Topic 12 - Superposition

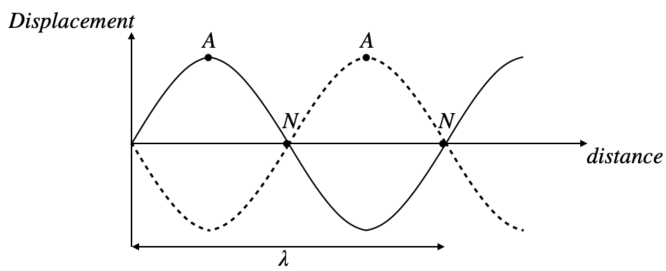
1 Principle of superposition

The **Principle of superposition** states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point

2 Stationary waves

A **Stationary wave** is the result of interference between two progressive waves of the same type, frequency, amplitude, and speed, travelling along the same line but in opposite directions

2.1 Properties of stationary wave



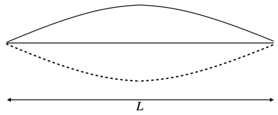
- The wave profile does not propagate
- Every particle of the wave oscillate about their respective equilibrium positions with the **same freqency** but **difference amplitude**
- The **antinode** is a point in a standing wave of maximum amplitude. The waves arrive **in phase**
- The **node** is a point of zero amplitude. The waves arrive **anti-phase** at the nodes
- Within two adjacent nodes, all particles oscillate **in phase** i.e. they reach their respective maxima / minima / equilibrium position at the same time
- particles in neighbouring segments vibrate π out of phase with each other
- distance between two adjacent nodes is $\frac{1}{2}\lambda$
- the **envelope** is a curve outlining the amplitudes of a standing wave

2.2 Comparing standing and progressive waves

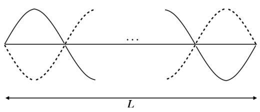
Property	Progressive wave	stationary wave
Waveform	propagates with the velocity of wave	does not propagate
Energy	transports energy	does not transport energy
Amplitude	all particles have the same amplitdue	Amplitude varies (from 0 to maximum)
Phase	All particles within one wave length have different phases	All particles between two adjacent nodes have the same phase. Particles in adjacent segments have $\phi = \pi$
Freqeuncy	All points vibrate in SHM with same frequency	All points vibrate in SHM with same frequency (except at nodes)
Wavelength	equals to the distance between consecutive points in phase	equal to the distance beteen two adjacent nodes or two adjacent anti-notes

2.3 Stationary waves in strings

when a string of length L that is fixed at both ends is plucked, standing waves of different frequencies are set up



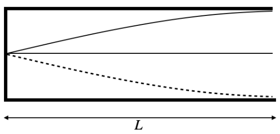
- mode of vibration: fundamental frequency
- wavelength
$$L = 1\left(\frac{\lambda_1}{2}\right)$$
$$\lambda_1 = 2L$$
- frequency
$$f = \frac{v}{2L}$$
- first harmonic



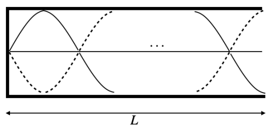
- mode of vibration: (n-1)th overtone
- wavelength
$$L = n\left(\frac{\lambda_n}{2}\right)$$
$$\lambda_n = \frac{2L}{n}$$
- frequency
$$f = n\frac{v}{2L}$$
- n^{th} harmonic

2.4 Stationary waves in air columns

A displacement node is formed at the closed end and displacement anti-node formed at the open end



- mode of vibration: fundamental frequency
- wavelength
$$L = 1\left(\frac{\lambda_1}{4}\right)$$
$$\lambda_1 = 4L$$
- frequency
$$f = \frac{v}{4L}$$
- first harmonic



- mode of vibration: (n-1)th overtone
- wavelength

$$L = (2n-1)\left(\frac{\lambda_{2n-1}}{4}\right)$$

$$\lambda_{2n-1} = \frac{4L}{2n-1}$$

- frequency

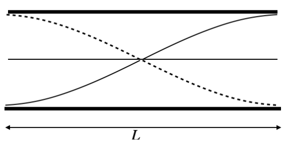
$$f = (2n-1)\frac{v}{4L}$$

- $(2n-1)^{th}$ harmonic

2.5 Stationary waves in pipes

Displacement anti-node at both ends

- displacement antinodes are **pressure nodes** i.e. least variation in pressure
- displacement nodes are **pressure anti-nodes**, largest variations in pressure



- mode of vibration: fundamental frequency
- wavelength

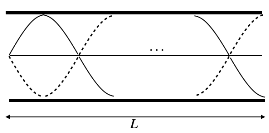
$$L = 1\left(\frac{\lambda_1}{2}\right)$$

$$\lambda_1 = 2L$$

- frequency

$$f = \frac{v}{2L}$$

- first harmonic



- mode of vibration: (n-1)th overtone
- wavelength

$$L = 2\left(\frac{\lambda_n}{2}\right)$$

$$\lambda_n = \frac{2L}{n}$$

- frequency

$$f = n\frac{v}{2L}$$

- n^{th} harmonic

2.6 End correction

the displacement antinodes at open ends of pipes are located slightly outside the pipe, hence when calculating wavelength, substitute

$$L = L_{actual} + c \ or \ L = L_{actual} + 2c$$

3 Diffraction

Diffraction is the bending or spreading of waves after passing through an aperture or round an obstacle

- Diffraction is pronouned when the wavelength of thewave i of the same order of magnitude as the width of the aperture or obstacle

4 Coherence and Interference

4.1 Coherence

Waves or sources are **coherent** if they have a constant phase difference

4.2 Interference

Interference is the **superposition** of two or more coherent waves to give a resultant wave whos resultant amplitdue is given by the principle of superposition

- constructive interference: when two waves arrive with a phase difference of zero
- destructive interference: when two waves arrive with a phase difference of πrad

4.3 Two source interference

For a two source interference to be observable

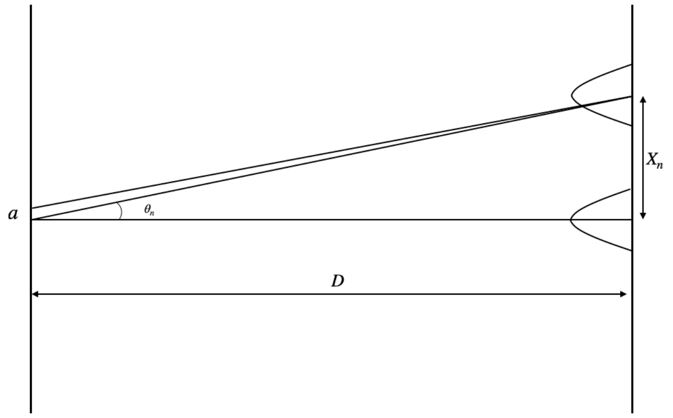
- sources must be coherent
- waves must have similar amplitude
- waves must overlap and be of the same type
- Transverse waves must be unploarised or polarised in the same plane

4.4 Path difference

Path difference is the difference in distance each wave travels from its source to the point where the two waves meet

	2 sources in phase	2 sources π out of phase
Constructive interference	$\Delta L = n\lambda$	$\Delta L = (n + \frac{1}{2})\lambda$
Destructive interference	$\Delta L = (n + \frac{1}{2})\lambda$	$\Delta L = n\lambda$

5 Double slit experiment



Assuming $a << D$, and hence light rays almost parallel, then

$$\text{path difference} = \Delta x = asin\theta_n$$

For constructive interference at the n^{th} bright fringe

$$asin\theta_n = n\lambda$$

$$sin\theta_n = \frac{n\lambda}{a}$$

Additionally,

$$tan\theta_n = \frac{X_n}{D}$$

Assuming $a >> lambda$, θ is small

$$sin\theta_n \cong tan\theta_n n$$

$$\frac{n\lambda}{a} \cong \frac{X_n}{D}$$

Hence constructive intereference takes place at

$$X_n = \frac{n\lambda D}{a}$$

Fringe separation is thus

$$x = X_n - X_{n-1} = \frac{\lambda D}{a}$$

6 Diffraction grating

For a diffraction grating of N lines per meter, slit separation is

$$d = \frac{1}{N}$$

Path difference is

$$\Delta x = dsin\theta_n$$

Assuming $d << D$, constructive interference occur

$$dsin\theta_n = n\lambda$$

since $\theta_n < 90°$

$$sin\theta_n < 1$$

$$\frac{n\lambda}{d} < 1$$

$$n < \frac{d}{\lambda}$$

7 Single slit interference

Huygens' Principle states that at any instant, all points on a wavefront could be recarded as secondary sources giving rise to their own outward spreading circular wavelets.

the envelope of wavefronts produced by each secondary source gives the new position of the wavefront

For a single slit diffraction with slit separation b , for $b << D$, position of the first minima is given by

$$bsin\theta = \lambda$$

$$sin\theta = \frac{\lambda}{b}$$

using arc length where

$$s = r\theta$$

for small θ the width, y of the central bright fringe is

$$y = D(2\theta) = \frac{2D\lambda}{b}$$

8 Rayleigh criterion

The **resolving power** of a single slit is the ability to distinguish between closely spaced objects

Rayleigh's criterion states that two objects are just resolved when the central maximum of one iamge falls on the first minimum of thee other image. The criterion is satisfied when the mimimum angular separation of the sources θ_{min} is

$$\theta_{min} = \frac{\lambda}{b}$$