

Physics 598

Homework 7

Cunwei Fan

cfan11

Problem 1

The first problem askes to simulate Sedov blast wave with initial condition that particles inside a specific radius have uniform pressure P_0 and the whole fluid has uniform density ρ_0 and thus its inverse V_0 is also constant. Although the problem is a one dimensional problem, the volume form is 3D and specifically spherical volume form $4\pi R^2 dR$. Thus, to make the volume equal for each particle or ρ constant for each particle, we have to give up the uniform distributed particle intially. Or more specifically, I use the equation

$$\pi(R_i + R_{i+1})^2(R_{i+1} - R_i) = V_i \quad \text{and} \quad R_0 = 0$$

to recursively find the R_i for specific V_i . In this problem I choose $V_i = 0.1$.

In this problem, I run for 3 different cases, the first one is with initially just 1 particle constrained in the shock front. the second case is with 2 particles constrained in the shock front, and the third case is with 5 particles constrained in the shock front. Inside the shock front, the pressure is set to be $P_0 = 1$ and outside the shock front, the pressure is set to be $P_0 = 0$ to make the simulation can go long enough by setting the shock pressure is always much bigger than the pressure in the outer region.

Here are the profiles for the three different cases.

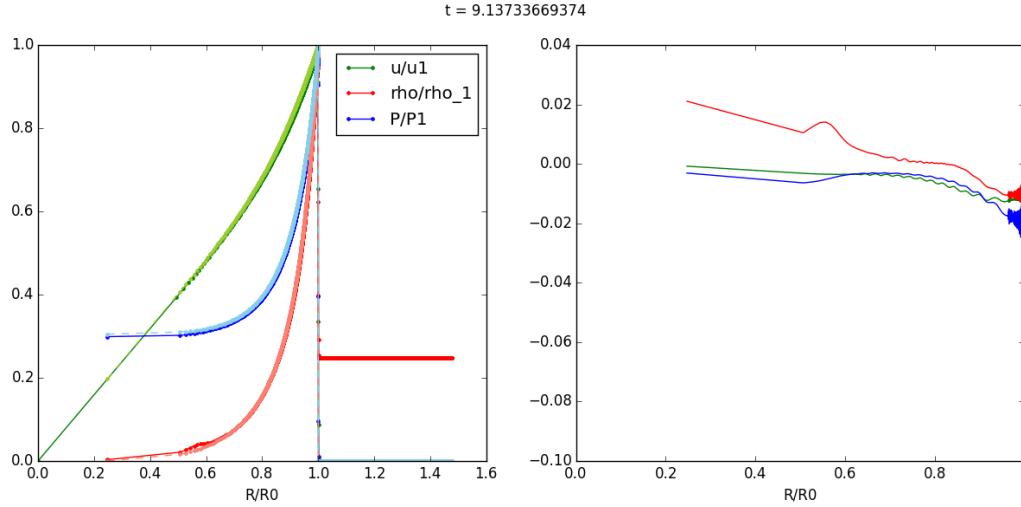


Figure 1: Profiles for 1 particle source

It can be seen that the profiles agree with the Sedov blast wave solution soon after the simulation begins. It needs around 9 units of time for the system to stablize to Sedov's solution. The plot on the right is the absolute error between the simulated profiles and the Sedov solution. The horizontal axis is $R/R_s = \eta$. As we can see the error is smaller than 0.02 at $t = 9$. Thus it is faster for one particles to stablize.

For two particle source the plot is like:

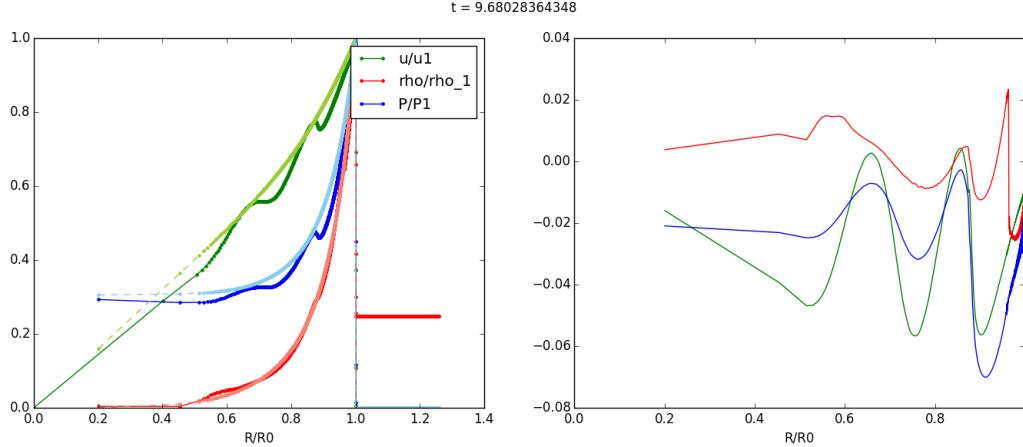


Figure 2: Profiles for 2 particle source

In the 2 particle case, it can be seen that there is some oscillations in the velocity profile and the pressure profile. At similar time, although the absolute error is around the same magnitude as the 1 particle source case, it definitely needs more time to stabilize and due to time constraint, I cannot run for another time for longer simulation. But the idea is clear, it definitely needs more time to stabilize.

For five particle source the plot is like:

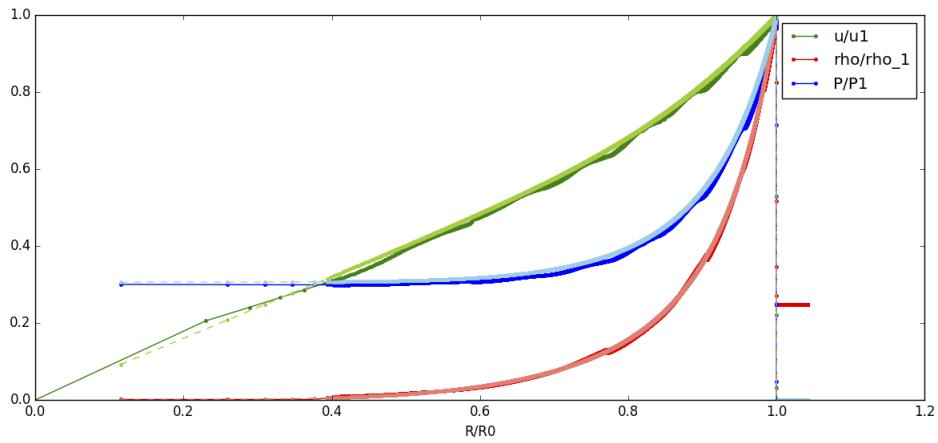


Figure 3: Profiles for 5 particle source profile at time $t = 393.62$

As can be seen, for five particle source, even at very late state, $t \sim 400$, there still exist some small oscillations around the analytical soluiton, although the oscillation is negligible at this stage. Since the oscillation is damped, if the code runs for a long time, the oscillation will diminish. However, that may take another 100 or 200 units of time. Below is the error plots for this profile.

In the error, plot, it may be some misleading to observe that the profiles are not oscillating at $\eta < 0.5$ regions. However, this may not be case. Since due to the shock, most particles are concentrated near the shock surface, the oscillation is clearer to see at that region. However, for small η , there are small number of particles, the oscillation is not easy to see. This does not mean that there is no oscillation for sure in that region. However, the error is definitely smaller at that regions.

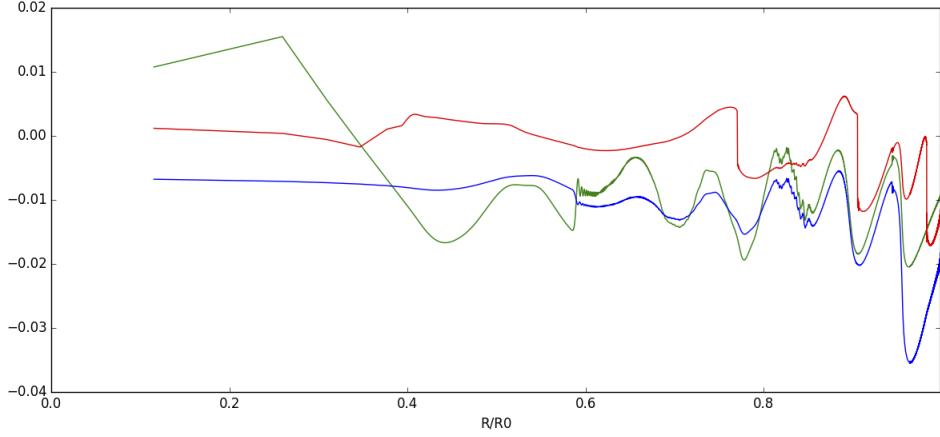


Figure 4: Profiles for 5 particle source error at time $t = 393.62$

From the tail of the velocity profile, we can propose that the oscillation in the profile is due to the oscillation of the left most particle whose density is very low. Due to its low density, it is more sensitive to the pressure gradients and thus it is harder to diminish the osicllation of it and therefore, we need a long time to stablize the system.

To monitor the accuracy of the simulation of the system, the conservation of energy is checked. For the energy check, the fomula used for the calculation is

$$E = \int \left(\frac{1}{2} \rho u^2 + \frac{P}{\gamma - 1} \right) 4\pi R^2 \, dR = \sum \left(\frac{3}{2} P_i V_i + \frac{1}{2} u_i^2 \right)$$

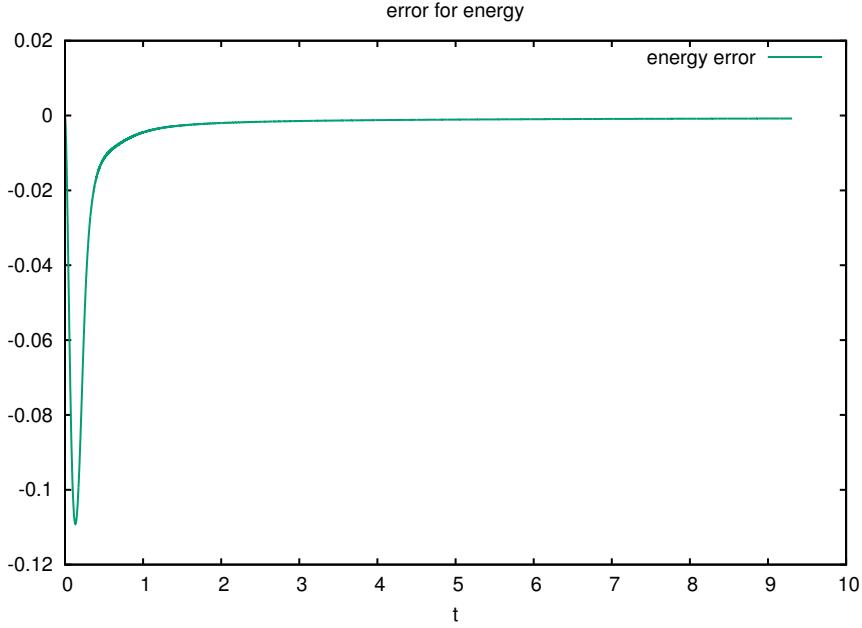


Figure 5: The $(E - E_0)/E_0$ is plotted with time as the horizontal axis for 1 particle case

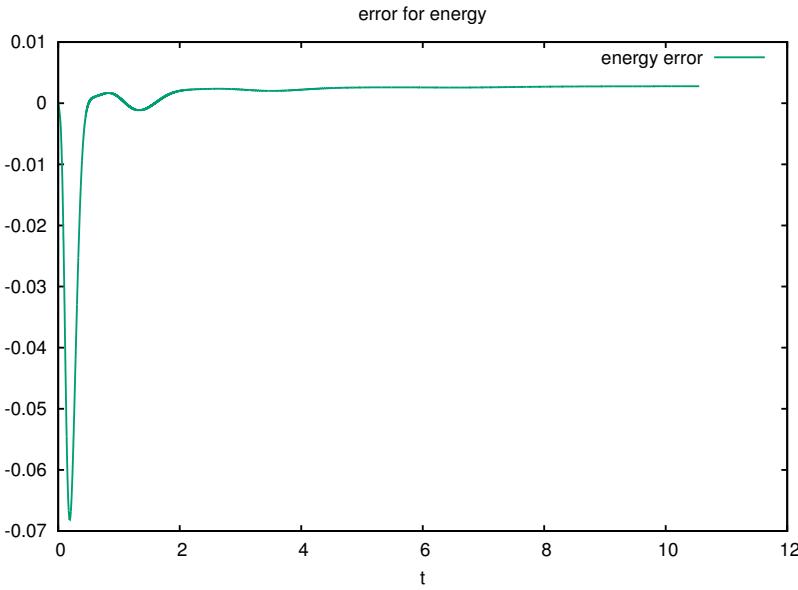


Figure 6: The $(E - E_0)/E_0$ is plotted with time as the horizontal axis for 2 particle case

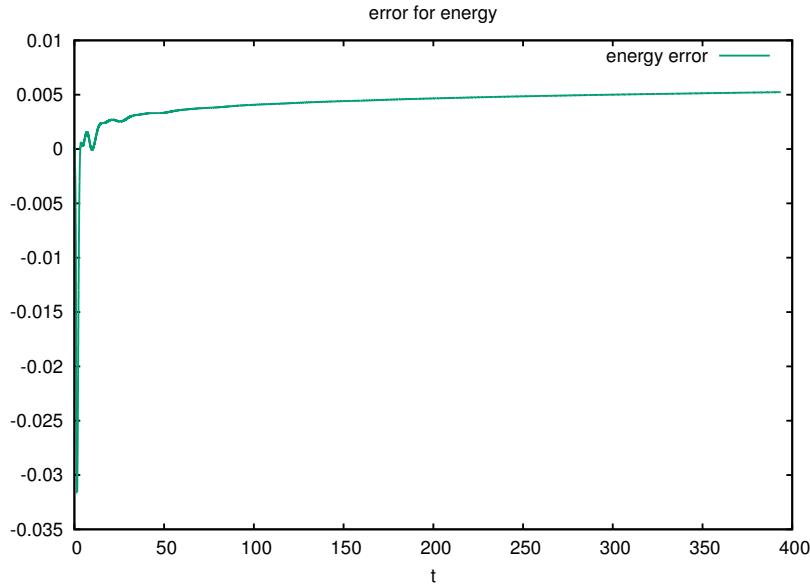


Figure 7: The $(E - E_0)/E_0$ is plotted with time as the horizontal axis for 5 particle case

As shown in the above plots, the error dips to a very big value and then gets back to around its initial energy. This is because, at the beginning only very small number of particles are perturbed. Thus, the sum of the energy is not accurate enough, since the accurate energy should be calculated from integrations. As time goes on, more particles are perturbed and thus the sum is more like an integration and then the error gets back to very low value. This argument is confirmed as we increase the number of particles contained in the shock surface, the absolute value of biggest error goes down, from 10 percent to 3 percent. The reason that the more particles included in the initial shock the more accurate the energy is calculated is that as more particles inside the shock initially, the more quickly that more particles will be perturbed into the shock.

The shock position check for the three cases are below:

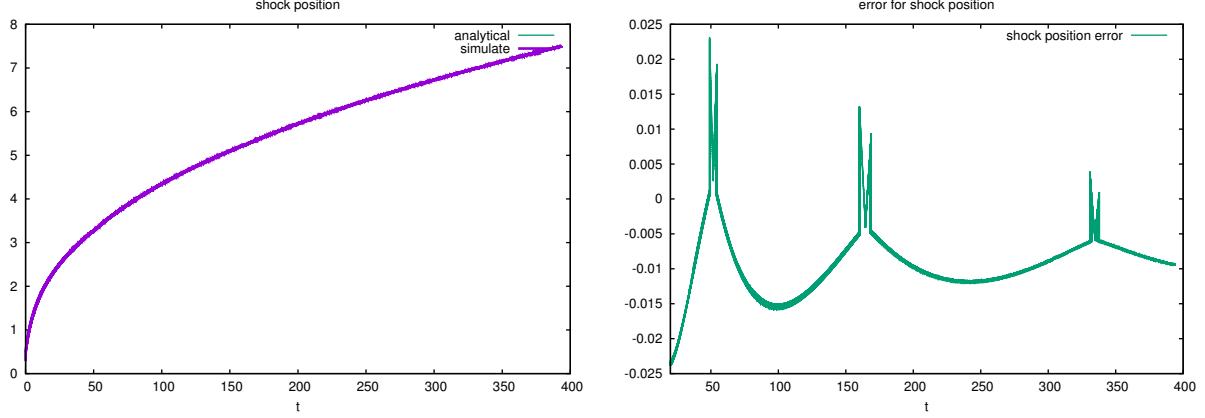


Figure 8: The shock position and error for 5 cell case (LEFT: position, RIGHT: error)

For the peak, I choose the maximum of density as the shock front. However, for this choice, as can be seen, there are some peaks in the error which means for some time, the choice for peak fronts is not accurate enough. And the periodic oscillation of errors may be due to the oscillation mentioned above in the velocity profile.

For the first 400 particles, the history of each particle is plotted and the red line in the plot is the Sedov predicted shock position over time.

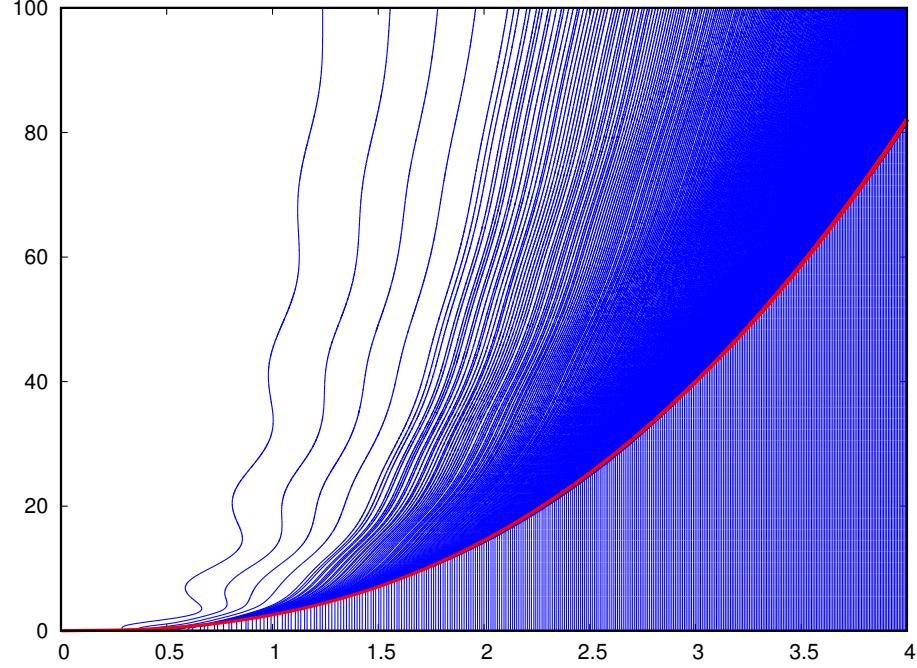


Figure 9: the horizontal axis is the radial coordinates of each particle and the vertical axis is the time axis

Problem 2

In this problem, similar situation is considered, but different initial condition is imposed. In this problem, the initial situation is setted as:

$$u(R) = \begin{cases} \frac{R}{R_0} u_{max} & R \leq R_0 \\ 0 & R > R_0 \end{cases}$$

In this problem, I set $u_{max} = 0.725$ to make the initial energy the same as the energy in the previous 5 particle case. Below is the profile plot for this case:

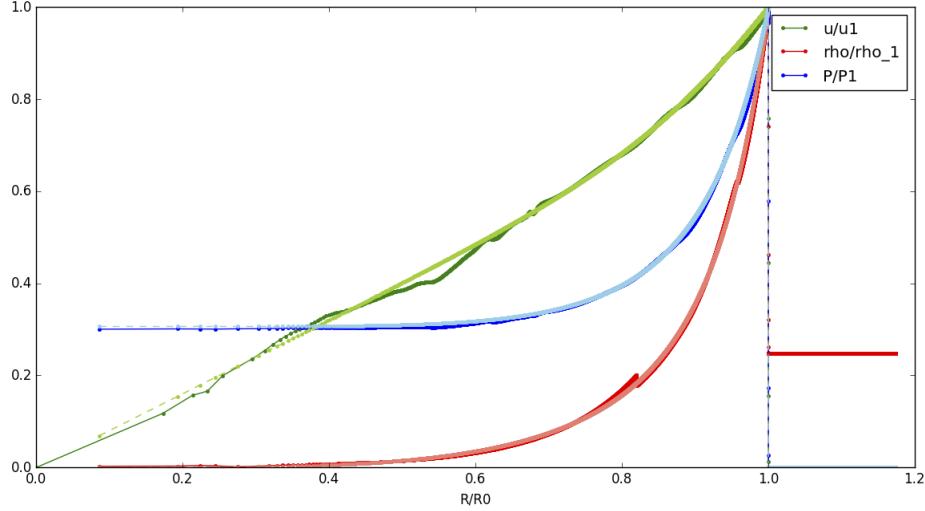


Figure 10: The profile for the shock 5-particle case at $t = 260$

The error for the profile is as shown

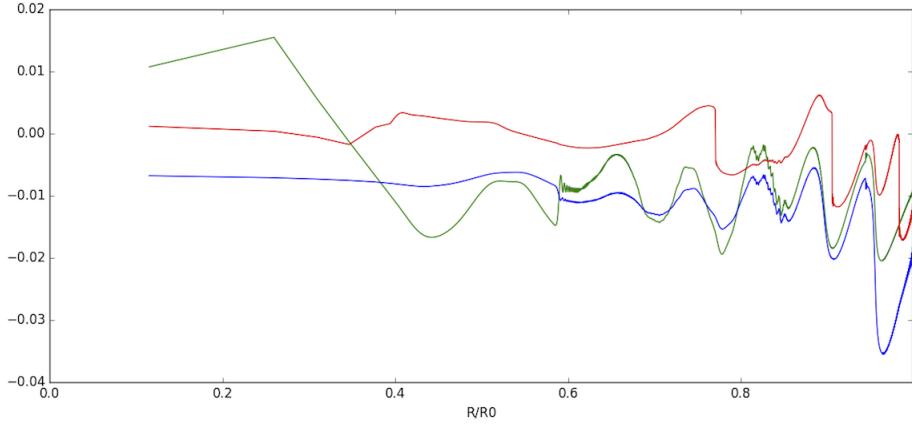


Figure 11: The error for the shock 5-particle case at $t = 260$

As seen in the plot, the profile error is within 4 percent which is good considerred that the system has not already stablized as the time here is only $t = 260$ and there are some ripples in the velocity profile plot. However, the energy conservation check is not good. The energy blows up to 15 percent and stablized there. The origin of the extra energy is mysterious, maybe from the cpu heat (just joke here). There may be a

bug in the code as there is a warning that the \sqrt{PV} has some invalid value which means each P or V has a negative value for some particle (probably the first particle). Then, when calculating the dt for later time the `nan` cell was omitted. Therefore, the chosen dt for this cell may be too big and then the great error is induced. All this is just postulate. To confirm this, I need more time to check what is wrong with this. As for the reason, I checked the total volume of the system over time. I found that the total volume is decreasing by 2 percent. However, as the pressure is big at first, thus, the increased error may be due to the decreased volume. As the pressure goes down, the pressure work done due to the error goes down and then the energy stabilized around the error value and is still increasing in a very slow pace.

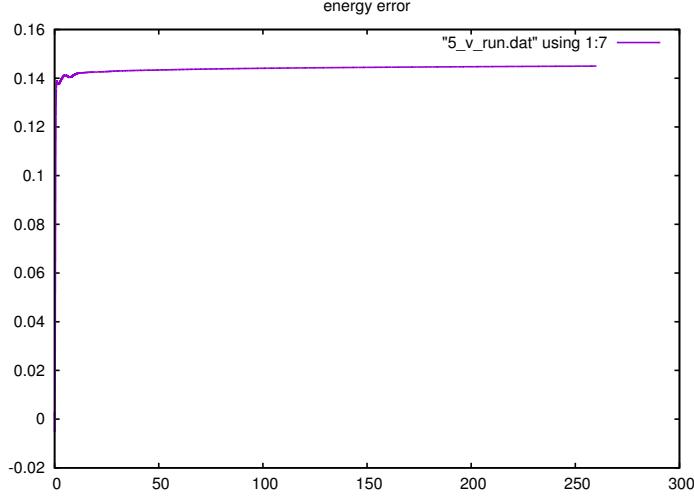


Figure 12: The energy error for the shock 5-particle case

For the shock position and its error:

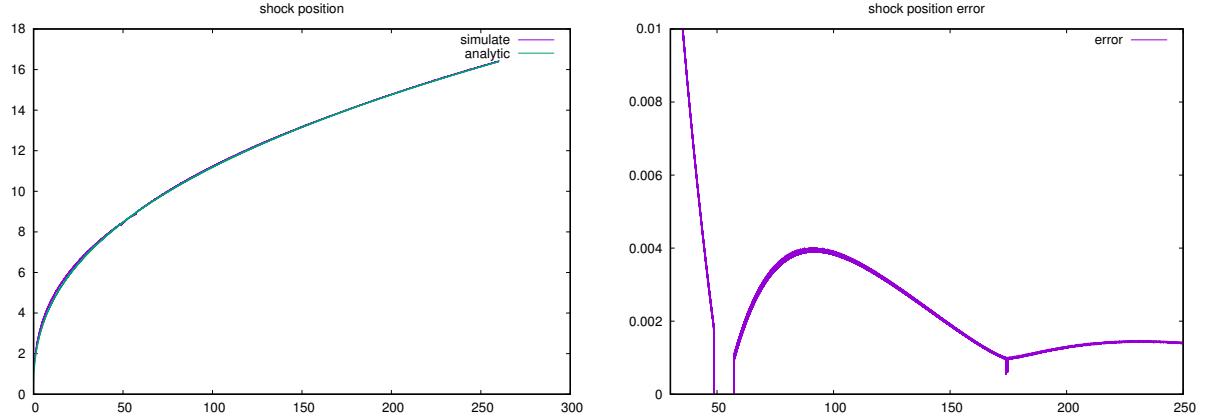


Figure 13: The shock position and error for 5 cell case (LEFT: position, RIGHT: error)

Problem 3

In this problem, the physical situation is that a ball of fluids expanding in a vacuum space, as described in the problem set that the density is zero outside the sphere. Thus, to deal with the problem, a rarefaction model is perfect. (I tried to use designate the last particle with very big volume to model the problem, however, this does not work for some reason). The rarefaction is applied to the last particle of the system

and the speed has to be bigger than the expansion speed which is $2C_0/(\gamma + 1)$. As ρ is 10 and $P_0 = 1.0$, the expansion speed is around 1 and for safety, I choose my rarefaction speed as 20. Here is the plot for the profile.

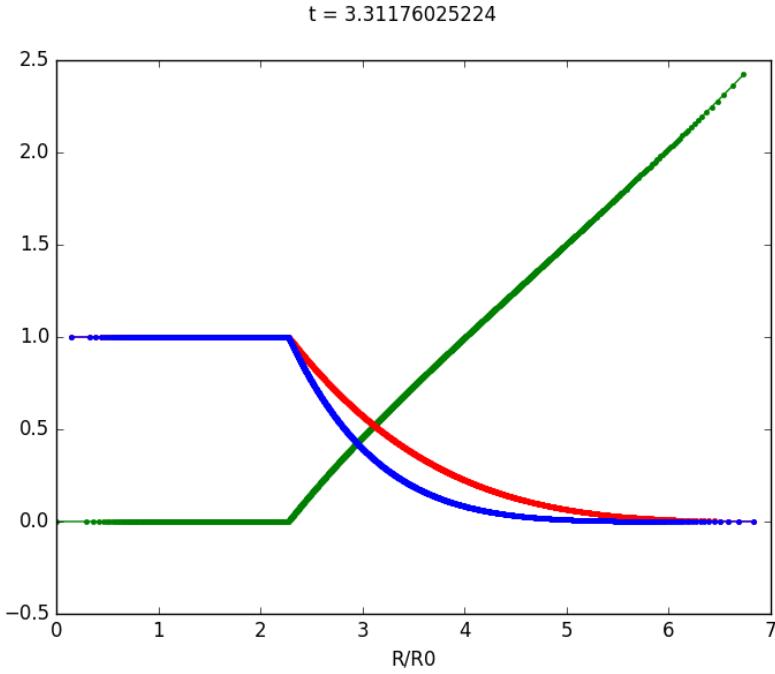


Figure 14: The profile for spherical rarefaction at $t = 3.311$, all the values are normalized by its initial value: u/C_0 , ρ/ρ_0 and P/P_0

And the energy conservation is plotted as

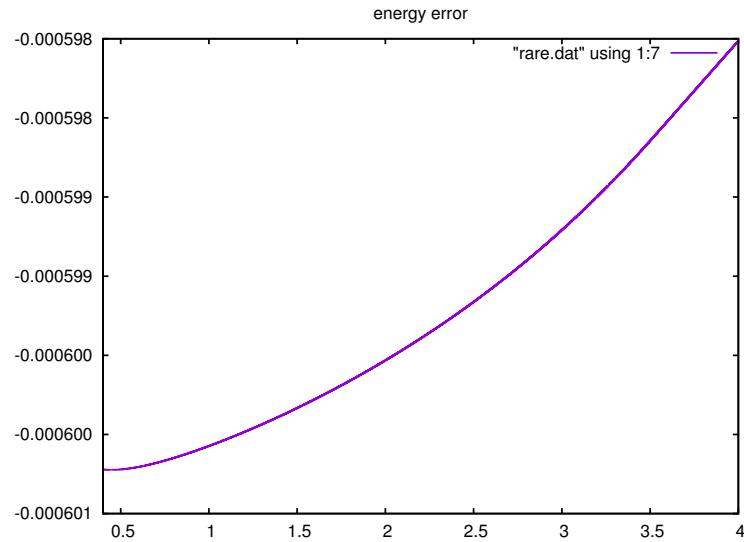


Figure 15: The profile for spherical rarefaction at $t = 3.311$

As the rarefaction information touches the center, the profiles look as

$t = 20.3042807006$

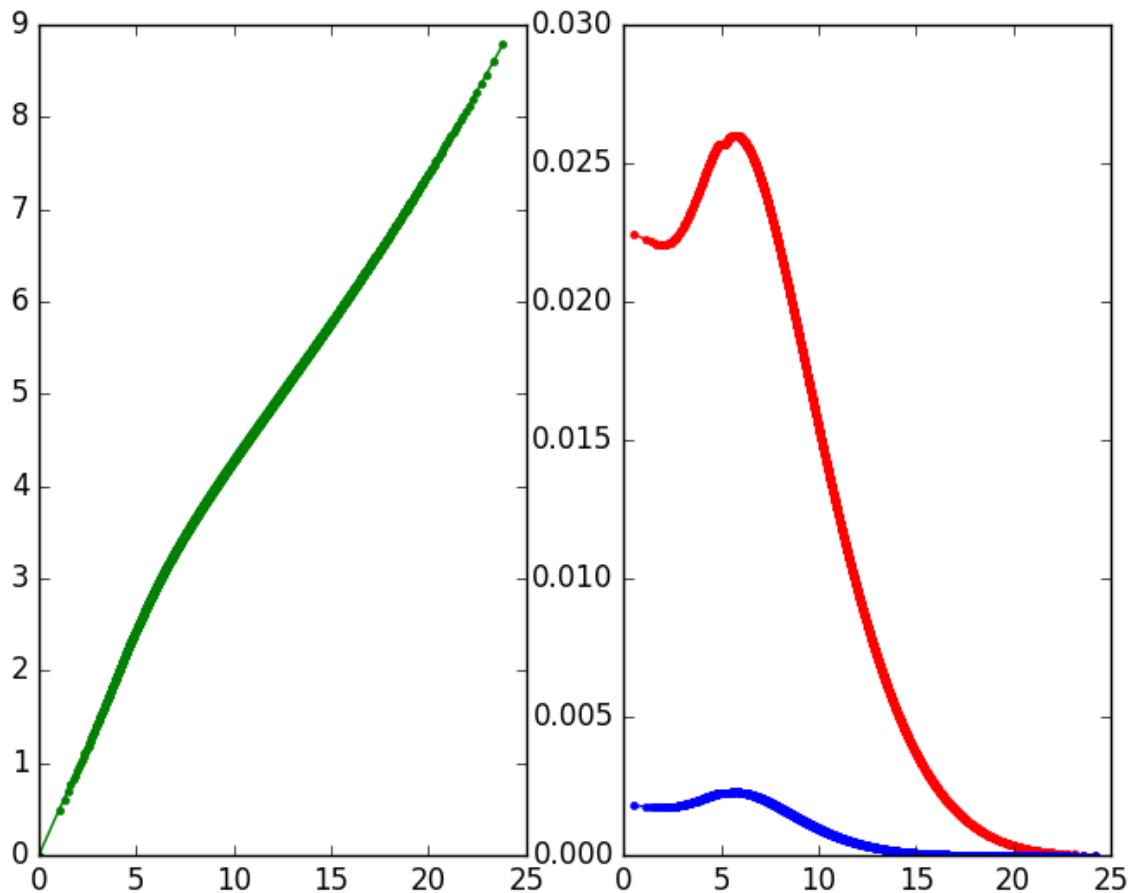


Figure 16: The profile for spherical rarefaction at $t = 20$