3-15 Add a spherical wrist to the three-link cylindrical arm of Problem 3-13 and write the complete inverse kinematics solution.

3-16 Add a spherical wrist to the Cartesian manipulator of Problem 3-14 and write the complete inverse kinematics solution.

3-17 Write a computer program to compute the inverse kinematic equations for the elbow manipulator using Equations (3.64)–(3.69). Include procedures for identifying singular configurations and choosing a particular solution when the configuration is not singular. Test your routine for various special cases, including singular configurations.

3-18 The Stanford manipulator of Example 3.5 has a spherical wrist. Given a desired position o and orientation R of the end effector,

1. Compute the desired coordinates of the wrist center o_c^0.
2. Solve the inverse position kinematics, that is, find values of the first three joint variables that will place the wrist center at o_c. Is the solution unique? How many solutions did you find?
3. Compute the rotation matrix R_3^0. Solve the inverse orientation problem for this manipulator by finding a set of Euler angles corresponding to R_6^3 given by Equation (3.52).

3-19 Repeat Problem 3-18 for the PUMA 260 manipulator of Problem 3-10, which also has a spherical wrist. How many total solutions did you find?

3-20 Find all other solutions to the inverse kinematics of the elbow manipulator of Example 3.9.

3-21 Modify the solutions θ_1 and θ_2 for the spherical manipulator given by Equations (3.47) and (3.49) for the case of a shoulder offset.

NOTES AND REFERENCES

The Denavit-Hartenberg convention for assigning coordinate frames was introduced in the fifties, and is described in [57] and [27]. Since then, many articles have been written on the topics of forward and inverse kinematics. Seminal articles that deal with forward kinematics include [19], [29], [74], [75], [103], [57], and [138]. Inverse kinematics problems are considered in [6], [45], [53], [75], [76], [103], [105], [113], and [134]. In the late seventies and

4-9 Show that $S^3(k) = -S(k)$. Use this and Problem 4-8 to verify Equation (4.18).

4-10 Given any square matrix A, the exponential of A is a matrix defined as

$$e^A = I + A + \frac{1}{2}A^2 + \frac{1}{3!}A^3 + \cdot$$

Given $S \in so(3)$ show that $e^S \in SO(3)$.

Use the acts that $e^A e^B = e^{A+B}$ provided that A and B commute, that is, $AB = BA$, and the fact that $\det(e^A) = e^{Tr(A)}$.

4-11 Show that $R_{k,\theta} = e^{S(k)\theta}$ for k a unit vector.

Hint: Use the series expansion for the matrix exponential together with Problems 4-8 and 4-9. Alternatively use the fact that $R_{k,\theta}$ satisfies the differential equation

$$\frac{dR}{d\theta} = S(k)R.$$

4-12 Use Problem 4-11 to show the converse of Problem 4-10, that is, if $R \in SO(3)$ then there exists $S \in so(3)$ such that $R = e^S$.

4-13 Given the Euler angle transformation

$$R = R_{z,\psi}R_{y,\theta}R_{z,\phi}$$

show that $\frac{d}{dt}R = S(\omega)R$ where

$$\omega = \{c_\psi s_\theta \dot{\phi} - s_\psi \dot{\theta}\}i + \{s_\psi s_\theta \dot{\phi} + c_\psi \dot{\theta}\}j + \{\dot{\psi} + c_\theta \dot{\phi}\}k$$

The components of i, j, k, respectively, are called the **nutation**, **spin**, and **precession.**

4-14 Repeat Problem 4-13 for the Roll-Pitch-Yaw transformation. In other words, find an explicit expression for ω such that $\frac{d}{dt}R = S(\omega)R$, where R is given by Equation (2.38).

4-15 Two frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogeneous transformation

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity $v_1(t) = [3, 1, 0]^T$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?

4-16 For the three-link planar manipulator of Example 4.6, compute the vector o_c and derive the manipulator Jacobian matrix.

4-17 Compute the Jacobian J_{11} for the 3-link elbow manipulator of Example 4.9 and show that it agrees with Equation (4.98). Show that the determinant of this matrix agrees with Equation (4.99).

4-18 Compute the Jacobian J_{11} for the three-link spherical manipulator of Example 4.10.

4-19 Use Equation (4.102) to show that the singularities of the SCARA manipulator are given by Equation (4.104).

4-20 Find the 6×3 Jacobian for the three links of the cylindrical manipulator of Figure 3.7. Find the singular configurations for this arm.

4-21 Repeat Problem 4-20 for the Cartesian manipulator of Figure 3.28.

4-22 Complete the derivation of the Jacobian for the Stanford manipulator from Example 4.7.

4-23 Verify Equation (4.81) by direct computation.

4-24 Show that $B(\alpha)$ given by Equation (4.87) is invertible provided $s_\theta \neq 0$.

4-25 Suppose that $\dot{q}$ is a solution to Equation (4.110) for $m < n$.

1. Show that $\dot{q} + (I - J^+J)b$ is also a solution to Equation (4.110) for any $b \in \mathbb{R}^n$.
2. Show that $b = 0$ gives the solution that minimizes the resulting joint velocities.

4-26 Verify Equation (4.114).

4-27 Verify Equation (4.120).

4-28 Verify Equation (4.121).