DA 605 - Assignment 2 - Problem Set 1

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(1) **Show that in general:**

$$A^T A \neq AA^T$$

Before showing in general, we'll show a simple counterexample:

```
> matrixA <- matrix(c(0,1,2,0), nrow = 2, ncol = 2)
> matrixA

     [,1] [,2]
[1,]    0    2
[2,]    1    0

> aTranspose <- t(matrixA)
> aTranspose

     [,1] [,2]
[1,]    0    1
[2,]    2    0

> matrixA %*% aTranspose

     [,1] [,2]
[1,]    4    0
[2,]    0    1

> aTranspose %*% matrixA

     [,1] [,2]
[1,]    1    0
[2,]    0    4
```

Now, we'll show in general (for two 2 x 2 matrices):

$$A^T =$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

and therefore

$$A^T =$$

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$A^T A =$$

$$\begin{pmatrix} a^2 + b^2 & ac + bd \\ ca + cd & c^2 + d^2 \end{pmatrix}$$

$$AA^T =$$

$$\begin{pmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{pmatrix}$$

which shows they are not equal.

(2) **For a special type of square matrix A, we get**

$$AA^T == A^T A$$

. Under what conditions could this be true? (Hint: The Identity matrix I is an example of such a matrix).

This is true whenever the matrices are symetric (As is the case with the identity matrix).