Deep Learning Theory and Applications

Feedforward Networks

Yale

CPSC/AMTH 663

Outline

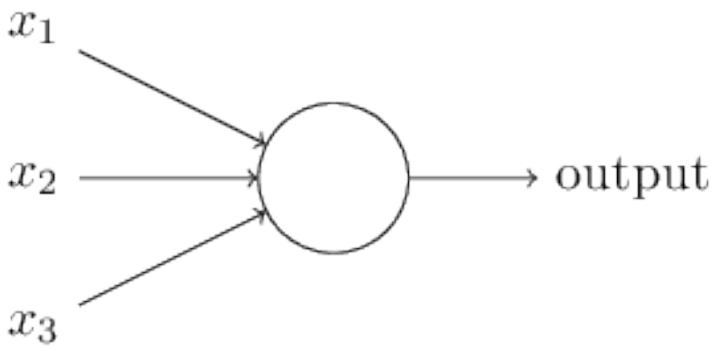
1. Feedforward network overview
   - Sigmoid neuron
   - Architecture of a neural network

Handwritten digit recognition

Handwritten digit recognition

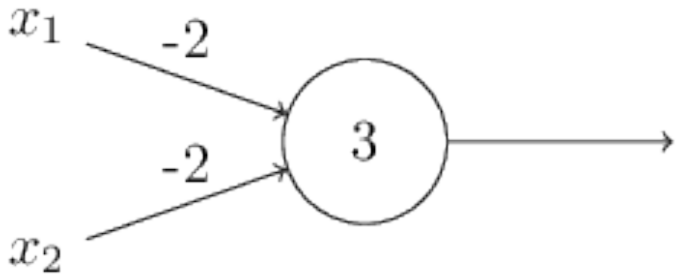
Recall a Perceptron

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$



Logic circuits with perceptrons

- NAND gates can be constructed from perceptrons
- NAND gates are universal for computation
  - Any computation can be built from NAND gates
  - Therefore, perceptrons are universal for computation



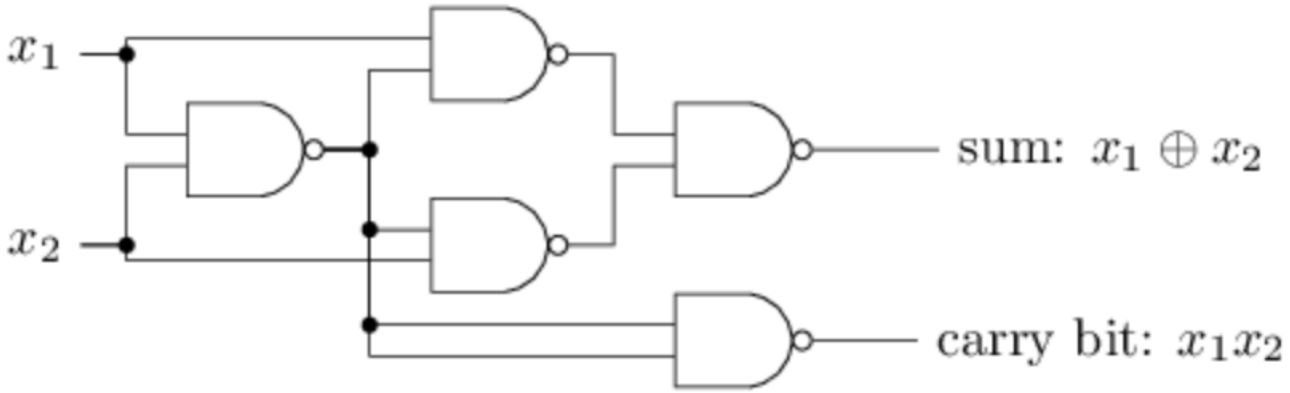
Nielsen, 2015

Adders

- How do you create an AND?
- How do you create an OR?
- Creating an ADDER

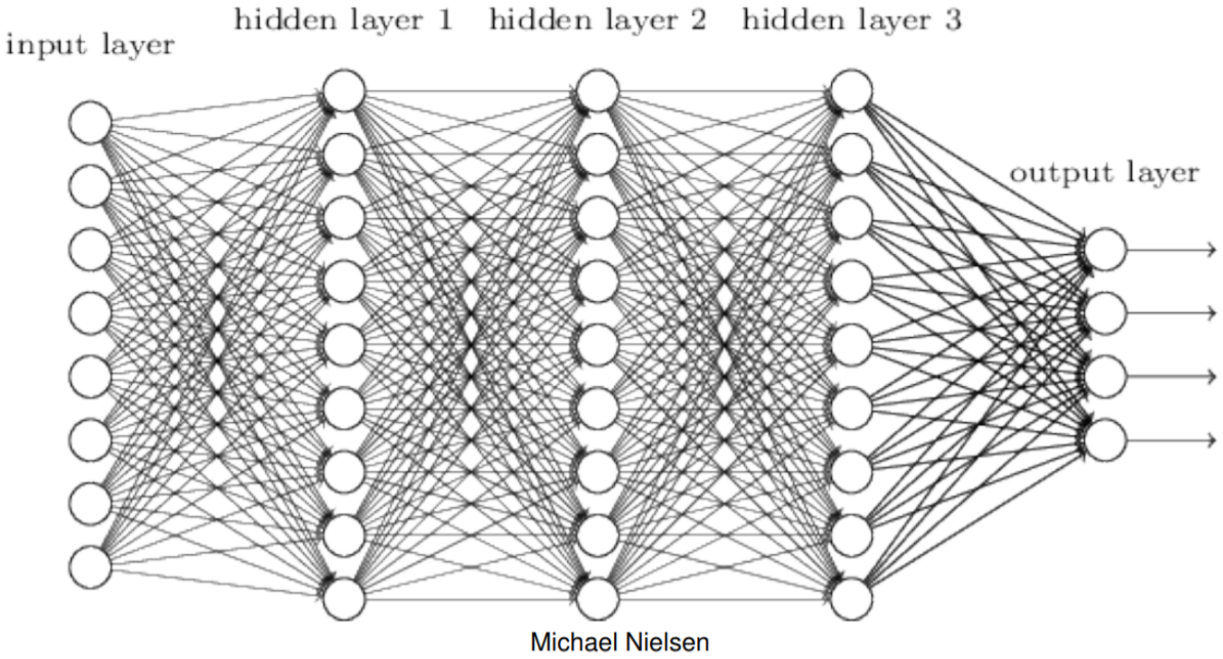
sum: $x_1 \oplus x_2$

$x_1 x_2$



Feedforward Network

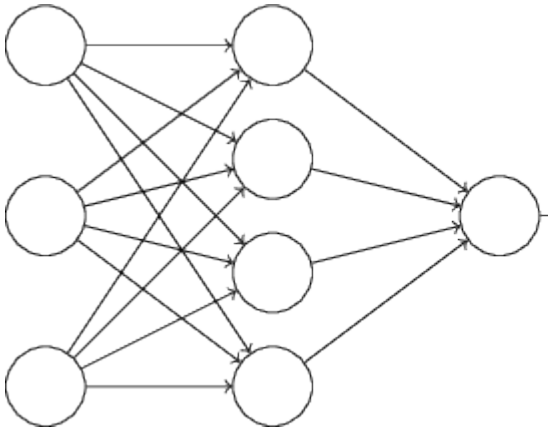
- We can create *learning algorithms* that automatically tune the weights and biases
  - Tuning occurs in response to external stimuli and w/o direct intervention
  - Creates a circuit designed for the problem at hand



Learning the weights and biases

- Goal: use a network of perceptrons to learn to solve a problem
- Example: learn the weights and biases in a network to perform handwritten digit recognition

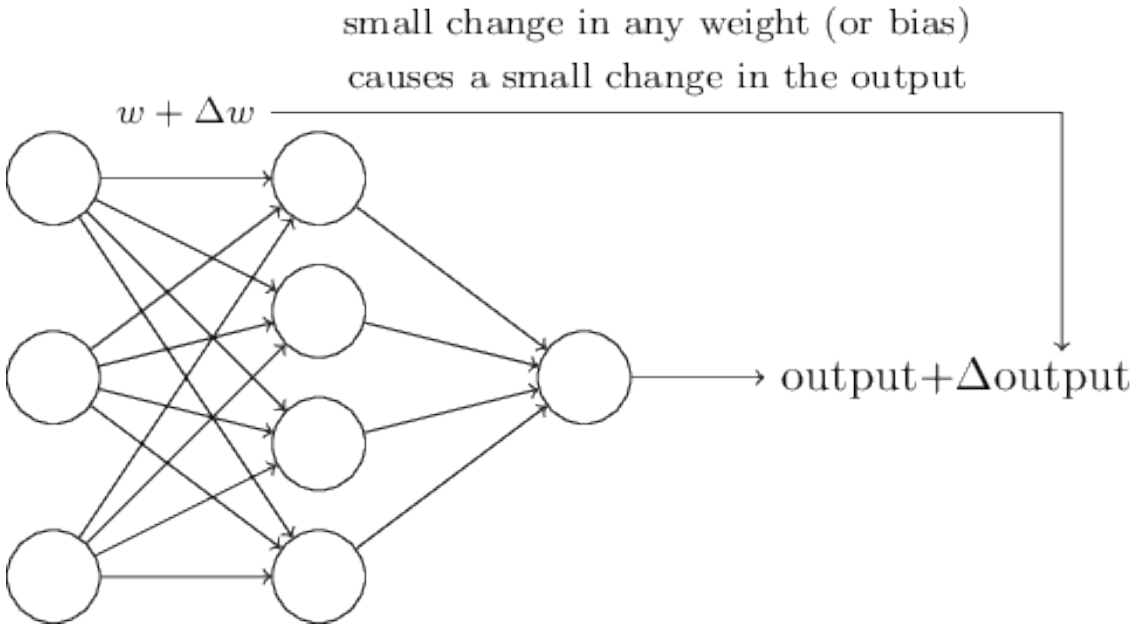
Input



Digit Classification

Learning the weights and biases

- Want small changes in weights or biases to result in small changes in output

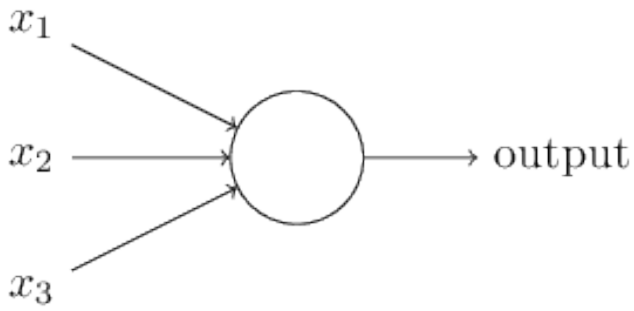


Learning the weights and biases

- Want small changes in weights or biases to result in small changes in output
- Example: suppose network misclassifies an “8” as a “9”
  - Make small changes in weights and biases to get network a little closer to correct classification
  - Repeat to produce better and better output (i.e. learn)
- **Problem**: this doesn’t happen with perceptron networks
  - Small changes in weights and bias of a single perceptron may cause output to flip (big change)
  - This flip may create complicated changes in the network
  - So even if we now correctly classify the “9”, the network behavior on all other images may be drastically different
  - Difficult to learn appropriate weights and biases

Sigmoid Neuron

- A ***sigmoid neuron*** is a modified perceptron so that small changes to weights and biases ⇒ small changes in output

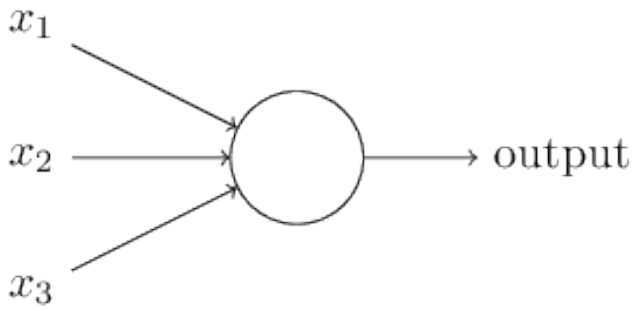


- Inputs may be *between* 0 and 1
  - E.g. 0.638 is a valid input
- Weights w_i for each input and bias b
- Output is $\sigma(w \cdot x + b)$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Sigmoid Neuron

- A ***sigmoid neuron*** is a modified perceptron so that small changes to weights and biases ⇒ small changes in output



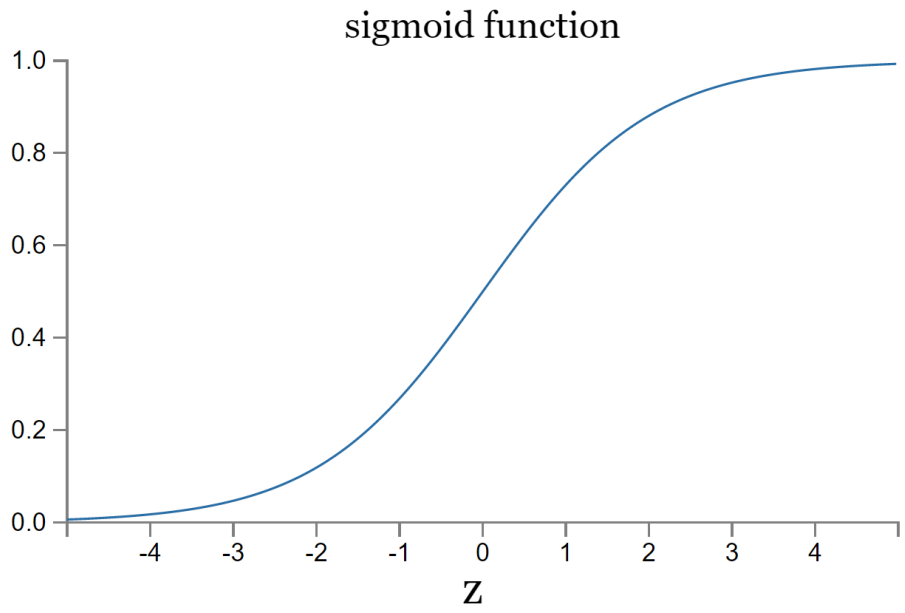
- Output is

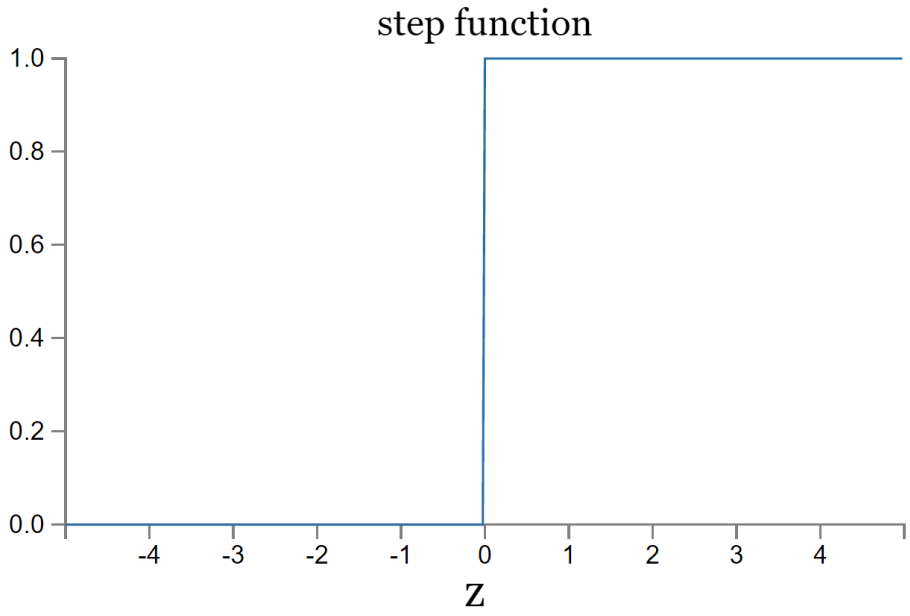
$$\sigma(w \cdot x + b) = \frac{1}{1 + \exp(-\sum_j w_j x_j - b)}$$

Relationship between sigmoid neuron and perceptrons

- $\sigma(z) = \frac{1}{1+e^{-z}}$
- Suppose $z = w \cdot x + b$ is large and positive
  - $e^{-z} \approx 0 \Rightarrow \sigma(z) \approx 1$
  - I.e. if $z = w \cdot x + b$ is large and positive, the sigmoid neuron output is approximately 1 (like a perceptron)
- Suppose $z = w \cdot x + b$ is large and negative
  - $e^{-z} \to \infty \Rightarrow \sigma(z) \approx 0$
  - I.e. if $z = w \cdot x + b$ is large and negative, the sigmoid neuron output is approximately 0 (like a perceptron)
- Some deviation for modest values of $z = w \cdot x + b$

Relationship between sigmoid neuron and perceptrons





- For the perceptron, $\sigma =$ the step function
- Smoothness of the sigmoid function $\Rightarrow$ small changes in weights and biases produce small change in output

Weights, bias, and output

- Change in weight Δw_j, change in bias Δb

$$\Delta\text{output} \approx \sum_j \frac{\partial \text{output}}{\partial w_j} \Delta w_j + \frac{\partial \text{output}}{\partial b} \Delta b$$

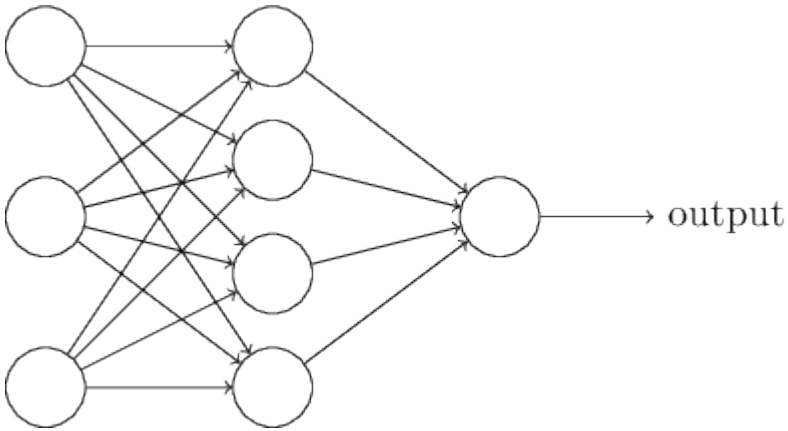
- Δoutput is a *linear function* of the changes Δw_j and Δb in the weights and bias
  - Linearity makes it easier to choose small changes in weights and biases to achieve the desired small change in output
- Thus sigmoid neurons have similar behavior as perceptrons but are easier to use

Activation functions

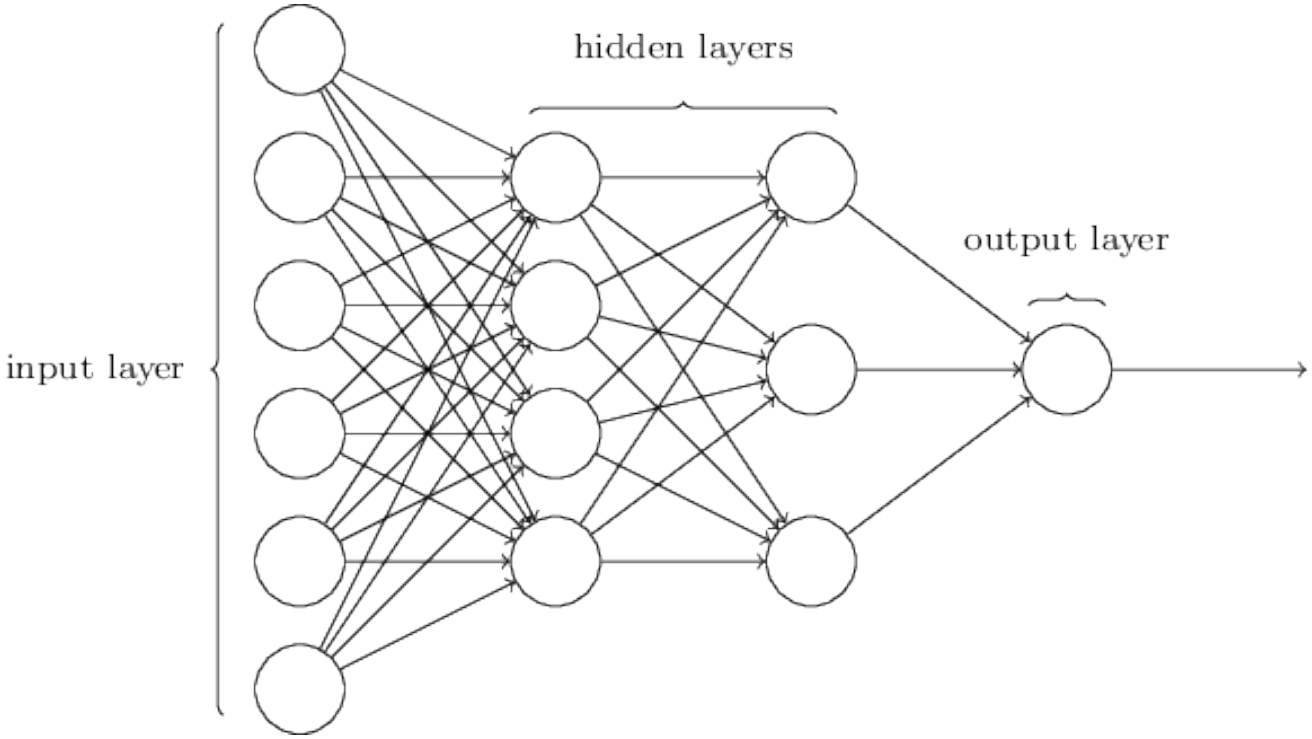
- If smoothness is what we need, why the sigmoid function σ?
- Using σ simplifies the algebra
- Other ***activation functions*** $f(w \cdot x + b)$ are used in practice
  - Depends on the application
  - More on these later in the semester

Classification with sigmoid neurons

- Sigmoid neuron output = any real number between 0 and 1
- How do we do classification?
- Threshold the final output
  - E.g., a value above 0.5 indicates a “9” while a value below 0.5 does not

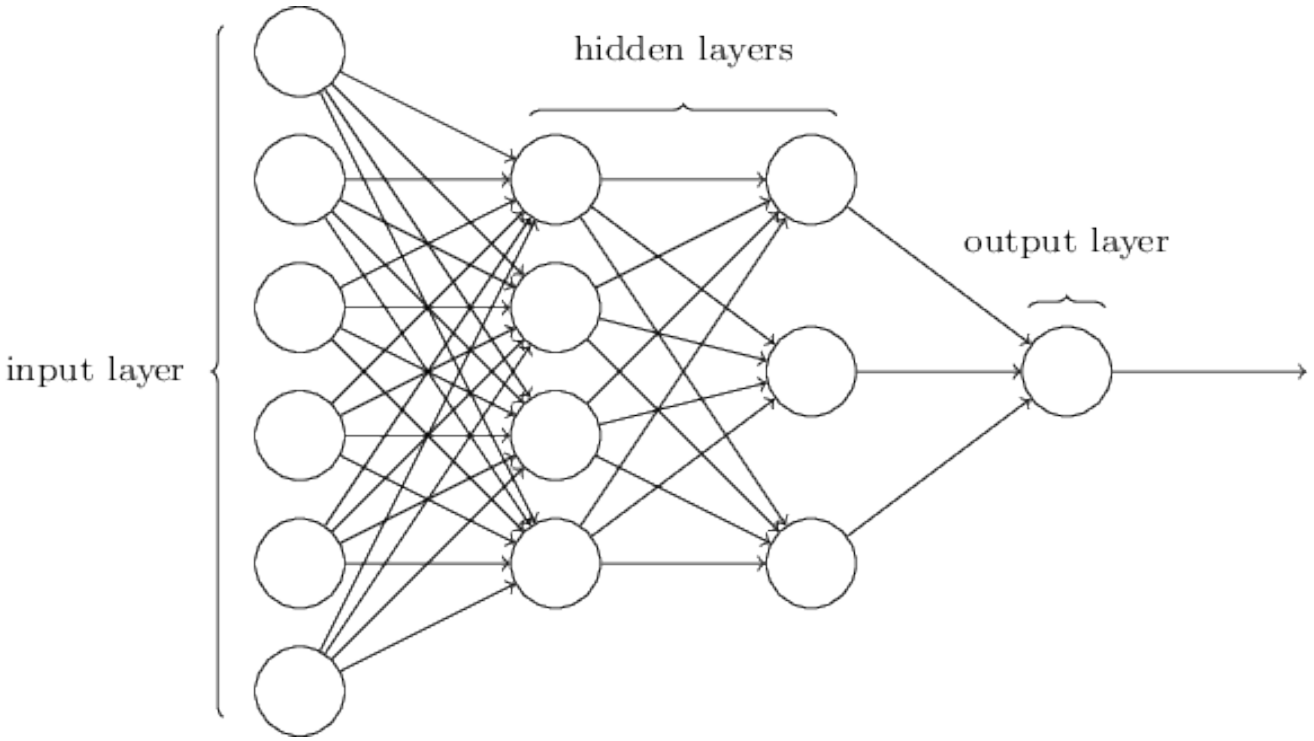


Architecture of Neural Networks



- Sometimes called multi-layer perceptron (although sigmoid neurons are used)
- Output from one layer is used as input for the next (feedforward network)

Architecture of Neural Networks



- Input and output layers are easily designed
- Hidden layers are harder
  - Many heuristics exist (which we'll cover later)

Handwritten digit recognition

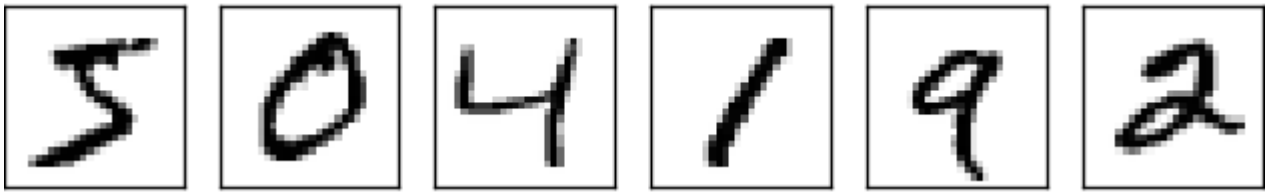
Two problems

1. Segmentation
2. Digit recognition

Handwritten digit recognition

Two problems

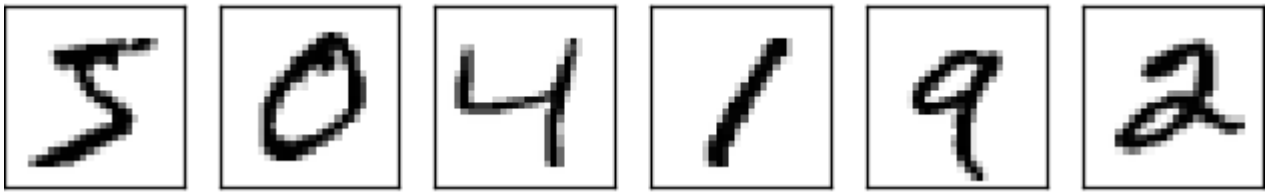
1. Segmentation
2. Digit recognition



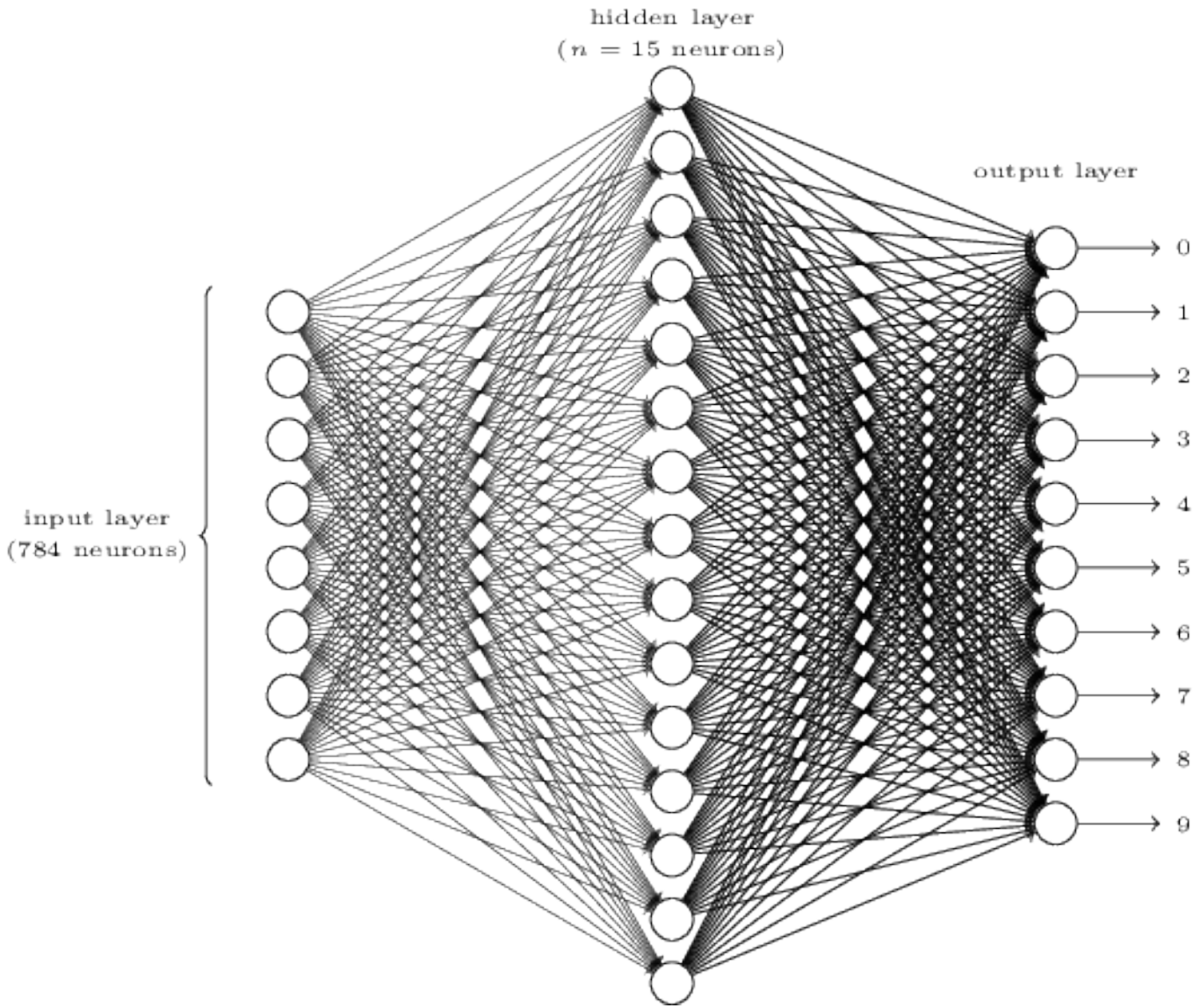
- We'll focus on digit recognition

Image segmentation approach

- Run digit classifier on multiple segmentations
  - High score if classifier is confident in all segments
  - Low score if the classifier fails in one or more segments
- I.e., if the classifier is struggling, it's due to a poor segmentation



A simple network



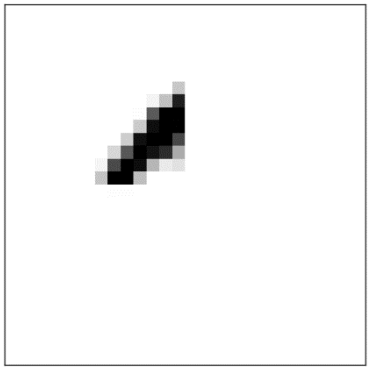
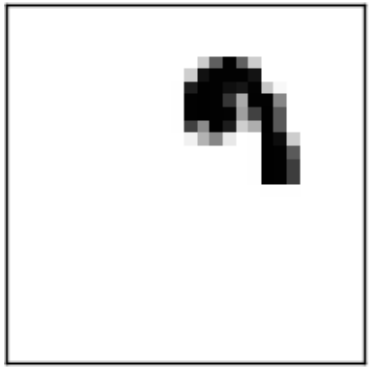
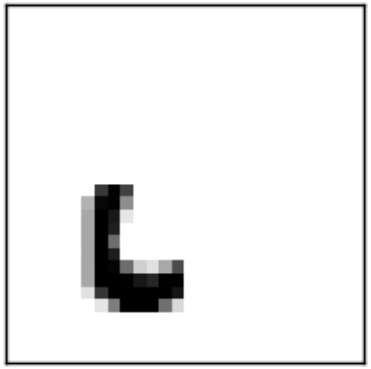
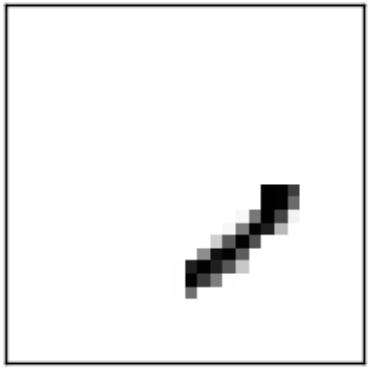
Number of output neurons

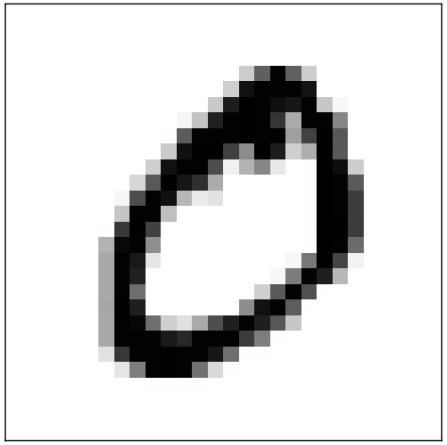
- Why use 10?
- Four bits are enough to encode the answer
  - Using more seems inefficient

- Empirically, 10 works better
  - Why?

Number of output neurons

- Possible heuristic:

$\sum$    

$=$ 

Further Reading

- Nielsen, chapter 1
- Goodfellow et al., chapter 6