COMP 8700 Fall 2020 Take-home Exam/Assignment 3

Due: Jan 6., 11:59pm

Wen Dong #110057395

7.4 Which of the following are correct?
a. False |= True.
b. True |= False.
c. $(A \land B) \models (A \Leftrightarrow B)$.
d. $A \Leftrightarrow B \models A \lor B$.
e. $A \Leftrightarrow B \models \neg A \lor B$.
f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$.
g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$.
h. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$.
i. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$.
j. $(A \lor B) \land \neg (A \Rightarrow B)$ is satisfiable.
k. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable.
l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

Correct answers are:

a, e, g, h, j, k

7.5 Prove each of the following assertions:
a. α is valid if and only if True |= α.
Answer:
According to the definition, True |= α means in all models where True is true, then also α is true. Since True is true in all models, so α is true in all models too, so α is valid as per the definition of validity.
On the other hand, if α is valid, it is true in all worlds, so anything entails α, thus True entails α

b. For any α, False |= α.
Answer:
False |= α means, in all the worlds where False is true, α is also true, since there is no model in which False is true, then it is uncontroversial that α is true in an empty set of models, because the target model set is empty.

c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

Answer:
$\alpha \models \beta$ means in all worlds where α is true, then also β is true, so $\alpha \models \beta$ holds as α and β are both true. True => True
if sentence $(\alpha \Rightarrow \beta)$ is true, then α is False, or α is True and β is True as below:

α	β	$\alpha \models \beta$
False	False	True
True	True	True
False	True	True

In all the three cases, $\alpha \models \beta$ is true according to the definition of entailment explained above.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.
Answer:
$\alpha \equiv \beta$ means $\alpha \models \beta$ and $\beta \models \alpha$, from preceding question c, we have proved $\alpha \models \beta$ iff sentence $(\alpha \Rightarrow \beta)$ is valid, so $\alpha \equiv \beta$ means $(\alpha \Rightarrow \beta)$ and $(\beta \Rightarrow \alpha)$, i.e. $(\alpha \Leftrightarrow \beta)$

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.
Answer:
$(\alpha \wedge \neg\beta)$
$\equiv \neg(\neg\alpha \vee \beta)$ # De Morgan
$\equiv \neg(\alpha \Rightarrow \beta)$ # reverse implication elimination
$\alpha \models \beta$ means in all worlds where α is true, then also β is true, whereas sentence $\neg(\alpha \Rightarrow \beta)$ is $\neg$True thus False, so unsatisfiable given $\alpha \models \beta$
on the other hand, $\neg(\alpha \Rightarrow \beta)$ is unsatisfiable means $(\alpha \Rightarrow \beta)$ is always valid due to the definition of negation, thus $\alpha \models \beta$

7.6 Prove, or find a counterexample to, each of the following assertions:
a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$
Answer:
$\alpha \models \gamma$ or $\beta \models \gamma$
$\equiv (\alpha => \gamma) \vee (\beta => \gamma)$
$\equiv (\neg\alpha \vee \gamma) \vee (\neg\beta \vee \gamma)$
$\equiv \neg\alpha \vee \gamma \vee \neg\beta$
$\equiv \neg\alpha \vee \neg\beta \vee \gamma$
$\equiv \neg(\alpha \wedge \beta) \vee \gamma$
$\equiv (\alpha \wedge \beta) => \gamma$
So, it proves $(\alpha \wedge \beta) \models \gamma$

b. If $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.
Answer:
$\alpha \models (\beta \wedge \gamma)$

$\equiv \alpha \Rightarrow (\beta \wedge \gamma)$
$\equiv \neg\alpha \vee (\beta \wedge \gamma)$ #implication elimination
$\equiv (\neg\alpha \vee \beta) \wedge (\neg\alpha \vee \gamma)$ #distributivity of ∨ over ∧
$\equiv (\alpha \Rightarrow \beta) \wedge (\alpha \Rightarrow \gamma)$
So it proves $\alpha \models \beta$ and $\alpha \models \gamma$

c. If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).
Answer:
$\alpha \models (\beta \vee \gamma)$
$\equiv \alpha \Rightarrow (\beta \vee \gamma)$
$\equiv \neg\alpha \vee (\beta \vee \gamma)$
$\equiv (\neg\alpha \vee \beta) \vee \gamma \equiv (\alpha \Rightarrow \beta) \vee \gamma$ # equation 1
$\equiv (\neg\alpha \vee \gamma) \vee \beta \equiv (\alpha \Rightarrow \gamma) \vee \beta$ # equation 2
Then we can infer:

β	γ	α => β	α => γ
True	True	Valid as βis true	Valid as γ is true
True	False	Valid as βis true	
False	True		Valid as γ is true
False	False	Valid, as γ is false, according to equation 1	Valid, as β is false, according to equation 2

So, from the table, it proves $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.7 Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?
a. $B \vee C$.
Answer: 12 models

b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.
Answer: 15 models

c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

Answer: 0 models
$(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

$\equiv (\neg A \vee B) \wedge A \wedge \neg B \wedge C \wedge D$

$\equiv (\neg A \wedge A \wedge \neg B \wedge C \wedge D) \vee (B \wedge A \wedge \neg B \wedge C \wedge D)$

So, it’s unsatisfiable, as in the first disjunct, $\neg A \wedge A$ is there, and in the second $B \wedge \neg B$, both are conjunctions of complementary variables.

7.10 Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 (page 249).

a. Smoke ⇒ Smoke

Answer: valid, as by implication elimination it becomes following sentence, which is always true

¬Smoke ∨ Smoke

As there must be one true between a pair of complementary literals.

b. Smoke ⇒ Fire

Answer: not valid, but satisfiable

Implication elimination: ≡ ¬Smoke ∨ Fire

Answer: valid

Smoke	Fire	¬Smoke	¬Smoke ∨ Fire
T	T	F	T
F	T	T	T
F	F	T	T
T	F	F	F

c. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

Answer: not valid, but satisfiable

≡ (¬Smoke ∨ Fire) => (¬Smoke ⇒ ¬Fire) implication elimination

≡ (¬Smoke ∨ Fire) => (Smoke ∨ ¬Fire) contraposition

≡ ¬ (¬Smoke ∨ Fire) ∨(Smoke ∨ ¬Fire) implication elimination

≡ (Smoke ∧ ¬Fire) ∨(Smoke ∨ ¬Fire)

≡ (Smoke ∨Smoke ∨ ¬Fire) ∧ (¬Fire ∨ Smoke ∨ ¬Fire) distributivity of ∨ over ∧

≡ (Smoke ∨ ¬Fire) ∧ (¬Fire ∨ Smoke)

≡ (Smoke ∨ ¬Fire)

Smoke	Fire	¬Fire	Smoke ∨¬Fire
T	T	F	T
F	T	F	F
F	F	T	T
T	F	T	T

d. Smoke ∨ Fire ∨ ¬Fire

Answer: valid

≡ Smoke ∨ (Fire ∨ ¬Fire) associativity of ∨

≡ Smoke ∨ True

Which is always true.

e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Answer: valid

≡(¬ (Smoke ∧ Heat) ∨ Fire) ⇔ ((¬ Smoke ∨ Fire) ∨ (¬ Heat ∨ Fire)) implication elimination

≡(¬ Smoke ∨ ¬ Heat ∨ Fire) ⇔ ((¬ Smoke ∨ Fire) ∨ (¬ Heat ∨ Fire)) De Morgan

≡(¬ Smoke ∨ ¬ Heat ∨ Fire) ⇔ (¬ Smoke ∨ ¬ Heat ∨ Fire ∨ Fire) Associativity of ∨

≡(¬ Smoke ∨ ¬ Heat ∨ Fire) ⇔ (¬ Smoke ∨ ¬ Heat ∨ Fire)

f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)

Answer: valid

≡(¬ Smoke ∨ Fire) ⇒ (¬ (Smoke ∧ Heat) ∨ Fire) implication elimination

≡(¬ Smoke ∨ Fire) ⇒ ((¬Smoke ∨ ¬Heat) ∨ Fire) De Morgan

≡(¬ Smoke ∨ Fire) ⇒ (¬Smoke ∨ Fire ∨ ¬Heat) associativity of∨

≡ ¬ (¬ Smoke ∨ Fire) ∨ (¬Smoke ∨ Fire ∨ ¬Heat) implication elimination

≡ (Smoke ∧ ¬Fire) ∨ (¬Smoke ∨ Fire ∨ ¬Heat) De Morgan

≡ (Smoke ∨ ¬Smoke ∨ Fire ∨ ¬Heat)∧(¬Fire ∨ ¬Smoke ∨ Fire ∨ ¬Heat) distributivity

≡ (Smoke ∨ ¬Smoke ∨ Fire ∨ ¬Heat)∧(¬Fire ∨ Fire ∨ ¬Smoke ∨ ¬Heat)

≡ true ∧ true

≡ true

g. Big ∨ Dumb ∨ (Big ⇒ Dumb)

Answer: valid

≡Big ∨ Dumb ∨ (¬Big ∨ Dumb)

≡Big ∨¬Big ∨ Dumb ∨ Dumb associativity of ∨

$\equiv Big \vee \neg Big \vee Dumb$

$\equiv true$

7.12 Use resolution to prove the sentence $\neg A \wedge \neg B$ from the clauses in Exercise 7.20.

Answer:

From 7.20 we know
S1: $A \Leftrightarrow (B \vee E)$.
S4: $E \Rightarrow B$
According to Modus Ponens
$A \Leftrightarrow (B \vee E)$.
$\equiv A \Leftrightarrow (B \vee B)$ because $E \Rightarrow B$
$\equiv A \Leftrightarrow B$
$\equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$

$\equiv (\neg A \vee B) \wedge (\neg B \vee A)$

$\equiv ((\neg A \vee B) \wedge (\neg B)) \vee ((\neg A \vee B) \wedge A))$

$\equiv (\neg A \wedge \neg B) \vee (B \wedge A)$

A	B	$(\neg A \wedge \neg B)$	$(B \wedge A)$	$(\neg A \wedge \neg B) \vee (B \wedge A)$
T	T	F	T	T
T	F	F	F	F
F	T	F	F	F
F	F	T	F	T

From 3, 5,6 in 7.20, we know

S3: $C \wedge F \Rightarrow \neg B$ and S5: $B \Rightarrow F$ and S6: $B \Rightarrow C$

$\equiv (\neg C \vee \neg F \vee \neg B) \wedge (\neg B \vee F) \wedge (\neg B \vee C)$

$\equiv ((\neg C \vee \neg F \vee \neg B) \wedge (\neg B) \vee (\neg C \vee \neg F \vee \neg B) \wedge F)) \wedge (\neg B \vee C)$

$\equiv ((\neg C \wedge (\neg B) \vee \neg F \wedge (\neg B)) \vee (\neg C \wedge F \vee \neg B \wedge F)) \wedge (\neg B \vee C)$

$\equiv ((\neg C \wedge (\neg B) \vee \neg F \wedge (\neg B)) \vee (\neg C \wedge F \vee \neg B \wedge F)) \wedge (\neg B) \vee$
$((\neg C \wedge (\neg B) \vee \neg F \wedge (\neg B)) \vee (\neg C \wedge F \vee \neg B \wedge F)) \wedge C$

$\equiv ((\neg C \wedge (\neg B) \vee \neg F \wedge (\neg B)) \vee (\neg C \wedge F \wedge (\neg B) \vee \neg B \wedge F)) \vee$
$((\neg F \wedge (\neg B) \wedge C) \vee (\neg B \wedge F \wedge C))$

$\equiv (\neg B) \wedge (\neg C \vee \neg F \vee (\neg C \wedge F \vee F) \vee (\neg F \wedge C) \vee (F \wedge C))$

According to And-elimination $(\neg B)$ is True, then B is False

So, $(\neg A \land \neg B) \lor (B \land A)$

$\equiv (\neg A \land \neg B) \lor False$

$\equiv (\neg A \land \neg B)$

Proved.

7.20 Convert the following set of sentences to clausal form.

Answer:

$\equiv (A \Rightarrow (B \lor E)) \land ((B \lor E) \Rightarrow A)$

$\equiv(\neg A \lor B \lor E) \land ((\neg B \land \neg E) \lor A)$ implication elimination

$\equiv(\neg A \lor B \lor E) \land ((\neg B \lor A) \land (\neg E \lor A))$ distributivity of $\lor$ over $\land$

S2: $E \Rightarrow D$.

Answer:

$\equiv \neg E \lor D$

S3: $C \land F \Rightarrow \neg B$.

Answer:

$\equiv \neg (C \land F) \lor \neg B$

$\equiv \neg C \lor \neg F \lor \neg B$ De Morgan

S4: $E \Rightarrow B$.

$\equiv \neg E \lor B$

S5: $B \Rightarrow F$.

$\equiv \neg B \lor F$

S6: $B \Rightarrow C$

$\equiv \neg B \lor C$

Give a trace of the execution of DPLL on the conjunction of these clauses.

$(\neg A \lor B \lor E) \land (\neg B \lor A) \land (\neg E \lor A)$

$\neg E \lor D$

$\neg C \lor \neg F \lor \neg B$

$\neg E \lor B$

$\neg B \lor F$

$\neg B \vee C$

Set E to false, then simplified as:

$(\neg A \vee B) \wedge (\neg B \vee A)$

$\neg C \vee \neg F \vee \neg B$

$\neg B \vee F$

$\neg B \vee C$

Set B to false:

$\neg A$

Set A to false:

true

8.2 Consider a knowledge base containing just two sentences: P(a) and P(b). Does this knowledge base entail $\forall x\ P(x)$? Explain your answer in terms of models.

Answer: No

Suppose a model contains 3 or more elements in the domain, the relation referred to by P holds solely for a and b, but not for the other element. So, it does not entail $\forall x\ P(x)$.

8.6 Which of the following are valid (necessarily true) sentences?

a. $(\exists x\ x=x) \Rightarrow (\forall y\ \exists z\ y=z)$.

Answer:

valid, if we do not impose unique-names assumption.

b. $\forall x\ P(x) \vee \neg P(x)$.

Answer: valid.

c. $\forall x\ Smart(x) \vee (x=x)$.

Answer: valid, because (x=x) is always true.

8.9 This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate

logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3)

is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.
(i) $In(Paris \land Marseilles, France)$.
Answer: (2)
Syntactically invalid thus meaningless.

(ii) $In(Paris, France) \land In(Marseilles, France)$.
Answer: (1)

(iii) $In(Paris, France) \lor In(Marseilles, France)$.
Answer: (3)

b. There is a country that borders both Iraq and Pakistan.
(i) $\exists\, c\ Country(c) \land Border(c, Iraq) \land Border(c, Pakistan)$.
Answer: (1)

(ii) $\exists\, c\ Country(c) \Rightarrow [Border(c, Iraq) \land Border(c, Pakistan)]$.
Answer: (3)

(iii) $[\exists\, c\ Country(c)] \Rightarrow [Border(c, Iraq) \land Border(c, Pakistan)]$.
Answer: (2)

(iv) $\exists\, c\ Border(Country(c), Iraq \land Pakistan)$.
Answer: (2)

c. All countries that border Ecuador are in South America.
(i) $\forall c\ Country(c) \land Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$.
Answer: (1)

(ii) $\forall\, c\ Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$.
Answer: (1)

(iii) $\forall\, c\ [Country(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$.
Answer: (3)

(iv) $\forall c\ Country(c) \land Border(c, Ecuador) \land In(c, SouthAmerica)$.
Answer: (3)

d. No region in South America borders any region in Europe.
(i) $\neg[\exists\, c, d\ In(c, SouthAmerica) \land In(d, Europe) \land Borders(c, d)]$.
Answer: (1)

(ii) $\forall\, c, d\ [In(c, SouthAmerica) \land In(d, Europe)] \Rightarrow \neg Borders(c, d)]$.
Answer: (1)

(iii) $\neg \forall\, c\ In(c, SouthAmerica) \Rightarrow \exists d\ In(d, Europe) \land \neg Borders(c, d)$.

Answer: (3)

(iv) $\forall c\ In(c, SouthAmerica) \Rightarrow \forall d\ In(d, Europe) \Rightarrow \neg Borders(c, d)$.
Answer: (2)

e. No two adjacent countries have the same map color.
(i) $\forall x, y\ \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee$
$\neg (MapColor(x) = MapColor(y))$.
Answer: (1)

(ii) $\forall x, y\ (Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg (x = y)) \Rightarrow$
$\neg (MapColor(x) = MapColor(y))$.
Answer: (1)

(iii) $\forall x, y\ Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge$
$\neg (MapColor(x) = MapColor(y))$.
Answer: (1)

(iv) $\forall x, y\ (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow MapColor(x \_= y)$.

Answer: (2)

8.28 Consider a first-order logical knowledge base that describes worlds containing people,
songs, albums (e.g., “Meet the Beatles”) and disks (i.e., particular physical instances of CDs).
The vocabulary contains the following symbols:
CopyOf (d, a): Predicate. Disk d is a copy of album a.
Owns(p, d): Predicate. Person p owns disk d.
Sings(p, s, a): Album a includes a recording of song s sung by person p.
Wrote(p, s): Person p wrote song s.
McCartney, Gershwin, BHoliday, Joe, EleanorRigby, TheManILove, Revolver :

Constants with the obvious meanings. Express the following statements in first-order logic:
a. Gershwin wrote “The Man I Love.”
Answer: Wrote(Gershwin, TheManILove)

b. Gershwin did not write “Eleanor Rigby.”
Answer: $\neg$ Wrote(Gershwin, EleanorRigby)

c. Either Gershwin or McCartney wrote “The Man I Love.”
Answer: Wrote(Gershwin, TheManILove) $\vee$ Wrote(McCartney, TheManILove)

d. Joe has written at least one song.
$\exists$s Wrote(Joe, s)

e. Joe owns a copy of *Revolver*.
Owns(Joe, *Revolver*)

f. Every song that McCartney sings on *Revolver* was written by McCartney.
$\forall$s [Sings(McCartney, s, Revolver) => Wrote(McCartney, s)]

g. Gershwin did not write any of the songs on *Revolver*.
$\neg$ $\exists$s[Wrote(Gershwin, s) $\wedge$ $\exists$p Sings(p, s, Resolver)]

h. Every song that Gershwin wrote has been recorded on some album. (Possibly different
songs are recorded on different albums.)
$\forall$s Wrote(Gershwin, s) => $\exists$p, $\exists$a Sings(p, s, a)

i. There is a single album that contains every song that Joe has written.
$\exists$1 a $\forall$s Wrote(Joe, s) =>$\exists$p Sings(p, s, a)

j. Joe owns a copy of an album that has Billie Holiday singing “The Man I Love.”
$\exists$a $\exists$d Owns(Joe, *d*) $\wedge$ CopyOf(d, a) $\wedge$ Sings(Billie, TheManILove, a)

k. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)
$\forall$a $\exists$s Sings(McCartney,s,a) => $\exists$d CopyOf(d, a) $\wedge$ Owns(Joe, d)

l. Joe owns a copy of every album on which all the songs are sung by Billie Holiday.

$\exists$a $\exists$d CopyOf(d, a) $\wedge$ Owns(Joe, d) $\wedge$ $\forall$s Sings(Billie, s, a)

13.8 Given the full joint distribution shown in Figure 13.3, calculate the following:

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

a. **P**(toothache) .
= 0.108 + 0.012 + 0.016 + 0.064 = 0.2

b. **P**(Cavity) .
= 0.108 + 0.012 + 0.072 + 0.008 = 0.2

c. **P**(Toothache **|** cavity) .
= **P**(Toothache ∧ cavity) / P(cavity)
= (0.108 + 0.012) / 0.19
= 0.12 / 0.2
= 0.6

d. **P**(Cavity **|** toothache ∨ catch) .

= **P**(Cavity ∧ (toothache ∨ catch)) / **P** (toothache ∨ catch)

= (0.108 + 0.012 + 0.072) / 0.416

= 0.192 / 0.416

= 0.46

13.17 Show that the statement of conditional independence
P(X, Y | Z) = **P**(X **|** Z)**P**(Y **|** Z)
is equivalent to each of the statements
P(X **|** Y,Z) = **P**(X |Z) and **P**(Y **|** X,Z) = **P**(Y **|** Z) .

Answer:

P(X, Y | Z) = P(X | Z)P(Y | Z)

=> P(X, Y, Z)/P(Z) = P(X | Z) P(Y, Z)/P(Z)

=> P(X, Y, Z) = P(X | Z) P(Y, Z)

=> P(X, Y, Z) / P(Y, Z) = P(X | Z)

=> P(X, Y, Z) / P(Y, Z) = P(X | Z)

=> P(X | Y, Z) = P(X | Z)

First statement proved.

P(X, Y | Z) = P(X | Z)P(Y | Z)

=> P(X, Y, Z)/P(Z) = P(Y | Z) P(X, Z)/P(Z)

=> P(X, Y, Z) = P(Y | Z) P(X, Z)

=> P(Y, X, Z) / P(X, Z) = P(Y | Z)

=> P(Y, X, Z) / P(X, Z) = P(Y | Z)

=> P(Y | X, Z) = P(Y | Z)

Second statement proved.

Exercise 14.14 (a) (b) (c)iuj

14.14 Consider the Bayes net shown in Figure 14.23.
a. Which of the following are asserted by the network *structure*?
(i) **P**(B, I,M) = **P**(B)**P**(I)**P**(M).
(ii) **P**(J |G) = **P**(J | G, I).
(iii) **P**(M | G,B, I) = **P**(M | G,B, I, J).

Answer: (ii) and (iii) are true

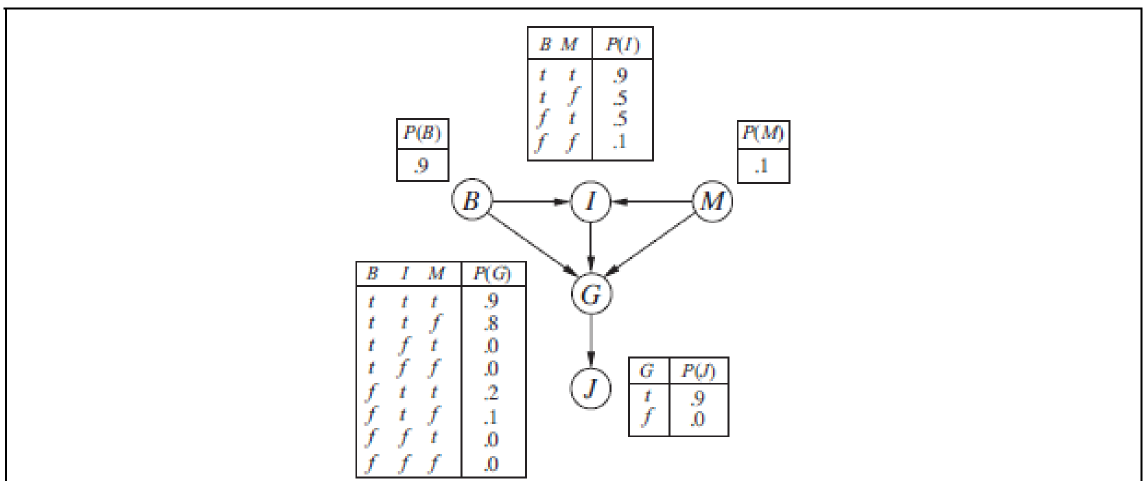

Figure 14.23 A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

b. Calculate the value of P(b, i, ¬ m, g, j).

= P(b) P(¬ m) P(i | b, ¬ m) P(g | b, ¬ m,i) P(j | g)
= 0.9 * (1- 0.1) * 0.5 * 0.8 * 0.9
= 0.2916

c. Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.

$P(j \mid b, i, m)$

$= \Sigma_{g1} P(j \mid b, i, m, g1)$

$= \Sigma_{g1} P(j, b, i, m, g1) / \Sigma_{j1, g1} P(j1, b, i, m, g1)$

$= \Sigma_{g1} P(b) P(m) P(i \mid b,m) P(g1 \mid b,m,i) P(j \mid g1)) / \Sigma_{j1, g1} P(b) P(m) P(i \mid b,m) P(g1 \mid b,m,i) P(j1 \mid g1))$

$= \Sigma_{g1} P(g1 \mid b,m,i) P(j \mid g1)) / \Sigma_{j1, g1} P(g1 \mid b,m,i) P(j1 \mid g1))$

$= (0.9 * 0.9 + (1-0.9) * 0) / 0.9 * 0.9 + (1-0.9) * 0 + 0.9 * 0.1 + 0.1 * 1$

$= 0.81$