Algorithm

Chapter 1

Hardness of Problems

1.1 Computational Problems and Algorithms

Definition 1.1. A **computational problem** is a relation between two sets, i.e.,

$$R \subseteq X \times Y,$$

where X is called the set of **instances** and Y is called the sets of **solutions**.

Definition 1.2. A **computational model** is a model which defines a set of basic operations, each transforming a state of computation into another.

Example. A popular computational model is called the **random-access machine** (RAM) model. In this model, we have an infinite sequence of cells (the ith cell is denoted $M[i]$), and we allow operations of the following forms, where i, j, k are positive integers.

- $M[i] \leftarrow j$.
- $M[i] \leftarrow M[j] + M[k]$.
- $M[i] \leftarrow M[j] - M[k]$.
- $M[i] \leftarrow M[M[j]]$.
- $M[M[i]] \leftarrow M[j]$.
- If $M[i] > 0$, jump to operation numbered with j.

Definition 1.3. Given a computational model, an **algorithm** is defined as a finite sequence of basic operations (defined in that computational model) that transforms a given input into a unique output.