



4A7 Aviation and the Environment [Lecture 2]

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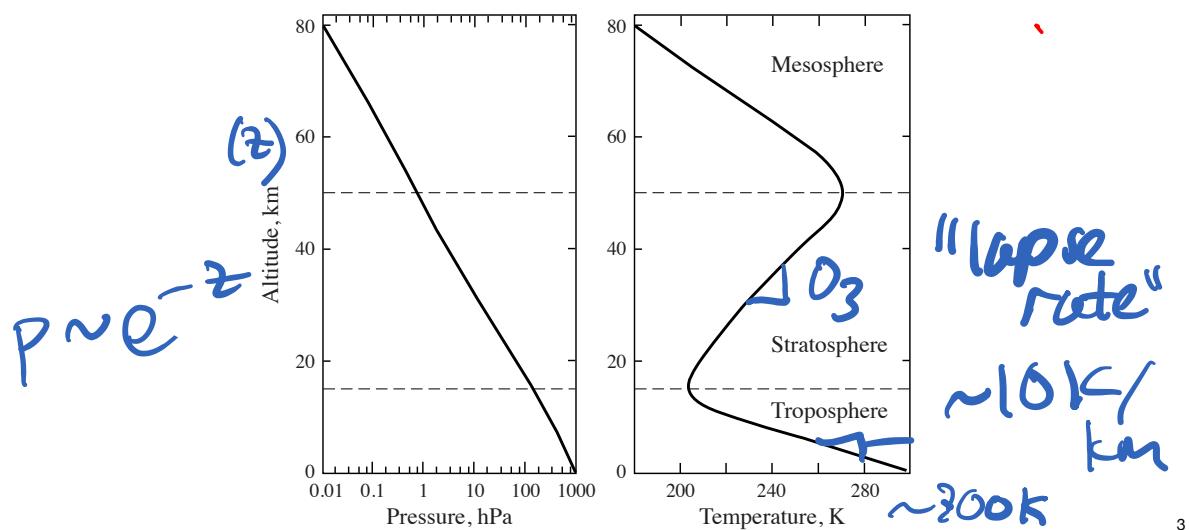
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Topics

- Structure of the atmosphere
- Chemical composition of the atmosphere
- Aircraft thrust requirements
- Aircraft range
- Phases of flight

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Vertical pressure profile and thermal structure



Turbulence

atmospheric

ABL

- In the planetary boundary layer (or mixing layer) - the lowest ~1 km of the atmosphere - there is significant mechanical and buoyancy-generated turbulence
- The stratosphere is still
- Turbulence levels in the free troposphere vary, and strongly related to water vapor concentrations

The International Standard Atmosphere

- Default assumed for preliminary design purposes

$$p_{sl} = 1.01325 \text{ bar} = 760 \text{ mm Hg}$$

$$T_{sl} = 15^\circ\text{C} = 288.15 \text{ K}$$

$$\rho_{sl} = 1.225 \text{ kg/m}^3$$

$$v_{sl} = \frac{\mu_{sl}}{\rho_{sl}} = 14.64 \times 10^{-6} \text{ m}^2/\text{s}$$

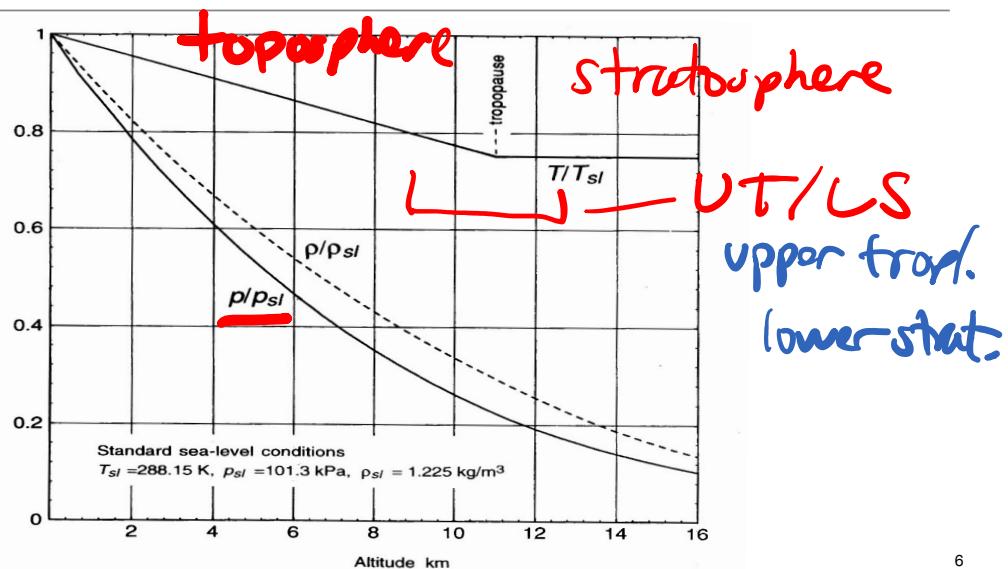
$$a_{sl} = \sqrt{\gamma R T_{sl}} = 340 \text{ m/s}$$

- Aircraft fly at pressure altitudes - e.g. 31, 35, 39 kft E-W, 33, 37, 41 kft W-E

Height m	$\frac{T}{T_{sl}}$	$\frac{p}{p_{sl}}$	$\frac{\rho}{\rho_{sl}}$	$\frac{V}{V_{sl}}$
0	1.0000	1.0000	1.0000	1.0000
1000	0.9774	0.8870	0.9075	1.0826
2000	0.9549	0.7846	0.8217	1.1739
3000	0.9324	0.6920	0.7422	1.2753
4000	0.9098	0.6085	0.6689	1.3881
5000	0.8873	0.5334	0.6012	1.5138
6000	0.8648	0.4660	0.5389	1.6543
7000	0.8423	0.4057	0.4816	1.8117
8000	0.8198	0.3519	0.4292	1.9887
9000	0.7973	0.3040	0.3813	2.1881
10000	0.7748	0.2615	0.3376	2.4137
11000	0.7523	0.2240	0.2978	2.6697
12000	0.7519	0.1915	0.2546	3.1206
13000	0.7519	0.1636	0.2176	3.6514
14000	0.7519	0.1398	0.1860	4.2722
15000	0.7519	0.1195	0.1590	4.9983
16000	0.7519	0.1022	0.1359	5.8476
17000	0.7519	0.0873	0.1162	6.8408
18000	0.7519	0.0747	0.0993	8.0023
19000	0.7519	0.0638	0.0849	9.3606
20000	0.7519	0.0546	0.0726	10.9488
21000	0.7551	0.0467	0.0618	12.9031
22000	0.7585	0.0399	0.0527	15.2021
23000	0.7620	0.0342	0.0449	17.8964
24000	0.7654	0.0293	0.0383	21.0515
25000	0.7689	0.0252	0.0327	24.7434
26000	0.7723	0.0216	0.0280	29.0603
27000	0.7758	0.0186	0.0239	34.1039
28000	0.7792	0.0160	0.0205	39.9926
29000	0.7826	0.0137	0.0175	46.8626
30000	0.7861	0.0118	0.0150	54.8714

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The International Standard Atmosphere



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$$N_A = 6.022 \times 10^{23} \frac{\text{molec.}}{\text{mol}}$$

Units of atmospheric composition

Mixing ratio ("concentration") $C_X = \text{mol of } X \text{ per mol of air}$

$$\frac{\text{mol}}{\text{mol}} / \frac{V}{V} / \frac{\text{ppm}}{\text{ppmv}} / \text{ppb}$$

Mass concentration

$$\frac{\text{mass}}{\text{volume}} / \frac{\text{mg}}{\text{m}^3}$$

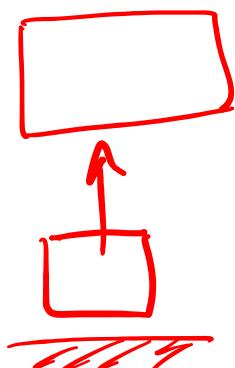
Number concentration

$$[X] / \frac{\text{molec.}}{\text{Vol.}} / \frac{\text{molec.}}{\text{cm}^3}$$

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Concept question

When a parcel of air with a certain mass concentration of substance X rises in the atmosphere, what happens to the mass concentration of X?



expands as p↓

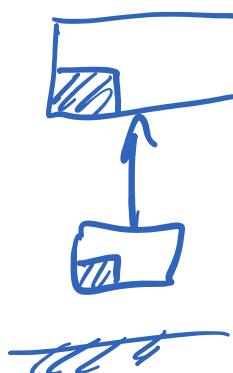
$$\frac{mg}{m^3} \downarrow$$

*mass conc. is
not conserved*

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Concept question

When a parcel of air with a certain mixing ratio of substance X rises in the atmosphere, what happens to the mixing ratio of X?



$$V/V = \text{Const}$$

$$\text{Mass. Conc.} = \frac{\text{mixing ratio}}{\text{Molar mass}} \times \frac{\text{Mol air}}{\text{Vol}}$$

$$\frac{mg}{m^3} = \frac{\text{mol}}{\text{mol air}} \times \frac{g}{\text{mol}} \times \frac{\text{Mol air}}{m^3} (\times 10^6)$$

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Chemical composition of the atmosphere

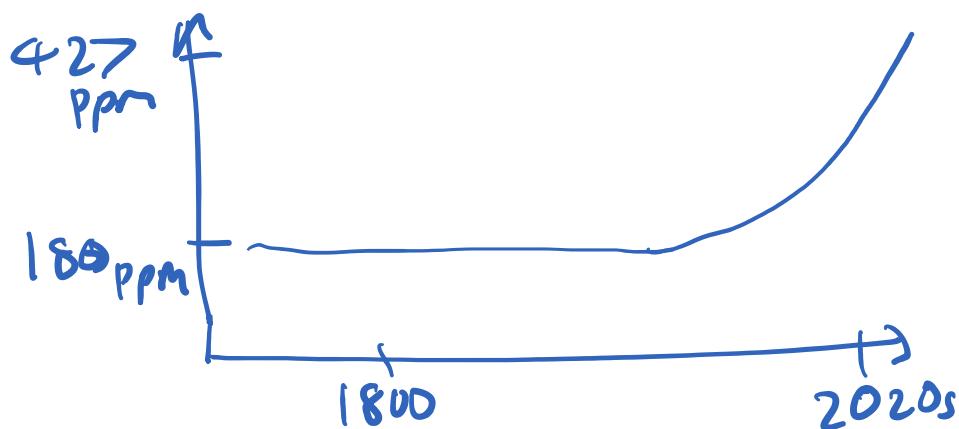
Gas	Mixing ratio (mol/mol)
Nitrogen (N_2)	0.78
Oxygen (O_2)	0.21
Argon (Ar)	0.0093
Carbon dioxide (CO_2)	427 ppm
Neon (Ne)	18×10^{-6}
Ozone (O_3)	$0.01-10 \times 10^{-6}$
Helium (He)	5.2×10^{-6}
Methane (CH_4)	1.7×10^{-6}
Krypton (Kr)	1.1×10^{-6}
Hydrogen (H_2)	500×10^{-9}
Nitrous oxide (N_2O)	320×10^{-9}

} const.
 + 27 ppm
 → highly variable
 } trace gases

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CO_2 over time

CO_2 concentrations have increased since the industrial revolution, as have concentrations of several other human-influenced greenhouse gases (GHGs) and air pollutants



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Selecting the engine thrust

- The aerodynamically most demanding phase of flight is top-of-climb
- The mechanically most demanding phase of flight is takeoff
- Thrust falls with density, so an engine that satisfies TOC is fine for takeoff
- At TOC, an aircraft needs to maintain a climb rate of 300 fpm or 1.5 m/s
- For an aircraft that is climbing at an angle θ

$$L = w \cos \theta \approx w$$

$$F_N - D = w \sin \theta$$

$$\Rightarrow F_N/w = D/w + \sin \theta$$

$$\Rightarrow F_N/w = \frac{1}{L/D} + \sin \theta$$

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A320 thrust needed at TOC

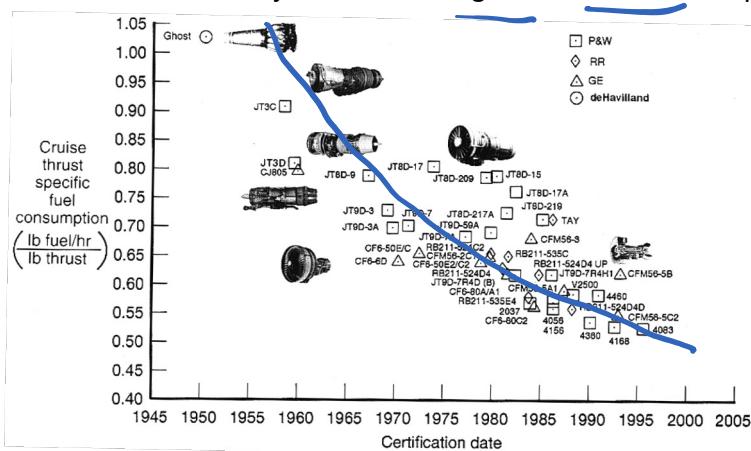
- At 31 kft at the start of cruise an A320 needs to have a minimum climb performance of 1.5 m/s (this maps to 0.34°)
- Assuming L/D = 18 and an initial cruise weight of 72 t, what minimum thrust/engine is needed?

$$\begin{aligned}
 F_N &= w \left[\frac{1}{L/D} + \sin \theta \right] \\
 &= 72 \times 10^3 \times 9.81 \times \left[\frac{1}{18} + \sin 0.34^\circ \right] \\
 &= 43.4 \text{ kN (total)} \\
 &\quad (\text{or } 5000 \text{ lbf per engine})
 \end{aligned}$$

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Thrust specific fuel consumption (sfc) (tsfc)

- sfc is the conventional way to relate fuel use to thrust
- It is the fuel flow rate divided by the thrust in kg/s/N or lb/hr/lb or equivalent



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How do we maximize range?

- Maximizing L/D gives you the maximum aircraft weight for a given thrust
- What if you want to maximize range for a given amount of energy?

$$\frac{\text{dist}}{\text{energy}} \propto \frac{\text{velocity} \times \text{time}}{\text{drag} \times \text{time}} \propto \frac{\text{velocity}}{\text{drag}}$$

- For a given lift (=weight), drag is proportional to $1/(L/D)$

$$\frac{\text{dist}}{\text{energy}} \propto V \frac{L}{D}$$

$$a = \sqrt{\sigma RT}$$

$$M = V/a$$

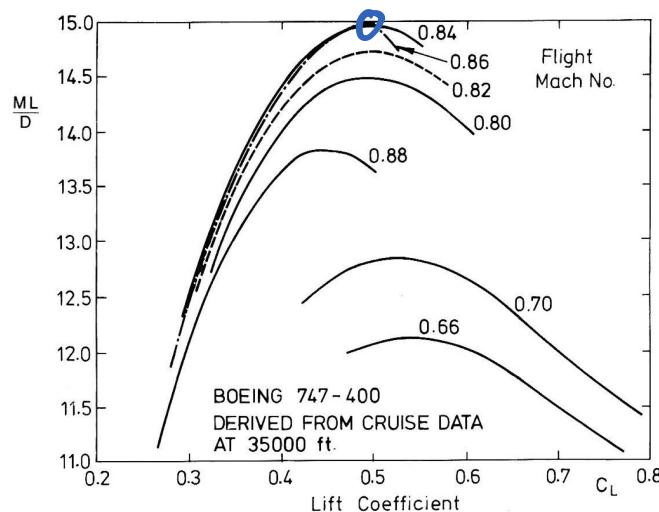
- But for the narrow range of altitudes aircraft fly in, approximately

$$\frac{\text{dist}}{\text{energy}} \propto M L/D$$

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ML/D as a function of lift coefficient

Maximum range at M0.86 (then rapidly falls) and a lift coefficient of 0.5



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The Breguet range equation

During flight, the aircraft weight changes at the rate the fuel is burnt:

$$\frac{dw}{dt} = -g \cdot sfc \cdot \text{net thrust} \rightarrow \text{drag} = \frac{w}{CL/D}$$

$$= -g \frac{sfc \cdot w}{CL/D}$$

$$\int_{w_{start}}^{w_{end}} \frac{dw}{w} = -\frac{g sfc}{VL/D} \int_0^R ds$$

$$dt = ds/V$$

Taking VL/D as constant and integrating:

$$\Rightarrow R = -\frac{VL/D}{g sfc} \ln\left(\frac{w_{end}}{w_{start}}\right) \leftarrow \text{BRE}$$

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Linearized Breguet range equation

The Breguet range equation is weakly non-linear, and especially for shorter missions it can be linearized.

$$\begin{aligned} w_{\text{start}} &= w_{\text{end}} + w_{\text{fuel}} \\ \Rightarrow \ln\left(\frac{w_{\text{start}} - w_{\text{fuel}}}{w_{\text{start}}}\right) &= \ln\left(1 - \frac{w_{\text{fuel}}}{w_{\text{start}}}\right) \approx \frac{-w_{\text{fuel}}}{w_{\text{start}}} \\ \Rightarrow w_{\text{fuel}} &\approx \frac{g \text{ sfc } w_{\text{start}} R}{V L/D} \end{aligned}$$

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Range of an A320

$$1.61 \times 10^{-5} \frac{\text{kg/s}}{N}$$

Assume TSFC = 0.57 lb/hr/lb, MTOGW = 78 t, MLW = 62.4 t, V = 922 km/hr (256 m/s)

$$\frac{w_{\text{start}}}{w_{\text{end}}} = \frac{78}{62.4} = 1.25$$

$$\Rightarrow R = \frac{256 \times 18}{9.81 \times 1.61 \times 10^{-5}} \times \ln(1.25) = 6510 \text{ km}$$

$$R_{\text{linearized}} = 5835 \text{ km}$$

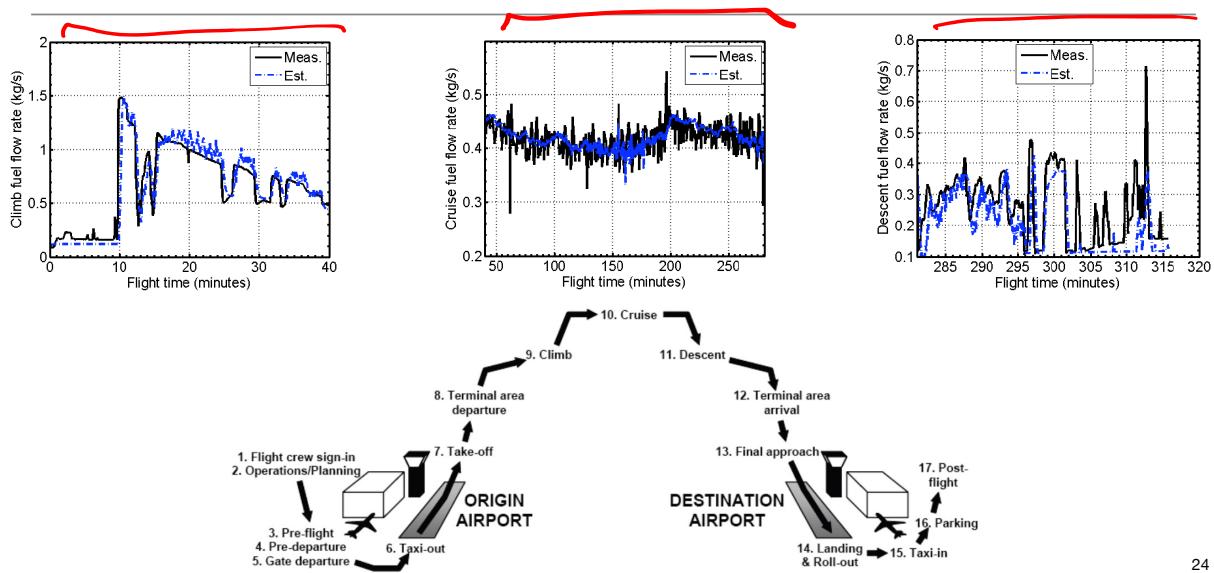
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Phases of flight



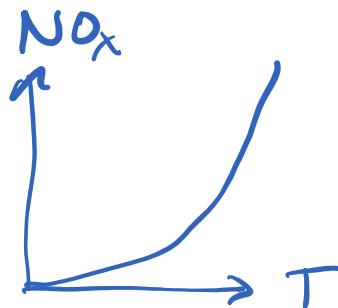
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Takeoff and climb-out

F_{00}

Int'l Civil Aviation Org. (ICAO)

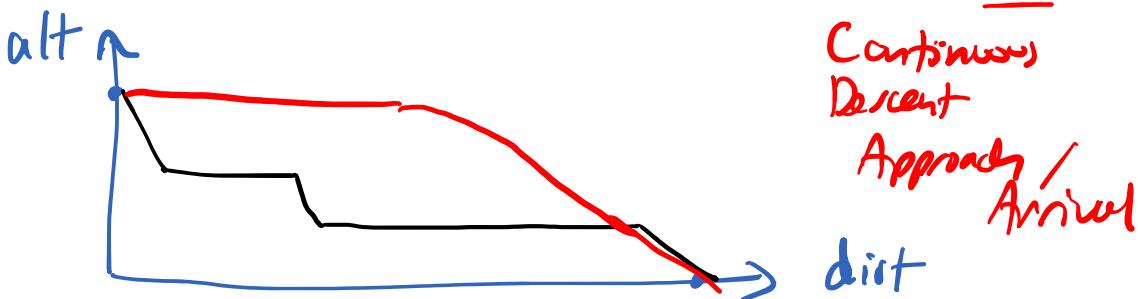
- Takeoff occurs at 100% of maximum rated thrust (ICAO default)
- In practice reduced thrust or de-rated takeoffs occur, with typical takeoff thrusts of 85%
- This has the net effect of:
 - Reducing maintenance cost
 - Increasing the takeoff roll and/or climb-out time
 - Increasing fuel burn and CO₂ emissions (usually)
 - Decreasing NO_x emissions



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Approach and landing

- The ICAO default approach thrust is 30% F_{00} , but in practice this varies
- A typical approach and (more) ideal approach vary, with the latter being a CDA:



- Note that there may also be holding

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Taxi

- The ICAO default taxi thrust is 7% F_{00} , but in practice this may be more like 4%
- Fuel burn and most emissions from taxiing are a small percentage of an overall aircraft mission, with the exception of HC emissions
*and CO
(low T → unburned fuel)*

- Measures to decrease taxi fuel burn include:

- Single engine taxiing
- Gate-hold programs (e.g. N-control)

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LTO cycle

- LTO = landing and takeoff cycle
- Includes all operations below 3000 ft *AGL*
- This has traditionally been regulated on the basis of ICAO default thrust settings and ICAO default times in mode (TIM)

TIME in mode

	Flight phase	TIM (s)	Thrust setting (F/F_{00}), %
<u>Arrival</u>	Approach	240	30
	Taxi in	420	7
	Taxi out	1140	7
<u>Departure</u>	Takeoff	42	100
	Climb	132	85

→

- The ICAO TIMs are also not always an accurate reflection of a real aircraft's operation

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