How Should Central Bank Issue Digital Currency?

Fangfang Gong*

September 14, 2025

Abstract

This paper develops a micro-founded general equilibrium model of payments to study the optimal design of a retail central bank digital currency (CBDC) where both currency and bank deposits are used in exchange. In particular, I investigate the impact of a CBDC holding limit on equilibrium allocations, private bank intermediation, and welfare. If the holding limit is set within an intermediate range of values, then the CBDC coexists with physical currency and deposits at the intensive margin, crowds out deposits at a slower rate, and improves welfare. A calibration to the United States economy suggests this range lies between 37% and 82% of the optimal amount of CBDC held without distortion.

JEL Classification: E41,E58

Keywords: Central Bank Digital Currency; CBDC Designs; Payment Choices; Distermediation

* School of Economics, Singapore Management University. E-mail: ffgong.2021@phdecons.smu.edu.sg

1 Introduction

With advancements in mobile payment technology, the growth of online retailing, and shocks such as the COVID-19 pandemic, the decline in cash use by the public has become both inevitable and pronounced. This declining demand for cash as a means of payment is matched by a rising demand for digital monies—such as bank deposits, digital wallets, and stablecoins—provided by the private sector. The dominant role of private monies in the payment landscape can hinder financial inclusion—disadvantaging the unbanked and underbanked[1]—and increase the payment system's vulnerability during times of disruptions. These concerns, along with other considerations such as supporting monetary sovereignty and expanding the central bank's monetary policy toolkit, have prompted central bankers to consider issuing retail central bank digital currencies (CBDCs), including European Central Bank (ECB), Bank of England (BoE), and People's Bank of China (PBoC) [2]. A retail CBDC is an electronic form of central bank money that is accessible to the general public for retail transactions. While issuing retail CBDCs can help central banks regain their influence in the digital era's payment landscape, it may also lead to undesirable consequences, with disintermediating private banks being one of the most prominent concerns. A retail CBDC can potentially crowd out bank deposits by raising the funding costs for commercial banks. To mitigate such potential adverse effects, the design of a retail CBDC must be approached with careful consideration. One design feature proposed by central bankers to help retain bank deposits is imposing a cap on the amount of CBDC that households can hold or use. More than two thirds of 86 central banks surveyed by Bank for International Settlement (BIS) in 2023 consider a (potential) retail CBDC that is subject to holding limits (see Di Iorio et al., 2024) . For instance, the ECB has considered a €3,000 to €4,000 limit per individual for digital euro holdings (Panetta, 2022). The BoE has suggested a per capita cap between £10,000 and £20,000 for the digital pound (Cunliffe, 2023), while the majority of commercial bank respondents in a 2023 consultation survey favoured lower limits between £3,000 and £5,000 (see Bank of England and HM Treasury, 2024). Meanwhile, the PBoC has introduced multiple limits for the digital yuan, including a single payment limit,

[1]The legal tender status of cash means that cash must be accepted for debt repayment. However, retail transactions are not always considered as debt repayment. They are private agreements between buyers and sellers. In many countries, including Sweden, the UK, and the US, businesses can refuse cash if they state their policy upfront. Businesses may increasingly opt to do so if cash is less often used in an attempt to streamline the transaction process.

[2]As of July 2025, 65 countries are exploring a retail CBDC, according to Atlantic Council, a think tank. Three countries have issued their own retail CBDCs, which are the Bahamas, Jamaica and Nigeria.

a daily cumulative limit, and a balance cap, which vary depending on the level of anonymity the digital wallet provides (Mu, 2022). For example, the most anonymous level-four wallet has a single transaction limit of 2,000 RMB (approximately 279 USD).

Taking this design feature into account, this paper studies how the design of a retail CBDC along three dimensions affects equilibrium allocations, private bank intermediation, and welfare in an environment where the CBDC competes with cash and private bank deposits as a medium of exchange. The CBDC can pay interest, can be held subject to limit, and will be costly to use. The three design features, therefore, are the interest payment, the holding limit, and the adoption cost. Specifically, this paper aims to answer the following questions. When a CBDC is introduced, will it be adopted, and if so, under what conditions? If the CBDC is adopted, then can it coexist with cash and bank deposits, and if so, what form would this coexistence take? Will a holding limit on individuals' CBDC balances help mitigate the disintermediation effect? Can introducing a CBDC improve welfare, and if so, what will be the design? This paper addresses these questions by developing a model of banking and means of payment, featuring two perfectly segmented meetings, each with distinct payment methods available. Cash and bank deposits can only be used in Type 1 and Type 2 meetingss respectively, while the CBDC, if introduced, can be used universally. Households make their portfolio decisions based on different characteristics of available assets for exchange. Therefore, the model allows for an investigation of the effects of different CBDC designs on paper currency and private banks.

My main finding is that the CBDC holding limit affects both households' adoption of the CBDC and, when adopted, households' portfolio choices, as well as the rate at which private bank deposits are crowded out. I show that when the CBDC pays a higher interest rate than cash and deposits, then if the holding limit is set within an intermediate range of values, the CBDC coexists with physical currency at the intensive margin in Type 1 meetings, and coexists with deposits both at the intensive and extensive margin in Type 2 meetings, where there is intensive margin coexistence of two means of payment when households hold both media of exchange, and there is extensive margin coexistence of two means of payment when some households accumulate one medium of exchange and others accumulate the other. A sufficiently high limit makes households want to pay the adoption cost and hold the CBDC, and meanwhile, a sufficiently low limit induces households to top up with the other means of payment available. If the holding limit is, instead, relatively large, then cash vanishes and the CBDC coexists with deposits only at the extensive margin. Moreover, in both cases where the CBDC and deposits coexist at the intensive and extensive margin in Type 2 meetings in equilibrium, by setting a higher holding limit on the CBDC, bank deposits are more crowded out, albeit at different rates.

Calibrating my model to the United States economy, I find that a CBDC will be adopted by both buyers in both Type 1 and Type 2 meetingss, coexist with cash and deposits at the intensive margin, crowd out deposits at a slower rate, and improve welfare if the holding limit is between 37% and 82% of the optimal amount of the CBDC held without distortion.

To the best of my knowledge, this paper is the first one to examine the theoretical impact of CBDC's holding limit design feature in the New Monetarist tradition. Therefore, this paper establishes a theoretical basis for existing empirical literature and discussions documenting the appropriate size of the cap on CBDC holdings. For example, Li et al. (2024) show that a large holding limit of 25,000 digital Canadian dollars could effectively mitigate disruptions to the financial system within a heterogeneous-agent framework. Bidder et al. (2025) investigates, in a dynamic stochastic general equilibrium (DSGE) model, how a binding limit affects financial stability, economic outcomes, and welfare. They suggest an optimal holding limit ranging between €1,500 and €2,500 under different parameterization.

My work contributes to the strand of literature assessing the macroeconomic effects of introducing a CBDC, including Brunnermeier and Niepelt (2019). A large part of this literature focuses primarily on CBDC's interest-bearing feature, such as Barrdear and Kumhof (2022), Davoodalhosseini (2022), Williamson (2022), Hua and Zhu (2021), and Dong and Xiao (2021). Within this stream, a subset of papers evaluates the potential crowding-out effect of a CBDC on private bank deposits, under different assumptions about banks' market power. Keister and Sanches (2022) and Chiu and Davoodalhosseini (2023) adopt a perfectly competitive setting, Andolfatto (2021) assumes a monopolistic bank, and Chiu et al. (2023) use data to discipline the level of competitiveness quantitatively. My model builds on Keister and Sanches (2022), and incorporates a holding limit and an adoption cost for the CBDC. In Keister and Sanches (2022), they discuss the desirability of a universal digital currency, which is conceptually aligned with the CBDC featured in my model. However, their discussions center around the optimal interest rate and examines how credit frictions and the scarcity of productive projects shape the optimal policy, whereas I focus on the equilibrium impact of the CBDC holding limit, and how its interaction with an interest rate and a adoption cost for the CBDC alters equilibrium allocations, investment and welfare. Last but not least, this paper aligns closely with the literature on the optimal design of a retail CBDC, where privacy is mostly discussed. Agur et al. (2022) investigate the optimal level of anonymity a CBDC should provide in the presence of network effects, both with and without interest payments. Similarly, Wang (2023) examines the optimal privacy design of a CBDC, but in the context of money laundering. This paper abstracts from the issue of privacy, and it is implicitly assumed that a CBDC can be as anonymous as cash while also as transparent as deposits.

The rest of the paper is organized as follows. Section 2 builds the model. Section 3 and Section 4 characterize equilibrium without CBDC and with CBDC respectively. Section 5 discusses the welfare impact of different CBDC designs. Section 6 calibrates the model and assesses its quantitative implications. Section 7 discusses different forms of balance-contingent transfer on CBDC. Section 8 concludes.

2 The model

The model is based on the frameworks of Lagos and Wright (2005) and Keister and Sanches (2022). Time is discrete and with infinite horizon. Each period is divided into two subperiods: a frictional decentralized market (DM) and a Walrasian centralized market (CM). There are two perishable goods which are produced and consumed in the two subperiods respectively: the DM good and the CM good. Section 2.1 discusses the types of agents in the economy. Section 2.2 elaborates on the assets for exchange and the flow of payments and goods. Section 2.3 discusses households' demand for assets, while Section 2.4 addresses the supply side. Section 2.5 defines the stationary equilibrium.

2.1 Agents

There are four types of agents: a unit measure each of buyers and sellers, a continuum of bankers, and the central bank.

Buyers and sellers live forever and discount across periods with factor $\beta \in (0,1)$. In the DM, the first subperiod, buyers and sellers meet bilaterally, where buyers consume what sellers produce and not vice versa. By consuming q units of the DM good, a buyer's utility is $u(q)$ with $u'(0) = \infty$, $u' > 0$, and $u'' < 0$. Sellers incur a linear cost in producing q units of the DM good with $c(q) = q$. Hence, the efficient amount of DM trade q^*, the one maximizing the total trade surplus, solves $u'(q^*) = 1$. Then in the second subperiod, the CM, both buyers and sellers can work and consume the CM good. By working h hours, they produce h units of the CM good with disutility $-h$. Their preferences for CM consumption is $U(\cdot)$ with $U'(0)=\infty$, $U'>0$, and $U''<0$. The equilibrium CM consumption, X^*, solves $U'(X^*)=1$. In summary, buyers' and sellers' period utilities are

$$U^b(q,X,h) = u(q)+U(X)-h,$$
$$U^s(q,X,h) = -q+U(X)-h,$$

where X is consumption of the CM good and h is labor input.

There are overlapping generations of bankers and a new cohort of bankers is born each CM. They become old the next period and die at the end of next CM. They are active only in the CM and only consume when they are old. Young bankers are endowed with an indivisible investment project but they have no internal funds to finance it. Therefore, they must borrow from households by issuing deposits. Investment projects take one unit of current CM good as input and have heterogeneous returns in the next CM. There is a total measure η of bankers whose project returns are known in advance and are uniformly distributed in the support $[0,\bar{\gamma}]$. Bankers have limited commitment, and therefore their project returns are not fully pledgeable. Young bankers can credibly pledge only a fraction $\varepsilon\in[0,1]$ of their project returns to depositors, as they can abscond with the remainder. Moreover, bankers are subject to a reserve requirement so that they can only invest a fraction $1-\mu$ of issued deposits into projects. The rest becomes reserves at the central bank.

The central bank issues (potentially) three types of liabilities: cash, reserves, and CBDC if introduced. Cash and CBDC are liquid in that they can be used to facilitate exchanges, whereas reserves are illiquid.

2.2 Assets and Exchange

Agents other than the central bank lack commitment and there is no record-keeping technology among buyers and sellers so DM trade must be quid pro quo. I use take-it-or-leave-it (TIOLI) offers made by buyers as the trading protocol in the DM. Buyers will have to use a means of payment in order to consume in the DM. Furthermore, households are perfectly and permanently [3]segmented into two types of meetings where different means of payment can be used for transactions before they make their portfolio choices: a fraction λ_1 of households engages in Type 1 meetings, while the remaining fraction $\lambda_2\equiv 1-\lambda_1$ participates in Type 2 meetings. Before a CBDC is introduced, households can use only

[3]Results are the same when the segmentation is random each period as long as the proportions of households engaging in Type 1 and Type 2 meetings remain the same.

cash in Type 1 meetings and only deposits in Type 2 meetings. We can think of Type 1 meetings as point-of-sale transactions where sellers do not have the technology to verify bank deposits, and Type 2 meetings as transactions where sellers cannot verify physical currency. When a CBDC is introduced, it is assumed that the CBDC can be verified and used in both types of meetings. The three possible media of exchange are embedded with different characteristics. First, they have different rates of return. Returns on cash and the CBDC are controlled by the central bank through targeting the inflation rate and interest payment on the CBDC whereas the deposit rate is an equilibrium object. Second, there is a cap on the amount of the CBDC that buyers can hold, which is denoted as $\bar{e}$. Third, there are fixed costs associated with using deposits and the CBDC incurred by buyers, which are summarized as f and δ respectively. The CBDC fixed cost, δ, has two components: δ_1 and δ_2. The first component, δ_1, is a user cost, as is f; the second, δ_2, represents the central bank's service charge. For the economy as a whole, the former reflects a true welfare cost, whereas the latter is merely a transfer cost. User costs include resources, both human and physical, invested in adopting the underlying technology that supports a specific payment method. For instance, users of deposits and CBDC need to devote time to opening and managing their bank accounts and CBDC wallets. Aside from it, there are potential fees paid by households to the central bank for keeping their CBDC accounts active. One motivation for the central bank to charge CBDC account holders is to ensure the sustainability of CBDC issuance, as the costs for maintaining the CBDC infrastructure can be substantial (Koonprasert et al., 2024). Nonetheless, δ_2 can take on negative values, which implies that the central bank subsidizes households' use of the CBDC. Overall, I assume that buyers' adoption cost of the CBDC, δ, is strictly positive given that the fixed cost of using the CBDC is greater than that of cash[4].

The flow of payments and goods is as follows. In the CM, buyers in both types of meetings make their portfolio choices, i.e., which means of payment to bring into the next DM for trade. When bank deposits and the CBDC are accumulated, fixed costs are payable at this stage. Young bankers issue deposits to finance their investment. In the following DM, decentralized trade takes place, where money and goods change hands. In the next CM, sellers redeem deposits received with now old bankers and consume. Old bankers consume what is left after repaying deposits and interests. And a new cohort of young

[4]For buyers, the fixed cost of using cash is normalized to zero; therefore, δ represents the excess fixed cost of using the CBDC relative to cash. To use the CBDC, buyers have to set up their CBDC wallets, learn how to make payments with them, and sacrifice privacy for transactions.

bankers are born with investment projects. Buyers receive lump-sum transfers and adjust their balances. Figure 1 summarizes activities carried out by private agents in a timeline.

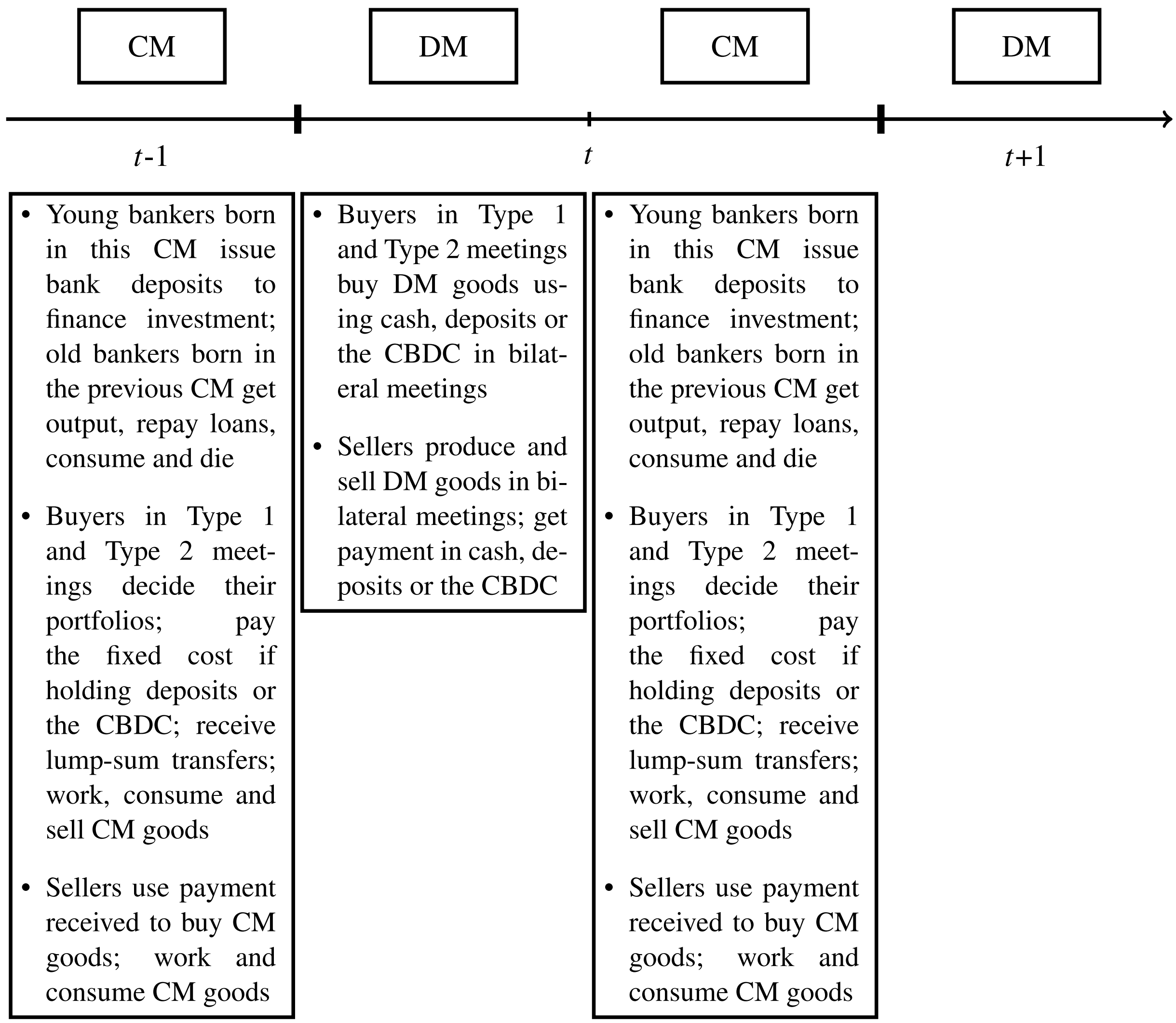

Figure 1: Timeline.

2.3 Asset Demand

In this section, I solve buyers' portfolio problems in both Type 1 and Type 2 meetings within a generalized setup to derive households' demand for the three possible means of payment: cash, deposits and the CBDC.

In what follows, time subscripts on period-t variables are omitted, while variables from $t-1$ are labeled with subscript -1, and variables from $t+1$ with subscript $+1$. Define $\vec{a} =$

(c,d,e) as the portfolio vector of real balances of cash, deposits and CBDC accumulated by a buyer. Denote the price of money in terms of the CM good in period t as ϕ. The net nominal interest rate on CBDC balances is i. Let $\vec{R}=(R^c,R^d,R^e)$ be the vector of real gross returns on cash, deposits and the CBDC, where $R^c=\frac{\phi}{\phi_{-1}}$, $R^e=\frac{(1+i)\phi}{\phi_{-1}}$, and R^d is determined in equilibrium. Here it is implicitly assumed that there is one-to-one exchange rate between cash and the CBDC, as they have the same price ϕ. See Section 2.4.1 for a detailed discussion. Let W_j and V_j denote buyers' CM and DM value functions in meetings $j\in\{1,2\}$, where 1 refers to Type 1 meetings and 2 Type 2 meetings.

In the CM, a buyer chooses her consumption of the CM good X^b, labor h, and portfolio $\vec{a}_{+1}=(c_{+1},d_{+1},e_{+1})$ carried into the next DM, and

$$
\begin{aligned}
W_j(\vec{a}) = & \max_{(X^b,h,\vec{a}_{+1})} \quad U(X^b)-h+\beta V_j(\vec{a}_{+1}) \\
\text{subject to} \quad & X^b+\vec{1}\cdot\vec{a}_{+1}=h+\vec{R}\cdot\vec{a}+T-\mathbf{1}_{\{d_{+1}>0\}}\times f-\mathbf{1}_{\{e_{+1}>0\}}\times\delta, \\
\text{and} \quad & e_{+1}\le\bar{e},
\end{aligned}
$$

where $\vec{1}=(1,1,1)$, "$\cdot$" is the inner product, and T is the lump-sum receipt in real terms.

Assuming an interior solution for h and substituting h from the budget constraint, the CM value function can be rewritten as:

$$
\begin{aligned}
W_j(\vec{a}) = & R\cdot\vec{a}+T+\max_{X^b}\left\{U(X^b)-X^b\right\} \\
& +\max_{\vec{a}_{+1}}\left\{-\vec{1}\cdot\vec{a}_{+1}+\beta V_j(\vec{a}_{+1})-\mathbf{1}_{\{d_{+1}>0\}}\times f-\mathbf{1}_{\{e_{+1}>0\}}\times\delta\right\}, \qquad (1) \\
\text{subject to} \quad & e_{+1}\le\bar{e}.
\end{aligned}
$$

One standard result of the Lagos-Wright model is $\frac{\partial W_j(\vec{a})}{\partial a}=R^a$ for $a=c,d,e$.

The buyer's DM value function is

$$
\begin{aligned}
V_j(\vec{a}) &= u(q_j)+W_j(\vec{a}-\vec{p}) \\
&= u(q_j)+R\cdot(\vec{a}_j-\vec{p}_j)+W_j(0),
\end{aligned}
$$

where $\left(\vec{p}_j\equiv\left(p_j^c,p_j^d,p_j^e\right),q_j\right)$ are the terms of trade, representing payment in cash, deposits and the CBDC, and the amount of DM goods traded in Type j meetings.

Given that a buyer brings portfolio $\vec{a}=(c,d,e)$ into the DM in Type j meetings, the terms of trade assuming TIOLI offer trading protocol solves the following problem:

$$\begin{aligned} \max_{q_j, \vec{p}_j} \quad & S^b \\ \text{subject to:} \quad & S^s \geq 0, \qquad (2) \\ & 0 \leq \vec{p}_j \leq f_j(\vec{a}), \qquad (3) \end{aligned}$$

where $S^b = u(q_j) + W_j(\vec{a} - \vec{p}) - W_j(\vec{a}) = u(q_j) - R \cdot \vec{p}_j$, and $S^s = -q_j + R \cdot \vec{p}_j$ are buyers' and sellers' surplus from DM trade respectively. Specifically, a buyer's trade surplus is her gain from DM consumption minus the value of payment. Sellers' participation constraint is (2), and (3) is a feasibility constraint that says buyers cannot ask for money from sellers, nor can they offer to transfer more units of money than they possess that can be used in Type j meetings. The function f_j requires the buyer to pay with the types of money that are accepted in Type j meetings. For example, if a CBDC is introduced, we have $f_1(\vec{a}) = (c, 0, e)$, and $f_2(\vec{a}) = (0, d, e)$.

Solutions to the above problem is

$$q_j = R \cdot \vec{p}_j = \begin{cases} R \cdot f_j(\vec{a}) & \text{for } R \cdot f_j(\vec{a}) < q^*, \\ q^* & \text{for } R \cdot f_j(\vec{a}) \geq q^*. \end{cases} \qquad (4)$$

Buyers can obtain the socially-efficient quantity q^* if they bring sufficient money to compensate sellers for the disutility of producing q^*. Otherwise, buyers consume only what their available portfolios allow.

With the above solutions, the buyer's CM value function can be reexpressed as

$$\begin{aligned} W_j(\vec{a}) = & R \cdot \vec{a} + T + \max_{X^b} \left\{ U(X^b) - X^b \right\} + \beta W_{j,+1}(0) \\ & + \max_{\vec{a}_{+1}} \left\{ -\vec{1} \cdot \vec{a}_{+1} + \beta \left[u(q_{j,+1}) + R \cdot \vec{a}_{+1} - R \cdot \vec{p}_{j,+1} \right] - \mathbf{1}_{\{d_{+1} > 0\}} \times f - \mathbf{1}_{\{e_{+1} > 0\}} \times \delta \right\}, \end{aligned}$$

subject to $\quad e_{+1} \leq \bar{e}$.

The Lagrangian $\mathscr{L}$ for the buyer's portfolio problem above is

$$\mathscr{L} = -\vec{1} \cdot \vec{a}_{+1} + \beta \left[u(q_{j,+1}) + R \cdot \vec{a}_{+1} - R \cdot \vec{p}_{j,+1} \right] - \mu (e_{+1} - \bar{e}).$$

Define function $\Lambda(L) \equiv \max\{u'(L) - 1, 0\}$, where L is available liquidity. $\Lambda(L)$ is thus the marginal benefit of liquidity. Recall that buyers know the types of meetings they will enter in the next DM when making their portfolio choices, so buyers in Type 1 meetings will not hold deposits and buyers in Type 2 meetings will not accumulate cash. The first-order conditions dictating households' demand for cash, deposits and the CBDC are

$$\Lambda\left(R \cdot f_1(\vec{a})\right) \leq \frac{1}{\beta R^c} - 1, \tag{5}$$

$$\Lambda\left(R \cdot f_2(\vec{a})\right) \leq \frac{1}{\beta R^d} - 1, \tag{6}$$

$$\Lambda\left(R \cdot f_j(\vec{a})\right) \leq \frac{1+\mu}{\beta R^e} - 1, \tag{7}$$

which hold with equality if and only if $a > 0$ for $a = c, d, e$. The complementary slackness conditions for $e_{+1} \leq \bar{e}$ are

$$\mu \geq 0, \tag{8}$$

$$\mu(e_{+1} - \bar{e}) = 0. \tag{9}$$

2.4 Asset Supply

2.4.1 Currency

I focus on stationary equilibria where the total supply of central bank monies, M—including both paper and digital forms—grows at a constant rate π, such that $\frac{M}{M_{-1}} = \frac{\phi_{-1}}{\phi} = \pi$. Here it is implicitly assumed that the central bank controls only the total supply of cash and CBDC, but not the composition, and households can exchange one for the other at a fixed exchange rate of one. Therefore, the central bank's budget constraint is

$$\phi(C+E) = \phi(C_{-1} + (1+i)E_{-1}) + \tau - (\lambda_1\lambda_1^e + \lambda_2\lambda_2^e)\delta_2,$$

where C and E are the amounts of cash and CBDC outstanding, τ is the real transfer to households, $(\lambda_1\lambda_1^e + \lambda_2\lambda_2^e)\delta_2$ is the service charge (subsidies) paid from (to) households using the CBDC, where λ_1^e and λ_2^e are fractions of Type 1 and Type 2 meetings where the CBDC is used.

2.4.2 Bank Deposits

As in Keister and Sanches (2022), the banking sector is perfectly competitive with moral hazard problem. Given a market deposit rate R^d, a banker with return $\gamma \in [0, \bar{\gamma}]$ is willing to issue deposits and invest only if

$$\frac{1}{1-\mu} R^d \leq \gamma + \frac{\mu}{(1-\mu)\pi}.$$

The right-hand side is the banker's investment income, which has two components. For $\frac{1}{1-\mu}$ units of deposits issued, one unit is invested in the project which generates γ units of the CM good next period. This explains the first term. The remaining $\frac{\mu}{1-\mu}$ units of issued deposits is invested in reserves with a rate of return of $\frac{1}{\pi}$, which is the second term. In addition, as bankers have limited commitment, they cannot promise to repay depositors more than the value of their collateral

$$\frac{1}{1-\mu} R^d \leq \varepsilon\gamma + \frac{\mu}{(1-\mu)\pi}, \tag{10}$$

where I assume bankers' asset holding of central bank reserves is fully pledgeable.

Let $\hat{\gamma}$ denote the productivity of the banker who satisfies the plegeability constraint (10) at equality. Thus I have

$$R^d = (1-\mu)\varepsilon\hat{\gamma} + \frac{\mu}{\pi}. \tag{11}$$

2.5 Market Clearing

Since cash is used only in Type 1 meetings and deposits are only used in Type 2 meetings, their market clearing conditions are

$$\phi C = \lambda_1 \lambda_1^c c + \mu \lambda_2 (\lambda_2^d d + \lambda_2^{mix} \tilde{d}), \tag{12}$$

$$\frac{\eta}{1-\mu}(\bar{\gamma} - \hat{\gamma}) = \lambda_2 (\lambda_2^d d + \lambda_2^{mix} \tilde{d}), \tag{13}$$

where λ_1^c is the fraction of Type 1 meetings where cash is used, λ_2^d is the fraction of Type 2 meetings where only deposits are used, and λ_2^{mix} where the mixture of deposits and the CBDC is used. d and $\tilde{d}$ are households' demand for deposits when they hold only deposits, and both deposits and the CBDC. I distinguish between the two, as the two portfolios can coexist in equilibrium. Reserves put in the central bank by private banks are held in the form of cash, hence the second term in the demand side for cash. Note that bankers who are able to issue deposits will issue $\frac{1}{1-\mu}$units of deposits so that they can invest one unit of CM good into their projects after the reserve requirement is met.

The market clearing condition for the CBDC is

$$\phi E = (\lambda_1 \lambda_1^e + \lambda_2 \lambda_2^e)\, e. \tag{14}$$

Definition 1. *A stationary equilibrium is a list of portfolios accumulated by buyers* $\{\vec{a}_j\}_{s=1,2}$*, terms of trade* $\{\vec{p}_j, q_j\}_{s=1,2}$*, fractions of buyers holding different portfolios* $\{\lambda_1^c, \lambda_1^e, \lambda_2^d, \lambda_2^e, \lambda_2^{mix}\} \in [0,1]$ *where* $\lambda_2^d + \lambda_2^e = 1$*, the deposit rate* R^d *and the cutoff bank's productivity* $\hat{\gamma}$ *that satisfy equations* (4)-(9), (11)-(14) *given policy parameters* $\{\pi, i, \delta, \bar{e}\}$.

In what follows, I will characterize the equilibrium in the benchmark economy where there is no CBDC issuance in Section 3. Then, I will introduce a CBDC into the economy and derive households' portfolio choices for both cases when the holding limit on the CBDC is not binding in Section 4.1 and is binding in Section 4.2.

3 Equilibrium without CBDC

Without a CBDC, buyers can only use cash in Type 1 meetings and deposits in Type 2 meetings. Therefore, the corresponding first-order conditions for cash and deposits will hold with equality:

$$\Lambda(R^c c(R^c)) = \frac{1}{\beta R^c} - 1, \tag{15}$$

$$\Lambda(R_0^d d(R_0^d)) = \frac{1}{\beta R_0^d} - 1, \tag{16}$$

where the subscript 0 indicates the deposit rate in the benchmark equilibrium without a CBDC.

Using the market clearing conditions, conditions (15)-(16) can be reexpressed as

$$\Lambda(\frac{c(R^c)}{\pi}) = \frac{\pi}{\beta} - 1, \tag{17}$$

$$\Lambda(R_0^d \frac{\eta(\bar{\gamma} - \frac{R_0^d - \frac{\mu}{\pi}}{(1-\mu)\varepsilon})}{\lambda_2}) = \frac{1}{\beta R_0^d} - 1. \tag{18}$$

To guarantee the existence and uniqueness of equilibrium, I make the following assumptions:

Assumption A1. *Preferences are such that:*

$$\text{(i) } d(R^d) \text{ is strictly increasing in } R^d \quad \text{and} \quad \text{(ii) } \lim_{R^d \to 0} \lambda_2 \frac{\lambda^{-1}\left(\frac{1}{\beta R^d}-1\right)}{R^d} < \eta(\bar{\gamma}-\hat{\gamma}(R^d)).$$

The two assumptions ensure that the supply of and demand for deposits intersect at a unique deposit rate, which is well-defined. The following proposition defines the equilibrium with no CBDC.

Proposition 1. *Assume A1 holds, in the benchmark economy without a CBDC, the unique equilibrium consists of portfolios* $\left\{\vec{a}_1 = (c(R^c),0,0), \vec{a}_2 = \left(0, d(R_0^d), 0\right)\right\}$*, terms of trade* $\{\vec{p}_j, q_j\}_{s=1,2}$*, and a deposit rate* R_0^d *satisfying equations* (4) *and* (15)-(18) *given parameter* π.

4 Equilibrium with CBDC

Once a CBDC is introduced, buyers can choose between two means of payment in both Type 1 and Type 2 meetingss since it is assumed that the CBDC can be used universally. Their choices depend critically on how the CBDC is designed and issued: the interest rate R^e, the holding limit $\bar{e}$, and the adoption cost δ. I will derive buyers' optimal portfolio choices given different designs of the CBDC in subsequent sections. In Section 4.1, the rate of return on the CBDC is sufficently low or the holding limit on the CBDC is sufficiently high that the amount of the CBDC that households carry, if they use it, does not exceed the limit. And the opposite in Section 4.2.

In Type 1 meetings, buyers choose the portfolio $\vec{a}_1 = (c, 0, e)$ that maximizes the following CM value function obtained by rearranging and collecting terms from expression (1).

$$W_1 = (\beta R^c - 1)c + (\beta R^e - 1)e + \beta\left(u(q_1^c + q_1^e) - q_1^c - q_1^e\right) - \mathbf{1}_{\{e>0\}} \times \delta + \Omega_1, \tag{19}$$

where $\Omega_1 = R \cdot \vec{a} + T + \max_{X^b}\{U(X^b) - X^b\} + \beta W_{1,+1}(0)$ is independent of buyers' choice between cash and the CBDC, q_1^c is the amount of DM goods consumed using cash and q_1^e using the CBDC, and δ is the CBDC adoption cost.

Similarly, in Type 2 meetings, buyers choose the portfolio $\vec{a}_2 = (0, d, e)$ that maximizes the following CM value function.

$$W_2 = (\beta R^d - 1)d + (\beta R^e - 1)e + \beta \left(u(q_2^d + q_2^e) - q_2^d - q_2^e \right) - \mathbf{1}_{\{d>0\}} \times f - \mathbf{1}_{\{e>0\}} \times \delta + \Omega_2, \tag{20}$$

where $\Omega_2 = R \cdot \vec{a} + T + \max_{X^b} \left\{ U(X^b) - X^b \right\} + \beta W_{2,+1}(0)$, q_2^d is the amount of DM goods consumed using deposits and q_2^e using the CBDC, and f is the deposit fixed cost.

4.1 Non-binding CBDC Holding Limit

In this section, the central bank sets $(R^e, \bar{e})$ such that $e(R^e) \leq \bar{e}$, where $e(R^e)$ is obtained from the first-order condition: $\Lambda\left(R^e e(R^e)\right) = \frac{1}{\beta R^e} - 1$.

We start with Type 1 meetings. The following lemma states that it will never be optimal for buyers to accumulate both media of exchange in a single portfolio since the only feasible case is when the two types of monies have the same rates of return (i.e., $R^c = R^e$), and when this is the case, buyers will optimally choose to bring only cash given a strictly positive fixed cost of using the CBDC. Proofs of all propositions and lemmas are contained in the Appendix.

Lemma 1. *Assume $(R^e, \bar{e})$ are such that $e(R^e) \leq \bar{e}$, then in Type 1 meetings, given $\delta > 0$, either $\vec{a}_1 = (c(R^c), 0, 0)$, or $\vec{a}_1 = (0, 0, e(R^e))$.*

Therefore, buyers choose between the cash-only portfolio and the CBDC-only portfolio, and pick whichever has a higher CM value. Denote W_1^c and W_1^e as the optimal CM value functions for holding only cash and only the CBDC, respectively. From (19), we have

$$W_1^c = (\beta R^c - 1)c(R^c) + \beta\left(u(q_1^c) - q_1^c\right) + \Omega_1, \tag{21}$$

$$W_1^e = (\beta R^e - 1)e(R^e) + \beta\left(u(q_1^e) - q_1^e\right) - \delta + \Omega_1. \tag{22}$$

With (21)-(22), I obtain a threshold of R^e, which depends on policy parameters R^c and δ, and I denote it as $R^e(R^c, \delta)$. $R^e(R^c, \delta)$ is the rate of return on the CBDC that makes it as desirable as cash to buyers, which strictly increases in R^c and δ. Given a strictly positive fixed cost of using the CBDC, the CBDC will have to pay a positive net nominal interest rate to incentivize buyers to switch from using cash to its digital counterpart. So were the

CBDC to be adopted, it should be such that $e(R^e) > c(R^c)$ and $q_1^e > q_1^c$. Hence, when deciding whether to adopt the CBDC, households face a tradeoff between more efficient DM exchange and a higher fixed cost. The following proposition dictates buyers' portfolio choices in Type 1 meetings when the rate of return on the CBDC takes different ranges of value.

Proposition 2. *Assume $(R^e, \bar{e})$ are such that $e(R^e) \leq \bar{e}$, then in Type 1 meetings, given $\delta > 0$, there exists a cutoff $R^e(R^c, \delta) > R^c$ such that*

(i) when $R^e < R^e(R^c, \delta)$, $\vec{a}_1 = (c(R^c), 0, 0)$, $q_1 = q_1^c$;

(ii) when $R^e > R^e(R^c, \delta)$, $\vec{a}_1 = (0, 0, e(R^e))$, $e(R^e) > c(R^c)$, $q_1 = q_1^e > q_1^c$;

(iii) when $R^e = R^e(R^c, \delta)$, $\vec{a}_1 = \begin{cases} (c(R^c), 0, 0) & w.p.\ \lambda_1^c \\ (0, 0, e(R^e)) & w.p.\ 1 - \lambda_1^c \end{cases}$, $q_1 = \begin{cases} q_1^c & w.p.\ \lambda_1^c \\ q_1^e & w.p.\ 1 - \lambda_1^c \end{cases}$, $e(R^e) > c(R^c)$, $q_1^e > q_1^c$.

Proof. *See the Appendix.*

Now, I turn to Type 2 meetings. It again will not be optimal for buyers to hold a mixed portfolio of deposits and the CBDC, as when they have the same rates of return, buyers will choose the one with a lower fixed cost. Lemma 2 formalizes this idea.

Lemma 2. *Assume $(R^e, \bar{e})$ are such that $e(R^e) \leq \bar{e}$, then in Type 2 meetings, given $f, \delta > 0$, either $\vec{a}_2 = \big(0, d(R^d), 0\big)$, or $\vec{a}_2 = (0, 0, e(R^e))$.*

Thus, buyers will choose between the deposit-only and CBDC-only portfolios whichever has a higher CM value. Denote $W_2^d(R^d)$ and W_2^e as the optimal CM value functions for holding only deposits at deposit rate R^d and holding only the CBDC, respectively.

$$W_2^d(R^d) = (\beta R^d - 1)d(R^d) + \beta\left(u\left(q_2^d(R^d)\right) - q_2^d(R^d)\right) - f + \Omega_2, \tag{23}$$

$$W_2^e = (\beta R^e - 1)e(R^e) + \beta\left(u(q_2^e) - q_2^e\right) - \delta + \Omega_2. \tag{24}$$

If the CBDC is not adopted in Type 2 meetings, then we have $R^d = R_0^d$, the equilibrium deposit rate in the benchmark economy without a CBDC. A key difference with Type 1 meetings is that when the CBDC offers a higher CM value than deposits with deposit rate R_0^d, the CBDC may not drive out deposits completely in Type 2 meetings. Instead, the deposit rate will increase until the CM value of holding deposits matches the CM value of holding the CBDC in equilibrium, given the following assumptions.

Assumption A2.

$$(i) \quad W_2^e \leq W_2^d(R^d = \frac{1}{\beta}) \quad and \quad (ii) \quad \bar{\gamma} > \hat{\gamma}(R^d).$$

The two assumptions ensure that deposits will always be used in equilibrium. Therefore, with (23) evaluated at R_0^d and (24) , I obtain a threshold of R^e, which depends on R_0^d and δ, and I denote it as $R^e(R_0^d, \delta)$. $R^e(R_0^d, \delta)$ is the rate of return on the CBDC that makes it as desirable as deposits in the benchmark economy to buyers, which strictly increases in R_0^d and δ. Proposition 3 specifies households portfolio choices in Type 2 meetings given different rates of return on the CBDC.

Proposition 3. *Assume $(R^e, \bar{e})$ are such that $e(R^e) \leq \bar{e}$, then in Type 2 meetings, there exists a cutoff $R^e(R_0^d, \delta)$ such that*

(i) when $R^e \leq R^e(R_0^d, \delta)$, $R^d = R_0^d$, $\vec{a}_2 = \big(0, d(R_0^d), 0\big)$, $q_2 = q_2^d(R_0^d)$;

(ii) when $R^e(R_0^d, \delta) < R^e \leq \frac{1}{\beta}$, $\vec{a}_2 = \begin{cases} \big(0, d(R^d), 0\big) & w.p.\ \lambda_2^d \in (0,1) \\ \big(0, 0, e(R^e)\big) & w.p.\ 1 - \lambda_2^d \end{cases}$, $q_2 = \begin{cases} q_2^d(R^d) & w.p.\ \lambda_2^d \in (0,1) \\ q_2^e & w.p.\ 1 - \lambda_2^d \end{cases}$, R^d is obtained from the indifference condition: $W_2^d(R^d) = W_2^e$, and $R^d > R_0^d$.

Proof. *See the Appendix.*

Proposition 4 gives some comparative statics regarding R^e.

Proposition 4. *Assume $(R^e, \bar{e})$ are such that $e(R^e) \leq \bar{e}$. If the CBDC is adopted in at least one type of meeting such that*

(i) $R^e > \max\big\{R^e(R^c, \delta), R^e(R_0^d, \delta)\big\}$, then as R^e increases, $q_1^e, q_2^e, R^d, q_2^d, \lambda_2^e$ also increase, while λ_2^d decreases;

(ii) $R^e(R^c, \delta) < R^e < R^e(R_0^d, \delta)$, then as R^e increases, q_1^e also increases, while R^d, q_2^d, λ_2^d remain unchanged;

(iii) $R^e(R_0^d, \delta) < R^e < R^e(R^c, \delta)$, then as R^e increases, $q_2^e, R^d, q_2^d, \lambda_2^e$ also increases, while λ_2^d decreases and q_1^e remains unchanged.

Proof. See the Appendix.

4.2 Binding CBDC Holding Limit

In this section, the central bank sets $(R^e, \bar{e})$ such that $\bar{e} < e(R^e)$. Recall that the policymaker has three CBDC-related policy tools at its disposal: the interest rate, the holding limit, and the cost. More importantly, buyers may choose to hold both means of payment available in a single portfolio. The idea is that when the rate of return on the CBDC is high and the fixed cost of holding the CBDC is small, buyers will start accumulating the CBDC up to the holding limit, $\bar{e}$, and when the limit is low, they will continue topping up with cash in Type 1 meetings, and with deposits in Type 2 meetings if not too costly, to have more DM trade from this extra liquidity.

4.2.1 Type 1 Meetings

In Type 1 meetings, there are three possible types of portfolios that buyers may choose. The following lemma outlines them.

Lemma 3. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. In Type 1 meetings, buyers may choose one of the following portfolios*

(i) $\vec{a}_1 = (c(R^c), 0, 0)$*;*

(ii) $\vec{a}_1 = (0, 0, \bar{e})$*;*

(iii) $\vec{a}_1 = (\tilde{c}, 0, \bar{e})$*, where* $\tilde{c} = c(R^c) - \frac{R^e \bar{e}}{R^c}$*. The mixed portfolio is feasible only if (a)* $R^e > R^c$ *and (b)* $R^e \bar{e} < R^c c(R^c)$*.*

Buyers may hold only cash, hold the CBDC up to the holding limit, or hold the CBDC up to the limit and supplement it with cash. Regarding the two feasibility conditions for the third portfolio $(\tilde{c}, 0, \bar{e})$, where cash coexists with the CBDC at the intensive margin in Type 1 meetings, the former one—$R^e > R^c$—implies that buyers prefer the CBDC at the margin as it has a higher rate of return, but the amount of the CBDC they are allowed to hold is sufficiently low such that the marginal gain from more DM trade exceeds the marginal cost of topping up with cash, as implied by the latter condition—$R^e \bar{e} < R^c c(R^c)$.

Buyers will choose to hold the portfolio that gives them the highest value when they make their portfolio decisions in the CM. The optimal CM value functions for these three types of portfolios are:

$$
\begin{aligned}
W_1^c =& (\beta R^c - 1)c(R^c) + \beta\left(u(q_1^c) - q_1^c\right) + \Omega_1, \\
W_1^{\bar{e}} =& (\beta R^e - 1)\bar{e} + \beta\left(u(q_1^{\bar{e}}) - q_1^{\bar{e}}\right) - \delta + \Omega_1, \\
W_1^{mix} =& (\beta R^e - 1)\bar{e} + (\beta R^c - 1)\tilde{c} + \beta\left(u(q_1^{mix}) - q_1^{mix}\right) - \delta + \Omega_1,
\end{aligned}
$$

where the superscript $\bar{e}$ indicates that buyers hold $\bar{e}$ units of the CBDC, $q_1^{\bar{e}}$ is the amount of DM goods purchased using $\bar{e}$ units of the CBDC and q_1^{mix} using both cash and the CBDC, and $\tilde{c}$ is the amount of cash held in the mixed portfolio.

Note two things here. One is that when the mixed portfolio is feasible, $W_1^{mix} > W_1^{\bar{e}}$, as there is no fixed cost for using cash and the amount of liquidity provided by CBDC is sufficiently low to make topping up with cash desirable. In other words, when cash can coexist with the CBDC at the intensive margin, buyers will never choose the CBDC-only portfolio in Type 1 meetings. Lemma 4 summarizes this result. The other is that the amount of liquidity and thereby the amount of DM goods traded is the same for the cash-only portfolio and the mixed portfolio—$R^e\bar{e} + R^c\tilde{c} = R^c c(R^c)$. However, by accumulating the CBDC to the limit and topping up with cash in the mixed portfolio, buyers, on one hand, hold fewer units of money and thereby save in their efforts to acquire money—$\tilde{c} + \bar{e} < c(R^c)$—since the CBDC pays a higher interest rate than cash; on the other hand, they incur a fixed cost for using the CBDC. Lemma 5 records these results. Whether it is optimal to do so depends on which one is greater: savings in labor to acquire money or the fixed CBDC adoption cost. If $(\frac{R^e}{R^c} - 1)\bar{e} > (<)\ \delta$, then the mixed portfolio is better (worse) than the cash-only portfolio.

Lemma 4. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. In Type 1 meetings, if $R^e > R^c$ and $R^e\bar{e} < R^c c(R^c)$, then buyers will never choose $\vec{a}_1 = (0, 0, \bar{e})$.*

Proof. *See the Appendix.*

Lemma 5. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. In Type 1 meetings, if $R^e > R^c$ and $R^e\bar{e} < R^c c(R^c)$, then $q_1^{mix} = q_1^c$, $R^e\bar{e} + R^c\tilde{c} = R^c c(R^c)$, $\tilde{c} + \bar{e} < c(R^c)$, and $W_1^{mix} = W_1^c + (\frac{R^e}{R^c} - 1)\bar{e} - \delta$.*

Proof. *See the Appendix.*

Next, I will derive buyers' optimal portfolio choices in Type 1 meetings. There are three cases. First, when the CBDC is not paying a higher interest rate than cash, the CBDC will not be adopted given its fixed cost. Second, if the CBDC pays a positive net nominal interest rate but its holding limit is too high for the mixed portfolio to be feasible, buyers will choose between the cash-only portfolio and the CBDC-only portfolio. Third, if the CBDC pays a positive net nominal interest rate and its holding limit is sufficiently low to make the mixed portfolio feasible, buyers will choose between the cash-only portfolio and the mixed portfolio. If the limit is sufficiently large such that it is worth paying the CBDC fixed cost to save on the cost of acquiring money, buyers will opt for the mixed portfolio. The following lemma defines some important cutoffs for the CBDC holding limit and shows how the cutoffs position relative to each other, which matter for the evolving of equilibrium allocations as the limit varies.

Lemma 6. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. I obtain three cutoffs—$\bar{e}_1, \bar{e}_2, \bar{e}_3$—such that:*

(i) $\bar{e}_1 \equiv \frac{\delta}{\frac{R^e}{R^c}-1}$, *and* $W_1^c = W_1^{mix}|_{\bar{e}=\bar{e}_1}$;

(ii) $\bar{e}_2 \equiv \bar{e}(R^c, R^e, \delta)$, *and* $W_1^c = W_1^{\bar{e}=\bar{e}_2}$;

(iii) $\bar{e}_3 \equiv \frac{R^c c(R^c)}{R^e}$, *and* $\tilde{c}(\bar{e}_3) = 0$.

And when $R^e > R^c$, there exists $\hat{\delta} > 0$ such that:

(i) if $0 < \delta < \hat{\delta}$, *then* $\bar{e}_1 < \bar{e}_2 < \bar{e}_3$;

(ii) if $\delta \geq \hat{\delta}$, *then* $\bar{e}_3 \leq \min\{\bar{e}_1, \bar{e}_2\}$, *where* $\hat{\delta} = (1 - \frac{R^c}{R^e})c(R^c)$.

Proof. *See the Appendix.*

$\bar{e}_1$ is the cutoff that the holding limit must exceed for the mixed portfolio to outperform the cash-only portfolio, $\bar{e}_2$ denotes the threshold that makes the CBDC-only portfolio as desirable as the cash-only portfolio, and $\bar{e}_3$ specifies the upper bound on the limit for the mixed portfolio to be feasible. If the limit surpasses $\bar{e}_3$, the amount of cash held in the mixed portfolio becomes negative, implying households will borrow cash in the mixed portfolio which is not feasible.

The following proposition consolidates these results and presents buyers' optimal portfolio choices in Type 1 meetings under different designs of the CBDC.

Proposition 5. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. In Type 1 meetings,*

1. $\vec{a}_1 = (c(R^c), 0, 0)$, $q_1 = q_1^c$ *when:*
   (i) $R^e \leq R^c$, *or*
   (ii) $R^e > R^c$, $0 < \delta < \hat{\delta}$, *and* $\bar{e} < \bar{e}_1$, *or*
   (iii) $R^e > R^c$, $\delta \geq \hat{\delta}$, *and* $\bar{e} < \bar{e}_2$;
2. $\vec{a}_1 = (\tilde{c}, 0, \bar{e})$, $q_1 = q_1^c$ *when* $R^e > R^c$, $0 < \delta < \hat{\delta}$, *and* $\bar{e}_1 < \bar{e} < \bar{e}_3$;
3. $\vec{a}_1 = (0, 0, \bar{e})$, $q_1 = q_1^{\bar{e}}$ *when* $R^e > R^c$, *and* $\bar{e} > \max\{\bar{e}_2, \bar{e}_3\}$.

Proof. *See the Appendix.*

Figures 2 and 3 summarize how buyers' optimal portfolio choices in Type 1 meetings change as the CBDC holding limit varies, for the two cases: (i) $R^e > R^c$, $0 < \delta < \hat{\delta}$; and (ii) $R^e > R^c$, $\delta \geq \hat{\delta}$. In Figure 2, $0 < \delta < \hat{\delta}$, so buyers switch from holding only cash, to holding a mixture of cash and the CBDC, and lastly to holding only the CBDC, as the limit relaxes. In Figure 3, $\delta \geq \hat{\delta}$, and therefore $\bar{e}_1 \geq \bar{e}_3$, which implies that the range of holding limits making the mixed portfolio optimal lies outside its feasible set. In other words, whenever the mixed portfolio is feasible, it is not optimal. Therefore, buyers switch from holding only cash to holding only the CBDC, as the limit increases.

4.2.2 Type 2 Meetings

Similarly, in Type 2 meetings, there are also three possible types of portfolios that buyers can hold. Buyers may hold only deposits, hold only the CBDC up to the holding limit, or hold a mixture of the two types of money. Let $\tilde{d}$ denote the amount of deposits in the mixed portfolio. The optimal CM value functions of the three types of portfolios are:

$$\begin{aligned}
W_2^d(R^d) &= (\beta R^d - 1)d(R^d) + \beta\left(u\left(q_2^d(R^d)\right) - q_2^d(R^d)\right) - f + \Omega_2, \\
W_2^{\bar{e}} &= (\beta R^e - 1)\bar{e} + \beta\left(u\left(q_2^{\bar{e}}\right) - q_2^{\bar{e}}\right) - \delta + \Omega_2, \\
W_2^{mix}(R^d) &= (\beta R^e - 1)\bar{e} + (\beta R^d - 1)\tilde{d} + \beta\left(u\left(q_2^{mix}(R^d)\right) - q_2^{mix}(R^d)\right) - \delta - f + \Omega_2.
\end{aligned}$$

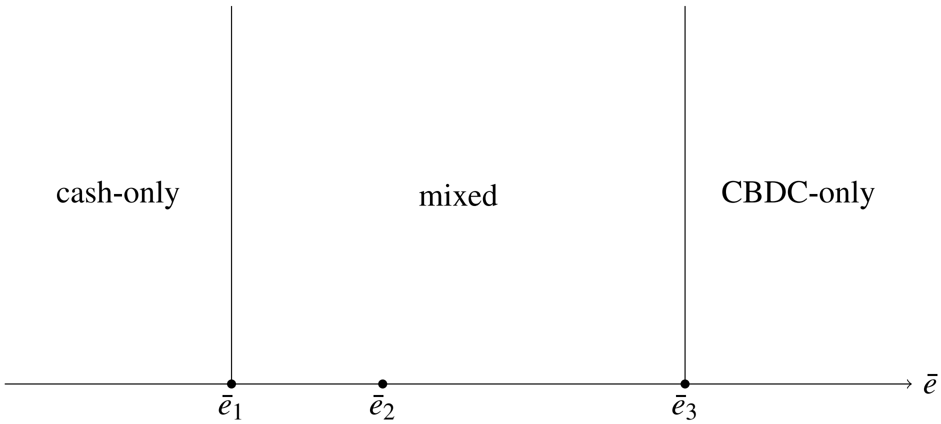

Figure 2: Optimal portfolio in Type 1 meetings when $R^e > R^c$ and $0 < \delta < \hat{\delta}$.

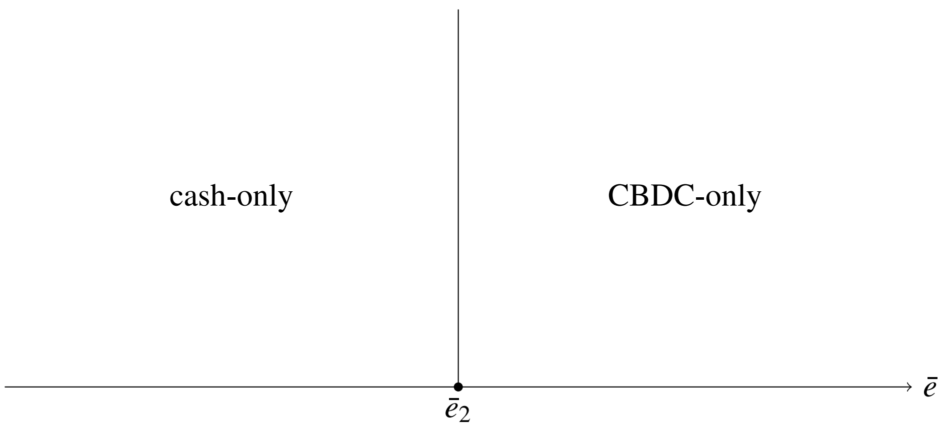

Figure 3: Optimal portfolio in Type 1 meetings when $R^e > R^c$ and $\delta \geq \hat{\delta}$.

The mixed portfolio $(0, \tilde{d}, \bar{e})$, is feasible only if

$$\text{(i)} \quad R^e > R^d \quad \text{and} \quad \text{(ii)} \quad R^e \bar{e} < R^d d(R^d). \tag{25}$$

When the mixed portfolio is feasible, compared with the deposit-only portfolio at the same deposit rate, the DM consumption is the same—$q_2^{mix} = q_2^d(R^d)$. However, the mixed portfolio saves on labor to acquire money by paying the fixed cost for using the CBDC—$\tilde{d} + \bar{e} < d(R^d)$. Lemma 7 documents these results.

Lemma 7. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. In Type 2 meetings, if $R^e > R^d$ and $R^e \bar{e} < R^d d(R^d)$, then $q_2^{mix} = q_2^d(R^d)$, $R^e \bar{e} + R^d \tilde{d} = R^d d(R^d)$, $\tilde{d} + \bar{e} < d(R^d)$, and $W_2^{mix}(R^d) = W_2^d(R^d) + (\frac{R^e}{R^d} - 1)\bar{e} - \delta$.*

Proof. *See the Appendix.*

To guarantee the monotonicity of equilibrium portfolios in the holding limit, I make the following assumption

Assumption A3.

$$\textit{(i)} \quad \frac{\partial \left(W_2^{\bar{e}} - W_2^{mix}(R^d)\right)}{\partial \bar{e}} > 0 \quad \textit{and} \quad \textit{(ii)} \quad \frac{\partial \tilde{d}}{\partial \bar{e}} < 0.$$

In general, there are two cases for households' optimal portfolio choices in Type 2 meetings. If the CBDC is not paying a higher interest rate than R_0^d, households either hold only deposits or hold only the CBDC. In this case, when the limit is set sufficiently low, the CBDC will not be adopted in Type 2 meetings. When the limit is set sufficiently large, it creates competitive pressure on banks to raise the deposit rate until households become indifferent between holding only deposits or only the CBDC. At this point, a nonnegative fraction of households switch to holding the CBDC from deposits as fewer banks can afford to issue deposits to finance their investment, and we have the extensive margin coexistence of deposits and the CBDC in Type 2 meetings. If, instead, the CBDC offers a higher interest payment, households can hold the mixed portfolio. To facilitate analysis, lemma 8 defines four cutoffs for $\bar{e}$ and shows their relative positions for the second case.

Lemma 8. *Assume* $(R^e, \bar{e})$ *are such that* $\bar{e} < e(R^e)$*. I obtain four cutoffs—*$\bar{e}'_1, \bar{e}'_2, \bar{e}'_3, \bar{e}'_4$*—such that:*

(i) $\bar{e}'_1 \equiv \frac{\delta}{\frac{R^e}{R_0^d} - 1}$, *and* $W_2^d(R_0^d) = W_2^{mix}(R_0^d)|_{\bar{e}=\bar{e}'_1}$*;*

(ii) $\bar{e}'_2 \equiv \bar{e}(R_0^d, R^e, \delta, f)$*, and* $W_2^d(R_0^d) = W_2^{\bar{e}=\bar{e}'_2}$*;*

(iii) $\bar{e}'_3 \equiv \bar{e}(R^e, \delta, f)$*, and* $W_2^{mix}(R^d)|_{\bar{e}=\bar{e}'_3} = W_2^{\bar{e}=\bar{e}'_3}$*;*

(iv) $\bar{e}'_4 \equiv \bar{e}(R^e, \delta)$*, and* $\tilde{d}(\bar{e}'_4) = 0$*.*

And when $R^e > R_0^d$ *and* $W_2^d(R_0^d, f = 0) > W_2^{\bar{e}=\bar{e}'_1}$*, there exist* $\hat{f} > 0$ *such that:*

(i) if $f > \hat{f}$*, then* $\bar{e}'_2 < \bar{e}'_1$*;*

(ii) if $0 < f < \hat{f}$*, then* $\bar{e}'_1 < \bar{e}'_2 < \bar{e}'_3 < \bar{e}'_4$*, where* $W_2^d(R_0^d, \hat{f}) = W_2^{\bar{e}=\bar{e}'_1}$*.*

Proof. *See the Appendix.*

When the equilibrium deposit rate is R_0^d, buyers are indifferent between holding the mixture and holding only deposits at $\bar{e}'_1$, while they are indifferent between holding only the CBDC and holding only deposits at $\bar{e}'_2$. Beyond $\bar{e}'_3$, buyers no longer find it optimal to top up with deposits after already accumulating the CBDC to the holding limit, as the extra amount of liquidity deposits provide can no longer compensate for its fixed cost, whereas beyond $\bar{e}'_4$, the mixed portfolio collapses to containing only the CBDC.

Proposition 6 records these results and shows buyers' optimal portfolio choices in Type 2 meetings under different designs of the CBDC.

Proposition 6. *Assume $(R^e, \bar{e})$ are such that $\bar{e} < e(R^e)$. In Type 2 meetings, the CBDC is adopted in three distinct ways:*

1. $\vec{a}_2 = \big(0, d(R_0^d), 0\big)$, $q_2 = q_2^d(R_0^d)$, *and* $R^d = R_0^d$ *when:*
   - *(i)* $R^e \leq R_0^d$ *and* $\bar{e} \leq \bar{e}'_2$, *or*
   - *(ii)* $R^e > R_0^d$ *and* $\bar{e} \leq \min\{\bar{e}'_1, \bar{e}'_2\}$;
2. $\vec{a}_2 = \begin{cases} \big(0, d(R^d), 0\big) & w.p.\ \lambda_2^d \in (0,1) \\ \big(0, \tilde{d}, \bar{e}\big) & w.p.\ \lambda_2^{mix} \in (0,1) \end{cases}$, *such that* $\lambda_2^d + \lambda_2^{mix} = 1$, $q_2 = q_2^d(R^d) = q_2^{mix} > q_2^d(R_0^d)$, $R^d = \frac{R^e \bar{e}}{\delta + \bar{e}} > R_0^d$ *when* $R^e > R_0^d$, $0 < f < \hat{f}$, *and* $\bar{e}'_1 < \bar{e} < \bar{e}'_3$;
3. $\vec{a}_2 = \begin{cases} \big(0, d(R^d), 0\big) & w.p.\ \lambda_2^d \in (0,1) \\ (0, 0, \bar{e}) & w.p.\ \lambda_2^e \in (0,1) \end{cases}$, $q_2 = \begin{cases} q_2^d(R^d) & w.p.\ \lambda_2^d \in (0,1) \\ q_2^{\bar{e}} & w.p.\ \lambda_2^e \in (0,1) \end{cases}$, *such that* $\lambda_2^d + \lambda_2^e = 1$, R^d *is obtained from the indifference condition:* $W_2^d(R^d) = W_2^{\bar{e}}$ *with* $R^d > R_0^d$ *when:*
   - *(i)* $R^e \leq R_0^d$ *and* $\bar{e} > \bar{e}'_2$, *or*
   - *(ii)* $R^e > R_0^d$ *and* $\bar{e} > \max\{\bar{e}'_2, \bar{e}'_3\}$.

Proof. *See the Appendix.*

Figures 4 and 5 summarize how buyers' optimal portfolio choices in Type 2 meetings change as the CBDC holding limit varies, for the two cases: (i) $R^e > R_0^d$, $0 < f < \hat{f}$; and (ii) $R^e > R_0^d$, $f \geq \hat{f}$. In Figure 4, where $0 < f < \hat{f}$, buyers initially do not adopt the CBDC. As the holding limit relaxes, they first become indifferent between holding only deposits and holding the mixed portfolio, and eventually become indifferent between holding only deposits and holding only the CBDC as it is no longer worth paying the fixed cost of deposits and holding the mixture. In Figure 5, where $f \geq \hat{f}$, we have $\bar{e}_1' \geq \bar{e}_2'$, suggesting that the fixed cost for using deposits is too large to make it optimal for households to hold both deposits and the CBDC regardless of the value of the limit. Consequently, buyers optimally choose between the deposit-only portfolio and the CBDC-only portfolio.

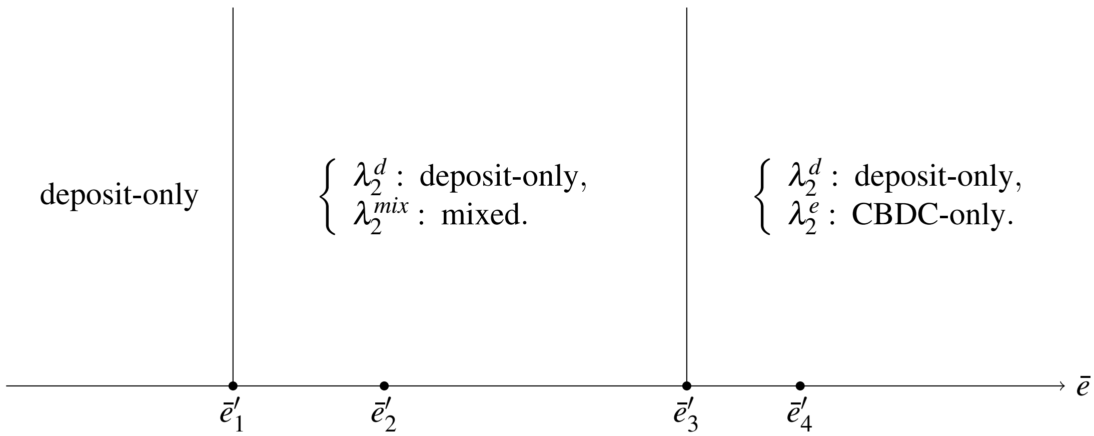

Figure 4: Optimal portfolio in Type 2 meetings when $R^e > R_0^d$ and $0 < f < \hat{f}$.

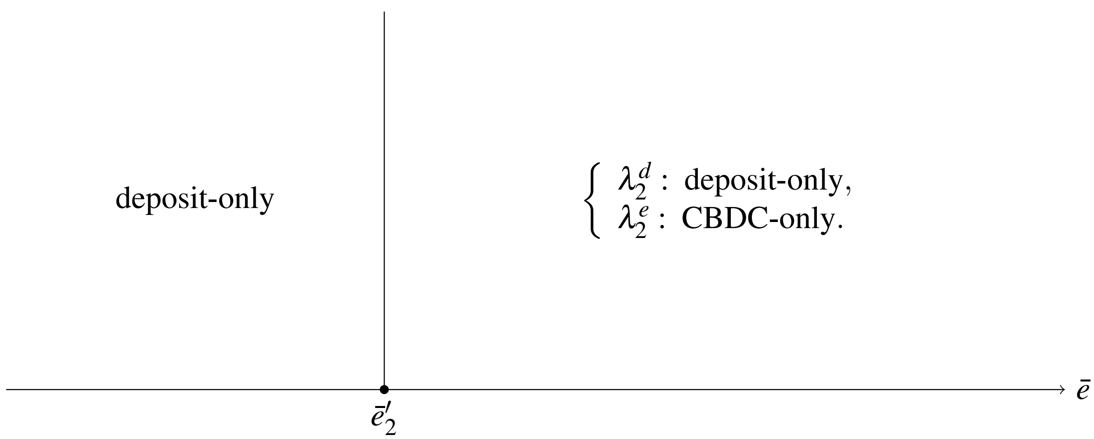

Figure 5: Optimal portfolio in Type 2 meetings when $R^e > R_0^d$ and $f \geq \hat{f}$.

If the CBDC is adopted in Type 2 meetings, a higher CBDC holding limit until it stops binding always results in a higher deposit rate, which makes fewer banks able to issue deposits and invest. However, the rate at which a higher CBDC holding limit raises the deposit rate—thereby crowding out deposits and investment—differs depending on how the CBDC coexists with deposits, as the equilibrium deposit rate is determined in different ways. In Section 6, I conduct a quantitative analysis and investigate the difference.

Proposition 7 integrates the results for both Type 1 and Type 2 meetingss.

Proposition 7. *Assume $(R^e,\bar{e})$ are such that $\bar{e} < e(R^e)$. If $R^e > \max\{R^c, R_0^d\}$, $R_0^d > R^c > \hat{R}^c$, $W_2^d(R_0^d, f=0) > W_2^{\bar{e}=\bar{e}_1'}$, $0 < \delta < \hat{\delta}$, $0 < f < \hat{f}$, then we have $0 < \bar{e}_1 < \bar{e}_1' < \bar{e}_3' < \bar{e}_3 < e(R^e)$ such that:*

(i) when $\bar{e} \in (0,\bar{e}_1)$, $\vec{a}_1 = (c(R^c),0,0)$, $\vec{a}_2 = \big(0,d(R_0^d),0\big)$, $q_1 = q_1^c$, $q_2 = q_2^d(R_0^d)$, and $R^d = R_0^d$;

(ii) when $\bar{e} = \bar{e}_1$, $\vec{a}_1 = \begin{cases} (c(R^c),0,0) & w.p.\ \lambda_1^c - \lambda_1^e \\ (\tilde{c},0,\bar{e}) & w.p.\ \lambda_1^e \end{cases}$, $q_1 = q_1^c$, such that $\lambda_1^c = 1$, $\lambda_1^e \in [0,1]$, $\vec{a}_2 = \big(0,d(R_0^d),0\big)$, $q_1 = q_1^c$, $q_2 = q_2^d(R_0^d)$, and $R^d = R_0^d$;

(iii) when $\bar{e} \in (\bar{e}_1,\bar{e}_1']$, $\vec{a}_1 = (\tilde{c},0,\bar{e})$, $\vec{a}_2 = \big(0,d(R_0^d),0\big)$, $q_1 = q_1^c$, $q_2 = q_2^d(R_0^d)$, and $R^d = R_0^d$;

(iv) when $\bar{e} \in (\bar{e}_1',\bar{e}_3')$, $\vec{a}_1 = (\tilde{c},0,\bar{e})$, $\vec{a}_2 = \begin{cases} \big(0,d(R^d),0\big) & w.p.\ \lambda_2^d \\ (0,\tilde{d},\bar{e}) & w.p.\ \lambda_2^{mix} \end{cases}$, $q_1 = q_1^c$, $q_2 = q_2^d(R^d) > q_2^d(R_0^d)$, and $R^d = \frac{R^e\bar{e}}{\delta+\bar{e}} > R_0^d$;

(v) when $\bar{e} = \bar{e}_3'$, $\vec{a}_1 = (\tilde{c},0,\bar{e})$, $\vec{a}_2 = \begin{cases} \big(0,d(R^d),0\big) & w.p.\ \lambda_2^d \\ (0,\tilde{d},\bar{e}) & w.p.\ \lambda_2^{mix} \\ (0,0,\bar{e}) & w.p.\ \lambda_2^e - \lambda_2^{mix} \end{cases}$, $q_1 = q_1^c$, $q_2 = \begin{cases} q_2^d(R^d) & w.p.\ \lambda_2^d + \lambda_2^{mix} \\ q_2^{\bar{e}} & w.p.\ \lambda_2^e - \lambda_2^{mix} \end{cases}$, and $R^d = \frac{R^e\bar{e}_3'}{\delta+\bar{e}_3'} > R_0^d$;

(vi) when $\bar{e} \in (\bar{e}_3',\bar{e}_3]$, $\vec{a}_1 = (\tilde{c},0,\bar{e})$, $\vec{a}_2 = \begin{cases} \big(0,d(R^d),0\big) & w.p.\ \lambda_2^d \\ (0,0,\bar{e}) & w.p.\ \lambda_2^e \end{cases}$, $q_1 = q_1^c$, $q_2 = \begin{cases} q_2^d(R^d) & w.p.\ \lambda_2^d \\ q_2^{\bar{e}} & w.p.\ \lambda_2^e \end{cases}$, and $W_2^d(R^d) = W_2^{\bar{e}}$ with $R^d > R_0^d$;

(vii) when $\bar{e} \in (\bar{e}_3, e(R^e)]$, $\vec{a}_1 = (0,0,\bar{e})$, $\vec{a}_2 = \begin{cases} \big(0,d(R^d),0\big) & w.p.\ \lambda_2^d \\ (0,0,\bar{e}) & w.p.\ \lambda_2^e \end{cases}$, $q_1 = q_1^{\bar{e}} > q_1^c$, $q_2 = \begin{cases} q_2^d(R^d) & w.p.\ \lambda_2^d \\ q_2^{\bar{e}} & w.p.\ \lambda_2^e \end{cases}$, and $W_2^d(R^d) = W_2^{\bar{e}}$ with $R^d > R_0^d$,

where $\bar{e}_3(\hat{R}^c) = \bar{e}'_3$.

Proof. *See the Appendix.*

Figure 6 presents the results stated in Proposition 7, showing how equilibrium portfolios evolve as the CBDC holding limit increases, when R^e is high, and both f and δ are not too large. Note three things here. One is that the CBDC is firstly adopted in Type 1 meetings, then in Type 2 meetings—when $R^d_0 > R^c$—as the marginal saving in the cost of holding money is greater for buyers in Type 1 meetings when using the CBDC. Second, when $\bar{e} \in (\bar{e}_1, \bar{e}'_1]$, $q_1 = R^c c(R^c)$ and $q_2 = R^d_0 d(R^d_0)$ do not depend on $\bar{e}$. The independence of q_2 and R^d from $\bar{e}$ is immediate, as the CBDC is not used in Type 2 meetings at all. What is interesting is the result that $\bar{e}$ does not affect the amount of liquidity and hence DM consumption in Type 1 meetings where buyers hold a mix of cash and the CBDC. This is because after accumulating the CBDC to the limit, buyers continue topping up with cash until the marginal benefit of liquidity equals the marginal cost of holding cash—$\frac{1}{\beta R^c} - 1$, as indicated by (5). Hence, $R^c \tilde{c} = q_1 - R^e \bar{e}$ adjusts to guarantee that the liquidity provided by cash fills the gap between payment required to trade q_1 and the liquidity provided by the CBDC. When the liquidity provided by the CBDC increases (decreases), the liquidity provided by cash decreases (increases). In other words, in this regime, changes in CBDC holding limits are neutral. This result aligns with the concept of credit neutrality discussed in Gu et al. (2016), in which real money balances adjust to changes in the credit limit so that the total amount of liquidity remains unchanged. Last but not least, if the CBDC is designed to have a smaller fixed cost or a higher interest rate, then it will be adopted at a lower holding limit.

In Figure 6, as $\bar{e}$ increases, $\tilde{c}$ decreases for $\bar{e}_1 < \bar{e} < \bar{e}_3$; $q_1 (= q^c_1)$ first remains constant for $\bar{e} \leq \bar{e}_3$, then $q_1 (= q^{\bar{e}}_1)$ increases with $\bar{e}$. The amount of DM exchange in Type 2 meetings is given by $q_2 = q^d_2(R^d_0)$ for $\bar{e} \leq \bar{e}'_1$, after which, we have two groups of buyers holding different portfolios. For $\bar{e}'_1 < \bar{e} < \bar{e}'_3$, some households—with fraction λ^{mix}_2—hold a mix of deposits and the CBDC, while others—with fraction λ^d_2—hold only deposits. In this region, $q^{mix}_2 = q^d_2(R^d) > q^d_2(R^d_0)$, with R^d, q^{mix}_2, $q^d_2(R^d)$, and λ^{mix}_2 strictly increasing in $\bar{e}$. After $\bar{e}'_3$, some buyers hold only deposits while others hold only the CBDC. Note that at $\bar{e}'_3$, buyers are indifferent between holding a mix of deposits and the CBDC, and holding only the CBDC. Holding both deposits and the CBDC achieves more efficient DM exchange but incurs the fixed cost of using deposits, compared with holding only the CBDC. However, we have $q_2^{\bar{e}=\bar{e}'_3} < q_2^{mix}|_{\bar{e}=\bar{e}'_3}$, meaning that the amount of DM goods consumed exhibits a discrete drop if households choose to hold only the CBDC due to a sudden reduction in available liquidity. The same pattern applies to λ_2^e. At $\bar{e}'_3$, λ_2^e drops as the equilibrium shifts from one in which households using the CBDC hold the CBDC as part of a mixed portfolio to one in which the CBDC is held on its own. The difference is the fraction of households who switches back to holding only deposits, and this ensures that aggregate demand for deposits remain unchanged. Nonetheless, R^d, $q_2^d(R^d)$, $q_2^{\bar{e}}$ and λ_2^e continue to rise as $\bar{e}$ increases after $\bar{e}'_3$.

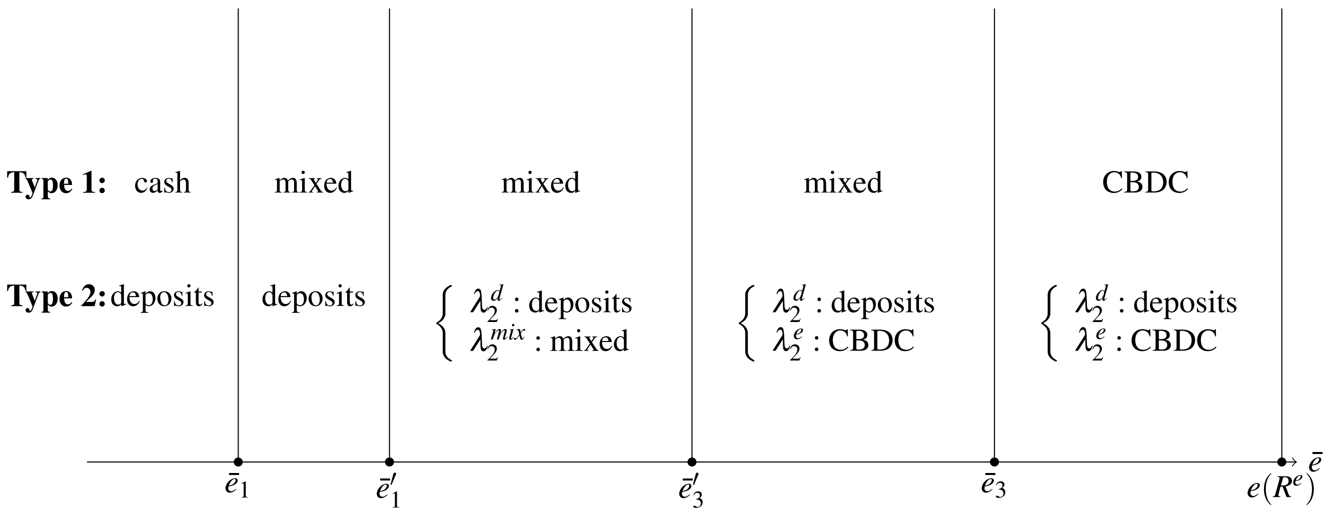

Figure 6: Optimal portfolios.

Notes. This figure shows the equilibrium portfolios under different ranges of $\bar{e}$ if $R^e > \max\{R^c, R_0^d\}$, $R_0^d > R^c > \hat{R}^c$, $W_2^d(R_0^d, f=0) > W_2^{\bar{e}=\bar{e}'_1}$, $0 < \delta < \hat{\delta}$, $0 < f < \hat{f}$.

5 CBDC Design

For discussions of optimal designs of the CBDC, welfare is defined as the sum of the utilities of all agents. Aggregate welfare is then

$$\begin{aligned}\mathscr{W} = {} & 2U(X^*) - H + \lambda_1 \left[\beta\left(u(q_1) - q_1\right)\right] \\ & + \lambda_2 \left\{ (\lambda_2^d + \lambda_2^{mix}) \left[\beta\left(u\left(q_2^d(R^d)\right) - q_2^d(R^d)\right)\right] + (\lambda_2^e - \lambda_2^{mix}) \left[\beta\left(u(q_2^e) - q_2^e\right)\right] \right\}, \quad (26)\end{aligned}$$

where H is aggregate working hours, which is obtained with the CM goods market-clearing condition,

$$2X^* + \lambda_1 \lambda_1^e \delta_1 + \lambda_2 \left[(\lambda_2^d + \lambda_2^{mix}) f + \lambda_2^e \delta_1 \right] + \eta (\bar{\gamma} - \hat{\gamma}) = H + \eta \int_{\hat{\gamma}}^{\bar{\gamma}} \gamma d\gamma, \tag{27}$$

where the left-hand side is expenditure in CM goods, including households' optimal CM consumption, aggregate user costs for using deposits and the CBDC, and bankers' investment; the right-hand side is output from labor and bankers' investment projects.

Substituting H obtained from (27) into (26), aggregate welfare then takes the following expression:

$$\begin{aligned}\mathscr{W} &= 2\left(U(X^*) - X^*\right) + \eta \int_{\hat{\gamma}(R^d)}^{\bar{\gamma}} \left(\gamma - \frac{1}{\beta} \right) d\gamma + \lambda_1 \left[\beta \left(u(q_1) - q_1 \right) - \lambda_1^e \delta_1 \right] \\ &+ \lambda_2 \Bigg\{ \lambda_2^d \left[\beta \left(u\left(q_2^d(R^d) \right) - q_2^d(R^d) \right) - f \right] + \lambda_2^{mix} \left[\beta \left(u\left(q_2^d(R^d) \right) - q_2^d(R^d) \right) - f - \delta_1 \right] \\ &+ (\lambda_2^e - \lambda_2^{mix}) \left[\beta \left(u(q_2^e) - q_2^e \right) - \delta_1 \right] \Bigg\}. \end{aligned} \tag{28}$$

In Section 5.1, I discuss properties of the welfare function when the CBDC holding limit is nonbinding, and in Section 5.2 when binding.

5.1 Non-binding CBDC Holding Limit

As $\bar{e} \geq e(R^e)$, the central bank can only affect equilibrium allocations and thereby welfare through varying R^e and δ. Note here that the central bank can make the CBDC more (less) costly to adopt by setting a higher (lower) service charge fee, δ_2. The CBDC user cost—δ_1—cannot be directly adjusted by the central bank, as it depends on user-specific fixed effects. On one hand, a higher R^e and a lower δ will make the CBDC more likely to be adopted, as the two conditions from Section 4.1—$R^e > R^e(R^c, \delta)$, $R^e > R^e(R_0^d, \delta)$—are more likely to be satisfied. On the other hand, given that the CBDC is used in both Type 1 and Type 2 meetingss, a higher interest payment on the CBDC will lead to more efficient DM exchange across the board (i.e., q_1, q_2^d, q_2^e increase) while at the same time crowding out bank deposits and investment; a lower CBDC adoption cost results in higher DM trade only in Type 2 meetings where only deposits are used (i.e., only q_2^d increases) while at the same time disintermediating private banks. Therefore, if an increase in R^e and a decrease in δ cause the same extent of bank disintermediation, the former policy leads to a higher welfare. This result provides a justification for the CBDC to be interest-bearing and is formally presented in the following proposition.

Proposition 8. *When $\bar{e} \geq e(R^e)$, if an increase in R^e ($\Delta R^e > 0$) and a decrease in δ ($\Delta\delta < 0$) produce the same change in R^d (ΔR^d), then the welfare gain from the increase in R^e exceeds that from the decrease in δ:*

$$\Delta\mathscr{W}(\Delta R^e) > \Delta\mathscr{W}(\Delta\delta).$$

In addition, changes in R^e affect aggregate welfare in a continuous way except when $R^e = R^e(R^c, \delta)$. When $R^e = R^e(R^c, \delta)$, buyers in Type 1 meetings are indifferent between the cash-only portfolio and the CBDC-only portfolio, however, welfare is different for the two types of equilibria when $\lambda_1^c = 1$, $\lambda_1^e = 0$, and when $\lambda_1^c = 0$, $\lambda_1^e = 1$. The reason is because households are indifferent between the two portfolios when they make their portfolio decisions in the CM and at this point they take the lump-sum transfer as given. Although compared with holding cash, holding the CBDC promotes more efficient DM exchange, the increase in DM consumption is exactly offset by the increase in CM working hours, as holding the CBDC entails a fixed cost, and this creates households' indifference between cash and the CBDC. Yet, the lump-sum transfer from the central bank is different when $\lambda_1^c = 1$, $\lambda_1^e = 0$, and when $\lambda_1^c = 0$, $\lambda_1^e = 1$. This is because when the aggregate demand—hence the supply—for different types of central bank money changes, central bank's seigniorage revenue changes, and hence the lump-sum transfer to households. Difference in the lump-sum transfer in the two types of equilibria implies that difference in DM consumption cannot be perfectly offset by difference in CM working hours. Thus, welfare is different by the difference in lump-sum transfer. Let T^c and T^e denote the lump-sum transfers when $\lambda_1^c = 1$, $\lambda_1^e = 0$, and when $\lambda_1^c = 0$, $\lambda_1^e = 1$ at $R^e = R^e(R^c, \delta)$ respectively. The following proposition formalizes this discussion.

Proposition 9. *When $\bar{e} \geq e(R^e)$, at $R^e = R^e(R^c, \delta)$, $W_1^c = W_1^e$, but $\mathscr{W}(\lambda_1^c = 0, \lambda_1^e = 1) - \mathscr{W}(\lambda_1^c = 1, \lambda_1^e = 0) = T^e - T^c = \lambda_1\left((1-R^e)\,e\,(R^e) - (1-R^c)\,c\,(R^c) + \delta_2\right)$.*

5.2 Binding CBDC Holding Limit

In this section, I focus on the welfare impact of the CBDC holding limit—$\bar{e}$. On one hand, a higher $\bar{e}$ increases the likelihood of CBDC adoption; on the other hand, after the CBDC is adopted in both types of meeting, a higher $\bar{e}$ may or may not increase q_1—depending on whether the CBDC in Type 1 meetings is adopted alongside cash or as a standalone means of payment—but certainly increases q_2 while at the same time leading to a higher R^d, making it harder and more expensive for private banks to issue deposits and invest. Therefore, the central bank faces a tradeoff when choosing a higher or lower value of $\bar{e}$. Next, I will discuss how aggregate welfare behaves at two critical cutoffs of $\bar{e}$: $\bar{e}_1$ and $\bar{e}'_3$.

First, when $R^e > R^c$, and $0 < \delta < \hat{\delta}$, at $\bar{e}_1$, buyers in Type 1 meetings are indifferent between holding only cash and holding a mix of cash and the CBDC, as

$$W_1^{mix}|_{\bar{e}=\bar{e}_1} = W_1^c,$$

where the CBDC adoption cost δ is exactly offset by savings in labor to acquire money and the lump-sum transfer received is the same. However, for the two types of equilibria when $\lambda_1^c = \lambda_1^e = 1$ and when $\lambda_1^c = 1, \lambda_1^e = 0$, the quantity and composition of real money balances differ. Consequently, the central bank's seigniorage revenue, and thus the lump-sum transfer to buyers, also differ. Let $\tilde{c}(\bar{e}_1)$ denote the amount of cash held in the mixed portfolio with $\bar{e}_1$ units of the CBDC. We have $\tilde{c}(\bar{e}_1) + \bar{e}_1 < c(R^c)$. Denote T^{mix} as the lump-sum transfer when $\lambda_1^c = \lambda_1^e = 1$. Proposition 10 shows how welfare changes when buyers in Type 1 meetings switch from holding only cash to holding both cash and the CBDC at $\bar{e}_1$. As it turns out, aggregate welfare changes by the amount of user costs incurred for adopting the CBDC.

Proposition 10. *When $\bar{e} < e(R^e)$, $R^e > R^c$, and $0 < \delta < \hat{\delta}$, at $\bar{e}_1$, $W_1^c = W_1^{mix}$, but $\mathscr{W}(\lambda_1^c = \lambda_1^e = 1) - \mathscr{W}(\lambda_1^c = 1, \lambda_1^e = 0) = T^{mix} - T^c = -\lambda_1 \delta_1$.*

Second, when $R^e > R_0^d$, and $0 < f < \hat{f}$, at $\bar{e}'_3$, households in Type 2 meetings are indifferent among the three types of portfolios: $(0, d(R^d), 0), (0, \tilde{d}, \bar{e}'_3), and (0, 0, \bar{e}'_3)$. However, aggregate welfare is different when $\lambda_2^{mix} = \lambda_2^e > 0$ where the CBDC is held alongside deposits at the intensive margin and when $\lambda_2^{mix} = 0$ where the CBDC is held alongside deposits at the extensive margin. As the equilibrium switches from the former to the latter, since deposits no longer coexist with the CBDC at the intensive margin, λ_2^d increases to ensure that aggregate demand for deposits remain unchanged. As a consequence, the fraction of buyers holding the CBDC decreases, meaning that real balances of the CBDC drop. Thus, the lump-sum transfer from the central bank differs across the two equilibria. Denote λ_2^{md} as the fraction of buyers switching from holding the mixed portfolio to holding only deposits. Proposition 11 shows how welfare responds when the equilibrium changes from $\lambda_2^{mix} = \lambda_2^e > 0$ to $\lambda_2^{mix} = 0$ at $\bar{e}'_3$.

Proposition 11. *When $\bar{e} < e(R^e)$, $R^e > R_0^d$, and $0 < f < \hat{f}$, at $\bar{e}_3'$, $W_2^d(R^d) = W_2^{\bar{e}} = W_2^{mix}(R^d)$, but $\mathscr{W}(\lambda_2^{mix} = 0) - \mathscr{W}(\lambda_2^{mix} = \lambda_2^e > 0) = -\lambda_2 \lambda_2^{md}((1 - R^e)\bar{e} + \delta_2)$.*

6 Quantitative Analysis

Theoretically, a CBDC can coexist at the intensive margin with both cash and deposits, and improve welfare, if the holding limit is neither too small nor too large. It remains to be answered empirically, however, how large this range is and how significant the impact of a CBDC can be. To answer these questions, I use data on the United States economy to first calibrate the model parameters without a CBDC, and then conduct a counterfactual analysis to study the effects of introducing a CBDC.

6.1 Calibration

I modify the model in two aspects. First, I introduce search frictions in the DM so the probability that a buyer meets a seller is α. Second, the terms of trade in the DM is determined by the Kalai bargaining, with bargaining power $1 - \theta$ to the seller. This captures the fact that sellers enjoy substantial markups. The two modifications do not affect the results qualitatively, but can matter quantitatively.

Consider an annual model. CM and DM utility functions take the logarithmic form $U(X) = Alog(X)$ and CRRA form $u(q) = \frac{(q+B)^{1-\sigma} - B^{1-\sigma}}{1-\sigma}$ respectively. The parameter B is set to 0.001, which ensures that $u(0) = 0$. The measure of banks, η, is normalized to one without loss of generality. There are 11 parameters to calibrate: $(\beta, \lambda_1, \lambda_2, \mu, \varepsilon, \alpha, A, \sigma, \theta, \bar{\gamma}, f)$. There is a direct match for the first five parameters: $(\beta, \lambda_1, \lambda_2, \mu, \varepsilon)$. The rest has to be calibrated internally. Parameters are primarily calibrated using data from 1987-2008; for data not available in this range, observations from alternative years are used.

The data used in my calibration exercise come from seven sources: (1) data from the Survey of Consumer Payment Choice (SCPC) and the Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; (2) data from the survey of costs of payments in Denmark; (3) haircuts applied to high-quality liquid assets (HQLA) in the Basel Framework; (4) new M1 series from Lucas and Nicolini (2015); (5) call report data from the Federal Financial Institutions Examination Council; (6) several time series on macro variables, reserves, and retail value added from Federal Reserve Economic Data (FRED). The calibration of several key parameters is discussed briefly below.

I obtain the fractions of Type 1 and Type 2 meetings, λ_1 and λ_2, from the SCPC (Greene and Stavins 2018) and the DCPC (Premo, 2018). From the SCPC, I estimate the fraction of transactions conducted online, and the DCPC provides information on the perceived percentage of transactions that reject cash or debit/credit card payments at the point of sale. I use data from surveys in 2016, and the numbers are similar in 2015 and 2017. In the context of my model, λ_1 is the fraction of transactions that accept only cash, which are point-of-sale transactions where cards are not accepted, and λ_2 is the fraction that accepts only deposits, including both online transactions and point-of-sale transactions where cash is not accepted. The fraction of online transactions is estimated at 25.37% from the SCPC. According to the DCPC, at the point of sale, 6.14% of transactions do not accept debit or credit cards, while 1.44% do not accept cash. Hence, the fraction of cash-only transactions is $6.14\% \times (1 - 25.37\%) = 4.58\%$, and the fraction of deposit-only transactions is $25.37\% + 1.44\% \times (1 - 25.37\%) = 26.44\%$. With normalization, $\lambda_1 = \frac{4.58}{4.58+26.44} = 14.76\%$, and $\lambda_2 \equiv 1 - \lambda_1 = 85.24\%$.

Next, I calibrate the DM trading probability (α), the two utility function parameters (A, σ), and the bargaining power (θ) jointly to match the money demand curve, the retail value-added share of output, and a 20% retail markup. I use the new M1 series from Lucas and Nicolini (2015) for the aggregate money demand. The deposit rate used to calculate the money demand in the model is obtained from the interest expenses and balances on transactional deposits series in the call report data, which was also used in Drechsler et al. (2017). The retail value-added in the model is the value of payment in the DM. I use data from 1987 to 2008 for this calibration. The retail value-added share of output takes the average from 2005 to 2008, since data on retail value-added is not available prior to 2005. It is reasonable to assume that this share does not fluctuate significantly over the 1987–2008 period, as it has remained stable from 2005 to 2024.

To calibrate f, the resource cost of using deposits, I construct an auxiliary variable, $\hat{f}$, which is the per period service charge. There is no service charge on deposit account in the model, as I assume perfect competition, however, in the data there is, as it is not perfect competition. I set $\hat{f}$ to match service charges per dollar on deposit accounts, using series from the call report data in 1987-2008. Service charges on deposit accounts, among other components, include periodic maintenance fees. Deposit accounts consist of both transaction and non-transaction accounts in the data, where only the former one is discussed in the paper. However, in the model, it is the relative size of the fixed costs of private bank deposits and CBDC that affects households' portfolio decisions; therefore, this caveat does not undermine the quantitative results. Then, f is chosen to match the ratio between the time-related costs of checking account statements and the annual subscription fees for international debit cards in Denmark in 2009. The ratio is $\frac{19.6}{45} = 0.44$. Hence, $f = 0.44 \times \hat{f}$.

Table 1 lists all calibrated parameter values with their targets. Figure 7 presents the money demand curve predicted by the model against its empirical data counterpart spanning 1987 to 2008.

Table 1: Calibration Results

Parameters	Notation	Value	Calibration Targets
Calibrated externally			
Discount factor	β	0.96	Standard in literature
Reserve requirement	μ	2.4%	1987-2008 avg. required reserves/trans. balances
Bank's pledgeability constraint	ε	0.85	Haircut applied to level 2A assets in Basel III
Frac. of Type 2 meetings	λ_1	14.76%	SCPC2016, DCPC2016
Frac. of Type 1 meetings	λ_2	85.24%	SCPC2016, DCPC2016
Calibrated internally			
Prob. of DM trading	α	0.51	Retail value-added 2005-08
Coeff. on CM consumption	A	3.57	Money demand 1987-2008
Curv. of DM consumption	σ	0.42	Money demand 1987-2008
Buyer's bargaining power	θ	0.76	Retailer markup 20%
Upper bound of bank's productivity	$\bar{\gamma}$	1.87	1987-2008 avg. interest rate on transactional deposits
Resource cost of deposits	f	0.0024	Resource cost/subscription fee 2009
Account fee of deposits	$\hat{f}$	0.0054	1987-2008 avg. service charges per dollar deposit

6.2 Effects of a CBDC with Holding Limit

I now introduce a CBDC that pays twice the net nominal interest rate of deposits, has the same user cost as deposits with no service charge, and conduct counterfactual analysis. I am particularly interested in how the CBDC affects equilibrium allocations, investment, and welfare with different holding limits. Figure 8 shows the results. In all figures, the horizontal axis represents the CBDC holding limit in real terms, which is bounded above by $e(R^e)$. The first row displays the quantity of DM trade in Type 1 and Type 2 meetings, and the fractions of households in Type 2 meetings holding only deposits, the mixture, and only the CBDC. The second row shows the amount of deposits in the mixed portfolio, the

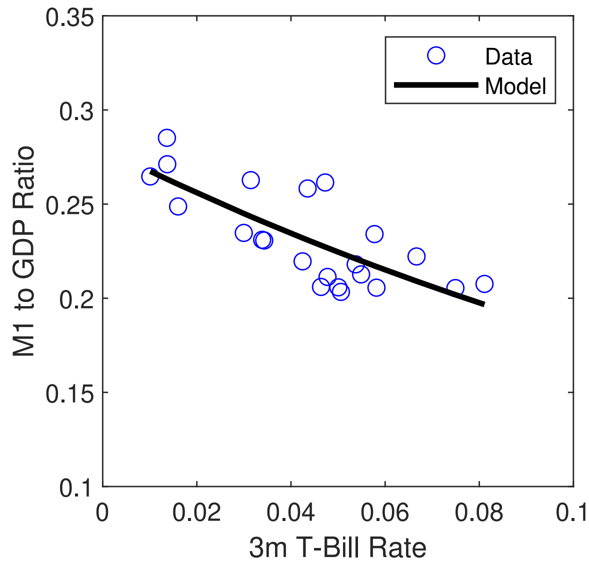

Figure 7: Money Demand under Calibrated Parameters.

equilibrium gross real deposit rate, as well as aggregate welfare.

First note that if the CBDC holding limit $\bar{e}$ is too tight—lower than 19% of $e(R^e)$ under this particular design—then the CBDC will not be adopted; this is reflected in the flat areas of the figures.

As the holding limit increases, the CBDC will first be adopted in Type 1 meetings, then in Type 2 meetings, with households' portfolio choices evolving as shown in Figure 6. In Type 1 meetings, if $\bar{e}$ is between 19% and 93% of the optimal amount of the CBDC held without distortion, buyers in Type 1 meetings accumulate both cash and the CBDC, and hence the CBDC coexists at the intensive margin with cash. Should $\bar{e}$ exceed this range, cash would fall out of use. We next move to Type 2 meetings. If $\bar{e}$ is between 37% and 82% of $e(R^e)$, then the CBDC coexits with deposits both at the intensive and extensive margin, as some use only deposits and others use both. The amount of deposits held in the mixed portfolio $\tilde{d}$ decreases as $\bar{e}$ increases. If $\bar{e}$ surpasses this range, those holding the mixed portfolio will stop doing so. Some of them acquire only the CBDC, while others switch back to using deposits. As such, at $\bar{e} = 0.72$, there is a sudden increase in the fraction of deposit-only meetings.

Next, we focus on investment. If the deposit rate rises, fewer banks can issue deposits to finance their investment, and therefore, changes in the deposit rate correspond to changes in investment. On one hand, once the CBDC is adopted by buyers in Type 2 meetings, a larger holding limit always crowds out private bank deposits by introducing more competition. On the other hand, the rate at which the CBDC disintermediates banks is slower when the limit lies in an intermediate range where the mixed portfolio is used.

Lastly, we look at the welfare change relative to the equilibrium without a CBDC. Note that there are two welfare drops: one when the CBDC is adopted in Type 1 meetings; another when buyers in Type 2 meetings stop holding a mix of deposits and the CBDC. In the first case, welfare drops by $\lambda_1 \delta_1$—the utility cost for adopting the CBDC; in the second case, the amount of liquidity falls, and hence the drop in welfare.

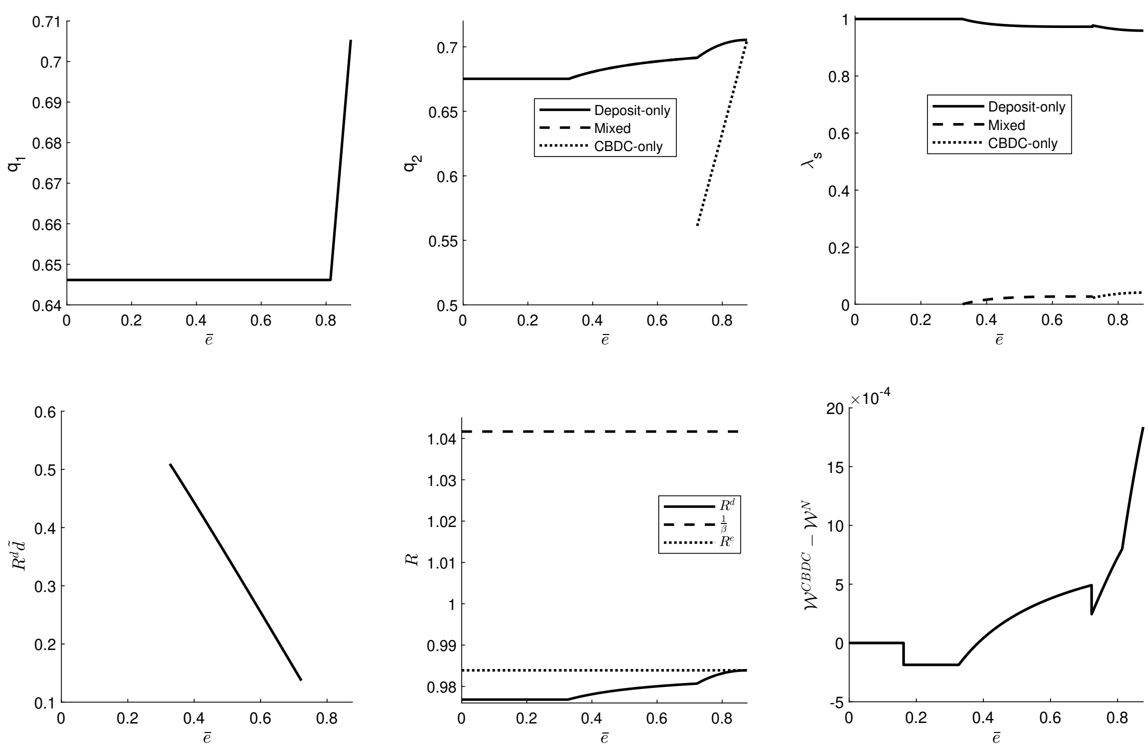

Figure 8: Effects of the Holding Limit on CBDC.

7 Balance-contingent Interest Rate

One motivation for the central bank to issue an interest-bearing CBDC is to decouple the opportunity cost of holding public money from the inflation rate. By paying a higher nominal interest rate on CBDC, the central bank can raise the real balances held by households. A relevant question is how the mechanism of interest payment on CBDC should be designed—whether in a linear or nonlinear fashion—and how this choice affects bank deposits in equilibrium.

Davoodalhosseini (2022) shows that the monetary policy of balance-contingent transfer, which is made available with CBDC, can achieve the efficient level of DM exchange, q^*. Furthermore, in this section, I show that whether the transfer scheme is linear or nonlinear in CBDC balances can have different implications on bank intermediation.

If the transfer scheme is linear, as in Section 4.1, a higher interest payment on CBDC induces more efficient DM exchange in Type 2 meetings while crowding out more bank deposits. These are the results shown in Proposition 4.

If, instead, the transfer scheme is nonlinear in the following fashion,

$$\hat{R^e} = \begin{cases} R^c & \text{for } e < \kappa \\ R^e > R^c & \text{for } e \geq \kappa, \end{cases}$$

where κ denotes the minimum CBDC balance eligible for interest payment, and $\kappa > e(R^e)$, then a higher minimum balance requirement κ can crowd in deposits in equilibrium while at the same time incentivizing households to hold more CBDC. The following proposition presents conditions under which the CBDC will be adopted in both Type 1 and Type 2 meetingss at the minimum balance requirement.

Proposition 12. *If* $R^e > \max\left\{R^e(R^c,\delta), R^e(R_0^d,\delta)\right\}$*,* $W_2^d(R_0^d) > W_1^c - \delta$*, and* $e(R^e) < \kappa \leq \min\left\{\underline{\kappa}_1(R^c,R^e,\delta), \underline{\kappa}_2(R_0^d,R^e,\delta,f)\right\}$*, then* $\vec{a}_1 = (\kappa,0,0)$*,* $\vec{a}_2 = \begin{cases} \left(0,d(R^d),0\right) & \text{w.p. } \lambda_2^d \in (0,1) \\ (0,0,\kappa) & \text{w.p. } \lambda_2^e \in (0,1) \end{cases}$*, such that* $\lambda_2^d + \lambda_2^e = 1$.

Proof. *See the Appendix.*

Under conditions in Proposition 12, as κ increases, $q_1^e, q_2^e, \lambda_2^d$ also increase, while R^d, q_2^d, λ_2^e decrease. Unlike under a linear transfer scheme, here R^d and q_2^d move in the opposite direction to $q^e_{s\in\{1,2\}}$.

Since linear and nonlinear balance-contingent transfers on CBDC affect bank intermediation differently, a key policy implication is that central banks can tailor the design of transfers to the prevailing level of investment efficiency. To promote DM exchange efficiency in CBDC meetings, central banks can adopt a nonlinear transfer scheme and adjust the minimum balance requirement when investment is inefficiently low, whereas a linear transfer scheme is preferable when investment is inefficiently high.

8 Conclusion

The introduction of a CBDC entails both benefits and potential costs. Aside from providing the public with a safe digital payment option, allowing central banks to conduct monetary policies effectively, and promoting financial inclusion, one of the frequently raised concerns of issuing a CBDC is the potential negative impact on private financial institutions. While policymakers in some countries have come up with the idea of imposing a cap on households' CBDC wallet so as to mitigate disruptions to private banks, the equilibrium implications of such a design remain unexplored. Can other tools, like the interest payment, do the same job? Can central banks incentivize households to hold more CBDC by subsidizing the use of a certain amount of CBDC while not undermining the deposit creation of private banks?

My analysis shows how a relatively standard framework of banking and payment within the New Monetarist tradition can shed light on these issues. The findings suggest that a holding limit on the CBDC can effectively mitigate disintermediation risks, and enables the CBDC to coexist with physical currency and bank deposits at the intensive margin.

The CBDC design proposed in this paper can inform the regulation of stablecoins. If stablecoins were to gain widespread use as both a means of payment and a store of value, they could crowd out bank deposits. This disintermediation risk can be mitigated by imposing caps on stablecoin holdings by households and firms.

A Proofs of Propositions and Lemmas

A.1 Proof of terms of trade

The sellers' participation constraint (2) must hold at equality since otherwise buyers could increase their surplus by slightly reducing the amount they offer to pay sellers so that sellers would still find the offer acceptable. In addition, the constraint, $0 \leq \vec{p}_j$, is never binding because autarky is never optimal. Thus, the terms of trade solves the following problem

$$\max_{q_j, \vec{p}_j} \quad u(q_j) - R \cdot \vec{p}_j \quad \text{s.t.} \quad -q_j + R \cdot \vec{p}_j = 0, \vec{p}_j \leq \vec{a}_j$$

$\Leftrightarrow \max_{q_j, \vec{p}_j} \quad u(q_j) - q_j \quad \text{s.t.} \quad \vec{p}_j \leq \vec{a}_j$. When $\vec{p}_j < \vec{a}_j$, $u'(q_j) - 1 = 0$ pins down q_j, which is q^*; when $\vec{p}_j = \vec{a}_j$, $q_j = R \cdot \vec{a}_j$.

A.2 Proof of Lemma 1

For buyers in Type 1 meetings to hold both cash and the CBDC, the following first-order conditions for cash and CBDC holdings will hold with equality only if $R^c = R^e$.

$$\lambda\left(R \cdot f_1(\vec{a})\right) \leq \frac{1}{\beta R^c} - 1, \tag{A.1}$$

$$\lambda\left(R \cdot f_1(\vec{a})\right) \leq \frac{1}{\beta R^e} - 1. \tag{A.2}$$

However, when $R^c = R^e \Rightarrow W_1^c > W_1^e$, as $\delta > 0$. So buyers will only hold cash.

A.3 Proof of Lemma 2

For buyers in Type 2 meetings to hold both deposits and the CBDC, the following first-order conditions for deposits and CBDC holdings will hold with equality only if $R^d = R^e$.

$$\lambda\left(R \cdot f_2(\vec{a})\right) \leq \frac{1}{\beta R^d} - 1, \tag{A.3}$$

$$\lambda\left(R \cdot f_2(\vec{a})\right) \leq \frac{1}{\beta R^e} - 1. \tag{A.4}$$

However, as $R^d = R^e$ and $f + \delta > \max\{f, \delta\}$, buyers obtain the same amount of DM surplus subject to the same cost of holding money when holding both deposits and the CBDC compared with when holding only one of them, while paying two fixed costs. So, buyers will choose to hold only one type of money.

A.4 Proof of Proposition 2

By setting $W_1^c = W_1^e \Leftrightarrow (\beta R^c - 1)c(R^c) + \beta(u(q_1^c) - q_1^c) + \Omega_1 = (\beta R^e - 1)e(R^e) + \beta(u(q_1^e) - q_1^e) - \delta + \Omega_1 \Leftrightarrow (\beta R^c - 1)c(R^c) + \beta(u(R^c c(R^c)) - R^c c(R^c)) = (\beta R^e - 1)e(R^e) + \beta(u(R^e e(R^e)) - R^e e(R^e)) - \delta \Rightarrow R^e(R^c, \delta)$.

A.5 Proof of Proposition 3

By setting $W_2^d(R_0^d) = W_2^e \Leftrightarrow R^e(R_0^d, \delta)$. When $R^e > R^e(R_0^d, \delta)$, the deposit rate R_0^d will increase to R_1^d until $W_2^d(R_1^d) = W_2^e$. The fraction of buyers holding deposits, $\lambda_2^d(R_1^d)$, is pinned down by the deposit market clearing condition: $\lambda_2^d = \frac{\eta(\bar{\gamma}-\hat{\gamma}(R_1^d))}{\lambda_2 d(R_1^d)}$. $\bar{\gamma} - \hat{\gamma}(R_1^d(R^e)) = 0 \Rightarrow \bar{\gamma}(R^e)$, and when $\bar{\gamma} > \bar{\gamma}(R^e)$, $\lambda_2^d > 0$.

A.6 Proof of Lemma 3

A.7 Proof of Lemma 4

When holding only $\bar{e}$ units of the CBDC, the FOC is $\lambda(R^e\bar{e}) < \frac{1}{\beta R^e} - 1 < \frac{1}{\beta R^c} - 1$, so it is optimal to top up with cash until $\lambda(R^e\bar{e} + R^c\tilde{c}) = \frac{1}{\beta R^c} - 1$.

A.8 Proof of Lemma 5

If $R^e > R^c$, then $\tilde{c} + \bar{e} = c(R^c) + (1 - \frac{R^e}{R^c})\bar{e} < c(R^c)$.

A.9 Proof of Lemma 6

1. First, when $0 < \delta < \hat{\delta}$, $\bar{e}_1 < \bar{e}_3$;

2. Second, when $\bar{e}_1 < \bar{e}_3$, $W_1^{\bar{e}_1} < W_1^{mix}|_{\bar{e}=\bar{e}_1} = W_1^c = W_1^{\bar{e}_2} = W_1^c < W_1^{mix}|_{\bar{e}=\bar{e}_3} = W_1^{\bar{e}_3}$. $W_1^{\bar{e}}$ is strictly increasing in $\bar{e}$, so $\bar{e}_1 < \bar{e}_2 < \bar{e}_3$.

A.10 Proof of Lemma 7

If $R^e > R^d$, then $\tilde{d} + \bar{e} = d(R^d) + (1 - \frac{R^e}{R^d})\bar{e} < d(R^d)$.

A.11 Proof of Proposition 5

1. When one of the following conditions is true, $W_1^c > \max\{W_1^{\bar{e}}, W_1^{mix}\}$:

   (i) $R^e \leq R^c$, or

   (ii) $R^e > R^c$, $0 < \delta < \hat{\delta}$, $\bar{e} < \bar{e}_1$, or

   (iii) $R^e > R^c$, $\delta \geq \hat{\delta}$, $\bar{e} < \bar{e}_2$;

2. when $R^e > R^c$, $\bar{e} > \max\{\bar{e}_2, \bar{e}_3\}$, $W_1^{\bar{e}} > \max\{W_1^c, W_1^{mix}\}$;

3. when $R^e > R^c$, $0 < \delta < \hat{\delta}$, and $\bar{e}_1 < \bar{e} < \bar{e}_3$, $W_1^{mix} > \max\{W_1^c, W_1^{\bar{e}}\}$.

A.12 Proof of Lemma 8

1. $W_2^d(R_0^d, f) = W_2^{\bar{e}'_1} \Rightarrow \hat{f}$ such that $\bar{e}'_1 < \bar{e}'_2$ if $f < \hat{f}$, and $\bar{e}'_1 \geq \bar{e}'_2$ if $f \geq \hat{f}$. And when $W_2^d(R_0^d, f = 0) > W_2^{\bar{e}'_1}$, $\hat{f} > 0$;

2. $f < \hat{f} \Rightarrow \bar{e}'_1 < \bar{e}'_2 \Rightarrow W_2^{\bar{e}'_1} < W_2^{\bar{e}'_2} = W_2^d(R_0^d) = W_2^{mix}(R_0^d(\bar{e}'_1))$. Note $W_2^{\bar{e}'_3} = W_2^{mix}(R^d(\bar{e}'_3))$. With Assumption A3, we thus have $\bar{e}'_1 < \bar{e}'_3 \Rightarrow R^d(\bar{e}'_3) > R_0^d(\bar{e}'_1) \Rightarrow W_2^d(R_0^d) = W_2^{\bar{e}'_2} < W_2^{\bar{e}'_3} = W_2^{mix}(R^d(\bar{e}'_3)) = W_2^d(R^d(\bar{e}'_3)) \Rightarrow \bar{e}'_2 < \bar{e}'_3$;

3. $W_2^{\bar{e}'_4} > W_2^{mix}(R^d(\bar{e}'_4))$ if $f > 0$; $W_2^{\bar{e}'_3} = W_2^{mix}(R^d(\bar{e}'_3))$. With Assumption A3, we thus have $\bar{e}'_3 < \bar{e}'_4$;

4. Taken together, when $0 < f < \hat{f}$, we have $\bar{e}'_1 < \bar{e}'_2 < \bar{e}'_3 < \bar{e}'_4$.

A.13 Proof of Proposition 6

1. When one of the following conditions is true, $W_2^d(R_0^d) > \max\{W_2^{\bar{e}}, W_2^{mix}(R^d)\}$:
   (i) $R^e \leq R_0^d$ and $\bar{e} \leq \bar{e}'_2$, or
   (ii) $R^e > R_0^d$ and $\bar{e} \leq \min\{\bar{e}'_1, \bar{e}'_2\}$;

2. when $R^e > R_0^d$, $\bar{e}'_1 < \bar{e} < \bar{e}'_3$, $W_2^{mix}(R^d) > \max\{W_2^{\bar{e}}, W_2^d(R_0^d)\}$, so the deposit rate will increase until $W_2^{mix}(R^d) = W_2^d(R^d)$, where households are indifferent between holding only deposits or the mix;

3. when one of the following conditions is true, $W_2^{\bar{e}} > \max\{W_2^d(R_0^d), W_2^{mix}(R^d)\}$, so the deposit rate will increase until $W_2^d(R^d) = W_2^{\bar{e}}$, where households are indifferent between holding only deposits or only the CBDC:
   (i) $R^e \leq R_0^d$ and $\bar{e} > \bar{e}'_2$, or
   (ii) $R^e > R_0^d$ and $\bar{e} > \max\{\bar{e}'_2, \bar{e}'_3\}$.

A.14 Proof of Proposition 7

When $R_0^d > R^c$, $0 < \bar{e}_1 < \bar{e}'_1$; when $R^c > \hat{R}^c$, $\bar{e}'_3 < \bar{e}_3$; when $R^e > R^c$, $\bar{e}_3 < e(R^e)$.

A.15 Proof of Proposition 9

First, from the aggregate welfare function (28), I have

$$\Delta\mathscr{W} = \lambda_1\left(\beta\Delta u - \beta\Delta q - \delta_1\right). \tag{A.5}$$

Substituting the following equilibrium conditions into (A.5), I have $\Delta\mathscr{W} = \lambda_1\left((1-R^e)\,e\left(R^e\right) - (1-R^c)\,c\left(R^c\right)\right.$

$$\begin{aligned}\beta\Delta u &= -c(R^c) + e(R^e) + \delta,\\ \beta\Delta q &= R^e e(R^e) - R^c c(R^c).\end{aligned}$$

Second, denote the price of money as ϕ_1 and ϕ_2, aggregate supply of cash and CBDC as C_1, E_1 and C_2, E_2 for the equilibrium when $\lambda_1^c = \lambda_1$, $\lambda_1^e = 0$, and when $\lambda_1^c = 0$, $\lambda_1^e = \lambda_1$ respectively. Then I have

$$\Delta T = \phi_2\left(C_2 + E_2 - C_{2,-1} - (1+i)E_{2,-1}\right) - \phi_1\left(C_1 + E_1 - C_{1,-1} - (1+i)E_{1,-1}\right) + \lambda_1\delta_2. \tag{A.6}$$

Substituting the following market-clearing conditions into (A.6), I obtain $\Delta T = \lambda_1\left((1-R^e)\,e\left(R^e\right) - (1-R^c)\,c\right.$

$$\begin{aligned}\phi_1 C_1 &= \lambda_1 c + \mu\lambda_2\lambda_2^d d,\\ \phi_2 C_2 &= \mu\lambda_2\lambda_2^d d,\\ \phi_1 E_1 &= \lambda_2\lambda_2^e e(R^e),\\ \phi_2 E_2 &= \left(\lambda_1 + \lambda_2\lambda_2^e\right) e(R^e).\end{aligned}$$

Hence, $\Delta\mathscr{W} = \Delta T$.

A.16 Proof of Proposition 10

As the amount of DM trade, portfolios in Type 2 meetings and investment remain unchanged, change in aggregate welfare solely comes from changes in hours worked by households in Type 1 meetings.

$$\Delta\mathscr{W} = -\Delta h = -\lambda_1\left(\tilde{c} + \bar{e} - c(R^c) + \delta\right) + T^{mix} - T^c. \tag{A.7}$$

For the equilibrium where households in Type 1 meetings use only cash, denote the price of money, and aggregate supply of cash and CBDC as ϕ^c, C and E; for the equilibrium where households in Type 1 meetings hold the mixed portfolio, denote the price of money, and aggregate supply of cash and CBDC as ϕ^{mix}, C^{mix} and E^{mix}. Then I have

$$\begin{aligned}
T^c &= \phi^c(C+E) - \phi^c(C_{-1}+E_{-1}),\\
T^{mix} &= \phi^{mix}\left(C^{mix}+E^{mix}\right) - \phi^{mix}\left(C^{mix}_{-1}+(1+i)E^{mix}_{-1}\right)+\lambda_1\delta_2.
\end{aligned}$$

Together with the following market-clearing conditions

$$\begin{aligned}
\phi^c C &= \lambda_1 c(R^c) + \mu\lambda_2(\lambda_2^d d + \lambda_2^{mix}\tilde{d}),\\
\phi^{mix}C^{mix} &= \lambda_1\tilde{c} + \mu\lambda_2(\lambda_2^d d + \lambda_2^{mix}\tilde{d}),\\
\phi E &= \lambda_2\lambda_2^e \bar{e},\\
\phi^{mix}E^{mix} &= (\lambda_1+\lambda_2\lambda_2^e)\,\bar{e},
\end{aligned}$$

and the relationship $c(R^c) = \tilde{c} + (1+i)\bar{e}$, I have

$$T^{mix} - T^c = \lambda_1(\tilde{c}+\bar{e}-c(R^c)+\delta_2). \tag{A.8}$$

Substituting (A.8) into (A.7), I arrive at $\Delta\mathscr{W} = -\lambda_1\delta_1$.

A.17 Proof of Proposition 11

First, from the aggregate welfare function (28), I have

$$\Delta\mathscr{W} = \lambda_2\left(\lambda_2^e\,(\beta\Delta u - \beta\Delta q + f) + \lambda_2^{md}\delta_1\right). \tag{A.9}$$

Substituting the following equilibrium conditions into (A.9), I have $\Delta\mathscr{W} = -\lambda_2\lambda_2^{md}\left((1-R^e)\,\bar{e}+\delta_2\right)$.

$$\begin{aligned}
-\beta\Delta u &= \tilde{d}+f,\\
-\beta\Delta q &= R^d\tilde{d},\\
\lambda_2^e\tilde{d} &= \lambda_2^{md}(d-\tilde{d}) = \lambda_2^{md}(\delta+\bar{e}),\\
R^d &= \frac{R^e\bar{e}}{\delta+\bar{e}}.
\end{aligned}$$

Second, denote the price of money and aggregate supply of CBDC as ϕ_1 and E_1 for the equilibrium where $\lambda_2^{mix} = \lambda_2^e > 0$, and as ϕ_2 and E_2 for the equilibrium where $\lambda_2^{mix} = 0$.

Then I have

$$\Delta T = \phi_2 \left(E_2 - (1+i) E_{2,-1} \right) - \phi_1 \left(E_1 - (1+i) E_{1,-1} \right) - \lambda_2 \lambda_2^{md} \delta_2. \tag{A.10}$$

Substituting the following market-clearing conditions into (A.10), I obtain $\Delta T = -\lambda_2 \lambda_2^{md} \left((1 - R^e) \bar{e} + \delta_2 \right)$.

$$\phi_1 E_1 = \left(\lambda_1 \lambda_1^e + \lambda_2 \lambda_2^{mix} \right) \bar{e},$$
$$\phi_2 E_2 = \left(\lambda_1 \lambda_1^e + \lambda_2 \lambda_2^e \right) \bar{e}.$$

Hence, $\Delta \mathscr{W} = \Delta T$.

A.18 Proof of Proposition 12

$W^{\kappa}_{s \in \{1,2\}} = (\beta R^e - 1)\kappa + \beta(u(q^e_s) - q^e_s) - \delta + \Omega_1$, which is strictly decreasing in κ as $\kappa > e(R^e)$. By setting $W^c_1 = W^{\kappa}_1 \Leftrightarrow \underline{\kappa}_1(R^c, R^e, \delta)$; $W^d_2(R^d_0) = W^{\kappa}_2 \Leftrightarrow \underline{\kappa}_2(R^d_0, R^e, \delta, f)$. So if $e(R^e) < \kappa \leq \min\left\{ \underline{\kappa}_1(R^c, R^e, \delta), \underline{\kappa}_2(R^d_0, R^e, \delta, f) \right\}$, $W^{\kappa}_{s \in \{1,2\}} > \max\left\{ W^c_1, W^d_2(R^d_0) \right\}$, and the CBDC will be adopted in both types of meetings.

References

Agur, I., Ari, A., and Dell'Ariccia, G. (2022). Designing central bank digital currencies. *Journal of Monetary Economics*, 125(C):62–79.

Andolfatto, D. (2021). Assessing the Impact of Central Bank Digital Currency on Private Banks. *The Economic Journal*, 131(634):525–540.

Bank of England and HM Treasury (2024). Response to the Bank of England and HM Treasury Consultation Paper – The Digital Pound: A New Form of Money for Households and Businesses?

Barrdear, J. and Kumhof, M. (2022). The macroeconomics of central bank digital currencies. *Journal of Economic Dynamics and Control*, 142(C).

Bidder, R., Jackson, T., and Rottner, M. (2025). CBDC and Banks: Disintermediating Fast and Slow. BIS Working Papers No. 1280.

Brunnermeier, M. K. and Niepelt, D. (2019). On the equivalence of private and public money. *Journal of Monetary Economics*, 106(C):27–41.

Chiu, J. and Davoodalhosseini, S. M. (2023). Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-Like Design. *Management Science*, 69(11):6708–6730.

Chiu, J., Davoodalhosseini, S. M., Jiang, J., and Zhu, Y. (2023). Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment. *Journal of Political Economy*, 131(5):1213–1248.

Cunliffe, J. (2023). The Digital Pound. Speech at UK Finance.

Davoodalhosseini, S. M. (2022). Central bank digital currency and monetary policy. *Journal of Economic Dynamics and Control*, 142(C).

Di Iorio, A., Kosse, A., and Mattei, I. (2024). Embracing Diversity, Advancing Together – Results of the 2023 BIS Survey on Central Bank Digital Currencies and Crypto. Bank for International Settlements Papers No. 147.

Dong, M. and Xiao, S. (2021). Central Bank Digital Currency: A Corporate Finance Perspective. Mimeo.

Drechsler, I., Savov, A., and Schnabl, P. (2017). The deposits channel of monetary policy*. *The Quarterly Journal of Economics*, 132(4):1819–1876.

Greene, C. and Stavins, J. (2018). The 2016 and 2017 Surveys of Consumer Payment Choice: Summary Results. Federal Reserve Bank of Atlanta Discussion Paper No. 18–3.

Gu, C., Mattesini, F., and Wright, R. (2016). Money and credit redux. *Econometrica*, 84(1):1–32.

Hua, J. and Zhu, Y. (2021). Monetary Policy Pass-Through with Central Bank Digital Currency. Staff Working Papers 21-10, Bank of Canada.

Keister, T. and Sanches, D. (2022). Should Central Banks Issue Digital Currency? *The Review of Economic Studies*, 90(1):404–431.

Koonprasert, T. T., Kanada, S., Tsuda, N., and Reshidi, E. (2024). Central bank digital currency adoption: Inclusive strategies for intermediaries and users. *Fintech Notes*, 2024(005):A001.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484.

Li, J., Usher, A., and Zhu, Y. (2024). Central Bank Digital Currency and Banking Choices. Staff Working Papers 24-4, Bank of Canada.

Lucas, R. E. and Nicolini, J. P. (2015). On the stability of money demand. *Journal of Monetary Economics*, 73:48–65.

Mu, C. (2022). Balancing Privacy and Security: Theory and Practice of the E-CNY's Controllable Anonymity. People's Bank of China.

Panetta, F. (2022). The Digital Euro and the Evolution of the Financial System. Introductory Statement at the Committee on Economic and Monetary Affairs of the European Parliament, Brussels.

Premo, J. L. (2018). Guide to the 2016 Diary of Consumer Payment Choice. Federal Reserve Bank of Boston: Consumer Payments Research Center.

Wang, Z. (2023). Money laundering and the privacy design of central bank digital currency. *Review of Economic Dynamics*, 51:604–632.

Williamson, S. (2022). Central Bank Digital Currency: Welfare and Policy Implications. *Journal of Political Economy*, 130(11):2829–2861.