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Superfluidity

Abstract

Superfluids are an example of a macroscopic quantum state, much like Bose-Einstein condensates. The properties of superfluid ^{4}He are presented and explained alongside a brief history of the field of superfluids. The fountain effect, zero viscosity capillary flow, and other phenomena are explained using the "two-fluid model" of superfluids. Finally, we provide applications and touch on other related areas of study.

Bose-Einstein condensates are well known as a state of matter in which cooling a gas of bosons to very low temperatures causes a macroscopic amount of the bosons to pile into the ground state.[1] Bose-Einstein condensation has been observed in a variety of atoms and particles, from Rubidium atoms in 1995 to, most recently, photons.[2] Superfluidity is the liquid analogue to Bose-Einstein condensation, and can be defined by the unique properties superfluids possess, including zero entropy and zero viscosity, as well as an enormous thermal conductivity. This paper will focus primarily on superfluidity within the most common isotope of Helium, ^{4}He, a classic example of a superfluid. Note that the entire Helium atom has zero angular momentum and is thus a boson.

Just as we have defined BEC above, we can define superfluids as a state of matter in which a macroscopic amount of bosons are in the ground state of the fluid. In 1938, Fritz London proposed this very idea. He claimed that superfluid helium "forms a macroscopic liquid matter wave" as a consequence of BEC.[3] Yet, as French physicist Sébastien Balibar argues, this "happened to be very difficult to prove." [3] It is true that both BEC and superfluids form only under a certain critical temperature, and we cannot ignore the fact that bosons pile into the ground state in both of these cases. But Schmets and Montfrooji point out that "the main difference between BEC and superfluidity is that BEC is a property of the ground state, while superfluidity is a property of the excited states." [4] The latter is reminiscent of superconductivity, where the presence of an energy gap causes the system to have zero electrical resistance. In superfluidity, the presence of an energy gap causes the system to display zero viscosity, as well as many other interesting properties and phenomena that we will explore.

Helium II displays zero viscosity when flowing through thin capillaries.[4-6] Lev Landau, a Russian physicist who won the Nobel Prize in 1962 for his work on superfluidity, developed a theory that focused on quantized vibrational excitations called "rotons." We can make use of this theory to understand how superfluids display zero viscosity. Assume a superfluid is moving through a capillary with velocity **v**. If we move into the rest frame of the fluid, the capillary walls move at a velocity –**v**. Assume that the friction between the walls of the capillary and the fluid generate an excitation through the liquid with energy E_{ex} and momentum **p**. Then, assuming M is the mass of the liquid, we can write the energy of the fluid in the moving frame as

$$E_{liquid} = E_0 + E_{ex}$$

where E_0 is the ground state energy. If we move back into the rest frame of the capillary, then the expression for the total energy becomes

$$E_{liquid} = E_0 + \frac{1}{2}Mv^2 + E_{ex} + \mathbf{p}\cdot\mathbf{v}$$

where the third term comes from a Doppler shift in the excitation frequency. Note that the total energy of the liquid must decrease as friction causes the liquid to dissipate energy, so the terms in this equation due to the excitation must be negative. Simple algebra and handling directions of vectors leads to the result

$$v > \frac{E_{ex}}{|\mathbf{p}|}$$

Thus, excitations only form if the relative velocity between the liquid and the walls of the capillary obeys the above inequality. We manipulate this further to arrive at a critical velocity,

$$v_c = \min_p \frac{E_{ex}}{|\mathbf{p}|}$$

This is sometimes called Landau's criteria of superfluidity.[7] If the velocity of the liquid is lower than this minimum velocity, excitations will not be generated. The liquid will flow through the capillary without resistance due to friction from the walls. In other words, the liquid will display zero viscosity.[5]

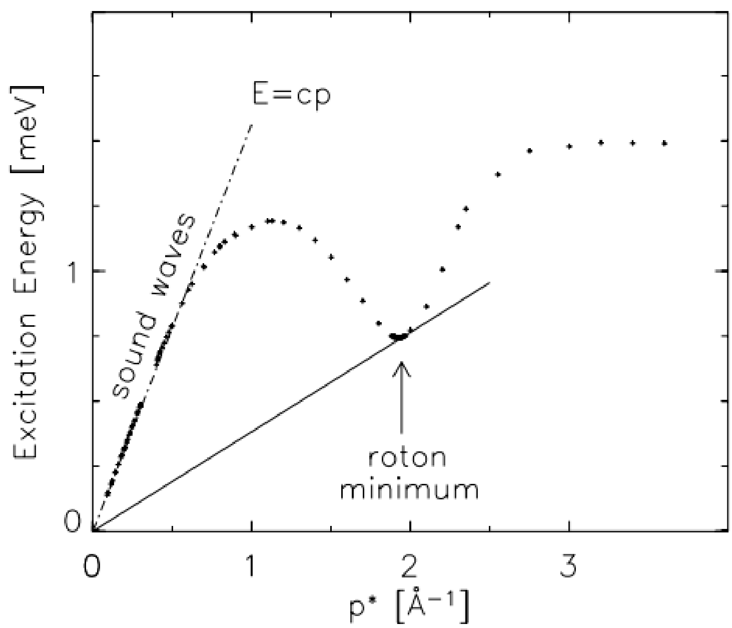

Figure 1: Excitation spectrum for superfluid ^4He. This plot is similar for all strongly interacting Bose liquids. The energy required to create a disturbance of momentum p = (*hbar*)(p*) is plotted against p*. Taken from [4].

Of course, this depends not only on the temperature, which causes the velocity to change, but also on the values for the energy and momentum of the excitation. If there exists a positive, finite minimum excitation group velocity $\frac{E_{ex}}{|\mathbf{p}|}$, then there will exist a critical velocity below which the fluid will turn into a superfluid. We can visualize this by plotting experimental data for the excitation energy vs. the momentum of the excitation (Fig. 1). In this figure, the slope of a line from the origin to any point gives the group velocity of the excitation (rise over run). There exists a minimum at $p^* \cong 2$, where the wavelength of the excitations is comparable to the distance between atoms in the liquid. The minimum gap between the ground state and the excited state is called the roton minimum, or just the roton. Furthermore, the critical velocity, or minimum group velocity achievable, is the slope of the line from the origin to the roton minimum,† as marked in the figure. In normal fluids, the minimum slope is zero, so the velocity can never go low enough that the fluid flows without friction or drag. Going back to what we stated earlier, the difference between BEC and superfluidity is that the latter is primarily a phenomenon of the excited states. Now, we can see why. The existence of the roton gap allows ^{4}He to become a superfluid, as the corresponding minimum excitation group velocity or critical velocity is positive and finite. So if the liquid is cooled down far enough that the velocity dips below this critical velocity, it will exhibit zero viscosity in capillary flow, thus becoming a superfluid.

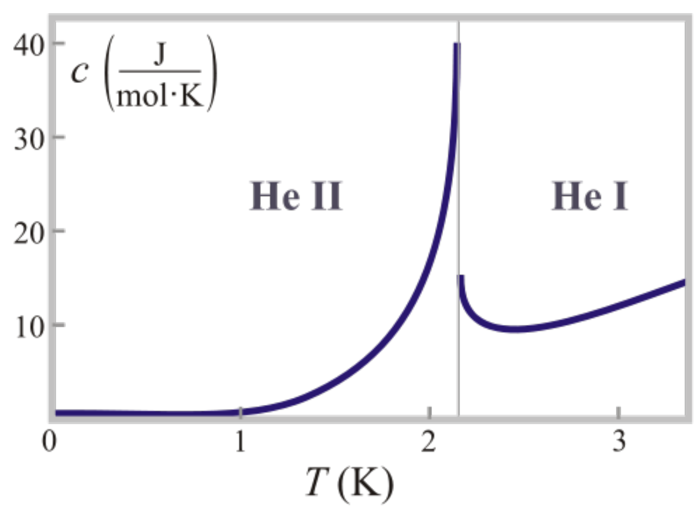

Figure 2: Specific heat curve for ^{4}He. The singularity at $T_\lambda = 2.17$ K represents a phase change from Helium I to Helium II. Taken from [8].

† While the excitation corresponding to the minimum is called the roton, the excitation corresponding to the maximum at $p^* \cong 1$ is called the maxon.[10]

The temperature corresponding to this critical velocity for ^4He is $T_\lambda = 2.17$ K, and is usually referred to as the lambda point due to the shape of the specific heat curve (Fig. 2). Above the lambda point, ^4He exists as liquid Helium I, which behaves more or less like a normal liquid. It does have several outstanding properties, however. As Leitner remarks, "the zero point [ground state] energy is relatively more important [in Helium I] than in any other liquid," [6] so the spacing between atoms is very large. This leads to a very low density and refractive index: 0.125 g/cm3 and 1.025, respectively. Furthermore, because He is a noble gas, the Van der Waals force between helium atoms is very small. Taking all of this into account, we can see that Helium I will not freeze at low temperatures and will remain a liquid (Fig. 3).

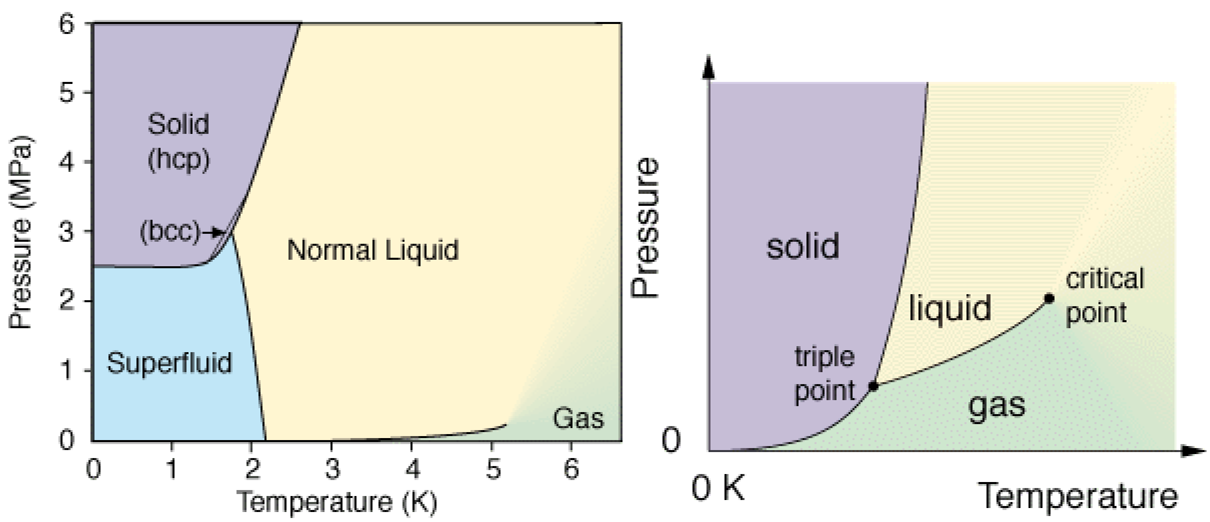

Figure 3: ^4He phase diagram (left) and a typical phase diagram (right). Note that liquid ^4He does not turn into a solid at low pressures. Taken from [9].

The physical transition between Helium I and Helium II is remarkable. As Helium I is cooled down, usually via pumping off vapor, it boils more and more violently. At the lambda point, the liquid suddenly stops boiling altogether. The surface of the liquid helium in the superfluid phase is calm, even as the temperature continues to fall and evaporation continues. This can be attributed to the extremely large thermal conductivity of Helium II. Because the heat is conducted away with negligible resistance, the walls of the vessel containing the liquid helium drop in temperature, and are no longer warmer than the liquid.[3] In fact, the "heat conductivity is measured to increase by a factor of about one million below the lambda point," [11] making Helium II have the highest thermal conductivity of any known material. Of course, it may not be so viable to have a near absolute zero material as a heat sink for your computer, so the applications of this high thermal conductivity are limited.

Going back to the zero viscosity property of superfluid helium, the reader may have noticed that the previous statement had a qualifier attached: Helium II displays zero viscosity *when flowing through thin capillaries*. In fact, it can be shown that Helium II does not display "zero viscosity" in another experiment.[6] Imagine Helium I, or any non-superfluid, placed in a tube with a cylindrical motor inside. The motor is then turned on, and spins rapidly. Because the liquid in the tube has nonzero viscosity, the motor pulls on the closest layer of liquid, which then pulls on another layer of liquid, and so on until the entire liquid is rotating in the direction of the motor's rotation. Then, if we put a paddle inside the tube while this is happening, we would be able to observe the viscosity as the fluid pushes the paddle, making it spin. When this experiment is done with Helium II, the paddle still spins, indicating a nonzero viscosity.

This was the subject of much debate in the late 1930s and early 1940s. The Hungarian physicist László Tisza proposed a theory,[†] which Landau subsequently refined, based on the idea that Helium II was not completely superfluidic. Instead, Tisza argued, Helium II was comprised of a zero viscosity superfluid component and a viscous normal component that behaves like a normal liquid.[3] This theory came to be known as the "two fluid model" of Helium II. It explained the vastly different results gotten from capillary flow and from bulk flow in an open space. In the former, the superfluid component exhibits zero viscosity, easily flowing through

[†] Interestingly, the first experimental evidence of nonclassical behavior for ^4He under 2.2K was published in January 1938. In April of that year, London's article on the relationship between superfluidity and BEC was published, and just one month later, Tisza proposed the two fluid model in an article one column long.

the bulk flow experiment, the motor is able to push on the normal component, making it rotate the paddle. The zero viscosity superfluid component does not interact with the motor or the paddle and contributes nothing to the paddle's motion. Tisza went further, claiming that the normal component was made up of atoms in excited states while the superfluid component was made up of only atoms in the ground state. He also stated that the ratio of normal to superfluid component was dependent on temperature. Only the normal component is present at the lambda point and only the superfluid component is present at absolute zero.

This two fluid model holds up well in terms of entropy as well. A phenomenon of Helium II called the fountain effect very nicely illustrates the zero entropy property of the superfluid component. We wish to show that the superfluid component possesses no entropy. To do this, we must first separate the superfluid component from the normal component, which we assume carries heat and thus has nonzero entropy. We have seen how to do this in both the capillary flow and bulk flow discussions above, but here capillary flow is more appropriate as it physically selects only the superfluid component. Thus, by constructing a large amount of small capillaries, we can separate the two components.

But, then, how do we actually test the entropy of a substance? Thermodynamics tells us that entropy is a measure of chaos, and that thermal energy always flows from areas of higher temperature to those of lower temperature. In other words, the Clausius statement of the second law of thermodynamics states that "no process is possible whose sole result is the transfer of heat from a cooler to a hotter body." [12] So while liquids with nonzero entropy must flow from hot to cold, a liquid with zero entropy would be able to violate this law, and would flow from a region of low temperature to a region of higher temperature.

If we put these two conditions together—namely, the separation of the superfluid

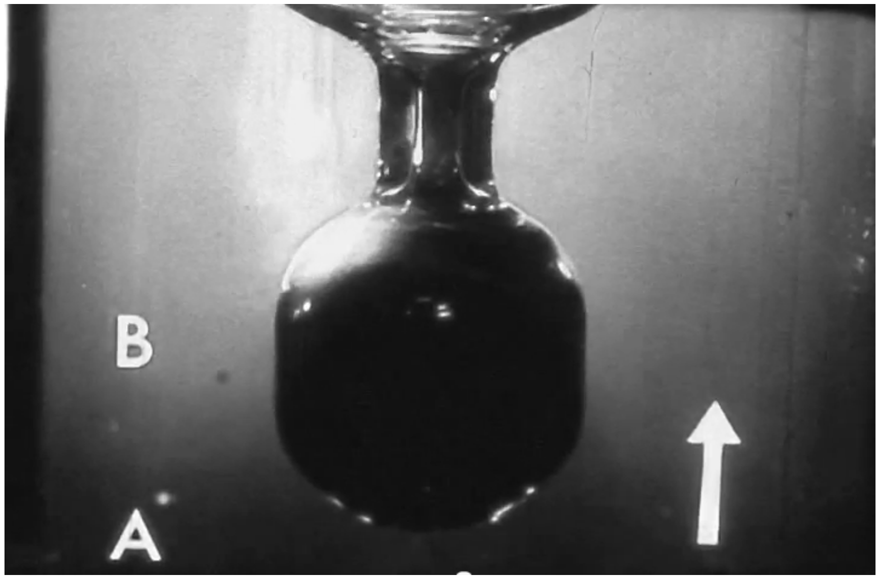

Figure 4: Helium II flowing upwards through fine capillary channels in the powder. A laser is pointed just above point B. Taken from [6].

component and the test for entropy, we get an experiment like the one shown in Figure 4, done by Alfred Leitner in 1963. The entire apparatus shown is submerged in Helium II. The bulb in the middle is packed with a fine powder that provides capillary channels less than one micron in width, effectively blocking the normal component of the liquid helium from entering. A laser shines on the glass tube just above point B, increasing the temperature there. If the other side of this tube opens into a spout, the end result is, as the name of the "fountain effect" suggests, a fountain of liquid helium. The superfluid component moves up from a lower temperature region to a higher temperature region, so it must have zero entropy. Furthermore, the heat from the laser is transferred to the superfluid component, "converting it into the normal component." [6]

It can be shown that this difference in entropy leads to the viscosity condition using quasi-particles, or elementary excitations, associated with the normal component. As Yamamoto argues, quasi-particles can be thermally excited, as Leitner did with a laser. These quasi-particles can collide with the walls of the container and "dissipate their energies and momenta," thus slowing the liquid down. Thus, because the superfluid component does not carry heat, it cannot have viscosity.[7]

Furthermore, this quasi-particle argument can be extended. As we have done above, we can assume that the fluid is flowing through a capillary. The relative velocity between the fluid and the walls is **v**, and each quasi-particle carries energy E_{ex} and momentum **p**. Then, as Yamamoto points out, we can use the Bose-Einstein distribution function to estimate the quasi-particle population:

$$N_p = \frac{1}{\exp\left[\frac{E_{ex} + \mathbf{p} \cdot \mathbf{v}}{k_B T}\right] - 1}$$

When, then, do quasi-particles exist? When does the normal component exist? For N_p to be a positive number, the exponential term must be greater than 1. Thus the inequality $E_{ex} + \mathbf{p} \cdot \mathbf{v} > 0$ must be satisfied. Manipulating this further allows us to define a condition for a positive quasi-particle population:

$$v < \min_p \frac{E_{ex}}{|\mathbf{p}|}$$

But wait—this is the same expression as the Landau criteria for superfluidity! What does this mean? Firstly, from the current argument, we can conclude that below the critical velocity, quasi-particles will exist within the fluid, and thus a normal component will exist. On the other hand, the Landau criterion tells us that under the same conditions, Helium II will be able to flow without viscosity through thin capillaries. Thus, we conclude that the normal component and the superfluid component must both be present in Helium II below the critical velocity.

The reader may realize that our initial derivation of Landau's criteria was purely classical. In fact, Landau purposely made no reference to BEC in his 1941 article because he believed that the two concepts were unrelated. Even London, who had proposed a link between BEC and superfluidity, said that the BEC-superfluid connection was a ridiculous assumption: "[this] model … is so far away from reality that it simplifies liquid helium to an ideal gas." [3] Yet developments in the 1950s and 1960s made it clearer and clearer that the two model system could not be taken as a complete theory for superfluidity. Instead, as Leitner suggests, we should "look at [Helium II] as one liquid system, capable of two different types of motion simultaneously," [6] a zero viscosity, zero entropy superfluid flow and a viscous heat-carrying normal flow. The distinction between this and having two "components" is subtle. The classical model assumes that the two components are separate—that is, the superfluid and normal motions actually act on different helium atoms. In actuality, it does both.

Furthermore, the model must take into account the connection to BEC, but it is not very simple. In regular BEC experiments, a single exposure can map out an entire condensate, and experimentalists can essentially "see" and measure BEC directly. In superfluidity, however, "there is no experimental evidence for the existence of a condensate in liquid helium." [3] Balibar's 2003 talk mentions that the most recent experimental progress at the time came from "Deep Inelastic Neutron Scattering" experiments. These experiments assume the existence of a condensate and depend on the shape of the distribution function for quasi-particles to achieve numerical results.[3]

Since then, advances have been made in the field of superfluidity and superconductivity, the two of which overlap significantly, as superfluids with charged particles are superconductors. 2004 saw the experimental realization of ultracold superfluid Fermi gas. This is possible because fermions pair together at very low temperatures, creating "Cooper pairs" with integral spin.[13] This also occurs in ^{3}He, and superfluid ^{3}He can be made at very low temperatures. In

2011, NASA observed rapid cooling inside a neutron star, direct evidence for superfluids inside the core of the star.[14] In Balibar's 2003 talk, Balibar urges physicists to explore the metastable regions of high pressure for superfluids. Looking back to figure 3, we can see that superfluid helium can turn into a solid at large enough pressures. Indeed, much research has been done in "supersolids" and "superglass" as well as in extremely pressurized superfluids. While some of the basic properties of superfluid Helium II have been explained, more applications of superfluidity are bound to exist. In the meantime, there are many areas that require further study.

This paper has covered and explained several of the more well-known properties of superfluid Helium II, but several interesting facets of Helium II have been omitted due to their heavily mathematical nature or their redundancy with other examples already brought up. Here, we shall make a brief mention of these phenomena and properties. Firstly, Helium II conducts heat in the form of waves rather than by diffusion. The speed of these "second sound" waves can be measured, and even reflections and resonance of the heat waves can be observed.[15] Secondly, the Rollin film phenomenon is an illustrative example of superfluidity's strange properties. Helium II can creep up along the sides of a vessel to even out its level. This is especially striking when Helium II is placed inside a cup hanging above a pool of liquid helium. The helium is able to climb out of the cup and drip down into the pool below.[6,11] Thirdly, superfluids circulate about quantized vortex lines. A detailed explanation of this property can be found in [4] and [16].

Superfluidity is a phenomenon that can be defined in many different ways. Superfluids display zero viscosity when flowing through small capillaries, yet show viscous behavior in bulk flow experiments. A superfluid is able to carry no heat, yet it also carries heat. The "two fluid model" was created to explain these discrepancies, and while it is not entirely correct to think of

Helium II as two separate components, it does give us physical intuition for the underlying quantum mechanics. As we have seen, superfluidity is not a dead subject. Superfluids have been discovered in neutron stars, are viable superconductors, and have shown up in many other places. The applications of current superfluid theory are many, and this theory is not yet complete. Some open areas of research include the connection between BEC and superfluidity and high pressure superfluids. Furthermore, ultracold Fermi gas superfluids were made not even a decade ago. Many of the shocking properties of superfluids have been discovered and explained, but superfluidity as a field and concept is still open.

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