

# FNCE-1018 Notes

November 7, 2025

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# 1 Introduction

# 2 Measurement and Structure of the National Economy

## 2.1 National Income Accounting: Production, Income, Expenditure

**National income accounting** states that except for incomplete or misreported data, *the following three approaches give identical measurements of the amount of current economic activity*

- Product approach: the amount of final output produced
- Income approach: the incomes received by producers of output
- Expenditure approach: the amount of spending by final purchasers of output

## 2.2 Gross Domestic Product

**Definition 2.1. Gross Domestic Product (Product Approach)** is defined as the market value of final goods and services newly produced within a nation during a fixed period of time.

**Remarks. Market value:** prices at which goods and services are sold

- (+): Allows adding of production of different goods and services
- (-): Some goods or services are not sold in formal markets, denoted **nonmarket goods and services**.

**Remarks. Newly produced:** GDP only includes goods and services produced within current period.

**Remarks. Final:** Goods and services are final if they are not intermediate. Goods and services are intermediate if they are used up in the production of others **in the same period they were produced**. By this definition, **inventories** are a final good.

**Definition 2.2. Gross National Product** is defined as the market value of final goods and services newly produced *by domestic factors of production* during the current period.

**Definition 2.3. NFP or Net factor payments from abroad** is defined as the income paid to domestic factors of production by the rest of the world minus income paid to foreign factors of production by the domestic economy.

$$GDP = GNP - NFP$$

**Definition 2.4. Income-Expenditure Identity:** The **Gross Domestic Product** (Expenditure Approach) is defined as the total spending on final goods and services produced within a nation during a specified period of time.

$$Y = C + I + G + NX$$

Where

$$\begin{aligned} Y = GDP &= \text{total output / production} \\ &= \text{total income} \\ &= \text{total expenditure} \end{aligned}$$

$$C = \text{consumption}$$

$$I = \text{investment}$$

$$G = \text{government purchases of goods and services}$$

$$NX = \text{net exports of goods and services}$$

**Remarks. Consumption** is spending by households on final goods and services, **including those produced abroad**. It comprises of

- consumer durables
- nondurables
- services

**Remarks. Investment** includes spending on

- fixed investment: spending on new capital goods
  - business fixed investment
  - residential investment
- inventory investment: spending on inventory holdings

Note that goods that are **produced** but **unsold** are considered investments in inventory.

Note also that investment includes spending on foreign-produced goods.

**Remarks. Government purchases** of goods and services include expenditure by government for a currently produced good or service, foreign or domestic.

Note that  $G$  excludes

- transfers
- interest payment on national debt

Government spending on fixed investments or inventory is counted under  $G$ , not  $I$ .

**Remarks. Net Exports** are exports minus imports.

- Exports: goods and services produced within a country purchased by foreigners, **added to total spending** because they represent spending on final goods and services within the country
- Imports: vice versa

**Definition 2.5. National Income** is defined as the sum of

- Compensation of employees
- Proprietors' income
- Rental income of persons
- Corporate profits
- Net interest
- Taxes on production and imports
- Business current transfer payments
- Current surplus of government enterprises

**GDP (Income Approach)** is defined as National Income plus

- statistical discrepancy
- depreciation: consumption of fixed capital

**Remarks.**

$$\underbrace{\underbrace{\text{National Income} + \text{Statistical Discrepancy}}_{\text{Net National Product}} + \text{Depreciation} + \text{NFP}}_{\text{Gross Domestic Product}} = \text{Gross National Product}$$

**Definition 2.6. Private disposable income** is the amount of income that the private sector has available to spend.

$$\text{private disposable income} = Y + NFP + TR + INT - T$$

where

$$Y = \text{GDP}$$

$$NFP = \text{net factor payment from abroad}$$

$$TR = \text{transfers received from the government}$$

$$T = \text{taxes}$$

**Definition 2.7. Net government income** is taxes paid by private sector, minus payments from government to the private sector

$$\text{net government income} = T - TR - INT$$

## 2.3 Savings and Wealth

**Saving** is the excess of current income over current needs.

**Definition 2.8. Private saving** is defined as the difference of private disposable income and consumption

$$\begin{aligned} S_{pvt} &= \text{private disposable income} - \text{consumption} \\ &= (Y + NFP - T + TR + INT) - C \end{aligned}$$

**Remarks.** Note that **investments** is not subtracted from private disposable income even though it constitutes private spending, because it *does not satisfy current needs*.

**Definition 2.9. Government saving** is defined as net government income, less government purchases of goods and services

$$\begin{aligned} S_{govt} &= \text{net government income} - \text{government purchases} \\ &= (T - TR - INT) - G \end{aligned}$$

**Remark.** Government purchase is technically divided into government consumption (to meet current needs) and government investment (on long-lived assets). For most purposes, including the above definition,  $G$  is assumed to be entirely made up of government consumption.

**Remark. Government budget surplus** is defined as government receipts less outlays

$$\begin{aligned} \text{budget surplus} &= \text{government receipts} - \text{government outlays} \\ &= T - (G + TR + INT) \\ &= S_{govt} \end{aligned}$$

**Definition 2.10. National saving** is defined as the sum of private saving and government saving

$$\begin{aligned} S &= S_{pvt} + S_{govt} \\ &= (Y + NFP - T + TR + INT - C) + (T - TR - INT - G) \\ &= Y + NFP - C - G \end{aligned}$$

**Definition 2.11. Current Account Balance** is defined as payments received from abroad in exchange for currently produced goods and services, minus payments made to foreigners by the domestic economy for currently produced goods and services and net unilateral transfers.

$$CA = NX + NFP + NUT$$

**Remark.**  $CA, NX, NFP, NUT$  all follow the money.

- money coming in:  $CA > 0$
- money leaving:  $CA < 0$

**Definition 2.12.** The **Financial Account balance** is defined as the additive inverse of the current account.

$$FA = -CA$$

on the assumption that any domestic currency spent on foreign goods in excess of what foreigners need to purchase domestic goods will be spent purchasing domestic financial assets.

**Remark.** A company is a **net creditor** (i.e. lending abroad) if

$$FA < 0 \Leftrightarrow CA > 0 \Leftrightarrow NX > 0 \text{ assuming } NFP = NUT = 0$$

**Theorem 2.13. Uses-of-saving identity** The economy's private saving is used in 3 ways

$$S_{pvt} = I + (-S_{govt}) + CA$$

- Investment: firms borrow from savers to finance investment
- Government budget deficit: government finances deficits by borrowing from private savers
- Current account balance: foreigners borrow from US private savers to fund payments to the US

**Proof.** We begin by substituting  $Y = C + I + G + NX$  in the definition of national saving

$$\begin{aligned}
 S &= Y + NFP - C - G \text{ by definition} \\
 &= (C + I + G + NX) + NFP - C - G \text{ by substituting } Y \\
 &= I + (NX + NFP) \\
 &= I + (CA) \text{ by definition} \\
 S_{pvt} + S_{govt} &= I + CA \\
 S_{pvt} &= I + (-S_{govt}) + CA
 \end{aligned}$$

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**Definition 2.14. National Wealth** is the total wealth of residents in a country, consisting of

- domestic physical assets, i.e. capital goods and land
- net foreign assets, comprising of
  - (+) foreign physical and financial assets
  - (-) foreign physical and financial liabilities

**Remark.** Domestic financial assets held by domestic residents are **not** part of national wealth because each domestic financial asset is matched by a domestic financial liability.

**Remark.** National wealth can change through

- Change in value of existing assets and liabilities
  - National saving
    - Since  $S = I + CA$ , every dollar in savings increases domestic capital stock or domestic net foreign assets.
- Increases in national saving increase national wealth dollar for dollar.*

## 2.4 Real GDP, Price Indexes, Inflation

**Definition 2.15.** A **real** variable is one that measures the physical quantity of economic activity using the prices of a base year.

**Definition 2.16.** A **price index** is a measure of the average level of prices for some specified set of goods and services, relative to the prices in a specified base year.

**Definition 2.17.** The **GDP deflator** is a price index that measures the overall level of prices of goods and services included in GDP.

$$\text{GDP deflator} = 100 \times \frac{\text{nominal GDP}}{\text{real GDP}}$$

**Definition 2.18.** The **Consumer Price Index**, or **CPI**, measures the prices of consumer goods, calculated as 100 times the current cost of a specific basket of consumer items divided by the cost of the same basket in the reference base period.

**Remark.** The CPI tends to overstate increases in cost of living because of

- improvements in quality of goods and services
- substitution bias
  - CPI doesn't account for substitution away from the specified basket of goods

**Definition 2.19. Inflation rate** for a given period is defined as the percentage change in the price index in the same period.

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} = \frac{\Delta P_{t+1}}{P_t}$$

**Remark.** The FED's preferred inflation measure is the **Personal Consumption Expenditures (PCE)** price index, which measures consumer prices in the national income and product accounts.

- Headline inflation: overall change in PCE price Index
- Core inflation: change in PCE excluding **food and energy**

Differences between PCE and CPI

- PCE is based on actual household expenditure, avoids substitution bias
- PCE is broader than CPI

- PCE can be revised when better data is available
- PCE is chain-weighted
- CPI is based on average spending habits of **urban population**

## 2.5 Interest rates

In general, interest rate is a rate of return promised by a borrower to a lender.

**Definition 2.20.** The **real interest rate** on an asset is the rate at which the real value or purchasing power of the asset increases over time.

The **nominal interest rate** on an asset is the rate at which the nominal value or purchasing power of the asset increases over time.

$$\begin{aligned}\text{real interest rate} &= \text{nominal interest rate} - \text{inflation} \\ &= i - \pi\end{aligned}$$

At the time of lending, the nominal interest rate is known, but the real interest rate is unknown because the rate of inflation over the loan period is unknown.

**Definition 2.21.** **Expected real interest rate** is the nominal interest rate minus expected rate of inflation.

$$r = i - \pi^e$$

# 3 Productivity, Output and Employment

The output of an economy depends on (1) the quantities of inputs (labor, capital, raw materials) and (2) the productivity of inputs.

## 3.1 The production function

The textbook / FNCE 1018 develops a two-factor model of production.

**Definition 3.1.** The **production function** states that

$$Y = AF(K, N)$$

where

$Y$  = real output produced in a given period of time

$A$  = total factor productivity

$K$  = capital stock

$N$  = labor, or number of workers

$F$  = some function relating  $Y$  to  $K, N$

**Example.** The **Cobb-Douglas** production function states that

$$Y = AK^a N^{1-a}$$

Empirically,

$$Y = AK^{0.3} N^{0.7}$$

**Remarks.** Under Cobb-Douglas, holding  $N$  constant and varying  $K$ :

- $Y$  increases with  $K$ , but at a decreasing rate (diminishing marginal product of capital).
- and vice versa.

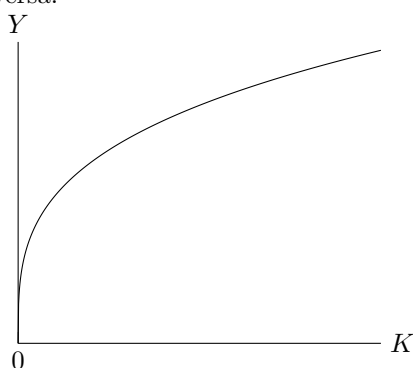


Fig -  $Y$  as a function of  $K$ , fixed  $N$

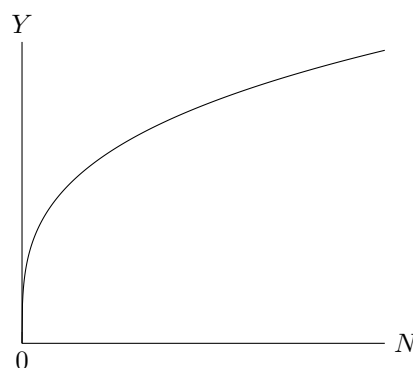


Fig -  $Y$  as a function of  $N$ , fixed  $K$

**Definition 3.2.** The **Marginal product of capital** is the increase in output produced that results from a one-unit increase in capital stock.

$$MPK = \frac{\Delta Y}{\Delta K}$$

**Definition 3.3.** The **Marginal product of labor** is the increase in output produced that results from a one-unit increase in labor.

$$MPN = \frac{\Delta Y}{\Delta N}$$

**Remarks.** The  $MPK$  and  $MPN$

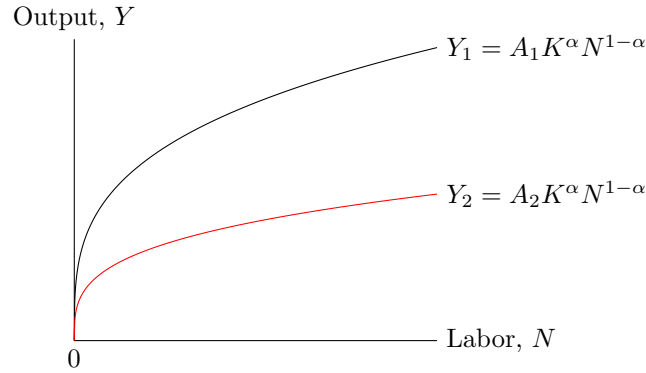
- are positive
- are decreasing in  $K / N$ , due to diminishing marginal product



## 3.2 Adverse productivity shock

**Remarks.** Given a decrease in  $A$ ,

- the marginal product decreases for every value of  $N$
- the amount of output decreases for every value of  $N$



**Fig - Decrease in  $A$ , holding  $K$  fixed**

Note that

$$\begin{aligned}
 A_2 &< A_1 \\
 \Rightarrow Y_2 &< Y_1 \\
 \Rightarrow MPN_2 &= A_2 K^\alpha (1 - \alpha) N^{-\alpha} < A_1 K^\alpha (1 - \alpha) N^{-\alpha} = MPN_1
 \end{aligned}$$

## 3.3 Labor Demand

Assumptions for the labor market model:

- all workers alike
- firms view wages as being determined by competitive labor market
- firms aim to maximize profits

The marginal product of labor measures the benefit of employing additional worker in terms of extra output produced.

**Definition 3.4. Marginal revenue product of labor** is the benefit of employing an additional worker in terms of the extra revenue produced.

$$MRPN = P \times MPN$$

The quantity of labor demanded is

- in nominal terms: equal to the  $MRPN$
- in real terms: equal to  $MPN$

**Definition 3.5.** Real wage refers to the wage measured in terms of units of output.

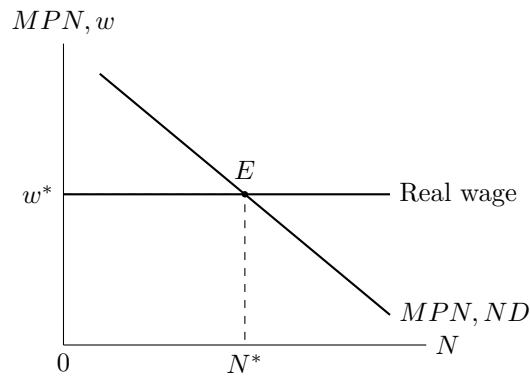
$$w = \frac{W}{P}$$

**Remarks.** Note that firms will want to increase employment under the following condition, all 4 statements are equivalent

$$\begin{aligned}
 MPN &> w \\
 \Leftrightarrow MPN &> \frac{W}{P} \\
 \Leftrightarrow P \times MPN &> W \\
 \Leftrightarrow MPRN &> W
 \end{aligned}$$

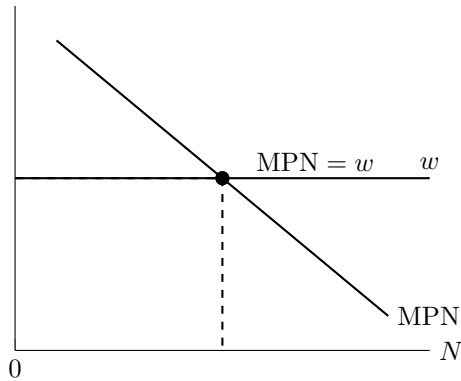
Vice versa for condition under which the firm will want to decrease employment

**Remarks.** For a single firm, the wage is set by the market

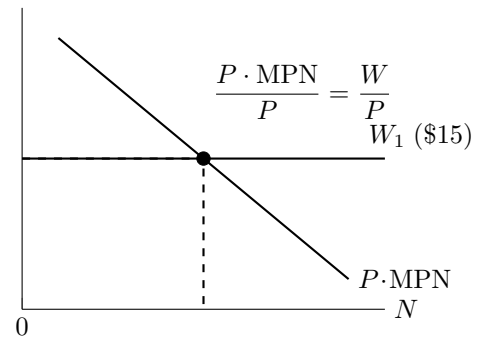


**Remark.** The following intercepts are mathematically equivalent.

Real wage,  $w$  (goods/hour)



$W, P \times MPN$

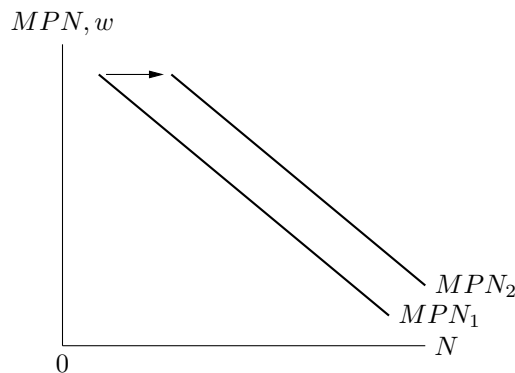


### 3.3.1 Factors that shift labor demand

**Remark.** Factors that affect labor demand must change the amount of labor that firms want to employ *at any given level of the real wage*.

The labor demand increases in response to

- $A \uparrow$ , productivity improvements / positive supply shock
- $K \uparrow$ , increase in capital supply



### 3.3.2 Aggregate labor demand

**Definition 3.6.** Aggregate labor demand is the sum of labor demands of all the firms in an economy.

**Remark.** The aggregate labor demand looks the same as individual firm labor demand.

## 3.4 Labor supply

**Definition 3.7.** The aggregate labor supply is the sum of labor supplied by everyone in the economy.

### 3.4.1 Income-leisure trade-off and real wages

The marginal benefit of work is the utility gained from extra income. The marginal cost of work is the utility lost from reducing leisure.

**Definition 3.8.** An individual worker seeks to maximize his or her utility function

$$\max_N U(\mathcal{L}, \mathcal{C})$$

subject to

$$\mathcal{C} = w(T - \mathcal{L}) + Z$$

Where

$T$  = total number of hours available

$Z$  = non-labor income

$w$  = real wage rate

$U$  = utility function with inputs  $\mathcal{L}, \mathcal{C}$

$\mathcal{C}$  = consumption

$\mathcal{L} = T - N$  = leisure

**Definition 3.9.** The **substitution effect** refers to an increase in the opportunity cost of leisure causing workers to substitute away from leisure towards work.

**Remark. Pure substitution effect:** one-day rise in real wage,  $NS \uparrow$ .

**Definition 3.10.** The **income effect** refers to workers being better off and hence working less.

**Remark. Pure income effect:** changes in  $Z$ , e.g. winning the lottery, or higher expected future real wages,  $NS \downarrow$

**Remark. Income and substitution effect** work in opposite directions on labor supply.

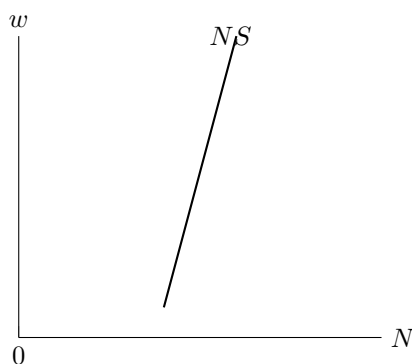
An increase in real wages

- raises the marginal benefit of work, increases labor supply, by **substitution effect**
- increases workers' wealth, decreases labor supply, by **income effect**

**Remark.** Empirically, labor supply

- rises when there is a **temporary increase in real wage**
- falls when there is a **permanent increase in real wage**

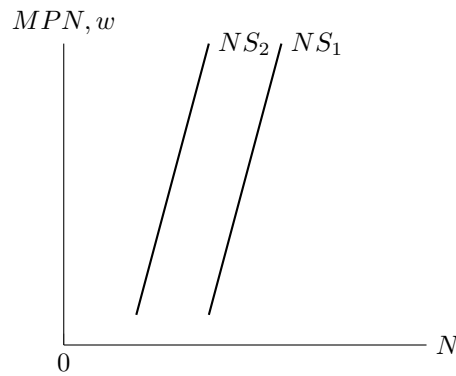
In aggregate, the labor supply is **rising in real wages**.



### 3.4.2 Factors that affect labor supply

**Remark.** The labor supply shifts left in response to

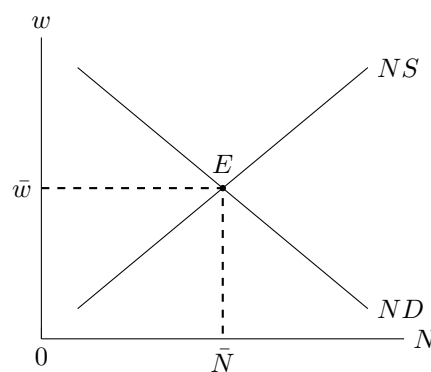
- increases in wealth,  $NS \downarrow$
- increases in expected future real wage,  $NS \downarrow$
- decrease in working age population
- decrease in participation rate



## 3.5 Labor Market Equilibrium

Under classical assumptions, real wage adjusts reasonably quickly to bring labor demand and supply into equilibrium.

**Definition 3.11. Full-employment level of employment,**  $\bar{N}$  is defined as the equilibrium level of employment. The corresponding market-clearing real wage is  $\bar{w}$ .



### 3.5.1 Full employment output

**Definition 3.12. Full-employment output,**  $\bar{Y}$ , also called **potential output**, is the level of output that firms in the economy supply when wages and prices have fully adjusted.

$\bar{Y}$  is achieved when aggregate employment reaches its full-employment level,  $\bar{N}$

$$\bar{Y} = AF(K, \bar{N})$$

**Remark.** A decrease in  $A$  reduces  $\bar{Y}$  in two ways

- $A \downarrow \rightarrow \bar{Y} \downarrow$  directly
- $A \downarrow \rightarrow MPN \downarrow \rightarrow ND \downarrow \rightarrow \bar{N} \downarrow \rightarrow \bar{Y} \downarrow$

## 3.6 Unemployment

Under classical assumptions, all workers who are willing to work at the prevailing wage find jobs.

**Remark.** The Bureau of Labor Statistics surveys 60,000 households monthly. Each person over 16 is assigned to

- $E$ , employed, if person worked full-time or part-time during the past week
- $U$ , unemployed, if person didn't work during the past week but looked for work in the past four weeks
- $NLF$ , not in labor force, if the person didn't work and didn't look for work in the past 4 weeks
  - discouraged workers, people who become discouraged and move from  $U$  to  $NLF$

$$\text{labor force} = LF = E + U$$

$$\text{adult population} = LF + NLF$$

$$\text{participation rate} = \frac{LF}{LF + NLF}$$

$$\text{employment ratio} = \frac{E}{LF + NLF}$$

**Remark.** US unemployment is characterized by two contradictory statements

- most unemployment spells are short ( $< 2$  months)

- most unemployed are experiencing unemployment spells with a long duration

**Remark.** Sources of unemployment

- **frictional unemployment:** arises as workers search for suitable jobs and firms search for suitable workers
- **structural unemployment:** long-term and chronic unemployment that exists even when the economy is not in a recession
  - unskilled, low skilled workers
  - relocation of labor from shrinking industries / depressed regions

**Definition 3.13.** The **natural rate of unemployment**,  $\bar{u}$  is the rate of unemployment that prevails when output and employment are at the full-employment level.

The difference between actual unemployment and natural unemployment is **cyclical unemployment**

$$\text{cyclical unemployment} = u - \bar{u}$$

## 3.7 Okun's Law

**Theorem 3.14. Okun's Law** states that the gap between full-employment output and actual output increases by 2 percent for each percent increase in unemployment

$$\frac{\bar{Y} - Y}{\bar{Y}} = 2(u - \bar{u})$$

Alternatively, the percentage change in real output is roughly 3 percent minus two times the change in unemployment

$$\frac{\Delta Y}{Y} = \frac{\Delta \bar{Y}}{\bar{Y}} - 2\Delta u$$

**Proof.**

$$\begin{aligned} \frac{\bar{Y} - Y}{\bar{Y}} &= 1 - \frac{Y}{\bar{Y}} = c(u - \bar{u}) \\ \frac{Y}{\bar{Y}} - 1 &= c(\bar{u} - u) \end{aligned}$$

Taking percentage change on both sides, we get

$$\begin{aligned} \Delta \left( \frac{Y}{\bar{Y}} \right) &= c(\Delta \bar{u} - \Delta u) \\ \frac{Y + \Delta Y}{\bar{Y} + \Delta \bar{Y}} - \frac{Y}{\bar{Y}} &= c(\Delta \bar{u} - \Delta u) \\ \frac{\bar{Y}\Delta Y - Y\Delta \bar{Y}}{\bar{Y}(\bar{Y} + \Delta \bar{Y})} &= c(\Delta \bar{u} - \Delta u) \end{aligned}$$

Multiplying by  $\left( \frac{\bar{Y} + \Delta \bar{Y}}{\bar{Y}} \right) \approx 1$ ,

$$\begin{aligned} \frac{\bar{Y}\Delta Y - Y\Delta \bar{Y}}{\bar{Y}Y} &\approx c(\Delta \bar{u} - \Delta u) \\ \frac{\Delta Y}{Y} - \frac{\Delta \bar{Y}}{\bar{Y}} &\approx c(\Delta \bar{u} - \Delta u) \\ \frac{\Delta Y}{Y} &\approx \frac{\Delta \bar{Y}}{\bar{Y}} + c(\Delta \bar{u} - \Delta u) \end{aligned}$$

Taking  $c$  to be 2 and  $\Delta \bar{u}$  to be 0, we get

$$\frac{\Delta Y}{Y} = \frac{\Delta \bar{Y}}{\bar{Y}} - 2\Delta u$$

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# 4 Consumption, saving and investment

## 4.1 Consumption and saving

The aggregate level of desired consumption,  $C^d$  is the sum of of the desired consumption of all households. The desired national saving,  $S^d$  is the level of national saving that occurs when aggregate consumption is at  $C^d$ . In a closed economy,

$$S^d = Y - C^d - G$$

### 4.1.1 Inter-temporal Consumption Model

**Definition 4.1.** An individual's **Present Value of Lifetime Resources**,  $PVLR$  is defined as

$$PVLR = a + y + \frac{y^f}{1+r}$$

Where

$a$  = assets, current period

$y$  = income, current period

$y^f$  = future income, current period

**Definition 4.2.** An individual's **Present Value of Lifetime Consumption**,  $PVLC$  is defined as

$$PVLC = c + \frac{c^f}{1+r}$$

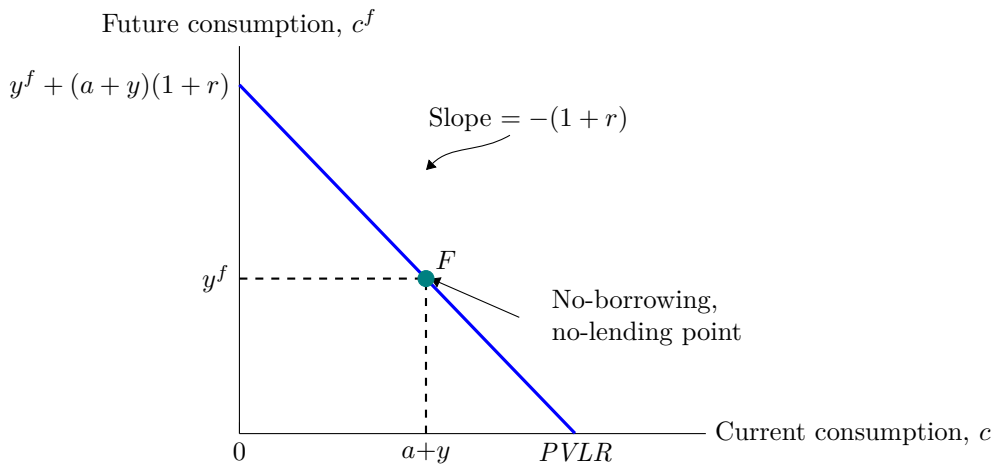
Where

$c$  = consumption, current period

$y$  = consumption, future period

**Definition 4.3.** An individual's **budget constraint** is given by

$$\begin{aligned} PVLC &= PVLR \\ c + \frac{c^f}{1+r} &= (a + y) + \frac{y^f}{1+r} \\ c^f &= (a + y - c)(1+r) + y^f \\ &= \underbrace{(a + y)(1+r)}_{\text{intercept}} + \underbrace{y^f - (1+r)c}_{\text{slope}} \end{aligned}$$



**Remark.** We can classify individuals as lending or borrowing

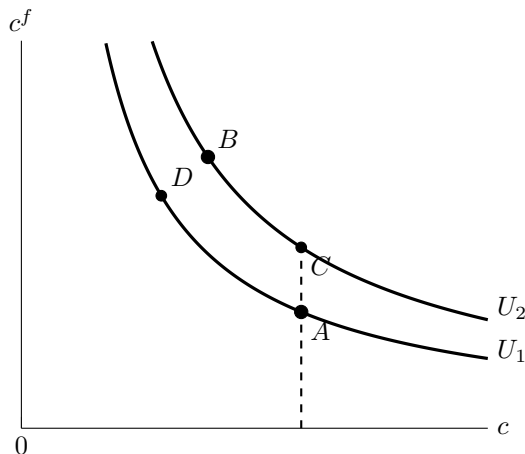
- lending, if  $c < a + y \Leftrightarrow a + y - c > 0$
- borrowing if  $c > a + y \Leftrightarrow a + y - c < 0$

**Remark.** We can classify individuals as saving or dissaving

- saving, if  $y > c$
- dissaving, if  $y < c$ 
  - borrowing, if  $c > y + a \Leftrightarrow a + y - c < 0$

**Dissaving  $\neq$  Borrowing.**

**Remark.** In choosing between  $c$  and  $c^f$ , consumers face diminishing marginal utility. The indifference curve has the shape



To convince ourselves that  $B$  is preferred to  $A$

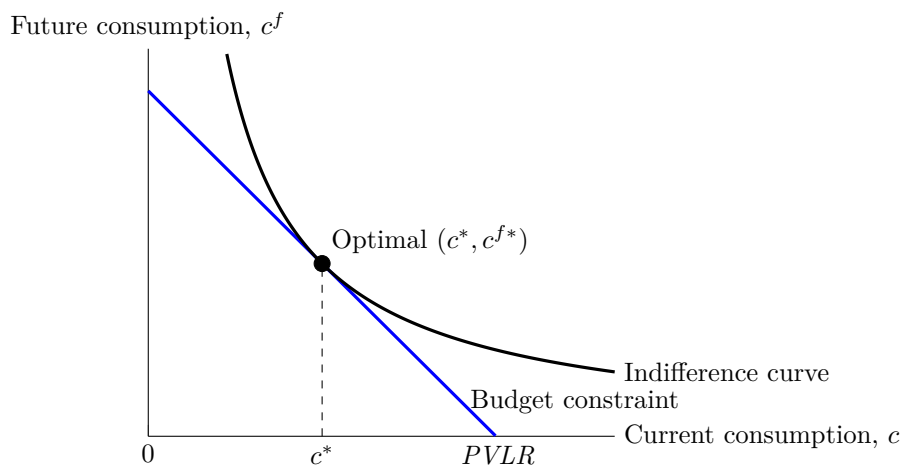
- $C$  has more  $c^f$  than  $A$  and the same  $c$ , hence

$$A \prec C \sim B$$

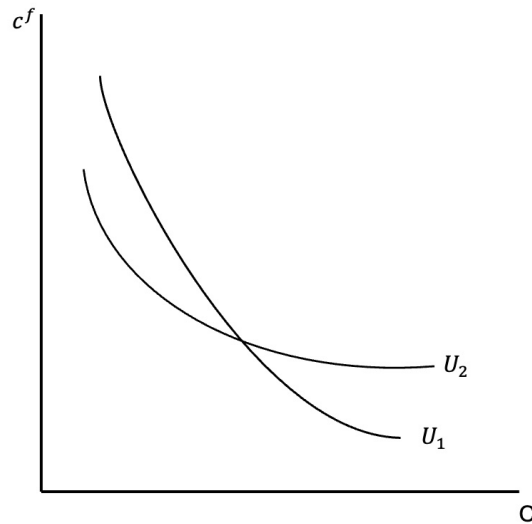
- $A$  is equally desirable as  $D$ , which has less  $c$  and  $c^f$  than  $B$

$$A \sim D \prec B$$

**Remark.** Optimal consumption is the point of tangency of the budget constraint and the indifference curve.



**Remark.** Slope of indifference curve



- $U_1$ : **present-oriented**, steeper, values consumption today, require a lot of consumption to give up a unit of consumption today
- $U_2$ : **future-oriented**, flatter, require a lot of consumption today to give up a unit of consumption tomorrow

#### 4.1.2 Effects of an increase in income, wealth, and expected future income

**Definition 4.4.** The **income effect** on consumption refers to an increase in consumption arising from an increase in current income, assets or expected future income.

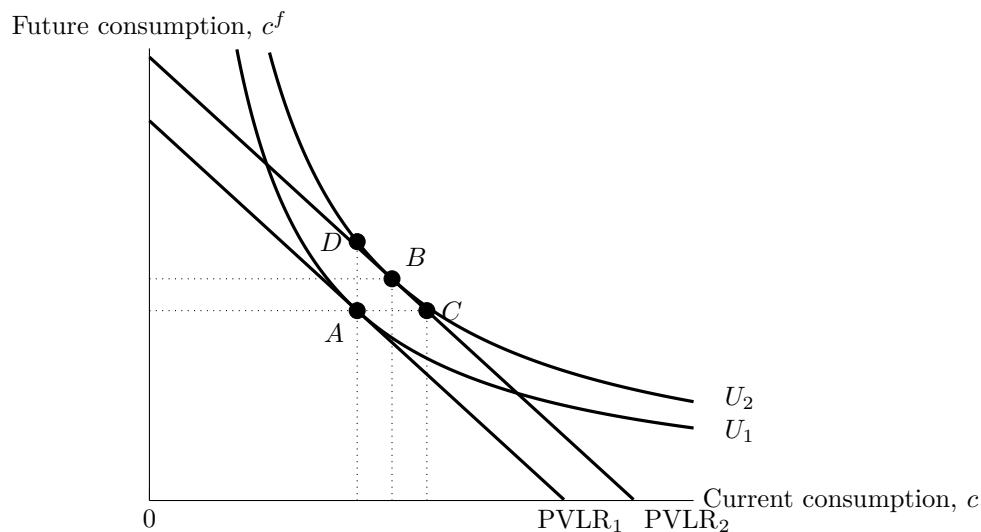
**Remarks.** Income effect occurs when

- $a \uparrow$
- $y \uparrow$
- $y^f \uparrow$

As a result

- $PVLR \uparrow$
- $r$  unchanged

**Income effect** operates through  $PVLR$  with unchanged  $r$ .



Note that the following points represent

- $A$ : initial optimal consumption
- $B$ : consumption smoothing, spending some and saving some
- $C$ : spending it all
- $D$ : saving it all



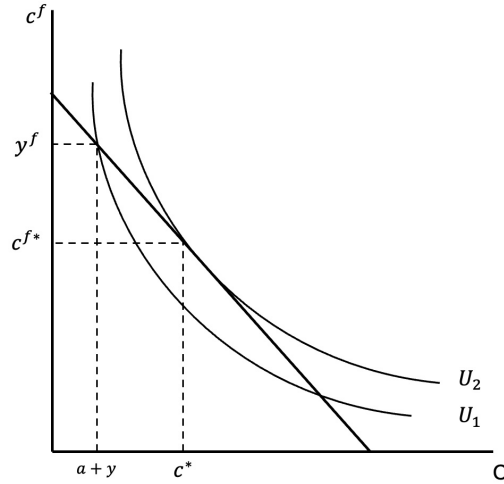
### 4.1.3 Riccardian Equivalence

**Theorem 4.5.** The **Riccardian Equivalence Proposition** states that tax cuts do not affect desired consumption or national saving because in the long run, because all government purchases must be paid for by taxes.

**Remark.** Empirically, tax cuts increases current consumption. Riccardian Equivalence is reconciled with empirical observations under assumptions of borrowing constraint.

Suppose an individual faces borrowing constraints and cannot borrow. The individual wants to spend up to  $c^*$  but can only attain  $c \leq a + y$ .

A one-time tax rebate can be seen as a relaxing of the borrowing constraints from  $a + y$  up to some  $a + \tilde{y}$ .



If

- $a + \tilde{y} < c^*$  increase  $c$  by the full amount of the tax rebate
- $a + \tilde{y} > c^*$  increase  $c$  only up to  $c^*$

#### Remarks. Riccardian Assumptions

1. Assuming REP does not hold
  - People spend some and save some, i.e. *consumption smoothing*

$$T \downarrow \implies c \uparrow, s \uparrow$$

2. Assuming REP holds and **there are borrowing constraints**

- Borrowers facing constraints increase consumption,

$$T \downarrow \implies c \uparrow, s \downarrow$$

3. Assuming REP holds and there are no borrowing constraints

$$T \downarrow \implies c, s \text{ constant}$$

### 4.1.4 Effects of an increase in interest rate

**Definition 4.6.** The **expected after-tax real interest rate** is the after-tax nominal interest rate minus expected rate of inflation

$$r_{a-t} = (1 - t)i - \pi^e$$

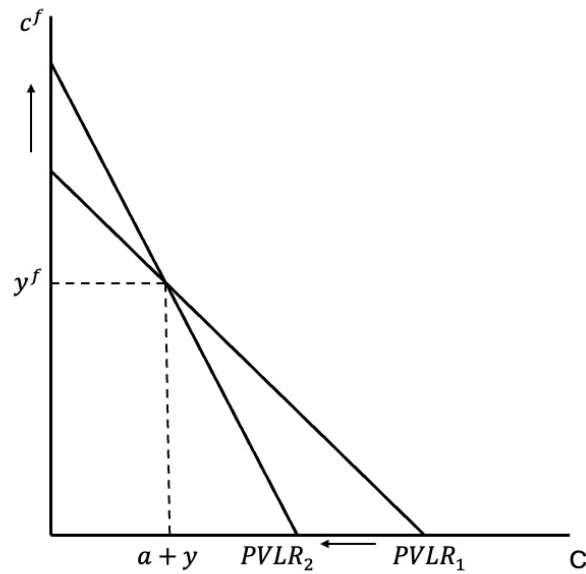
Where

$$\begin{aligned} i &= \text{nominal interest rate} \\ t &= \text{tax rate on interest income} \\ \pi^e &= \text{expected inflation} \end{aligned}$$

#### Remarks. Effect of an increase in interest rate on budget constraint

- $PVLR \downarrow$ : present value of future income decreases

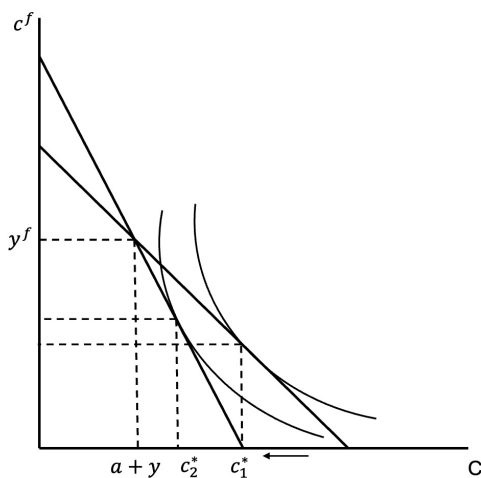
- vertical intercept  $\uparrow$ : future value of present income and assets increases
- no-borrowing no-lending point remains the same
- $c^*$ : depends



**Remarks. Effect of an increase in interest rate**

**For borrowers**

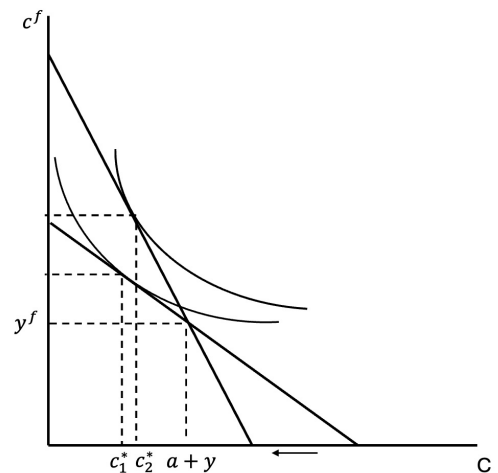
- Substitution effect:  $r \uparrow \Rightarrow s \uparrow c \downarrow$
- Income effect:  $r \uparrow \Rightarrow s \uparrow c \downarrow$



Borrowers **unequivocally consume less**.

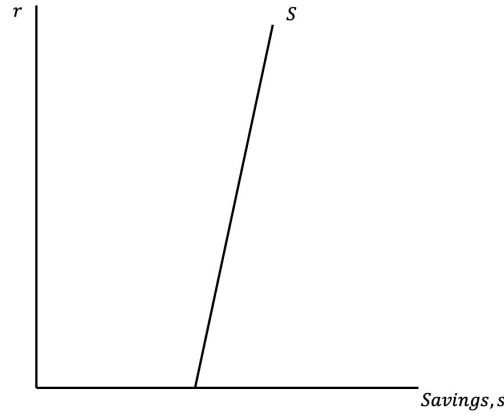
**For lenders**

- Substitution effect:  $r \uparrow \Rightarrow s \uparrow c \downarrow$
- Income effect:  $r \uparrow \Rightarrow s \downarrow c \uparrow$



Theory alone cannot predict the behavior of lenders when  $r$  increases.

**Remark. Empirically**, an increase in interest rate causes a moderate decrease in consumption and a moderate increase in savings.



#### 4.1.5 Effects of government purchases and taxes

Assumign no  $NFP$  or  $NX$ ,

$$S = Y - C - G$$

$$S_{priv} = \underbrace{(Y - T)}_{\text{Disposable Income}} - C$$

$$S_{govt} = T - G$$

**Remarks.** Effect of a tax cut, assuming REP does not hold

$$\bar{Y} = C + I + G$$

$$T \downarrow \Rightarrow (\bar{Y} - T \downarrow) \uparrow \Rightarrow C \uparrow \text{ by part}$$

$$S = (\bar{Y} - C \uparrow - G) \downarrow$$

$$\bar{Y} = C \uparrow + I \downarrow + G$$

**Remarks.** Effect of increase in government purchases

$$\bar{Y} = C + I + G \uparrow$$

$$S = (\bar{Y} - C - G \uparrow) \downarrow$$

$$\bar{Y} = C \downarrow \text{ a little } + I \downarrow + G \uparrow$$

#### 4.1.6 Summary of factors affecting consumption

**Remark.** Summary of factors affecting consumption.

Change	$\Delta C$	$\Delta S$
$y \uparrow$	$c \uparrow$	$s \uparrow$
$a \uparrow$	$c \uparrow$	$s \downarrow$
$y^f \uparrow$	$c \uparrow$	$s \downarrow$
$r \uparrow$	$c \downarrow$	$s \uparrow$

## 4.2 Investment

**Definition 4.7.** A firm's **desired capital stock** is the profit-maximizing amount of capital for the firm.

**Remark.** The profit-maximizing level of capital is achieved when the expected future marginal benefit, *expected future marginal product of capital*,  $MPK^f$  is equal to the expected future marginal cost, *user cost of capital*.

**Definition 4.8.** The **user cost of capital** is the expected real cost of a unit of capital for a specific period of time.

$$uc = (r + d)p_K$$

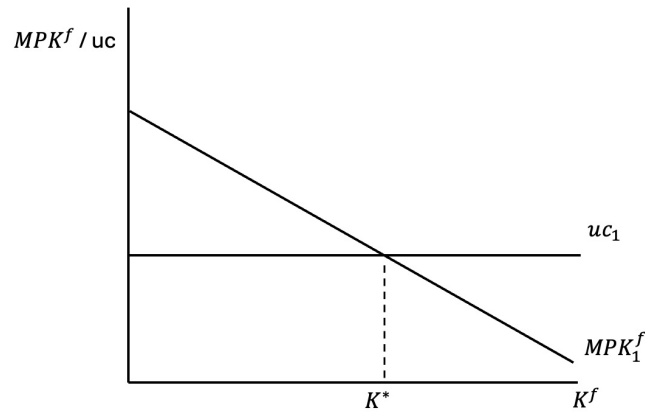
Where

$p_K$  = real price of capital goods

$d$  = capital depreciation rate

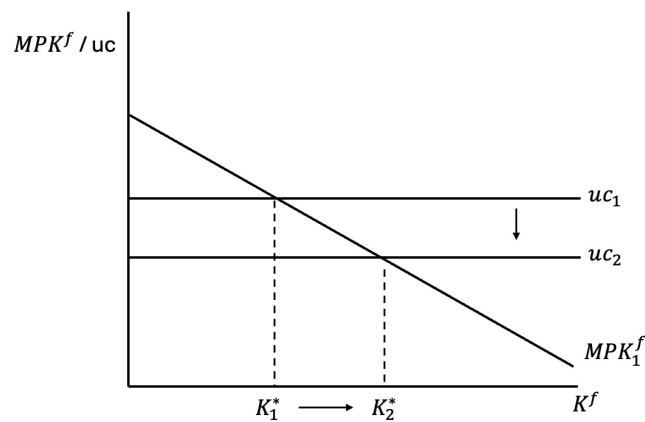
$r$  = expected real interest rate

**Remark. Desired Capital Stock**

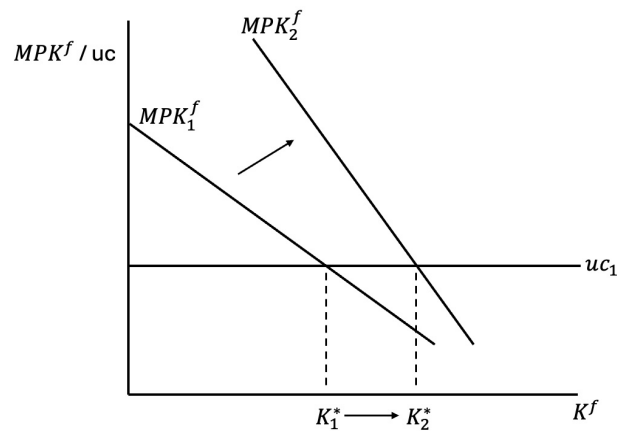


#### 4.2.1 Changes in desired capital stock

**Remark. Effect of decrease in user cost of capital**



**Remark. Effect of an increase in marginal product of capital**



#### 4.2.2 Effects of taxes on desired capital stock

**Definition 4.9.** The **tax-adjusted user cost of capital** is the user cost of capital divided by  $1 + \tau$  where  $\tau$  is the tax rate on firm revenues.

$$\text{tax-adjusted user cost of capital} = \frac{uc}{1 - \tau} = \frac{(r + d)p_K}{1 - \tau}$$

Firms facing corporate taxes earn an after-tax future marginal product of capital

$$(1 - \tau)MPK^f$$

Profit max capital stock occurs when

$$(1 - \tau)MPK^f = uc \implies MPK^f = \frac{uc}{1 - \tau} = \frac{(r + d)p_K}{1 - \tau}$$

### 4.2.3 Desired capital stock and investment

**Definition 4.10.** Gross investment is defined as the total purchase or construction of new capital goods.

**Definition 4.11.** Net investment is defined as the difference between gross investment and depreciation.

$$\underbrace{K_{t+1} - K_t}_{\text{net investment}} = \underbrace{I_t}_{\text{gross investment}} - \underbrace{dK_t}_{\text{depreciation}}$$

Where

$$\begin{aligned} I_t &= \text{gross investment in year } t \\ K_t &= \text{capital stock in the beginning of year } t \\ K_{t+1} &= \text{capital stock at the beginning of year } t + 1 \end{aligned}$$

Assuming firms seek to match  $K_{t+1}$  to  $K^*$ ,

$$I_t = \underbrace{K^* - K_t}_{\text{desired net increase in capital stock}} + \underbrace{dK_t}_{\text{investment required to replace worn out capital}}$$

## 4.3 Goods Market Equilibrium

**Definition 4.12.** The **goods market equilibrium condition**, assuming a closed economy, states that the quantity of goods demanded is the sum of desired consumption, desired investment and government purchases

$$Y = C^d + I^d + G$$

**Remark.** The goods market equilibrium is **not the same** as the income-expenditure identity for a closed economy.

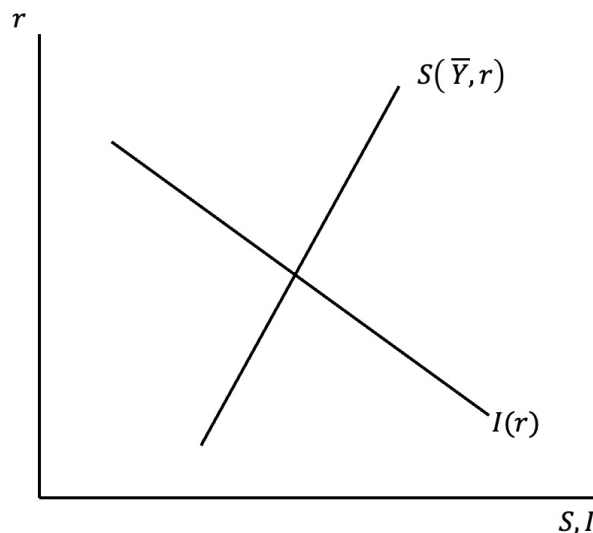
The income-expenditure identity relates actual income to actual spending and is always satisfied.

The goods market equilibrium condition may not always be satisfied. If firms produce more than consumers want to purchase

- inventory increases,
- income-expenditure identity remains true, increase in  $Y$  matched by increase in  $I$  (inventory spending)
- goods-market equilibrium no longer holds,  $Y > C^d + I^d + G$

**Corollary 4.13.** Goods market equilibrium implies that national saving is equal to desired investment.

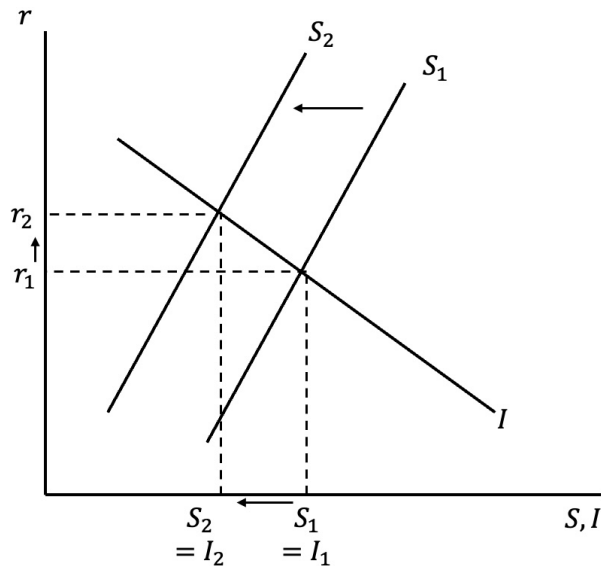
$$\begin{aligned} Y &= C^d + I^d + G \\ Y - C^d - G &= I^d \\ S^d &= I^d \end{aligned}$$



### 4.3.1 Factors affecting savings curve

**Remark.** Savings decreases due to

- decrease in current income
- increase in expected future income
- increase in wealth
- increase in current taxes
- increase in government purchases



**Remark. Effect of an adverse productivity shock**

An adverse productivity shock lowers labor demand and quantity of labor

$$A \downarrow \implies NS \downarrow \bar{N} \downarrow$$

Also, an adverse productivity shock lowers output

$$A \downarrow \implies A \downarrow F(K, N) = Y \downarrow$$

Since  $S = Y - C - G$ , and  $\bar{Y} \downarrow$ , the decrease in income lowers  $C$  and  $S$  by part via consumption smoothing.

**Remark. Effect of an increase in wealth**

Given an increase in wealth, individuals consume more due to consumption smoothing.

$$C \uparrow \implies S = (Y - C \uparrow - G) \downarrow$$

**Remark.** Summary of factors affecting goods-market equilibrium

Change	Description	$\Delta C$	$\Delta S$
$A \downarrow$	Total factor productivity	$\downarrow$	$\downarrow$
$G \uparrow$	Government purchases	$\downarrow$ a little	$\downarrow$
Wealth $\uparrow$	Increase in household wealth	$\uparrow$	$\downarrow$
$T \downarrow$	Taxes on households	$\uparrow$ followed by $\downarrow$ a little	$\downarrow$
$\tau \downarrow$	Business (corporate) tax rate	$\downarrow$ a little	$\uparrow$ a little

# 5 Saving and Investment in the Open Economy

## 5.1 Balance of Payments Accounting

**Definition 5.1.** (Current Account) The **current account** measures a country's trade in currently produced goods and services, along with unilateral transfers between countries.

$$CA = NX + NFP + NUT$$

Where

- $NX$ : net exports of goods and services
- $NFP$ : net income from abroad (primary income), approximated by  $NFP$
- $NUT$ : net unilateral transfers (secondary income)

**Remark.**  $CA$  is equal to the amount of funds that a country has available for net foreign lending

**Definition 5.2.** (Capital and financial account): The **capital and financial account** consists of

- capital account: unilateral transfers of assets
- financial account: transactions involving flow of assets

**Definition 5.3.** (Official settlements balance): **Official reserve assets** are assets, other than domestic money or securities, held by central banks and which can be used in making international payments.

The **official settlements balance**, or **balance of payments**, is the net increase in a country's official reserve assets.

**Result 5.4.** In each period, the current account balance and the capital and financial account balance must sum to zero.

$$CA + KFA = 0$$

Every international transaction involves a swap of goods and services for assets between countries. The two sides of the swap have offsetting effects on  $CA + KFA$

**Example.** The following are equivalent

- $CA$  surplus of \$10M
- $KFA$  deficit of \$10M
- net **acquisition** of foreign assets of \$10M
- net foreign lending of \$10M
- net exports of \$10M, assuming  $NFP = NUT = 0$

## 5.2 Goods market equilibrium in an open economy

**Theorem 5.5.** (National income accounting identity) Assuming  $NUT = 0$

$$S = I + CA = I + (NX + NFP)$$

i.e. national saving ( $S$ ) can be used to

1. increase capital stock through  $I$
2. increase stock of net foreign assets by **lending to foreigners**

**Theorem 5.6.** (Goods market equilibrium for an open economy) At goods market equilibrium, actual national saving and investment match their desired levels.

i.e. The desired amount of national saving is equal to the desired amount of domestic investment plus the amount lent abroad

$$S^d = I^d + CA = I^d + (NX + NFP)$$

For simplicity, assuming  $NFP = 0$ , the goods market equilibrium condition is

$$S^d = I^d + NX$$

This is equivalent to

$$Y = C^d + I^d + G + NX$$

and

$$NX = Y - (C^d + I^d + G)$$

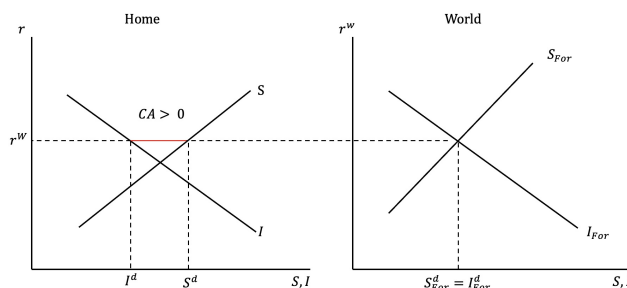
The last equation can be interpreted as: at goods market equilibrium, the amount of net exports equals total output less desired absorption (spending by domestic residents).

- output > absorption  $\Rightarrow NX > 0$
- output < absorption  $\Rightarrow NX < 0$

## 5.3 Small Open Economy

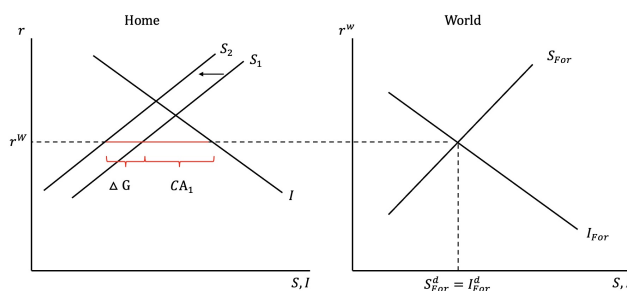
**Definition 5.7.** (Small open economy): A **small open economy** is defined as an economy too small to affect world real interest rate.

**Remark.** In a small open economy, the interest rate is fixed at  $r^w$ . Changes to  $S$  and  $I$  changes  $CA$ .

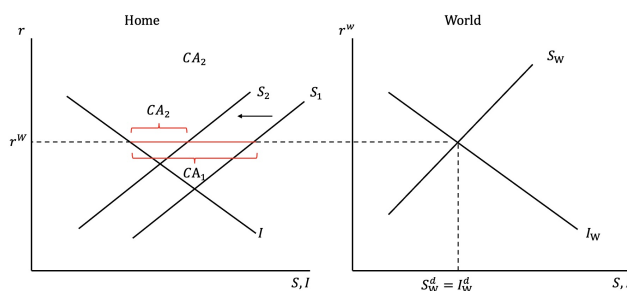


**Example.** Change in  $G$

- $G \uparrow \Rightarrow S = (Y - C - G \uparrow) \downarrow \Rightarrow NX \downarrow = CA \downarrow$



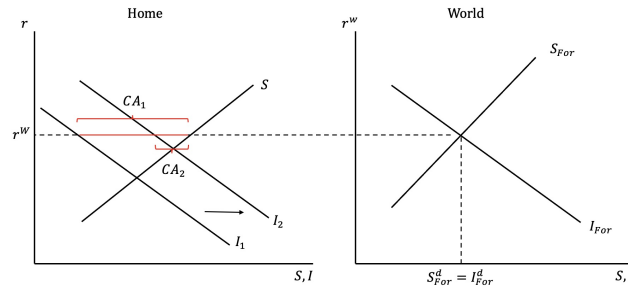
**Example.** Temporary adverse supply shock in small open economy



**Example.** Increase in  $MPK^f$

- $I \uparrow \Rightarrow CA \downarrow$





## 5.4 Large Open Economy

**Definition 5.8.** (Large open economy): A **large open economy** affects world interest rates.

The large open economy model is analogous to a two-economy model consisting of a domestic and a world economy.

**Remark.** In a large open economy / two economy model, the world interest rate is the interest rate such that desired international lending by one economy matches desired international borrowing by the other economy.

We know that world supply of goods matches world demand for goods

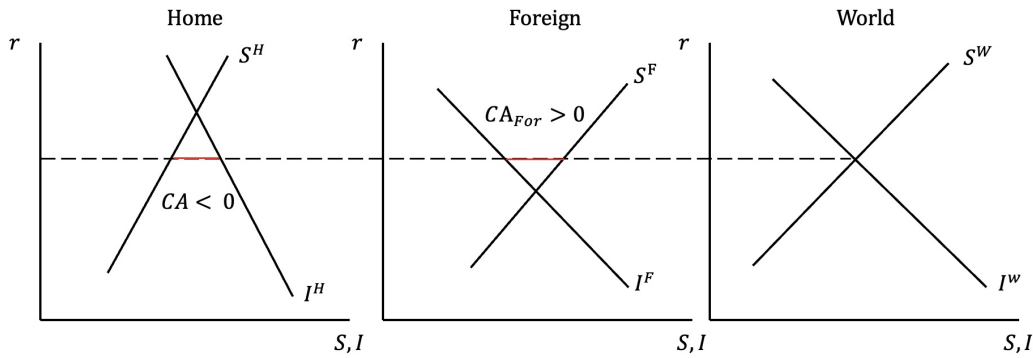
$$Y + Y_{For} = C^d + I^d + G + C_{For}^d + I_{For}^d + G_{For}$$

Hence, world saving matches world investment

$$\underbrace{(Y - G^d - G)}_{S^d} + \underbrace{(Y_{For} - C_{For}^d - G_{For})}_{S_{For}^d} = I^d + I_{For}^d$$

Hence, desired international borrowing matches desired international lending

$$\underbrace{(S^d - I^d)}_{CA} + \underbrace{(S_{For}^d - I_{For}^d)}_{CA_{For}} = 0$$

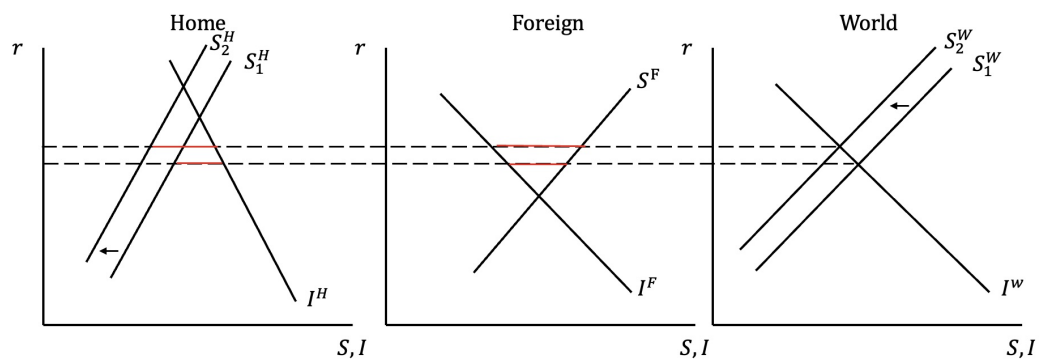


**Example.** Increase in government spending

- $G^H \uparrow \Rightarrow S^H = (\bar{Y}^H - C^H - G^H \uparrow) \downarrow$
- domestic savings curve shifts left
- world savings curve shifts left
  - world savings shifts by the manitude such that  $r^W$  brings  $NX^H$  and  $NX^F$  into equilibrium, i.e.

$$NX^H = NX^F \Leftrightarrow CA^H = CA^F$$

- composition of GDP:  $\bar{Y}^H_{no\ change} = C_{a\ little} \downarrow + I^H \downarrow + G^H \uparrow + NX^H \downarrow$



## 5.5 Fiscal policy and Current Account

**Proposition 5.9.** An increase in government budget deficit will raise the current account deficit only if the increase in budget deficit reduces national saving.

# 6 Long Run Economic Growth

## 6.1 Growth Accounting

**Definition 6.1.** (Capital-labor ratio) The capital-labor ratio is the amount of capital stock per worker, denoted

$$k_t = \frac{K_t}{N_t}$$

**Definition 6.2.** (Solow Growth Steady State) The Solow steady state is a situation in which the economy's output per worker, consumption per worker, and capital stock per worker are constant.

**Result 6.3.** (Growth accounting equation) From the production function

$$Y = AF(K, N)$$

The relationship between output, input and productivity growth is

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + a_K \frac{\Delta K}{K} + a_N \frac{\Delta N}{N}$$

Where

- $a_K$  = elasticity of output wrt capital
- $a_N$  = elasticity of output wrt labor

## 6.2 Solow Growth Model

**Result 6.4.** (Solow Growth Model)

Denote

- $N_t$ : the population in year  $t$
- $Y_t$ : output in year  $t$
- $I_t$ : gross investment in year  $t$
- $C_t$ : consumption in year  $t$
- $S_t$ : saving in year  $t$
- $y_t = \frac{Y_t}{N_t}$ : output per worker in year  $t$
- $c_t = \frac{C_t}{N_t}$ : consumption per worker in year  $t$
- $k_t = \frac{K_t}{N_t}$ : capital stock per worker in year  $t$

Assume

- $N_t$  grows at fixed rate  $n$
- economy is closed and  $G = 0$ , which implies output is either consumed or invested to grow capital stock, then

$$C_t = Y_t - I_t$$

- $S_t$  is proportional to current income for some fixed saving rate

$$S_t = sY_t$$

The per-worker production function is

$$c_t = Af(k_t) - (n + d)k_t$$

At steady state, capital grows at  $n$ , hence

$$I_t = (n + d)K_t$$

Steady state consumption is therefore

$$C_t = Y_t - (n + d)K_t$$

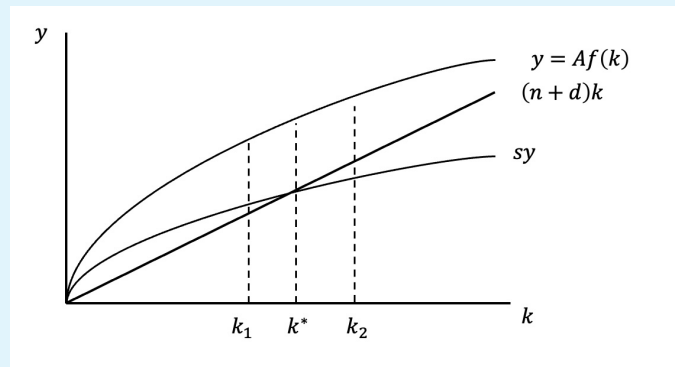
In per worker terms,

$$c_t = Af(k_t) - (n + d)k_t$$

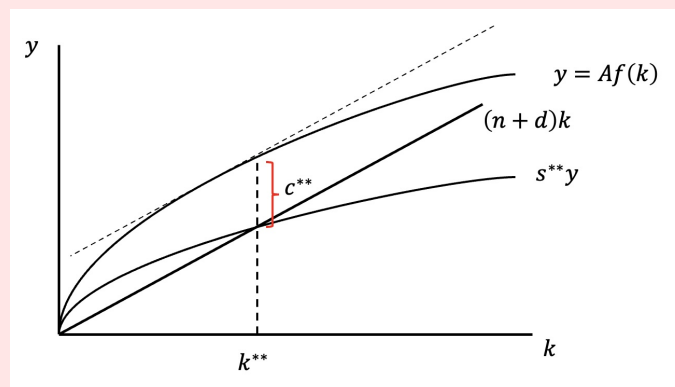
Since national savings equal investment

$$S_t = sY_t = (n + d)K_t \implies fAf(k_t) = (n + d)k_t$$

At equilibrium,

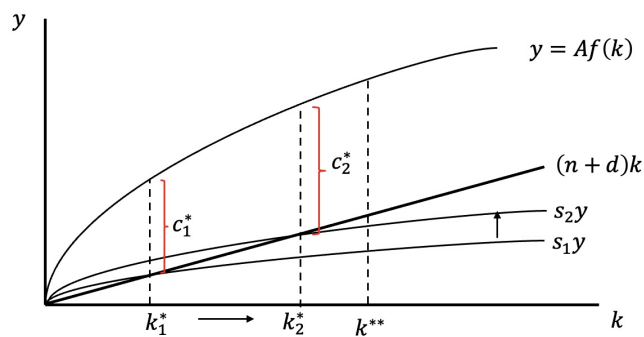


**Definition 6.5.** (Golden Rule capital labor ratio): The golden rule capital labor ratio is the capital-labor ratio that maximizes consumption.

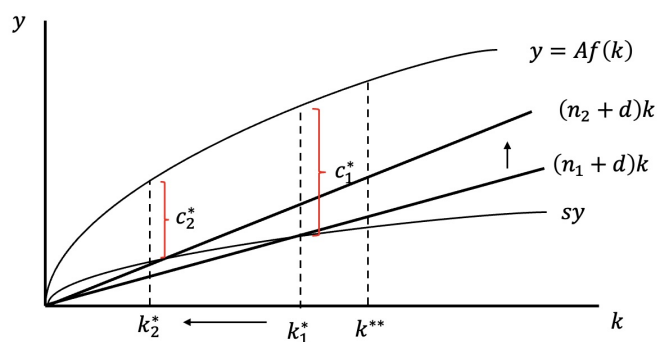


**Remark.** There is no equilibrating mechanism to bring  $k$  to  $k^{**}$ .

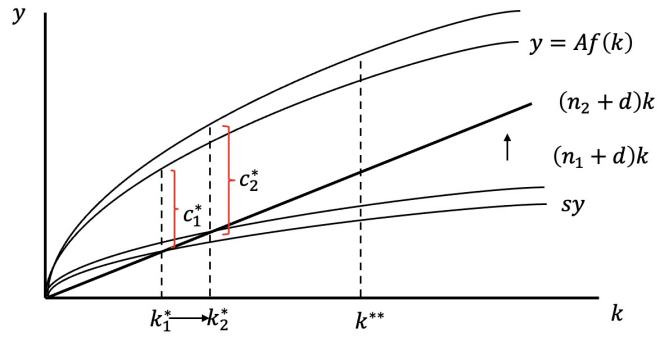
**Remark.** Effects of change in  $s$ , saving rate



**Remark.** Effects of change in  $n$ , population growth



**Remark.** Effects of change in  $A$ , productivity growth



## 6.3 Endogenous Growth Theory

**Remark.** According to Solow Growth Model, sustained growth in output per capital can only be achieved through sustained productivity growth, which is exogenous.

Endogenous growth theory allows for endogenous productivity growth.

**Example.** One endogenous model is

$$Y = AK$$

Note that in this model, the marginal product of capital is  $A$ , and the marginal product of capital does not depend on  $K$  (no diminishing marginal products).

Assume that national saving is a constant fraction of output

$$S = sY = sAK$$

Also,

$$I = \Delta K + dK$$

Hence

$$\begin{aligned} I &= S \\ \Rightarrow \Delta K &= dK = sAK \\ \Rightarrow \frac{\Delta K}{K} &= sA - d \\ \Rightarrow \frac{\Delta Y}{Y} &= sA - d \end{aligned}$$