Topic 3 - Dynamics

1 Newton's first law of motion

Newton's First law of motion: a body continues in its state of rest or uniform motion in a straight line unless acted upon by a resultant external force

1.1 Inertia

The inertia of a body is its reluctance to a change in motion

The mass m of a body is an intrinsic property which resists change in motion

1.2 Equilirbium

the conditions necessary for equilibrium of a body are

- resultant force acting on body is 0
- resultant torque on the body about any axis is 0

2 Newton's second law of motion and Momentum

Newton's second law of motion: rate of change of momentum of a body is proportional to the resultant force acting on it and occurs in the direction of the force

$$F_{net} \propto \frac{d(p)}{dt}$$

2.1 Momentum

Momentum of a body is defined as the product of its mass m and velocity v, ie

$$p = mv$$

2.2 Newton's second law

$$\begin{aligned} F_{net} &\propto \frac{d(p)}{dt} \\ F_{net} &= k\frac{d(p)}{dt}, \text{ and since k = 1 when quantities in S.I. units,} \\ F_{net} &= \frac{d(p)}{dt} = \frac{d(mv)}{dt} \\ &= m\frac{dv}{dt} + v\frac{dm}{dt} \end{aligned}$$

Hence

- when mass is constant, $F_{net} = m\frac{dv}{dt}$
- when velocity is constamt, $F_{net} = v\frac{dm}{dt}$

3 Newton's third law of motion

Newton's Third law of motion: if body A exerts a force on body B, then body B exerts an equal but opposite force on A

$$|F_{by\ A\ on\ B}| = |F_{by\ B\ on\ A}|$$
$$F_{AB} = -F_{BA}$$

4 Impulse and momentum change

Impulse is equal to the change in momentum, defined as the product of a force F acting on an object and the time Δt for which the force acts

$$F = \frac{\Delta p}{\Delta t}$$
$$\Delta p = F\Delta t$$

Impulse can be found as the area under the $F - t$ graph

5 Collisions and Conservation of momentum

By Newton's Third Law, when A collides with B,

$$F_{BA} = -F_{AB}$$

since time interval dt is the same, impulse on A is opposite to impulse on B

$$\int_{t_i}^{t_f} F_{BA} dt = -\int_{t_i}^{t_f} F_{AB} dt$$

since impulse = change in momentum

$$\Delta p_a = \Delta p_b$$
$$m_A v_A - m_A u_A = -(m_B v_B - m_B u_B)$$
$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

net initial momentum of bodies = net final momentum of bodies

The principle of conservation of linear momentum: when bodies in a system interact, total momentum of the system remains constant, provided no net external force acts on the system

$$\sum m_i u_i = \sum m_i v_i$$

Types of collision

- **Elastic collision** total KE conserved
- **Inelastic collision** total KE not conserved
- **Completely inelastic collision** particles have the same final velocity (particles stick together and move off together)

for elastic collision, by principle of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_1 v_2$$

and since that total KE remains constant

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

rearranging the expressions

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

dividing the equations to remove m_1 and m_2

$$\frac{u_1^2 - v_1^2}{u_1 - v_1} = \frac{u_2^2 - v2^2}{v_2 - u_2}$$

$$\frac{(u_1 + v_1)(u_1 - v_1)}{u_1 - v_1} = \frac{(v_2 + u_2)(v_2 - u_2)}{v_2 - u_2}$$

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1$$

relative speed of approach = relative speed of separation