Homework Assignment #7 (7 points)

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive (+) for a serious disease (known as disease "X"). The accuracy of the test is as follows:

> The probability of testing positive (+) given that you have disease X is 0.95
>
> The probability of testing negative (-) given that you don't have disease X is 0.87.
>
> The good news is that disease X is rare, striking only one in 9,500 people.

1) Use the MAP procedure to determine what prognosis the doctor should give (do you have disease X, or not). (4 pts)

Hint: Start by writing the above English description in probabilistic form (each of the above sentences describes P(___) or P(___ | ___)).

2) Try again, using the ML procedure instead. Does this change your answer? (1pt)

3) Calculate the actual probability that you have the disease, given these test results. (Hint: Use Bayes' Rule.) (2 pts)

On all of the above, SHOW YOUR WORK!

① $P(+|X) = 0.95$, $P(-|\neg X) = 0.87$, $P(X) = 1/9500$

$P(-|X) = 0.05$ $P(+|\neg X) = 0.13$ $P(\neg X) = 9499/9500$

$h_1: P(+|X)P(X) = 0.95\left(\frac{1}{9500}\right) = 0.0001$

$h_2: P(+|\neg X)P(\neg X) = 0.13\left(\frac{9499}{9500}\right) = 0.129986$

$h_{MAP} = \neg X$

The doctor should give you about you don't have X.

② $h_1 : P(+|X) = 0.95$

$h_2 : P(+|\neg X) = 0.13$

$h_{ML} = X$

If changes my answer.

③ $P(X|+) = \dfrac{P(+|X)P(X)}{P(+)}$

$= \dfrac{P(+|X)P(X)}{P(+|X)P(X) + P(+|\neg X)P(\neg X)}$

$= \dfrac{0.95(0.0001)}{0.95(\frac{1}{9500}) + 0.13(\frac{9499}{9500})}$

$= 0.000769$