DA 605 - Assignment 13

Dan Fanelli

NUMERICAL INTEGRATION AND DIFFERENTIATION

- Write a program to compute the derivative of f(x) = x^3 + 2x^2 at any value of x.
- Your function should take in a value of x and return back an approximation to the derivative of f(x) evaluated at that value.
- You should not use the analytical form of the derivative to compute it. * Instead, you should compute this approximation using limits.

```
library(boot)

deriv_at <- function(x0){
  func_value_1 <- function(x){
    return (x * x * x + 2 * x * x)
  }
  y0 <- func_value_1(x0)
  x1 <- x0 + 0.0000001
  y1 <- func_value_1(x1)
  value <- (y1-y0)/(x1-x0)
  return (value)
}
```

Test our Derivatives:

in our heads, we work out the deriv to 3x^2 + 4x…so:

```
# at 2 it should be 20..
deriv_at(2)
```

```
## [1] 20
```

```
# at 4 it should be 64..
deriv_at(4)
```

```
## [1] 64
```

```
# at -1 it should be -1..
deriv_at(-1)
```

```
## [1] -1
```

```
# at 0 it should be 0..
deriv_at(0)
```

```
## [1] 2e-07
```

- Now, write a program to compute the area under the curve for the function 3x^2+4x in the range x = [1, 3].
- You should first split the range into many small intervals using some really small delta-x value (say 1e-6) and then compute the approximation to the area under the curve.

```
integral_of <- function(x_left, x_right){
  func_value_2 <- function(x){
    return (3 * x * x + 4 * x)
  }
  num_blocks <- 1000
  block_width <- (x_right - x_left) / num_blocks
  # go from left to right-1 (left sum)
  x_list <- seq(from = x_left, to = (x_right-block_width), by = block_width)
  block_heights <- func_value_2(x_list)

  return (block_width * sum(block_heights))
}

integral_of(1, 3)
```

```
## [1] 41.968
```

```
# which checks out by hand - ie - 42 is the exact value...
```

Scanned Problems:

Dan Fanelli – DA605 – Assignment #13

$\int \sin x \cos x$

$u = \sin x \qquad v = \sin x$

$du = \cos x \qquad dv = \cos x$

$\int \sin x \cos x = \sin^2 x - \int \sin x \cos x$

$2 \cdot \int \sin x \cos x = \sin^2 x$

$\int \sin x \cos x = \frac{1}{2} \sin^2 x$

$\int x^2 e^x$

$u = x^2 \qquad v = e^x$

$du = 2x \qquad dv = e^x$

$\int x^2 e^x = x^2 e^x - \int e^x \cdot 2x$

pull out the constant

$u = x \qquad v = e^x$

$du = 1 \qquad dv = e^x$

$\int x^2 e^x = x^2 e^x - 2(\underset{u}{x}\,\underset{v}{e^x} - \int e^x)$

$\int x^2 e^x = x^2 e^x - 2xe^x + 2e^x$

$= e^x(x^2 - 2x + 2)$

$$\frac{d}{dx}(x\cos x) = 1\cdot\cos x + x\cdot(-\sin x)$$

$$= \cos x - x\sin x$$

$$\frac{d}{dx}\left(e^{x^4}\right)$$

let $u = x^4$, so $(e^u)' = e^u du = e^{x^4}\cdot 4x$