phi\dot\theta & 0 \end{bmatrix}$$

— serial chain — closed chain

$$S(\omega)R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$R_{11} = (\dot\phi + C_\theta \dot\psi)(-S_\psi C_\phi C_\theta - S_\phi C_\psi) + (C_\phi S_\theta)(-S_\phi S_\theta \dot\psi - C_\phi \dot\theta)$$

Match with $\frac{d}{dt} R_{11}$ ✓

$$R_{12} = (-\dot\phi - C_\theta \dot\psi)(C_\psi C_\phi C_\theta - S_\phi S_\psi) + (C_\phi S_\theta \dot\psi - S_\phi \dot\theta)(C_\phi S_\theta)$$

match with $\frac{d}{dt} R_{12}$ ✓

$$R_{13} = (S_\phi C_\theta \dot\psi + C_\phi \dot\theta)(C_\psi C_\phi C_\theta - S_\psi S_\phi) + (-C_\phi S_\theta \dot\psi + S_\phi \dot\theta)(-S_\psi C_\phi C_\theta - S_\phi C$$

Match with $\frac{d}{dt} R_{13}$ ✓

$$R_{21} = (\dot\phi + C_\theta \dot\psi)(-S_\psi C_\phi C_\theta - S_\phi C_\psi) + (S_\phi S_\theta)(-S_\phi S_\theta \dot\psi - C_\phi \dot\theta)$$

Match with $\frac{d}{dt} R_{21}$ ✓

$$R_{22} = (-\dot\phi - C_\theta \dot\psi)(C_\psi S_\phi C_\theta + C_\phi S_\psi) + (C_\phi S_\theta \dot\psi - S_\phi \dot\theta)(S_\phi S_\theta)$$

Match with $\frac{d}{dt} R_{22}$ ✓

$$R_{23} = (S_\phi S_\theta \dot\psi + C_\phi \dot\theta)(C_\psi S_\phi C_\theta + C_\phi S_\psi) + (-C_\phi S_\theta \dot\psi + S_\phi \dot\theta)(-S_\psi S_\phi C_\theta$$

Match with $\frac{d}{dt} R_{23}$ ✓

$$R_{31} = (\dot\phi + C_\theta \dot\psi)(S_\theta S_\psi) + (-S_\phi S_\theta \dot\psi - C_\phi \dot\theta)(C_\theta)$$

match with $\frac{d}{dt} R_{31}$ ✓

$$R_{32} = (\dot\phi - C_\theta \dot\psi)(-S_\theta C_\psi) + (-C_\phi S_\theta \dot\psi + S_\phi \dot\theta)(C_\theta)$$

match with $\frac{d}{dt} R_{32}$ ✓

$$R_{33} = (S_\phi S_\theta \dot\psi + C_\phi \dot\theta)(-S_\theta C_\psi) + (-C_\phi S_\theta \dot\psi + S_\phi \dot\theta)(S_\theta S_\psi)$$

Match with $\frac{d}{dt} R_{33}$ ✓

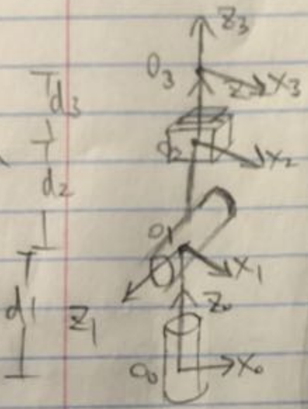
Therefore $\frac{d}{dt} R = S(\omega) R$!

4-15.

$$H_1^0 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad V_1(t) = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$V^0 = H_1^0 V_1 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

4-18.



LINK	a	α	d	θ
1	0	$\frac{\pi}{2}$	d_1	θ_1
2	0	$-\frac{\pi}{2}$	0	θ_2
3	0	0	d_3	0

$$J_{11} = \begin{bmatrix} z_0 \times (O_6) & z_1 \times (O_6 - O_1) & z_2 \end{bmatrix}$$

$$z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad H_1^0 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2^1 = \begin{bmatrix} C_{\theta_2} & 0 & -S_{\theta_2} & 0 \\ S_{\theta_2} & 0 & C_{\theta_2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0 H_2^1 = \begin{bmatrix} C_{\theta_1}C_{\theta_2} & -S_{\theta_1} & -C_{\theta_1}S_{\theta_2} & 0 \\ S_{\theta_1}C_{\theta_2} & C_{\theta_1} & -S_{\theta_1}S_{\theta_2} & 0 \\ S_{\theta_2} & 0 & C_{\theta_2} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_3^0 = H_2^0 H_3^2 = \begin{bmatrix} C_{\theta_1}C_{\theta_2} & -S_{\theta_1} & -C_{\theta_1}S_{\theta_2} & -C_{\theta_1}S_{\theta_2}d_3 \\ S_{\theta_1}C_{\theta_2} & C_{\theta_1} & -S_{\theta_1}S_{\theta_2} & -S_{\theta_1}S_{\theta_2}d_3 \\ S_{\theta_2} & 0 & C_{\theta_2} & C_{\theta_2}d_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore O_6 = O_3 = \begin{bmatrix} -C_{\theta_1}S_{\theta_2}d_3 \\ -S_{\theta_1}S_{\theta_2}d_3 \\ C_{\theta_2}d_3 + d_1 \end{bmatrix} \quad O_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix} \quad z_1^0 = \begin{bmatrix} S_{\theta_1} \\ -C_{\theta_1} \\ 0 \end{bmatrix} \quad z_2^0 = \begin{bmatrix} -C_{\theta_1} S_{\theta_2} \\ -S_{\theta_1} S_{\theta_2} \\ C_{\theta_2} \end{bmatrix}$$

$$z_0 \times (O_6) = \begin{array}{ccc} i & j & k \\ 0 & 0 & 1 \\ -C_{\theta_1} S_{\theta_2} d_3 & -S_{\theta_1} S_{\theta_2} d_3 & C_{\theta_2} d_3 + d_1 \end{array}$$

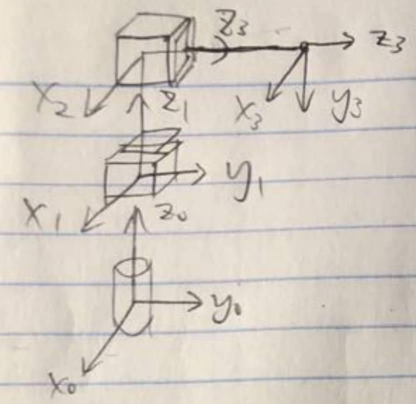
$$= \begin{bmatrix} S_{\theta_1} S_{\theta_2} d_3 \\ -C_{\theta_1} S_{\theta_2} d_3 \\ 0 \end{bmatrix}$$

$$z_1 \times (O_6 - O_1) = \begin{array}{ccc} i & j & k \\ S_{\theta_1} & -C_{\theta_1} & 0 \\ -C_{\theta_1} S_{\theta_2} d_3 & -S_{\theta_1} S_{\theta_2} d_3 & C_{\theta_2} d_3 \end{array}$$

$$= \begin{bmatrix} -C_{\theta_1} C_{\theta_2} d_3 \\ -S_{\theta_1} C_{\theta_2} d_3 \\ -S_{\theta_1}^2 S_{\theta_2} d_3 - C_{\theta_1}^2 S_{\theta_2} d_3 \end{bmatrix} = \begin{bmatrix} -C_{\theta_1} C_{\theta_2} d_3 \\ -S_{\theta_1} C_{\theta_2} d_3 \\ -S_{\theta_2} d_3 \end{bmatrix}$$

$$J_{11} = \begin{bmatrix} S_{\theta_1} S_{\theta_2} d_3 & -C_{\theta_1} C_{\theta_2} d_3 & -C_{\theta_1} S_{\theta_2} \\ -C_{\theta_1} S_{\theta_2} d_3 & -S_{\theta_1} C_{\theta_2} d_3 & -S_{\theta_1} S_{\theta_2} \\ 0 & -S_{\theta_2} d_3 & C_{\theta_2} \end{bmatrix}$$

4-20.



LINK	a	α	d	θ
1	0	0	d_1	θ_1
2	0	$-\frac{\pi}{2}$	d_2	0
3	0	0	d_3	0

$$H_1^0 = \begin{bmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & 0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_2^0 = H_1^0 H_2^1 = \begin{bmatrix} C_{\theta_1} & 0 & -S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & C_{\theta_1} & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_2^0 H_3^2 = \begin{bmatrix} C_1 & 0 & -S_1 & -S_1 d_3 \\ S_1 & 0 & C_1 & C_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} Z_0 \times O_3 & Z_1 & Z_2 \\ Z_0^0 & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$Z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad O_3 = \begin{bmatrix} -S_1 d_3 \\ C_1 d_3 \\ d_1 + d_2 \end{bmatrix} \quad Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad Z_2 = \begin{bmatrix} -S_{\theta_1} \\ C_{\theta_1} \\ 0 \end{bmatrix}$$

$$Z_0^0 \times O_3^0 = \begin{matrix} i & j & k \\ 0 & 0 & 1 \\ -S_1 d_3 & C_1 d_3 & d_1 + d_2 \end{matrix}$$

$$= \begin{bmatrix} -C_1 d_3 \\ -S_1 d_3 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -c_1 d_3 & 0 & -s_{\theta_1} \\ -s_1 d_3 & 0 & c_{\theta_1} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\det J_{11} = \det \begin{bmatrix} -c_1 d_3 & 0 & -s_1 \\ -s_1 d_3 & 0 & c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= c_1^2 d_3 + s_1^2 d_3 = d_3$$

$\det J_{11} = 0 \Rightarrow$ singular

$\Rightarrow \underline{d_3 = 0}$

$\Rightarrow$ If the third prismatic joint does not move, then only a z-axis movement can be achieve but lose x-y axis movement