

財務工程作業五(Monte Carlo Simulation)—學習歷程

Step1:

使用 Monte Carlo method：利用 Hull White Model 模擬出很多組 Short rate 路徑，程式部分套用 QuantLib 套件來計算

The Hull-White Model

- The Hull-White model is the following special case,

$$dr = (\theta(t) - ar) dt + \sigma dW.$$

- When the current term structure is matched,

$$\theta(t) = \frac{\partial f(0, t)}{\partial t} + af(0, t) + \frac{\sigma^2}{2a} (1 - e^{-2at}).$$

– Recall that $f(0, t)$ defines the forward rate curve.

QuantLib source: <http://gouthamanbalaraman.com/blog/hull-white-simulation-quantlib-python.html>

Step2:

使用 Monte Carlo method：將上一步計算出的 Short rate 帶入 Geometric Brownian Motion 模擬出一樣多組對應的股價路徑

Solving the SDE [edit]

For an arbitrary initial value S_0 the above SDE has the analytic solution (under Itô's interpretation):

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right).$$

The derivation requires the use of Itô calculus. Applying Itô's formula leads to

$$d(\ln S_t) = (\ln S_t)' dS_t + \frac{1}{2} (\ln S_t)'' dS_t dS_t = \frac{dS_t}{S_t} - \frac{1}{2} \frac{1}{S_t^2} dS_t dS_t$$

where $dS_t dS_t$ is the quadratic variation of the SDE.

$$dS_t dS_t = \sigma^2 S_t^2 dt + 2\sigma S_t^2 \mu dW_t dt + \mu^2 S_t^2 dt^2$$

程式部分參考上課資源：https://docs.google.com/presentation/d/e/2PACX-1vRRpiXAxT35pUTud9EzQt5Awoof0z0X03sDGe5kaVbt6CPJdyozZtortnBNUupMWHXaHafywKJ993sv/pub?start=false&loop=false&delayms=3000&slide=id.g8443ec3d2c_0_183

Step3:

利用到期日的股價計算到期日的選擇權 payoff，取期望值，並折現回現值，即可計算出選擇權價格

1. Call Option payoff = $\text{MAX}(\text{stock price} - \text{strike price}, 0)$
2. Put Option payoff = $\text{MAX}(\text{strike price} - \text{stock price}, 0)$