COMP 8700 Fall 2020 Take-home Exam/Assignment 3

Due: Jan 6., 11:59pm

**7.4** Which of the following are correct?

**a**. False |= True.

**b**. True |= False.

**c**. (A ∧ B) |= (A ⇔ B).

**d**. A ⇔ B |= A ∨ B.

**e**. A ⇔ B |= ￢A ∨ B.

**f**. (A ∧ B) ⇒ C |= (A ⇒ C) ∨ (B ⇒ C).

**g**. (C ∨ (￢A ∧ ￢B)) ≡ ((A ⇒ C) ∧ (B ⇒ C)).

**h**. (A ∨ B) ∧ (￢C ∨￢D ∨ E) |= (A ∨ B).

**i**. (A ∨ B) ∧ (￢C ∨￢D ∨ E) |= (A ∨ B) ∧ (￢D ∨ E).

**j**. (A ∨ B)∧ ￢(A ⇒ B) is satisfiable.

**k**. (A ⇔ B) ∧ (￢A ∨ B) is satisfiable.

**l**. (A ⇔ B) ⇔ C has the same number of models as (A ⇔ B) for any fixed set of

proposition symbols that includes A, B, C.

Correct answers are:

a, e, g, h, j, k

**7.5** Prove each of the following assertions:

**a**. α is valid if and only if True |= α.

True |= α means in all models where True is true, then also a α is true. Since True is true in all models, so α is true in all models too, so α must be true as only true is valid in all worlds.

Answer:

True |= a

True => a

-True ∨ a

False ∨ a

a

So from the inference, we can see “True |= a” entails “a” true, and since there is only one disjunctive clause “a” logically equivalent to “True |= a”, so a must be true.

**b**. For any α, False |= α.

Answer:

False |= a

False => a

-False ∨ a

True ∨ a

So from the result “True ∨ a”, we can see it must be true as True is one of the disjunctive clauses.

**c**. α |= β if and only if the sentence (α ⇒ β) is valid.

**d**. α ≡ β if and only if the sentence (α ⇔ β) is valid.

**e**. α |= β if and only if the sentence (α ∧ ￢β) is unsatisfiable.

**7.6** Prove, or find a counterexample to, each of the following assertions:

**a**. If α |= γ or β |= γ (or both) then (α ∧ β) |= γ

**b**. If α |= (β ∧ γ) then α |= β and α |= γ.

**c**. If α |= (β ∨ γ) then α |= β or α |= γ (or both).

**7.7** Consider a vocabulary with only four propositions, A, B, C, and D. How many models

are there for the following sentences?

**a**. B ∨ C.

**b**. ￢A∨￢B ∨￢C ∨ ￢D.

**c**. (A ⇒ B) ∧ A∧ ￢B ∧ C ∧ D.

**7.10** Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify

your decisions using truth tables or the equivalence rules of Figure 7.11 (page 249).

**a**. Smoke ⇒ Smoke

**b**. Smoke ⇒ Fire

**c**. (Smoke ⇒ Fire) ⇒ (￢Smoke ⇒ ￢Fire)

**d**. Smoke ∨ Fire ∨ ￢Fire

**e**. ((Smoke ∧ Heat ) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

**f**. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat ) ⇒ Fire)

**g**. Big ∨ Dumb ∨ (Big ⇒ Dumb)

**7.12** Use resolution to prove the sentence ￢A∧￢B from the clauses in Exercise 7.20.

**7.20** Convert the following set of sentences to clausal form.

S1: A ⇔ (B ∨ E).

S2: E ⇒ D.

S3: C ∧ F ⇒ ￢B.

S4: E ⇒ B.

S5: B ⇒ F.

S6: B ⇒ C

Give a trace of the execution of DPLL on the conjunction of these clauses.

**8.2** Consider a knowledge base containing just two sentences: P(a) and P(b). Does this

knowledge base entail ∀x P(x)? Explain your answer in terms of models.

**8.6** Which of the following are valid (necessarily true) sentences?

**a**. (∃x x=x) ⇒ (∀ y ∃z y =z).

**b**. ∀x P(x) ∨￢P(x).

**c**. ∀ x Smart(x) ∨ (x=x).

**8.9** This exercise uses the function MapColor and predicates In(x, y), Borders(x, y), and

Country(x), whose arguments are geographical regions, along with constant symbols for

various regions. In each of the following we give an English sentence and a number of candidate

logical expressions. For each of the logical expressions, state whether it (1) correctly

expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3)

is syntactically valid but does not express the meaning of the English sentence.

**a**. Paris and Marseilles are both in France.

(i) In(Paris ∧ Marseilles, France ).

(ii) In(Paris, France ) ∧ In(Marseilles, France ).

(iii) In(Paris, France ) ∨ In(Marseilles, France ).

**b**. There is a country that borders both Iraq and Pakistan.

(i) ∃ c Country(c) ∧ Border (c, Iraq) ∧ Border (c, Pakistan).

(ii) ∃ c Country(c) ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)].

(iii) [∃ c Country(c)] ⇒ [Border (c, Iraq) ∧ Border (c, Pakistan)].

(iv) ∃ c Border (Country(c), Iraq ∧ Pakistan).

**c**. All countries that border Ecuador are in South America.

(i) ∀c Country(c) ∧ Border (c,Ecuador ) ⇒ In(c, SouthAmerica).

(ii) ∀ c Country(c) ⇒ [Border (c,Ecuador ) ⇒ In(c, SouthAmerica)].

(iii) ∀ c [Country(c) ⇒ Border (c,Ecuador )] ⇒ In(c, SouthAmerica).

(iv) ∀c Country(c) ∧ Border (c,Ecuador ) ∧ In(c, SouthAmerica).

**d**. No region in South America borders any region in Europe.

(i) ￢[∃ c, d In(c, SouthAmerica) ∧ In(d, Europe) ∧ Borders(c, d)].

(ii) ∀ c, d [In(c, SouthAmerica) ∧ In(d, Europe)] ⇒ ￢Borders(c, d)].

(iii) ￢∀ c In(c, SouthAmerica) ⇒ ∃d In(d, Europe)∧ ￢Borders(c, d).

(iv) ∀ c In(c, SouthAmerica) ⇒ ∀d In(d, Europe) ⇒ ￢Borders(c, d).

**e**. No two adjacent countries have the same map color.

(i) ∀ x, y ￢Country(x) ∨ ￢Country(y)∨ ￢Borders(x, y) ∨

￢(MapColor (x) = MapColor (y)).

(ii) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y) ∧ ￢(x = y)) ⇒

￢(MapColor (x) = MapColor (y)).

(iii) ∀ x, y Country(x) ∧ Country(y) ∧ Borders(x, y) ∧

￢(MapColor (x) = MapColor (y)).

(iv) ∀ x, y (Country(x) ∧ Country(y) ∧ Borders(x, y)) ⇒ MapColor (x \_= y).

**8.28** Consider a first-order logical knowledge base that describes worlds containing people,

songs, albums (e.g., “Meet the Beatles”) and disks (i.e., particular physical instances of CDs).

The vocabulary contains the following symbols:

CopyOf (d, a): Predicate. Disk d is a copy of album a.

Owns(p, d): Predicate. Person p owns disk d.

Sings(p, s, a): Albuma includes a recording of song s sung by person p.

Wrote(p, s): Person p wrote song s.

McCartney, Gershwin, BHoliday, Joe, EleanorRigby, TheManILove, Revolver :

Constants with the obvious meanings.

**13.8** Given the full joint distribution shown in Figure 13.3, calculate the following:

**a**. **P**(toothache) .

**b**. **P**(Cavity) .

**c**. **P**(Toothache | cavity) .

**d**. **P**(Cavity | toothache ∨ catch) .

**13.17** Show that the statement of conditional independence

**P**(X, Y | Z) = **P**(X | Z)**P**(Y | Z)

is equivalent to each of the statements

**P**(X | Y,Z) = **P**(X |Z) and **P**(B | X,Z) = **P**(Y | Z) .

Exercise 14.14 (a) (b) (c)iuj

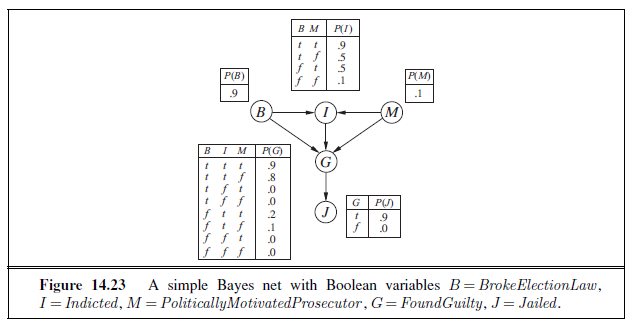
**14.14** Consider the Bayes net shown in Figure 14.23.

**a**. Which of the following are asserted by the network *structure*?

(i) **P**(B, I,M) = **P**(B)**P**(I)**P**(M).

(ii) **P**(J |G) = **P**(J | G, I).

(iii) **P**(M | G,B, I) = **P**(M | G,B, I, J).



**b**. Calculate the value of P(b, i,￢m, g, j).

**c**. Calculate the probability that someone goes to jail given that they broke the law, have

been indicted, and face a politically motivated prosecutor.