2.2, 3.3, 4.1

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证：$\mathcal{L}(x,y,\lambda)=\alpha\ln(x-x_0)+\beta\ln(y-y_0)+\lambda[I-px-qy]$

$$\begin{cases}\dfrac{\partial\mathcal{L}}{\partial x}=\dfrac{\alpha}{x-x_0}-\lambda p=0 \quad (1) \rightarrow \dfrac{\alpha}{\lambda p}+x_0=x \quad (4)\\ \dfrac{\partial\mathcal{L}}{\partial y}=\dfrac{\beta}{y-y_0}-\lambda q=0 \quad (2) \rightarrow \dfrac{\beta}{\lambda q}+y_0=y \quad (5)\\ I-px-qy=0 \quad (3)\end{cases}$$

由(4)(5)式

$$I-\frac{\alpha}{\lambda}-px_0-\frac{\beta}{\lambda}-qy_0=0$$

$$I-p_0x_0-qy_0=\frac{\alpha+\beta}{\lambda}$$

$$\frac{1}{\lambda}=\frac{I-px_0-qy_0}{\alpha+\beta}$$

代入(4) $\dfrac{\alpha}{\alpha+\beta}(I-px_0-qy_0)+p\cdot x_0=px$

即 $\alpha I+\beta px_0-\alpha qy_0=px$

同理可证 $\beta I-\beta px_0+\alpha qy_0=qy$

得证

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3.3. $L = F(X) + \lambda [C - G(X)]$

$F(x) = \sum [a_j x_j - \frac{1}{2}\beta x_j^2]$，$G(x) = \sum x_j$

$\begin{cases} L_j(x) = a_j - \beta_j x_j - \lambda \le 0 \\ x_j \ge 0 \end{cases}$

$\begin{cases} L_j(\lambda) = C - G(X) \ge 0 \\ \lambda \ge 0 \end{cases}$

(i) 总有未使用 即证 $C - G(x) > 0$

下面仅讨论 $G(x)$ 最大情况，即 $x_j > 0$

$\therefore x_j = \frac{a_j - \lambda}{\beta_j}$

$\sum x_j = H - \lambda \cdot K$

又 $C > H$

$\therefore C > \sum x_j$ 即 $C - G(x) > 0$

(ii) 由(i) $C > H$ 时 $x_j = \frac{a_j}{\beta_j}$ 即为正值

$\therefore$ 下面讨论 $C < H$ 情况

此时 $\lambda > 0$，$C = \sum x_j$

又 $H - \sum x_j - \lambda K \le 0$

$\lambda \ge \frac{H - C}{K}$

$x_j \ge \frac{a_j - \lambda}{\beta_j}$

当 $a_j > \lambda$ 时，$x_j > 0$

此时 $\lambda = \frac{H-C}{K}$

又 $a_j > \frac{H-C}{K}$

$\therefore x_j > 0$，每个项目都能获得

(iii) 计入为最优条件下平衡

易知，当 $x_j = 0$ 时

$a_j < \lambda$

当 $x_j > 0$ 时

$a_j' = \lambda + \beta x_j > \lambda$

因此 $a_j < a_j'$ 得证

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$$L = W(U^1(x_{11},\cdots,x_{1G}),\cdots,U^C(x_{C1},\cdots x_{CG})) + \sum_g \pi_g[\phi_g - \sum x_{cg}] + \sum_f \lambda_f [Z_f - \sum_g z_{fg}]$$

(1) 此时 $\frac{\partial L}{\partial x_{cg}} = \left(\frac{\partial W}{\partial U_c}\right)\left(\frac{dU^c}{dx_{cg}}\right) - \pi_g$

$$\frac{\partial L}{\partial z_{fg}} = \pi_g \cdot \frac{\partial \phi_g}{\partial z_{fg}} - w_f$$

一阶偏导条件相同　π_g 代表商品g的边际价值
w_f 代表要素f的边际价值

(2) 可以分散化

由 $\pi_g \frac{\partial \phi_g}{\partial z_{fg}} - w_f = 0$ 得

企业~~田~~仅保证边际产出等于边际要素成本，即可达到~~整~~社会中的帕累托最优

(3) 总收入：$\sum \pi_g x_g$

$$I_c = \pi_1 x_{c1} + \pi_2 x_{c2} \cdots \pi_G x_{cG}$$

$$\sum I_c = \sum_{g=1}^{G} \pi_g \cdot \sum_{c=1}^{C} x_{cg}$$

由例题可知
在最优下 $\sum_{c=1}^{C} x_{cg} = x_g$
因此 $I_c = \sum \pi_g x_g$