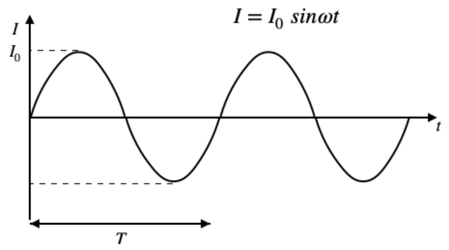
Topic 18 - Alternating Currents

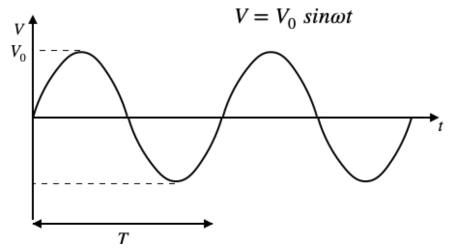
1 Characteristics of AC

An alternating current **varies periodically** with time in magnitude and direction.

Term	Definition
Period, T	Time taken for one complete cycle
Frequency, F	Number of complete cycles per unit time
Peak current, I_0	Amplitude of current

2 Sinusoidal a.c.





3 Power in a.c. circuit

For a simple resistive circuit with resistor R, and an a.c. source,

$$V_{ac} = I_{ac}R$$

$$P_{ac} = I_{ac}V_{ac} =_{ac}^2 R = \frac{V_{ac}^2}{R}$$

- Energy dissipated is the area under the $P_{ac} - t$ graph
- Power at any moment is equal to the product of I and V, hence P varies periodically with time,
- $P_{min} = 0$ when $I = V = 0$
- $P_0 = I_0 V_0$

3.1 Mean power, $\langle P_{ac} \rangle$

$$\langle P_{ac} \rangle = \frac{\text{Total energy dissipated in time t}}{t}$$

$$\langle P_{ac} \rangle = \frac{\text{area under } P_{ac} - t \text{ graph in time } t}{t}$$

$$\langle P_{ac} \rangle = (I_{rms})^2 R$$

The r.m.s. value of an alternating current or voltage is the value of a **steady** direct current or voltage that would produce thermal energy at the **same rate** in a given resistor

$$\langle P \rangle = I_{rms}^2 R = \frac{V_{rms}^2}{R} = I_{rms}V_{rms}$$

The rms value can be found by

1. Squaring the instantaneous current I

2. finding **mean** value of I^2

3. Taking the **square root** of the mean value

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\langle I^2 \rangle}$$

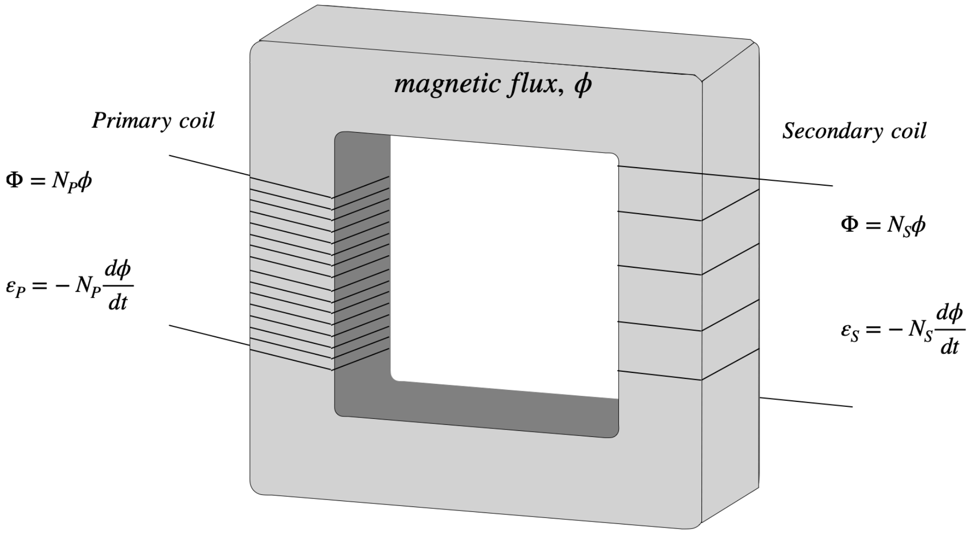
For a sinusoidal current and voltage

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\langle P \rangle = I_{rms} V_{rms} = \left(\frac{I_0}{\sqrt{2}}\right)\left(\frac{V_0}{\sqrt{2}}\right) = \frac{P_0}{2}$$

4 Transformer



- **Function**: to use mutual electromagnetic induction to step up or step down voltage
- a common iron core is used to concentrate magnetic flux through both coils
- AC current flows in primary coil, setting up an alternating magnetic flux in the iron core
- According to Faraday's law, the alternating magnetic flux linkage through both coils induces an alternating e.m.f. across each turn in both coils

4.1 Derivation of results

since the magnetic flux ϕ is the same **through each turn** for both coils, the induced emf is the same through each turn

$$\frac{\varepsilon_P}{N_P} = \frac{\varepsilon_S}{N_S}$$

$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$$

Assuming 0 resistance for an ideal transformer,

$$\frac{V_S}{V_P} = \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P}$$

Assuming no power loss for an ideal transformer, input power = output power hence

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

4.2 Power loss and design features

Cause of power loss	Design features
Joule heating of cooper wires	Thick copper wires of low resistance used
Heating due to **eddy currents** in iron core	The iron core is made of laminated sheets curring across path of eddy currents, to increase resistance to current flow
Hysteresis loss	Soft iron is used, which can be easily magnetised and demagnetised
Magnetic flux leakage	Iron core maximises flux linkage, E-I shaped iron core is used

5 Transmission of electrical power

For a supply power P, transmitting at voltage V and current I, and total cable resistance R, the power loss is

$$P_{loss} = I^2R = \left(\frac{P}{V}\right)^2 R$$

- hence a high voltage is used to minimise power loss

Alternatively, the power loss can be found by first finding p.d. across cable

$$V_{cable} = IR = \left(\frac{P}{V}\right) R$$

Power loss is the power dissipated in cable

$$P_{loss} = I^2R = \frac{V_{cable}^2}{R} = \left(\frac{P}{V}\right)^2 R$$

6 Rectification with diode