Problem 4

1. The perceptron rule is

$$output = \begin{cases} 0, if\ w \cdot x + b \leq 0 \\ 1, if\ w \cdot x + b > 0 \end{cases}$$

Now multiply all weights and biases by c, where c > 0,

$$cw \cdot x + cb = c(w \cdot x + b)$$

Therefore, if $cw \cdot x + cb \leq 0$, then $w \cdot x + b \leq 0$, thus output = 0. Likewise, if $cw \cdot x + cb > 0$, then $w \cdot x + b > 0$, thus output = 1.

The behavior of the network doesn't change.

2. The output of a sigmoid neuron is

$$\frac{1}{1 + \exp(-\sum_j w_j x_j - b\,)}$$

Now multiply all weights and biases by c, where c > 0,

$$\frac{1}{1 + \exp(-\sum_j cw_j x_j - cb\,)}$$

$$= \frac{1}{1 + \exp\{(\text{-c})\cdot(\sum_j w_j x_j + b)\,\}}$$

As $c \to \infty$, if $w \cdot x + b \leq 0$, then (-c)·$(\sum_j w_j x_j + b) \to\ +\infty$, thus output = 0.

Likewise, if $w \cdot x + b > 0$, then (-c)·$(\sum_j w_j x_j + b) \rightarrow -\infty$, thus output = 1.

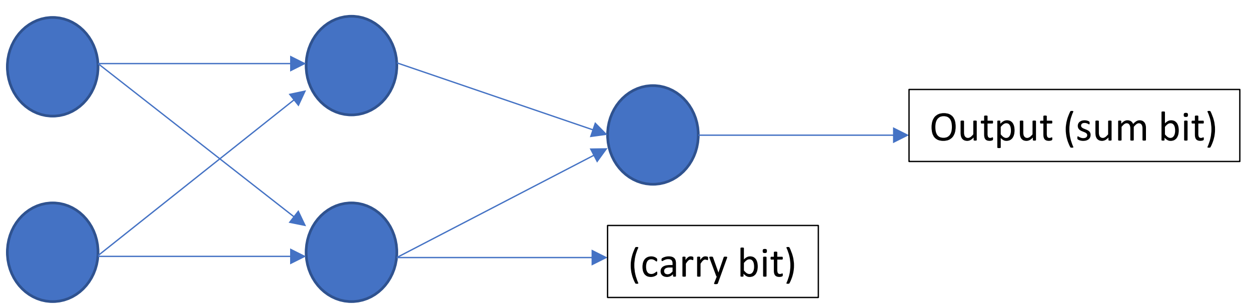
The behavior of the network of sigmoid neurons is exactly the same as the network of perceptrons.

When $w \cdot x + b = 0$, $output = \frac{1}{2}$ always, thus it differs from a perceptron neuron.

3. See notebook.

4. See notebook.

5.



x_1	x_2	a_{11}	a_{12}	output
0	0	1	0	0
0	1	0	0	1
1	0	0	0	1
1	1	0	1	0

Using perceptrons and the perceptron rule, with the following weights and biases:

$$z_{11} = -x_1 - x_2 + 0.9$$
$$z_{12} = 0.7x_1 + 0.7x_2 - 1, \text{ and}$$

$$z_{21} = -a_{11} - a_{12} + 0.5$$

The sum bit is the output of an XOR gate, and the carry bit is the output of an AND gate.