Network Transport Layer:
Network Resource Allocation Framework

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http://zoo.cs.yale.edu/classes/cs433/

11/15/2018

Admin.

- Programming assignment 4 status
  - Part 1: Code checkpoint: 11:55 pm, Nov. 15, 2018
    - Receive should added handling Protocol.TRANSPORT_PKT
    - Send: should at least be able to generate a segment
  - Part 2: Design discussion with instructor or TF checkpoint: Nov. 27; Complete code and report due: 1:30 pm, Nov. 29, 2018.

Recap: TCP/Reno Throughput Modeling (Random Loss Model)

$$\Delta W = \begin{cases} \frac{1}{W} & \text{if the packet is not lost} \\ -\frac{W}{2} & \text{if packet is lost} \end{cases}$$

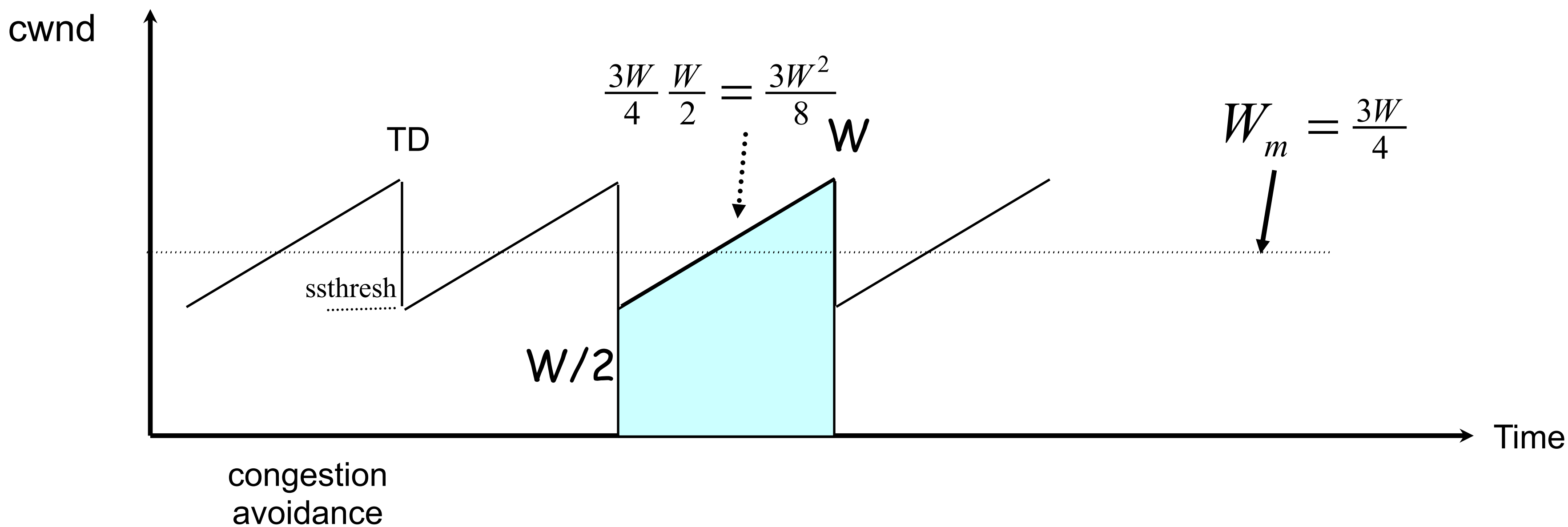
$$\textit{mean of } \Delta W = (1-p)\frac{1}{W} + p(-\frac{W}{2}) = 0$$

$$\Rightarrow \quad \textit{mean of } W = \sqrt{\frac{2(1-p)}{p}} \approx \frac{1.4}{\sqrt{p}}, \text{ when } p \text{ is small}$$

$$\Rightarrow \quad \boxed{\textit{throughput} \approx \frac{1.4S}{RTT\sqrt{p}}, \text{ when } p \text{ is small}}$$

This is called the TCP throughput sqrt of loss rate law.

Recap: TCP/Reno Throughput Modeling: (Periodical Loss Model)



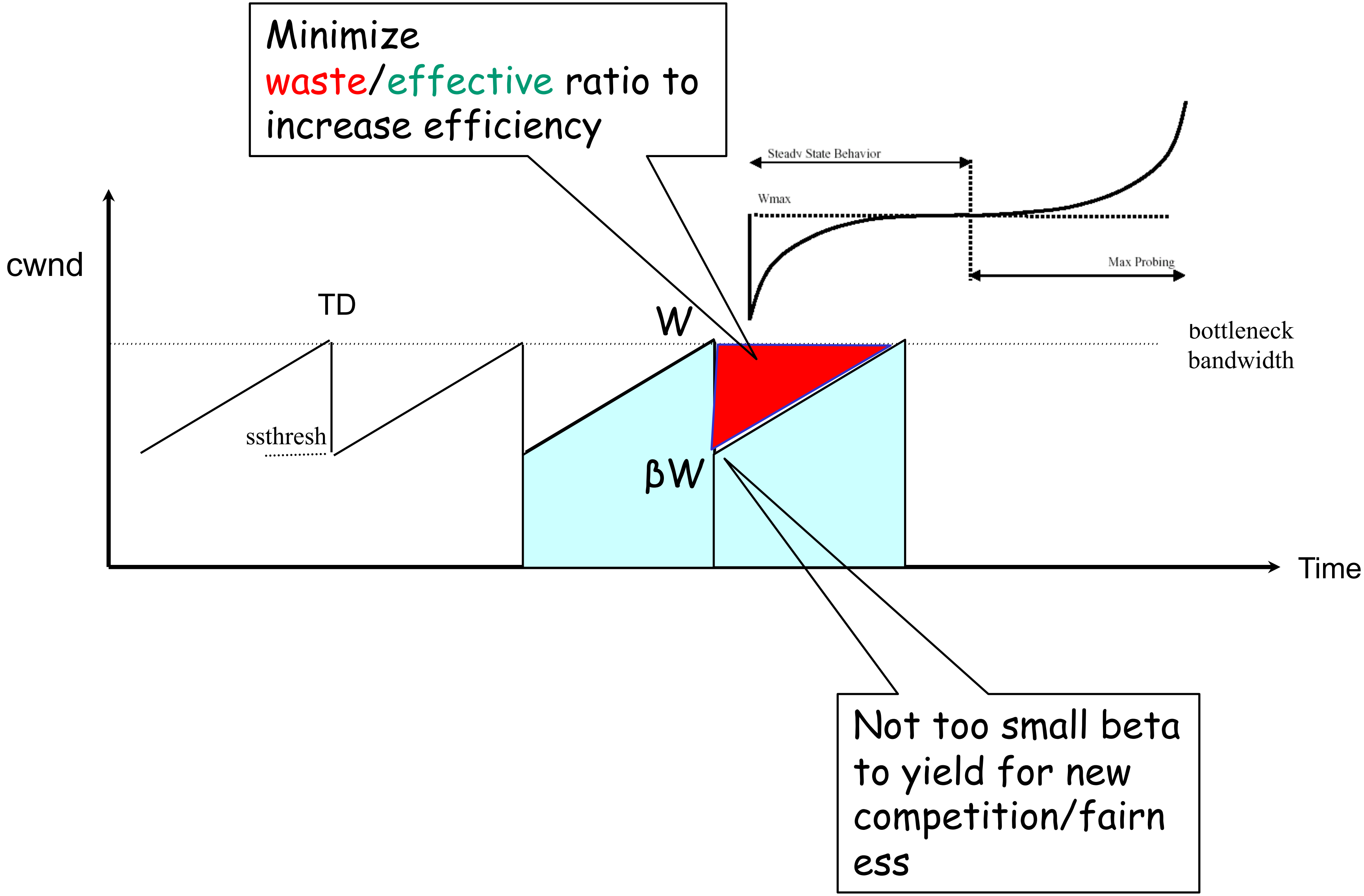
Total packets sent per cycle = (W/2 + W)/2 * W/2 = $3W^2/8$

Assume one loss per cycle => p = $1/(3W^2/8) = 8/(3W^2)$

$$=> W = \frac{\sqrt{8/3}}{\sqrt{p}} = \frac{1.6}{\sqrt{p}}$$

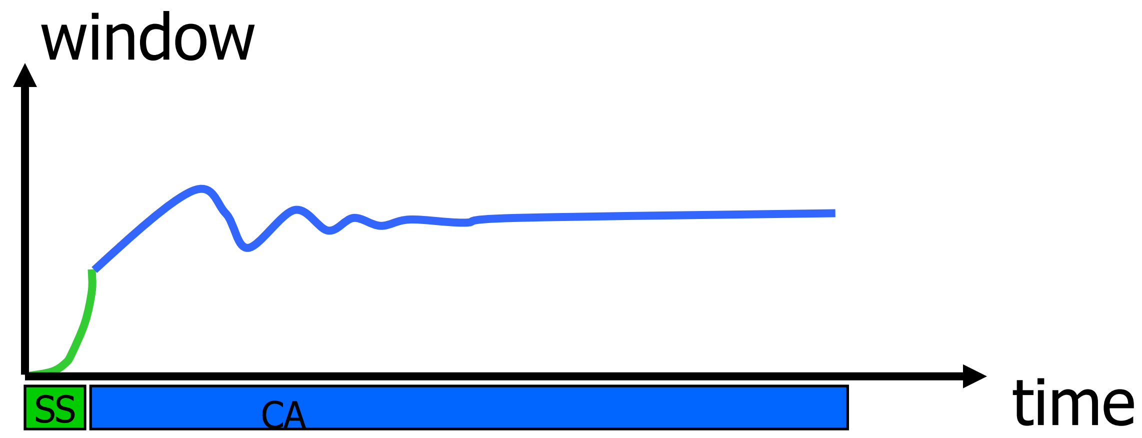
$$\Rightarrow throughput = \frac{S}{RTT}\frac{3}{4}\frac{1.6}{\sqrt{p}} = \boxed{\frac{1.2S}{RTT\sqrt{p}}}$$

Recap: Design for Limited Buffer



Recap: TCP/Vegas CA algorithm

Delay, instead of loss as signaling: maintain a *constant* number of packets in the bottleneck buffer



for every RTT

{ if $W - W/RTT\ RTT_{min} < \alpha$ then W ++

if $W - W/RTT\ RTT_{min} > \alpha$ then W --

}

for every loss

W := W/2

queue size

Outline

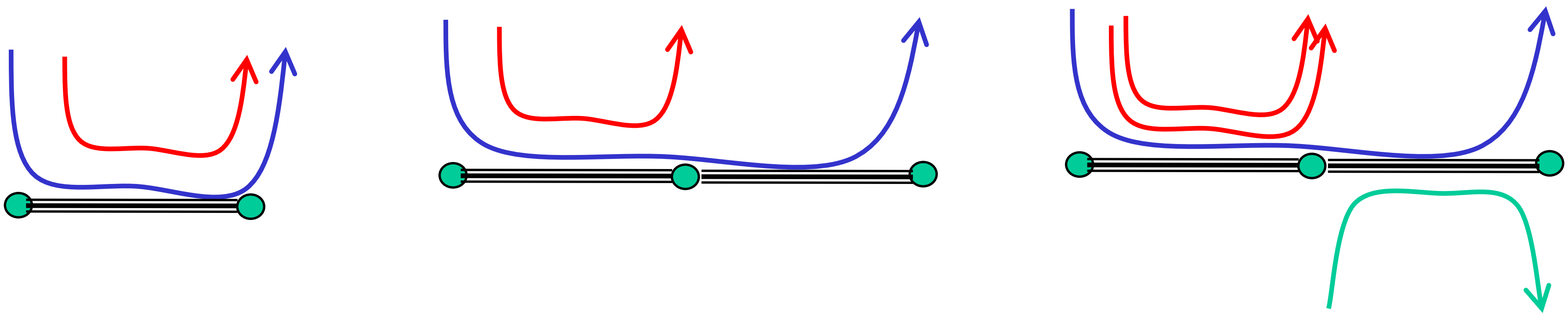
- Admin and recap
- Transport congestion control
  - what is congestion (cost of congestion)
  - basic congestion control alg.
  - TCP/Reno congestion control
  - TCP Cubic
  - TCP/Vegas
  - network-wide/global resource allocation
    - general framework

Motivation

- So far our discussion is implicitly on a network with a single bottleneck link; this simplifies design and analysis:
  - efficiency/optimality (high utilization)
    - fully utilize the bandwidth of the link
  - fairness (resource sharing)
    - each flow receives an ***equal*** share of the link's bandwidth

Network Resource Allocation

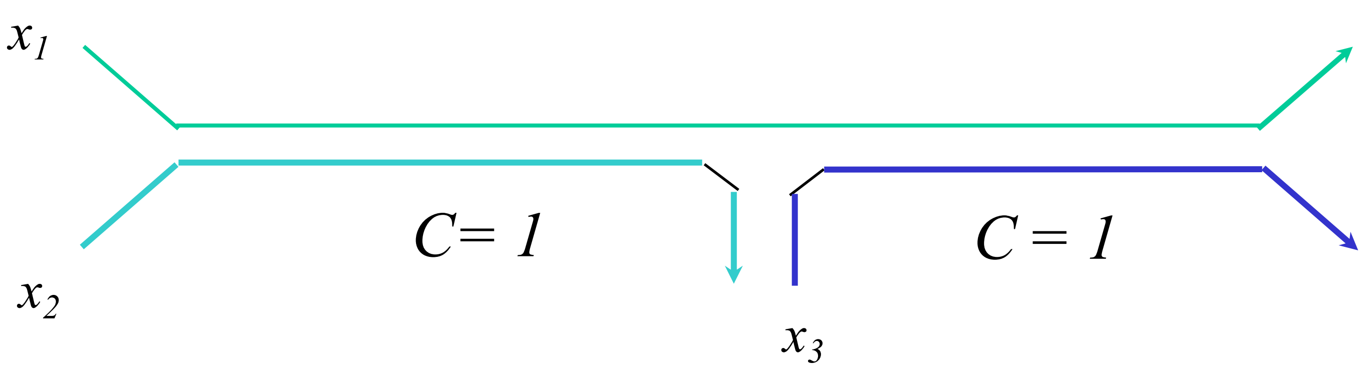
- It is important/helpful to understand and design protocols for a general network topology
  - how will TCP allocate resource in a general topology?
  - how should resource be allocated in a general topology?



Example: TCP/Reno Rates

- Rates: $x_1 = \frac{1}{1+2\sqrt{2}} = 0.26$

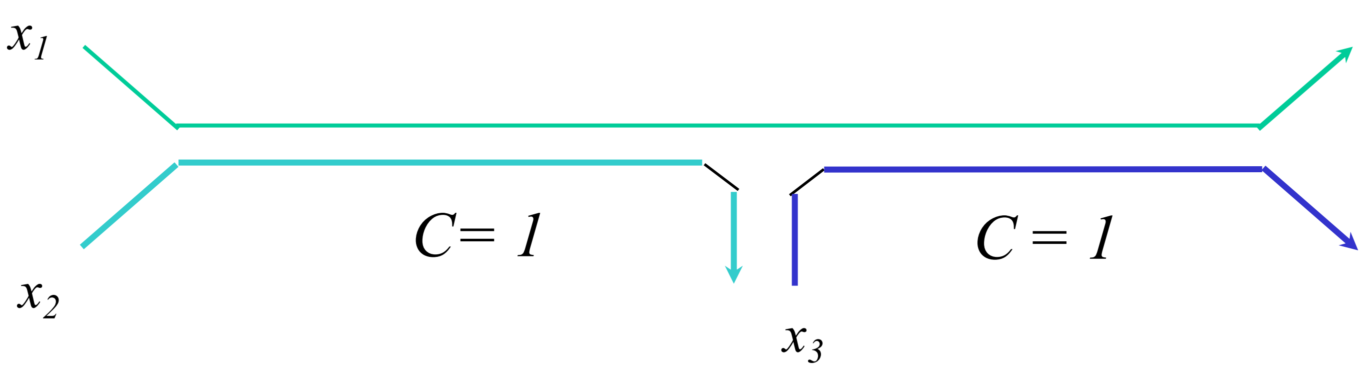
  $x_2 = x_3 = 0.74$



Example: TCP/Vegas Rates

- Rates : $x_1 = 1/3$

  $x_2 = x_3 = 2/3$



Example: Max-min Fairness

- Max-min fairness: maximizes the throughput of the flow receiving the minimum (of resources)
    - Justification: John Rawls, *A Theory of Justice* (1971)
        - http://en.wikipedia.org/wiki/John_Rawls
    - This is a resource allocation scheme used in ATM and some other network resource allocation proposals

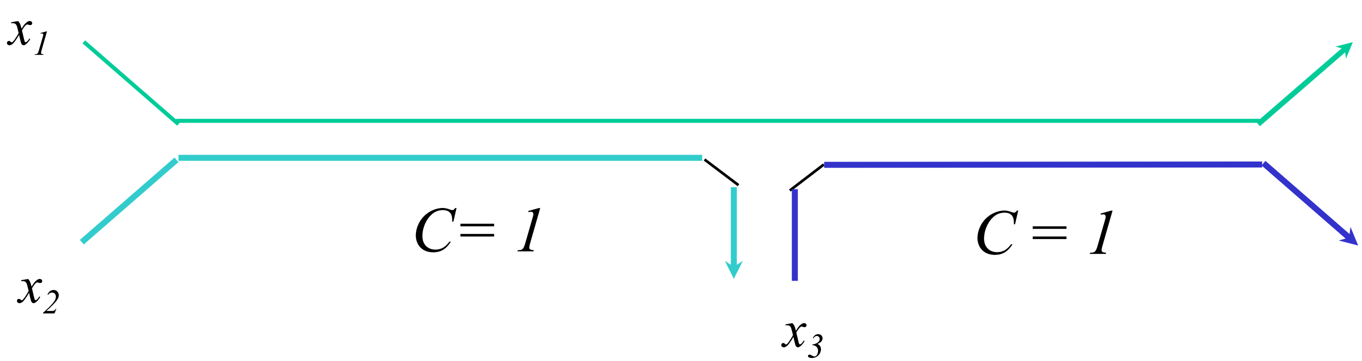
Example: Max-Min

$$\max_{x_f \geq 0} \quad \min\{x_f\}$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

- Rates: $x_1 = x_2 = x_3 = 1/2$



Framework: Network Resource Allocation Using Utility Functions

- A set of flows F
- Let x_f be the rate of flow f, and the utility to flow f is $U_f(x_f)$.
- Maximize aggregate utility, subject to capacity constraints

$$
\begin{array}{ll}
\text{max} & \displaystyle\sum_{f \in F} U_f\left(x_f\right) \\
\text{subject to} & \displaystyle\sum_{f:f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\
\text{over} & x \ge 0
\end{array}
$$

Example: Maximize Throughput

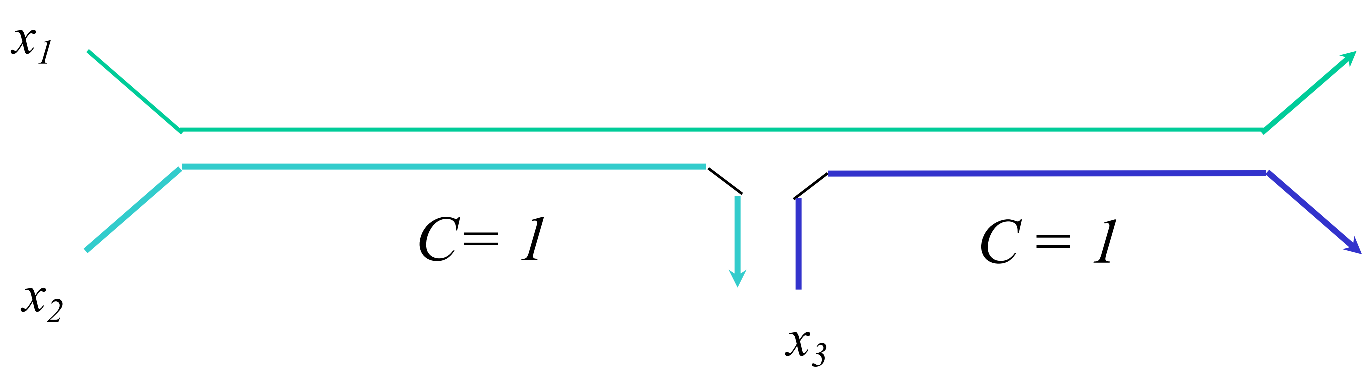
$$\max_{x_f \geq 0} \quad \sum_f x_f$$

$$U_f(x_f) = xf$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

- Optimal: $x_1 = 0$
  $x_2 = x_3 = 1$



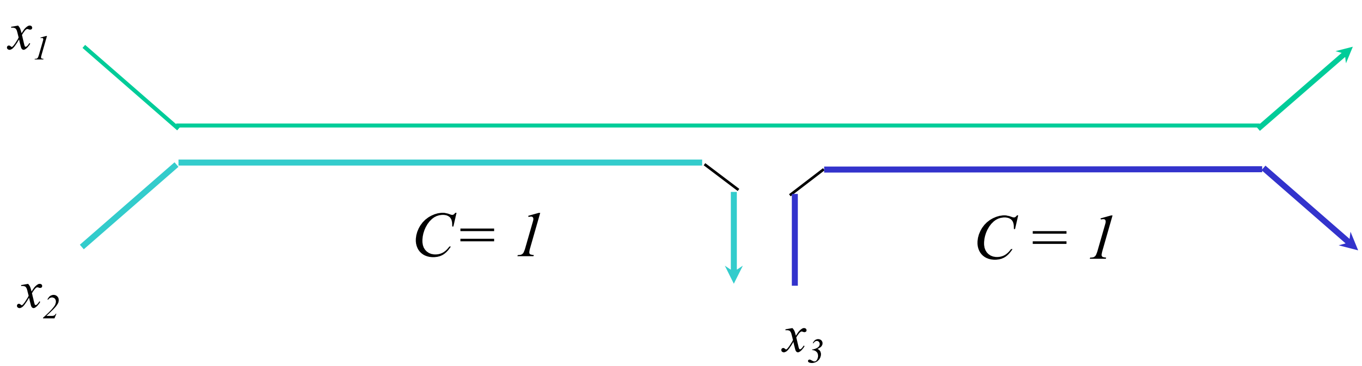
Example: Proportional Fairness

$$\max_{x_f \geq 0} \quad \sum_f \log x_f \qquad U_f(x_f) = \log(x_f)$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

- Optimal: $x_1 = 1/3$
  $x_2 = x_3 = 2/3$



Example 3: a "Funny" Utility Function

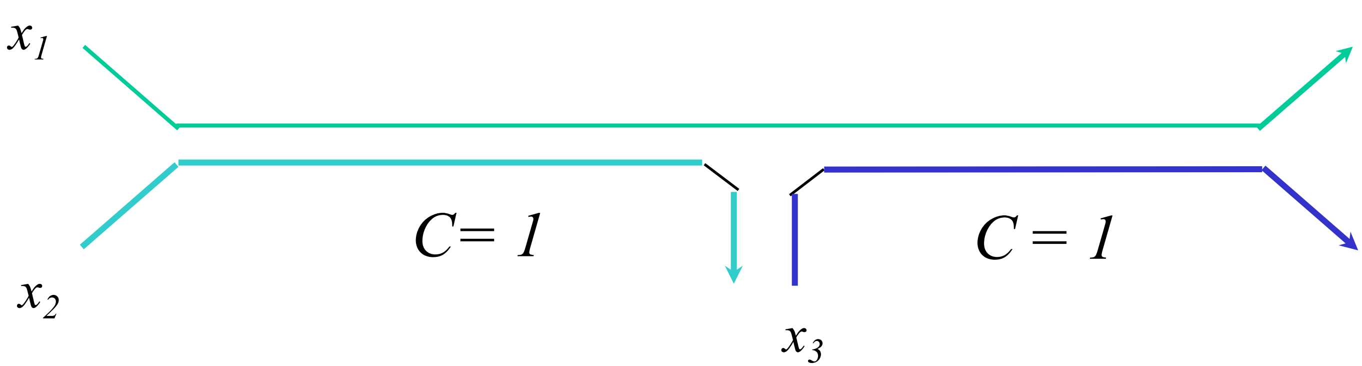
$$\max_{x_f \geq 0} \quad -\frac{1}{4x_1} - \frac{1}{x_2} - \frac{1}{x_3}$$

$$\text{subject to} \quad x_1 + x_2 \leq 1$$

$$x_1 + x_3 \leq 1$$

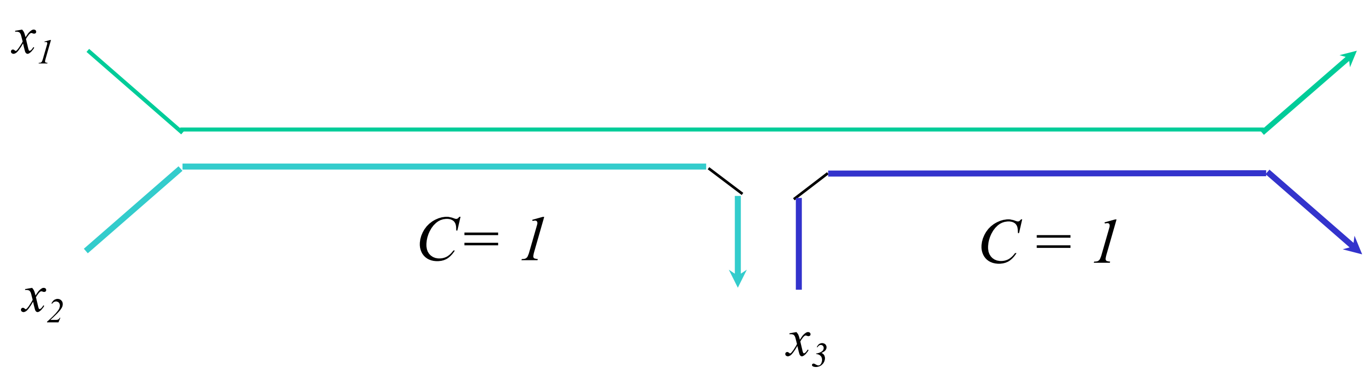
$$U_f(x_f) = -\frac{1}{RTT^2 x_f}$$

- Optimal: $x_1 = \frac{1}{1+2\sqrt{2}} = 0.26$
  $x_2 = x_3 = 0.74$



Summary of Examples

Protocol/Objective	Allocation (x1, x2, x3)		
TCP/Reno	0.26	0.74	0.74
TCP/Vegas	1/3	2/3	2/3
Max Throughput	0	1	1
Max-min	½	½	½
Max sum log(x)	1/3	2/3	2/3
Max sum of -1/(RTT^2 x)	0.26	0.74	0.74



Questions

$$\begin{array}{ll} \max & \sum_{f \in F} U_f(x_f) \\ \text{subject to} & \sum_{f:f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\ \text{over} & x \ge 0 \end{array}$$

- **Science/reverse understanding:** what do TCP/Reno, TCP/Vegas achieve? Are they related with log(x), -1/(RTT2x) utilities?
- **Forward design:** systematically
  - design objective function
  - design distributed alg to achieve objective

Objective	Allocation (x1, x2, x3)		
TCP/Reno	0.26	0.74	0.74
TCP/Vegas	1/3	2/3	2/3
Max throughput	0	1	1
Max-min	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Max sum log(x)	1/3	2/3	2/3
Max sum of $-1/(RTT^2\, x)$	0.26	0.74	0.74

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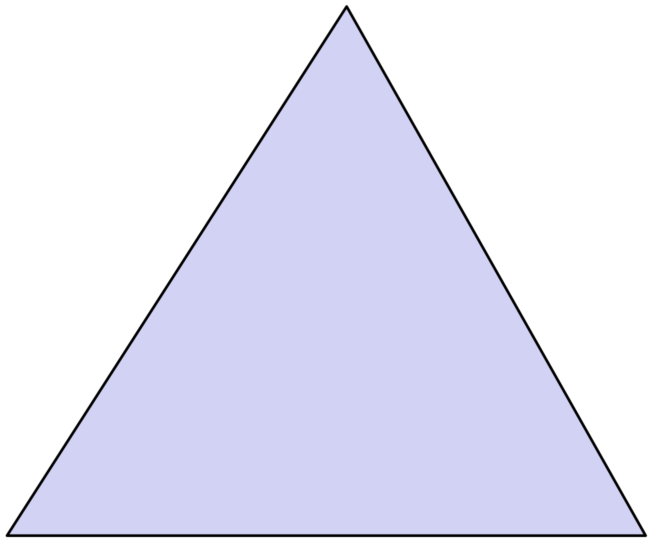
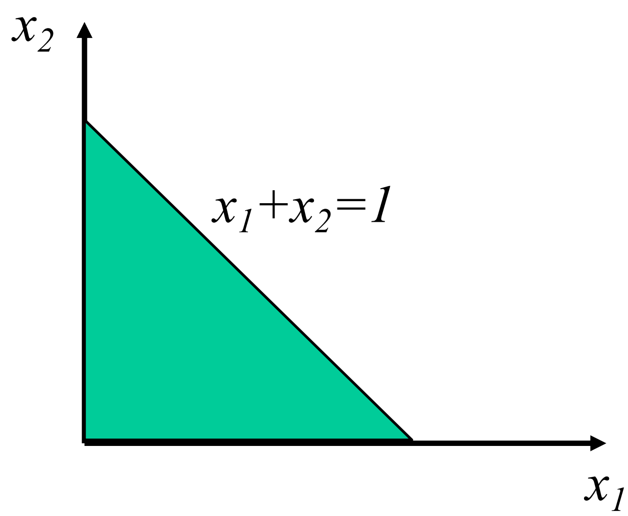
  - TCP/Vegas
  - network wide resource allocation
    - general framework
    - objective function: an example of an axiom derivation of network-wide objective function

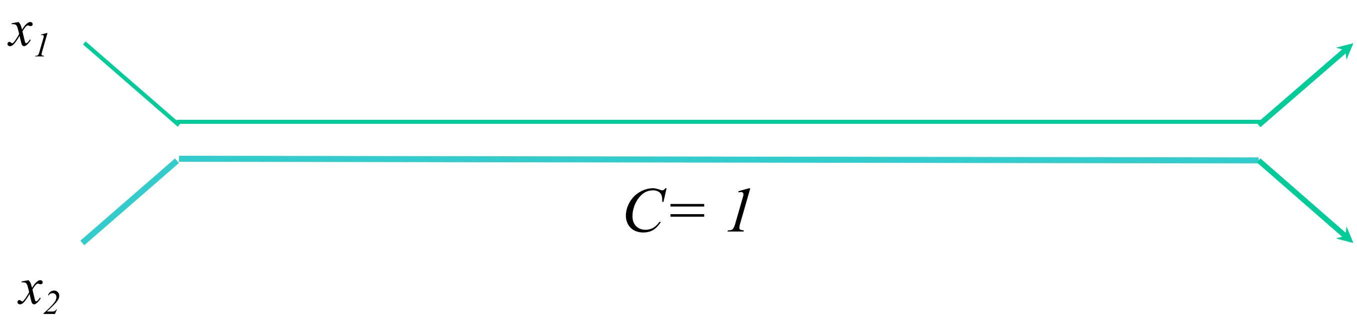
Network Bandwidth Allocation



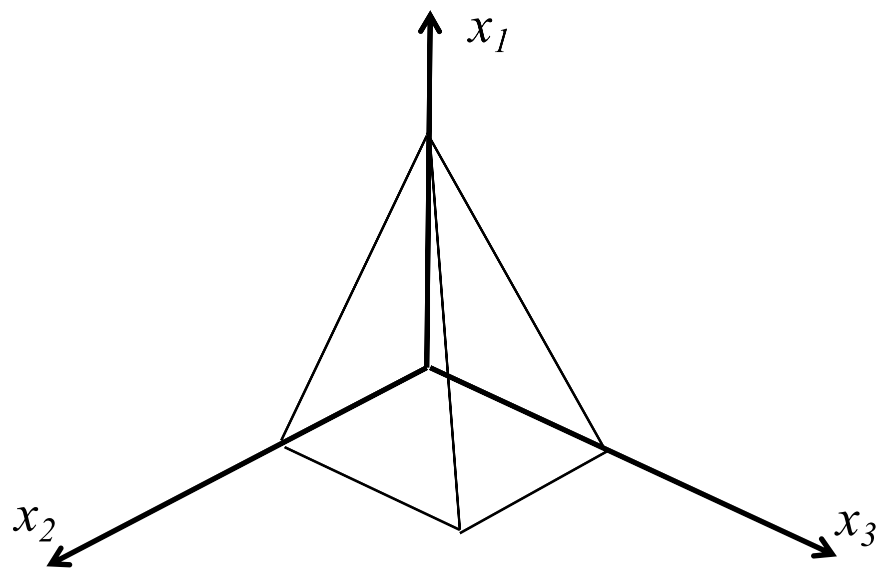
- High level picture
  - given the feasible set of bandwidth allocation, we want to pick an allocation point that is efficient and fair
- The determination of the allocation point should be based on "first principles" (axioms)

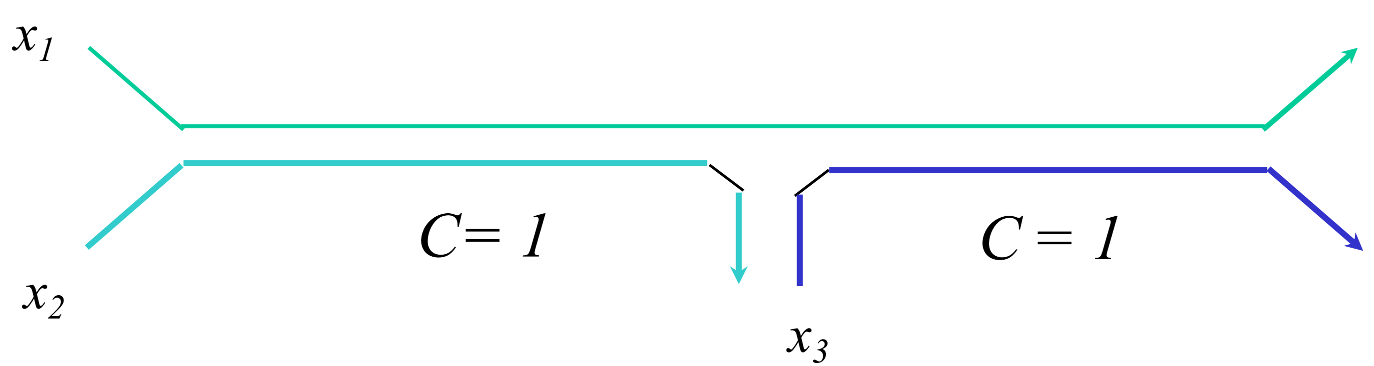
Network Bandwidth Allocation: Feasible Region





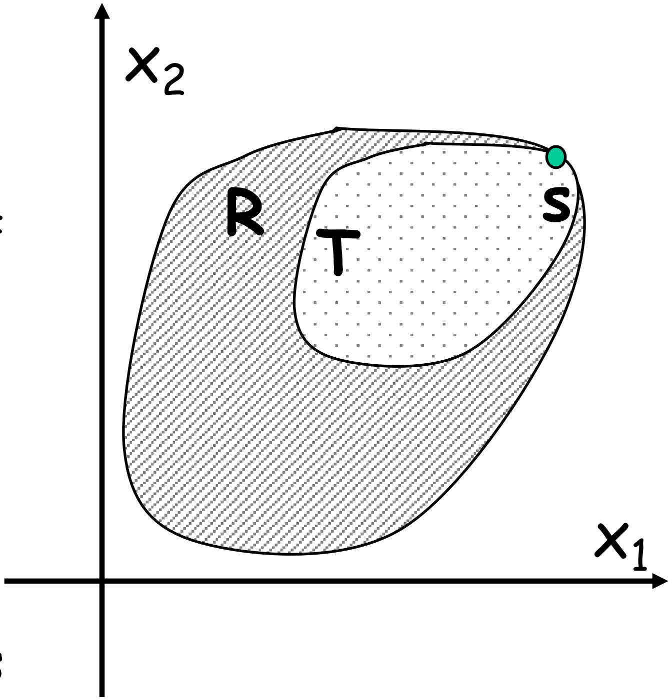
Network Bandwidth Allocation: Feasible Region





Network Bandwidth Allocation: Axioms

- Assume a finite, convex feasible set in the first quadrant
- Axioms
  - Pareto optimality
    - impossibility of increasing the rate of one user without decreasing the rate of another
  - symmetry
    - a symmetric feasible set yields a symmetric outcome
  - invariance of linear transformation
    - the allocation must be invariant to linear transformations of users' rates
  - independence of irrelevant alternatives
    - assume s is an allocation when feasible set is R, $s \in T \subset R$, then s is also an allocation when the feasible set is T



Nash Bargain Solution (NBS)

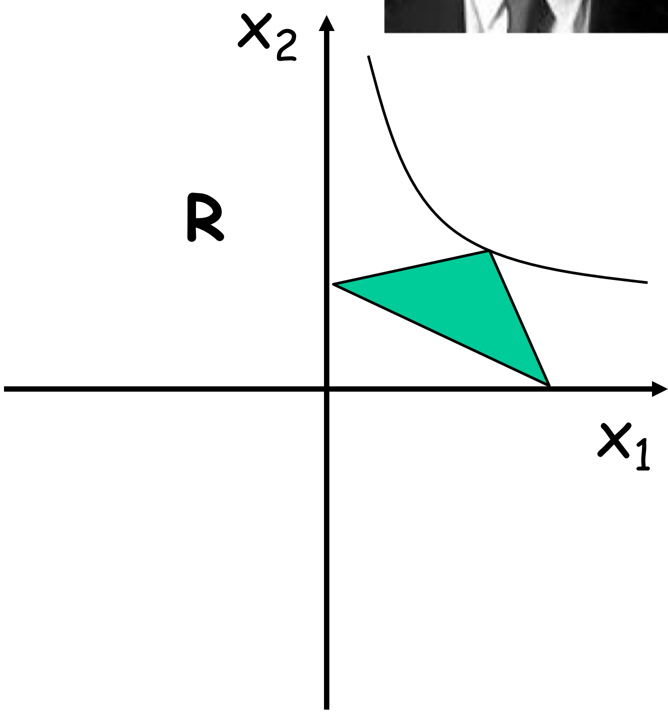
- Surprising result by John Nash (1951)
  - the rate allocation point satisfying the axioms is the feasible point which maximizes

$$x_1 x_2 \cdots x_F$$

- This is equivalent to maximize

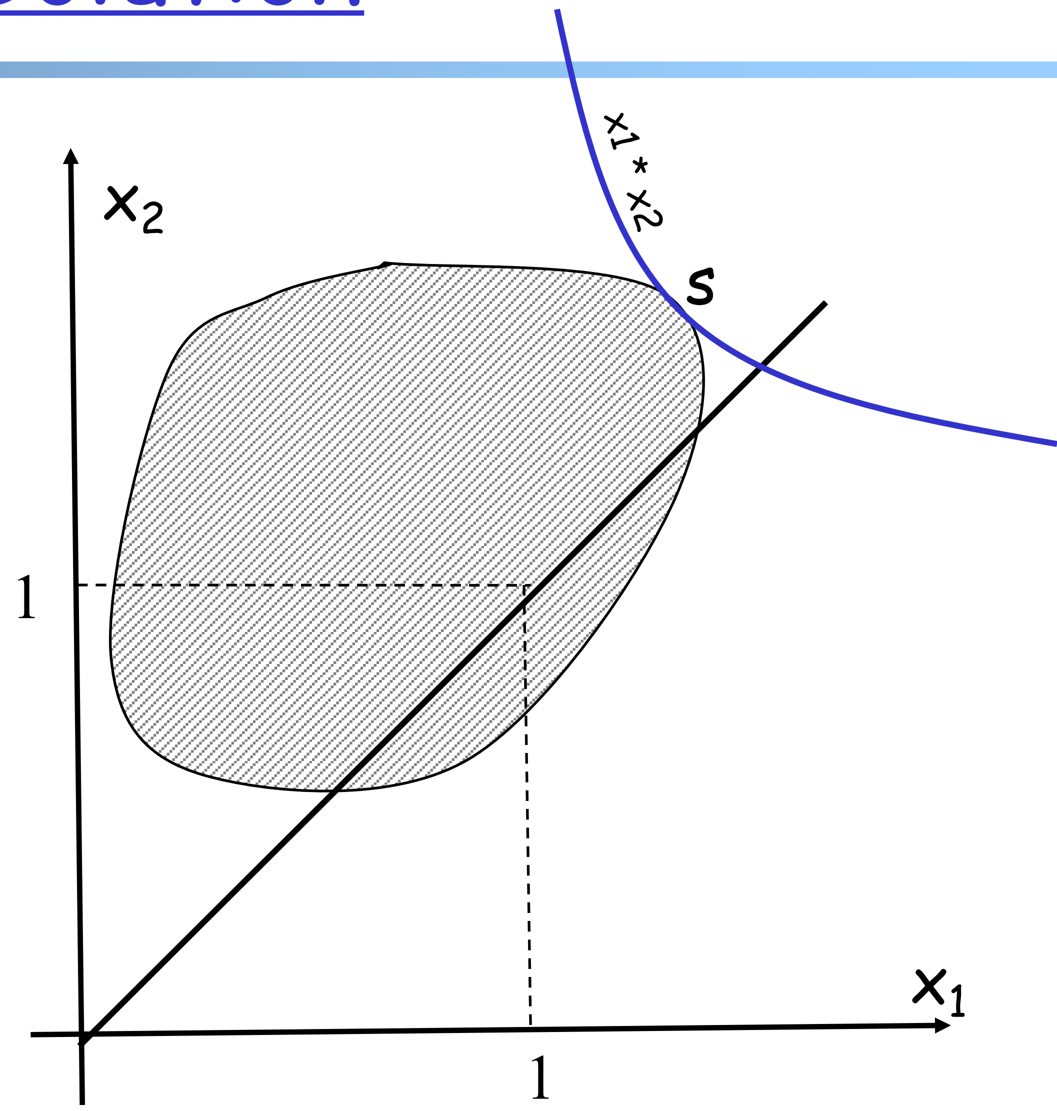
$$\sum_f \log(x_f)$$

- In other words, assume each flow f has utility function $\log(x_f)$



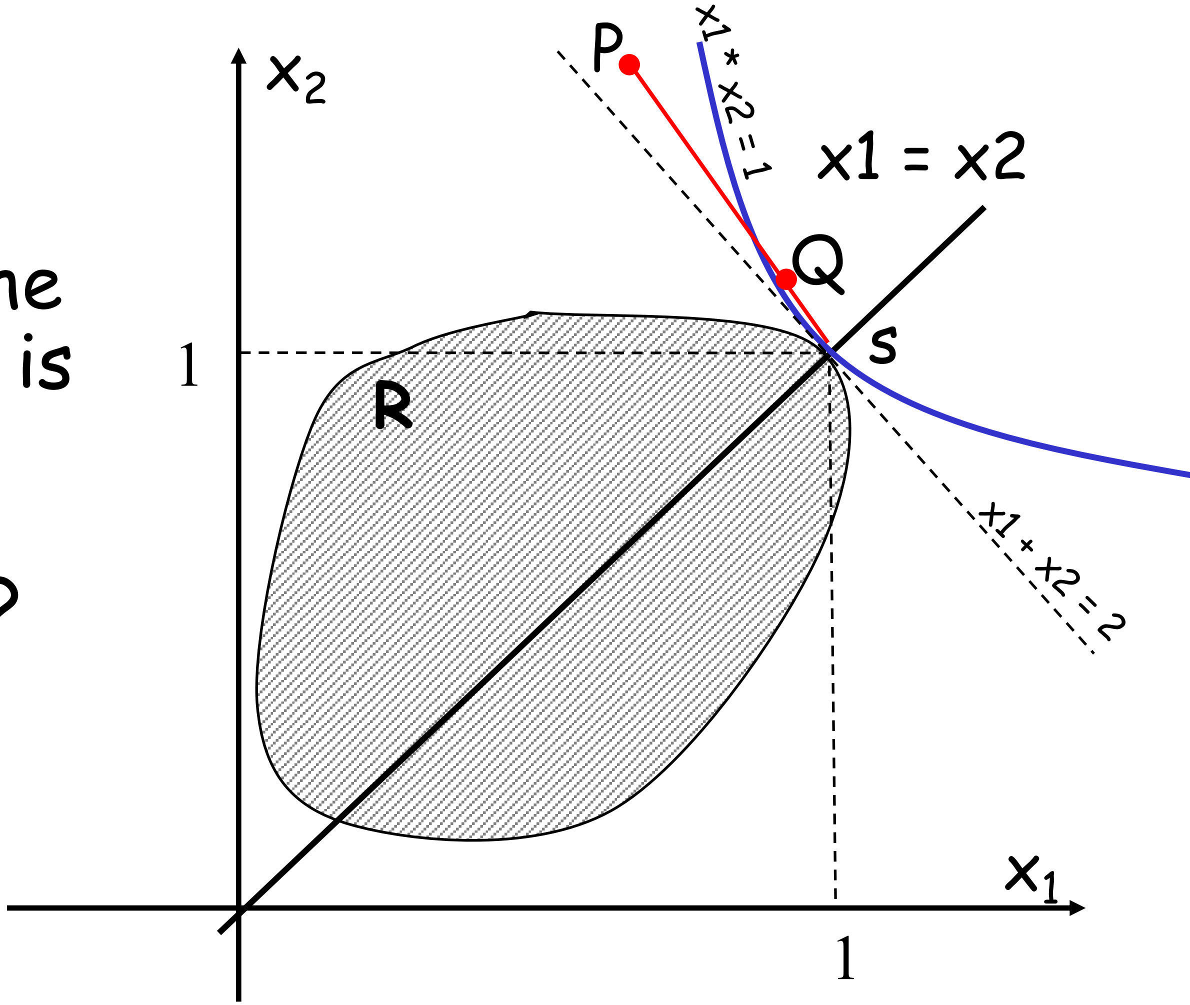
Nash Bargain Solution

- We will give a proof for F = 2
  - think about F > 2
- Assume s is the feasible point which maximizes
  x1 * x2
- Scale the feasible set so that s is at (1, 1)
  - how?



Nash Bargain Solution

Question: after the transformation, is there any feasible point with x1 + x2 > 2?

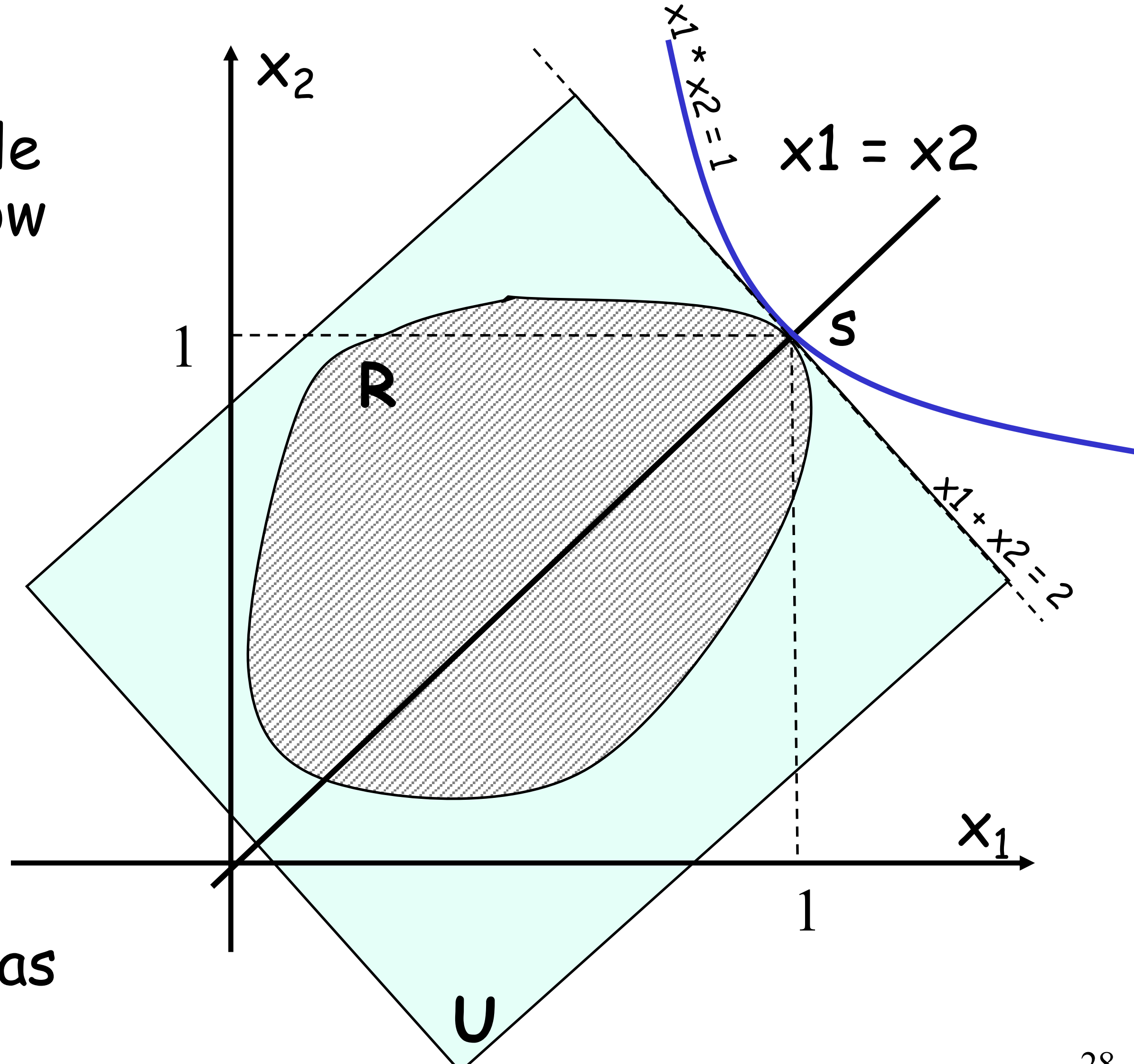


Nash Bargain Solution

- Consider the symmetric rectangle U containing the now feasible set

-> According to symmetry and Pareto, s is the allocation when feasible set is U

- According to independence of irrelevant alternatives, the allocation of R is s as well.



NBS ⇔ Proportional Fairness

- Allocation is proportionally fair if for any other allocation, aggregate of proportional changes is non-positive, e.g. if x_f is a proportional-fair allocation, and y_f is any other feasible allocation, then require

$$\sum_f \frac{y_f - x_f}{x_f} \leq 0$$

Offline Question to Think About

- Vary the axioms and see if you can derive any objective functions

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  - TCP/Reno congestion control
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  - TCP/Vegas
  - network wide resource allocation
    - general framework
    - objective function: an example axiom derivation of network-wide objective function
    - algorithm: general distributed algorithm framework

Recall: Resource Allocation Framework

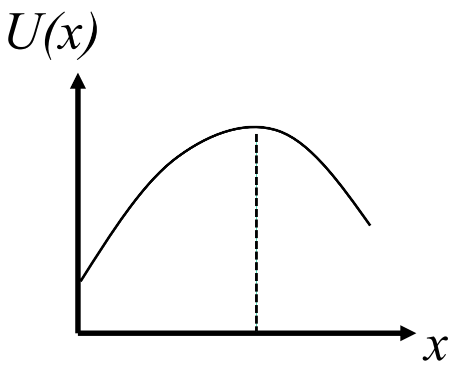
- The Resource-Allocation Problem:

$$
\begin{array}{ll}
\max & \sum_{f \in F} U_f\left(x_f\right) \\
\text{subject to} & \sum_{f : f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\
\text{over} & x \ge 0
\end{array}
$$

- Goal: Design a distributed alg to solve the problem.

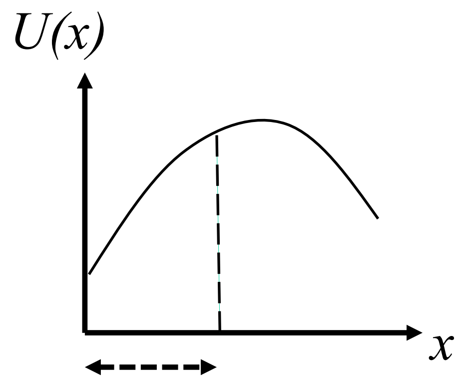
Review

- What are typical approaches to optimize an objective function U(x) ?



The gradient algorithm

$$\Delta x \sim dU(x)/dx$$



range

$$\Delta x \sim dU(x)/dx$$

make sure x does not go out of range (e.g., projection)

Recall: Resource Allocation Framework

- The Resource-Allocation Problem:

$$
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\max & \sum_{f \in F} U_f\left(x_f\right) \\
\text{subject to} & \sum_{f : f \text{ uses link } l} x_f \le c_l \text{ for any link } l \\
\text{over} & x \ge 0
\end{array}
$$

- Goal: Design a distributed alg to solve the problem.

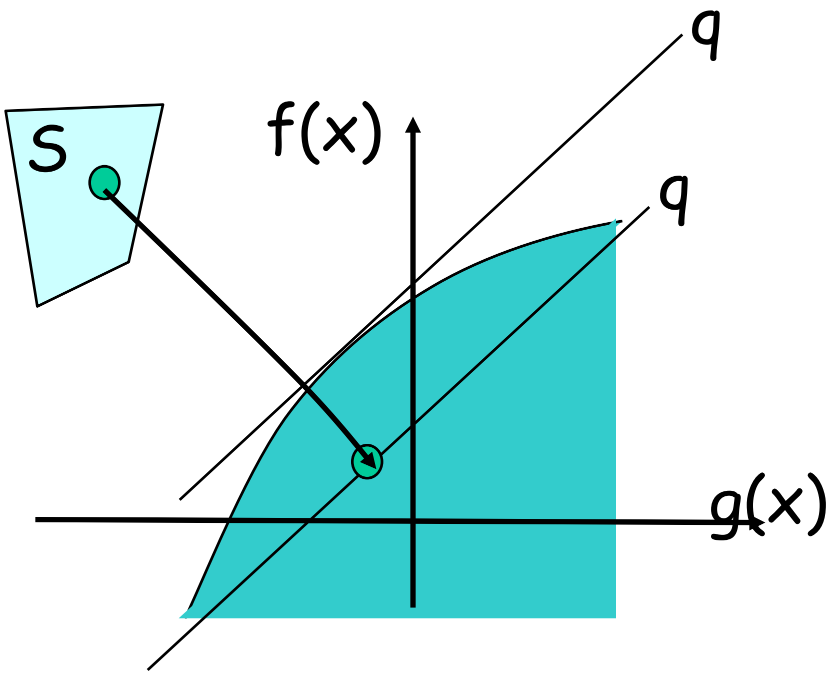
- Discussion: What are issues to apply the gradient alg to solve the Resource-Allocation program?

A Two-Slide Summary of Constrained Convex Optimization Theory

$$\begin{array}{ll} \max & \sum_{f \in F} U_f(x_f) \\ \text{subject to} & Ax \le C \\ \text{over} & x \ge 0 \end{array}$$

$$\begin{array}{ll} \max & f(x) \\ \text{subject to} & g(x) \le 0 \\ \text{over} & x \in S \end{array}$$

f(x) concave
g(x) linear
S is a convex set



-Map each x in S, to [g(x), f(x)]

-Top contour of map is concave

-Easy to read solution from contour

-For each slope q (>=0), computes f(x) – q g(x) of all mapped [f(x), g(x)]

$$D(q) = \max_{x \in S} \left(f(x) - q g(x) \right)$$

A Two-Slide Summary of Constrained Convex Optimization Theory

$$\begin{array}{ll} \max & f(x) \\ \text{subject to} & g(x) \le 0 \\ \text{over} & x \in S \end{array}$$

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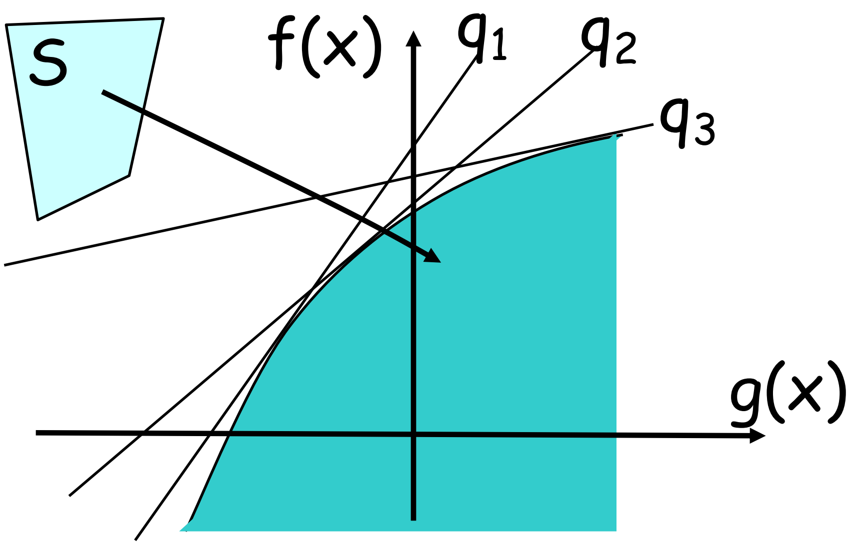
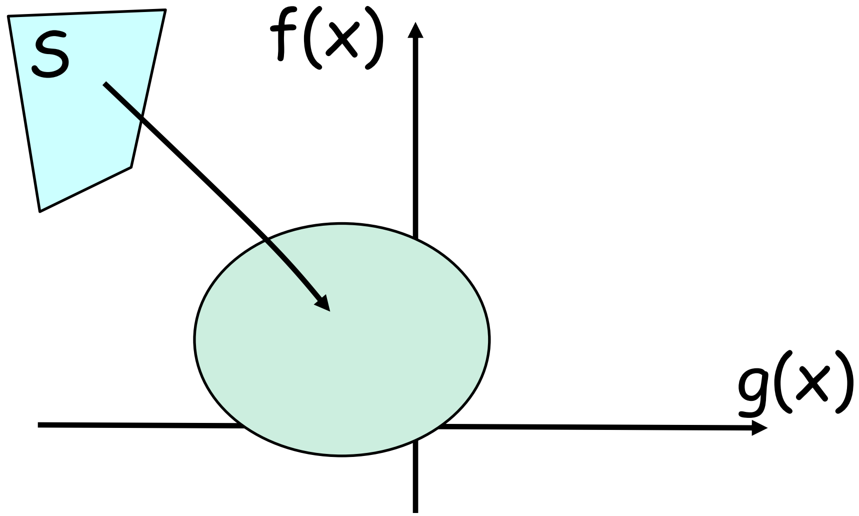


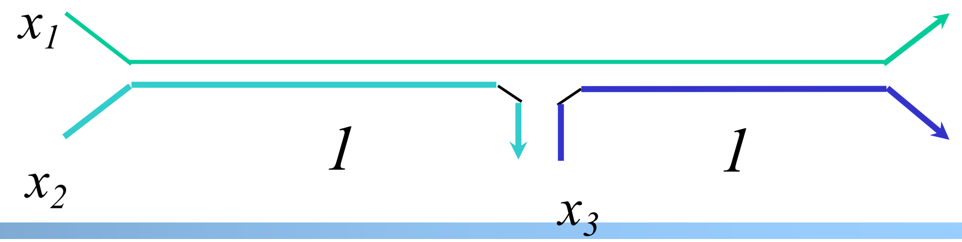
$$D(q) = \max_{x \in S}\left(f(x) - q g(x)\right)$$

-D(q) is called the dual;
q (>= 0) are called prices in economics
-D(q) provides an upper bound on obj.
- According to optimization theory: when D(q) achieves minimum over all q (>= 0), then the optimization objective is achieved.

Exercise



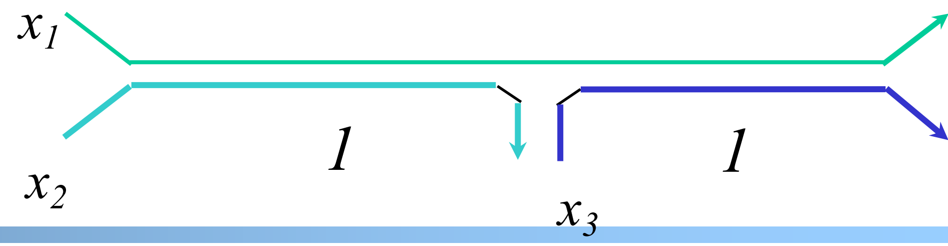
Dual of the Primal



max	$\sum_{f \in F} U_f\left(x_f\right)$
subject to	$\sum_{f : f \text{ uses link } l} x_f \le c_l$ for any link l
over	$x \ge 0$

$$D(q) = \max_{x_f \ge 0} \left(\sum_f U_f(x_f) - \sum_l q_l \Big(\sum_{f:\text{uses } l} x_f - c_l \Big) \right)$$

Dual of the Primal



$$D(q) = \max_{x_f \geq 0} \left(\sum_f U_f(x_f) - \sum_l q_l \left(\sum_{f:\text{uses } l} x_f - c_l \right) \right)$$

$$= \max_{x_f \geq 0} \sum_f \left(U_f(x_f) - x_f \sum_{l:\text{f uses } l} q_l \right) + \sum_l q_l c_l$$

$$= \sum_f \max_{x_f \geq 0} \left(U_f(x_f) - x_f \sum_{l:\text{f uses } l} q_l \right) + \sum_l q_l c_l$$

Distributed Optimization: Flow Problem

- Given p_f (=sum of dual var q_l along the path) flow f chooses rate x_f to maximize:

$$\max_{x_f} \quad U_f(x_f) - x_f p_f$$
$$\text{over} \quad x_f \geq 0$$

- Using the aggregated signals (p), the optimization problem of each flow is independent of each other!

Distributed Optimization: Flow Problem

$$\max_{x_f} \quad U_f(x_f) - x_f p_f$$
$$\text{over} \quad x_f \geq 0$$

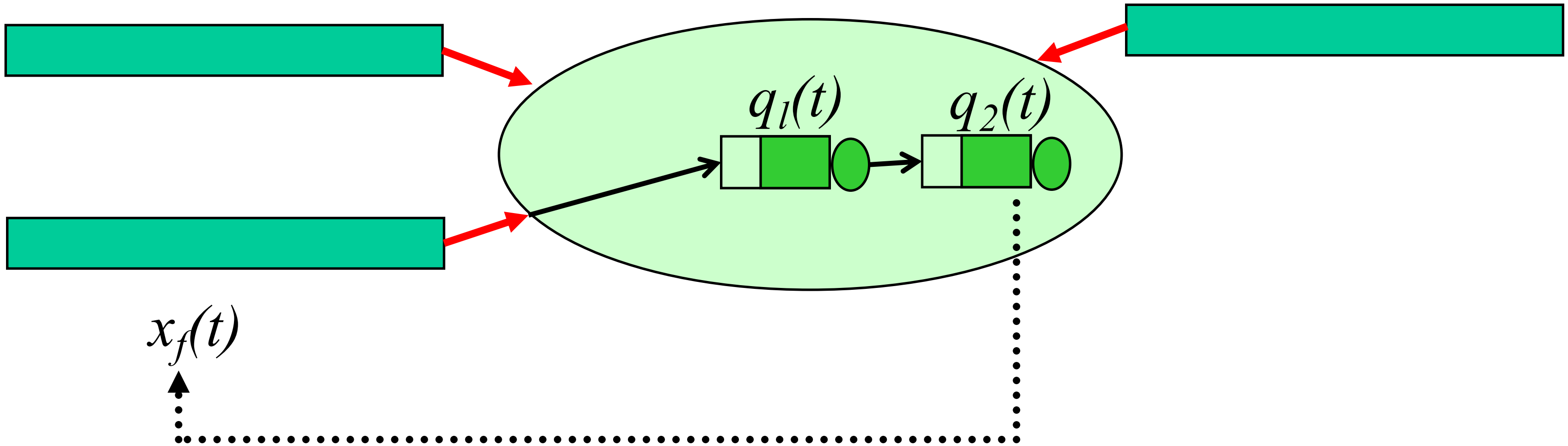
How should flow f adjust x_f locally?

$$\boxed{\Delta x_f \propto U'_f(x_f) - p_f}$$

At equilibrium (i.e., at optimal), x_f satisfies:

$$U'_f(x_f) - p_f = 0$$

Interpreting Congestion Measure



$$p_f = \sum_{\text{f uses l}} q_l$$

$$\Delta x_f \propto U'_f(x_f) - p_f$$

Distributed Optimization: Network Problem

$$D(q) = \sum_f \max_{x_f \geq 0} \left(U_f(x_f) - x_f \sum_{l:\text{f uses } l} q_l \right) + \sum_l q_l c_l$$

The network (i.e., link l) adjusts the link signals q_l (assume after all flows have picked their optimal rates given congestion signal)

$$\min_{q \geq 0} \widetilde{D}(q) = \sum_l q_l (c_l - \sum_{f:\text{f uses } l} x_f)$$

Distributed Optimization: Network Problem

$$\min_{q \geq 0} D(q) = \sum_{l} q_l (c_l - \sum_{f:\text{ f uses } l} x_f)$$

How should link l adjust q_l locally?

$$\Delta q_l \propto -\frac{\partial D(q)}{q_l}$$

$$\frac{\partial}{\partial q_l} D(q) = c_l - \sum_{f:\text{uses } l} x_f$$

$$\Delta q_l \propto \sum_{f:\text{uses } l} x_f - c_l$$

System Architecture

- SYSTEM(U):

$$\begin{array}{ll} \max & \sum_{f \in F} U_f\left(x_f\right) \\ \text{subject to} & \sum_{f:f \text{ uses link } l} x_f \leq c_l \text{ for any link } l \\ \text{over} & x \geq 0 \end{array}$$

- FLOW:

$$\Delta x_f \propto U'_f\left(x_f\right) - p_f$$

$$\begin{array}{ll} \max\limits_{x_f} & U_f\left(x_f\right) - x_f p_f \\ \text{over} & x_f \geq 0 \end{array}$$

- NETWORK:

$$\Delta q_l \propto -\frac{\partial D(q)}{q_l}$$

$$\min_{q \geq 0} \widetilde{D}(q) = \sum_l q_l (c_l - \sum_{f:\text{f uses } l} x_f)$$

Decomposition Theorem

- There exist vectors **p** , **w** and **x** such that
  1. $w_f = p_f x_f$ for $f \in F$
  2. w_f solves $USER_f(U_f; p_f)$
  3. x solves NETWORK(w)

- The vector x then also solves SYSTEM(U).

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    - objective function: an example axiom derivation of network-wide objective function
    - algorithm: a general distributed algorithm framework
    - application: TCP/Reno and TCP/Vegas revisited

TCP/Reno Dynamics

$$\boxed{\Delta x_f \propto U'_f\left(x_f\right) - p_f}$$

$$\Delta W_{pkt} = (1-p)\frac{1}{W} - p\,\frac{W}{2}$$

$$\Delta W_{RTT} = \Delta\, W_{RTT} * W = (1-p) \;-\; p\,\frac{W^2}{2} \cong 1 - p\,\frac{W^2}{2}$$

$$\Delta x \;=\; \frac{\Delta W_{RTT}}{RTT} = \frac{1}{RTT} \;-\; \frac{RTT}{2}\,p\,x^2$$

$$= \frac{RTT}{2}\,x^2\left(\frac{2}{x^2 RTT^2} - p\right)$$

TCP/Reno Dynamics

$$\boxed{\Delta x_f \propto U'_f(x_f) - p_f}$$

$$\Delta x = \frac{RTT}{2} x^2 \boxed{\left(\frac{2}{x^2 RTT^2} - p\right)}$$

$$U'_f(x_f) - p_f$$

$$\Rightarrow U'_f(x_f) = \left(\frac{\sqrt{2}}{x_f RTT}\right)^2 \qquad \Rightarrow U_f(x_f) = -\frac{2}{RTT^2 x_f}$$

TCP/Vegas Dynamics

$$\boxed{\Delta x_f \propto U'_f\left(x_f\right) - p_f}$$

$$\Delta \mathrm{w}_{\mathrm{RTT}} \approx -(w - xRTT_{\min} - \alpha)$$

$$\Delta x = \frac{\Delta W_{RTT}}{RTT} = -\left(\frac{w}{RTT} - \frac{x}{RTT}RTT_{min} - \frac{\alpha}{RTT}\right)$$

$$= -\frac{w}{RTT} + \frac{x}{RTT}RTT_{min} + \frac{\alpha}{RTT}$$

$$= -x + \frac{x}{RTT}RTT_{min} + \frac{\alpha}{RTT}$$

$$= \frac{x}{RTT}\left(-RTT + \mathrm{RTT}_{\min} + \frac{\alpha}{x}\right)$$

$$= \frac{x}{RTT}\left(\frac{\alpha}{x} - (RTT - \mathrm{RTT}_{\min})\right)$$

TCP/Vegas Dynamics

$$\boxed{\Delta x_f \propto U'_f\left(x_f\right) - p_f}$$

$$\Delta x = \frac{x}{RTT}\left(\boxed{\frac{\alpha}{x} - (RTT - \mathrm{RTTmin}}\right)\right)$$

$$U'_f\left(x_f\right) - p_f$$

$$\Rightarrow U'_f(x_f) = \frac{\alpha}{x} \qquad \Rightarrow U_f(x_f) = \alpha \log(x_f)$$

Summary: TCP/Vegas and TCP/Reno

- Pricing signal is queueing delay $T_{queueing}$

$$x_f = \frac{\alpha}{T_{queueing}}$$

$$U_f'(x_f) = T_{queueing}$$

$$\Rightarrow U_f'(x_f) = \frac{\alpha}{x_f}$$

$$\Rightarrow U_f(x_f) = \alpha \log(x_f)$$

- Pricing signal is loss rate p

$$x_f = \frac{\alpha}{RTT\sqrt{p}}$$

$$U_f'(x_f) = p$$

$$\Rightarrow U_f'(x_f) = \left(\frac{\alpha}{x_f RTT}\right)^2$$

$$\Rightarrow U_f(x_f) = -\frac{\alpha'}{RTT^2 x_f}$$

Summary: Resource Allocation Frameworks

- Forward (design) engineering:
  - how to determine objective functions
  - given objective, how to design effective, distributed alg

$$
\begin{array}{ll}
\text{max} & \sum_{f \in F} U_f(x_f) \\
\text{subject to} & Ax \le C \\
\text{over} & x \ge 0
\end{array}
$$

- Reverse (understand) engineering:
  - understand current protocols (what are the objectives of TCP/Reno, TCP/Vegas?)
- Additional pointers:
  - http://www.statslab.cam.ac.uk/~frank/pf/

Summary

- Many aspects of TCP can be studied, for example
  - Transport security
  - TCP under wireless (LTE)
  - Multipath TCP
  - ...