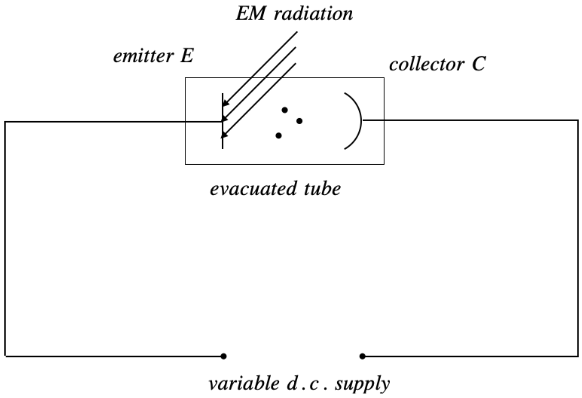
Topic 19 - Quantum Physics

1 Photoelectric effect

The **photoelectric effect** is a process in which electrons are emitted from a metal surface when an electromagnetic radiation of sufficiently high freqeuency is incident on the surface
A **photoelectron** is an eelctron emitted from the surface of a material due to the incident electromagnetic radiation

1.1 The photoelectric experiment



when collector C is sufficiently positive wrt E

$$V_{CE} = +ve$$

- all photoelectrons attracted to C
- rate of emission of photoelectrons = rate of electrons reaching C
- ammeter reads constant **saturated photocurrent,** i_0

$$\frac{dN_e}{dt} = \frac{i_0}{e}$$

when collector C is sufficiently negative wrt E

$$V_{CE} = -ve$$

- most energetic photoelectrons do not have sufficient energy to reach C
- photocurernt i is zero

$$KE_{max} = eV_s$$

Stopping potential is the negative potential of colelctor wrt emitter which prevents the most energetic photoelectrons from reaching the collector and hence resulting in **zero photocurrent**

1.2 Discrepencies with classical wave theory

By classical wave theory, the intensity of a wave is defined as the energy incident per unit area per unit time

$$I = \frac{E}{tA}$$

1. Absence of time lag

   By classical wave theory, electrons should absorb energy over a period of time before it gains enough energy.

2. Existence of threshold frequency

   Since Energy of a wave is dependednt on the square of its amplitude, photoelectrons should be emitted if radiation of sufficient intensity is used, and no threshold frequency should be observed

3. Max K.E. is independent of intensity but varies with frequency

   By classical wave theory, the stopping potential V_s should increase with intensity of light, since light of higher intensity should eject photoelectrons with greater KE.

1.3 Quantum theory of light

Light and other forms of EM radiation are emitted in discrete packets of energy called 'quantum', and the energy E in each quantum emitted is given by

$$E = hf$$

where $h = 6.63 \times 10^{-34} Js$

A **photon** is a quantum of electromagnetic energy.

By substituting $c = f\lambda$, the energy of a photon is

$$E = hf = f\frac{c}{\lambda}$$

where $c = 3.00 \times 10^{8} ms^{-1}$

Hence a monochromatic beam of light containing N photons has total energy

$$E_{total} = Nhf = Nh\frac{c}{\lambda}$$

in relation to the photoelectric experiment,

- a stream of photons bombard surface of metal
- free electrons near the surface could be struck by a photon and gains the **whole amount of energy**.
- if the gain in energy is sufficient, the electorns can leave the plate as a photoelectron
- the energy of a photon must be completely absorbed by a **single electron**, otherwise it is reflected or transmitted.

1.4 Einstein's photoelectric equation

the **work function energy** Φ of a metal is defined as the **minimum amount of energy** necessary for an electron to escape from the surface of a metal

Einstein's photoelectric equation states that the

$$\text{photon energy} = \text{work function energy} + \text{maximum KE of photoelectron}$$

$$hf = \Phi + \frac{1}{2}m_e v_{max}^2$$

1.5 Einstein's photoelectric equation applied to the photoelectric experiment

1. Max KE independent of intensity and varies linearly with frequency

   Photoelectrons with max KE come from surface of the metal. Those below the surface lose energy due to collision with atoms and are emitted with **lower KE**

   KE varies up to a maximum given by
   $$KE_{max} = hf - \Phi$$

2. Existance of threshold frequency f_0

   When KE is zero, no electrons can escape
   $$\Phi = hf_0$$
   Hence KE is only greater than zero for $f > f_0$

> **Threshold frequency** is the minimum frequency of EM radiation below which no emission of photoelectrons occur

3. Absence of time lag

   photoelectrons are emitted immediately after gaining energy from a photon. All of a photon's energy is transferred immediately upon collision. Hence emission has no time lag

1.6 Photoelectric equation applied to stopping potential V_s

For the most energetic photoelectrons

$$\text{decrease in KE} = \text{increase in EPE}$$

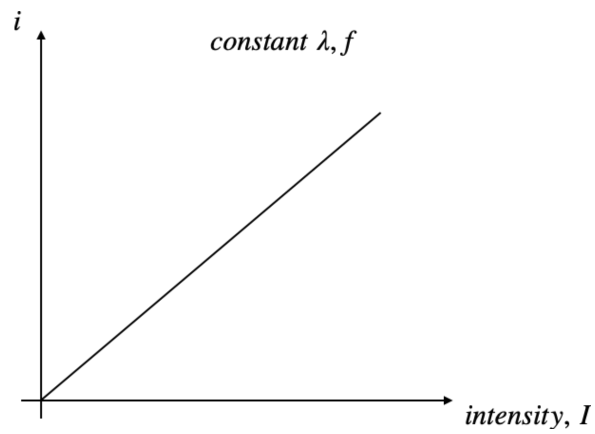
$$\frac{1}{2}m_e v_{max}^2 - 0 = eV_s$$
$$KE_{max} = eV_s$$

Rewriting Einstein's photoelectric equation

$$KE_{max} = hf - \Phi$$
$$eV_s = hf - \Phi = hf - hf_0$$
$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$

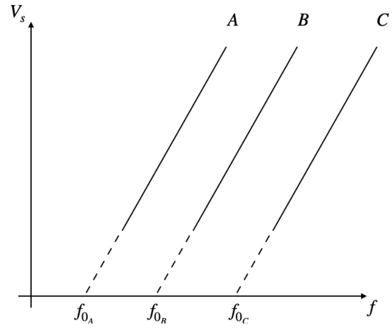
1.7 Associated graphical representations of photoelectric results

1.7.1 intensity - current graphs



$$\frac{dN_e}{dt} \propto \frac{dN_p}{dt}$$
$$intensity = \frac{P}{A} = \frac{E}{tA} = nhf$$
$$I \propto \frac{dN_p}{dt} \propto \frac{dN_e}{dt}$$

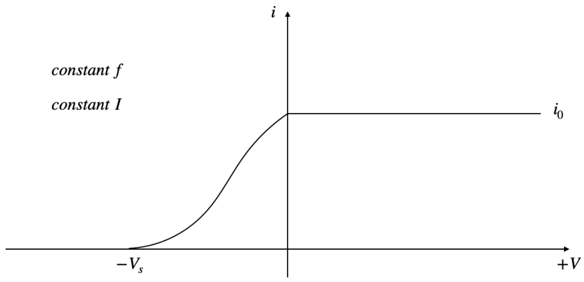
1.7.2 Stopping potential - frequency graphs



$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$

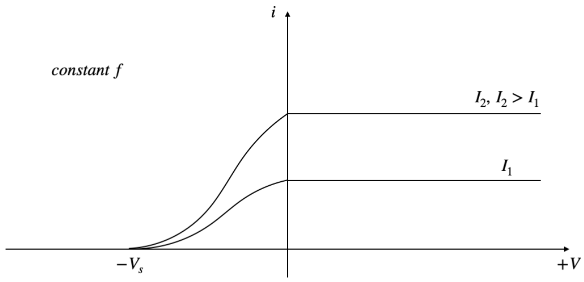
A similar graph of KE_{max} against f could be obtained

1.8 Potential difference - current graphs



- when V_{CE} is positive, all emitted electrons reach collector, hence **photcurrent, i is at maximum,** i_0
- As V_{CE} becomes more negative, more electrons are repelled from collector, hence i decreases
- i reaches 0 when $V_{CE} = V_s$, and **not even the most energetic photoelectrons** can reach collector. stopping potential is given by

$$eV_s = KE_{max}$$

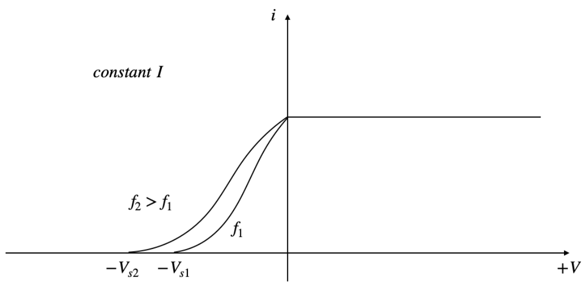


$$i_0 \propto I$$

Since

$$KE_{max} = eV_s = hf - \Phi$$

V_s constant



$$V_s \propto f$$

2 Wave-particle duality

The theory of wave-particle duality states that matter and waves have particle-like and wave-like properties
The **De Broglie wavelength** is the wavelength associated with wave-like properties of a particle

For a particle with momentum $\mathbf{p} = \mathbf{mv}$, the associated De Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

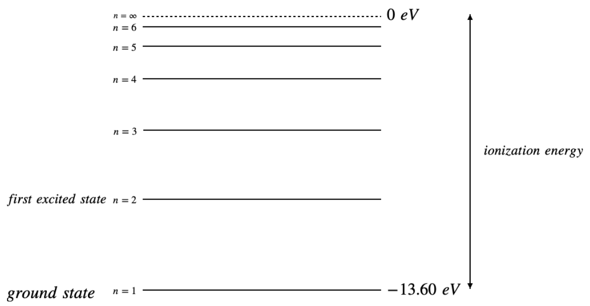
For EM radiation with wavelength λ, the radiation exhibits particle behaviour with associated momentum

$$p = \frac{h}{\lambda} = \frac{hf}{c}$$

	Evidence for wave-like properties	Evidence for particle-like properties
Light	Interference, diffraction	Photoelectric effect, emission spectra
Electrons	Diffraction	Electrons have mass and charge, and undergo collision

3 Energy levels and line spectra

3.1 Quantisation of energy levels and Bohr's atomic model



Bohr postulated that

- There are only certain allowed orbits in the atom
- Electrons are in stable state or **ground state** when they occupy orbits corresponding to **lowest** energy levels
- An atom radiates energy when elcetron transits from a more energetic state to a lower energy state. The energy is emitted as **one quantum**

$$E_i - E_f = hf$$

3.2 Excitation and de-excitation of atomss

Ionisation is the process of creating charged particles

Excitation is the process where atoms absorb energy without ionisation. Excitation occurs due to

1. **Particle collision**

   A high speed particle collides and imparts its energy to an electron. It can transfer **part or whole** of its energy.

   Energy transferred must be sufficient for electron to **transit to higher energy level** but **needs not match** the difference in energy levels

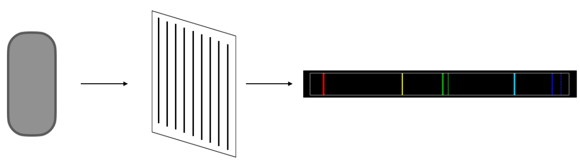
2. **Photons**

   If a photon with energy **exactly equal** to the energy difference between 2 levels in the atom collides with an orbital electron, the photon will be absorbed.

   If photon's energy is not exactly equal, it will not be absorbed

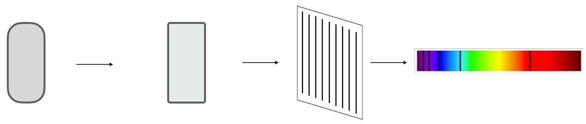
During **De-excitement**, excited orbital electrons return to a lower energy state and give off excess energy in the form of photons

3.3 Emission line spectrum



- A gas is placed in a discharge tube at **low pressure** and **high voltage**
- When gas is heated / bombarded with electrons, the electrons in gas atoms are excited to higher energy levels
- The excited electrons remain in higher energy state momentarily before de-exciting to lower energy.
- Upon de-exciting, photons are emitted with energy corresponding to the energy difference
- The gas starts to glow and light is emitted from discharge tube
- When light is exxamined through diffraction grating or spectrometer, a spectrum of distinct, well-defined lines is observed.

3.4 Absorption line spectrum

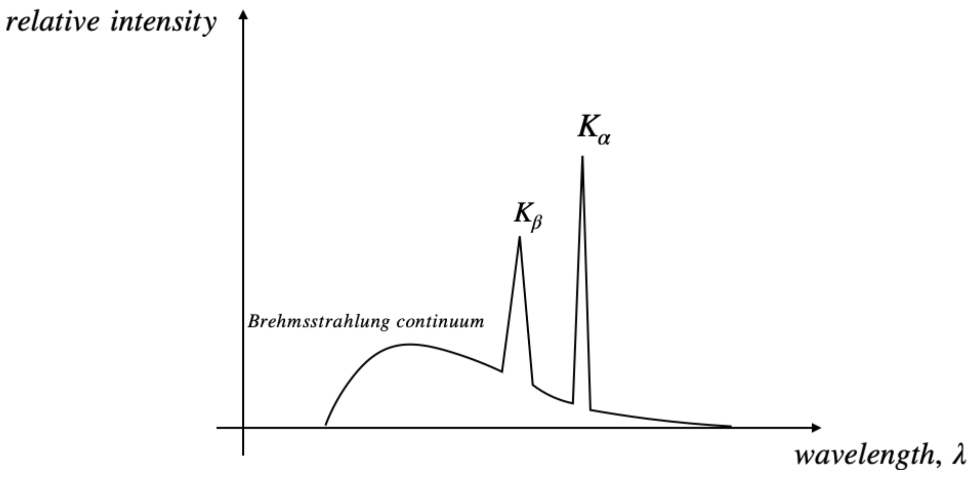


- When white light containing all visible frequencies passes through a **cool gass**, the atoms of cool gas absorb photons of certain frequencies to jump to a higher energy level.
- After excitation, the atoms eventually return to lower energy state by emitting the same photon it absorbed.
- Such emission occurs in all directions and therefore have **lower intensities**. When light emerging from discharge tube is passed through a dirffraction grating, light of these intensities **appear to be missing**
- a spectrum of dark lines on bright background is observed

4 X-ray spectrum

4.1 Production of X-ray

X-rays are produced when **high speed electrons are suddenly slowed down**, for example when a metal target is struck by electrons accelerated through a high potential difference



Note that

- Below a certain wavelength, no X-ray produced
- The existence of two distinct components, the continuous spectrum and the characteristic x-rays

4.2 The continuous spectrum

The broad spectrum, high energy EM radiation produced when highly energetic electrons are decelerated. It is characterized by a minimum wavelength λ_{min} corresponding to photon emitted by the loss of maximum energy of an electron

The **continuous spectrum** is produced due to EM radiation emitted by high speed electrons when they are slowed down to **different extent** due to interaction with nuclei of target atoms

In an extreme case, an electron with energy eV loses all its energy, hence its entire KE is transformed into the energy of the X-ray photon, given by

$$eV = \frac{1}{2}m_e v_e^2 = hf_{max} = h\frac{c}{\lambda_{min}}$$

Hence,

$$\lambda_{min} = \frac{hc}{eV}$$

4.3 The characteristic X-rays

The sharp spectrum, high energy EM radiation produced when electrons from the higher shell de-excite to a vacancy in the lower shell, created by highly energetic electrons

X-ray photons are produced when accelerated electron collides into an electron orbiting in the K-shell. If sufficient energy is transferred, the latter electron is ejected from the target atom.

The wavelength of characteristic X-rays produced depend on the difference in energy level

$$hf = \frac{hc}{\lambda} = E_n - E_1, \text{for } n \in 2,\, 3$$

When the vacancy in K-shell is filled by an electron from the L-shell, an X-ray photon of the K_α **characteristic X-ray** is emitted

When the vacancy in K-shell is filled by an electron from the M-shell, an X-ray photon of the K_β **characteristic X-ray** is emitted
Note that

- The intensity of K_α and K_β characteristic X-rays are high because the **rates of emission** are high
- K_α characteristic X-rays have higher intensity because
  - L-shells are nearer to K-shells
  - The vacancy in K-shells are filled by an electron from L-shell with greater probability

5 Heisenberg uncertainty principle

The **Heisenberg Uncertainty Principle** states that if a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x-component of momentum is made with uncertainty Δp_x, the product of the two uncertainties can never be smaller than $\frac{\bar{h}}{2}$

$$\Delta x \Delta p_x \geq \frac{\bar{h}}{2}$$

where $\bar{h} = \frac{h}{2\pi}$ is the reduced Planck constant.

It can be approximated that

$$\Delta x \Delta p_x \geq h$$

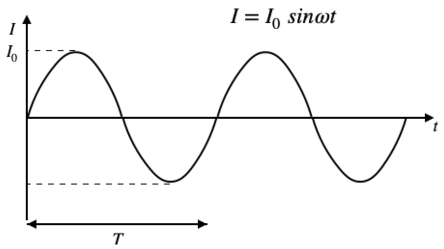
Topic 18 - Alternating Currents

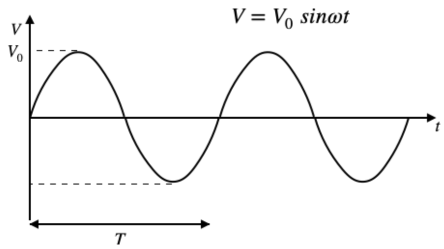
1 Characteristics of AC

An alternating current **varies periodically** with time in magnitude and direction.

Term	Definition
Period, T	Time taken for one complete cycle
Frequency, F	Number of complete cycles per unit time
Peak current, I_0	Amplitude of current

2 Sinusoidal a.c.





3 Power in a.c. circuit

For a simple resistive circuit with resistor R, and an a.c. source,

$$V_{ac} = I_{ac} R$$

$$P_{ac} = I_{ac} V_{ac} = I_{ac}^2 R = \frac{V_{ac}^2}{R}$$

- Energy dissipated is the area under the $P_{ac} - t$ graph
- Power at any moment is equal to the product of I and V, hence P varies periodically with time,
- $P_{min} = 0$ when $I = V = 0$
- $P_0 = I_0 V_0$

3.1 Mean power, $\langle P_{ac} \rangle$

$$\langle P_{ac} \rangle = \frac{\text{Total energy dissipated in time t}}{t}$$

$$\langle P_{ac} \rangle = \frac{\text{area under } P_{ac} - t \text{ graph in time } t}{t}$$

$$\langle P_{ac} \rangle = (I_{rms})^2 R$$

The r.m.s. value of an alternating current or voltage is the value of a **steady** direct current or voltage that would produce thermal energy at the **same rate** in a given resistor

$$\langle P \rangle = I_{rms}^2 R = \frac{V_{rms}^2}{R} = I_{rms} V_{rms}$$

The rms value can be found by

1. Squaring the instantaneous current I

2. finding **mean** value of I^2
3. Taking the **square root** of the mean value

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\langle I^2 \rangle}$$

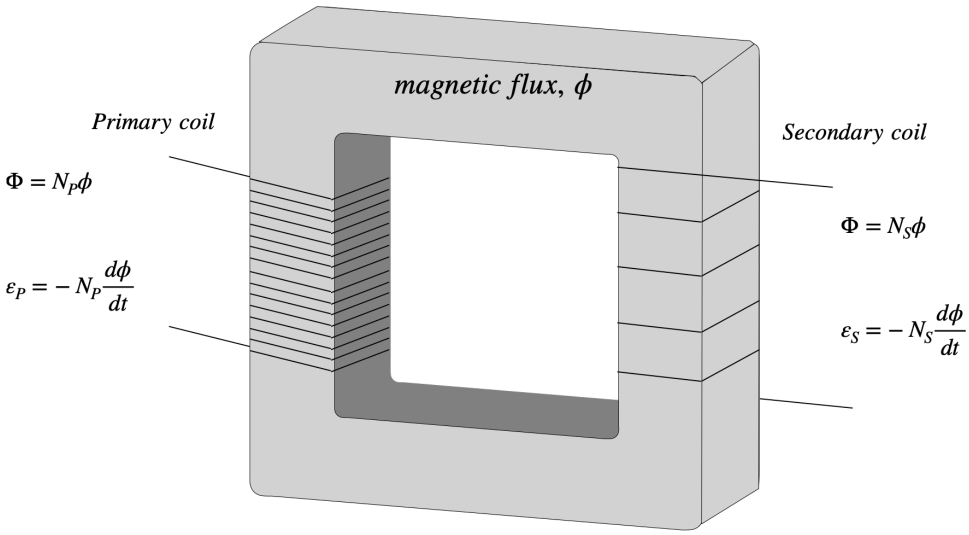
For a sinusoidal current and voltage

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\langle P \rangle = I_{rms} V_{rms} = \left(\frac{I_0}{\sqrt{2}}\right)\left(\frac{V_0}{\sqrt{2}}\right) = \frac{P_0}{2}$$

4 Transformer



- **Function**: to use mutual electromagnetic induction to step up or step down voltage
- a common iron core is used to concentrate magnetic flux through both coils
- AC current flows in primary coil, setting up an alternating magnetic flux in the iron core
- According to Faraday's law, the alternating magnetic flux linkage through both coils induces an alternating e.m.f. across each turn in both coils

4.1 Derivation of results

since the magnetic flux ϕ is the same **through each turn** for both coils, the induced emf is the same through each turn

$$\frac{\varepsilon_P}{N_P} = \frac{\varepsilon_S}{N_S}$$

$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$$

Assuming 0 resistance for an ideal transformer,

$$\frac{V_S}{V_P} = \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$$

Assuming no power loss for an ideal transformer, input power = output power hence

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

4.2 Power loss and design features

Cause of power loss	Design features
Joule heating of cooper wires	Thick copper wires of low resistance used
Heating due to **eddy currents** in iron core	The iron core is made of laminated sheets curring across path of eddy currents, to increase resistance to current flow
Hysteresis loss	Soft iron is used, which can be easily magnetised and demagnetised
Magnetic flux leakage	Iron core maximises flux linkage, E-I shaped iron core is used

5 Transmission of electrical power

For a supply power P, transmitting at voltage V and current I, and total cable resistance R, the power loss is

$$P_{loss} = I^2 R = \left(\frac{P}{V}\right)^2 R$$

- hence a high voltage is used to minimise power loss

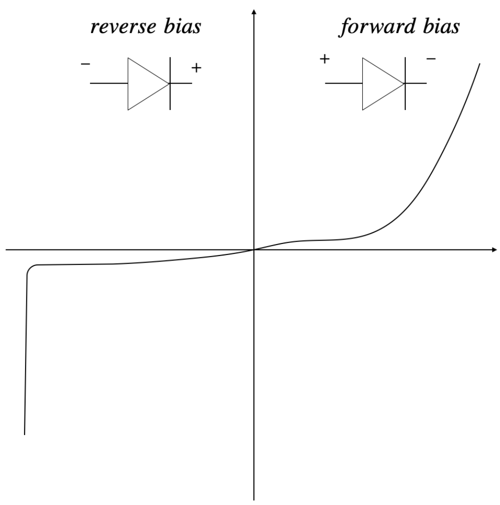
Alternatively, the power loss can be found by first finding p.d. across cable

$$V_{cable} = IR = \left(\frac{P}{V}\right) R$$

Power loss is the power dissipated in cable

$$P_{loss} = I^2 R = \frac{V_{cable}^2}{R} = \left(\frac{P}{V}\right)^2 R$$

6 Rectification with diode



Topic 17 - Electromagnetic Induction

1 Faraday's law of electromagnetic induction

Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage

$$\text{induced e.m.f.} = -\frac{d\Phi}{dt}$$

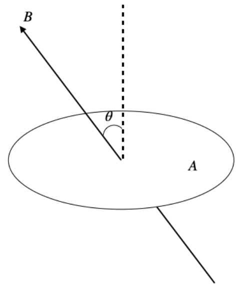
where Φ is the **magnetic flux linkage**

2 Magnetic flux and magnetic flux linkage

2.1 Magnetic flux

Magnetic flux, ϕ is defined as the product of an area and the component of the magnetic flux density perpendicular to that area

For an area A where a uniform magnetic field with magnetic flux density B passes at an angle θ



$$\phi = BAcos\theta$$

2.2 Magnetic flux linkage

The **magnetic flux linkage,** Φ of a coil is the product of the magnetic flux through the voil and the numebr of turns of the coil

For a coil of N turns with uniform cross sectional area A

$$\Phi = N\phi$$

3 Determining direction of induced current

NOTE: there will only be an induced current if there is an induced e.m.f. and **the circuit is closed**

3.1 Fleming's right hand rule

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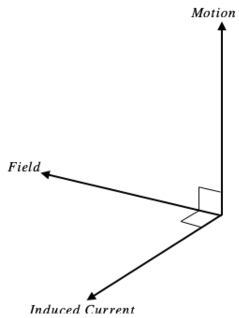


3.2 Lenz's law

Lenz's law states that the direction of induced e.m.f. is such as to cause effects to oppose the change producing it

- a result of conservation of energy

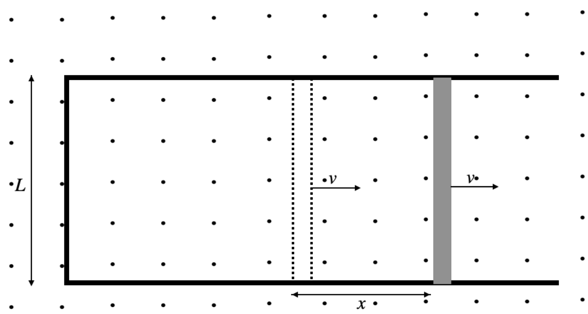
3.3 By first principles

- consider movement of free electrons inside a conductor
- consider direction of conventional current due to movement of conductor and hence electrons
- determine force on free electrons due to **Fleming's left hand rule**
- electrons will tend towards one end, while positive charge tends towards the other
- the separation of cahrge sets up an electric field.

NOTE THAT

- outside an e.m.f. source, current flows from **high to low** potential
- inside an e.m.f. source, current flows from **low to high** potential

4 Metal rod moving across uniform magnetic field



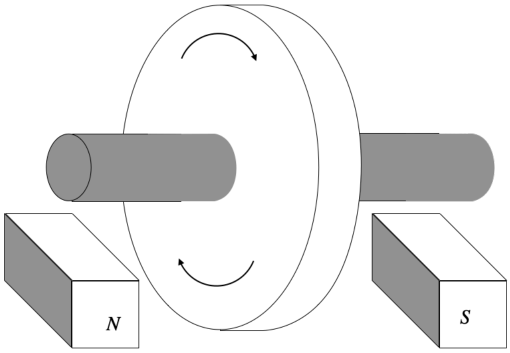
- the distance travelled by rod in time t is x
- the magnetic flux linkage in time t is

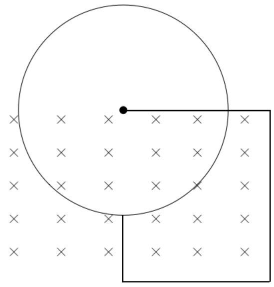
$$\Phi = BA = BLx$$

- The magnitude of induced e.m.f. is given by Faraday's law

$$|E| = \left| -\frac{d\Phi}{dt} \right| = BL\frac{dx}{dt} = BLv$$

5 Rotating disc in uniform magnetic field

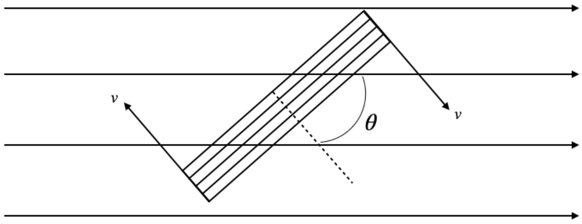




- the magnitude of induced e.m.f. is given by Faraday's law

$$|E| = \left|-\frac{d\Phi}{dt}\right| = B\frac{dA}{dt} = B\frac{\pi r^2}{T} = BAf = \frac{1}{2}Br^2\omega$$

6 Rotating coil in uniform magnetic field



- the magnetic flux linkage is given by

$$\Phi = NBAcos\theta = NBAcos\omega t$$

- By Faraday's law

$$E = -\frac{d\Phi}{dt}$$

$$= -\frac{d(NBAcos\omega t)}{dt}$$

$$= -NBA\frac{dcos\omega t}{dt}$$

$$= NBA\omega sin\omega t$$

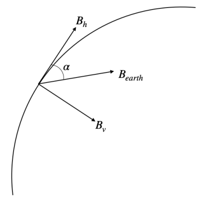
Topic 16 - Electromagnetism

1 Magnetic field

A **magnetic field** is a region in space where a **moving charge** or a **charge carrying conductor** or any ferromagnetic object will experience a magnetic force when it is placed in it

- the strength of magnetic field is expressed by a quantity called **magnetic flux density**, with units in **tesla, T**
- only **moving charge** in magnetic field experiences a magnetic force
- note that when representing a magnetic field, field lines point **away** from north and towards the south pole

Earth's magnetic field



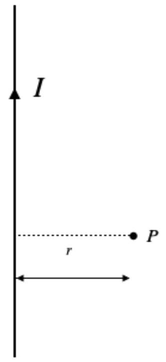
The magnetic field of earth can be resolved into two components

$$B_H = B_{earth} cos\alpha$$

$$B_v = B_{earth} sin\alpha$$

2 Magnetic fields due to currents

2.1 Magnetic force due to long straight wire

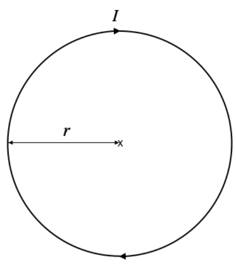


- the diretion of magnetic field is given by **Maxwell's right-hand grip rule**
- at point P, a distance r away from the conductor, B is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$

2.2 Magnetic fields due to circular coil



- direction given by right-hand grip rule
- the magnetic flux density at the center of the coil is given by

$$B = \frac{\mu_0 NI}{2r}$$

2.3 Magnetic field due to solenoid

- direction given by right-hand grip rule
- the magnetic flux density at the center of the solenoid is given by

$$B = \mu_0 n I$$

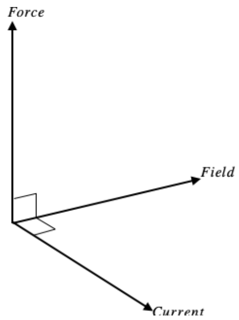
- the magnetic flux density at either end is

$$B = \frac{1}{2}\mu_0 n I$$

where n is the number of turns per unit length

3 Force on a current carrying conductor

The directions of current, field and force are given by **Fleming's left-hand rule**



3.1 Magnetic flux density, B

the **magnetic flux density** of a magnetic field is numerically equal to the **force per unit length** of a long straight conductor carrying a unit current at right angle to a uniform magnetic field

$$B = \frac{F}{IL}$$

hence,

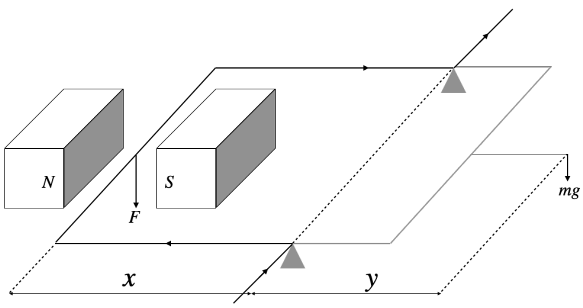
$$F = BIL$$

For case in which magnetic field is at an angle θ to the current, the force on conductor is given by

$$F = BILsin\theta$$

the S.I. unit is tesla
one tesla is the uniform magnetic flux density which, acting normally to a long straight wire carrying a current of $1A$, causes a force per unit legnth of $1Nm^{-1}$

3.2 Measuring magnetic flux density with a current balance

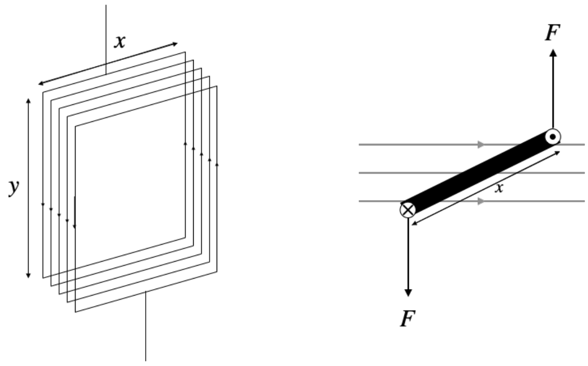


By principle of moments,

$$\text{sum of CW moments} = \text{sum of ACW moments}$$

$$mgy = BILx$$
$$B = \frac{mgy}{ILx}$$

3.3 torque on current carrying coil in a magnetic field



- the forces on vertical sides give rise to turning effect. the magnitude of each force is given by

$$F = NBIy$$

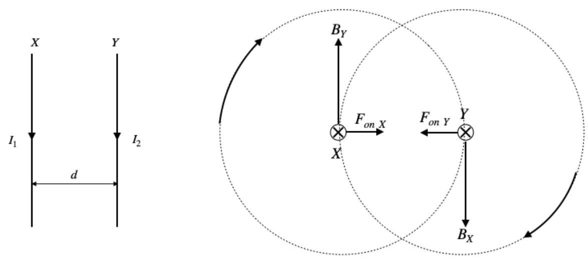
- since the force on each vertical side are **equal in magnitude** and **opposite in direction**, the constitute a couple.
- the force on the vertical sides remain **constant in magnitude** throughout the rotation
- the torque of a couple is given by

$$\tau = Fd$$

  where d is the perpendicular distance between the lines of action of the forces
- the torque τ due to the conducting coil is thus

$$\begin{aligned}\tau &= Fd \\ &= NBIy \times xcos\theta \\ &= NBIyxcos\theta \\ &= NBIAcos\theta\end{aligned}$$

4 Force between current-carrying conductor



The current in X produces a magnetic field B_X, whose magnitude at Y is given by

$$B_X = \frac{\mu_0 I_1}{2\pi d}$$

Wire Y thus experiences a magnetic force **towards X** and its magnitude is given by

$$F_{XonY} = B_X I_2 L = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 L$$

Likewise, X experiences a force **towards Y**, given by

$$F_{YonX} = B_Y I_1 L = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 L$$

The two wires with currents in the same direction **attract** each other, and the **force per unit length** on each wire is

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

In general

For 2 parallel current-carrying conductors

- the force per unit length is
$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$
- currents in the same direction attract, current in opposite directions repel

5 Force on moving charge

For a charged particle travelling distance L in time t,

$$v = \frac{L}{t}$$

The moving charge constitutes a current where

$$I = \frac{q}{t}$$

Hence force on charge is given by

$$F = BIL = \left(\frac{BqL}{t}\right) = Bqv$$

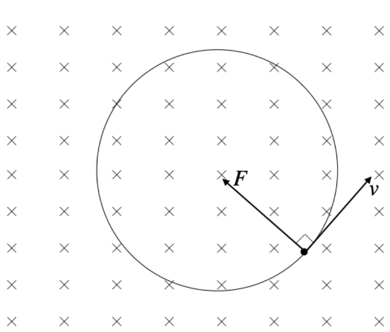
If velocity and field are inclined to each other by angle θ,

$$F = Bqvsin\theta$$

Note that direction of current is that of **conventional current**

5.1 Motion of charged particle in magnetic field

For a charged particle projected at right angle into a magnetic field



the **magnetic force** on moving charge provides for centripetal force

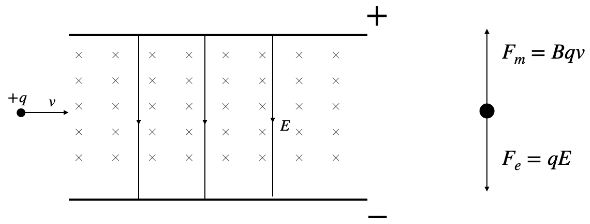
$$Bqv = \frac{mv^2}{2}$$

For a charged particle projected at some angle θ,

$$v_{parallel} = v\ cos\theta$$

$$v_{perpendicular} = v\ sin\theta$$

6 the velocity selector



Particles deflect upwards or downwards depending on F_m and F_e
Particles experience no deflection when

$$Bqv = qE$$

$$v = \frac{E}{B}$$

Topic 15 - DC Circuits

1 Series and parallel arrangement

1.1 Equivalent resistance

It is possible to replace a **combination of resistors** in any given circuit with a single resistor without altering the p.d. and the current across the terminals of the combination. The resistance of the single resistor is called the **equivalent resistance** of the combination

1.2 Resistor in series

The current I through each resistor is the same

$$R = R_1 + R_2 + R_3 + ...R_n$$

1.3 Resistor in parallel

The pd.d V through each resistor is the same

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...\frac{1}{R_n}$$

1.4 Combination of E.M.F. cells in series

$$E_{tota} = E_1 + E_2$$

1.5 Combination of E.M.F. cells in parallel

For two cells with emf E_1

$$E_{total} = E_1$$

2 Potential divider

For a circuit with emf V_s and 2 external loads R_1 and R_2 and current I
The current is given by

$$I = \frac{V_s}{R_1 + R_2}$$

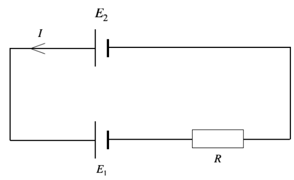
The potential difference across R_1 is thus

$$V_\circ = IR_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s$$

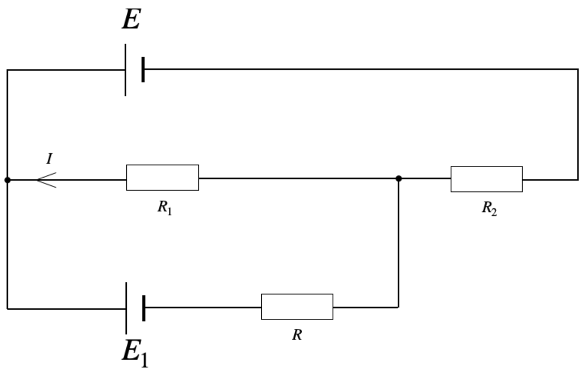
3 Potentiometer

3.1 Working principles of a potentiometer

For the circuit below, $I = 0$ when $E_1 = E_2$



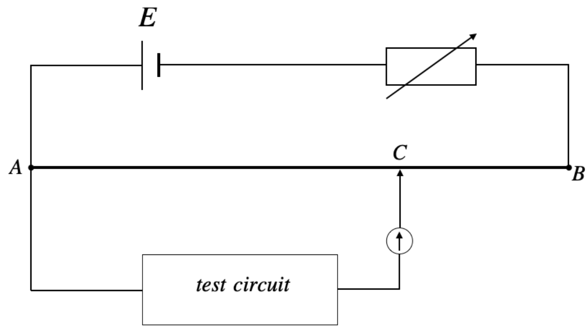
Replacing E_2 with a potential divider circuit,



By varying R_1 and R_2, no current flows through E_1 when the *p.d.* across R_1 is equal to E_1

3.2 Null point, balance point and balance length

By replacing R_1 and R_2 with a resistance wire AB and a sliding contact at point C, the galvanometer shows no reflection at **null point** or **balance point**. AC is the **balance length**.



Assuming

- Wire AB has uniform cross sectional area
- Potential difference across wire remains constant with time

For a **uniform wire** of length L,

$$R = \frac{\rho L}{A}$$

$$R \propto L$$

The potential difference across AC is

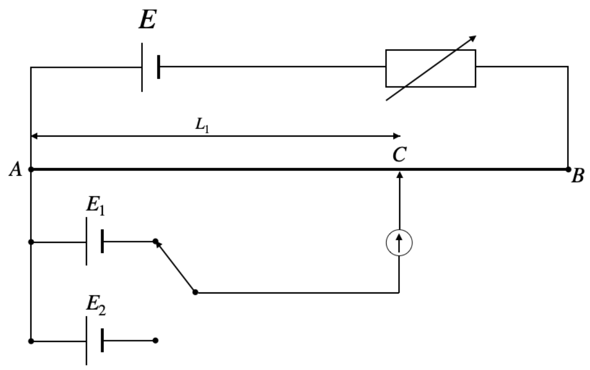
$$V_{AC} = \frac{R_{AC}}{R_{AB}} \times V_{AB}$$

$$V_{AC} = \frac{L_{AC}}{L_{AB}} \times V_{AB}$$

3.3 Voltmeter vs potentiometer

- for potentiometer, no errors are introduced by internal resistance of the cells (since no current flows through the cells at balance length
- real voltmeter has **finite resistance**, and hence draws a current from the cell, and lowers the terminal p.d. of the cell when connected
- for potentiometer, since **no current flows at balance point**, the potentiometer can be considered to be a voltmeter with **infinite resistance**

3.4 Application 1 - Comparison of e.m.fs



Since at balance length,

$$E_i = \frac{R_{AC}}{R_{AB}} \times V_{AB}$$

Since V_{AB} and R_{AB} constant,

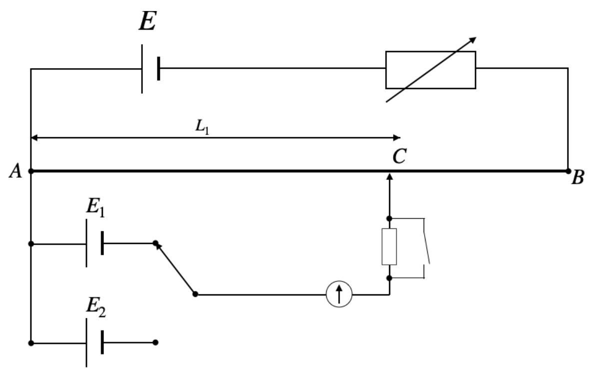
$$E_i = kL_i$$

hence

$$\frac{E_1}{E_2} = \frac{kL_1}{kL_2} = \frac{L_1}{L_2}$$

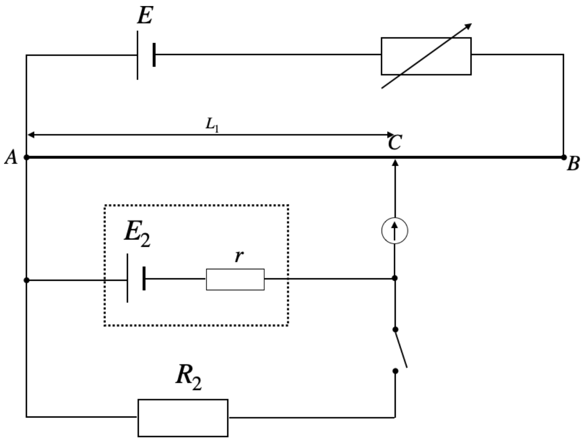
NOTE - shorting key as a protection for galvanometer

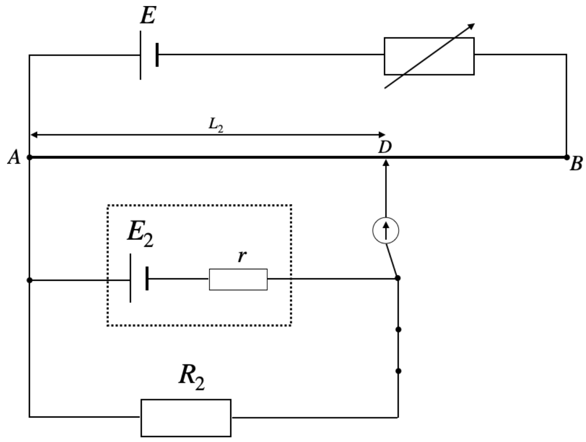
To ensure precision of potentiometer, the galvanometer used is very sensitive. The galvanometer should be protected from **high currents** that flows through in **off-balance** situations, by using a large resistance in series.



- Before approximate null point is reached, the shorting key is **left open**, to protect the galvanometer from large currents
- nearing the null point, the key is **closed**, to allow for full current to flow through, so that an accurate balance point can be found

3.5 Application 2 - Measuring e.m.f. and internal resistance of a cell





- when **switch is open**, E_2 can be found with L_1

$$E_2 = \frac{L_1}{L_{AB}} \times V_{AB}$$

- when switch is closed, there is a current flowing through E_2

$$\text{terminal p.d. across } E_2 = \text{terminal p.d. across } R_2 = \frac{L_2}{L_{AB}} \times V_{AB}$$

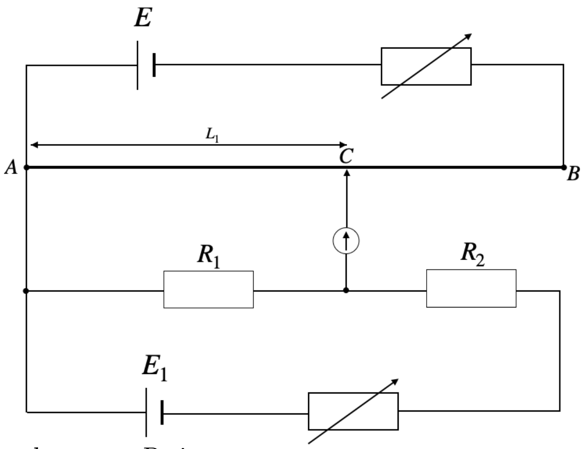
- the current flowing through R_2 is

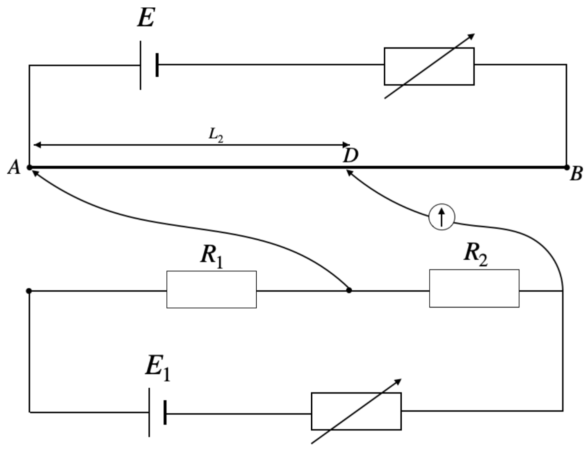
$$I = \frac{V}{R_2} = \frac{L_2}{L_{AB}} \times V_{AB} \times \frac{1}{R_2}$$

- the internal resistance can be found by

$$V = E_2 - Ir$$

3.6 Application 3 - Comparison of resistance of two resistors





p.d. across R_1 is,

$$V_1 = \frac{L_1}{L_{AB}} \times V_{AB}$$

p.d. across R_2 is

$$V_2 = \frac{L_2}{L_{AB}} \times V_{AB}$$

hence in the test circuit

$$IR_1 = \frac{L_1}{L_{AB}} \times V_{AB}$$

$$IR_R = \frac{L_R}{L_{AB}} \times V_{AB}$$

since current I is the same

$$\frac{R_1}{R_2} = \frac{L_1}{L_2}$$

Topic 14 - Current of electricity

1 Electric Current and charge

1.1 Electric Current

Electric current is defined as the rate of flow of charge

$$I = \frac{dQ}{dt}$$

The S.I. unit for current is Ampere

1.2 Charge

Electical charge is a fundamental property of matter which causes a charged particle to experience a force when placed in an electric field.

When a current I flows through a cross-section of a conductor for duration t, the amount of **electrical charge** passing through is given by

$$Q = It$$

One **coulomb** is defined as the amount of charge that passes through a point in one second due to a current of one ampere

1.3 Drift velocity

For a conductor with cross sectional area A and n charge carriers per unit volume, and a current I

In the time interval Δt , each charge moves a distance of

$$v_D(\Delta T), \text{ where } v_D \text{ is the drift velocity}$$

The number of charge carriers passing through a point in duration Δt is thus

$$nAv_D\Delta t$$

total charge passing through is

$$nAv_d\Delta tq$$

current is thus

$$I = \frac{nAv_d\Delta t}{\Delta t} = nAv_Dq$$

2 Potential difference

The **potential difference** between two points in a circuit is defined as the amount of electrical energy per unit charge that is converted to other forms of energy when charge passes from one point to another

$$V = \frac{W}{Q}$$

The SI units for p.d. is volt
one volt is the potential difference between two points when one joule of electrical energy is converted to other forms of energy as one coulomb of charge passes from one point to another

3 Electromotive Force

the **electromotive force** of a source is defined as the amount of electrical energy per unit charge that is converted from other forms of energy when charge passes through the source

$$E = \frac{W}{Q}$$

4 Resistance and resistivity

The **resistance** of a circuit component is defined as the ratio of the potential difference across it to the current flowing through it

$$R = \frac{V}{I}$$

one Ω is the resitance of a conductor when a potential difference of one volt across it causes a current of one ampere to flow through

4.1 Ohm's law

Ohm's law states that the potential difference across a conductor is proportial to the electric current passing through it, provided that its temperature remains constant

4.2 Resistivity ρ

The resistance of a unifom conductor is

- directly proportional to its length, l
- inversely proportional to its cross sectional area, A

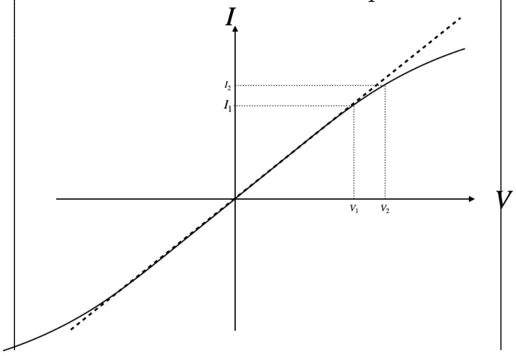
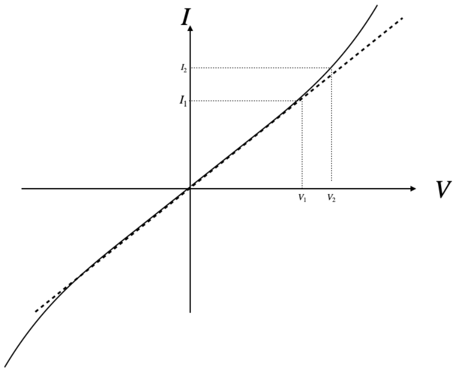
hence

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A}$$

4.3 Effect of temperature changes

- Effect 1: increase in density of free electrons. More electrons able to break free from atoms, more mobile electrons
- Effect 2: lattice ions vibrate faster with greater amplitudes. Electrons lose KE to fixed ions during collision, hence emperature increases. Ions then vibrate faster with greater amplitudes, making it more difficult for electrons to pass through the lattice

materials	metallic conductors	semiconductors - NTC thermistors
Effect 1	Electrons are already mobile at room temperature. No appreciable increase in number of conducting electrons	Semiconductors are poor conductors at low temperatures. Conductivity increases significantly with temperature
Effect 2	Significantly greater vibration of lattice ions, resistance increases	Significantly greater vibration of lattice ions.
Change in resistance	Effect 1 insignificant, effect 2 is present, resistance increases with temperature	Effect 1 more significant than effect 2. Resistance decreases with temperature
		

5 Electrical power

When a charge Q moves through a p.d. V, the amount of electrical energy converted to other forms is

$$W = QV$$

the rate of energy convertion is

$$P = \frac{dW}{dt} = \frac{d(QV)}{dt} = \frac{dQ}{dt}V = IV$$

5.1 Power of an ideal source

For an ideal source, work done is

$$W = QE$$

The power supplied is

$$P = \frac{dW}{dt} = \frac{dQ}{dt}E$$
$$P = IE$$

5.2 Power dissipated by a resistor

The p.d. across a resistor is

$$V = IR$$

The rate of energy dissipation is

$$P = IV = I^2R = \frac{V^2}{R}$$

6 Internal resistance of a source

For a real battery with internal resistance r connected to a circuit with resistor R

The power supplied by source is

$$P_s = IE$$

Power dissipated in internal resistance is

$$P_r = I^2r$$

Power dissipated in external load is

$$P_R = I^2R$$

By principle of conservation of energy

$$P_S = P_r + P_R$$

$$IE = I^2 r + I^2 R$$

$$E = I(r + R)$$

$$E = Ir + V_R$$

$$V_R = E - Ir$$

Hence terminal p.d. V decreases with increasing I

6.1 Power output and max power theorem

since

$$E = I(r + R)$$

$$I = \frac{E}{r + R}$$

heat transferred to external load R is

$$P_R = I^2 R = \left(\frac{E}{r + R}\right)^2 R$$

$$P_R = \frac{E^2 R}{(r + R)^2}$$

Max power delivered to external load R can be found by taking the derivative

$$\begin{aligned}
\frac{dP_R}{dR} &= 0 \\
0 &= \frac{E^2(r + R) - 2E^2 R}{(r + R)^3} \\
E^2(r + R) - 2E^2 R &= 0 \\
r + R - 2R &= 0 \\
r &= R
\end{aligned}$$

Hence a source of emf delivers the maximum amount of power to an external load when the resistance of the load is equal to the internal resistance of the source

Topic 13 - Electric Fields

1 Electric Force

1.1 Coulomb's law

Coulomb's law states that the force between two point charges is proportional to the product of the charges and inversely proportional to the square of the distance between them

$$F \propto \frac{Qq}{r^2}$$

For two charges Q and q separated by a distance r,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

where $\varepsilon = 8.85 \times 10^{-12} Fm^{-1}$

Electric force is a **vector** quantity

2 Electrical field strength

2.1 Eletric field

An **electric field** is a region of space in which a charge placed in that region experiences an electric force

2.2 Electric field lines

- the **direction of an electric field line** indicates the direction of the electric force acting on a **small positive test charge** placed at that point. The direction of the electric field strength is tangent to the electric field lines at any point
- the **density of field lines** indicates the relative magnitude of electric field strength
- Electric field lines **originate from positive charges** and **terminate on negative charges**

Note that for a conductor, placed inside an electric field, free electrons in an isolated solid or hollow conductor will redistribute until electrostatic equilibrium is achieved, hence

- no electric field within conductor
- electric field lines must start or en dat the surface of the conductor, and be perpendicular to the surface of the conductor
- **potential within the conductor is constant**: there is no potential difference between any points

2.3 Electric field strength, E

the **electric field strength** at a point is defined as the electric force exerted per unit **positive** test charge

$$E = \frac{F}{q}$$

The electric field strength due to charge Q at a distance r is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Electric field strength is a **vector** quantity

2.4 Electric field strength of a charged conducting sphere

For a **charged hollow or solid conducting sphere**,

- outside the sphere, electric field behaves as if all its charges are concentrated at its center, i.e. assumed to be point charge
- inside the sphere, $E = 0$ because all the charges are uniformly distributed on the surface of the sphere. The potential within the sphere is the same as that on the surface of the sphere

3 Electric Potential and Electric potential energy

The **electrical potential** at a point in an electric field is defined as the work done **per unit positive test charge** by an external force in bringing a small test charge from infinity to that point

$$V = \frac{W}{q}$$

The electric potential V at a distance r away from charge Q is

$$V = \frac{W}{q} = \frac{1}{q}\int_{\infty}^{r} F_{ext}dr = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

The electrical potential at a point due to multiple charges can be found by **adding** the individual potentials at that point due to each charge

The **electrical potential energy** of a charge at a point in an electric field is definend as the work done by an external force in bringing the charge from infinity to that point

$$U = W = qV$$

Hence, the electrical potential energy of a system of two point charges separated by r is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

3.1 Electron volt eV

The **electron-volt** is the energy gained by an electron when it is accelerated through a potential difference of one volt

$$1\ eV = 1.60 \times 10^{-19} J$$

3.2 Potential difference

$$\Delta V = V_{final} - V_{initial}$$

The work done and change in potential energy is

$$W = \Delta U = q\Delta V$$

3.3 Relationships between F, E, V, and U

$$E = -\frac{dV}{dr}$$

The electric field strength at a point is numerically equal to the potential gradient at that point.

Electric field strength points in the direction of decreasing potential

$$F = -\frac{dU}{dr}$$

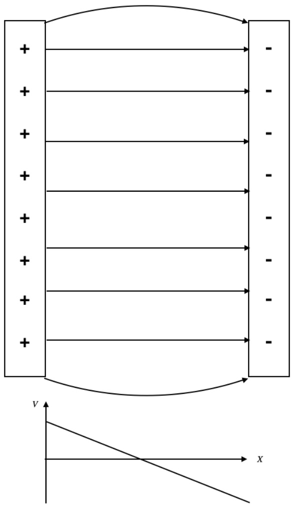
The elctric force at a point is numerically equal to the potentialenergy gradient
The electric force points in the direction of decreasing potential

4 Equipotential lines and surfaces

- equipotential lines are perpendicular to electric field lines
  - hence there is no component of electric field strength acting on a charge that moves along an equipotential line
$$W = 0$$
- movement along an equipotential line or surface requires no work since $\Delta V = 0$
$$W = q\Delta V = q(0) = 0$$
- Dashed lines are used to represent equipotential lines

5 Uniform electric field

- the electric field strength at any point has the **same magnitude** and **direction**
- the electric field lines are **parallel** and **equally spaced**



Since electric field is uniform, the potential gradient at any points is constant

$$\frac{dV}{dx} = -E$$

Therefore, the magnitude of electric field strength is

$$|E| = \left|\frac{\Delta V}{d}\right|$$

The magnitude of F on a charge q is

$$F = qE$$

Topic 12 - Superposition

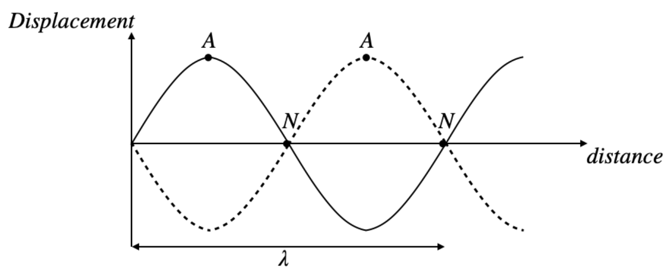
1 Principle of superposition

The **Principle of superposition** states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point

2 Stationary waves

A **Stationary wave** is the result of interference between two progressive waves of the same type, frequency, amplitude, and speed, travelling along the same line but in opposite directions

2.1 Properties of stationary wave



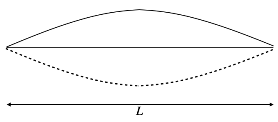
- The wave profile does not propagate
- Every particle of the wave oscillate about their respective equilibrium positions with the **same freqency** but **difference amplitude**
- The **antinode** is a point in a standing wave of maximum amplitude. The waves arrive **in phase**
- The **node** is a point of zero amplitude. The waves arrive **anti-phase** at the nodes
- Within two adjacent nodes, all particles oscillate **in phase** i.e. they reach their respective maxima / minima / equilibrium position at the same time
- particles in neighbouring segments vibrate π out of phase with each other
- distance between two adjacent nodes is $\frac{1}{2}\lambda$
- the **envelope** is a curve outlining the amplitudes of a standing wave

2.2 Comparing standing and progressive waves

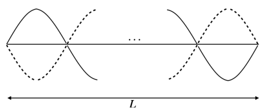
Property	Progressive wave	stationary wave
Waveform	propagates with the velocity of wave	does not propagate
Energy	transports energy	does not transport energy
Amplitude	all particles have the same amplitdue	Amplitude varies (from 0 to maximum)
Phase	All particles within one wave length have different phases	All particles between two adjacent nodes have the same phase. Particles in adjacent segments have $\phi = \pi$
Freqeuncy	All points vibrate in SHM with same frequency	All points vibrate in SHM with same frequency (except at nodes)
Wavelength	equals to the distance between consecutive points in phase	equal to the distance beteen two adjacent nodes or two adjacent anti-notes

2.3 Stationary waves in strings

when a string of length L that is fixed at both ends is plucked, standing waves of different frequencies are set up



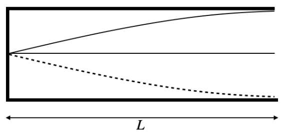
- mode of vibration: fundamental frequency
- wavelength
$$L = 1\left(\frac{\lambda_1}{2}\right)$$
$$\lambda_1 = 2L$$
- frequency
$$f = \frac{v}{2L}$$
- first harmonic



- mode of vibration: (n-1)th overtone
- wavelength
$$L = n\left(\frac{\lambda_n}{2}\right)$$
$$\lambda_n = \frac{2L}{n}$$
- frequency
$$f = n\frac{v}{2L}$$
- n^{th} harmonic

2.4 Stationary waves in air columns

A displacement node is formed at the closed end and displacement anti-node formed at the open end



- mode of vibration: fundamental frequency
- wavelength
$$L = 1\left(\frac{\lambda_1}{4}\right)$$
$$\lambda_1 = 4L$$
- frequency
$$f = \frac{v}{4L}$$
- first harmonic

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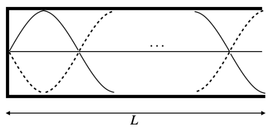

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- mode of vibration: (n-1)th overtone
- wavelength

$$L = (2n-1)\left(\frac{\lambda_{2n-1}}{4}\right)$$

$$\lambda_{2n-1} = \frac{4L}{2n-1}$$

- frequency

$$f = (2n-1)\frac{v}{4L}$$

- $(2n-1)^{th}$ harmonic

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2.5 Stationary waves in pipes

Displacement anti-node at both ends

- displacement antinodes are **pressure nodes** i.e. least variation in pressure
- displacement nodes are **pressure anti-nodes**, largest variations in pressure

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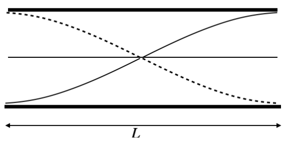

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- mode of vibration: fundamental frequency
- wavelength

$$L = 1\left(\frac{\lambda_1}{2}\right)$$

$$\lambda_1 = 2L$$

- frequency

$$f = \frac{v}{2L}$$

- first harmonic

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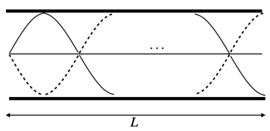

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- mode of vibration: (n-1)th overtone
- wavelength

$$L = 2\left(\frac{\lambda_n}{2}\right)$$

$$\lambda_n = \frac{2L}{n}$$

- frequency

$$f = n\frac{v}{2L}$$

- n^{th} harmonic

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2.6 End correction

the displacement antinodes at open ends of pipes are located slightly outside the pipe, hence when calculating wavelength, substitute

$$L = L_{actual} + c \text{ or } L = L_{actual} + 2c$$

3 Diffraction

Diffraction is the bending or spreading of waves after passing through an aperture or round an obstacle

- Diffraction is pronouned when the wavelength of thewave i of the same order of magnitude as the width of the aperture or obstacle

4 Coherence and Interference

4.1 Coherence

Waves or sources are **coherent** if they have a constant phase difference

4.2 Interference

Interference is the **superposition** of two or more coherent waves to give a resultant wave whos resultant amplitdue is given by the principle of superposition

- constructive interference: when two waves arrive with a phase difference of zero
- destructive interference: when two waves arrive with a phase difference of πrad

4.3 Two source interference

For a two source interference to be observable

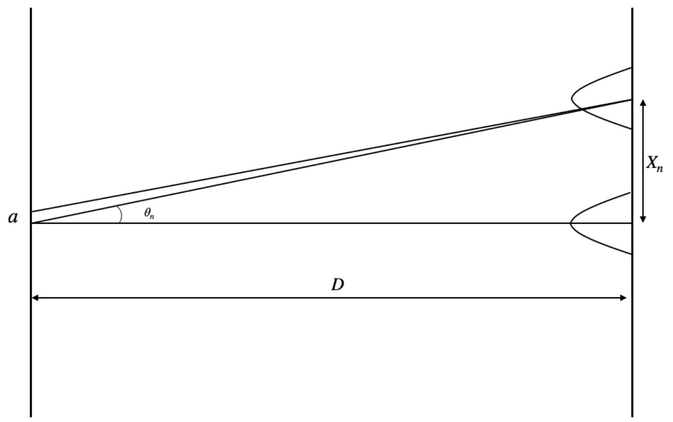
- sources must be coherent
- waves must have similar amplitude
- waves must overlap and be of the same type
- Transverse waves must be unploarised or polarised in the same plane

4.4 Path difference

Path difference is the difference in distance each wave travels from its source to the point where the two waves meet

	2 sources in phase	2 sources π out of phase
Constructive interference	$\Delta L = n\lambda$	$\Delta L = (n + \frac{1}{2})\lambda$
Destructive interference	$\Delta L = (n + \frac{1}{2})\lambda$	$\Delta L = n\lambda$

5 Double slit experiment



Assuming $a << D$, and hence light rays almost parallel, then

$$\text{path difference} = \Delta x = asin\theta_n$$

For constructive interference at the n^{th} bright fringe

$$asin\theta_n = n\lambda$$

$$sin\theta_n = \frac{n\lambda}{a}$$

Additionally,

$$tan\theta_n = \frac{X_n}{D}$$

Assuming $a >> lambda$, θ is small

$$sin\theta_n \cong tan\theta_n n$$

$$\frac{n\lambda}{a} \cong \frac{X_n}{D}$$

Hence constructive intereference takes place at

$$X_n = \frac{n\lambda D}{a}$$

Fringe separation is thus

$$x = X_n - X_{n-1} = \frac{\lambda D}{a}$$

6 Diffraction grating

For a diffraction grating of N lines per meter, slit separation is

$$d = \frac{1}{N}$$

Path difference is

$$\Delta x = dsin\theta_n$$

Assuming $d << D$, constructive interference occur

$$dsin\theta_n = n\lambda$$

since $\theta_n < 90°$

$$sin\theta_n < 1$$

$$\frac{n\lambda}{d} < 1$$

$$n < \frac{d}{\lambda}$$

7 Single slit interference

Huygens' Principle states that at any instant, all points on a wavefront could be recarded as secondary sources giving rise to their own outward spreading circular wavelets.

the envelope of wavefronts produced by each secondary source gives the new position of the wavefront

For a single slit diffraction with slit separation b , for $b << D$, position of the first minima is given by

$$bsin\theta = \lambda$$

$$sin\theta = \frac{\lambda}{b}$$

using arc length where

$$s = r\theta$$

for small θ the width, y of the central bright fringe is

$$y = D(2\theta) = \frac{2D\lambda}{b}$$

8 Rayleigh criterion

The **resolving power** of a single slit is the ability to distinguish between closely spaced objects

Rayleigh's criterion states that two objects are just resolved when the central maximum of one iamge falls on the first minimum of thee other image. The criterion is satisfied when the mimimum angular separation of the sources θ_{min} is

$$\theta_{min} = \frac{\lambda}{b}$$

Topic 11 - Wave Motion

A **progressive wave** is one that transfers energy from one point to another in the direction of wave propagation

- particles oscillate about their equilibrium positon

1 Wave terminology

- **Displacement** of a particle is the distance travelled in a specific direction from its equilibrium position
- **Amplitude, A** of a wave is the magnitude of maximum displacement of a particle from eqm position
- **Period, T** of a wave is the time taken for a particle to complete one oscillation
- **Frequency, F** is the numebr of oscillations per unit tiem made by a particle of a wave,

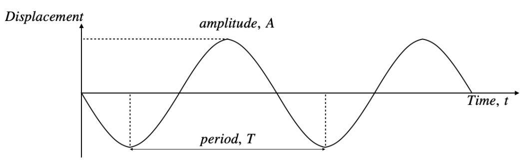
$$T = \frac{1}{f}$$

- **Wavelength** λ is the distance between two consecutive points which are in phase
- **Speed, v** is the distance travelled by a wave per unit time

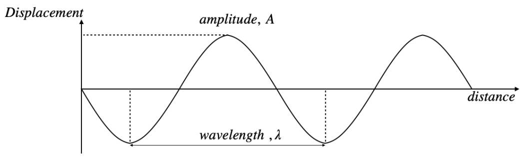
$$v = \lambda f = \frac{\lambda}{T}$$

- **Phase / Phase angle,** ϕ is a measure of the fraction of a cylce that has been completed by an on oscillating particle or by a wave.
- **Phase difference** between two partciles in a wave or between two waves at a point is the measure of the fraction of a cycle which one is ahead of the other
  - **In phase**: particles exercute the same motion at the same time, $\phi = 0$
  - **Out of phase**: partciles that are at different stages of motion
  - **Anti-phase**: execute motions that are out of phase by π rad
- **Wavefront** is a line or surface joining points on a wave that are in phase. Wave travels in a direction perpendicular to the wavefront

1.1 graphs of waves



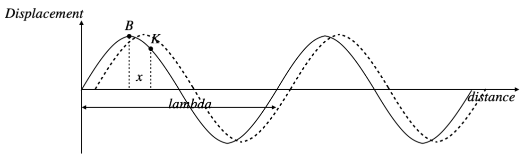
Displacemennt - time graph
graph represents displacement of **one** particle over time



Displacement - distance graph
graph represents the displacement of **all** particles at a particular instant

1.2 representation of phase difference

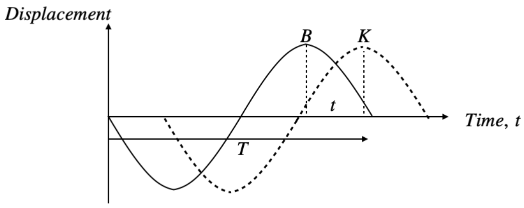
For a wave travelling to the right



Displacemennt - distance graph
B is on its way down and K is on its way up
K is lagging B by a phase differene of ϕ or (B is ahead of K by ϕ), $0 \leq \phi\pi$

$$\frac{x}{\lambda} = \frac{\phi}{2\pi}$$

$$\phi = 2\pi\frac{x}{\lambda}$$



Displacemennt - time graph
K is lagging B by a phase differene of ϕ or (B is ahead of K by ϕ), $0 \leq \phi\pi$

$$\frac{t}{T} = \frac{\phi}{2\pi}$$

$$\phi = 2\pi\frac{t}{T}$$

2 Wave Intensity

The **intensity** of a wave is defined as the rate of transfer of energy per unit area, where the area measured is perpendicular to the direction of energy transfer

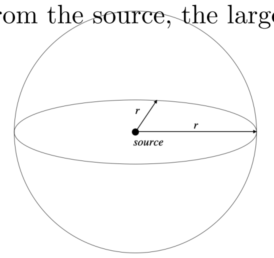
$$I = \frac{P}{S}$$

where P is the power of the source, and S is the surface area of the wave front

$$I = kA^2$$

2.1 Point source and inverse square

for a point source emitting waves that spread out radially, with a spherical wavefront, the further away from the source, the larger the area where energy is being distributed



$$I = \frac{P}{4\pi r^2}$$

Hence for a constant power, the inverse square law is known as

$$I = \frac{k}{r^2}$$

3 Transverse and longitudinal waves

A **transverse wave** is one in which particles oscillate in a direction perpendicular to the direction of energy transfer

A **longitudinal wave** is one in which the particles oscillate in a direction parallel to the direction of energy transfer

3.1 EM waves

Important properties

- Do not require medium to propagate, can move through vacuum
- EM waves travel at a speed of $c = 3.00 \times 10^8 ms^{-1}$ in vacuum
- EM waves are transverse waves and can hence be polarised

Radiation type	Radio	Microwave	Infrared	Visible	Ultraviolet	X-ray	Gamma ray
Wavelength (m)	10^3	10^{-2}	10^{-5}	0.5×10^{-6}	10^{-8}	10^{-10}	10^{-12}
Frequency(Hz)	10^4	10^8	10^{12}	10^{15}	10^{16}	$1o^{18}$	10^{20}

3.2 Sound waves

- **compression**: regions where particles are compressed together, regions of high pressure
- **rarefraction**: regions where particles are spread apart, regions of low pressure

4 polarisation

Polarisation is a phenomenon where the oscillations of the wave particles in a transverse wave are restricted to one direction only and this direction is perpendicular to the direction of wae propagation or energy transfer

- hence does not apply to longitudinal waves

4.1 Intensity

When unpolarised light is incident on an ideal polariser,

$$I = \frac{I_0}{2}$$

4.2 Malus' Law

Malus' Law states that the intensity of a beam of plane polarised light after passing through a plane polariser varies with the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity

$$I = I_0 cos^2\theta$$

When plane polarised light passes through a second polariser with its polarising axis at an angle of θ from the first polariser, the resultant amplitude is

$$A = A_0 cos\theta$$

Since

$$\begin{aligned} I &= kA^2 \\ &= k(A_0 cos\theta)^2 \\ &= kA_0^2 cos^2\theta \\ &= I_0 cos^2\theta \end{aligned}$$

Topic 10 - Oscillations

1 Simple Harmonic Motion

Simple harmonic motion is defined as the motion of a particle about a fixed point such that its acceleration is proportional to its displacement from the fixed point and is always directed towards the fixed point

$$a \propto -x$$
$$a = -\omega^2 x$$

where ω is the angular frequency of oscillation

- Magnitude of acceleration is the largest at maximum displacement, and zero at equilibrium position
- Magnitude of velocity is the largest at equilibrium position, and zero at maximum displacement

Angular frequency: is defined as the rate of change of phase angle of oscillation, and is equal to the product of 2π and its frequency

$$\omega = 2\pi f$$

Amplitude: is the magnitude of the maximum displacement of the particle from its equilibrium position
Period, T: the tie taken for one complete oscillation **Frequency, F**: the number of oscillations per unit time

$$T = \frac{2\pi}{\omega},\ \omega = \frac{2\pi}{T}$$

1.1 Equations for S.H.M.

For the case where particle is at maximum positive displacement initially, i.e. $x = x_0$ when $t = 0$:
displacement:

$$x = x_0 cos\omega t$$

velocity:

$$v = -x_0 \omega sin\omega t$$

aceeleration:

$$a = -x_0 \omega^2 cos\omega t$$

additionally,

$$\begin{aligned} v &= -x_0 \omega sin\omega t \\ &= -x_0 \omega \left(\pm\sqrt{1 - cos^2\omega t}\right) \\ &= \pm\omega\sqrt{x_0^2(1 - cos^2\omega t} \\ &= \pm\omega\sqrt{x_0^2 - x_0^2 cos^2\omega t} \end{aligned}$$

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

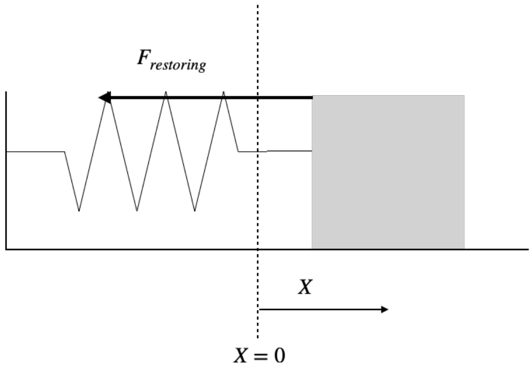
and

$$\begin{aligned} a &= -x_0 \omega^2 cos\omega t \\ &= -\omega^2 x \end{aligned}$$

1.2 Free oscillations and models for S.H.M.

Free oscillation occurs when an object oscillates with no resistive or driving forces, at its natural fequency. Its total energy and amplitude remains constant over time.

1.2.1 Horizontal Spring mass system



The restoring force is

$$F_{restoring} = -kx$$

By Newton's second law

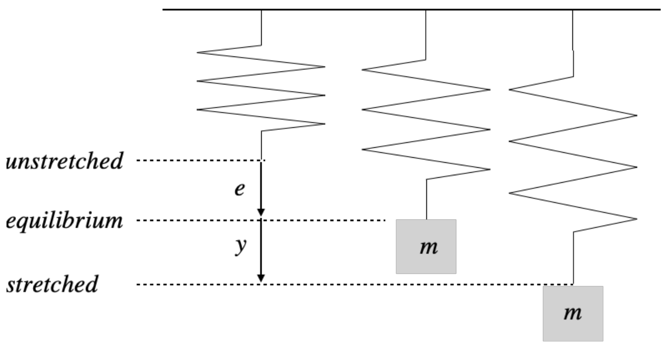
$$F_{restoring} = ma$$

$$-kx = ma$$

$$a = -\left(\frac{k}{m}\right)x$$

$$\omega = \sqrt{\frac{k}{m}},\ T = 2\pi\sqrt{\frac{k}{m}}$$

1.2.2 Vertical Spring mass system



The initial equilibrium is given by

$$mg = ke$$

when displaced by an initial y

$$F_{restoring} = -k(e+y) + mg$$

by Newton's second law

$$F_{restoring} = mg$$

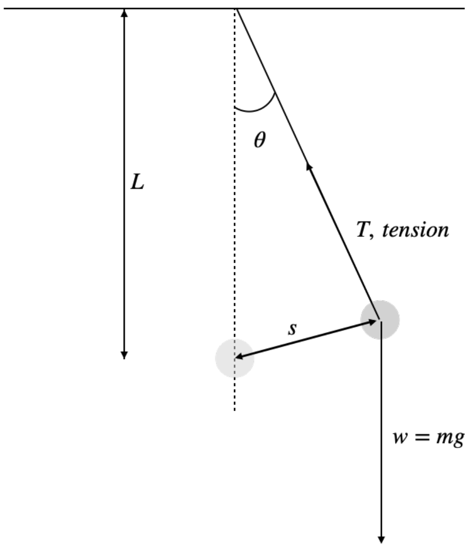
$$-k(e+y) + mg = ma$$

$$-ky = ma$$

hence,

$$a = -\left(\frac{k}{m}\right)y,\ \omega = \sqrt{\frac{k}{m}},\ T = 2\pi\sqrt{\frac{m}{k}}$$

1.2.3 Simple Pendulum



$$F_{restoring} = -mgsin\theta$$

For small θ $\sin\theta \cong \theta$, hence

$$F_{restoring} \cong -mg\theta = -mg\frac{s}{L}$$

By Newton's Second Law

$$F_{restoring} = ma$$
$$-mg\frac{s}{L} = ma$$

Hence,

$$a = -\left(\frac{g}{L}\right)s, \quad \omega = \sqrt{\frac{g}{L}}, \quad T = 2\pi\sqrt{\frac{L}{g}}$$

2 Energy in Simple Harmonic Motion

For a horizontal spring mass system
KE is given by

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}m\left(\pm\omega\sqrt{x_0^2 - x^2}\right)^2 \\ &= \frac{1}{2}m\omega^2\left(x_0^2 - x^2\right) \end{aligned}$$

PE is given by

$$E_p = \frac{1}{2}kx^2$$

and since $\omega = \sqrt{\frac{k}{m}}$,

$$E_p = \frac{1}{2}m\omega^2x^2$$

Total energy is thus

$$E = \frac{1}{2}m\omega^2x^2$$

2.1 Variations of energy with time

suppose $x = x_0 cos\omega t$, substituting $v = -x_0\omega sin\omega t$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_0^2 sin^2\omega t$$

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x_0^2 cos^2\omega t$$

3 Damping and forced oscillations

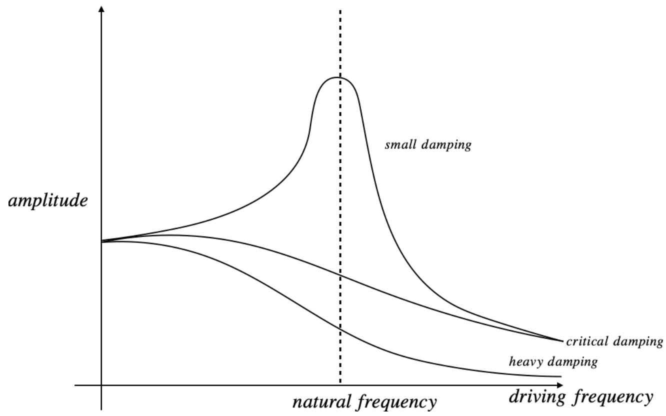
3.1 Damping

Damping is the process where energy is removed from an oscillating system

- **Light damping**: amplitude decays exponentially with time. Frequency decreases
- **Critical Damping**: results in no oscillation, and system returns to equilibrium position in the **shortest time**
- **Heavy Damping**: results in no oscillations and the system takes a long time to return to equilibrium position

3.2 Forced oscillation and resonance

Forced oscillations are produced when a body is subjected to a periodic external driving force



Resonance occurs when a system responds at maximum frequency to an external driving force. This occurs when the frequency of the driving force is equal to the natural frequency of the driving system

Circumstances where resonance is useful

- Radio receiver
- Acoustic resonance for musical instruments
- Magnet resonance in MRI

Circumstances where resonance is undesirable

- Mechanical damange to structures

Topic 9 - First Law of Thermodynamics

1 Specific heat capacity and specific latent heat

1.1 Heat capacity

The numerical value of the heat capacity of a body is the quanttity of heat required to raise the temperature of the body by one degree

$$C = \frac{Q}{\Delta T}$$

1.2 Specific heat capacity

the numerical value of the specific heat capacity of a substance is the quantity of heat required to raise the temperature of the unit mass of the substance by one degree

$$c = \frac{Q}{m\Delta T}$$

1.3 Determination of specific heat capacity

For solids, since

$$IVt = mc(T_f - T_i)$$

$$c = \frac{IVt}{m(T_f - T_i)}$$

For gases or liquids, to account for heat loss

$$\text{electrical energy supplied} = \text{heat transferred to liquid} + \text{HEAT LOSS to surrounding}$$

$$IV \times t = mc(T_{out} - T_{in}) + H$$

$$IV = \frac{m}{t}c(T_{out} - T_{in}) + \frac{H}{t}$$

rate of heat loss $\frac{H}{t}$ is proportional to the excess temperature of the apparatus.

using a different set of I, V and flow rate,

$$I'V' = \frac{m'}{t'}c(T_{out} - T_{in}) + \frac{H'}{t'}$$

using the two sets of equations,

$$IV - I'V' = \left(\frac{m}{t} - \frac{m'}{t'}\right)(T_{out} - T_{in})\ c$$

1.4 Kinetic model of matter

	solid	liquid	gas
packing arrangement of atoms/molecules	closely packed in lattice structure	slightly further apart than solids	very far apart
movement of atoms	vibrations about mean position	random motion throughout liquid	random motion at high speeds
intermolecular forces	strong intermolecular attractive and repulsive forces	attractive forces	negligible intermolecular forces

1.5 Change of phase

	Melting	Boiling
intermolecular interactions	lattice structure starts to break • molecules have enough energy to vibrate so violently that they are no longer held by attractive forces • the lattice structure collapses • change of state occurs	all bonds between atoms/molecules completely broken • thermal energy supplied is used to overcome the attractive forces between molecules • bonds are completely broken • change of state occurs
energy supplied	latent heat of fusion	latent head of vaporisation

standard qualitative questions

1. Why do melting and boiling take place without a change in temperature?
   - at melting: lattice structure collapses due to vibration of molecules and solid uner goes phase change
   - latent heat of fusion will not cause a change in temperature, and is used to overcome the attractive forces between atoms and causes the lattice structure to break
   - during boiling: intermolecular forces are completely broken
   - latent heat of evaporation used to overcome attractive forces between molecules
2. Why is specific latent heat of vaporisation higher than the specific latent heat of fusion for the same substance?
   - To melt a solid, some molecular bonds are broken
   - to vaporise a liquid, all remaining bonds must be broken
   - more bonds broken during vaporisation
   - furthermore, work is needed to do work against the external or atmospheric pressure for gas to expand, since gas occupies a much larger volume than liquid
3. Why is evaporation accompanied by cooling?
   - Kinetic theory supposes that molecules of liquids are at continual random motion and make frequent collisions with each other. During collision, some lose energy and some gain energy
   - if a molecule near the surface gains enough evergy, it will be able to escape from the attractive forces from the molecules below it
   - this results in a decrease in the average KE of the remaining molecule
   - since temperature is a measure of average KE, the liquid becomes coller

1.6 Latent heat

The numerical value of the specific latent heat of fusion is the quantity of heat required to convert unit mass of solid to liquid without any change in temperature

$$l_f = \frac{Q}{m}$$

The numerical value of specific latent heat of fusion is the quantity of ehat required to convert unit masss of liquid to gas without any change in temperature

$$l_v = \frac{Q}{m}$$

1.7 Determination of latent heat

For latent heat of fusion, to account for melting due to heat gained from surrounding

heat from electrical heater + heat gained from surrounding = latent heant of fusion

Let M be the total mass of ice melted and m be the mass of ice melted due to surroundings

$$IVt + ml_f = Ml_f$$

$$IVt = (M - m)l_f$$

$$l_f = \frac{IVt}{M - m}$$

For latent heat of vaporisation, to account for heat loss to the surrounding

$$IV = \frac{m}{t}l_v + \frac{H}{t}$$

$$I'V' = \frac{m'}{t'}l_v + \frac{H'}{t'}$$

$$l_v = \frac{IV - I'V'}{\frac{m}{t} - \frac{m'}{t'}}$$

2 Internal energy

> The internal energy of a system is determined by the state of the system and it is the sum of the random distribution of kinetic and potential energies associated with the molecules of the system
> The internal energy of a system is the sum of KE due to **random motion** of the molecules, and PE associated with the **intermolecular forces of the system**

Microscopic KE

$$\langle E_k \rangle = \frac{3}{2}kT$$

Microscopic PE
Molecules have PE due to intermolecular attraction and repulsion. Solids have the most negative PE
Effect of temperature A rise in temperature implies an increase in average KE, hence an increase in internal energy

2.1 Internal energy of an ideal gas

Since it is assumed that an ideal gas has **no intermolecular forces**, the internal energy of a gas is purely due to translational KE

for one molecule

$$\langle E_k \rangle = \frac{1}{1}m\langle c^2 \rangle = \frac{3}{2}kT$$

For N molecules

$$\text{total } E_k = \frac{1}{2}Nm\langle c^2 \rangle = \frac{3}{2}NkT$$

Since the internal energy U of an ideal gas is purely kinetic

$$U = \text{total } E_k - \frac{3}{2}NkT$$

$$U = \frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}pV$$

The **state** of a system is purely determined by p, V and T

- ΔU is solely dependent on T
- only a change in p or V which results in a change in T will cause a change in state

3 First law of thermodynamics

the **First law of thermodynamics** states that the increase in internal energy of a system is equal to the sum of the heat supplied to the system and the work done on the system, and the internal energy of a system depends only on its state

$$\Delta U = Q + W$$

	Positive (+)	Negative (-)
ΔU	increase in internal energy	decrease in internal energy
Q	Heat absorbed by the system	Heat loss by the system
W	Work done on system (compression)	Work done by system (expansion)

3.1 Work done, W

for a gas in an enclosed system with a frictionless, movable piston of area A, when gas expands, a force F is applied on the piston **by** the gas against external pressure p

work done on the gas is thus

$$W = -\int_{V_i}^{V_f} p dV$$

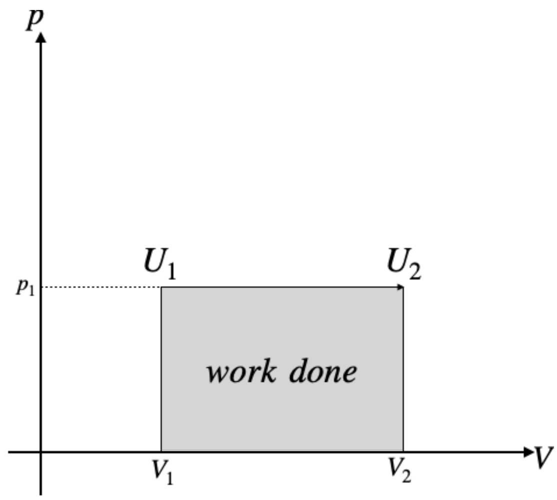
An increase in volume means work done on gas is negative

3.2 pressure-volume graphs

- Internal energy U is dependent on state and change in internal energy
- ΔU is independent of path taken
- Work done W is path dependent, and is the area under the P-V curve

3.3 Isobaric process

Volume changes at constant pressure. Area under curve is rectangular and magnitude of work done is $p\Delta V$



Isobaric expansion

Since volume increases, work done on the gas is negative
Since

$$T \propto pV$$

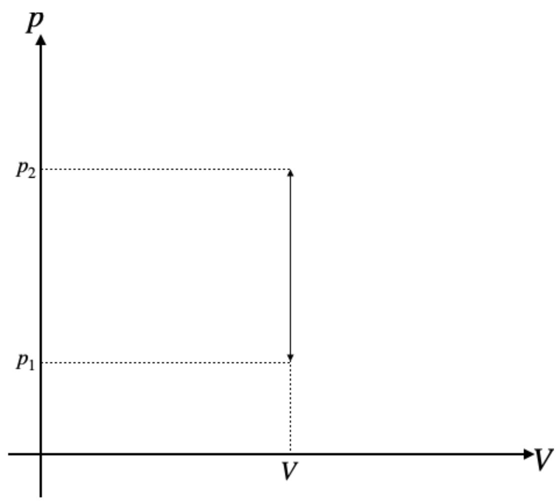
Temperature increases, and hence

$$\Delta U > 0$$

Hence heat is transferred into the system

3.4 Isovolumetric processes

Pressure changes without a change in volume



Isovolumetric process

Since V is constant, $W = 0$

$$\Delta U = Q + 0,\ \Delta U = Q$$

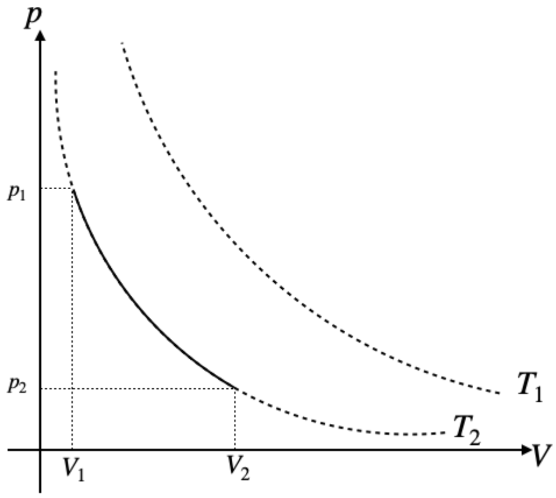
For an increase in pressure,

$$\Delta U > 0$$

$$Q > 0$$

3.5 Isothermal Process

For a system at constant temperature, and heat exchange happens slowly, such that pressure and volume changes occur without a change in temperature



Isothermal expansion

Along each isotherm, T constant, hence

$$\Delta U = 0$$

$$0 = Q + W$$

For an expansion, work is done by the system, hence

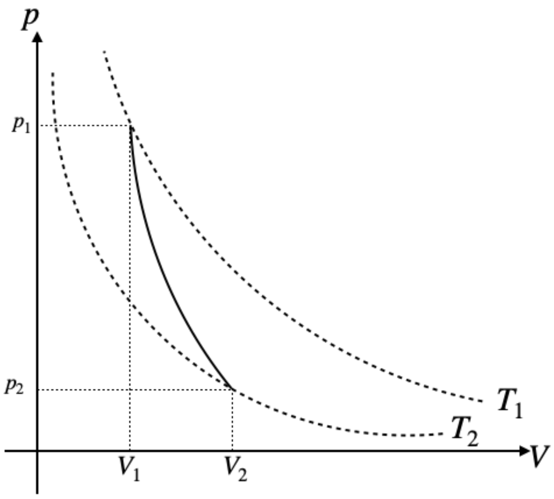
$$W < 0, \ hence \ Q > 0$$

3.6 Adiabetic Process

When a system under goes a change in pressure and volume with no heat being supplied to or lost from the system

This could happen if

- system is insulated
- the change in pressure and volume happens rapidly and hence no heat exchange with the surrouding occurs



For an expansion

$$W < 0$$

pressure decreases, T decreases, hence

$$\Delta U < 0$$

Topic 8 - Temperature and ideal gases

1 Temperature and thermal equilibrium

1.1 Heat

Heat is the energy transferred between objects because of the temperature difference between them

- when 2 objects are ni thermal contact, heat flows from an object of higher temperature to an object at a lower temperature until their temperatures equalize at some intermediate value

1.2 Thermal equilibrium

Two objects are in a state of **thermal equilibrium** when there is **no net flow of heat** between them
Two objects are in thermal equilibrium if and only if they are at the **same temperature**

1.3 Zeroth law of thermal dynamics

if object A and B are each separately in thermal equilibrium with object C, then A and B are also in thermal equilibrium with each other

2 Temperature scales

A temperature scale is a system of measuring temperature based on some thermometric property of a substance.
Criteria for a suitable thermometric property

1. Property **varies continuously and uniquely** with temperature
2. Change in property must be **large** enough to enable accurate measurements of temperature (ensure sensitivity of thermometer)
3. The value of thermometric property at any temperature within its working range must be **reproducible**

2.1 Centigrade scale

Empirical centigrade scale

- lower fixed point, 0°C : **ice point** (temperature of pure ice in equilibrium with water at standard atmospheric pressure of 101.3kPa)
- upper fixed point, 100°C : **steam point** (temperature of steam in equilibrium with boiling water at standard atmospheric pressure of 101.3kPa)
- divide the range into 100 degrees

An empirical scale is established by

1. taking the value of a thermometric property, X, at a ice and steam fized points, X_0 and X_{100}
2. dividing the range of values $(X_{100} - X_0)$ into a number of equal steps / degrees
3. assume that the thermometric property X vaies linearly with temperature, draw a calibration graph of X against temperature, θ
4. Measure thermometric property at unknown temperature θ and call it X_θ the unknown temperature θ in $^\circ C$ is then computed from

$$\theta = \frac{(X_\theta - X_0)}{(X_{100} - X_0)} \times 100^\circ C$$

2.2 Thermodynamic temperature scale

Absolute zero

> the absolute zero is defined as the zero pint (0K) of the thermodynamic temperature scale

Thermodynamic temperature scale Kelvin scale

- lower point: absolute zero
- upper point: triple point of water, (0.01°C, or 275.16K)

> The kelvin is defined as $\frac{1}{275.16}$ of the thermodynamic temperature of the triple point of water

3 Ideal gas equation

Ideal gas is an idealization of the real gas in which potential energy of intermolecular interaction is 0.

> An **ideal gas** is a gas which obeys the equation of state $pV = nRT$ at all pressures, volume, temperatures

3.1 equation of state / ideal gas equation

The ideal gas equation can be derived from

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$p_1V_1 = p_2V_2$$

Hence,

$$pV = nRT$$

where R is a molar gas constant $R = 8.314JK^{-1}mol^{-1}$
Avogadro's constant, N_A is defined as the number of atoms in a 12 grams of a carbon-12 sample

$$N_A = 6.023 \times 10^{23}$$

Alternative forms of the ideal gas equation

$$pV = \frac{N}{N_A}RT$$

$$pV = NkT$$

where $k = \frac{R}{N_A}$ is the Boltzmann constant, $k = 1.38 \times 10^{-23}$

4 Kinetic theory of gases

Molecular model of an ideal gas makes the following assumptions

1. molecules have **negligible volume**
2. molecules exert **no intermolecular forces** on one another, except during collisions
3. molecules move about in random motion in straight lines at constant speed
4. all collisions are completely elastic

5. the molecules of a particular gas are identical
6. there are sufficiently large number of molecules, so only average behaviour need to be considered

Derivation of model

For an ideal gas in a cubic container with length d
Let mass of each molecule be m. For one molecule with velocity c_X in the x-direction, upon colliding with the wall, its velocity changes from c_X to $-c_X$, the change in momentum is given by

$$\Delta p = (-mc_X) - mc_X = -2mc_X$$

Assuming no intermolecular collision, when the particle collides with the wall it would have travelled a distance of $2d$ in the x-direction. the time interval between collisions is thus given by

$$\Delta t = \frac{2d}{c_X}$$

By Newton's second law, the rate of change of momentum is given by

$$F_{\text{on molecule}} = \frac{\Delta P}{\Delta t} = -2mc_X \times \frac{c_X}{2d} = -\frac{mc_X^2}{d}$$

By Newton's third law, the force that the molecule acts on the wall is

$$F_{\text{on wall}} = -F_{\text{on molecule}} = \frac{mc_X^2}{d}$$

Total force is thus

$$F_{tot} = \frac{m}{d}(c_{X1}^2 + c_{X2}^2 + c_{X3}^2 ... c_{XN}^2)$$

The mean-square-speed in the x direction is defined as

$$\langle c_x^2 \rangle = \frac{c_{X1}^2 + c_{X2}^2 + c_{X3}^2 ... c_{XN}^2}{N}$$

Hence

$$F_{tot} = \frac{Nm}{d}\langle c_x^2 \rangle$$

Total pressure p on the wall is thus

$$p = \frac{F}{A} = \frac{F_{tot}}{d^2} = \frac{Nm}{d^3}\langle c_x^2 \rangle = \frac{Nm}{V}\langle c_x^2 \rangle$$

By Pythagoras' theorem

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

Taking the average

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle$$

$$\langle c_x^2 \rangle = \frac{1}{3}\langle c^2 \rangle$$

Substituting,

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

where N is the number of gas molecules
m is the mass of one molecule
$\langle c^2 \rangle$ is the mean-square-speed of the gas

Define **root-mean-square** speed as

$$c_{rms} = \sqrt{\langle c^2 \rangle} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 ... c_N^2}{N}}$$

Combining $pV = NkT$ and $pV = \frac{1}{3} Nm\langle c^2 \rangle$

$$pV = NkT = \frac{1}{3} Nm\langle c^2 \rangle$$

$$\frac{1}{2} m\langle c^2 \rangle = \frac{3}{2} kT$$

The LHS is the average translational KE of one gas molecule

$$\boxed{\langle E_k \rangle = \frac{1}{2} m\langle c^2 \rangle = \frac{1}{2} mc_{rms}^2 = \frac{3}{2} kT}$$

Topic 7 - Gravitational field

1 Gravitational force

Newton's law of gravitation states that two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

$$F = \frac{GMm}{r^2}$$

where G is the gravitational constant with a value of $6.67 \times 10^{-11} Nm^2kg^{-2}$

2 Gravitational field strength

2.1 Gravitational field

A gravitational field is a region of space in which a mass experiences a gravitational force

2.2 Gravitational field strength

The **gravitational field strength** at a point in space is defined as the gravitational force experienced per unit mass at that point

$$g = \frac{F}{m}$$

$$g = \frac{GM}{r^2}$$

note that the resultant gravitational field strength at a point due to more than one mass is the **vector sum** of the individual gravitational field strengths due to each mass

2.3 Gravitational field strength of a uniform sphere

According to the **Shell Theorem**

1. a spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point in its center (behaves like **point mass**
2. If the body is a spherically symmetric shell, no net gravitational force is exerted by the shell on any objects inside the shell
3. for a solid sphere of constant density, gravitational force **varies linearly** with distance from the center

henec for a uniform solid sphere of radius R, the variations of g with distance from the center, r is as follows

- Inside the sphere, for $r \leq R$:
  - mass of the inner sphere of radius r is $M = \rho V = \rho \frac{4}{3} \pi r^3$
  - g due to inne sphere is thus

$$g = \frac{GM}{r^2} = \frac{G}{r^2} \frac{4}{3} \rho \pi r^3 = \frac{4}{3} G \rho \pi r$$

$$g \propto r$$

- Outside the sphere, for $r \geq R$:

$$g = \frac{GM}{r^2}$$

2.4 Gravitational field strength of earth and apparent weight

For an object of mass m held by a force T provided by a spring balance

At polar region	At equator
$F_R = F_g - T$	$F_R = F_g - T$
There is no circular motionm, $F_R = 0$	To provide for centripetal force, $F_R = F_c$
$T = F_g$ spring balance indicates true weight of object	$T = F_g - F_c$ spring balance indicates apparent weight which is smaller than true weight of object
$mg_{freefall} = mg$	$mg_{freefall} = mg - ma_c$

3 Gravitational Potential Energy

the **gravitational potential energy** of a mass at a point in a gravitational field is the work done by an external force in bringing the mass from infinity to that point

$$U = -\frac{GMm}{r}$$

- GPE at infinity is defined as 0
- work done by external force in bringing an object from infinity to r is

$$W = \int_{\infty}^{r} F_{ext} dr = \int_{\infty}^{r} \frac{GMm}{r^2} = -\frac{GMm}{r}$$

- U and ϕ are negative because gravitational force is attractive, hence to bring a mass from infinity to a point a field, the direction of the external force is opposite to the direction of displacement of mass, henec negative work is done by the external force

3.1 Relationship between force and potential energy

$$F = -\frac{dU}{dr}$$

4 Gravitational Potential

the **gravitational potential** at a point in a gravitational field is defined as the work done per unit mass by an external force in bringing a small test mass from infinity to that point

$$\phi = \frac{U}{m} = -\frac{GM}{r}$$

4.1 relationship between field strength and potential

$$g = -\frac{d\phi}{dr}$$

5 Escape Velocity

Escape velocty is the minimum speed needed for the object to escape the gravitational influence of earth

At infinity,

$$E_p = 0,$$

if an object has sufficient energy to just reach infinity, $E_k = 0$, hence

$$E_T = E_p + E_k = 0$$

By principle of conservation of energy, an object with total energy of zero will be able to just reach infinity

For an object of mass m with initial velocity v, its initial energy is given by

$$E_p = U = -\frac{GMm}{r_{earth}}$$

$$E_k = \frac{1}{2}mv^2$$

Energy required to reach infinity is such that

$$E_T \geq 0$$

$$E_p + E_k \geq 0$$

$$-\frac{GMm}{r_{earth}} + \frac{1}{2}mv^2 \geq 0$$

Hence escape velocity is

$$v \geq \sqrt{\frac{2GM}{R_{earth}}}$$

$$v \geq \sqrt{2gR_{earth}}$$

6 Circular Orbits

recall that for an object in circular motion with mass m and linear velocity v and angular velocity ω

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

and that the time for one complete revolution is the period T, given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

6.1 Planetary motion and Kepler's third law

for a planet in orbit of the Sun, gravitational force F_g provides for centripetal force F_c hence

$$F_g = F_c$$

$$\frac{GMm}{r^2} = mr\omega^2$$

$$\frac{GMm}{r^2} = mr\left(\frac{2\pi}{T}\right)^2$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T^2 \propto r^3$$

6.2 Satellite motions

Energy of a satellite
Since gravitational force provides for centripetal force

$$F_g = F_c$$
$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$
$$v^2 = \frac{GM}{r}$$

hence KE is given by

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$$

GPE is given by

$$E_p = -\frac{GMm}{r}$$

Total energy is thus

$$E_p + E_k = -\frac{GMm}{2r}$$

Geostationary orbits is one that remains at a fixed position in the sky as viewed from any location on earth's surface, satistying the following conditions

1. Its orbital period is the same as that of Earth about its axis of rotation (24 hours)
2. Its direction of rotation is the same as that of Earth (west to east)
3. its plane of orbit lies in the same plane as the equator

Based on these conditions, using

$$\frac{GMm}{r^2} = mr\left(\frac{2\pi}{T}\right)^2$$

$$r = \frac{T^2GM}{4\pi^2}^{\frac{1}{3}} = 42250km$$

Advantages	Disadvantages
1. continuous surveillance of the region underneath 2. easy for communicating with ground station as it is permanently in view. no adjustment of ground absed antenna necessary 3. due to high altitude, satellites can transmit and receive signals over a large range	distance from earth surface is large, leading to 1. significant loss of signal strengths 2. poorer resolution in imaging satellites 3. time lag in telecommunication

6.3 Binary star systems

For two stars of mass M and m, separated by a distance d, show that their period of circular motion is

$$T^2 = \frac{4\pi^2 d^3}{G(M+m)}$$

both stars revolve around some center of mass C, such that their respective radius of circular motion is r_1 and r_2, and they have the same angular velocity ω

the gravitational force between them is given by

$$F_g = \frac{GMm}{d^2}$$

hence

$$\frac{GMm}{d^2} = m\omega r_1^2,\ and\ \frac{GMm}{d^2} = M\omega r_2^2$$

$$mr_1 = Mr_2$$

$$r_2 = \frac{m}{M}r_2$$

$$d = r_1 + r_2 = r_1 + \frac{m}{M}r_1 = r_1\left(1 + \frac{m}{M}\right)$$

substituting $r_1 = \frac{d}{1+\frac{m}{M}}$

$$\frac{GMm}{d^2} = m\left(\frac{2\pi}{T}\right)^2 r_1$$

$$T^2 = \frac{4\pi^2 d^2}{GM}\frac{d}{1+\frac{m}{M}} = \frac{4\pi^2 d^3}{G(M+m)}$$

Topic 6 - Circular Motion

1 Kinematics of circular motion

Angular displacement is the angle which an object makes with reference to a line

- the unit of angular displacement is **radian**
- the radian is the angle subtended by an arc length equal to the radius of the arc
- any angle θ measured in radians is defined by

$$\theta = \frac{s}{r}$$

where s is the arc length and r is the radius of circle

Angular velocity ω of a body is defined as the rate of change of angular velocty wrt time

$$\omega = \frac{d\theta}{dt}$$

Period, T of an object in circular motion is the time taken for one complete revolution **Frequency, F** of an object in circular motion is the number of complete revolutions made per unit time

$$f = \frac{i}{T}$$

Relationship between angular velocity and linear velocity
For an object in uniform circular motion at angular velocity ω with radius r and linear velocity v at any point in its path

since

$$s = r\theta$$

differetiating wrt to t on both sides

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

hence

$$v = rw$$

relationship between period, angular velocity and frequency
since time taken for one complete revoltion is period T,
and the angular displacement for one complete revolution is 2π

$$\omega = \frac{2\pi}{T} = 2\pi f$$

relationship between linear speed and period

$$v = \frac{\text{circumference of the circle}}{\text{period}} = \frac{2\pi r}{T}$$

2 Dynamics of circular motion

For an object in uniform circular motion

- **object experiences a force because** although object is moving at constant speed, its velocity is constantly changing because its direction of motion is constantly changing. Hence it means object is undergoing acceleration and hence it must be acted upon by a force
- the force or acceleration is **perpendicular to the motion of the object towards the center of the circle**, since the object is moving at constant speed there must not be any component of the force in the direction of the motion of the object, otherwise it will increase or decrease the speed of the object. Hence any force / acceleration must be perpendicular to the direction of motion

Centripetal acceleration and centripetal force
the magnitude of centripetal acceleration is

$$a = v\omega = r\omega^2 = \frac{v^2}{r}$$

and since $F = ma$, the centripetal force acting on the object is

$$F = ma = mv\omega = mr\omega^2 = \frac{mv^2}{r}$$

Topic 5 - Work, Energy, Power

1 Work

1.1 Work done by a constant force

For a constant force F that acts at angle θ to the horizontal and displaces an object horizontally to the right over a displacement s, the work done by the constant force is given by

$$W = Fs\ cos\theta$$

Work done by a constant force is the product of the force and the displacement in the direction of the force

- Work is a **scalar** quantity, and its S.I. unit is the joule(J)
- 1 J is the work done by a force of 1 N when an object is displaced by 1 m in the direction of the force

1.2 Work done by a variable force

Work done by a variable force is equal to the **area under the force-displacement graph**

$$W = \int_{s_1}^{s_2} F ds$$

1.3 Work done by an external force on spring

By Hooke's Law, the external force needed to produce an extension or compression x in a spring (that has not exceeded its limit of proportionality) is $F = kx$
wor done in stretching the spring by x is the area under the force-extension graph is

$$W = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

1.4 Work done by a gas

For a system of gas in a cylinder with a frictionless piston, if the gas is heated such that it expands slowly at constant pressure, a force F is applied on the piston by the gas molecules to expand against external pressure

The work done by the gas is the work done by force F in displacing the piston of cross sectional area A through a small distance Δx

$$W_{gas} = F\Delta x = pA\Delta x$$

And since $A\Delta x = \Delta V$

$$W_{gas} = p\Delta V$$

- when the gas expands, ΔV is positive, and hence work done by gas is positive
- when the gas contracts, ΔV is negative and work done by gas is negative (work is hence done **on** gas)

2 Energy

2.1 Forms of energy

Name	Form of energy
Chemical potential energy	energy related to the structural arrangement of atoms or molecules in a substance
Nuclear energy	energy released from atomic nuclei
Electrical energy	energy possessed by charge carriers moving under the influence of a potential difference
Internal energy	the sum of microscopic KE (associated with random motion) and PE (associated with interatomic of intermolecular forces)
Gravitational PE	energy due to the position of a mass in a gravtitational field
Electrical PE	energy due to the position of a charge in an electric field of another charge / sytem of charges
Elastic PE	energy stored due to the stretching or compressing of an object
Kinetic energy	energy due to motion of a body

2.2 Kinetic energy

In general, the KE of a body of mass m moving with velocity v can be expressed as

$$E_k = \frac{1}{2}mv^2$$

Derivation of KE
Consider a body of mass m moving with initial velocity u and accelerated by a constant force F. The body under goes constant acceleration a to a final velocty v over displacement s

Since force is in the direction of displacement

$$W = Fs = (ma)s$$

Since $v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a}$$

substituting into $W = (ma)s$

$$W = ma\left(\frac{v^2 - u^2}{2a}\right)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = \Delta E_k = E_{k,\ final} - E_{k,\ initial}$$

This is known as the **work-energy theorem**, where the net work done by all forces acting on a body is equal to the change in the KE of the body

2.3 Potential energy

Potential energy, usually expressed as U is the energy due to the **position or shape** of an object. The calculation of PE ususally requires a reference point with is defined to have a PE of 0

2.4 Gravitational potential energy

The GPE of an object of mass m and at height h above the surface of the earth is

$$E_p = mgh$$

where g is the aceeleration of free fall near Earth's surface
Derivation of GPE
Consider an object being raised upwards at constant velocity, from height h_1 to height h_2

Since v constant, $a = 0$, and the force required is $F = wg$
Work done by F in displacing the body upwards is

$$\begin{aligned} F &= Fs \\ &= mg(h_2 - h_1) \\ &= mgh_2 - mgh_1 \end{aligned}$$

2.5 Elastic potential energy

From earlier, work done in compressing or extending a spring is

$$W = \frac{1}{2}kx^2$$

The elastic potential energy stored is thus

$$U_E = \frac{1}{2}kx^2$$

2.6 Relationship between force and potential energy

For a field of force, the relationship bewteen F and PE U for one dimensional motion is given by

$$F = -\frac{dU}{dx}$$

- the magnitude of a force at point x is equal to the **gradient of the PE curve at x**
- the direction of the force is in the direction of decreasing potential energy

3 Energy conversion and conservation

3.1 Law of conservation of energy

The **law of conservation of energy** states that energy cannot be created or destroyed, it can only be converted from one form to another

3.2 Total mechanical energy and work done on a system

In a non-isolated system where work is done by an external force, W_F, by law of conservation of energy

$$(E_p + E_k)_{initial} + W_F = (E_p + E_k)_{final}$$

- the sum of potential and kinetic energy is aclled mechanical energy

$$\text{Mechanical energy} = E_p + E_k$$

- in an isolated system, mechanical energy is conserved
- in a non-isolated system, W_F may be positive or negative

3.3 Efficiency

$$\text{Efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100$$

4 Power

Power is define as the rate of work or energy conversion wrt time

$$P = \frac{dW}{dt}$$

If total work ΔW is done over time interval Δt, then average power is

$$\langle P \rangle = \frac{\Delta W}{\Delta t}$$

4.1 Relationship between P, F, and v for a constant force

If a constnat force F is applied and does work by moving an object over displacement s parallel to the force in time t, then the power can be found by

$$P = \frac{dW}{dt} = \frac{d(FS)}{dt} = F\frac{ds}{dt}$$

$$P = Fv$$

4.2 Wind turbines and related calculations

KE removed from wind is converted into electrical energy
Mass of air that passes through the area swept by the blades per second is

$$\text{Mass per second} = \frac{dm}{dt} = \frac{d(\rho v)}{dt} = \rho\frac{d(Ax)}{dt} = \rho A v$$

Loss of KE per second is

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Topic 4 - Forces

1 Hooke's Law

A string or wire when stretched is found to obey Hooke's Law up to its limit of proportionality

Hooke's Law states that the extension of a body is proportional to the applied load if the limit of proportionality is not exceeded

$$F = kx$$

Energy stored in the spring = work done in stretching the spring = Area unnder the force-extension graph

$$\text{Energy stored} = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

2 Fluid Statics

2.1 Density

Density of a substance is defined as its **mass per unit volume**

$$\rho = \frac{m}{V}$$

2.2 Pressure

Pressure is defined as the force per unit area, where the force is acting at right angles to the area

generally,

$$p = \frac{F}{A}$$

Pressure in a fluid, or hydrostatic pressure is given by

$$p = \rho g h$$

total pressure at a given depth below the water is thus

$$P = p_{at} + \rho g h$$

2.3 Upthrust

For a cylindrical object of cross sectional area A submerged in a liquid of density ρ

The downward force on the top of the object due to fluid is

$$F_{top} = p_1 A = \rho g h_1 A$$

The upward force on the bottom of the object due to fluid is

$$F_{bottom} = p_2 A = \rho g (h_1 + H) A$$

Since upward force is greater than the downward force, there is thus a net force on the object

$$\begin{aligned} F_{net} &= F_{bottom} - F_{top} \\ &= \rho g A H \\ &= \rho g V \\ &= mg, \text{ since } \rho = \frac{m}{V} \text{ where } m \text{ is the mass of } \textbf{fluid} \text{ displaced} \end{aligned}$$

The force of upthrust is the vertical upward force exerted on a body by a fluid when it is fully or partially submerged in the fluid due to the difference in fluid pressure

Archimedes Principle states that the buoyang force or upthrust is equal in magnitude and opposite in direction to the weight of the fluid that is displaced by a submerged or floating object

3 Viscous force

Viscous forces arise due to the collisions of an object which is moving through a fluid with the molecules of the fluid.
Recall section on air resistance and net force on an object falling due to gravity from Topic 2 - Kinematics

4 Equilibrium of forces

For an object to be in equilibrium, it must be in both translational and rotational equilibrium

The conditions for equilibrium

1. Resultant force on the object is zero
2. Resultant moment on the object about any axis is zero

4.1 1st Condition - Translational Equilibrium

$$\sum F_x = 0$$

$$\sum F_y = 0$$

4.2 2nd Condition - Rotational Equilibrium

Principle of moments:
for any body in rotational equilibrium, the sum of all clockwise moment about any axis must be equal to the sum of all anticlockwise moment about the same axis

Turning effect of a force is called its moment

the **moment** of a force about a point is defined as the produt of the force and the perpendicular distance from the point to the line of action of the force

Torque of a couple

A couple consists of a pair of equal and opposite forces whose lines of action do not coincide
The torque of a couple is the product of one force and the perpendicular distance between the two forces

Note: when only three coplanar forces act on a body in equilibrium, their lines of action must either all be parallel or they meet at a point

5 Center of gravity

The **center of gravity** of a body is the point at which its weight or the resultant of the distributed gravitational attraction on the body appears to act.

Topic 3 - Dynamics

1 Newton's first law of motion

Newton's First law of motion: a body continues in its state of rest or uniform motion in a straight line unless acted upon by a resultant external force

1.1 Inertia

The inertia of a body is its reluctance to a change in motion

The mass m of a body is an intrinsic property which resists change in motion

1.2 Equilirbium

the conditions necessary for equilibrium of a body are

- resultant force acting on body is 0
- resultant torque on the body about any axis is 0

2 Newton's second law of motion and Momentum

Newton's second law of motion: rate of change of momentum of a body is proportional to the resultant force acting on it and occurs in the direction of the force

$$F_{net} \propto \frac{d(p)}{dt}$$

2.1 Momentum

Momentum of a body is defined as the product of its mass m and velocity v, ie

$$p = mv$$

2.2 Newton's second law

$$\begin{aligned}
F_{net} &\propto \frac{d(p)}{dt} \\
F_{net} &= k\frac{d(p)}{dt}, \text{ and since k = 1 when quantities in S.I. units,} \\
F_{net} &= \frac{d(p)}{dt} = \frac{d(mv)}{dt} \\
&= m\frac{dv}{dt} + v\frac{dm}{dt}
\end{aligned}$$

Hence

- when mass is constant, $F_{net} = m\frac{dv}{dt}$
- when velocity is constamt, $F_{net} = v\frac{dm}{dt}$

3 Newton's third law of motion

Newton's Third law of motion: if body A exerts a force on body B, then body B exerts an equal but opposite force on A

$$|F_{by\ A\ on\ B}| = |F_{by\ B\ on\ A}|$$

$$F_{AB} = -F_{BA}$$

4 Impulse and momentum change

Impulse is equal to the change in momentum, defined as the product of a force F acting on an object and the time Δt for which the force acts

$$F = \frac{\Delta p}{\Delta t}$$

$$\Delta p = F\Delta t$$

Impulse can be found as the area under the $F - t$ graph

5 Collisions and Conservation of momentum

By Newton's Third Law, when A collides with B,

$$F_{BA} = -F_{AB}$$

since time interval dt is the same, impulse on A is opposite to impulse on B

$$\int_{t_i}^{t_f} F_{BA} dt = -\int_{t_i}^{t_f} F_{AB} dt$$

since impulse = change in momentum

$$\Delta p_a = \Delta p_b$$

$$m_A v_A + m_A u_A = -(m_B v_B - m_B u_B)$$

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

net initial momentum of bodies = net final momentum of bodies

The principle of conservation of linear momentum: when bodies in a system interact, total momentum of the system remains constant, provided no net external force acts on the system

$$\sum m_i u_i = \sum m_i v_i$$

Types of collision

- **Elastic collision** total KE conserved
- **Inelastic collision** total KE not conserved
- **Completely inelastic collision** particles have the same final velocity (particles stick together and move off together)

for elastic collision, by principle of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_1 v_2$$

and since that total KE remains constant

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

rearranging the expressions

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

dividing the equations to remove m_1 and m_2

$$\frac{u_1^2 - v_1^2}{u_1 - v_1} = \frac{u_2^2 - v2^2}{v_2 - u_2}$$

$$\frac{(u_1 + v_1)(u_1 - v_1)}{u_1 - v_1} = \frac{(v_2 + u_2)(v_2 - u_2)}{v_2 - u_2}$$
$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1$$

relative speed of approach = relative speed of separation

Topic 2 - Kinematics

1 Basic quantities

Scalar	Vector
Distance, x the total length of path followed by object	**Displacement, s** The distance moved in a specified direction from a reference point
speed, v instantaneous speed is the rate of change of distance wrt time $v = \frac{dx}{dt}$	**velocity, v** instantaneous velocity is the rate of change of displacement wrt time $v = \frac{ds}{dt}$
average speed is the total distance travelled over total time taken $$\langle v \rangle = \frac{\Delta x}{\Delta t}$$	average velocity is the total change in displacement over total time taken $$\langle v \rangle = \frac{\Delta x}{\Delta t}$$
	Acceleration, a instantaneous acceleration is the rate of change of velocity wrt time $$a = \frac{dv}{dt}$$
	Average acceleration is the total change in velocity over total time $$\langle a \rangle = \frac{\Delta v}{\Delta t}$$

2 Equations for uniformly accelerated motion

$$v = u + at \quad (1)$$
$$s = \frac{1}{2}(u + v)t \quad (2)$$
$$s = ut + \frac{1}{2}at^2 \quad (3)$$
$$v^2 = u^2 + 2as \quad (4)$$

3 kinematics of free fall and the effect of air resistance

objects in the uniform gravitational field of earth undergo uniformly accelerated motion downwards, and experience a constant acceleration with magnitude

$$g = 9.81ms^{-2}$$

Objects experience air resistance, whose magnitude is proportional to velocity and whose direction is opposite to velocity

- on an object's way up, it experiences air resistance in the direction of downward acceleration due to gravity, hence
$$a_{up} > g$$
- on its way down, it experiences air resistance opposite gravity, hence
$$a_{down} < g$$

4 non-linear motion

Topic 1 - Measurement

1 S.I. Base quantities and units

Base quantities are fundamental physical quantities used to define other physical quantities.
The Systeme Internationale d'Unites is based on seven base quantities

base quantity	base unit	Symbol
time	second	s
legnth	metre	m
mass	kilogram	kg
current	ampere	A
temperate	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

2 Derived quantities and their units

Derived quantities are physical quantities formed by combining base quantities. Derived units are products and/or quotients of base units.
Derived quantities are formed from base quantities according to a defining equation.
For example

Derived quantity	Defining equation	Base units	Derived unit	Symbol
force	force = mass $\times$ acceleration	$kgms^{-2}$	newton	N

3 Homogeneity of Equations

Only quantities with the same base units can be added, subtracted or equated, i.e.

$$A(B + C) = DE$$

AB, AC, and DE have the same units

When each of the terms in an equation has the same base units, the equation is **homogeneous** or **dimensionally consistent**

An equation can be dimensionally consistent but not physically correct due to

- wrong coefficients / signs
- missing / additional terms

4 Errors and Uncertaintines

4.1 Measuring instruments and their associated uncertainties

The precision of a instrument is determined by the number of **significant figures** in its measurements, which is in turn determined by **the smallest scale division**

Physical Quantity	Instrument	Precision
Length	metre rule	0.1cm
	vernier calipers	0.01cm
	micrometer screw gauge	0.001cm
time	digital stopwatch	0.01s
mass	electronic balance	0.01g

4.2 Systematic and random errors

Systematic errors result in all readings being either always or above true value by a **fixed amount**

- can be eliminated if source of error is known
- such as accounting for zero error

random errors result in readings being scattered about true value, with erros having equal probability of being positive or negative
Random errors can be **reduced** by

- repeating the measurement and taking average value item plotting a graph and drawing line of best fit

4.3 Accuracy vs Precision

Accuracy

- degree of closeness of readings/mean reading to actual value
- affected by systematic error

Precision

- degree of agreement between repeated measurements
- affected by random error

	Accuracy	Precision
Instrument	Calibration of instrument	smallest scale division
Measurements	Closeness of mean to true value	closeness of measurements to one another

5 Derived uncertainties

For 2 independent measurements of X and Y, X_1 Y_1, each with uncertainty ΔX and ΔY let variable Z be $X \times Y$, find the associated uncertainty ΔZ

5.1 the upper-lower bound method of uncertainty propagation

$$\begin{aligned}\Delta Z &= \frac{Z_{max} - Z_{min}}{2} \\ &= \frac{X_{max} \times Y_{max} - X_{min} \times Y_{min}}{2}\end{aligned}$$

5.2 calcualtion of uncertainties of derived quantities

$$\begin{array}{r|c}
Q = aX \pm bY & \Delta Q = |a|\Delta X + |b|\Delta Y \\
Q = aX \times Y & \frac{\Delta Q}{Q} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y} \\
Q = a\frac{X}{Y} & \frac{\Delta Q}{Q} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y} \\
Q = aX^m \times bY^n & \frac{\Delta Q}{Q} = |m|\frac{\Delta X}{X} + |n|\frac{\Delta Y}{Y} \\
Q = \frac{aX^m}{bY^n} & \frac{\Delta Q}{Q} = |m|\frac{\Delta X}{X} + |n|\frac{\Delta Y}{Y}
\end{array}$$

absolute uncertainty ΔQ are expressed to **1sf**

fractional uncertainty $\frac{\Delta Q}{Q}$ are expressed to **2sf**

percentage uncertainty of x is

$$\frac{\Delta Q}{Q} \times 100\%$$