7-3 Find the moments of inertia and cross products of inertia of a uniform rectangular solid of sides a, b, c with respect to a coordinate system with origin at the one corner and axes along the edges of the solid.

7-4 Given the inertia matrix $D(q)$ defined by Equation (7.83) show that $\det D(q) \neq 0$ for all q.

7-5 Show that the inertia matrix $D(q)$ for an n-link robot is always positive definite.

7-6 Verify the expression (7.59) that was used to derive the Christoffel symbols.

7-7 Consider a 3-link cartesian manipulator,

(a) Compute the inertia tensor J_i for each link $i = 1, 2, 3$ assuming that the links are uniform rectangular solids of length 1, width $\frac{1}{4}$, and height $\frac{1}{4}$, and mass 1.

(b) Compute the 3×3 inertia matrix $D(q)$ for this manipulator.

(c) Show that the Christoffel symbols c_{ijk} are all zero for this robot. Interpret the meaning of this for the dynamic equations of motion.

(d) Derive the equations of motion in matrix form:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u$$

7-8 Derive the Euler-Lagrange equations for the planar RP robot in Figure 3.25.

7-9 Derive the Euler-Lagrange equations for the planar RPR robot in Figure 3.33.

7-10 Derive the Euler-Lagrange equations of motion for the three-link RRR robot of Figure 3.32. Explore the use of symbolic software, such as Maple or Mathematica, for this problem. See, for example, the *Robotica* package [96].

7-11 For each of the robots above, define a parameter vector, Θ, compute the regressor, $Y(q, \dot{q}, \ddot{q})$ and express the equations of motion as

$$Y(q, \dot{q}, \ddot{q})\Theta = \tau \tag{7.179}$$

7-12 Recall for a particle with kinetic energy $K = \frac{1}{2}m\dot{x}^2$, the **momentum** is defined as

$$p = m\dot{x} = \frac{dK}{d\dot{x}}$$

Therefore, for a mechanical system with generalized coordinates $q_1, \ldots, q_n$, we define the **generalized momentum** p_k as

$$p_k = \frac{\partial L}{\partial \dot{q}_k}$$

where L is the Lagrangian of the system. With $K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$ and $L = K - V$ prove that

$$\sum_{k=1}^{n} \dot{q}_k p_k = 2K$$

7-13 There is another formulation of the equations of motion of a mechanical system that is useful, the so-called **Hamiltonian** formulation. Define the Hamiltonian function H by

$$H = \sum_{k-1}^{n} \dot{q}_k p_k - L$$

(a) Show that $H = K + V$.

(b) Using the Euler-Lagrange equations, derive Hamilton's equations

$$\begin{aligned} \dot{q}_k &= \frac{\partial H}{\partial p_k} \\ \dot{p}_k &= -\frac{\partial H}{\partial q_k} + \tau_k \end{aligned}$$

where τ_k is the input generalized force.

(c) For two-link manipulator of Figure 7.8 compute Hamiltonian equations in matrix form. Note that Hamilton's equations are a system of first order differential equations as opposed to a second order system given by Lagrange's equations.

7-14 Given the Hamiltonian H for a rigid robot, show that

$$\frac{dH}{dt} = \dot{q}^T \tau$$

where τ is the external force applied at the joints. What are the units of $\frac{dH}{dt}$?