UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE & ENGINEERING

FINAL EXAMINATION, April, 2020

ECE470F - ROBOT MODELING AND CONTROL

Examiner: Prof. L. Scardovi

FAMILY NAME ______________________________

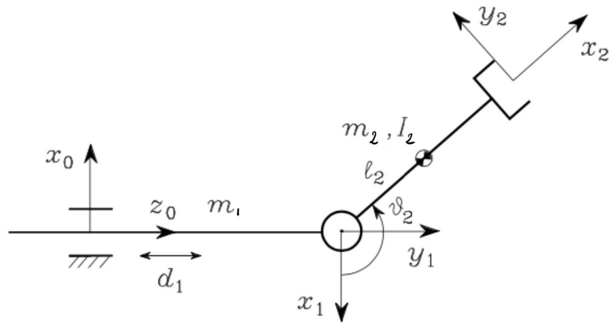
GIVEN NAME(S) ______________________________

STUDENT NUMBER ______________________________

Instructions:
- This is an open book exam.
- Time to enter your answers in the Quercus questionnaire: 3hrs.
- Time to upload your paperwork to Quercus: 4 hrs.
- There are **three problems** in this exam.

Problem	**Mark**
1	/20
2	/12
3	/18
Total:	/50

1. Consider the planar arm below.



Let m_1 and m_2 be the masses of the two links and l_2 the distance of the centre of mass of link 2 from the second joint axis. Let I_2 be the moment of inertia of link 2, calculated with respect to the axis of joint 2 centered at the center of mass. Choose as generalized coordinates $q_1 = d_1$ and $q_2 = \theta_2$. The robot is subject to the gravity force directed downwards.

(a) Find the dynamic model of the arm. Express the model in the form $D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$, where $\tau = [\tau_1, \tau_2]^T$ and τ_1, τ_2 are the force and torque applied by the motors to joints 1 and 2 respectively.

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Answers:

i. $D(q) = \begin{bmatrix} m_1 + m_2 & m_2 l_2 \cos(\theta_2) \\ m_2 l_2 \cos(\theta_2) & I_2 + m_2 l_2^2 \end{bmatrix}$, $C(q, \dot{q}) = \begin{bmatrix} 0 & -m_2 l_2 \sin\theta_2 \dot{\theta}_2 \\ m_2 l_2 \sin\theta_2 \dot{\theta}_2 & 0 \end{bmatrix}$,

$g(q) = \begin{bmatrix} 0 \\ m_2 l_2 g \sin(\theta_2) \end{bmatrix}$.

ii. $D(q) = \begin{bmatrix} m_1 + m_2 & m_2 l_2 \cos(\theta_2) \\ m_2 l_2 \cos(\theta_2) & I_2 + m_2 l_2^2 \end{bmatrix}$, $C(q, \dot{q}) = \begin{bmatrix} 0 & -m_2 l_2 \sin\theta_2 \dot{\theta}_2 \\ 0 & 0 \end{bmatrix}$, $g(q) = \begin{bmatrix} 0 \\ -m_2 l_2 g \sin(\theta_2) \end{bmatrix}$.

iii. $D(q) = \begin{bmatrix} m_1 + m_2 & m_2 l_2 \cos(\theta_2) \\ m_2 l_2 \cos(\theta_2) & I_2 + m_2 l_2^2 \end{bmatrix}$, $C(q, \dot{q}) = \begin{bmatrix} 0 & -m_2 l_2 \sin\theta_2 \dot{\theta}_2 \\ 0 & 0 \end{bmatrix}$, $g(q) = \begin{bmatrix} 0 \\ m_2 l_2 g \sin(\theta_2) \end{bmatrix}$.

iv. None of the above

v. I did not answer

(b) By using Feedback Linearization, design a controller making $q(t)$ asymptotically tend to the reference signal $q_r(t) = [0, \sin(t)]^T$. Design scalar controller gains K_p (proportional) and K_d (derivative) to achieve a rate of decay of the tracking error of at least e^{-5t}.

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Choose a range that contains the values of your gains:

i. $K_p < 0$, $K_d > 10$

ii. $K_p > 10$, $K_d < 10$

iii. $K_p > 0$, $K_d > 10$

iv. None of the above

v. I did not answer

(c) Rewrite the model (derived in the first question) in the form $Y(q, \dot{q}, \ddot{q})p = \tau$, where p is a suitable vector of parameters of the arm.

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Answers:

i. $Y(q, \dot{q}, \ddot{q}) = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 \cos q_2 + \ddot{q}_2 g \sin q_2 & 0 \\ 0 & \ddot{q}_2 \cos q_2 + \ddot{q}_2 \sin q_2 & \ddot{q}_2 \end{bmatrix}$

ii. $Y(q, \dot{q}, \ddot{q}) = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 \cos q_2 - \dot{q}_2^2 \sin q_2 & 0 \\ 0 & \ddot{q}_1 \cos q_2 + g \sin q_2 & \ddot{q}_2 \end{bmatrix}$

iii. $Y(q, \dot{q}, \ddot{q}) = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 \cos q_2 - \dot{q}_2 \sin q_2 & 0 \\ 0 & \ddot{q}_1 \cos q_2 + g \sin q_2 & \ddot{q}_2 \end{bmatrix}$

iv. None of the above

v. I did not answer

(d) Assume that the parameters are unknown. Design an adaptive controller making (q_1, q_2) track the reference signal $q_r(t) = [0, \sin(t)]^T$. Write the expression of your controller in full detail.

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The matrix $\bar{Y}(q, \dot{q}, a, v)$ derived in your adaptive controller is:

i. $\bar{Y}(q, \dot{q}, a, v) = \begin{bmatrix} a_1 & a_2 \cos q_2 + a_2 g \sin q_2 & 0 \\ 0 & a_2 \cos q_2 + a_2 \sin q_2 & a_2 \end{bmatrix}$

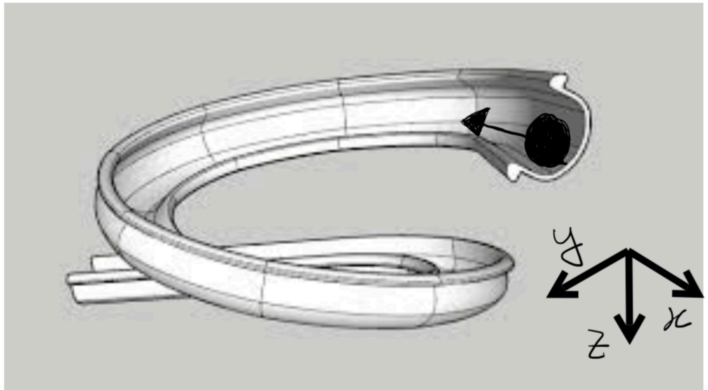
ii. $\bar{Y}(q, \dot{q}, a, v) = \begin{bmatrix} a_1 & a_2 \cos q_2 - \dot{v}_2^2 \sin q_2 & 0 \\ 0 & a_1 \cos q_2 + g \sin q_2 & a_2 \end{bmatrix}$

iii. $\bar{Y}(q, \dot{q}, a, v) = \begin{bmatrix} a_1 & a_2 \cos q_2 - v_2 \dot{q}_2 \sin q_2 & 0 \\ 0 & a_1 \cos q_2 + g \sin q_2 & a_2 \end{bmatrix}$

iv. None of the above

v. I did not answer

2. In this problem you are asked to model the motion of an object on a slide.



We represent the object as a point mass of 1Kg. The point is constrained to slide on the curve $c(z) = (f(z), h(z), z)$ representing the profile of the slide. The object is subject to the gravity force (directed along the z-axis).

(a) Using $q = z$ as generalized coordinate, find the Lagrangian function $\mathcal{L}(q, \dot{q})$.

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Answers:

i. $\mathcal{L} = \frac{1}{2}m\dot{z}^2 + mgz$

ii. $\mathcal{L} = \frac{1}{2}m\frac{\partial f}{\partial z}(z)\dot{z}^2$

iii. $\mathcal{L} = \frac{1}{2}m\left(\left(\frac{\partial f}{\partial z}(z)\right)^2 + \left(\frac{\partial h}{\partial z}(z)\right)^2\right)\dot{z}^2$

iv. None of the above

v. I did not answer

(b) Write the equations of motion in the form $D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = 0$.

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Answers:

i. $D(q) = m\left(\left(\frac{\partial f}{\partial z}(z)\right)^2 + \left(\frac{\partial h}{\partial z}(z)\right)^2\right)$, $C(q,\dot{q}) = \left(\frac{\partial f}{\partial z}(z)\frac{\partial^2 f}{\partial^2 z}(z) + \frac{\partial h}{\partial z}(z)\frac{\partial^2 h}{\partial^2 z}(z)\right)\dot{z}$, $g(q) = -mg$.

ii. $D(q) = m\left(\left(\frac{\partial f}{\partial z}(z)\right)^2 + \left(\frac{\partial h}{\partial z}(z)\right)^2 + 1\right)$, $C(q,\dot{q}) = \left(\frac{\partial f}{\partial z}(z)\frac{\partial^2 f}{\partial^2 z}(z) + \frac{\partial h}{\partial z}(z)\frac{\partial^2 h}{\partial^2 z}(z)\right)\dot{z}$, $g(q) = -mg$.

iii. $D(q) = m\left(\left(\frac{\partial f}{\partial z}(z)\right)^2 + \left(\frac{\partial h}{\partial z}(z)\right)^2 + 1\right)$, $C(q,\dot{q}) = \left(\frac{\partial f}{\partial z}(z)\frac{\partial^2 f}{\partial^2 z}(z) - \frac{\partial h}{\partial z}(z)\frac{\partial^2 h}{\partial^2 z}(z)\right)\dot{z}$, $g(q) = mg$.

iv. None of the above

v. I did not answer

(c) Suppose that wind imparts a force of 1 N in the direction of the y axis to the object. Modify your equations of motion to include the external force.

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(d) Assume that the slide profile is a helix defined by $c(z) = (\cos(z), \sin(z), z)$. Write the equations of motion (by taking into account the wind force introduced in the previous question).

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Answers:

(a) $\ddot{z} = g + \cos(z)$

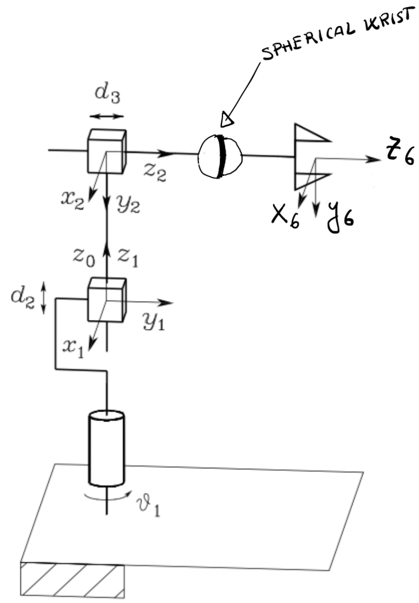
(b) $\ddot{z} = \dfrac{g + \cos(z)}{2}$

(c) $\ddot{z} = g + z$

(d) None of the above

(e) I did not answer

3. Consider the robot arm in the figure below



where the DH table of the spherical wrist is

a_i	α_i	d_i	θ_i
0	$-\pi/2$	0	θ_4
0	$\pi/2$	0	θ_5
0	0	d_6	θ_6

(a) Rewrite a detailed robot scheme that includes explicitly the joints of the spherical wrist. Assign the frames, and write the complete DH table of the robot.

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The first three rows of the DH table are:

i.

a_i	α_i	d_i	θ_i
0	0	0	θ_1
0	$\pi/2$	d_2	0
0	0	d_3	0

ii.

a_i	α_i	d_i	θ_i
0	0	0	θ_1
0	$-\pi/2$	d_2	0
0	0	d_3	0

iii.

a_i	α_i	d_i	θ_i
0	$\pi/2$	0	θ_1
0	$-\pi/2$	d_2	0
0	0	d_3	0

iv. None of the above

v. I did not answer

(b) Solve the forward kinematic problem, i.e. find the homogeneous transformation H_6^0.

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The 11-component of H_6^0 (first row-first column) is

i. $c_1\left(c_4c_5c_6 - s_4s_6\right) + s_1s_5c_6$

ii. $c_1\left(c_4c_5c_6 - s_4s_6\right) - s_5c_6$

iii. $\left(c_4c_5c_6 - s_4s_6\right) + s_5c_6$

iv. None of the above

v. I did not answer

(c) Solve the inverse kinematic problem, i.e., given a generic position r_d and orientation R_d of the end effector, find the joint variables $\theta_1, d_2, d_3, \theta_4, \theta_5$, and θ_6 as functions of r_d and R_d. Find the joint variables when R_d is the identity matrix and $r_d = [2, 1, 10]^T$.

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If R_d is the identity matrix and $r_d = [2, 1, 10]^T$ a solution for the joint variables d_3 and θ_5 is:

i. $d_3 = 5; \quad \theta_5 = \pi/2$

ii. $d_3 = \sqrt{5}; \quad \theta_5 = \pi/2$

iii. $d_3 = \sqrt{5}; \quad \theta_5 = 0$

iv. None of the above

v. I did not answer