# Topic 5 - Work, Energy, Power

# 1 Work

## 1.1 Work done by a constant force

For a constant force $F$ that acts at angle $\theta$ to the horizontal and displaces an object horizontally to the right over a displacement $s$, the work done by the constant force is given by

$$W = Fs\ cos\theta$$

**Work** done by a constant force is the product of the force and the displacement in the direction of the force

- Work is a **scalar** quantity, and its S.I. unit is the joule($J$)
- 1 $J$ is the work done by a force of 1 $N$ when an object is displaced by 1 $m$ in the direction of the force

## 1.2 Work done by a variable force

Work done by a variable force is equal to the **area under the force-displacement graph**

$$W = \int_{s_1}^{s_2} F ds$$

## 1.3 Work done by an external force on spring

By Hooke's Law, the external force needed to produce an extension or compression $x$ in a spring (that has not exceeded its limit of proportionality) is $F = kx$
wor done in stretching the spring by $x$ is the area under the force-extension graph is

$$W = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

## 1.4 Work done by a gas

For a system of gas in a cylinder with a frictionless piston, if the gas is heated such that it expands slowly at constant pressure, a force $F$ is applied on the piston by the gas molecules to expand against external pressure

The work done by the gas is the work done by force $F$ in displacing the piston of cross sectional area $A$ through a small distance $\Delta x$

$$W_{gas} = F\Delta x = pA\Delta x$$

And since $A\Delta x = \Delta V$

$$W_{gas} = p\Delta V$$

- when the gas expands, $\Delta V$ is positive, and hence work done by gas is positive
- when the gas contracts, $\Delta V$ is negative and work done by gas is negative (work is hence done **on** gas)

# 2 Energy

## 2.1 Forms of energy

| Name | Form of energy |
|---|---|
| Chemical potential energy | energy related to the structural arrangement of atoms or molecules in a substance |
| Nuclear energy | energy released from atomic nuclei |
| Electrical energy | energy possessed by charge carriers moving under the influence of a potential difference |
| Internal energy | the sum of microscopic KE (associated with random motion) and PE (associated with interatomic of intermolecular forces) |
| Gravitational PE | energy due to the position of a mass in a gravtitational field |
| Electrical PE | energy due to the position of a charge in an electric field of another charge / sytem of charges |
| Elastic PE | energy stored due to the stretching or compressing of an object |
| Kinetic energy | energy due to motion of a body |

## 2.2 Kinetic energy

In general, the KE of a body of mass $m$ moving with velocity $v$ can be expressed as

$$E_k = \frac{1}{2}mv^2$$

**Derivation of KE**
Consider a body of mass $m$ moving with initial velocity $u$ and accelerated by a constant force $F$. The body under goes constant acceleration $a$ to a final velocty $v$ over displacement $s$

Since force is in the direction of displacement

$$W = Fs = (ma)s$$

Since $v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a}$$

substituting into $W = (ma)s$

$$W = ma\left(\frac{v^2 - u^2}{2a}\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = \Delta E_k = E_{k,\ final} - E_{k,\ initial}$$

This is known as the **work-energy theorem**, where the net work done by all forces acting on a body is equal to the change in the KE of the body

## 2.3 Potential energy

Potential energy, usually expressed as $U$ is the energy due to the **position or shape** of an object. The calculation of PE ususally requires a reference point with is defined to have a PE of 0

### 2.4 Gravitational potential energy

The GPE of an object of mass $m$ and at height $h$ above the surface of the earth is

$$E_p = mgh$$

where g is the aceeleration of free fall near Earth's surface
**Derivation of GPE**
Consider an object being raised upwards at constant velocity, from height $h_1$ to height $h_2$

Since $v$ constant, $a = 0$, and the force required is $F = wg$
Work done by F in displacing the body upwards is

$$\begin{aligned} F &= Fs \\ &= mg(h_2 - h_1) \\ &= mgh_2 - mgh_1 \end{aligned}$$

### 2.5 Elastic potential energy

From earlier, work done in compressing or extending a spring is

$$W = \frac{1}{2}kx^2$$

The elastic potential energy stored is thus

$$U_E = \frac{1}{2}kx^2$$

### 2.6 Relationship between force and potential energy

For a field of force, the relationship bewteen $F$ and PE $U$ for one dimensional motion is given by

$$F = -\frac{dU}{dx}$$

- the magnitude of a force at point x is equal to the **gradient of the PE curve at x**
- the direction of the force is in the direction of decreasing potential energy

## 3 Energy conversion and conservation

### 3.1 Law of conservation of energy

The **law of conservation of energy** states that energy cannot be created or destroyed, it can only be converted from one form to another

### 3.2 Total mechanical energy and work done on a system

In a non-isolated system where work is done by an external force, $W_F$, by law of conservation of energy

$$(E_p + E_k)_{initial} + W_F = (E_p + E_k)_{final}$$

- the sum of potential and kinetic energy is aclled mechanical energy

$$\text{Mechanical energy} = E_p + E_k$$

- in an isolated system, mechanical energy is conserved
- in a non-isolated system, $W_F$ may be positive or negative

### 3.3 Efficiency

$$\text{Efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100$$

## 4 Power

Power is define as the rate of work or energy conversion wrt time

$$P = \frac{dW}{dt}$$

If total work $\Delta W$ is done over time interval $\Delta t$, then average power is

$$\langle P \rangle = \frac{\Delta W}{\Delta t}$$

### 4.1 Relationship between $P$, $F$, and $v$ for a constant force

If a constnat force $F$ is applied and does work by moving an object over displacement $s$ parallel to the force in time $t$, then the power can be found by

$$P = \frac{dW}{dt} = \frac{d(FS)}{dt} = F\frac{ds}{dt}$$

$$P = Fv$$

### 4.2 Wind turbines and related calculations

KE removed from wind is converted into electrical energy
Mass of air that passes through the area swept by the blades per second is

$$\text{Mass per second} = \frac{dm}{dt} = \frac{d(\rho v)}{dt} = \rho\frac{d(Ax)}{dt} = \rho A v$$

Loss of KE per second is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2$$