Wharton
University of Pennsylvania

The Idiosyncratic Volatility Puzzle (v. 2025)

FNCE 2370 Final Project

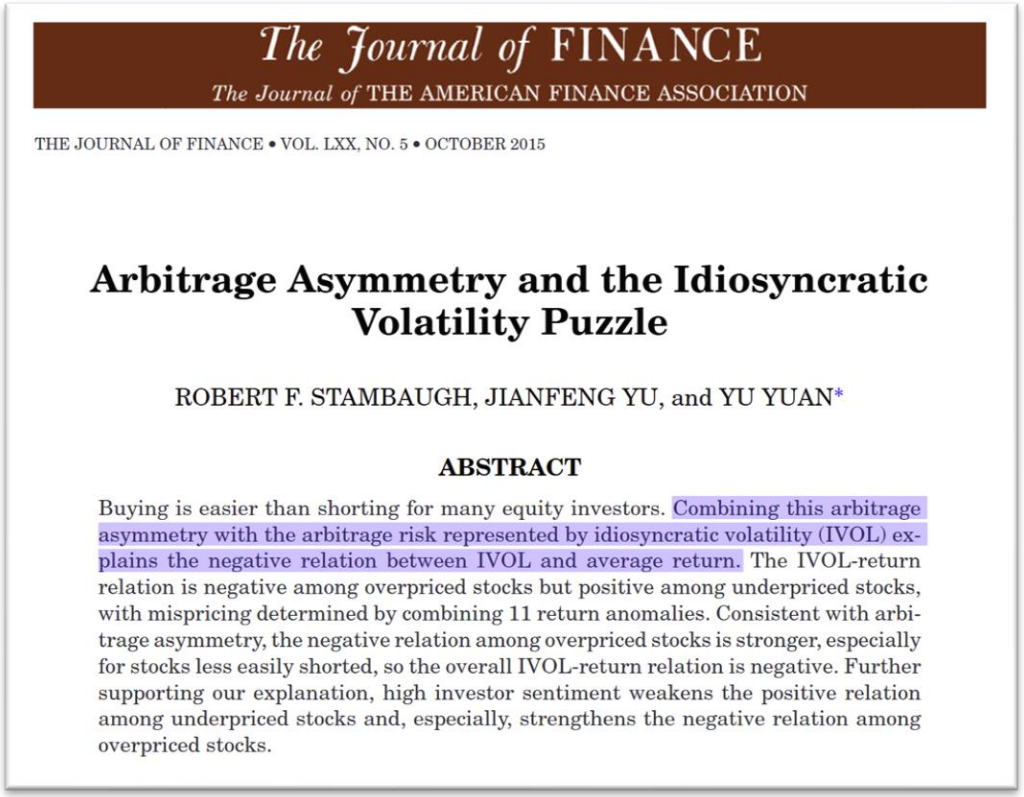
Frank Ma, XL Fu, San Thu Loon

Background of Study

Fama-French 3 Factor Regression: $R_{i,t} = a + bMKT_t + cSMB_t + dHML_t + \epsilon_{i,t}$

Idiosyncratic Volatility (IVOL) $= \sigma(\epsilon_{i,t})$

- Measures volatility not attributable to systematic risk (FF3 factors)
- **How does IVOL relate to expected return?**



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Arbitrage Asymmetry and the Idiosyncratic Volatility Puzzle

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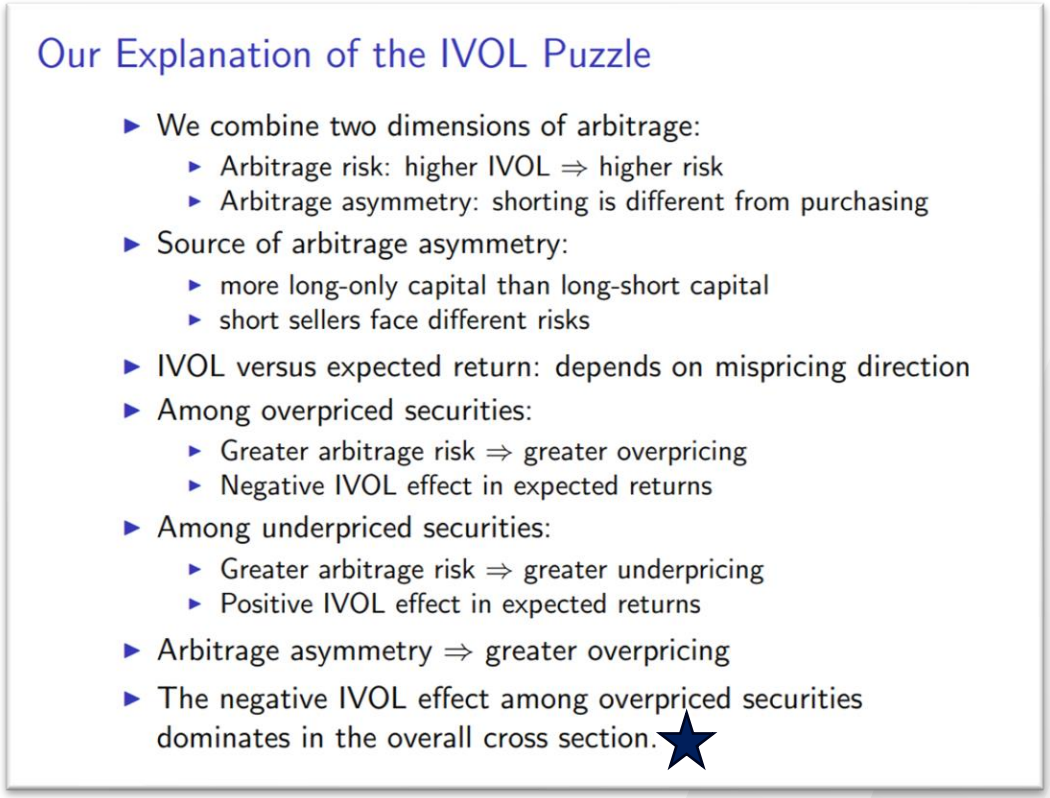
ABSTRACT

Buying is easier than shorting for many equity investors. Combining this arbitrage asymmetry with the arbitrage risk represented by idiosyncratic volatility (IVOL) explains the negative relation between IVOL and average return. The IVOL-return relation is negative among overpriced stocks but positive among underpriced stocks, with mispricing determined by combining 11 return anomalies. Consistent with arbitrage asymmetry, the negative relation among overpriced stocks is stronger, especially for stocks less easily shorted, so the overall IVOL-return relation is negative. Further supporting our explanation, high investor sentiment weakens the positive relation among underpriced stocks and, especially, strengthens the negative relation among overpriced stocks.



Our Explanation of the IVOL Puzzle

- We combine two dimensions of arbitrage:
  - Arbitrage risk: higher IVOL ⇒ higher risk
  - Arbitrage asymmetry: shorting is different from purchasing
- Source of arbitrage asymmetry:
  - more long-only capital than long-short capital
  - short sellers face different risks
- IVOL versus expected return: depends on mispricing direction
- Among overpriced securities:
  - Greater arbitrage risk ⇒ greater overpricing
  - Negative IVOL effect in expected returns
- Among underpriced securities:
  - Greater arbitrage risk ⇒ greater underpricing
  - Positive IVOL effect in expected returns
- Arbitrage asymmetry ⇒ greater overpricing
- The negative IVOL effect among overpriced securities dominates in the overall cross section.

Research Process

1 IVOL Analysis

- Identify universe of stocks
- Run FF3 regression to derive IVOL
- Conduct preliminary EDA on IVOL
- Analyze returns from L/S strategy on IVOL

2 Return Anomalies

- Extract data on 9 return anomalies, backed by Stambaugh et. al. (2015)
- Conduct EDA
- Rank all stocks by percentile and compute aggregate

3 Generate Portfolios

- Split all stocks into 5x5 portfolios - by their IVOL quintile (low to high IVOL) and overpriced extent
- Analyze portfolio returns

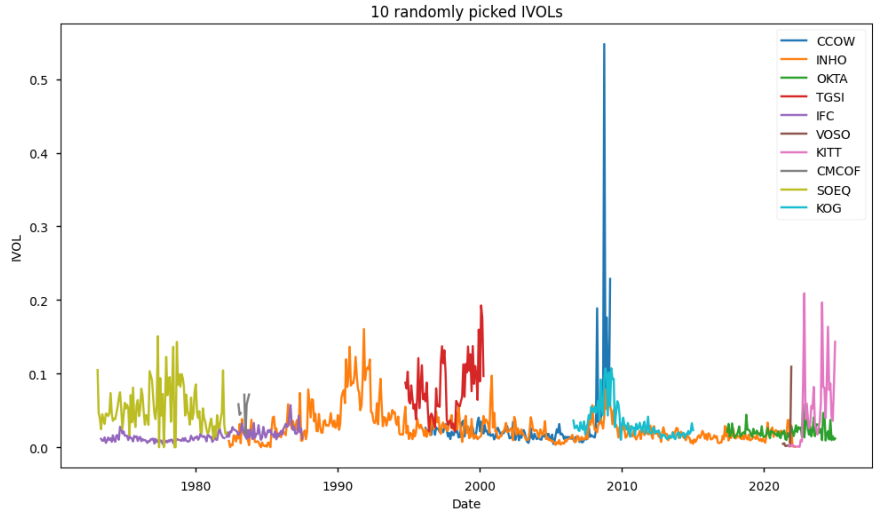
4 Compute Returns

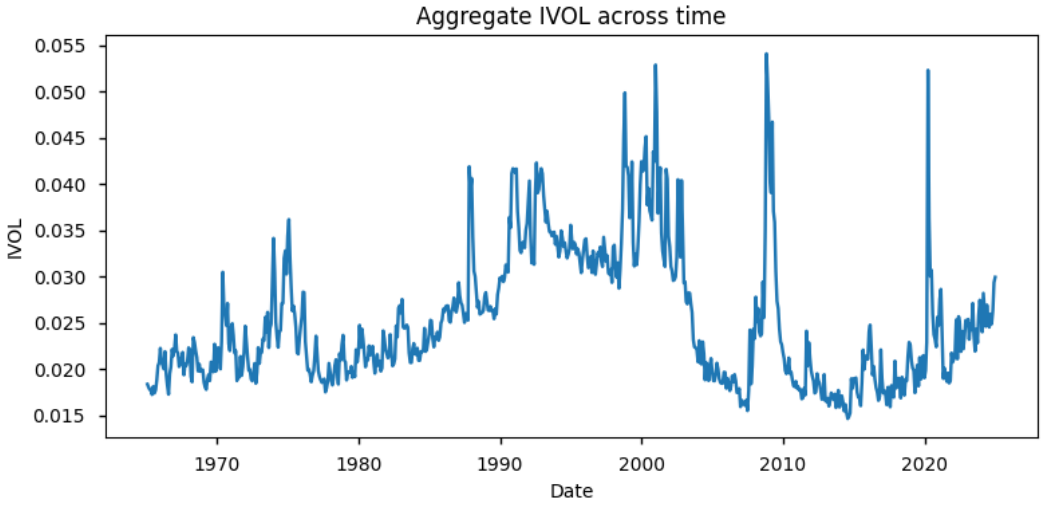
- Regress returns series against systematic factors to obtain excess returns
- Match patterns in excess returns to Stambaugh (2015)

Sources: wrds WHARTON RESEARCH DATA SERVICES The Global Standard for Business Research; CRSP; STANDARD & POOR'S

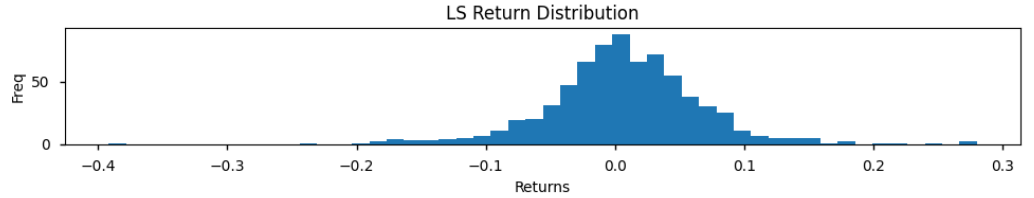
IVOL Analysis

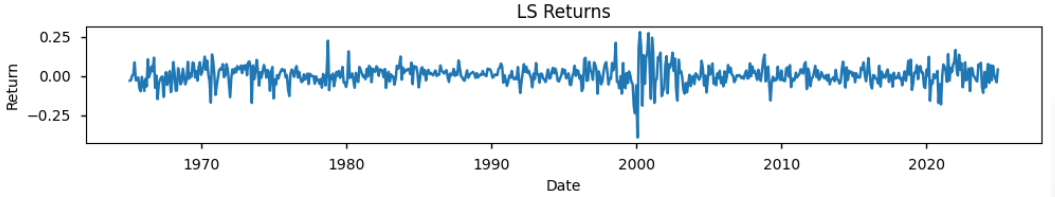
EDA

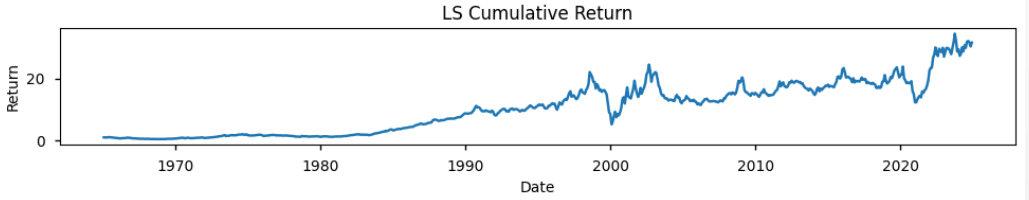


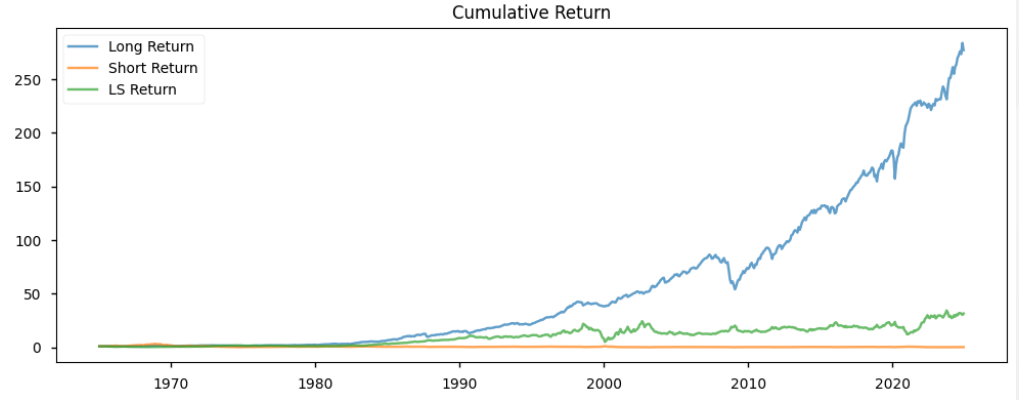


Long/Short Return









Return Anomalies

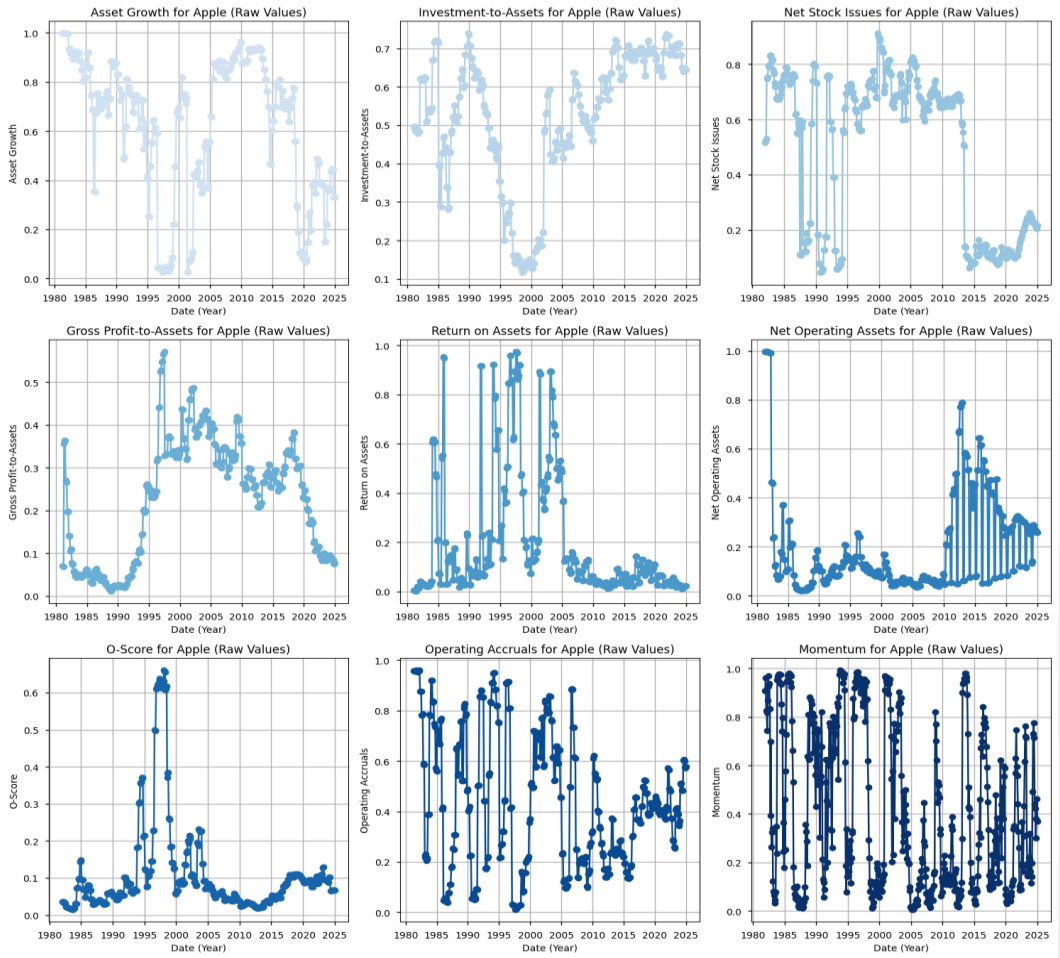
Set of 9 Anomalies

1. ~~Financial distress (Campbell, Hilscher, and Szilagyi (2008))~~
2. O-Score bankruptcy probability (Ohlson (1980))
3. Net stock issues (Ritter (1991), Loughran and Ritter (1995), Fama and French (2008))
4. ~~Composite equity issues (Daniel and Titman (2006))~~
5. Total accruals (Sloan (1996))
6. Net operating assets (Hirshleifer et al. (2004))
7. Momentum (Jegadeesh and Titman (1993))
8. Gross profitability (Novy-Marx (2013))
9. Asset growth (Cooper, Gulen, and Schill (2008))
10. Return on assets (Fama and French (2006), Chen, Novy-Marx, and Zhang (2010))
11. Investment-to-assets (Titman, Wei, and Xie (2004), Xing (2008))

{ Momentum, Gross Profitability, Return on Assets }: Higher value/percentile, higher expected returns, **lower overpricing**

{ Remaining 6 }: Higher value/percentile, lower expected returns, **higher overpricing**

Anomalies (Percentiles) Visualized for AAPL



Forecasting Next Month Returns using ML Methods

Approach: Regressing return for time t+1 (target) against return for t and each of the 9 anomalies in time t

① OLS

R-Squared: Terrible | MSE: 0.0174

+: Gross Profit-to-Assets
-: Time t return, Investment-to-Asset

```
Train shape: (1421382, 10)
Test shape: (355345, 10)

OLS Summary:
                            OLS Regression Results
==============================================================================
Dep. Variable:                 target   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     20.15
Date:                Tue, 22 Apr 2025   Prob (F-statistic):           7.86e-38
Time:                        03:53:16   Log-Likelihood:                8.5131e+05
No. Observations:             1421382   AIC:                           -1.703e+06
Df Residuals:                 1421371   BIC:                           -1.702e+06
Df Model:                          10
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0136      0.000     68.760      0.000       0.013       0.014
ret_t         -0.0037      0.001     -4.975      0.000      -0.005      -0.002
at_gr1     -3.902e-09   3.79e-08     -0.103      0.918   -7.81e-08    7.03e-08
capx_at       -0.0102      0.002     -6.230      0.000      -0.013      -0.007
chcsho_12m     4.052e-06   6.68e-06      0.606      0.544   -9.04e-06    1.71e-05
gp_at          0.0039      0.000     10.419      0.000       0.003       0.005
niq_at        -1.463e-05      0.000     -0.032      0.975      -0.001       0.001
...
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.04e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
OLS Test MSE: 0.017432376548565526
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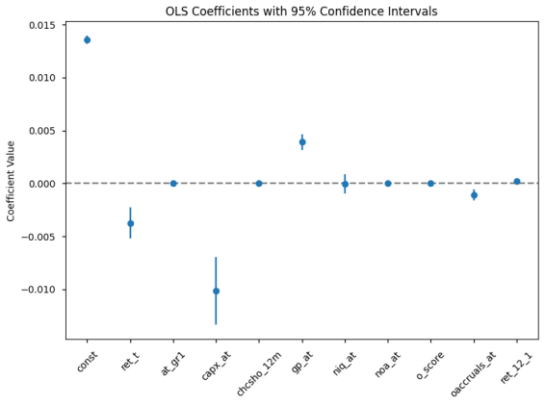
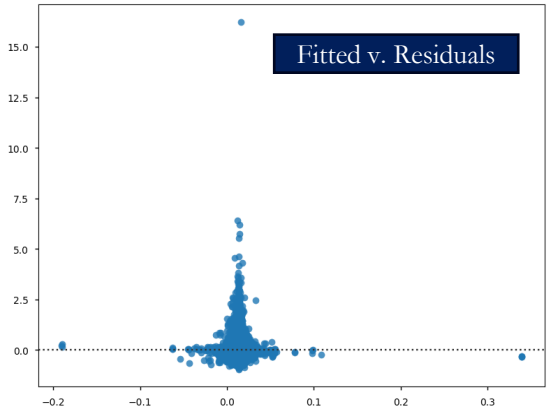
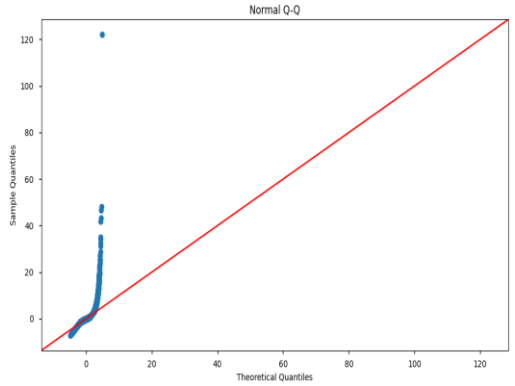
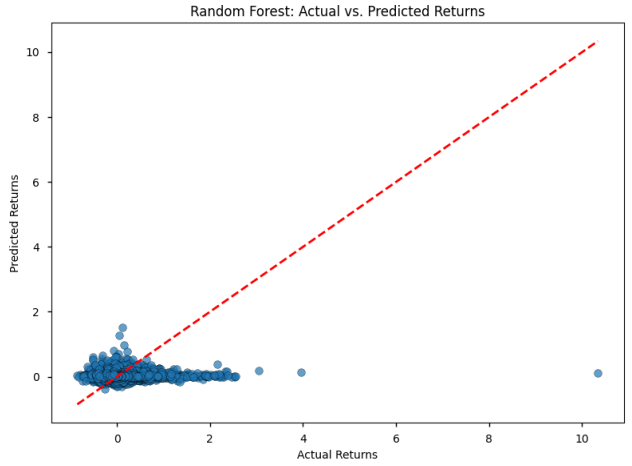




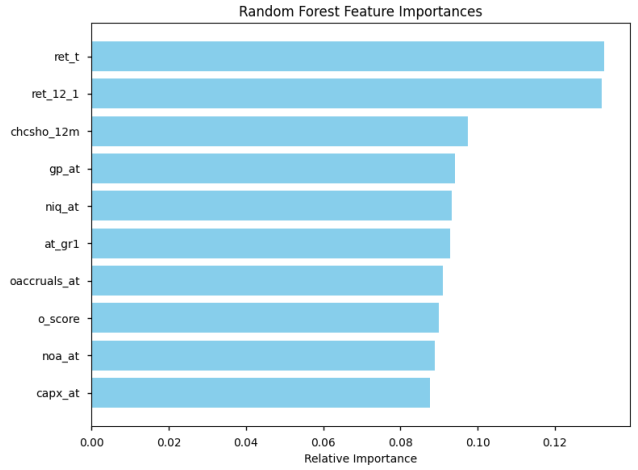


② Random Forest



Actual v. Predicted Returns:

- MSE: 0.0181 (slight improvement)
- RF tend to underpredict
- Still shows poor predictive power



Feature Importances

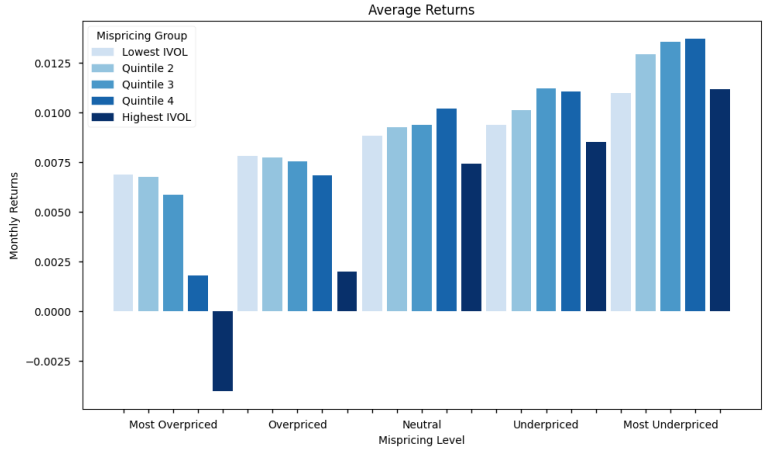
1. Return in time t
2. Returns momentum
3. Net stock issues

…

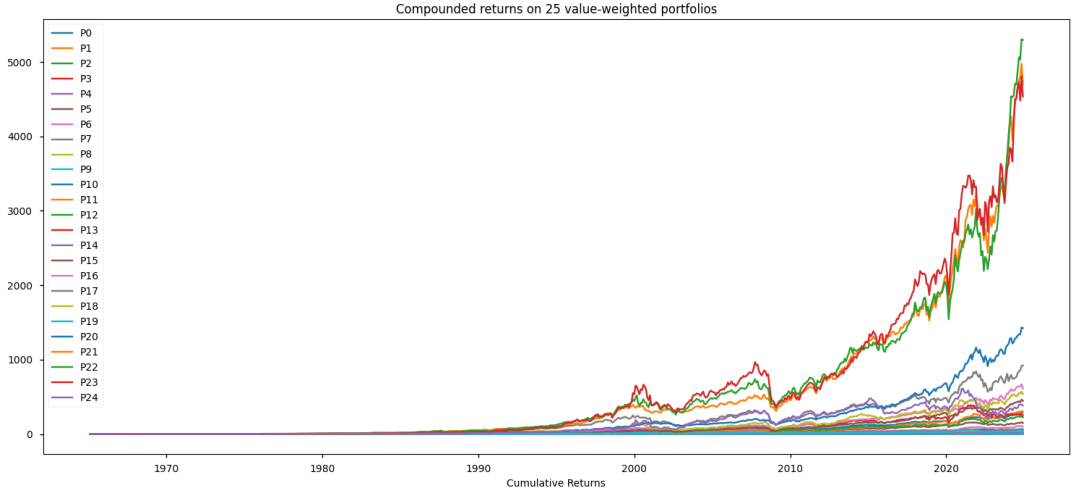
8. Net operating assets
9. Net investment-to-assets

Returns from 25 Portfolios (IVOL, Anomaly Sorted)

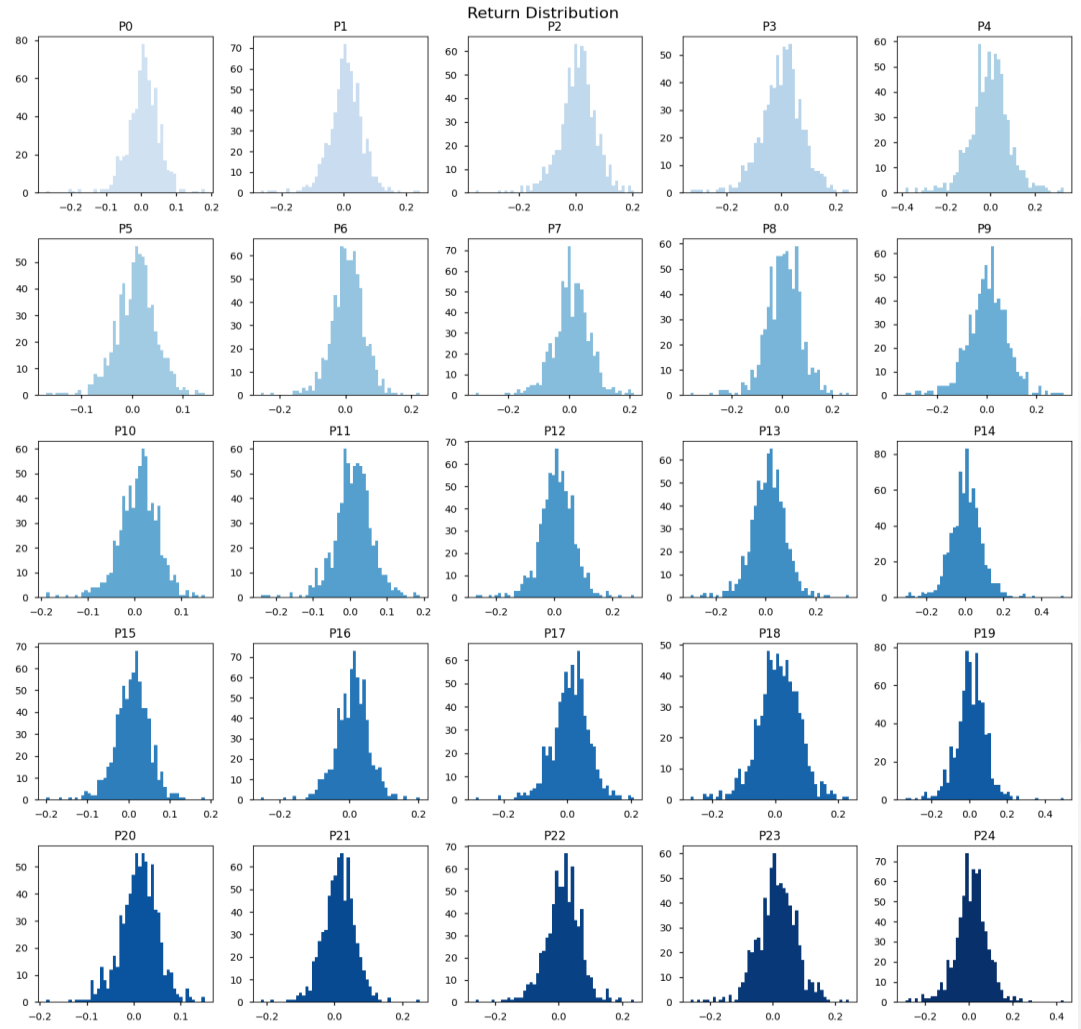
1 Average Returns



2 Compounded Returns

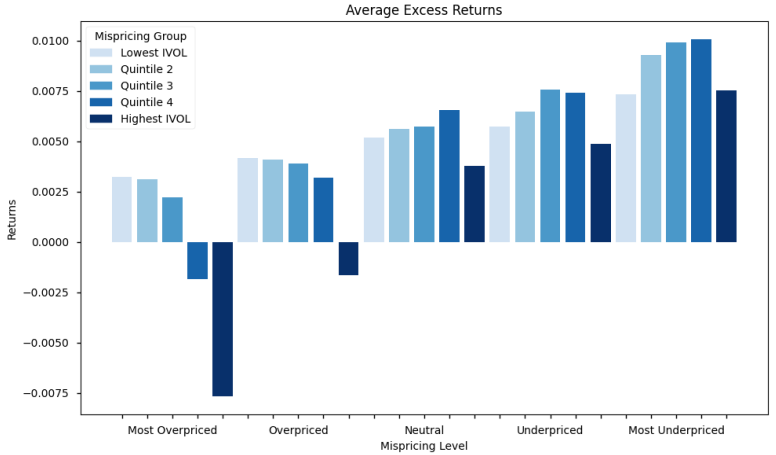


3 Returns Distribution

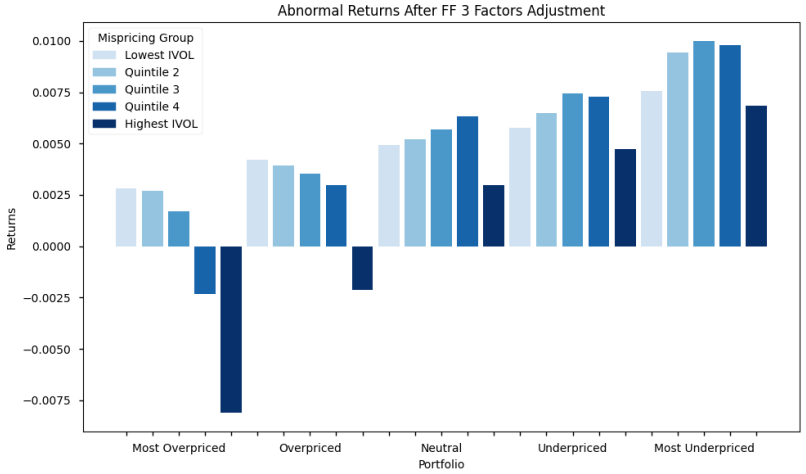


Excess and Abnormal Returns

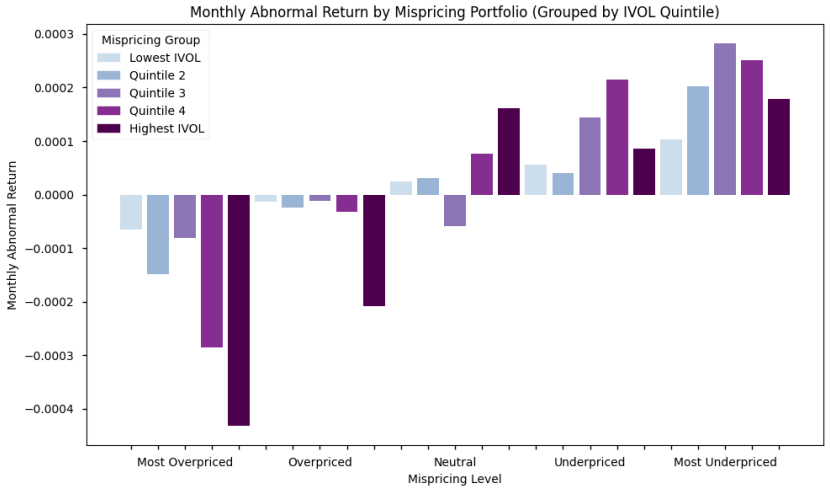
1 Average Excess Returns



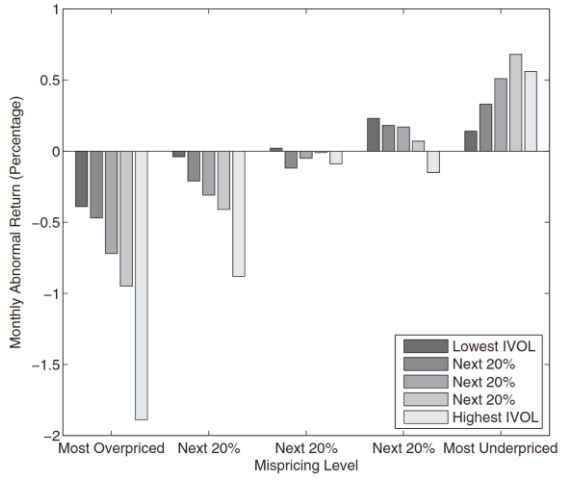
2 Abnormal Returns after FF3 Factors



3 Monthly Abnormal Returns



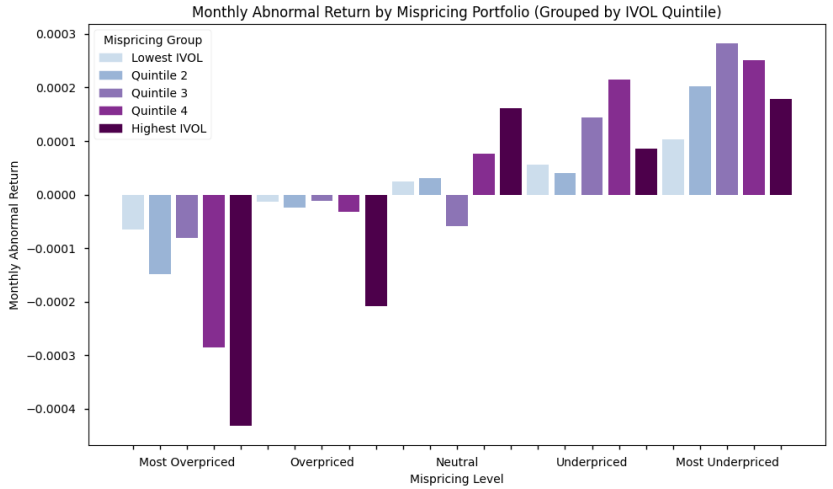
Stambaugh, Yu, and Yuan (2015)

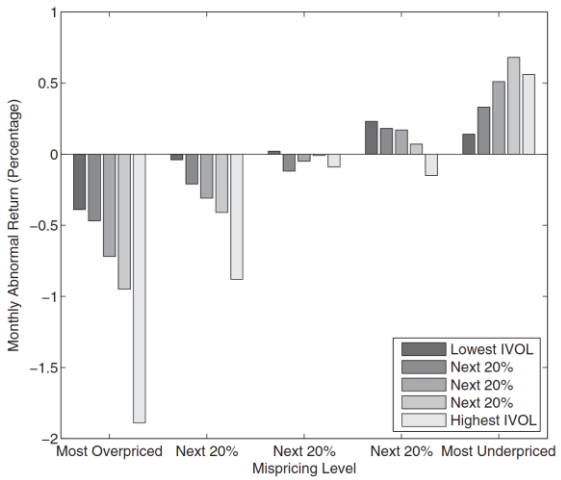


- "IVOL effect": relation between expected return and IVOL
- If arbitrage risk is important for mispricing, we expect
  - negative IVOL effect among overpriced stocks
  - positive IVOL effect among underpriced stocks
- If arbitrage asymmetry is important for mispricing, we expect the negative effect among overpriced stocks to be stronger.
  ⇒ Negative IVOL effect in overall cross section

Conclusion

1. With new data from 2013 to 2024, the same patterns are reproduced.

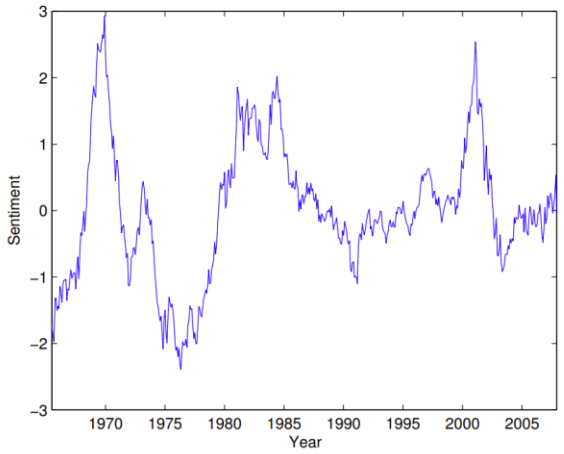


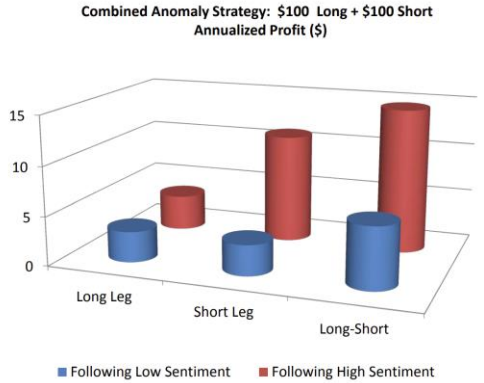


2. The negative correlation between IVOL and expected return (the "IVOL puzzle") persists.

Next Steps

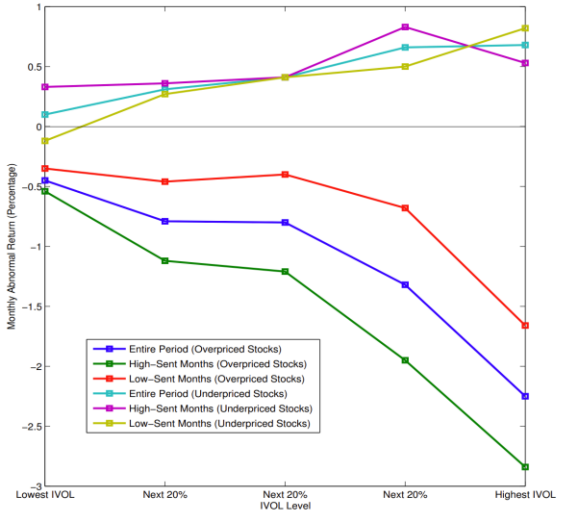
① Time Varying Mispricing





- Evidence of greater overpricing when sentiment is high
  - Stambaugh, Yu, and Yuan (*JFE*, 2012)
- Investor sentiment index (Baker-Wurgler)
  - Indicator of market-wide direction of mispricing
  - Principal component of six underlying measures:
    - closed-end fund discount
    - number of IPO's
    - first-day IPO returns
    - NYSE turnover
    - equity share of new issues
    - dividend premium (log B/M, payers minus nonpayers)

② Time Varying IVOL Effect



- If the degree and direction of mispricing vary over time, so should IVOL effects.
- We expect
  - (1) greater negative IVOL effect among overpriced stocks following high sentiment
  - (2) greater positive IVOL effect among underpriced stocks following low sentiment
- Arbitrage asymmetry ⇒ (1) should be stronger than (2)

THANKS FOR LISTENING!

Q&A