Topic 15 - DC Circuits

1 Series and parallel arrangement

1.1 Equivalent resistance

It is possible to replace a **combination of resistors** in any given circuit with a single resistor without altering the p.d. and the current across the terminals of the combination. The resistance of the single resistor is called the **equivalent resistance** of the combination

1.2 Resistor in series

The current I through each resistor is the same

$$R = R_1 + R_2 + R_3 + ...R_n$$

1.3 Resistor in parallel

The pd.d V through each resistor is the same

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...\frac{1}{R_n}$$

1.4 Combination of E.M.F. cells in series

$$E_{tota} = E_1 + E_2$$

1.5 Combiantion of E.M.F. cells in parallel

For two cells with emf E_1

$$E_{total} = E_1$$

2 Potential divider

For a circuit with emf V_s and 2 external loads R_1 and R_2 and current I
The current is given by

$$I = \frac{V_s}{R_1 + R_2}$$

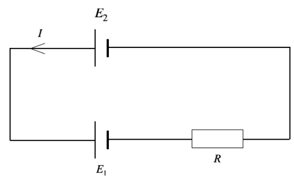
The potential difference across R_1 is thus

$$V_\circ = IR_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s$$

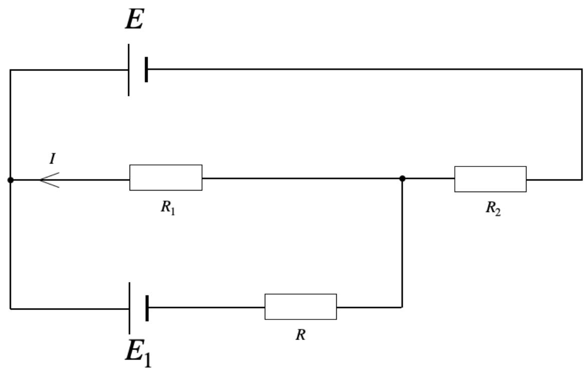
3 Potentiometer

3.1 Working principles of a potentiometer

For the circuit below, $I = 0$ when $E_1 = E_2$



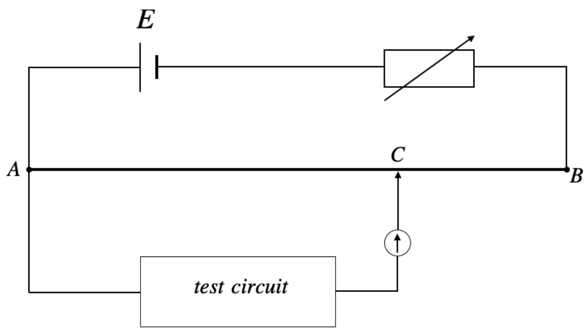
Replacing E_2 with a potential divider circuit,



By varying R_1 and R_2, no current flows through E_1 when the *p.d.* across R_1 is equal to E_1

3.2 Null point, balance point and balance length

By replacing R_1 and R_2 with a resistance wire AB and a sliding contact at point C, the galvanometer shows no reflection at **null point** or **balance point**. AC is the **balance length**.



Assuming

- Wire AB has uniform cross sectional area
- Potential difference across wire remains constant with time

For a **uniform wire** of length L,

$$R = \frac{\rho L}{A}$$

$$R \propto L$$

The potential difference aross AC is

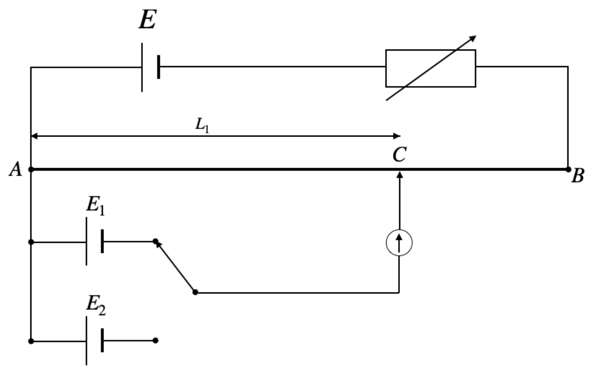
$$V_{AC} = \frac{R_{AC}}{R_{AB}} \times V_{AB}$$

$$V_{AC} = \frac{L_{AC}}{L_{AB}} \times V_{AB}$$

3.3 Voltmeter vs potentiometer

- for potentiometer, no errors are introduced by internal resistance of the cells (since no current flows through the cells at balance length
- real voltmeter has **finite resistance**, and hence draws a current from the cell, and lowers the terminal p.d. of the cell when connected
- for potentiometer, since **no current flows at balance point**, the potentiometer can be considered to be a voltmeter with **infinite resistance**

3.4 Application 1 - Comparison of e.m.fs



Since at balance length,

$$E_i = \frac{R_{AC}}{R_{AB}} \times V_{AB}$$

Since V_{AB} and R_{AB} constant,

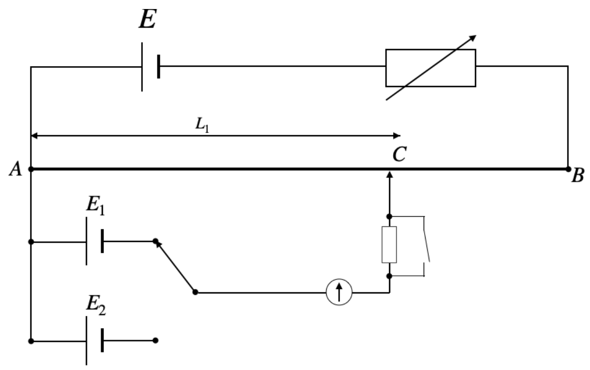
$$E_i = kL_i$$

hence

$$\frac{E_1}{E_2} = \frac{kL_1}{kL_2} = \frac{L_1}{L_2}$$

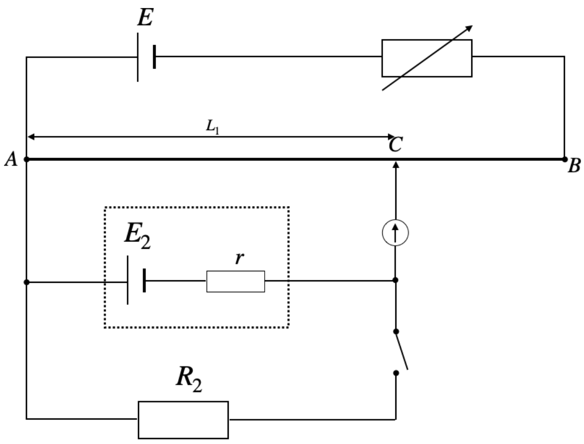
NOTE - shorting key as a protection for galvanometer

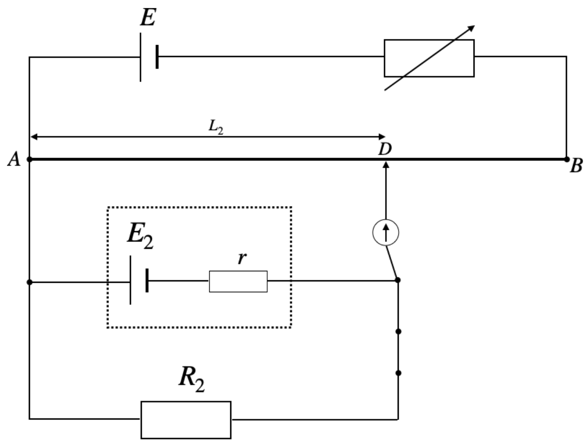
To ensure precision of potentiometer, the galvanometer used is very sensitive. The galvanometer should be protected from **high currents** that flows through in **off-balance** situations, by using a large resistance in series.



- Before approximate null point is reached, the shorting key is **left open**, to protect the galvanometer from large currents
- nearing the null point, the key is **closed**, to allow for full current to flow through, so that an accurate balance point can be found

3.5 Application 2 - Measuring e.m.f. and internal resistance of a cell





- when **switch is open**, E_2 can be found with L_1

$$E_2 = \frac{L_1}{L_{AB}} \times V_{AB}$$

- when switch is closed, there is a current flowing through E_2

$$\text{terminal p.d. across } E_2 = \text{terminal p.d. across } R_2 = \frac{L_2}{L_{AB}} \times V_{AB}$$

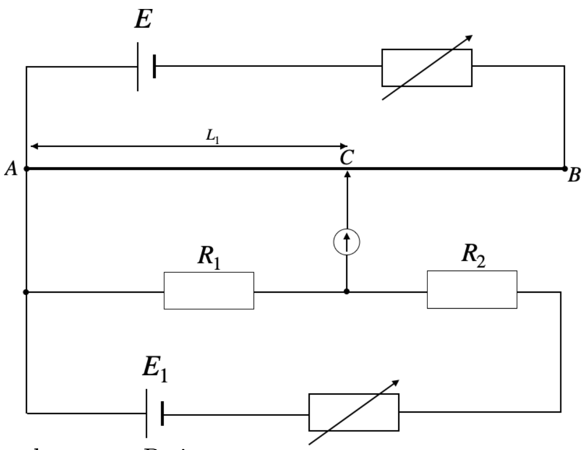
- the current flowing through R_2 is

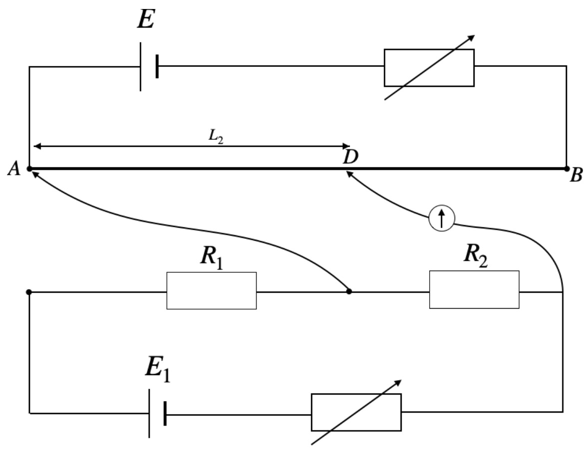
$$I = \frac{V}{R_2} = \frac{L_2}{L_{AB}} \times V_{AB} \times \frac{1}{R_2}$$

- the internal resistance can be found by

$$V = E_2 - Ir$$

3.6 Application 3 - Comparison of resistance of two resistors





p.d. across R_1 is,

$$V_1 = \frac{L_1}{L_{AB}} \times V_{AB}$$

p.d. across R_2 is

$$V_2 = \frac{L_2}{L_{AB}} \times V_{AB}$$

hence in the test circuit

$$IR_1 = \frac{L_1}{L_{AB}} \times V_{AB}$$

$$IR_R = \frac{L_R}{L_{AB}} \times V_{AB}$$

since current I is the same

$$\frac{R_1}{R_2} = \frac{L_1}{L_2}$$