Artificial Intelligence CPSC 470/570
PS 3: Logic and Inference
12 points (CPSC 470) or 18 points (CPSC 570)
Due Monday, Feb. 18th, 11:59:59 PM

Some reminders:

- **Grading contact:** Allan Wu (allan.wu@yale.edu) is the point of contact for initial questions about grading for this problem set.
- **Late assignments** are not accepted without a Dean's excuse.
- **Collaboration policy:** Remains the same as in PS2.
- **Submission:** You must upload your submission electronically to **Gradescope** before the cutoff deadline posted above. One way to do this is to print this assignment, write out your answers with pen or pencil ~~in the~~ spaces provided, and then upload images of each page of your assignment.
- **Students taking CPSC570:** Problem #4 is designed to be completed only by students in CPSC570. Students taking CPSC 470 do not need to do problem 4.

Problem 1 (3 points)

State each of the following in First-Order Predicate Calculus (FOPC), using only the list of provided predicates and functions. You may invent any variable or constant names that you desire. **If there is a single, unambiguous way to represent the statement, then just provide the FOPC representation. If there is any ambiguity in the sentence, the interpretation, or the representation, you should write 1-3 English sentences that describe the ambiguity and provide at least 2 FOPC sentences that are both accurate representations of the English statement.**

Allowed Predicates: *Likes(x,y)*, *Bird(x)*, *Ostrich(x)*, *Penguin(x)*, *Flies(x)*, *NeedsToLove(x,y)*. *Likes(x,y)* means "x likes y" and *NeedsToLove(x,y)* means "x needs to love y". The remaining have their obvious interpretations.

1.1 Everybody doesn't like something but nobody doesn't like Sara Lee.

$$\left(\forall x \exists y \neg Likes(x,y)\right) \wedge \left(\neg \exists z \neg Likes(z, Sara\ Lee)\right)$$

1.2 All birds except Ostriches and Penguins fly.

$\forall x \; Bird(x) \wedge \neg Ostrich(x) \wedge \neg Penguin(x) \Rightarrow Flies(x)$

1.3 Everybody needs somebody to love

Ambiguity is who loves who (somebody loves everybody, OR everybody loves somebody)

① $\forall x \; \exists y \; NeedsToLove(x, y)$

② $\forall x \; \exists y \; NeedsToLove(y, x)$

Problem 2 (3 points)

Using propositional logic, it is possible to prove theorems by simply enumerating all possible truth values of all variables and checking that the theorem holds. Demonstrate that each of the following is a valid theorem by filling in the provided truth table with **"T" for true and "F" for false.**

2.1 $(p \Rightarrow \neg p) \Rightarrow \neg p$

p	a: $\neg p$	b: $(p \Rightarrow \neg p)$	$b \Rightarrow a$
T	F	F	T
F	T	T	T

2.2 $((p \wedge q) \wedge r) \Rightarrow (p \wedge (q \wedge r))$

p	q	r	a: $p \wedge q$	b: $a \wedge r$	c: $q \wedge r$	d: $p \wedge c$	$b \Rightarrow d$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	F	T	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

2.3 $(p \wedge (q \vee r)) \Rightarrow ((p \wedge q) \vee (p \wedge r))$

p	q	r	a: $q \vee r$	b: $p \wedge a$	c: $p \wedge q$	d: $p \wedge r$	e: $c \vee d$	$b \Rightarrow e$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T	T
T	F	T	F	F	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	F	F	F	F	F	T
F	F	T	F	F	F	F	F	T
F	F	F	F	F	F	F	F	T

Problem 3 (6 points)

You are given the following facts:

1. Everyone who entered this country and who was not a diplomat was searched by a customs official.
2. William was a terrorist.
3. William entered this country.
4. William was searched by terrorists only.
5. No terrorist was a diplomat.

Show using first-order logic that:

Goal: There is a person who is both a terrorist and a customs official.

Your solutions should have the same format as slide 14 from the lecture on Inference (#9).

Hints:

- Start by translating the goal into FOPC and enter it into the line marked "goal".
- Line numbers 1-5 should be the FOPC statements that are equivalent to the English sentences 1-5 above.
- Use only the following predicates: *Entered(x)* meaning "x entered this country", *Diplomat(x)*, *CustomsOfficial(x)*, *Terrorist(x)*, and *Searched(x,y)* meaning that "x searched y".
- You may introduce any constants or variables that you need.
- The **Reasoning** column should contain references to an inference rule and the statements that you used to derive the new sentence. For example, "Existential elimination on 7" or "Modus ponens on 9 and 3" or "And-introduction on 1, 3, and 5" or "de Morgan's rules on 7".
- Your last line in the table should be the same FOPC statement as your goal.
- You may or may not need all of the lines in the table.

#	FOPC Sentence	Reasoning
	$\exists x\ Terrorist(x) \land CustomsOfficial(x)$	--- GOAL ---
1	$\forall x\, \exists y\ Entered(x) \land \neg Diplomat(x) \Rightarrow Searched(x, y) \land CustomsOfficial(y)$	given
2	$Terrorist(William)$	given
3	$Entered(William)$	given
4	$\forall x.\ Searched(William, x) \Rightarrow Terrorist(x)$	given
5	$\forall x.\ Terrorist(x) \Rightarrow \neg Diplomat(x)$	given
6	$Terrorist(William) \Rightarrow \neg Diplomat(William)$	universal elim. on 5.
7	$\neg Diplomat(William)$	Modus ponens 6, 2
8	$Entered(William) \land \neg Diplomat(William) \Rightarrow \exists y\ Searched(William, y) \land CustomOfficial(y)$	universal elim. on 1
9	$\exists y\ Searched(William, y) \land CustomOfficial(y)$	Modus ponens 8, 3, 7
10	$Searched(William, y) \Rightarrow Terrorist(y)$	universal elim. on 4
11	$\exists y\ Terrorist(y) \land CustomOfficial(y)$	Modus ponens 10, 9

Problem 4 (6 points) : GRADUATE STUDENTS ONLY

Consider the following (fictional) tale:

> Dorsey has been murdered. Angluin, Bhattacharjee, and Cai are suspects. Only one is guilty and the other two are innocent. The innocent ones told the truth to the police, but the guilty one may have lied.
>
> Angluin said that Bhattacharjee and Dorsey were friends and that Cai did not like Dorsey. Bhattacharjee said that he was not in town at the time of the murder, and moreover, he did not know Dorsey. Cai said that Angluin and Bhattacharjee were both with Dorsey just before Dorsey was murdered.

Your job is to prove that Bhattacharjee is the murderer (i.e., *murderer(B)*).

You should do this via a proof by contradiction. You should assume $\neg$ *murderer(B)* and show that this leads to a something of the form $P \wedge \neg P$, which is a contradiction since P cannot not be both true and false. (Your last line of the table should be something of the form $P \wedge \neg P$).

You should use the following predicates: *innocent(x), friends(x,y), murderer(x), likes(x,y), inTown(x), knows(x,y), with(x,y).*

#	FOPC Sentence	Reasoning
0	$\neg$ *murderer(B)*	assumption
1	$\forall x, \neg murder(x) \Rightarrow innocent(x)$	Given
2	$innocent(B) \Rightarrow \neg inTown(B)$	Given
3	$innocent(B) \Rightarrow \neg knows(B,D)$	Given
4	$inTown(D)$	Given
5	$\forall x, \neg inTown(x) \land inTown(D) \Rightarrow \neg with(x,D)$	Given
6	$\neg with(B,D) \Rightarrow \neg innocent(C)$	Given
7	$\neg innocent(C) \Rightarrow innocent(A)$	Given
8	$innocent(A) \Rightarrow friend(B,D)$	Given
9	$\forall x, \forall y\ friend(x,y) \Rightarrow knows(x,y)$	Given
10	$\neg murder(B) \Rightarrow innocent(B)$	universal elim on 1.
11	$innocent(B)$	modus ponens 10, 0
12	$\neg inTown(B)$	modus ponens 2, 11
13	$\neg knows(B,D)$	modus ponens 3, 11.
14	$\neg inTown(B) \land inTown(D) \Rightarrow \neg with(B,D)$	universal elim. on 5
15	$\neg with(B,D)$	modus ponens 14, 12, 4
16	$\neg innocent(C)$	modus ponens 6, 15
17	$innocent(A)$	modus ponens 7, 16
18	$friend(B,D)$	modus ponens 8, 17
19	$friend(B,D) \Rightarrow knows(B,D)$	universal elim. on 9
20	$knows(B,D)$	modus ponens 19, 18.
21	$knows(B,D) \land \neg knows(B,D)$	and introduction 20, 13.

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