COMP 8700 Fall 2020 Take-home Exam/Assignment 3

Due: Jan 6., 11:59pm

7.4 Which of the following are correct?

a. False $\models$ True.

b. True $\models$ False.

c. $(A \wedge B) \models (A \Leftrightarrow B)$.

d. $A \Leftrightarrow B \models A \vee B$.

e. $A \Leftrightarrow B \models \neg A \vee B$.

f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$.

g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.

h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.

i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.

j. $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.

k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

7.5 Prove each of the following assertions:

a. α is valid if and only if True $\models \alpha$.

b. For any α, False $\models \alpha$.

c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg \beta)$ is unsatisfiable.

7.6 Prove, or find a counterexample to, each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$

b. If $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.

c. If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.7 Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \vee C$.

b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.

c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

7.10 Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 (page 249).

a. $Smoke \Rightarrow Smoke$

b. $Smoke \Rightarrow Fire$

c. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

d. $Smoke \lor Fire \lor \neg Fire$

e. $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

f. $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \land Heat) \Rightarrow Fire)$

g. $Big \lor Dumb \lor (Big \Rightarrow Dumb)$

7.12 Use resolution to prove the sentence $\neg A \land \neg B$ from the clauses in Exercise 7.20.

7.20 Convert the following set of sentences to clausal form.

S1: $A \Leftrightarrow (B \lor E)$.

S2: $E \Rightarrow D$.

S3: $C \land F \Rightarrow \neg B$.

S4: $E \Rightarrow B$.

S5: $B \Rightarrow F$.

S6: $B \Rightarrow C$

Give a trace of the execution of DPLL on the conjunction of these clauses.

8.2 Consider a knowledge base containing just two sentences: $P(a)$ and $P(b)$. Does this knowledge base entail $\forall x\ P(x)$? Explain your answer in terms of models.

8.6 Which of the following are valid (necessarily true) sentences?

a. $(\exists x\ x=x) \Rightarrow (\forall y\ \exists z\ y=z)$.

b. $\forall x\ P(x) \lor \neg P(x)$.

c. $\forall x\ Smart(x) \lor (x=x)$.

8.9 This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and Country(x), whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.

(i) $In(Paris \land Marseilles, France)$.

(ii) $In(Paris, France) \land In(Marseilles, France)$.

(iii) $In(Paris, France) \lor In(Marseilles, France)$.

b. There is a country that borders both Iraq and Pakistan.

(i) $\exists\ c\ Country(c) \land Border(c, Iraq) \land Border(c, Pakistan)$.

(ii) $\exists\ c\ Country(c) \Rightarrow [Border(c, Iraq) \land Border(c, Pakistan)]$.

(iii) $[\exists\ c\ Country(c)] \Rightarrow [Border(c, Iraq) \land Border(c, Pakistan)]$.

(iv) $\exists\ c\ Border(Country(c), Iraq \land Pakistan)$.

c. All countries that border Ecuador are in South America.

(i) $\forall c\ Country(c) \land Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$.

(ii) $\forall\ c\ Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$.

(iii) $\forall\ c\ [Country(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$.

(iv) $\forall c\ Country(c) \land Border(c, Ecuador) \land In(c, SouthAmerica)$.

d. No region in South America borders any region in Europe.

(i) $\neg[\exists\ c, d\ In(c, SouthAmerica) \land In(d, Europe) \land Borders(c, d)]$.

(ii) $\forall\ c, d\ [In(c, SouthAmerica) \land In(d, Europe)] \Rightarrow \neg Borders(c, d)]$.

(iii) $\neg \forall\ c\ In(c, SouthAmerica) \Rightarrow \exists d\ In(d, Europe) \land \neg Borders(c, d)$.

(iv) $\forall\ c\ In(c, SouthAmerica) \Rightarrow \forall d\ In(d, Europe) \Rightarrow \neg Borders(c, d)$.

e. No two adjacent countries have the same map color.

(i) $\forall\ x, y\ \neg Country(x) \lor \neg Country(y) \lor \neg Borders(x, y) \lor$

$\neg(MapColor(x) = MapColor(y))$.

(ii) $\forall\ x, y\ (Country(x) \land Country(y) \land Borders(x, y) \land \neg(x = y)) \Rightarrow$

$\neg(MapColor(x) = MapColor(y))$.

(iii) $\forall\ x, y\ Country(x) \land Country(y) \land Borders(x, y) \land$

$\neg(MapColor(x) = MapColor(y))$.

(iv) $\forall\ x, y\ (Country(x) \land Country(y) \land Borders(x, y)) \Rightarrow MapColor(x \_= y)$.

8.28 Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., “Meet the Beatles”) and disks (i.e., particular physical instances of CDs). The vocabulary contains the following symbols:

$CopyOf(d, a)$: Predicate. Disk d is a copy of album a.

$Owns(p, d)$: Predicate. Person p owns disk d.

$Sings(p, s, a)$: Album$a$ includes a recording of song s sung by person p.

$Wrote(p, s)$: Person p wrote song s.

McCartney, Gershwin, BHoliday, Joe, EleanorRigby, TheManILove, Revolver :
Constants with the obvious meanings.

13.8 Given the full joint distribution shown in Figure 13.3, calculate the following:

a. **P**(toothache) .

b. **P**(Cavity) .

c. **P**(Toothache | cavity) .

d. **P**(Cavity | toothache ∨ catch) .

13.17 Show that the statement of conditional independence

P(X, Y | Z) = **P**(X | Z)**P**(Y | Z)

is equivalent to each of the statements

P(X | Y,Z) = **P**(X |Z) and **P**(B | X,Z) = **P**(Y | Z) .

Exercise 14.14 (a) (b) (c)iuj

14.14 Consider the Bayes net shown in Figure 14.23.
a. Which of the following are asserted by the network *structure*?

(i) **P**(B, I,M) = **P**(B)**P**(I)**P**(M).

(ii) **P**(J |G) = **P**(J | G, I).

(iii) **P**(M | G,B, I) = **P**(M | G,B, I, J).

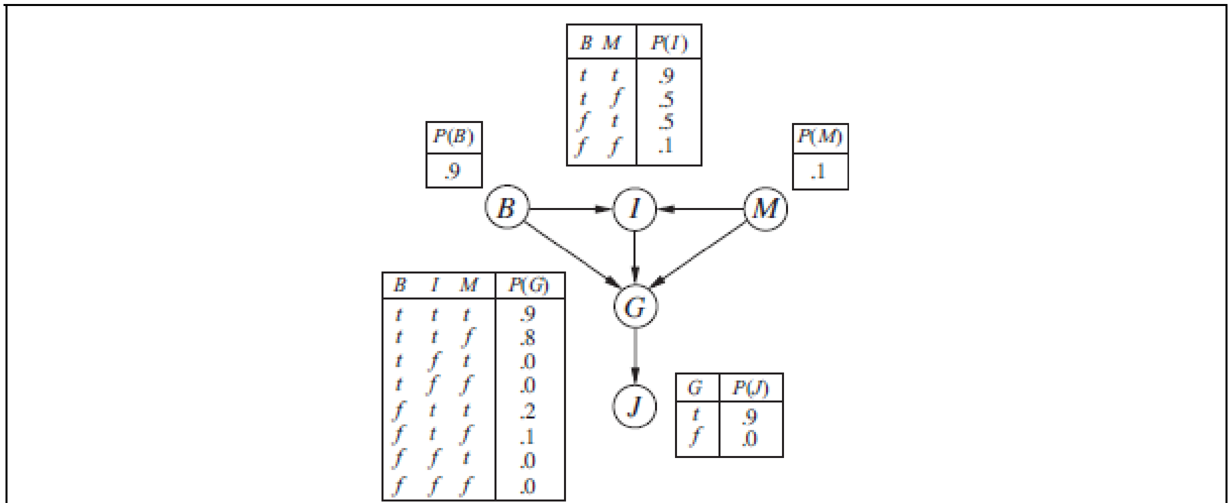

Figure 14.23 A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

b. Calculate the value of P(b, i, ¬m, g, j).

c. Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.