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CSE 3521

Homework #4

- a. `is_terminal()` and `utility(board, player)`

Testing board:

```
X
```

`is_terminal()` returns
false

`utility()` returns 0
(w.r.t. human player
selection)

Testing board:

```
O   X
is_terminal() returns
false
```

`utility()` returns 0
(w.r.t. human player
selection)

Testing board:

```
X   X   O
X   O   X
O
is_terminal() returns
false
```

`utility()` returns 0
(w.r.t. human player
selection)

Testing board:

```
X   X   O
```

```
X   O   X
X   O   O
is_terminal() returns
true
```

`utility()` returns 1
(w.r.t. human player
selection)

- b. Done
c. `tictactoe_minimax(board,cpu_player, cur_player)`

Turn 1:

```
X
Waiting on CPU player...
```

Complete. (Evaluated
55505 states, Expanded
29633 states)

CPU selects position 0
(Expected utility score
0)

Turn 2:

```
O
X
Waiting on human
player...
```

Player selects position
1

Turn 3:

```
O   X
X
Waiting on CPU player...
```

Complete. (Evaluated
1055 states, Expanded
582 states)

CPU selects position 7
(Expected utility score
0)

Turn 4:

O X
X
O

Waiting on human
player...

Player selects position
5

Turn 5:

O X
X X
O

Waiting on CPU player...

Complete. (Evaluated 47
states, Expanded 26
states)

CPU selects position 3
(Expected utility score
0)

Turn 6:

O X
O X X
O

Waiting on human
player...

Player selects position
6

Turn 7:

O X
O X X
X O

Waiting on CPU player...

Complete. (Evaluated 5
states, Expanded 3
states)

CPU selects position 2
(Expected utility score
0)

Turn 8:

O X O
O X X
X O

Waiting on human
player...

Player selects position
8

Turn 9:

O X O
O X X
X O X

Game complete.

Draw.

Turn 0:

Waiting on human
player...

Player selects position
2

Turn 1:

X
Waiting on CPU player...

Complete. (Evaluated
59705 states, Expanded
31973 states)

CPU selects position 4
(Expected utility score
0)

Turn 2:

X
O
Waiting on human
player...

Player selects position
6

Turn 3:

X
O
X
Waiting on CPU player...

Complete. (Evaluated
1053 states, Expanded
533 states)

CPU selects position 1
(Expected utility score
0)

Turn 4:

O X
O
X
Waiting on human
player...

Player selects position
7

Turn 5:

O X
O
X X
Waiting on CPU player...

Complete. (Evaluated 47
states, Expanded 26
states)

CPU selects position 8
(Expected utility score
0)

Turn 6:

O X
O
X X O
Waiting on human
player...

Player selects position
0

Turn 7:

X O X
O
X X O
Waiting on CPU player...

Complete. (Evaluated 5
states, Expanded 3
states)

CPU selects position 3
(Expected utility score
0)

Turn 8:

X O X
O O
X X O
Waiting on human
player...

Player selects position
5

Turn 9:

X O X

O O X
X X O
Game complete.

Draw.

Turn 0:

Waiting on human
player...

Player selects position
7

Turn 1:

X
Waiting on CPU player...

Complete. (Evaluated
63905 states, Expanded
34313 states)

CPU selects position 1
(Expected utility score
0)

Turn 2:

O
X
Waiting on human
player...

Player selects position
3

Turn 3:

O
X
X
Waiting on CPU player...

Complete. (Evaluated
1421 states, Expanded
805 states)

CPU selects position 6
(Expected utility score
0)

Turn 4:

O
X
O X
Waiting on human
player...

Player selects position
4

Turn 5:

O
X X
O X
Waiting on CPU player...

Complete. (Evaluated 51
states, Expanded 30
states)

CPU selects position 5
(Expected utility score
0)

Turn 6:

O
X X O
O X
Waiting on human
player...

Player selects position
2

Turn 7:

O X
X X O
O X
Waiting on CPU player...

Complete. (Evaluated 5
states, Expanded 3
states)

CPU selects position 0
(Expected utility score
0)

Turn 8:

O	O	X
X	X	O
O	X	

Waiting on human
player...

Player selects position
8

Turn 9:

O	O	X
X	X	O
O	X	X

Game complete.

Draw.

Our code plays the way as we
expected, and we cannot beat it.

The value of Evaluated and
Expanded are reducing dramatically
when we are having more moves. It
meets our expectation since when we
are making another movement, the
possible outcome is reduced.
However, the program still makes
some bad decisions, instead of trying
to win the game, it selected an
unprofitable path.

- d. Done
- e. `tictactoe_minimax_alphabeta(board,
cpu_player,cur_player,alpha,beta)`

Turn 0:

Waiting on human
player...

Player selects position
4

Turn 1:

X
Waiting on CPU player...

Complete. (Evaluated
2459 states, Expanded
1412 states)

CPU selects position 0
(Expected utility score
0)

Turn 2:

O
X
Waiting on human
player...

Player selects position
5

Turn 3:

O
X X
Waiting on CPU player...

Complete. (Evaluated 187
states, Expanded 105
states)

CPU selects position 3
(Expected utility score
0)

Turn 4:

O
O X X

Waiting on human
player...

Player selects position
6

Turn 5:

```

O
O   X   X
X

```

Waiting on CPU player...

Complete. (Evaluated 29
states, Expanded 18
states)

CPU selects position 2
(Expected utility score
0)

Turn 6:

```

O           O
O   X   X
X

```

Waiting on human
player...

Player selects position
1

Turn 7:

```

O   X   O
O   X   X
X

```

Waiting on CPU player...

Complete. (Evaluated 5
states, Expanded 3
states)

CPU selects position 7
(Expected utility score
0)

Turn 8:

```

O   X   O
O   X   X
X   O

```

Waiting on human
player...

Player selects position
8

Turn 9:

```

O   X   O
O   X   X
X   O   X

```

Game complete.

Draw.

Turn 0:

Waiting on human
player...

Player selects position
4

Turn 1:

```

X

```

Waiting on CPU player...

Complete. (Evaluated
2459 states, Expanded
1412 states)

CPU selects position 0
(Expected utility score
0)

Turn 2:

```

O
X

```

Waiting on human
player...

Player selects position
1

Turn 3:

O X
X

Waiting on CPU player...

Complete. (Evaluated 327
states, Expanded 195
states)

CPU selects position 7
(Expected utility score
0)

Turn 4:

O X
X
O

Waiting on human
player...

Player selects position
8

Turn 5:

O X
X
O X

Waiting on CPU player...

Complete. (Evaluated 43
states, Expanded 27
states)

CPU selects position 2
(Expected utility score
0)

Turn 6:

O X O
X
O X

Waiting on human
player...

Player selects position
3

Turn 7:

O X O
X X
O X

Waiting on CPU player...

Complete. (Evaluated 5
states, Expanded 3
states)

CPU selects position 5
(Expected utility score
0)

Turn 8:

O X O
X X O
O X

Waiting on human
player...

Player selects position
6

Turn 9:

O X O
X X O
X O X

Game complete.

Draw.

Turn 0:

Waiting on human
player...

Player selects position
2

Turn 1:

X
Waiting on CPU player...

Complete. (Evaluated
3648 states, Expanded
2143 states)

CPU selects position 4
(Expected utility score
0)

Turn 2:

X
O
Waiting on human
player...

Player selects position
8

Turn 3:

X
O
X
Waiting on CPU player...

Complete. (Evaluated 295
states, Expanded 171
states)

CPU selects position 5
(Expected utility score
0)

Turn 4:

X
O O
X
Waiting on human
player...

Player selects position
3

Turn 5:

X
X O O
X
Waiting on CPU player...

Complete. (Evaluated 47
states, Expanded 29
states)

CPU selects position 0
(Expected utility score
0)

Turn 6:

O X
X O O
X
Waiting on human
player...

Player selects position
7

Turn 7:

O X
X O O
X X
Waiting on CPU player...

Complete. (Evaluated 5
states, Expanded 3
states)

CPU selects position 6
(Expected utility score
0)

Turn 8:

O X


```

X    O    O
O    X    X
Waiting on human
player...

```

```

Player selects position
1

```

Turn 9:

```

O    X    X
X    O    O
O    X    X
Game complete.

```

Draw.

The value of Evaluated and Expanded are reducing dramatically when we are having more moves. It meets our expectation since when we are making another movement, the possible outcome is reduced. And the program is no-longer making unprofitable movements.

f. D

MiniMax:

Turn 0:

Waiting on CPU player...

Complete. (Evaluated 549946 states, Expanded 294778 states)

CPU selects position 0
(Expected utility score 0)

Turn 1:

```

X
Waiting on human
player...

```

Player selects position 4

Turn 2:

```

X
O
Waiting on CPU player...

```

Complete. (Evaluated 7332 states, Expanded 3864 states)

CPU selects position 1
(Expected utility score 0)

Turn 3:

```

X    X
O
Waiting on human
player...

```

Player selects position 2

Turn 4:

```

X    X    O
O
Waiting on CPU player...

```

Complete. (Evaluated 198 states, Expanded 104 states)

CPU selects position 6
(Expected utility score 0)

Turn 5:

```

X    X    O
O
X
Waiting on human
player...

```

Player selects position
3

Turn 6:

```
X   X   O
O   O
X
```

Waiting on CPU player...

Complete. (Evaluated 14
states, Expanded 8
states)

CPU selects position 5
(Expected utility score
0)

Turn 7:

```
X   X   O
O   O   X
X
```

Waiting on human
player...

Player selects position
8

Turn 8:

```
X   X   O
O   O   X
X           O
```

Waiting on CPU player...

Complete. (Evaluated 2
states, Expanded 1
states)

CPU selects position 7
(Expected utility score
0)

Turn 9:

```
X   X   O
```

```
O   O   X
X   X   O
Game complete.
```

Draw.

MiniMax w/ Alpha-Beta Pruning:

Turn 0:

Waiting on CPU player...

Complete. (Evaluated
20866 states, Expanded
12413 states)

CPU selects position 0
(Expected utility score
0)

Turn 1:

```
X
Waiting on human
player...
```

Player selects position
4

Turn 2:

```
X
O
Waiting on CPU player...
```

Complete. (Evaluated 871
states, Expanded 525
states)

CPU selects position 1
(Expected utility score
0)

Turn 3:

```
X   X
O
```

Waiting on human
player...

Player selects position
2

Turn 4:

```

X   X   O
O

```

Waiting on CPU player...

Complete. (Evaluated 64
states, Expanded 38
states)

CPU selects position 6
(Expected utility score
0)

Turn 5:

```

X   X   O
O
X

```

Waiting on human
player...

Player selects position
3

Turn 6:

```

X   X   O
O   O
X

```

Waiting on CPU player...

Complete. (Evaluated 10
states, Expanded 6
states)

CPU selects position 5
(Expected utility score
0)

Turn 7:

```

X   X   O
O   O   X
X

```

Waiting on human
player...

Player selects position
8

Turn 8:

```

X   X   O
O   O   X
X       O

```

Waiting on CPU player...

Complete. (Evaluated 2
states, Expanded 1
states)

CPU selects position 7
(Expected utility score
0)

Turn 9:

```

X   X   O
O   O   X
X   X   O

```

Game complete.

Draw.

Compare to two algorithms,
MiniMax w/ Alpha-Beta Pruning
have relatively few numbers of
Evaluated and Expanded states.
Results meet our expectation, but it
is unexpected that both algorithms
chose the exact same path.

MiniMax:

Turn 0:

Waiting on CPU player...

Complete. (Evaluated
549946 states, Expanded
294778 states)

CPU selects position 4
(Expected utility score
0)

Turn 1:

X
Waiting on human
player...

Player selects position
0

Turn 2:

O
X
Waiting on CPU player...

Complete. (Evaluated
6812 states, Expanded
3614 states)

CPU selects position 1
(Expected utility score
0)

Turn 3:

O X
X
Waiting on human
player...

Player selects position
7

Turn 4:

O X
X
O
Waiting on CPU player...

Complete. (Evaluated 258
states, Expanded 150
states)

CPU selects position 3
(Expected utility score
0)

Turn 5:

O X
X X
O
Waiting on human
player...

Player selects position
5

Turn 6:

O X
X X O
O
Waiting on CPU player...

Complete. (Evaluated 16
states, Expanded 10
states)

CPU selects position 2
(Expected utility score
0)

Turn 7:

O X X
X X O
O
Waiting on human
player...

Player selects position
6

Turn 8:

O X X

X X O
O O
Waiting on CPU player...

Complete. (Evaluated 2
states, Expanded 1
states)

CPU selects position 8
(Expected utility score
0)

Turn 9:

O X X
X X O
O O X
Game complete.

Draw.

MiniMax w/ Alpha-Beta Pruning:

Turn 0:

Waiting on CPU player...

Complete. (Evaluated
12826 states, Expanded
7365 states)

CPU selects position 4
(Expected utility score
0)

Turn 1:

X
Waiting on human
player...

Player selects position
0

Turn 2:

O

X
Waiting on CPU player...

Complete. (Evaluated 832
states, Expanded 497
states)

CPU selects position 1
(Expected utility score
0)

Turn 3:

O X
X
Waiting on human
player...

Player selects position
7

Turn 4:

O X
X
O
Waiting on CPU player...

Complete. (Evaluated 105
states, Expanded 67
states)

CPU selects position 3
(Expected utility score
0)

Turn 5:

O X
X X
O
Waiting on human
player...

Player selects position
5

Turn 6:

```

O    X
X    X    O
O
Waiting on CPU player...

```

Complete. (Evaluated 12 states, Expanded 8 states)

CPU selects position 2
(Expected utility score 0)

Turn 7:

```

O    X    X
X    X    O
O
Waiting on human
player...

```

Player selects position 6

Turn 8:

```

O    X    X
X    X    O
O    O
Waiting on CPU player...

```

Complete. (Evaluated 2 states, Expanded 1 states)

CPU selects position 8
(Expected utility score 0)

Turn 9:

```

O    X    X
X    X    O
O    O    X
Game complete.

```

Draw.

The new move_expand_order will slightly reduce the number of Evaluated, Expanded for MiniMax. However, the new move_expand_order significantly reduced the number of Evaluated, Expanded for MiniMax with Alpha-Beta Pruning (~8000 Evaluated, ~5000 Expanded).

MiniMax
Evaluated, Expanded
549946,294778
7332,3864
198,104
14,8
2,1

MiniMax w/ new
move_expand_order
Evaluated, Expanded
549946,294778
6812,3614
258,150
16,10
2,1

MiniMax w/ Alpha-Beta Pruning
Evaluated, Expanded
20866,12413
871,525
64,38
10,6
2,1

MiniMax w/ new
move_expand_order, Alpha-Beta
Pruning
Evaluated, Expanded
12826,7365
832,497
105,67

12,8
2,1