# Inference

CPSC 470 – Artificial Intelligence

Brian Scassellati

# Analogies to Programming

**Programming**

1. Choose a language
2. Write a program
3. Choose or write a compiler
4. Run your compiled program

**Knowledge Engineering**

1. Choose a logic
2. Build a Knowledge Base
3. Implement the proof theory
4. Infer new facts

How do we automate the process of inferring new facts?

# Review of Inference in Propositional Logic

# Inference Rules for Propositional Logic

- Modus Ponens (Implication-Elimination)
  - From an implication and its premise, infer conclusion

$$\frac{\alpha \Rightarrow \beta \ , \alpha}{\beta}$$

- And-Elimination
  - From a conjunction, you can infer any conjunct

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \alpha_3 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

# Inference Rules for Propositional Logic

- And-Introduction
  - From a list of sentences, you can infer the conjunct

$$\frac{\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$

- Or-Introduction
  - From a sentence, infer its disjunction with anything

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$

# Inference Rules for Propositional Logic

- Double-Negative Elimination
  - From a double negation, infer the positive sentence

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit Resolution
  - From a disjunction in which one is false, then you can infer the other is true

$$\frac{\alpha \vee \beta\ ,\ \ \neg\beta}{\alpha}$$

# Inference Rules for Propositional Logic

- Resolution
  - Since beta cannot be both true and false, one of the disjuncts must be true

$$\frac{\alpha \vee \beta \ , \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

  - Implication is transitive

$$\frac{\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

# First-Order Logic Requires Additional Rules of Inference

# Inference involving Quantifiers

- Remove variables using a substitution function

  $Subst(\theta,\alpha)$ : apply the binding list $\theta$ to the sentence $\alpha$

  $Subst(\{x/Tom, y/Jerry\}, Chases(x,y))=$

  $Chases(Tom,Jerry)$

# Inference involving Quantifiers

- Universal Elimination
  - For any sentence α, variable *v*, and ground term *g*

$$\frac{\forall v \ \alpha}{Subst(\{v/g\},\alpha)}$$

  - For example, from ∀x Likes(x,FrenchFries) we can substitute {x/Ben} to conclude Likes(Ben,FrenchFries)

# Inference involving Quantifiers

- Existential Elimination
  - For any sentence α, variable *v*, and constant symbol *k* that does not appear elsewhere

$$\frac{\exists v \ \alpha}{Subst(\{v/k\},\alpha)}$$

  - For example, ∃x Kill(x,Victim) we can infer Kill(Murderer,Victim)
  - ONLY works when Murderer does not already appear in the KB

# Inference involving Quantifiers

- Existential Introduction
  - For any sentence α, variable *v* that does **not** occur in α and ground term *g* that does appear in α

$$\frac{\alpha}{\exists v\ Subst(\{g/v\},\alpha)}$$

  - For example, from Speaks(Jim,German) we can infer ∃x Speaks(x,German)

# An Example Proof

It is a crime for an American to sell alcohol to a minor. Jimmy, a minor, has some beer. All of Jimmy's beer was sold to him by Nathan, an American.

Prove that Nathan is a criminal.

Goal: Criminal(Nathan)

1. $\forall$x,y,z American(x) $\wedge$ Alcohol(y) $\wedge$ Minor(z) $\wedge$ Sells(x,y,z) $\Rightarrow$ Criminal(x)
2. Minor(Jimmy)
3. $\exists$x Owns(Jimmy,x) $\wedge$ Beer(x)
4. $\forall$x Owns(Jimmy,x) $\wedge$ Beer(x) $\Rightarrow$ Sells(Nathan,x,Jimmy)
5. American(Nathan)
6. $\forall$x Beer(x)$\Rightarrow$Alcohol(x)

| # | FOPC statement | Reasoning |
| --- | --- | --- |
| | Criminal(Nathan) | -- GOAL -- |
| 1 | $\forall$x,y,z American(x) $\wedge$ Alcohol(y) $\wedge$ Minor(z) $\wedge$ Sells(x,y,z) $\Rightarrow$ Criminal(x) | given |
| 2 | Minor(Jimmy) | given |
| 3 | $\exists$x Owns(Jimmy,x) $\wedge$ Beer(x) | given |
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| 5 | American(Nathan) | given |
| 6 | $\forall$x Beer(x)$\Rightarrow$Alcohol(x) | given |
| 7 | Owns(Jimmy, B1) $\wedge$ Beer(B1) | Existential elim on 3 |

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| 9 | Beer(B1) $\Rightarrow$ Alcohol(B1) | Universal elim on 6 |

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| 11 | Owns(Jimmy,B1) $\wedge$ Beer(B1) $\Rightarrow$ Sells(Nathan,B1,Jimmy) | Universal elim on 4 |

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| 12 | Sells(Nathan,B1,Jimmy) | Modus ponens 11 7 |

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| 13 | American(Nathan) $\wedge$ Alcohol(B1) $\wedge$ Minor(Jimmy) $\wedge$ Sells(Nathan,B1,Jimmy) $\Rightarrow$ Criminal(Nathan) | Universal elim on 1 (x3) |

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| 14 | American(Nathan) ∧ Alcohol(B1) ∧ Minor(Jimmy) ∧ Sells(Nathan,B1,Jimmy) | And introduction 5, 10, 2, 12 |

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| 14 | American(Nathan) $\wedge$ Alcohol(B1) $\wedge$ Minor(Jimmy) $\wedge$ Sells(Nathan,B1,Jimmy) | And introduction 5, 10, 2, 12 |
| 15 | Criminal(Nathan) | Modus ponens 13, 14 |

# Can we perform inference as a search problem?

- Our example proof has 6 initial sentences and 9 additional proof steps
  - Initial state: 6 sentences
  - Operators: ~10 inference rules
    - But can be applied multiple ways
  - Goal: obtain the sentence Criminal(Nathan)
- Branching factor increases as the knowledge base grows
- Universal elimination can have a large branching factor on its own (any ground term can be used)

# Unification

- Unification is the process of finding substitutions that match a set of conditions
- The UNIFY algorithm takes two sentences and returns a unifier (a binding list) for them if one exists:

  UNIFY(p,q)=Θ where SUBST(Θ,p)=SUBST(Θ,q)

# Unification Examples

- Example using Knows(x,y)

  UNIFY( Knows(John, x), Knows(John, Jane) )

  = {x/Jane}

  UNIFY( Knows(John, x), Knows(y, Bill) )

  = {x/Bill, y/John}

  UNIFY( Knows(John, x), Knows(y, Mother(y)) )

  = {y/John, x/Mother(John)}

  UNIFY( Knows(John, x), Knows(x, Elizabeth) )

  = FAILURE

  UNIFY( Knows(John, $x_1$), Knows($x_2$, Elizabeth) )

  = {$x_1$/Elizabeth, $x_2$/John}

# Using Inference Rules

We can use these rules in two ways

- We can generate new inferences from existing sentences to expand the knowledge base
  - Used when a new sentence is added to KB
  - From premises to implications
  - Forward chaining
- We can try to prove a given sentence
  - From implications to premises
  - Backward chaining

# Forward Chaining

- Forward-Chaining:
  - Until no rule produces a new assertion
    - For each rule,
      - For each set of possible variable bindings
        - » Instantiate the consequent
        - » If the instantiated consequent is not already asserted, then assert it.
- Data-driven process (not driven toward any particular goal)

# Forward Chaining Example

1. $\forall x,y,z$ American(x) $\wedge$ Alcohol(y) $\wedge$ Minor(z) $\wedge$ Sells(x,y,z) $\Rightarrow$ Criminal(x)
2. Minor(Jimmy)
3. $\exists x$ Owns(Jimmy,x) $\wedge$ Beer(x)
4. $\forall x$ Owns(Jimmy,x) $\wedge$ Beer(x) $\Rightarrow$ Sells(Nathan,x,Jimmy)
5. American(Nathan)
6. $\forall x$ Beer(x)$\Rightarrow$Alcohol(x)

- Consider all the base terms: Nathan, Jimmy, B1, B2, ….
- Start instantiating #6

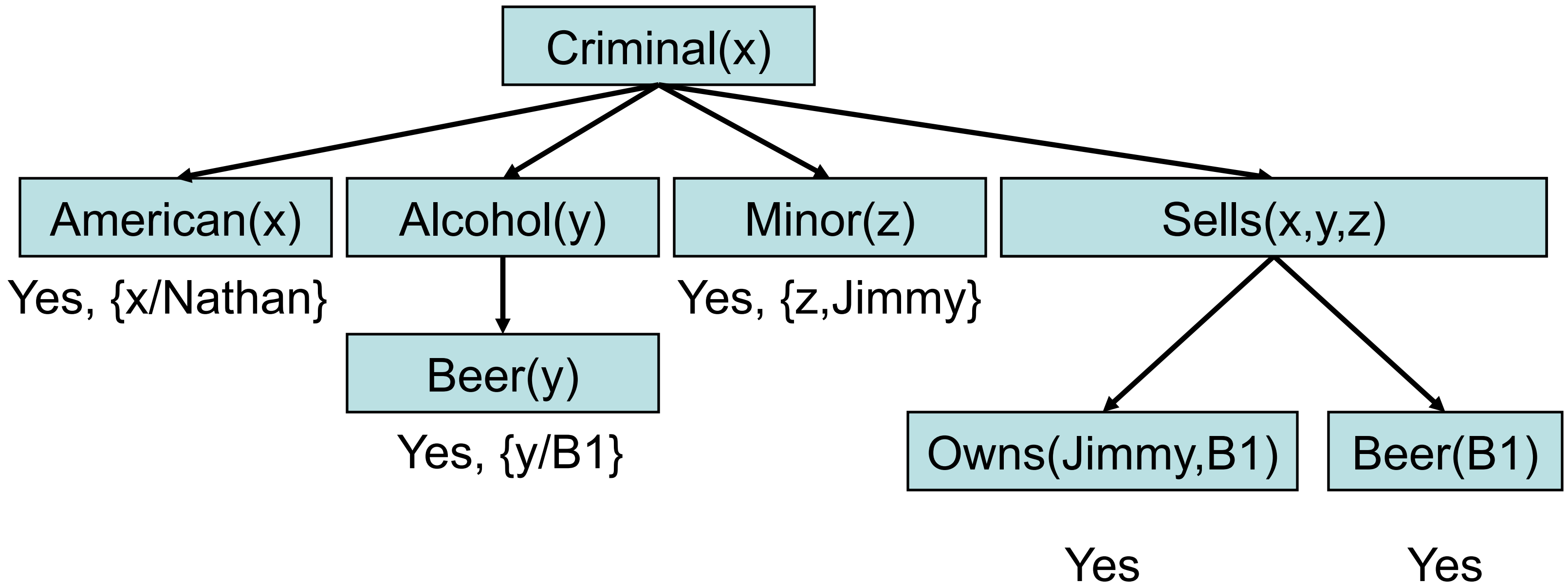
| | |
|---|---|
| Beer(Nathan) $\Rightarrow$ Alcohol(Nathan) | using {x/Nathan} |
| Beer(Jimmy) $\Rightarrow$ Alcohol(Jimmy) | using {x/Jimmy} |
| Beer(B1) $\Rightarrow$ Alcohol(B1) | using {x/B1} |
| Beer(B2) $\Rightarrow$ Alcohol(B2) | using {x/B2} |

Leads to a very disorganized (and full) knowledge base!

# Backward Chaining

- Goal-directed
- Starts with a goal state
- Moves backward through implications
- Attempts to construct a set of basic sentences in the KB

# Backward Chaining




1. $American(x) \wedge Alcohol(y) \wedge Minor(z) \wedge Sells(x,y,z) \Rightarrow Criminal(x)$
2. Minor(Jimmy)
3. Owns(Jimmy,B1)
4. Beer(B1)
5. $Owns(Jimmy,x) \wedge Beer(x) \Rightarrow Sells(Nathan,x,Jimmy)$
6. American(Nathan)
7. $Beer(x) \Rightarrow Alcohol(x)$

# Failures of Modus Ponens

- We've been using modus ponens as the primary tool for inference
  - But modus ponens does not allow us to deduce new implications, it only derives atomic conclusions

  $P(x) \Rightarrow Q(x)$ $Q(x) \Rightarrow S(x)$

  $\neg P(x) \Rightarrow R(x)$ $R(x) \Rightarrow S(x)$

  Show that $S(A)$ is true

  - Problem is that $\neg P(x) \Rightarrow R(x)$ cannot be converted into Horn form
- We need a more powerful inference rule!
  - Resolution!

# Resolution

- Since beta cannot be both true and false, one of the disjuncts must be true

$$\frac{\alpha \vee \beta \ , \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

- Implication is transitive

$$\frac{\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

# Modus Ponens is a Simplified version of Resolution

- Resolution

$$\frac{\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\alpha \Rightarrow \gamma}$$

- Modus Ponens

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta} \qquad is\ the\ same\ as \qquad \frac{True \Rightarrow \alpha, \alpha \Rightarrow \beta}{True \Rightarrow \beta}$$

- Thus, resolution is a more general (and more powerful) format than modus ponens

# Resolution has a Canonical Form

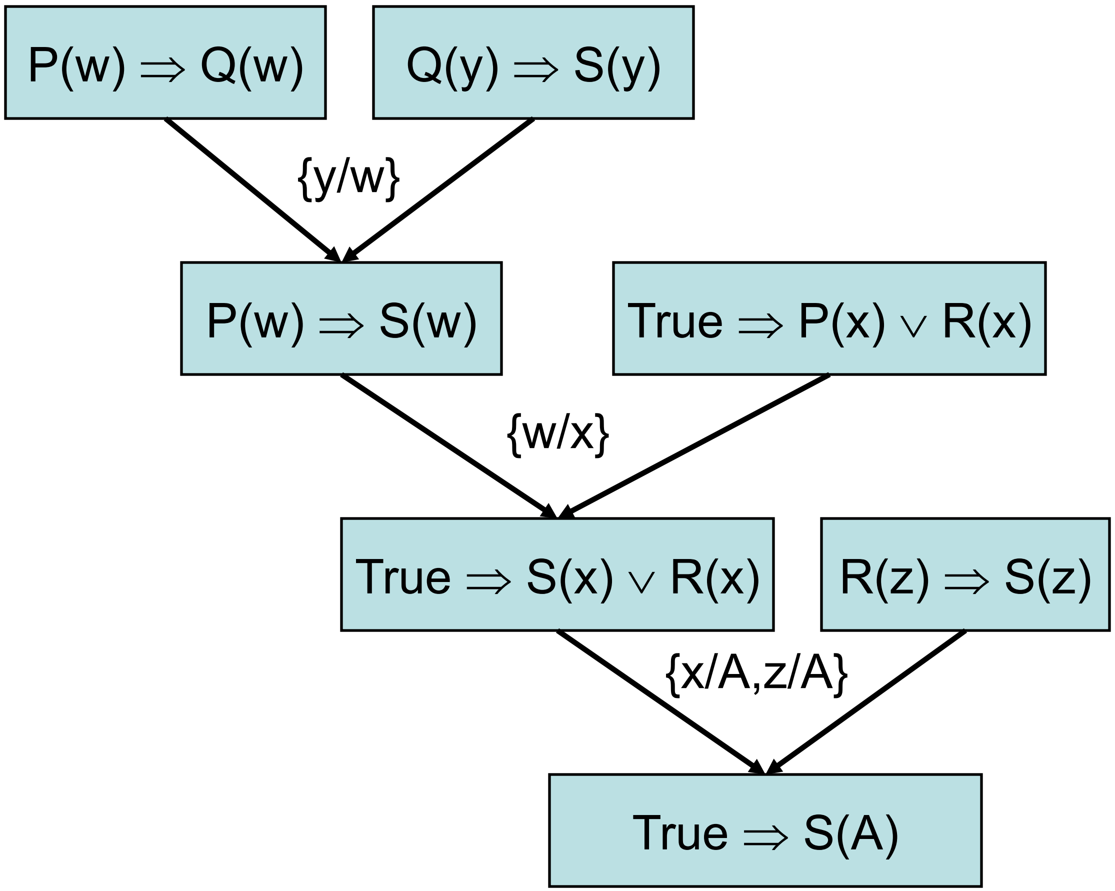
- Using this variant of resolution:

$$\frac{\alpha \vee \beta \,,\, \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

- We define the conjunctive normal form as the conjunction of all sentences in the knowledge base, where each sentence is a disjunction of literals

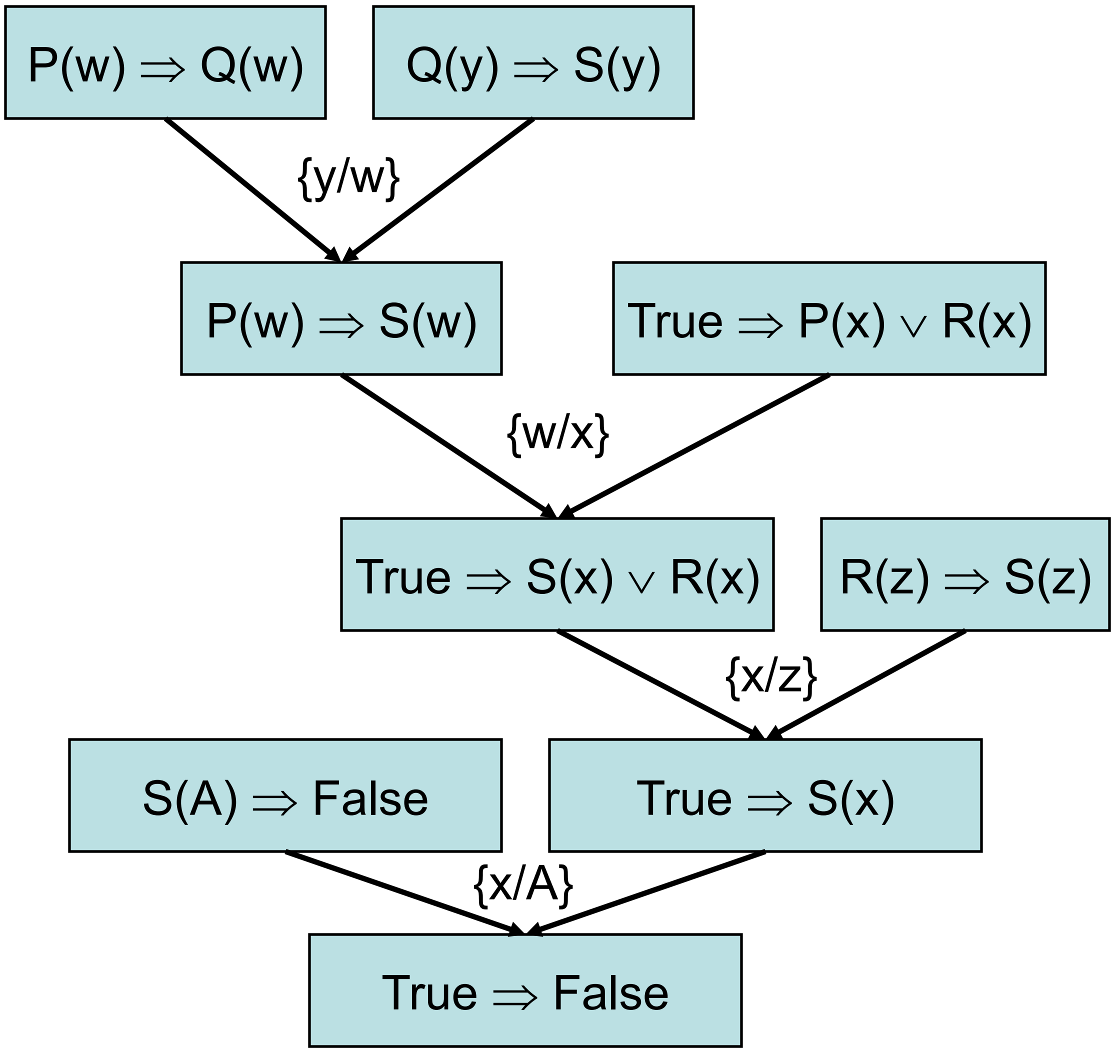
$$(p_1 \vee p_2 \vee \ldots \vee p_n) \wedge (q_1 \vee q_2 \vee \ldots \vee q_n) \wedge \ldots$$

# Resolution Proofs




- KB:
  - $P(w) \Rightarrow Q(w)$
  - $Q(y) \Rightarrow S(y)$
  - $\neg P(x) \Rightarrow R(x)$
  - $R(z) \Rightarrow S(z)$
- Could be rewritten in CNF
  - $\neg P(w) \vee Q(w)$
  - $\neg Q(y) \vee S(y)$
  - $P(x) \vee R(x)$
  - $\neg R(z) \vee S(z)$
- Or more simply as:
  - $P(w) \Rightarrow Q(w)$
  - $Q(y) \Rightarrow S(y)$
  - $True \Rightarrow P(x) \vee R(x)$
  - $R(z) \Rightarrow S(z)$
- Prove S(A) follows

# Resolution Proofs




- Some statements are valid, but cannot be shown using resolution
  - $P \vee \neg P$
- Refutation (proof by contradiction)
  - To prove S(A) is true, assume S(A) is false and show a contradiction

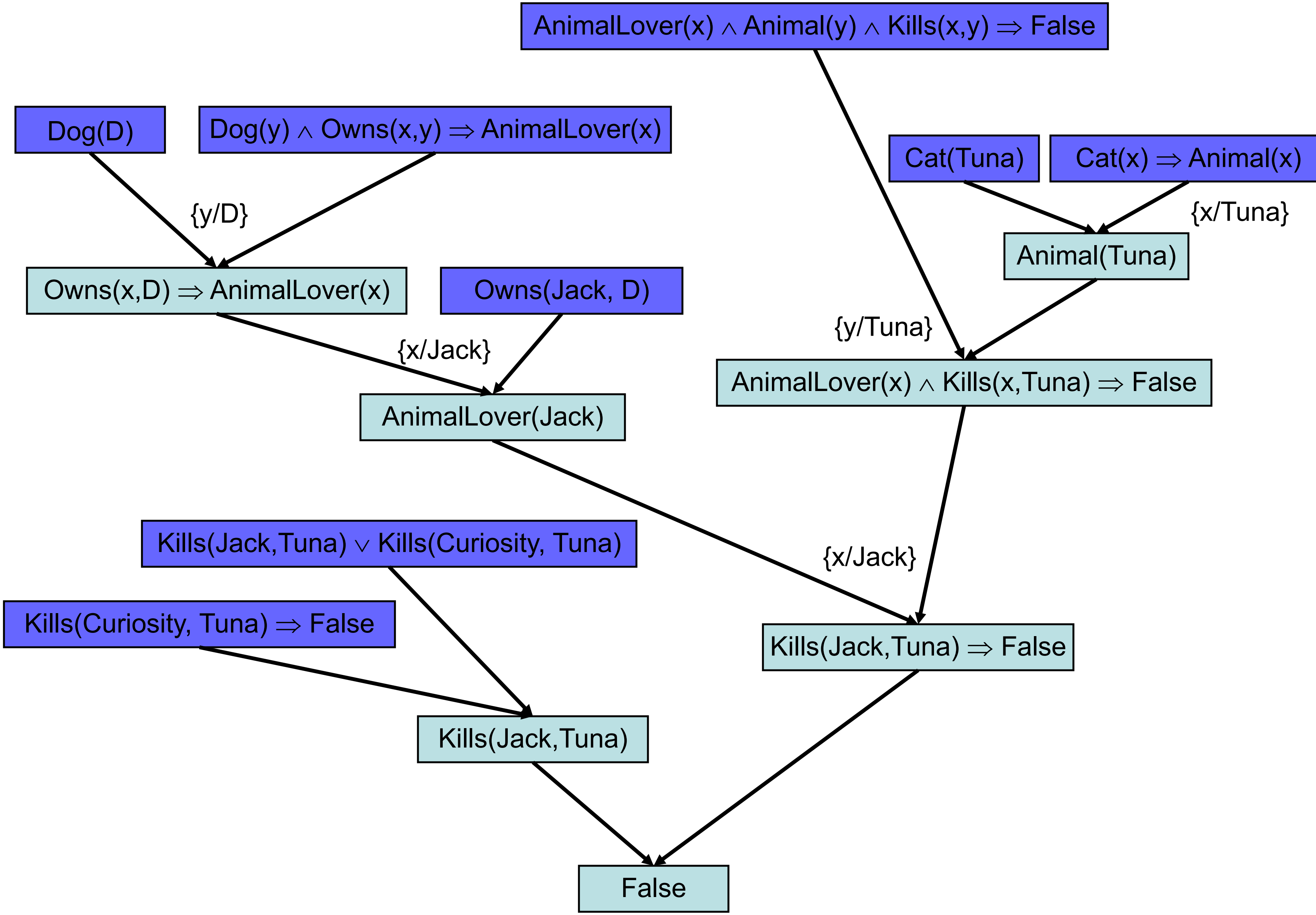
# One last Example

- Every dog owner is an animal lover
- No animal lover kills an animal
- Either Jack or Curiosity killed the cat, who is named Tuna
- A cat is an animal
- Jack owns a dog

- Did Curiosity kill the cat?

- $\text{Dog}(y) \wedge \text{Owns}(x,y) \Rightarrow \text{AnimalLover}(x)$
- $\text{AnimalLover}(x) \wedge \text{Animal}(y) \wedge \text{Kills}(x,y) \Rightarrow \text{False}$
- $\text{Kills}(\text{Jack},\text{Tuna}) \vee \text{Kills}(\text{Curiosity}, \text{Tuna})$
- Cat(Tuna)
- $\text{Cat}(x) \Rightarrow \text{Animal}(x)$
- Dog(D)
- Owns(Jack, D)

- $\text{Kills}(\text{Curiosity}, \text{Tuna}) \Rightarrow \text{False}$



AnimalLover(x) ∧ Animal(y) ∧ Kills(x,y) ⇒ False
Dog(D)
Dog(y) ∧ Owns(x,y) ⇒ AnimalLover(x)
Cat(Tuna)
Cat(x) ⇒ Animal(x)
{y/D}
{x/Tuna}
Animal(Tuna)
Owns(x,D) ⇒ AnimalLover(x)
Owns(Jack, D)
{y/Tuna}
{x/Jack}
AnimalLover(x) ∧ Kills(x,Tuna) ⇒ False
AnimalLover(Jack)
Kills(Jack,Tuna) ∨ Kills(Curiosity, Tuna)
{x/Jack}
Kills(Curiosity, Tuna) ⇒ False
Kills(Jack,Tuna) ⇒ False
Kills(Jack,Tuna)
False

# Coming Up

- Friday: Practical uses for Logical Inference Systems
- Problem Set #3 out today!
- Special Guest Lecture on Monday!