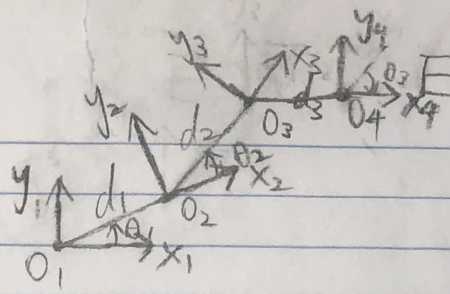
ECE 470 HW 2

3-2



LINK	a_i	α_i	d_i	θ_i
1	d_1	0	0	θ_1
2	d_2	0	0	θ_2
3	d_3	0	0	$-\theta_3$

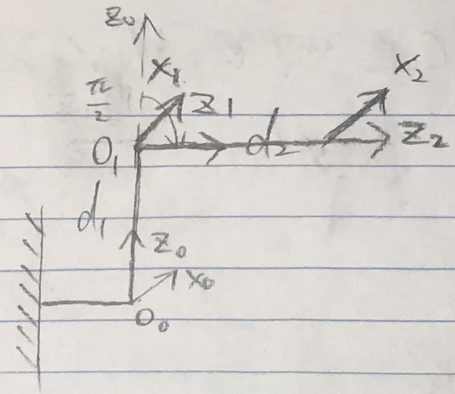
$$T_2^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & d_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & d_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & d_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & d_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 & d_3\cos\theta_3 \\ -\sin\theta_3 & \cos\theta_3 & 0 & -d_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^1 = T_2^1 \, T_3^2 \, T_4^3$$

3-3



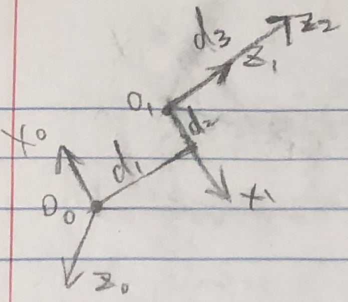
LINK	a_i	α_i	d_i	θ_i
1	0	$\frac{\pi}{2}$	d_1	0
2	0	0	d_2	0

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = T_1^0 T_2^1$$

3-4



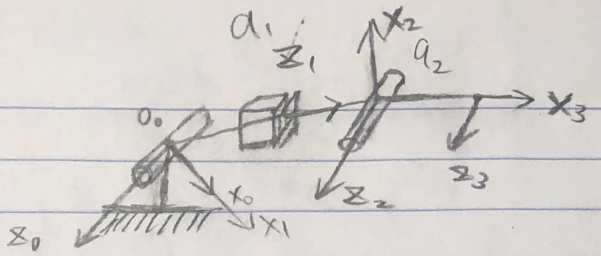
LINK	a_i	α_i	d_i	θ_i
1	d_2	$\frac{\pi}{2}$	d_1	π
2	0	0	d_3	0

$$T_1^0 = \begin{bmatrix} \cos\pi & -\sin\pi\cos\frac{\pi}{2} & \sin\pi\sin\frac{\pi}{2} & d_2\cos\pi \\ \sin\pi & \cos\pi\cos\frac{\pi}{2} & -\cos\pi\sin\frac{\pi}{2} & d_2\sin\pi \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = T_1^0 T_2^1$$

3-5



LINK	a_i	α_i	d_i	θ_i
1	0	$-\frac{\pi}{2}$	0	0
2	0	$-\frac{\pi}{2}$	a_1	π
3	a_2	0	0	$-\frac{\pi}{2}$

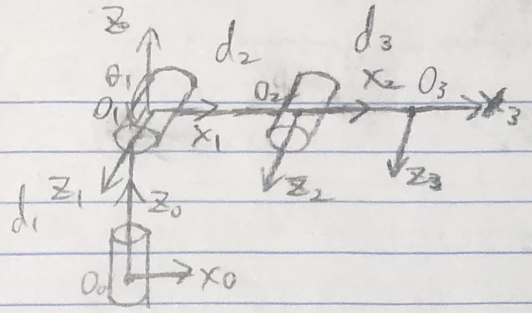
$$H_1^0 = \begin{bmatrix} C_0 & -S_0 C_{-\frac{\pi}{2}} & S_0 S_{-\frac{\pi}{2}} & 0 \\ S_0 & C_0 C_{-\frac{\pi}{2}} & -C_0 S_{-\frac{\pi}{2}} & 0 \\ 0 & S_{-\frac{\pi}{2}} & C_{-\frac{\pi}{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} C_\pi & -S_\pi C_{-\frac{\pi}{2}} & S_\pi S_{-\frac{\pi}{2}} & 0 \\ S_\pi & C_\pi C_{-\frac{\pi}{2}} & -C_\pi S_{-\frac{\pi}{2}} & 0 \\ 0 & S_{-\frac{\pi}{2}} & C_{-\frac{\pi}{2}} & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} C_{-\frac{\pi}{2}} & -S_{-\frac{\pi}{2}} C_0 & S_{-\frac{\pi}{2}} S_0 & a_2 C_{-\frac{\pi}{2}} \\ S_{-\frac{\pi}{2}} & C_{-\frac{\pi}{2}} C_0 & -C_{-\frac{\pi}{2}} S_0 & a_2 S_{-\frac{\pi}{2}} \\ 0 & S_0 & C_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

3-6



LINK	a_i	α_i	d_i	θ_i
1	0	θ_1	d_1	0
2	d_2	0	0	0
3	d_3	0	0	0

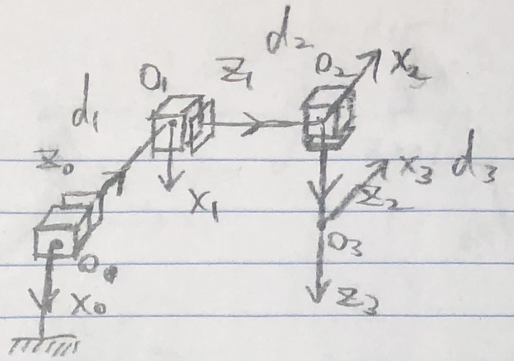
$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\theta_1} & -S_{\theta_1} & 0 \\ 0 & S_{\theta_1} & C_{\theta_1} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

3-7



LINK	a_i	α_i	d_i	θ_i
1	0	$\frac{\pi}{2}$	d_1	0
2	0	$\frac{\pi}{2}$	d_2	$\frac{\pi}{2}$
3	0	0	d_3	0

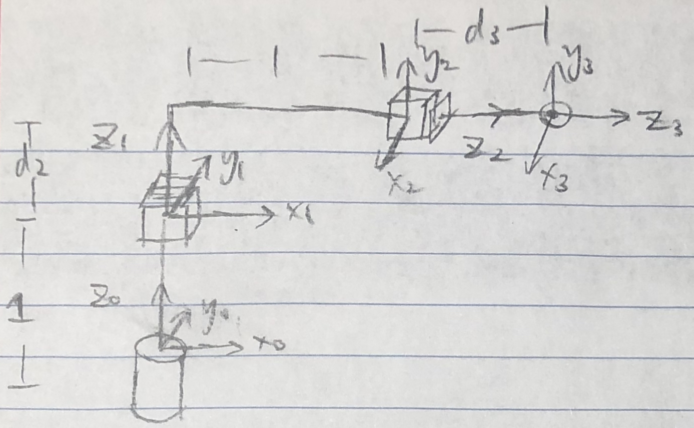
$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

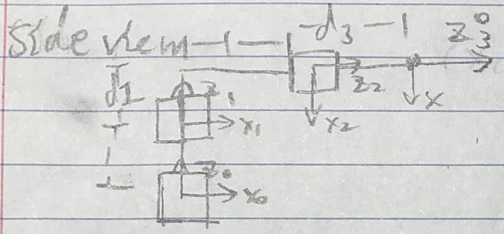
$$H_2^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = {}^0_3 = H_1^0 H_2^1 H_3^2$$

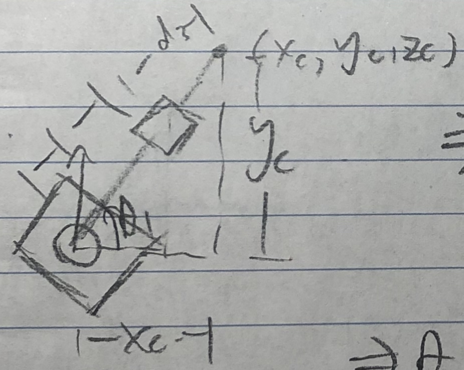
3-13





$\Rightarrow d_2 = z_c - 1$

Top view



$\Rightarrow d_3 = \sqrt{x_c^2 + y_c^2} - 1$

$\Rightarrow \theta_1 = \text{atan2}(y_c, x_c)$