

## Topic 8 - Temperature and ideal gases

### 1 Temperature and thermal equilibrium

#### 1.1 Heat

**Heat** is the energy transferred between objects because of the temperature difference between them

- when 2 objects are in thermal contact, heat flows from an object of higher temperature to an object at a lower temperature until their temperatures equalize at some intermediate value

#### 1.2 Thermal equilibrium

Two objects are in a state of **thermal equilibrium** when there is **no net flow of heat** between them

Two objects are in thermal equilibrium if and only if they are at the **same temperature**

#### 1.3 Zeroth law of thermal dynamics

if object A and B are each separately in thermal equilibrium with object C, then A and B are also in thermal equilibrium with each other

### 2 Temperature scales

A temperature scale is a system of measuring temperature based on some thermometric property of a substance.

Criteria for a suitable thermometric property

1. Property **varies continuously and uniquely** with temperature
2. Change in property must be **large** enough to enable accurate measurements of temperature (ensure sensitivity of thermometer)
3. The value of thermometric property at any temperature within its working range must be **reproducible**

#### 2.1 Centigrade scale

Empirical centigrade scale

- lower fixed point,  $0^{\circ}\text{C}$  : **ice point** (temperature of pure ice in equilibrium with water at standard atmospheric pressure of 101.3kPa)
- upper fixed point,  $100^{\circ}\text{C}$  : **steam point** (temperature of steam in equilibrium with boiling water at standard atmospheric pressure of 101.3kPa)
- divide the range into 100 degrees

An empirical scale is established by

1. taking the value of a thermometric property,  $X$ , at the ice and steam fixed points,  $X_0$  and  $X_{100}$
2. dividing the range of values  $(X_{100} - X_0)$  into a number of equal steps / degrees
3. assume that the thermometric property  $X$  varies linearly with temperature, draw a calibration graph of  $X$  against temperature,  $\theta$
4. Measure thermometric property at unknown temperature  $\theta$  and call it  $X_{\theta}$  the unknown temperature  $\theta$  in  $^{\circ}\text{C}$  is then computed from

$$\theta = \frac{(X_{\theta} - X_0)}{(X_{100} - X_0)} \times 100^{\circ}\text{C}$$

## 2.2 Thermodynamic temperature scale

### Absolute zero

the absolute zero is defined as the zero point (0K) of the thermodynamic temperature scale

### Thermodynamic temperature scale Kelvin scale

- lower point: absolute zero
- upper point: triple point of water, (0.01°C, or 273.16K)

The kelvin is defined as  $\frac{1}{273.16}$  of the thermodynamic temperature of the triple point of water

## 3 Ideal gas equation

Ideal gas is an idealization of the real gas in which potential energy of intermolecular interaction is 0.

An **ideal gas** is a gas which obeys the equation of state  $pV = nRT$  at all pressures, volume, temperatures

### 3.1 equation of state / ideal gas equation

The ideal gas equation can be derived from

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$
$$p_1 V_1 = p_2 V_2$$

Hence,

$$pV = nRT$$

where R is a molar gas constant  $R = 8.314 JK^{-1} mol^{-1}$

**Avogadro's constant**,  $N_A$  is defined as the number of atoms in a 12 grams of a carbon-12 sample

$$N_A = 6.023 \times 10^{23}$$

### Alternative forms of the ideal gas equation

$$pV = \frac{N}{N_A} RT$$

$$pV = NkT$$

where  $k = \frac{R}{N_A}$  is the Boltzmann constant,  $k = 1.38 \times 10^{-23}$

## 4 Kinetic theory of gases

Molecular model of an ideal gas makes the following assumptions

1. molecules have **negligible volume**
2. molecules exert **no intermolecular forces** on one another, except during collisions
3. molecules move about in random motion in straight lines at constant speed
4. all collisions are completely elastic

5. the molecules of a particular gas are identical

6. there are sufficiently large number of molecules, so only average behaviour need to be considered

### Derivation of model

For an ideal gas in a cubic container with length  $d$

Let mass of each molecule be  $m$ . For one molecule with velocity  $c_X$  in the x-direction, upon colliding with the wall, its velocity changes from  $c_X$  to  $-c_X$ , the change in momentum is given by

$$\Delta p = (-mc_X) - mc_X = -2mc_X$$

Assuming no intermolecular collision, when the particle collides with the wall it would have travelled a distance of  $2d$  in the x-direction. the time interval between collisions is thus given by

$$\Delta t = \frac{2d}{c_X}$$

By Newton's second law, the rate of change of momentum is given by

$$F_{\text{on molecule}} = \frac{\Delta P}{\Delta t} = -2mc_X \times \frac{c_X}{2d} = -\frac{mc_X^2}{d}$$

By Newton's third law, the force that the molecule acts on the wall is

$$F_{\text{on wall}} = -F_{\text{on molecule}} = \frac{mc_X^2}{d}$$

Total force is thus

$$F_{\text{tot}} = \frac{m}{d}(c_{X1}^2 + c_{X2}^2 + c_{X3}^2 \dots c_{XN}^2)$$

The mean-square-speed in the x direction is defined as

$$\langle c_x^2 \rangle = \frac{c_{X1}^2 + c_{X2}^2 + c_{X3}^2 \dots c_{XN}^2}{N}$$

Hence

$$F_{\text{tot}} = \frac{Nm}{d} \langle c_x^2 \rangle$$

Total pressure  $p$  on the wall is thus

$$p = \frac{F}{A} = \frac{F_{\text{tot}}}{d^2} = \frac{Nm}{d^3} \langle c_x^2 \rangle = \frac{Nm}{V} \langle c_x^2 \rangle$$

By Pythagoras' theorem

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

Taking the average

$$\begin{aligned} \langle c^2 \rangle &= \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle \\ \langle c_x^2 \rangle &= \frac{1}{3} \langle c^2 \rangle \end{aligned}$$

Substituting,

$$pV = \frac{1}{3} Nm \langle c^2 \rangle$$

where  $N$  is the number of gas molecules  
 $m$  is the mass of one molecule  
 $\langle c^2 \rangle$  is the mean-square-speed of the gas

Define **root-mean-square** speed as

$$c_{rms} = \sqrt{\langle c^2 \rangle} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 \dots c_N^2}{N}}$$

Combining  $pV = NkT$  and  $pV = \frac{1}{3}Nm\langle c^2 \rangle$

$$pV = NkT = \frac{1}{3}Nm\langle c^2 \rangle$$

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

The LHS is the average translational KE of one gas molecule

$$\langle E_k \rangle = \frac{1}{2}m\langle c^2 \rangle = \frac{1}{2}mc_{rms}^2 = \frac{3}{2}kT$$