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2.2

$$L = \alpha \ln(x - x_0) + \beta \ln(y - y_0) + \lambda(I - px - qy)$$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 0 \\ \dfrac{\partial L}{\partial y} = 0 \\ \dfrac{\partial L}{\partial \lambda} = 0 \end{cases} \quad 则: \begin{cases} \dfrac{\alpha}{x - x_0} - \lambda p = 0 \\ \dfrac{\beta}{y - y_0} - \lambda q = 0 \\ I = px + qy \end{cases}$$

则: $p\left(\dfrac{\alpha}{\lambda p} + x_0\right) + q\left(y_0 + \dfrac{\beta}{\lambda q}\right) = I$

$$\lambda = \frac{\alpha + \beta}{I - px_0 - qy_0} = \frac{1}{1 - px_0 - qy_0}$$

$$\therefore\ x = x_0 + \frac{\alpha}{\lambda p} = x_0 + \frac{\alpha(I - px_0 - qy_0)}{p}$$

$$y = y_0 + \frac{\beta}{\lambda q} = y_0 + \frac{\beta(I - px_0 - qy_0)}{q}$$

即: $px = px_0 + \alpha I - \alpha p x_0 - \alpha q y_0$

$$= \alpha I + (1 - \alpha) p x_0 - \alpha q y_0$$

$$= \alpha I + \beta p x_0 - \alpha q y_0$$

$$qy = \beta I + \alpha q y_0 - \beta p x_0.$$

即: 得证.

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3.3. $\max \sum (\alpha_j x_j - \frac{1}{2}\beta_j x_j^2)$.

$$L = \sum_j (\alpha_j x_j - \tfrac{1}{2}\beta_j x_j^2) + \lambda (C - \sum_j x_j)$$

s.t. $\sum_j x_j \le C$

$x_j \ge 0$.

$$\begin{cases} \dfrac{\partial L}{\partial x_j} = \alpha_j - \beta_j x_j - \lambda = 0. \\ \lambda (C - \sum x_j) = 0 \end{cases}$$

(1) 设 $\lambda = 0$，则: $x_j = \dfrac{\alpha_j - \lambda}{\beta_j} = \dfrac{\alpha_j}{\beta_j}$.

$\sum_{j=1}^{n} x_j = \sum_j \dfrac{\alpha_j}{\beta_j} = H < C$

故资金使用量小于总资金，$\lambda = 0$ 符合库-塔条件.

(2) $\lambda > 0$　$\sum_j x_j = C$.

$\because x_j = \dfrac{\alpha_j - \lambda}{\beta_j}$

$\therefore \sum_j x_j = \sum_j \dfrac{\alpha_j - \lambda}{\beta_j} = H - \lambda K = C$

则: $\lambda = \dfrac{H-C}{K}$

即: $\alpha_j > \lambda = \dfrac{H-C}{K}$　$\therefore x_j > 0$

则: 题设得证.

(3) $x_k = 0$　$\alpha_k \le \lambda$

$\forall x_j > 0$: $\alpha_j - \beta_j x_j = \lambda$

$\therefore \alpha_j > \lambda$

即: $\alpha_k \le \lambda < \alpha_j$　即: $x_k = 0$ 的 k, α_k 最小

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4.1 设消费者效用: $u_c(X_{cg})$

$$\max W(u_1,\cdots u_c)$$

$$\text{s.t.}\begin{cases} u_c = u_c(X_{cg}) \\ \sum_c X_{cg} = X_g = \phi_g(z_{1g},\cdots z_{fg}). \\ \sum_g z_{fg} \le z_f. \end{cases}$$

$$L = W + \sum_g \pi_g\left[\phi_g(z_{fg}) - \sum_c x_{cg}\right] + \sum_g \lambda_f\left(z_f - \sum_g z_{fg}\right)$$

$$\begin{cases} \frac{\partial L}{\partial x_{cg}} = 0 \\ \frac{\partial L}{\partial z_{fg}} = 0 \\ \frac{\partial L}{\partial \pi_g} = 0 \\ \frac{\partial L}{\partial \lambda_f} = 0 \end{cases}$$

则:

$$\begin{cases} \frac{\partial W}{\partial u_c}\cdot\frac{\partial u_c}{\partial x_{cg}} - \pi_g x_{cg} = 0. & ① \\ \pi_g\cdot\frac{\partial \phi}{\partial z_{fg}} - \lambda_f = 0. & ② \end{cases}$$

(1) 由①: 一阶条件未变

(2) 由②: ②为新增条件

(3): π_g: X_g 增加1单位带来的社会福利增加.

λ_f: f 增加1单位带来的社会福利增加.

(4) $\max\ P\cdot\phi_g - \sum \mu_f\cdot z_{fg} = V.$

μ_f 为租金. 隐价格.

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F.O.C: $p\cdot\frac{\partial\phi}{\partial z_{fg}} - \mu_f = 0$.

令 $p=\pi_g$, $\lambda_f=\mu_f$. 即可. 故可证.

(5). $\sum I_c = \sum \mu_f z_f$. $\equiv$ 总产出价值.