Topic 1 - Measurement

1 S.I. Base quantities and units

Base quantities are fundamental physical quantities used to define other physical quantities.
The Systeme Internationale d'Unites is based on seven base quantities

base quantity	base unit	Symbol
time	second	s
legnth	metre	m
mass	kilogram	kg
current	ampere	A
temperate	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

2 Derived quantities and their units

Derived quantities are physical quantities formed by combining base quantities. Derived units are products and/or quotients of base units.
Derived quantities are formed from base quantities according to a defining equation.
For example

Derived quantity	Defining equation	Base units	Derived unit	Symbol
force	force = mass × acceleration	$kgms^{-2}$	newton	N

3 Homogeneity of Equations

Only quantities with the same base units can be added, subtracted or equated, i.e.

$$A(B + C) = DE$$

AB, AC, and DE have the same units

When each of the terms in an equation has the same base units, the equation is ***homogeneous*** or ***dimensionally consistent***

An equation can be dimensionally consistent but not physically correct due to

- wrong coefficients / signs
- missing / additional terms

4 Errors and Uncertaintines

4.1 Measuring instruments and their associated uncertainties

The precision of a instrument is determined by the number of ***significant figures*** in its measurements, which is in turn determined by ***the smallest scale division***

Physical Quantity	Instrument	Precision
Length	metre rule	0.1cm
	vernier calipers	0.01cm
	micrometer screw gauge	0.001cm
time	digital stopwatch	0.01s
mass	electronic balance	0.01g

4.2 Systematic and random errors

Systematic errors result in all readings being either always or above true value by a ***fixed amount***

- can be eliminated if source of error is known
- such as accounting for zero error

random errors result in readings being scattered about true value, with erros having equal probability of being positive or negative
Random errors can be ***reduced*** by

- repeating the measurement and taking average value item plotting a graph and drawing line of best fit

4.3 Accuracy vs Precision

Accuracy

- degree of closeness of readings/mean reading to actual value
- affected by systematic error

Precision

- degree of agreement between repeated measurements
- affected by random error

	Accuracy	Precision
Instrument	Calibration of instrument	smallest scale division
Measurements	Closeness of mean to true value	closeness of measurements to one another

5 Derived uncertainties

For 2 independent measurements of X and Y, X_1 Y_1, each with uncertainty ΔX and ΔY let variable Z be $X \times Y$, find the associated uncertainty ΔZ

5.1 the upper-lower bound method of uncertainty propagation

$$\begin{aligned}\Delta Z &= \frac{Z_{max} - Z_{min}}{2} \\ &= \frac{X_{max} \times Y_{max} - X_{min} \times Y_{min}}{2}\end{aligned}$$

5.2 calcualtion of uncertainties of derived quantities

$$\begin{array}{c|c} Q = aX \pm bY & \Delta Q = |a|\Delta X + |b|\Delta Y \\ Q = aX \times Y & \frac{\Delta Q}{Q} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y} \\ Q = a\frac{X}{Y} & \frac{\Delta Q}{Q} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y} \\ Q = aX^m \times^n Y & \frac{\Delta Q}{Q} = |m|\frac{\Delta X}{X} + |n|\frac{\Delta Y}{Y} \\ Q = a\frac{X^m}{Y^n} & \frac{\Delta Q}{Q} = |m|\frac{\Delta X}{X} + |n|\frac{\Delta Y}{Y} \end{array}$$

absolute uncertainty ΔQ are expressed to ***1sf***

fractional uncertainty $\frac{\Delta Q}{Q}$ are expressed to ***2sf***

percentage uncertainty of x is

$$\frac{\Delta Q}{Q} \times 100\%$$