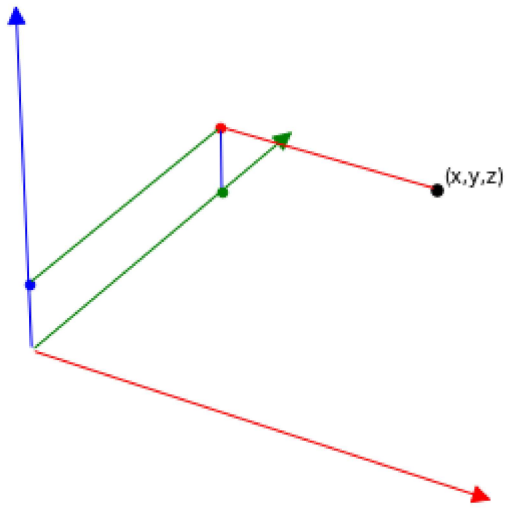
投影和重建

投影

笛卡尔坐标系

设在三维空间有一个向量，(x,y,z),然后这个空间有3个正交基(1,0,0),(0,1,0),(0,0,1)，那么我们将这个向量通过点乘的方法投影到这3个正交基上，分别得到3个系数，x，y，z 然后分别将这3个系数乘以三个基并且累加就可以还原这个向量的位置 = x*(1,0,0)*+y(0,1,0)+z*(0,0,1) = (x,y,z)

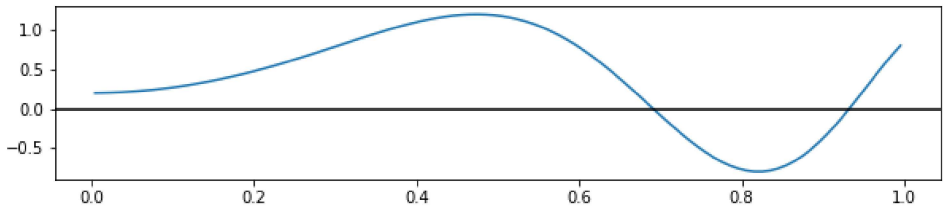


这个概念可以推广到N维空间

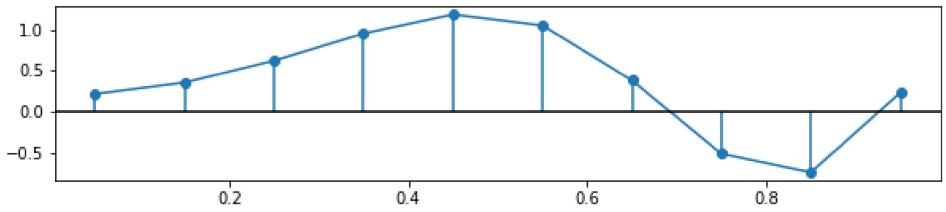
Vector Space

步骤如下:

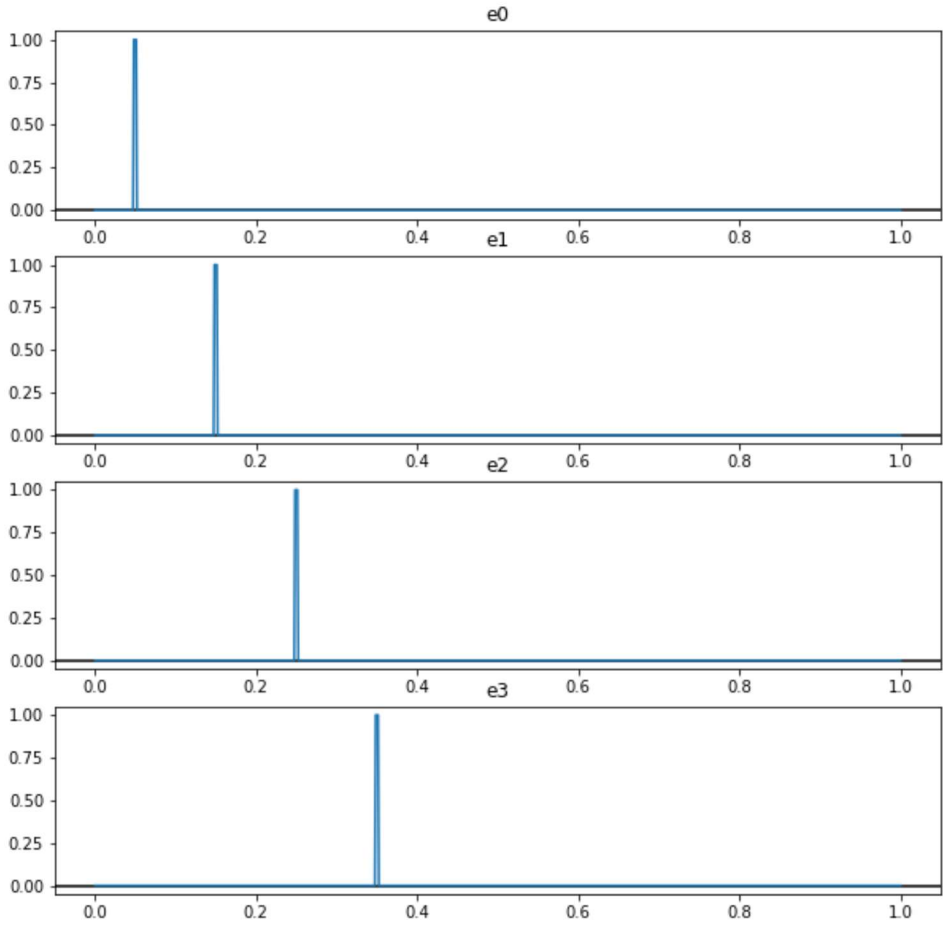
设有一个连续的函数F(x)



我们可以在一定区间内对其采样，得到N个sample，[x1,x2,x3,x4.....xn]，这样一个数组也可以将其看作一个N维的向量



设有一组基函数e(x)，他们只在采样点返回1，其余点返回0



对他们进行采样也能等到一组N维向量，基于定义我们得到的向量如下：

e0=[1,0,0,0,0,0,0...]

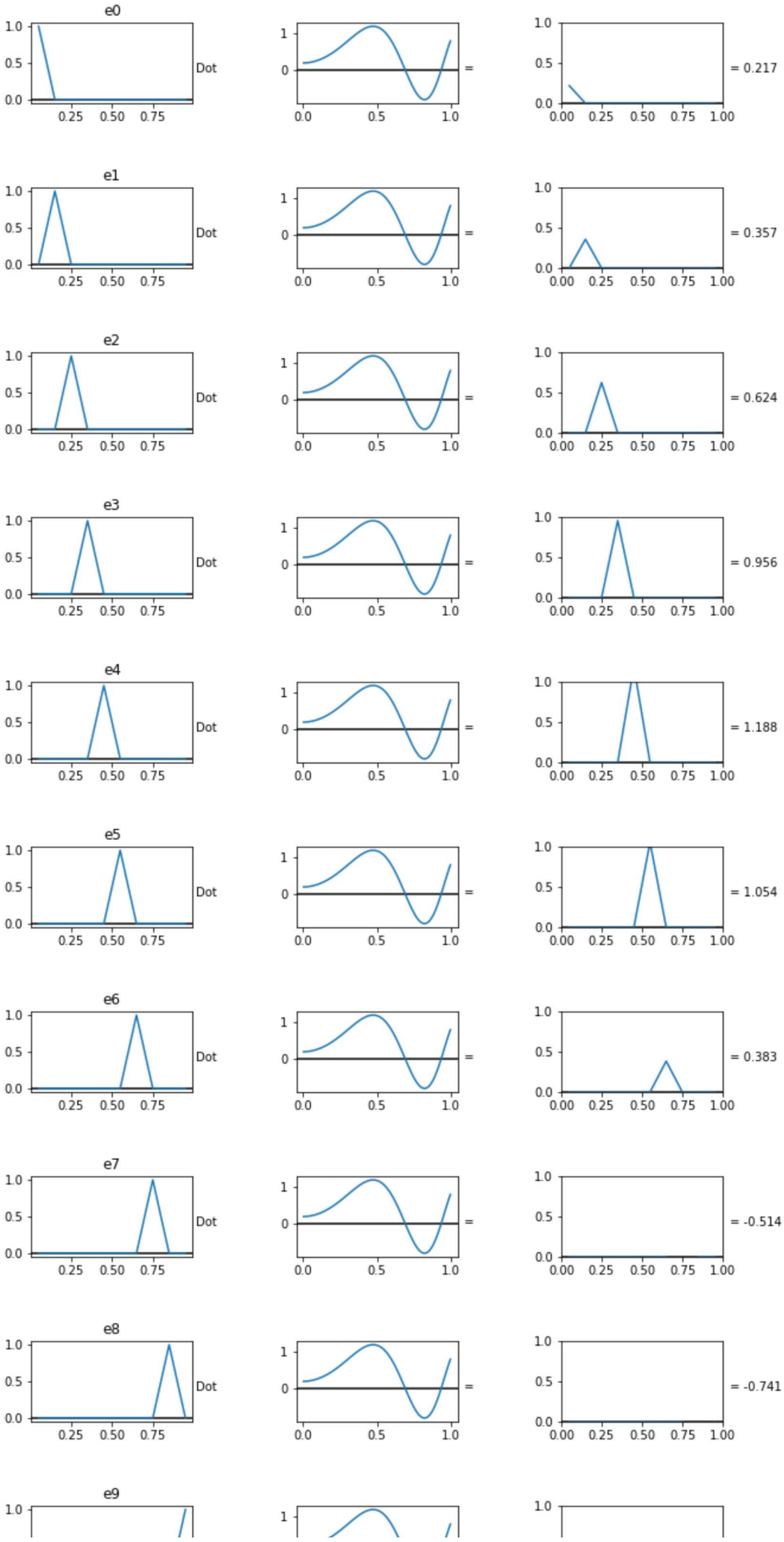
e1=[0,1,0,0,0,0,0...]

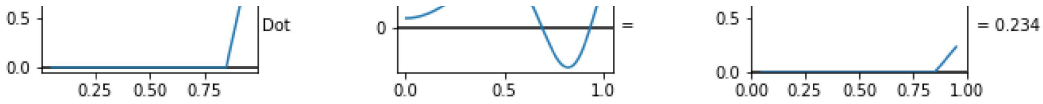
e2=[0,0,1,0,0,0,0...]

...

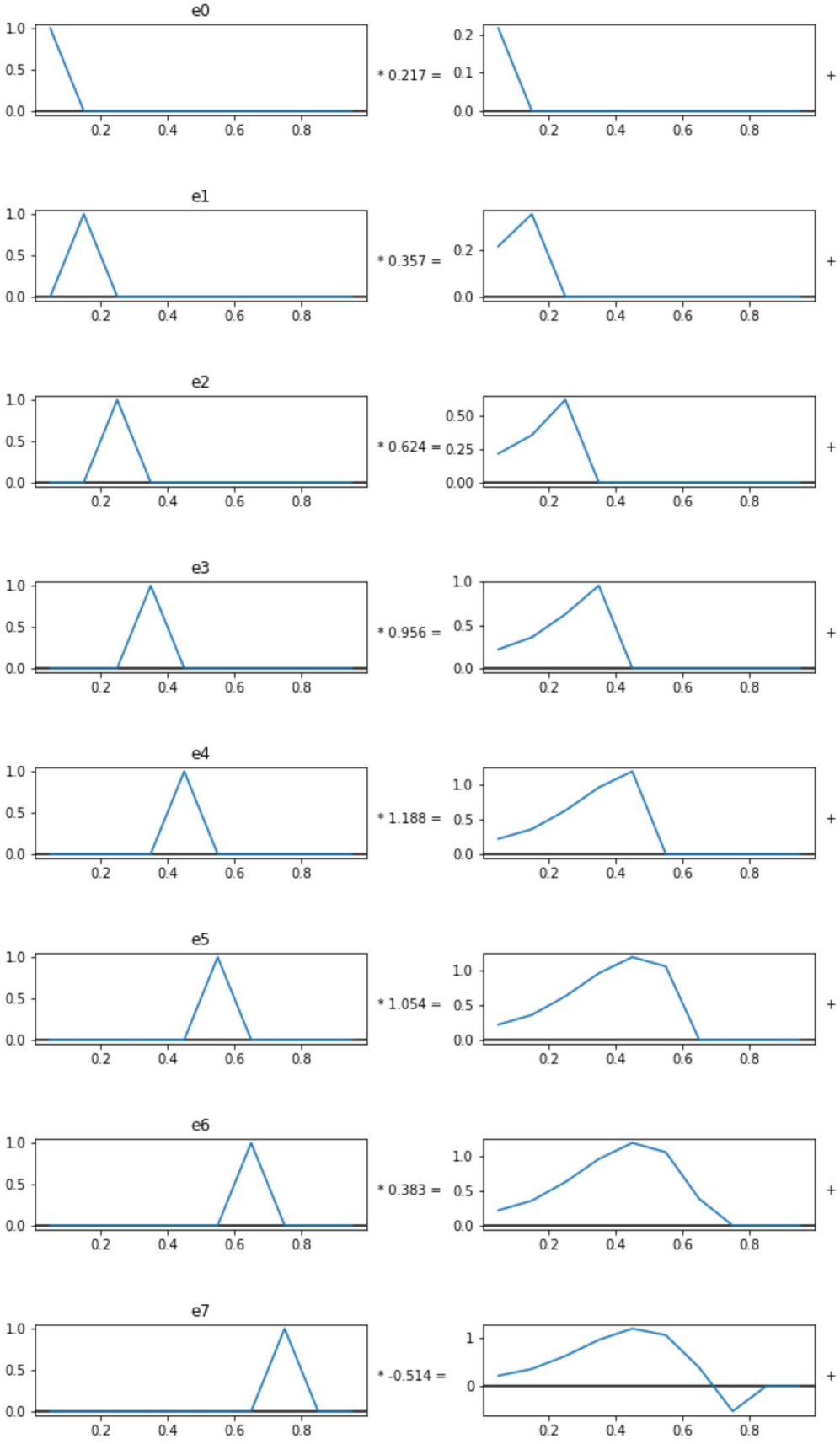
显然，他们两两正交

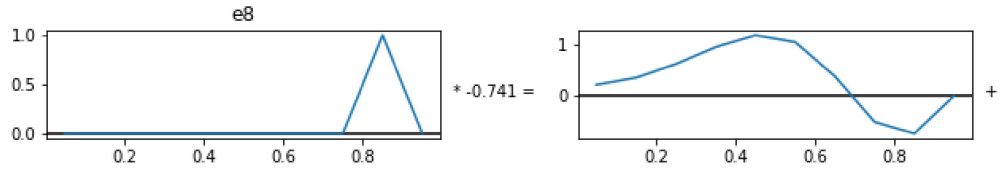
将刚才对原函数的采样得到的N维向量一次和这些基向量点乘，我们就完成了将原函数投影到一组正交基的过程

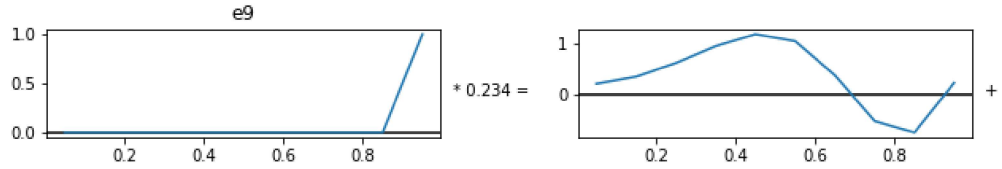

e0
Dot
=
= 0.217
e1
Dot
=
= 0.357
e2
Dot
=
= 0.624
e3
Dot
=
= 0.956
e4
Dot
=
= 1.188
e5
Dot
=
= 1.054
e6
Dot
=
= 0.383
e7
Dot
=
= -0.514
e8
Dot
=
= -0.741
e9

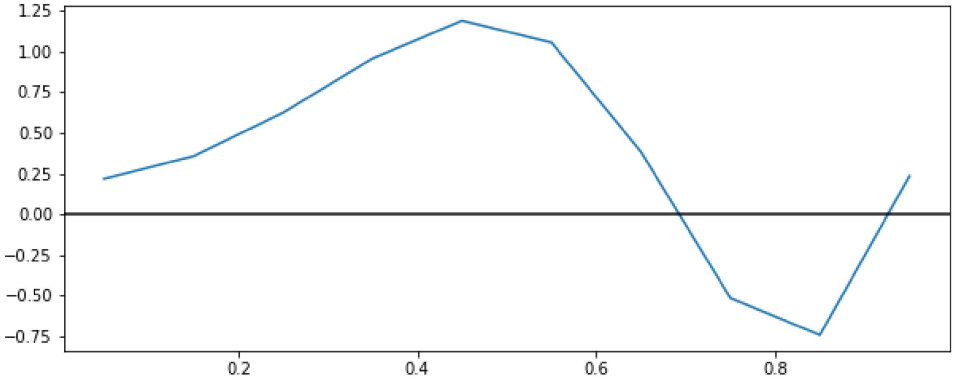


显然，如果我们过把N个sample逐个乘以基函数，然后将他们累加起来，就可以重建原函数




e8
1.0
0.5
0.0
0.2
0.4
0.6
0.8
* -0.741 =
1
0
+


e9
1.0
0.5
0.0
0.2
0.4
0.6
0.8
* 0.234 =
1
0
+


1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0.2
0.4
0.6
0.8