Propositional Logic

CPSC 470 – Artificial Intelligence

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Constraint Satisfaction Problems

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   T W O
+  T W O
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F O U R
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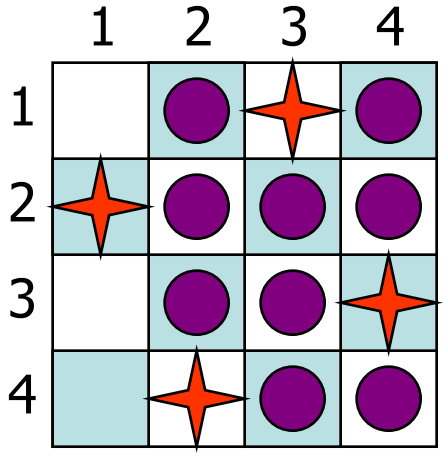
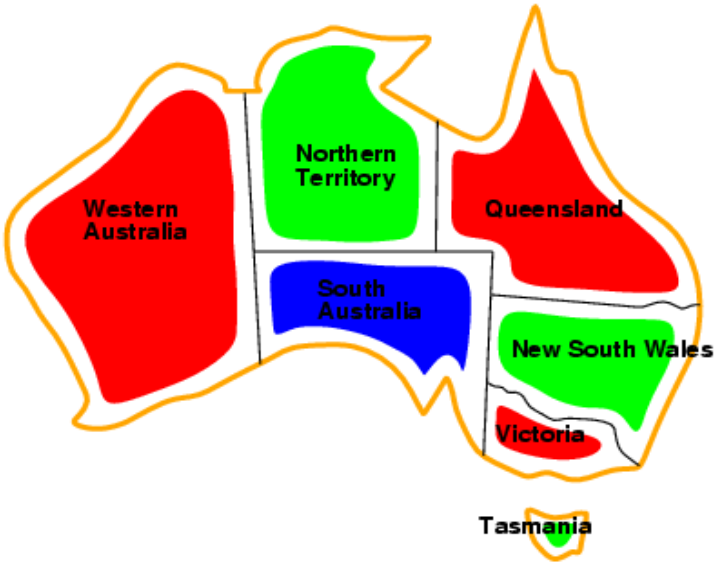


4						8		5
	3							
			7					
	2						6	
				8		4		
	4			1				
			6		3		7	
5		3	2		1			
1		4						



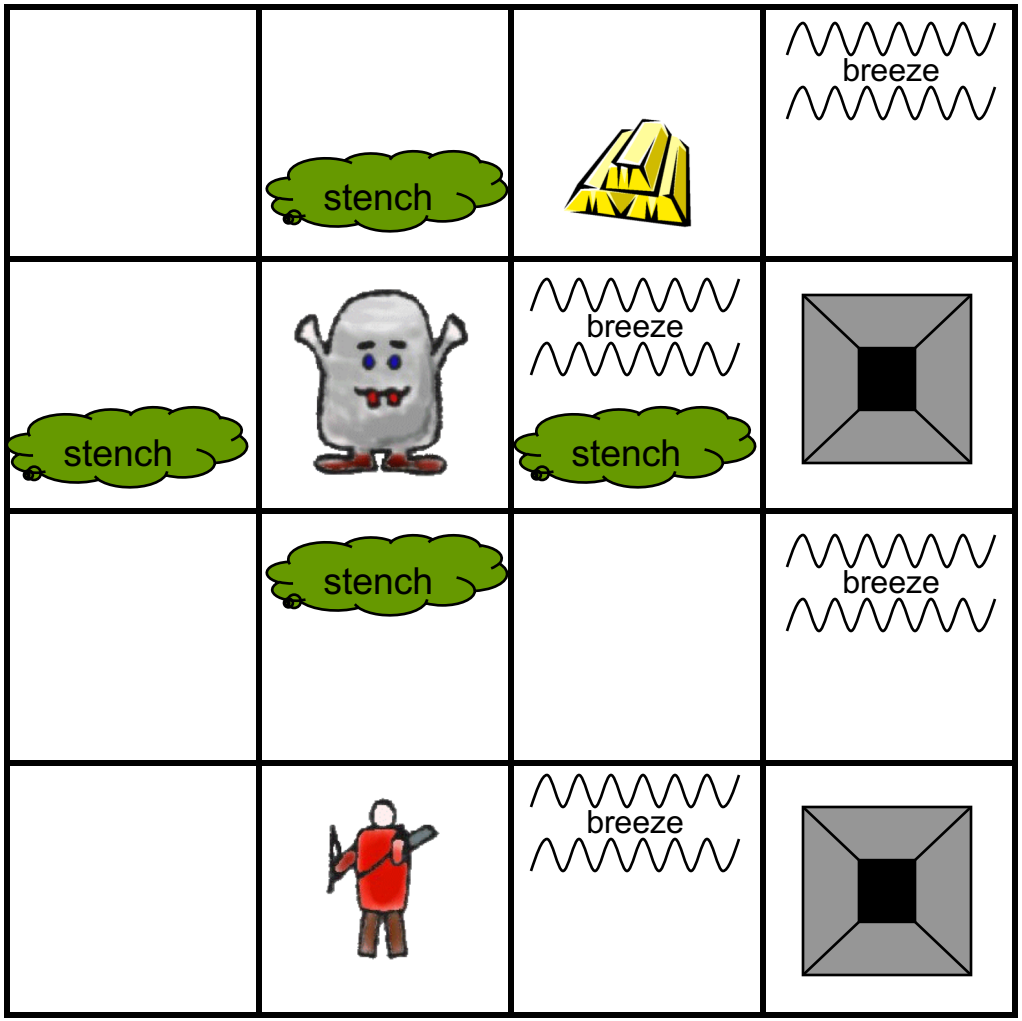
World Characterization

	Search	**CSP**
Fully Observable	Yes	Yes
Deterministic	Yes	Yes
Episodic	No	No
Static	Yes	Yes
Discrete	Yes	Mostly

World Characterization

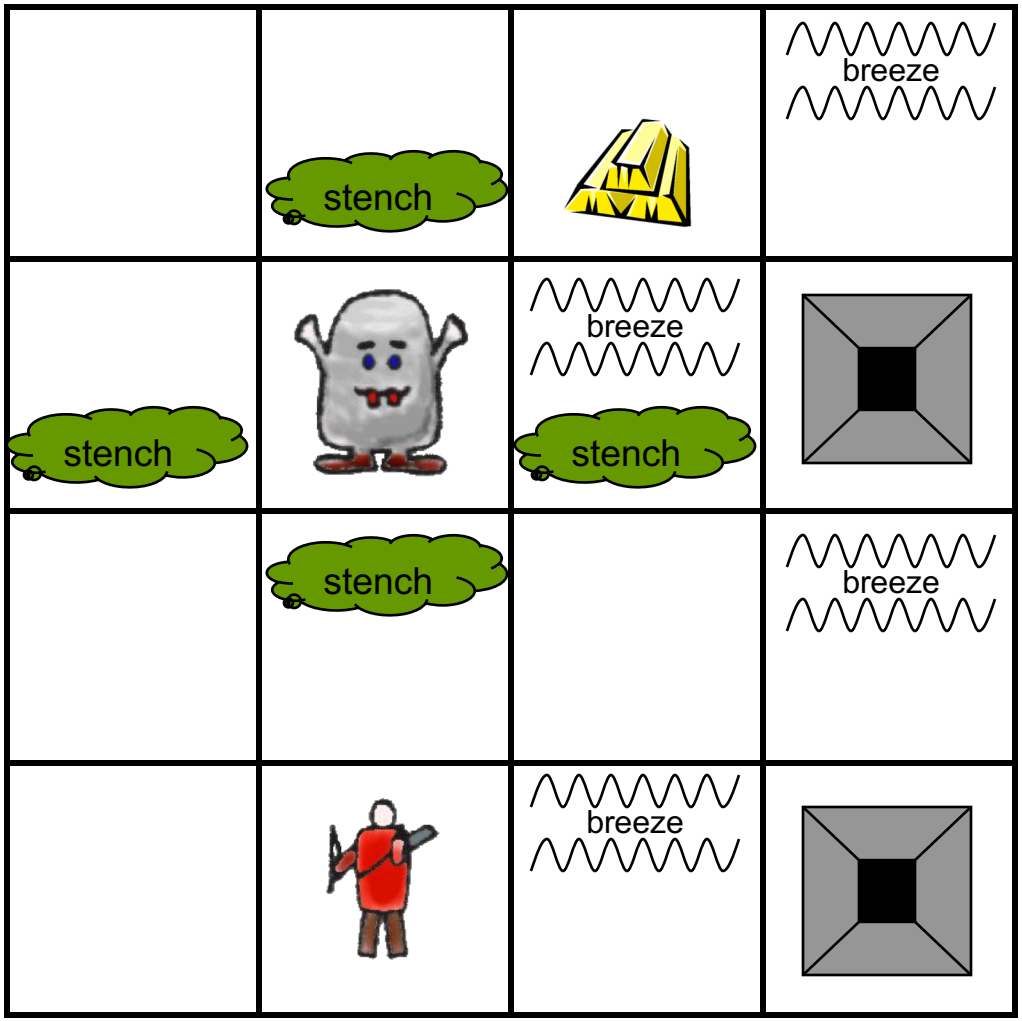
	Search	CSP	Today
Fully Observable	Yes	Yes	No
Deterministic	Yes	Yes	Yes
Episodic	No	No	No
Static	Yes	Yes	Yes
Discrete	Yes	Mostly	Yes

The Wumpus World



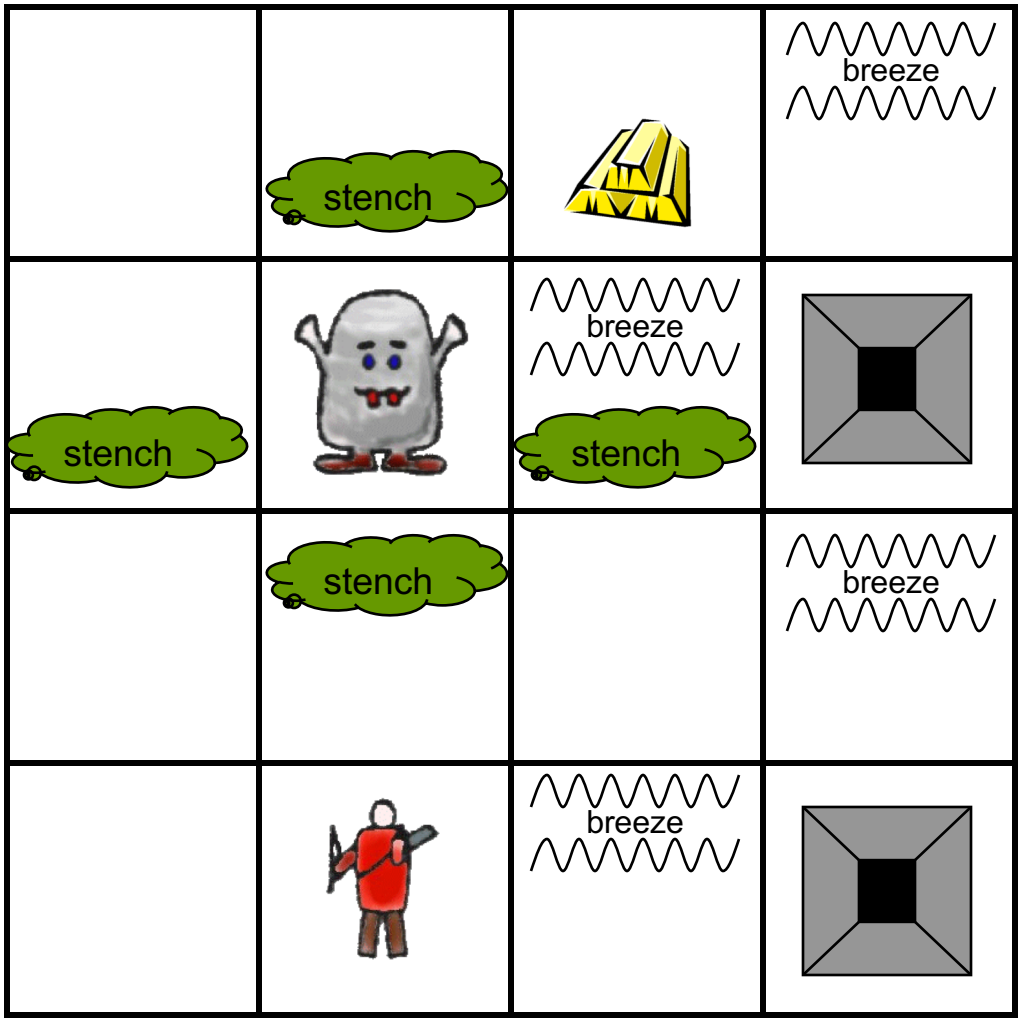
- Grid-like world
- Noble hero
- Horrible wumpus
- Bottomless pits
- Gold
- Breeze
- Stench

Actions in the Wumpus World



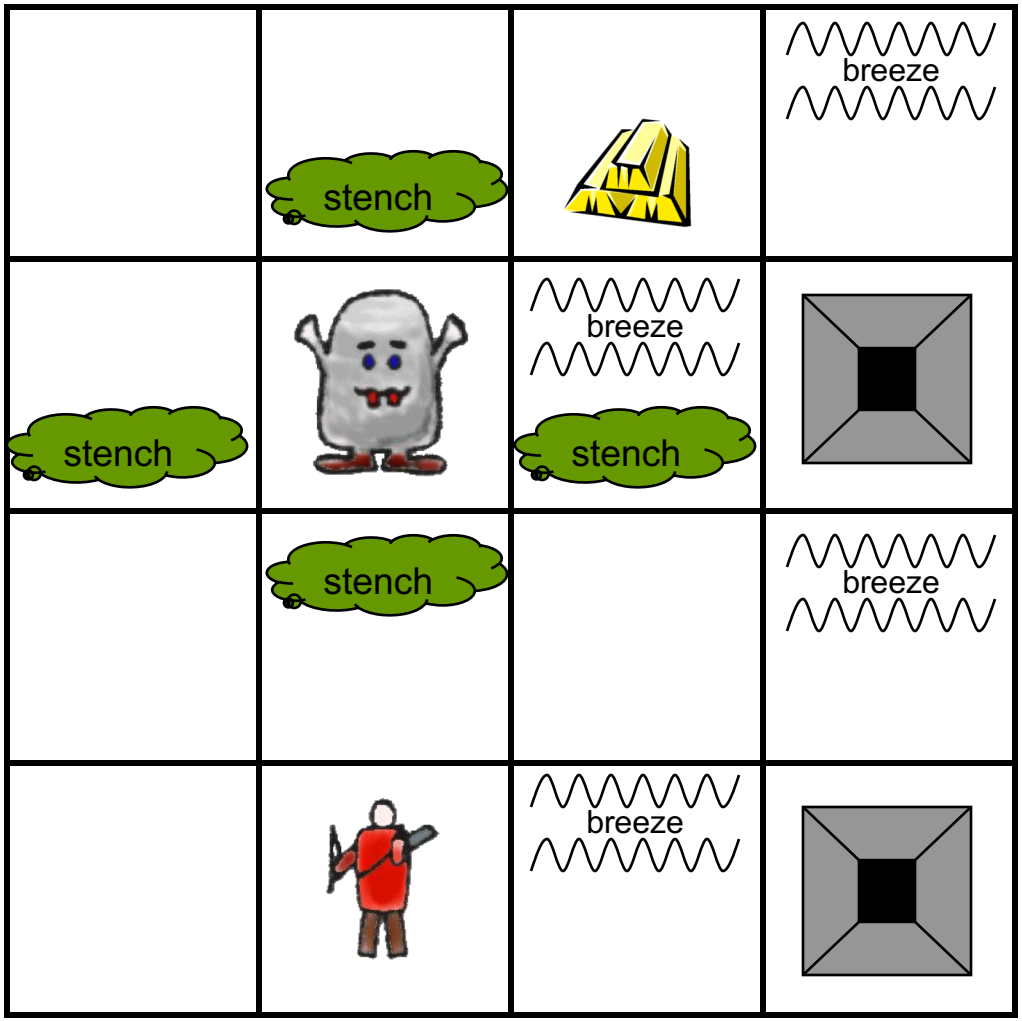
- Goals:
  - find the gold
  - kill the wumpus
  - go home
- Actions
  - Move N,S,E,W
  - Grab
  - Shoot(N,S,E,W)
    - Only one arrow!

The Wumpus World



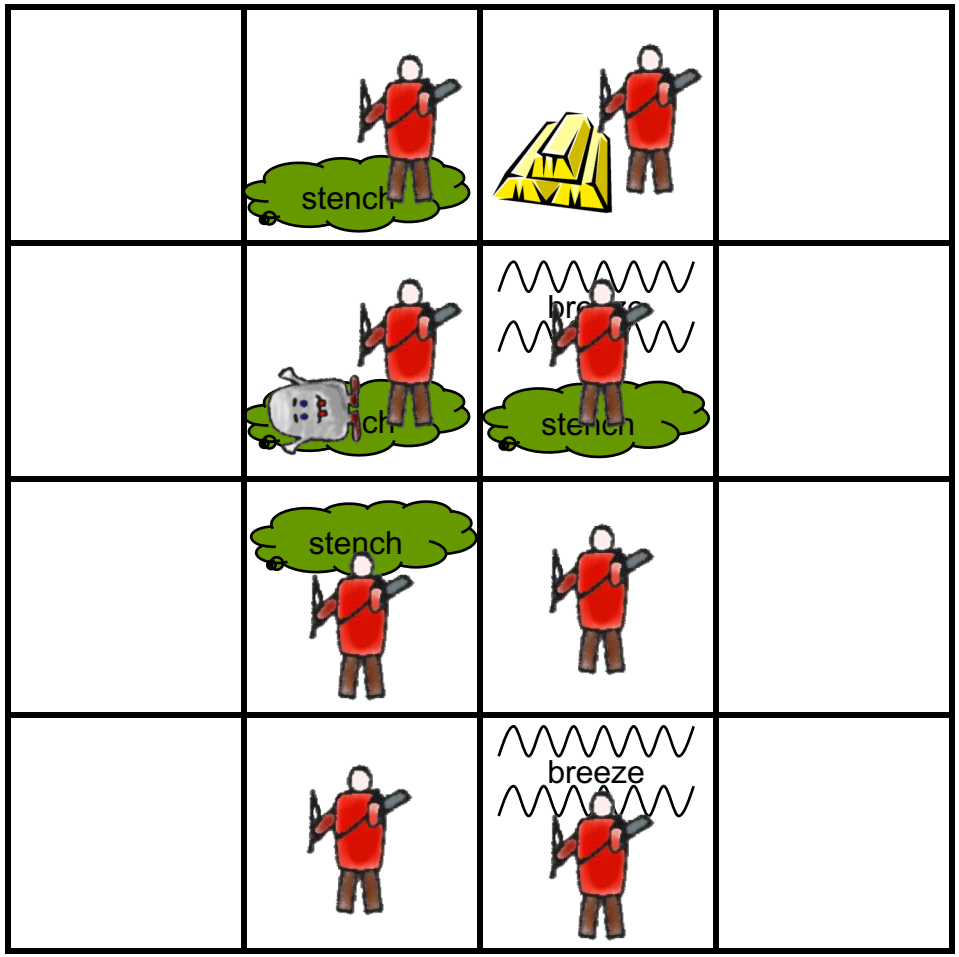
- If we had complete knowledge of the world, then we could simply build a search tree
- What if our perceptions are limited?

Incomplete Knowledge of the World



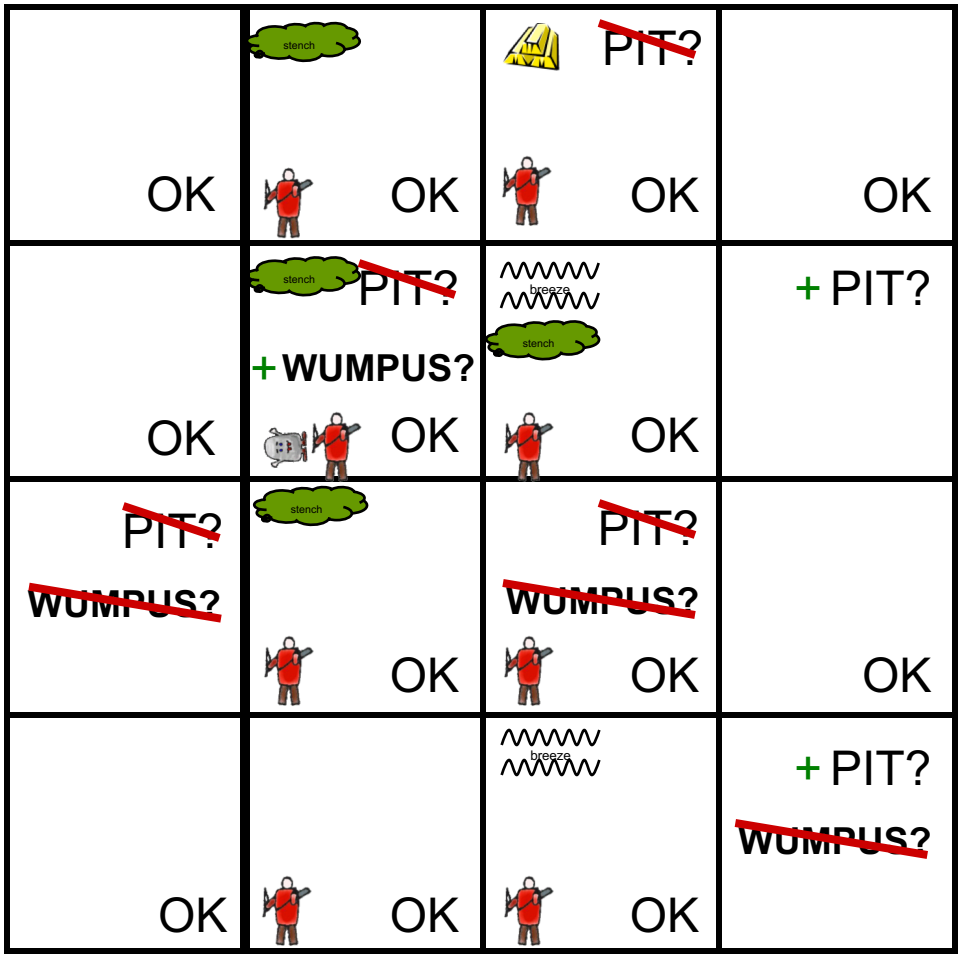
- Agent's percepts:
  - Stench
  - Breeze
  - Glitter
  - Bump
  - Scream
- Other than the agent, the world is static

Our First Wumpus Hunt



stench	breeze	glitter	bump	scream	
No	No	No	No	No	**South**
No	No	No	**Yes**	No	**East**
No	**Yes**	No	No	No	**West**
No	No	No	No	No	**North**
Yes	No	No	No	No	**East**
No	No	No	No	No	**North**
Yes	**Yes**	No	No	No	**Shoot(W)**
Yes	**Yes**	No	No	**Yes**	**West**
Yes	No	No	No	No	**North**
Yes	No	No	No	No	**East**
No	No	**Yes**	No	No	**Grab**

Annotated Wumpus Hunt



stench	breeze	glitter	bump	scream	
No	No	No	No	No	**South**
No	No	No	**Yes**	No	**East**
No	**Yes**	No	No	No	**West**
No	No	No	No	No	**North**
Yes	No	No	No	No	**East**
No	No	No	No	No	**North**
Yes	**Yes**	No	No	No	**Shoot(W)**
Yes	**Yes**	No	No	**Yes**	**West**
Yes	No	No	No	No	**North**
Yes	No	No	No	No	**East**
No	No	**Yes**	No	No	**Grab**

Today we will see how to build an agent that can perform this reasoning

Representing Beliefs

- In most programming languages, it is easy to specify statements like this…
  - *There is a pit in square [3,1]*
- But it is difficult to specify statements like these…
  - *There is a pit in either square [3,1] or [2,2]*
  - *There is no wumpus in square [2,2]*
  - *Because there was no breeze in square [1,2], there is a pit in square [3,1]*
- Require an agent that can represent this knowledge and perform the reasoning to infer new conclusions

Components of a Logic

- A formal system for representing the state of affairs
  - A sentence is a representation of a fact about the world
  - A syntax that describes how to make sentences
  - A semantics that gives constraints on how sentences relate to the state of affairs
  - A proof theory – a set of rules for deducing the entailments of a set of sentences

Entailment means that one thing follows from another

Properties of Logical Inference

- Inference is complete if it can find a proof for any sentence that is entailed
- A sentence is valid or necessarily true if and only if it is true under all possible interpretations in all possible worlds

  There is a stench in [1,1] or there is not a stench in [1,1]
- A sentence is satisfiable if and only if there is some interpretation in some world for which it is true

  There is a wumpus at [1,1]

Types of Commitment

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief 0…1
Fuzzy logic	degree of truth	degree of belief 0…1

- We make assumptions about
  - the world (ontological commitments)
  - the beliefs that an agent can hold (epistemological commitments)

Propositional Logic Syntax

- Basic Units (sentences)
  - *True* and *False*
  - Propositions *P, Q, …*
- Connectives

  $P \wedge Q$ and (conjunction)

  Returns true if both P and Q are true

  $P \vee Q$ or (disjunction)

  Returns true if either P or Q is true

  $P \Rightarrow Q$ implication

  If P is true then Q is also true

  $P \Leftrightarrow Q$ equivalence

  P is true exactly when Q is true

  $\neg P$ negation

  Returns true when P is false

Propositional Logic Grammar

- BNF (Backus-Naur form) for PL Grammar:

$$Sentence \rightarrow AtomicSentence \mid ComplexSentence$$

$$AtomicSentence \rightarrow True \mid False \mid P \mid Q \mid \ldots$$

$$ComplexSentence \rightarrow (Sentence) \mid$$

$$Sentence\ Connective\ Sentence \mid$$

$$\neg Sentence$$

$$Connective \rightarrow \wedge \mid \vee \mid \Rightarrow \mid \Leftrightarrow$$

- Also require an order of precedence

  From highest to lowest: $\neg \ \wedge \ \vee \ \Rightarrow \ \Leftrightarrow$

Propositional Logic Semantics

- Propositions can have any semantic meaning:

  P = "Paris is the capital of France"

  Q = "The wumpus is dead"

  R = "Bill Gates is the US President"

- Compound functions can be derived from a truth table:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

Validity and Inference

$$((P \vee H) \wedge \neg H) \Rightarrow P$$

P	H	$P \vee H$	$(P \vee H) \wedge \neg H$	$((P \vee H) \wedge \neg H) \Rightarrow P$
False	*False*			
False	*True*			
True	*False*			
True	*True*			

- Truth tables can also be used to test validity of a sentence
- Remember to read implications as conditionals:

  $P \Rightarrow Q$ is read as "if P then Q"

Inference Rules for Propositional Logic

- Modus Ponens (Implication-Elimination)
  - From an implication and its premise, infer conclusion

$$\frac{\alpha \Rightarrow \beta \ , \ \alpha}{\beta}$$

- And-Elimination
  - From a conjunction, you can infer any conjunct

$$\frac{\alpha_1 \wedge \alpha_2 \wedge \alpha_3 \wedge \ldots \wedge \alpha_n}{\alpha_i}$$

Inference Rules for Propositional Logic

- And-Introduction
  - From a list of sentences, you can infer the conjunct

$$\frac{\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n}{\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n}$$

- Or-Introduction
  - From a sentence, infer its disjunction with anything

$$\frac{\alpha_i}{\alpha_1 \vee \alpha_2 \vee \ldots \vee \alpha_n}$$

Inference Rules for Propositional Logic

- Double-Negative Elimination
  - From a double negation, infer the positive sentence

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit Resolution
  - From a disjunction in which one is false, then you can infer the other is true

$$\frac{\alpha \vee \beta\ ,\ \ \neg\beta}{\alpha}$$

Inference Rules for Propositional Logic

- Resolution
  - Since beta cannot be both true and false, one of the disjuncts must be true

$$\frac{\alpha \vee \beta \ , \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

  - Implication is transitive

$$\frac{\neg\alpha \Rightarrow \beta \quad , \quad \beta \Rightarrow \gamma}{\neg\alpha \Rightarrow \gamma}$$

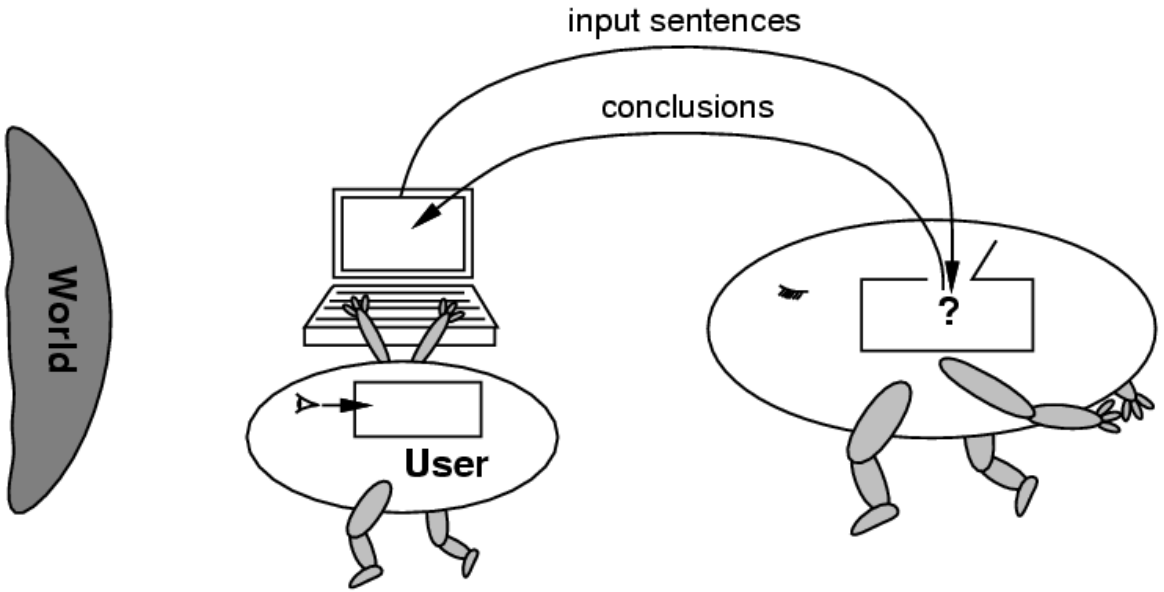
Truth Table for Resolution

α	β	γ	$\alpha \vee \beta$	$\neg\beta \vee \gamma$	$\alpha \vee \gamma$
False	*False*	*False*	*False*	*True*	*False*
False	*False*	*True*	*False*	*True*	*True*
False	*True*	*False*	*True*	*False*	*False*
False	*True*	*True*	*True*	*True*	*True*
True	*False*	*False*	*True*	*True*	*True*
True	*False*	*True*	*True*	*True*	*True*
True	*True*	*False*	*True*	*False*	*True*
True	*True*	*True*	*True*	*True*	*True*

- Truth tables can also be used to verify the inference rules

$$\frac{\alpha \vee \beta \ , \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

Logical Agents



- Input sentences can come from the user perceiving the world, or from a machine-readable representation of the world
- Infer new statements about the world that are valid

An Agent for the Wumpus World

- Convert perceptions into sentences:

  "In square [1,1], there is no breeze and no stench" … becomes…

  $\neg B_{11} \wedge \neg S_{11}$

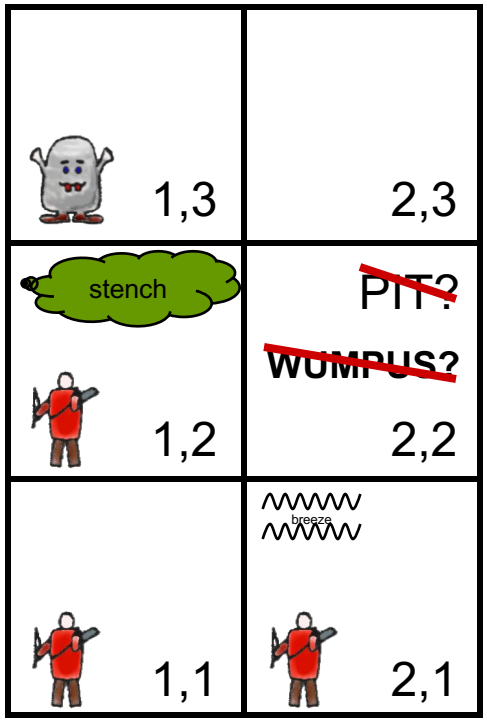
- Start with some knowledge of the world (in the form of rules)

  $R1 : \neg S_{11} \Rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

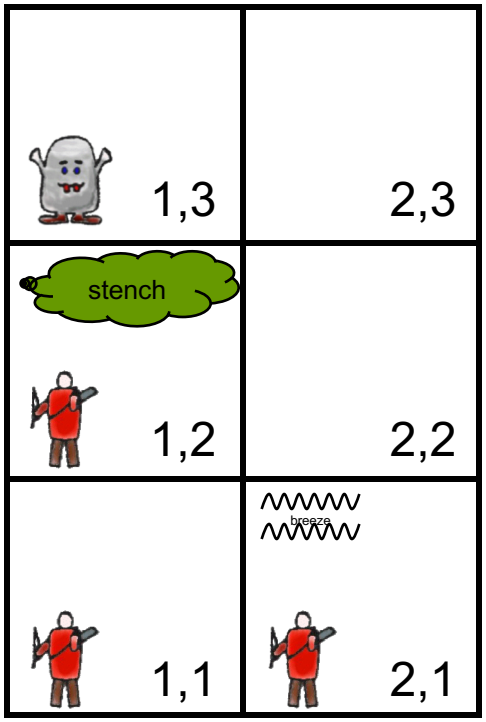
  $R2 : \neg S_{21} \Rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22}$

  ….

  $R4 : S_{12} \Rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$



Finding the Wumpus



Percepts:

$\neg S_{11}$

$\neg S_{21}$

S_{12}

1. Apply modus ponens and and-elimination to $\neg S_{11} \Rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$ to get

   $\mathbf{\neg W_{11}}$ $\mathbf{\neg W_{12}}$ $\mathbf{\neg W_{21}}$

2. Apply modus ponens and and-elimination to $\neg S_{21} \Rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22}$ to get

   $\mathbf{\neg W_{22}}$ $\mathbf{\neg W_{21}}$ $\mathbf{\neg W_{31}}$

3. Apply modus ponens to $S_{12} \Rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$ to get

   $\mathbf{W_{13} \vee W_{12} \vee W_{22} \vee W_{11}}$

4. Apply unit resolution to #3 and #1

   $\mathbf{W_{13} \vee W_{22}}$

5. Apply unit resolution to #4 and #2

   $\mathbf{W_{13}}$

The wumpus is in square [1,3]!!!

Problems with Propositional Logic

- Too many propositions!
  - How can you encode a rule such as “don’t go forward if the wumpus is in front of you”?
  - In propositional logic, this takes (16 squares * 4 orientations) = 64 rules!
- Truth tables become unwieldy quickly
  - Size of the truth table is 2^n where n is the number of propositional symbols

More Problems with Propositional Logic

- No good way to represent changes in the world
  - How do you encode the location of the agent?
- What kinds of practical applications is this good for?
  - Relatively little

Coming Up…

- More powerful logic!
  - First-order logic (also known as First Order Predicate Calculus)