IS 605 - Assignment 4

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Problem Set 1:

Given a 3 × 3 matrix A, write code in R to compute X = AAT and Y = ATA.

```
A <- matrix(c(-1,1,0,1,3,-2,5,0,1), ncol=3)
X <- A %*% t(A)
Y <- t(A) %*% A
```

EIGEN: "Then, compute the eigenvalues and eigenvectors of X and Y using the built-in commans in R."

```
eigen(X)$vectors
```

```
##            [,1]       [,2]       [,3]
## [1,] 0.99086430  0.0000000  0.1348627
## [2,] 0.07480836 -0.8320503 -0.5496326
## [3,] 0.11221254  0.5547002 -0.8244489
```

```
eigen(Y)$vectors
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.1747144  0.2223748  0.9591790
## [2,]  0.1889822  0.9636241 -0.1889822
## [3,]  0.9663129 -0.1482499  0.2103839
```

```
eigen(X)$values
```

```
## [1] 27.4907376 14.0000000  0.5092624
```

```
eigen(Y)$values
```

```
## [1] 27.4907376 14.0000000  0.5092624
```

SVD: "Then, compute the left-singular, singular values, and right-singular vectors of A using the svd command"

```
svd_obj <- svd(A)
svd_obj$u
```

```
##             [,1]          [,2]       [,3]
## [1,] -0.99086430 -2.103352e-17  0.1348627
## [2,] -0.07480836  8.320503e-01 -0.5496326
## [3,] -0.11221254 -5.547002e-01 -0.8244489
```

```
svd_obj$d
```

```
## [1] 5.2431610 3.7416574 0.7136263
```

```
svd_obj$v
```

```
##            [,1]       [,2]       [,3]
## [1,]  0.1747144  0.2223748 -0.9591790
## [2,] -0.1889822  0.9636241  0.1889822
## [3,] -0.9663129 -0.1482499 -0.2103839
```

"Examine the two sets of singular vectors and show that they are indeed eigenvectors of X and Y"

```
# svd u and eigen x are related:
svd_obj$u
```

```
##             [,1]          [,2]       [,3]
## [1,] -0.99086430 -2.103352e-17  0.1348627
## [2,] -0.07480836  8.320503e-01 -0.5496326
## [3,] -0.11221254 -5.547002e-01 -0.8244489
```

```
eigen(X)$vectors
```

```
##            [,1]       [,2]       [,3]
## [1,] 0.99086430  0.0000000  0.1348627
## [2,] 0.07480836 -0.8320503 -0.5496326
## [3,] 0.11221254  0.5547002 -0.8244489
```

```
# col 1: negatives of each other (adding yields zero)
round(svd_obj$u[,1] + eigen(X)$vectors[,1], digits = 4)
```

```
## [1] 0 0 0
```

```
# col 2: negatives of each other (adding yields zero)
round(svd_obj$u[,2] + eigen(X)$vectors[,2], digits = 4)
```

```
## [1] 0 0 0
```

```
# col 3: equal to each other (subtractinging yields zero)
round(svd_obj$u[,3] - eigen(X)$vectors[,3], digits = 4)
```

```
## [1] 0 0 0
```

```
# svd v and eigen y are related:
svd_obj$v
```

```
##            [,1]       [,2]       [,3]
## [1,]  0.1747144  0.2223748 -0.9591790
## [2,] -0.1889822  0.9636241  0.1889822
## [3,] -0.9663129 -0.1482499 -0.2103839
```

```
eigen(Y)$vectors
```

```
##            [,1]       [,2]       [,3]
## [1,] -0.1747144  0.2223748  0.9591790
## [2,]  0.1889822  0.9636241 -0.1889822
## [3,]  0.9663129 -0.1482499  0.2103839
```

```
# col 1: negatives of each other (adding yields zero)
round(svd_obj$v[,1] + eigen(Y)$vectors[,1], digits = 4)
```

```
## [1] 0 0 0
```

```
# col 2: equal to each other (subtractinging yields zero)
round(svd_obj$v[,2] - eigen(Y)$vectors[,2], digits = 4)
```

```
## [1] 0 0 0
```

```
# col 3: negatives of each other (adding yields zero)
round(svd_obj$v[,3] + eigen(Y)$vectors[,3], digits = 4)
```

```
## [1] 0 0 0
```

Problem Set 2:

Write function to computer matrix inverse using co-factors:

```
#
get_co_factor <- function(subM, r, c){
  sign <- (-1)^(r+c)
  det <- det(subM)
  return (sign * det)
}

get_co_factor_matrix <- function(M){
  cofactor_M <- M;
  for(c in 1:col(M)){
    for(r in 1:row(M)){
      cofactor_M[r,c] <- get_co_factor(M,r,c)
    }
  }
  return (cofactor_M)
}

myinverse <- function(A){
  cofactor_M <- get_co_factor_matrix(A)
  transpose_M <- t(cofactor_M)
  return (transpose_M / det(A))
}

in1 <- matrix(c(4,3,3,2), nrow=2)
out1 <- myinverse(in1)
in1 %*% out1
```

```
##      [,1] [,2]
## [1,]   -5  -18
## [2,]   -3  -13
```

```
in2 <- matrix(c(-24,20,-5,18,-15,4,5,-4,1), nrow=3)
out2 <- myinverse(in2)
in2 %*% out2
```

```
##      [,1] [,2] [,3]
## [1,]  325 -770  197
## [2,] -270  641 -164
## [3,]   72 -164   42
```