05_monte_carlo

December 18, 2025

1 Monte Carlo Simulation

To verify asymptotic power and size of the three tests, we conduct a MC simulation.

[1]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import torch
import sys
from scipy.stats import norm
from scipy.stats import kstest
import scipy.stats as stats
import seaborn as sns
sys.executable

sys.path.append('../')
from utils import utils
sys.executable
```

[1]: '/Users/fanghema/Desktop/aaSTAT_5200/STAT_5200_final_project/env/bin/python'

[2]:
```python
def simulate_dgp(N, K, R, T,
                 heterogeneity_strength=0.0,
                 sigma_u=0.2,
                 sigma_eps=0.5,
                 sigma_g=1.0,
                 seed=None):
    """
    Simulates data consistent with your estimator structure.

    N: industries
    K: observed factors (Mkt, SMB, ...)
    R: latent factors in your iterative_convergence
    T: number of periods

    heterogeneity_strength = 0  → null hypothesis
    heterogeneity_strength > 0 → alternative
    """
```

```python
    rng = np.random.default_rng(seed)

    G = rng.normal(0, sigma_g, size=(T, R))

    alpha = np.zeros(N)
    lambda_true = np.zeros((N, K))

    for i in range(N):
        lambda_true[i] = heterogeneity_strength * rng.normal(0, 1, size=K)

    beta_star_true = rng.normal(0, 1, size=(N, R))

    beta_true = np.zeros((N, K, T))
    for i in range(N):
        beta_i0 = rng.normal(0, 1, size=K)
        for t in range(T):
            beta_true[i,:,t] = beta_i0 + sigma_u * rng.normal(0,1,size=K)

    realized_cov = beta_true + sigma_u * rng.normal(0,1,size=(N,K,T))

    residuals = sigma_u * rng.normal(0,1,size=(N,K,T))

    r = np.zeros((N,T))
    for i in range(N):
        for t in range(T):
            mean_part = alpha[i] + lambda_true[i] @ beta_true[i,:,t]
            g_part    = beta_star_true[i] @ G[t]
            r[i,t] = mean_part + g_part + sigma_eps * rng.normal()

    return beta_true, r, realized_cov, residuals, G, beta_star_true, lambda_true
```

[3]:
```python
def run_mc(N, K, R, T, n_rep=200, verbose=False, heterogeneity_strength = 0):
    """
    Estimates empirical size of your tests:
    - full homogeneity
    - intercept homogeneity
    - slope homogeneity

    Simulates data under the TRUE null hypothesis (H0):
        - all lambda_i identical (here: all zeros)
        - all alpha_i identical (zeros)
    """

    gamma_a_lambda = np.empty_like(np.arange(n_rep))
    gamma_a = np.empty_like(np.arange(n_rep))
    gamma_lambda = np.empty_like(np.arange(n_rep))
```

```python
    for rep in range(n_rep):
        if verbose and (rep % 100) == 0:
            print(f"Processing {rep} out of {n_rep}")
        beta_true, r, realized_cov, residuals, G_true, beta_star_true, _ = \
            simulate_dgp(
                N=N, K=K, R=R, T=T,
                heterogeneity_strength=heterogeneity_strength
            )

        beta_hat = beta_true
        eta_hat, G_hat, beta_star_hat, _ = utils.iterative_convergence(
            beta_hat, r, N, K, R, T, n_iter=500
        )
        avar = utils.estimate_avar(
            beta_hat=beta_hat,
            excess_returns=r,
            eta=eta_hat,
            G=G_hat,
            beta_star=beta_star_hat,
            realized_covariance=realized_cov,
            residuals=residuals,
            N=N, K=K, R=R, T=T,
        )

        full = utils.full_homogeneity_test(eta_hat, avar, N, K, T)
        inta = utils.intercept_homogeneity_test(eta_hat, avar, N, K, T)
        slope = utils.slope_homogeneity_test(eta_hat, avar, N, K, T)

        gamma_a_lambda[rep] = full
        gamma_a[rep] = inta
        gamma_lambda[rep] = slope

    return {
        "gamma_a_lambda": gamma_a_lambda,
        "gamma_a": gamma_a,
        "gamma_lambda": gamma_lambda,
    }
```

[4]:

```python
alpha = 0.05
n_rep = 1000
print("SIZE TEST RESULTS:")

results = run_mc(N=20, K=3, R=1, T=200, n_rep=n_rep, verbose= True)
```

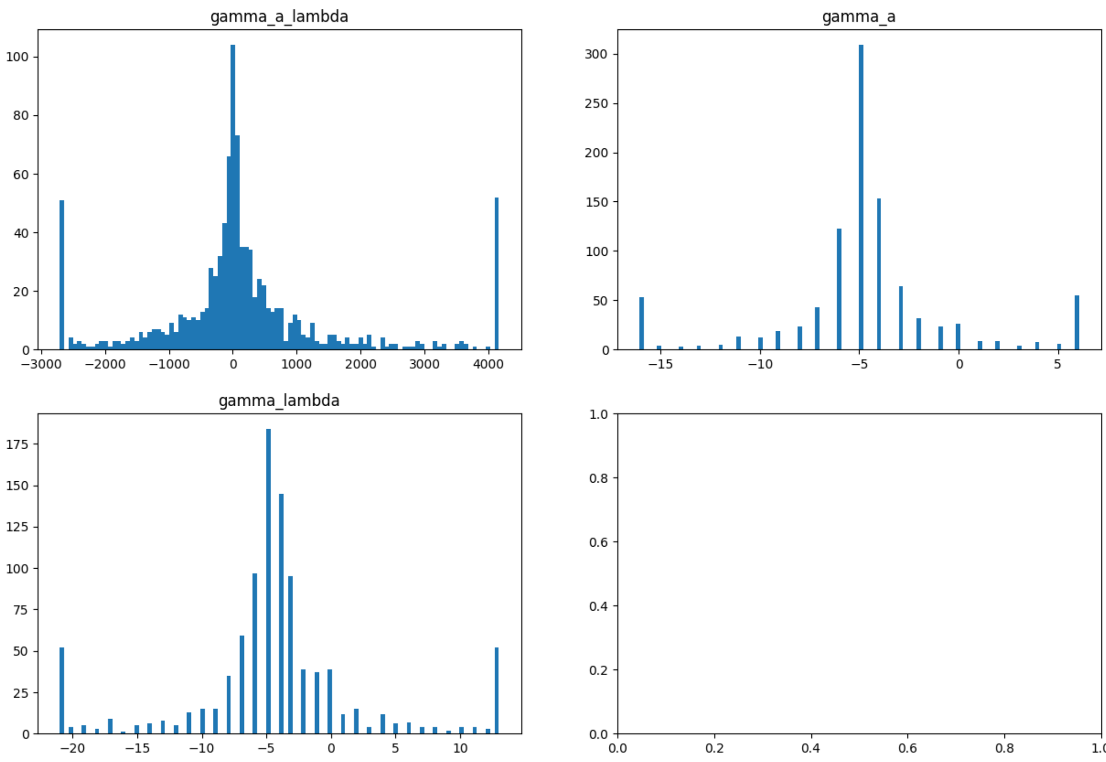
```
SIZE TEST RESULTS:
Processing 0 out of 1000
Processing 100 out of 1000
Processing 200 out of 1000
```

```
Processing 300 out of 1000
Processing 400 out of 1000
Processing 500 out of 1000
Processing 600 out of 1000
Processing 700 out of 1000
Processing 800 out of 1000
Processing 900 out of 1000
```

[5]:
```
critical = norm.ppf(1 - alpha/2)
print(f"Type I error rate - joint: ")
print(f"{(np.abs(results['gamma_a_lambda']) > critical).sum() / n_rep:.4f}")

print(f"Type I error rate - intercept: ")
print(f"{(np.abs(results['gamma_a']) > critical).sum() / n_rep:.4f}")

print(f"Type I error rate - slope: ")
print(f"{(np.abs(results['gamma_lambda']) > critical).sum() / n_rep:.4f}")
```

```
Type I error rate - joint:
0.9960
Type I error rate - intercept:
0.9420
Type I error rate - slope:
0.9120
```

[6]:
```
fig, axes = plt.subplots(2, 2, figsize = (15, 10))
axes = np.ravel(axes)

for i, test_statistic in enumerate(['gamma_a_lambda', 'gamma_a',␣
↪'gamma_lambda']):
    axes[i].hist(
        utils.clean(results[test_statistic]),
        bins = 100,
    )
    axes[i].set_title(test_statistic)
```

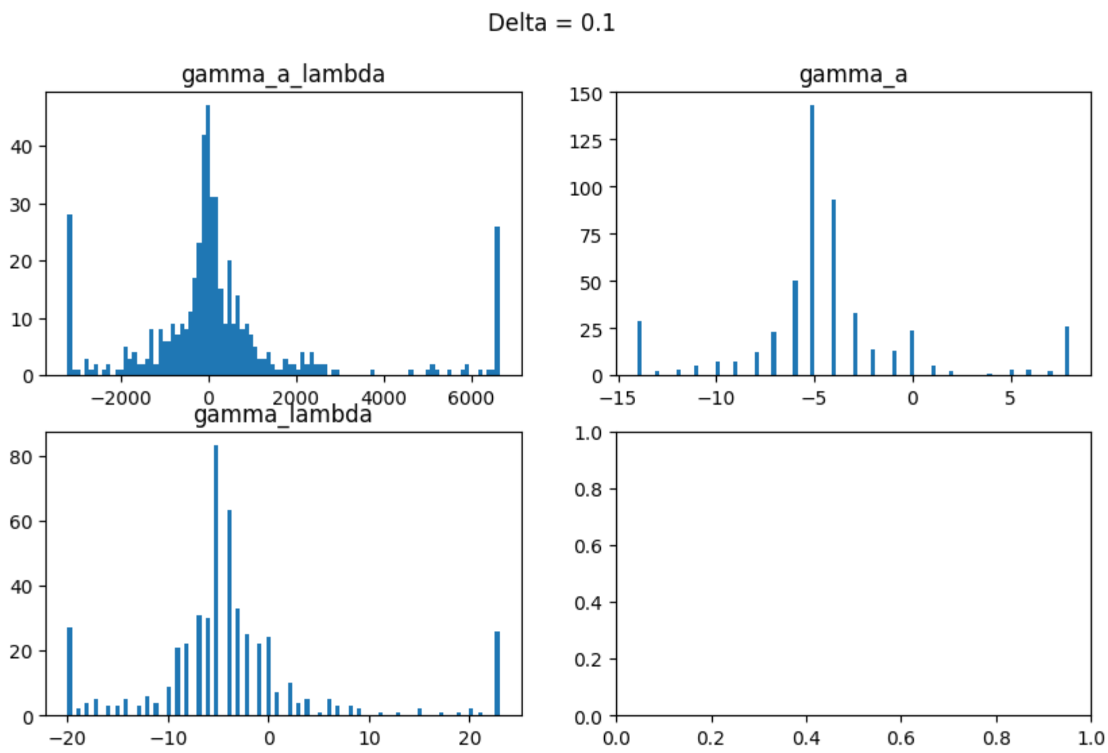


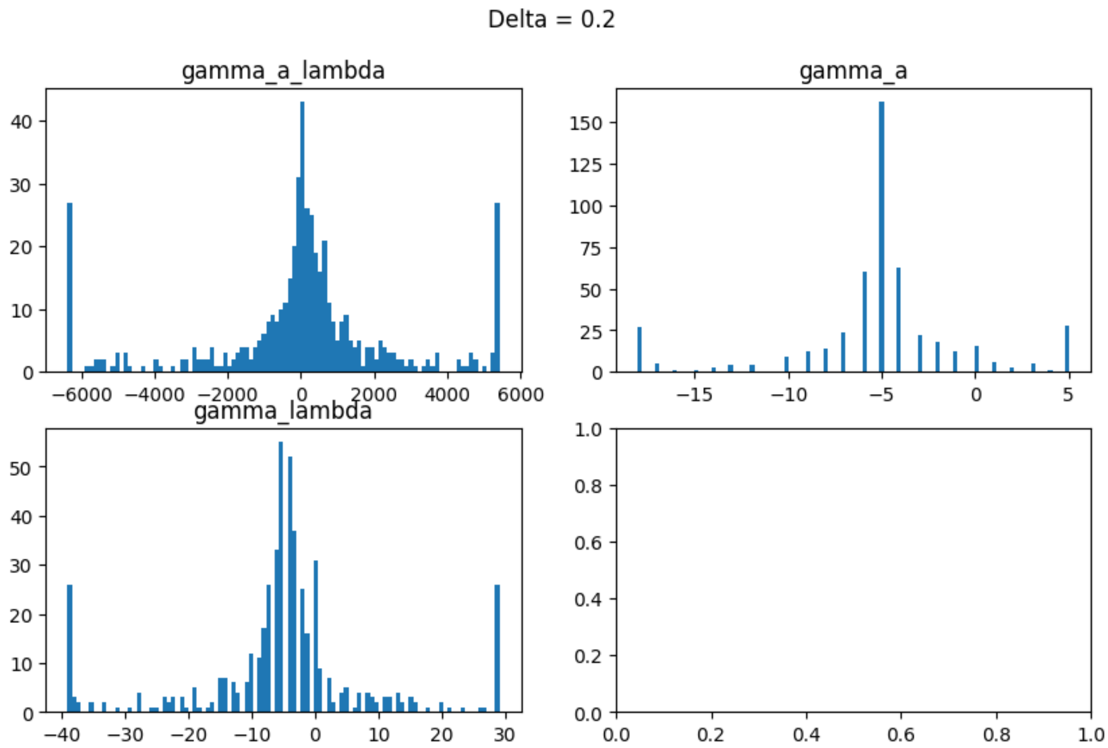
[7]:
```
power_results = {}
for delta in [0.1, 0.2, 0.5, 1.0]:

    print("====================================================================")
    print(f"POWER for heterogeneity_strength={delta}:")

    local_results = run_mc(N=20, K=3, R=1, T=200,
                    heterogeneity_strength=delta,
                    n_rep=500,
                    verbose = True)

    power_results[delta]= local_results

    critical = norm.ppf(1 - alpha/2)
    print(f"Power - joint")
    print(f"{(np.abs(results['gamma_a_lambda']) > critical).sum() / n_rep:.4f}")

    print(f"Power - intercept")
    print(f"{(np.abs(results['gamma_a']) > critical).sum() / n_rep:.4f}")

    print(f"Power - slope")
    print(f"{(np.abs(results['gamma_lambda']) > critical).sum() / n_rep:.4f}")
    print("====================================================================")
```

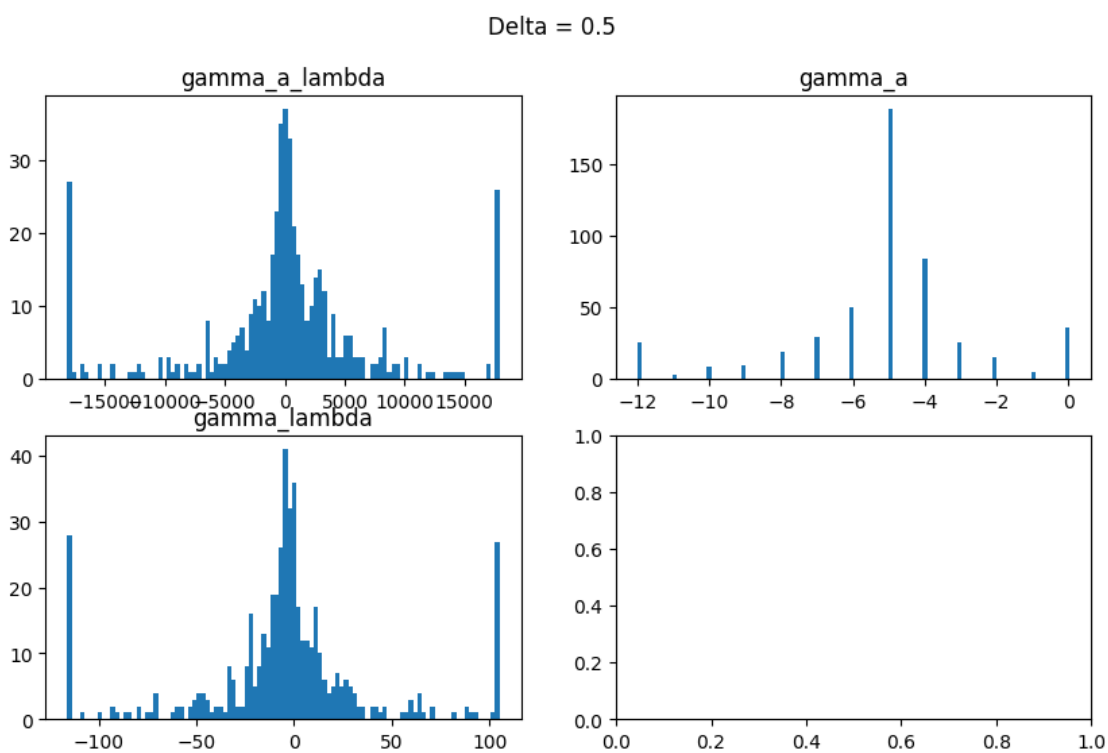
```
======================================================================
POWER for heterogeneity_strength=0.1:
Processing 0 out of 500
Processing 100 out of 500
Processing 200 out of 500
Processing 300 out of 500
Processing 400 out of 500
Power - joint
0.9960
Power - intercept
0.9420
Power - slope
0.9120
======================================================================
======================================================================
POWER for heterogeneity_strength=0.2:
Processing 0 out of 500
Processing 100 out of 500
Processing 200 out of 500
Processing 300 out of 500
Processing 400 out of 500
Power - joint
0.9960
Power - intercept
0.9420
Power - slope
0.9120
======================================================================
======================================================================
POWER for heterogeneity_strength=0.5:
Processing 0 out of 500
Processing 100 out of 500
Processing 200 out of 500
Processing 300 out of 500
Processing 400 out of 500
Power - joint
0.9960
Power - intercept
0.9420
Power - slope
0.9120
======================================================================
======================================================================
POWER for heterogeneity_strength=1.0:
Processing 0 out of 500
Processing 100 out of 500
```

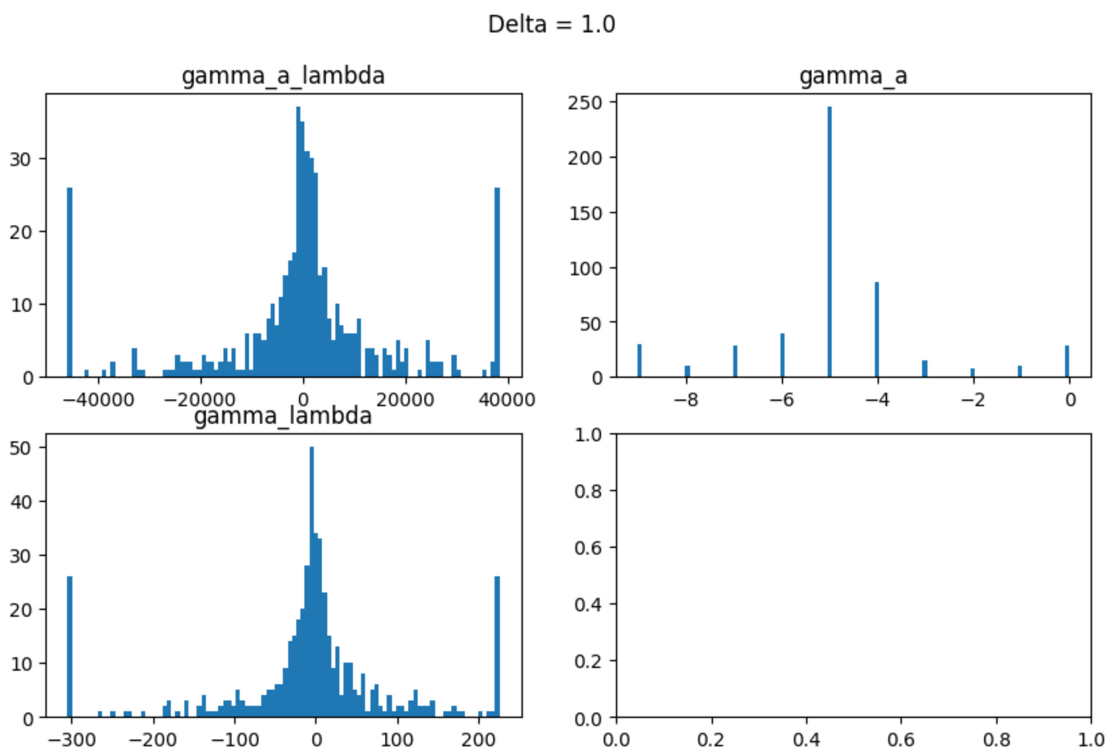
```
Processing 200 out of 500
Processing 300 out of 500
Processing 400 out of 500
Power - joint
0.9960
Power - intercept
0.9420
Power - slope
0.9120
=================================================================
```

[8]:
```
for delta in [0.1, 0.2, 0.5, 1.0]:
    fig, axes = plt.subplots(2, 2, figsize = (10, 6))
    axes = np.ravel(axes)

    for i, test_statistic in enumerate(['gamma_a_lambda', 'gamma_a',␣
 ↪'gamma_lambda']):
        axes[i].hist(
            utils.clean(power_results[delta][test_statistic]),
            bins = 100,
        )
        axes[i].set_title(test_statistic)
    fig.suptitle(f"Delta = {delta}")
```




Delta = 0.2
gamma_a_lambda
gamma_a
gamma_lambda
−6000 −4000 −2000 0 2000 4000 6000
−15 −10 −5 0 5
−40 −30 −20 −10 0 10 20 30
0.0 0.2 0.4 0.6 0.8 1.0


Delta = 0.5
gamma_a_lambda
gamma_a
gamma_lambda
−15000 −10000 −5000 0 5000 10000 15000
−12 −10 −8 −6 −4 −2 0
−100 −50 0 50 100
0.0 0.2 0.4 0.6 0.8 1.0



We examine the distribution of the test statistic under null

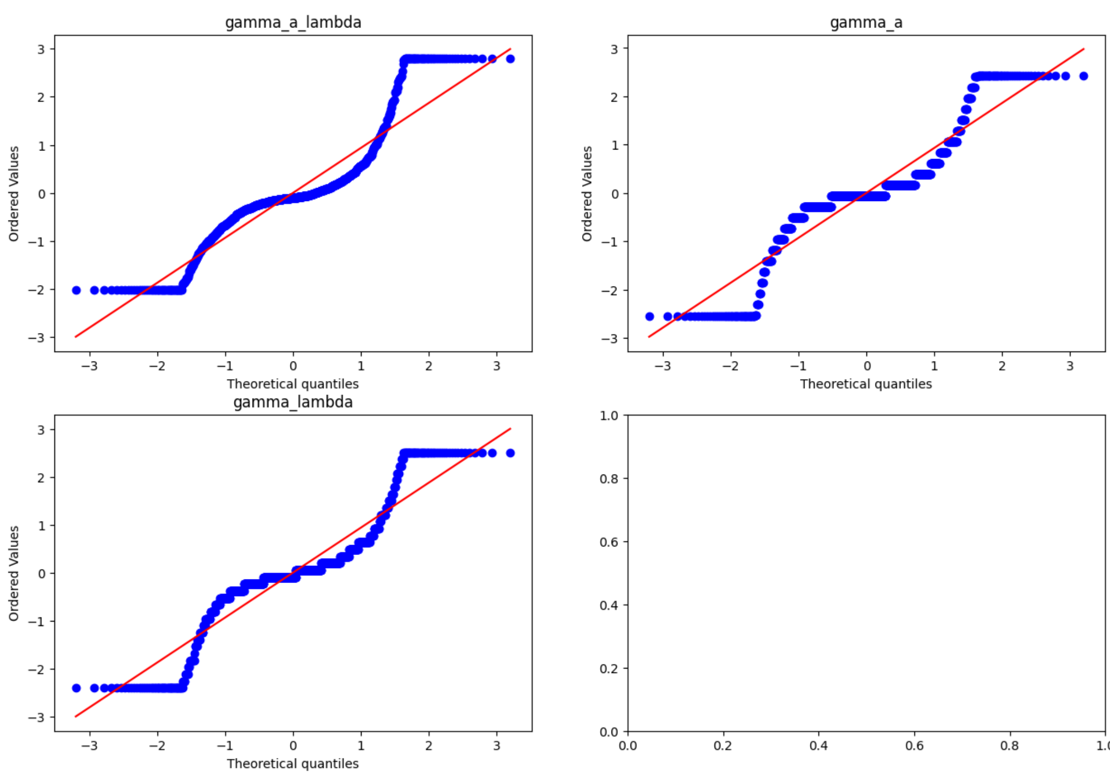
[9]:
```
print("MC parameters")
print("N=20")
print("K=3")
print("R=1")
print("T=200")
for k, v in results.items():
    print(f"==========" * 5)
    print(k)
    print(f"mean: {v.mean()}")
    print(f"std: {v.std()}")
    print(f"min: {v.min()}")
    print(f"max: {v.max()}")
```

```
MC parameters
N=20
K=3
R=1
T=200
==================================================
gamma_a_lambda
mean: 420.043
std: 24151.585248657095
```

```
min: -507281
max: 422577
==================================================
gamma_a
mean: -8.414
std: 75.15964744462285
min: -1923
max: 109
==================================================
gamma_lambda
mean: -0.58
std: 168.72552148385856
min: -1709
max: 4320
```

[10]:

```python
fig, axes = plt.subplots(2, 2, figsize=(15, 10))
axes = np.ravel(axes)

for i, test_statistic in enumerate(['gamma_a_lambda', 'gamma_a',␣
 ↪'gamma_lambda']):
    vals = utils.clean(results[test_statistic])
    z_vals = (vals- vals.mean()) / vals.std()
    stats.probplot(z_vals, dist="norm", plot=axes[i])
    axes[i].set_title(test_statistic)
```



1.1 Asymptotic properties of estimator

[ ]:
```python
def run_mc_eta_properties(
    N=20, K=3, R=1, T=200,
    n_rep=200,
    heterogeneity_strength=0.5,
    sigma_u=0.2,
    sigma_eps=0.5,
    sigma_g=1.0,
    seed=0
):
    """
    Monte Carlo experiment for sampling properties of the eta estimator.
    Returns:
        eta_hats: (n_rep, N, K+1)
        avars:    (n_rep, N*(K+1), N*(K+1))
        lambda_true: (N, K)
    """

    rng = np.random.default_rng(seed)

    p = N * (K+1)

    eta_hats = np.zeros((n_rep, N, K+1))
    avars    = np.zeros((n_rep, p, p))
    lambda_store = None  # to save true lambda

    for rep in range(n_rep):
        if(rep % 10) == 0:
            print(f"Processing rep {rep}")
        rep_seed = rng.integers(1, 1_000_000_000)

        (
            beta_true,          # (N, K, T)
            r,                  # (N, T)
            realized_cov,       # (N, K, T)
            residuals,          # (N, K, T)
            G_true,             # (T, R)
            beta_star_true,     # (N, R)
            lambda_true         # (N, K)
        ) = simulate_dgp(
            N=N, K=K, R=R, T=T,
            heterogeneity_strength=heterogeneity_strength,
            sigma_u=sigma_u,
            sigma_eps=sigma_eps,
```

```
            sigma_g=sigma_g,
            seed=rep_seed
        )

        if lambda_store is None:
            lambda_store = lambda_true.copy()

        beta_hat = beta_true

        # Estimate eta
        eta_hat, G_hat, beta_star_hat, objvals = utils.iterative_convergence(
            beta_hat=beta_hat,
            excess_returns=r,
            N=N, K=K, R=R, T=T,
            n_iter=2000
        )

        # Estimate AVAR
        avar_hat = utils.estimate_avar(
            beta_hat=beta_hat,
            excess_returns=r,
            eta=eta_hat,
            G=G_hat,
            beta_star=beta_star_hat,
            realized_covariance=realized_cov,
            residuals=residuals,
            N=N, K=K, R=R, T=T
        )

        eta_hats[rep] = eta_hat
        avars[rep] = avar_hat

    return eta_hats, avars, lambda_store

def analyze_eta_bias_variance(eta_hats, lambda_true, N, K, T):
    """
    eta_hats: (n_rep, N, K+1)
    lambda_true: (N, K)
    """
    n_rep = eta_hats.shape[0]
    Kp1 = K+1
    p = N*Kp1

    # build eta_true array
    eta_true = np.zeros((N, K+1))
    eta_true[:,0] = 0.0
    eta_true[:,1:] = lambda_true
```

```
    eta_true_vec = eta_true.reshape(p)

    # empirical mean
    eta_bar = eta_hats.mean(axis=0)
    eta_bias = eta_bar - eta_true

    print("\n==== BIAS OF ETA ESTIMATOR ====")
    print("Max abs bias:", np.max(np.abs(eta_bias)))
    print("Mean abs bias:", np.mean(np.abs(eta_bias)))

    eta_vec = eta_hats.reshape(n_rep, p)
    eta_centered = eta_vec - eta_true_vec

    emp_cov = (eta_centered.T @ eta_centered) / (n_rep - 1)

    return eta_true, eta_true_vec, emp_cov

def compare_empirical_to_avar(emp_cov, avars, T, N, K):
    Kp1 = K + 1
    p = N*Kp1

    avar_mean = avars.mean(axis=0)
    emp_var = np.diag(emp_cov)
    theo_var = np.diag(avar_mean) / T  # theoretical scaled variance

    print("\n==== VARIANCE COMPARISON ====")
    print("Empirical variance (first 10):", emp_var[:10])
    print("Theoretical variance (first 10):", theo_var[:10])

    return emp_var, theo_var


def check_z_normality(eta_hats, avars, eta_true_vec, T, param_index):
    """
    param_index: index in vec(eta) to test (0 to p-1)
    """

    n_rep = eta_hats.shape[0]
    p = eta_true_vec.shape[0]

    z_vals = []

    for rep in range(n_rep):
        eta_vec = eta_hats[rep].reshape(p)
        avar_hat = avars[rep]
```

```
        se = np.sqrt(avar_hat[param_index, param_index] / T)
        z = (eta_vec[param_index] - eta_true_vec[param_index]) / se

        z_vals.append(z)

    z_vals = np.array(z_vals)

    print("\n==== NORMALITY CHECK ====")
    print("mean(z):", z_vals.mean())
    print("std(z):", z_vals.std())
    print("KS test vs N(0,1):", kstest(z_vals, 'norm'))

    return z_vals
```

[ ]:
```
def coverage_probability(eta_hats, avars, eta_true_vec, param_index, T, alpha=0.
 ↪05):
    zcrit = norm.ppf(1 - alpha/2)

    n_rep = eta_hats.shape[0]
    p = len(eta_true_vec)
    covered = 0

    for rep in range(n_rep):
        eta_vec = eta_hats[rep].reshape(p)
        avar_hat = avars[rep]
        se = np.sqrt(avar_hat[param_index, param_index] / T)

        ci_low  = eta_vec[param_index] - zcrit*se
        ci_high = eta_vec[param_index] + zcrit*se

        if (eta_true_vec[param_index] >= ci_low) and (eta_true_vec[param_index]␣
 ↪<= ci_high):
            covered += 1

    coverage = covered / n_rep
    print("\n==== COVERAGE ====")
    print(f"Coverage probability: {coverage:.3f} (target = 0.95)")
    return coverage
```

[ ]:
```
eta_hats, avars, lambda_true = run_mc_eta_properties(
    N=20, K=3, R=1, T=200,
    heterogeneity_strength=0.5,
    n_rep=500
)

eta_true, eta_true_vec, emp_cov = analyze_eta_bias_variance(
    eta_hats, lambda_true, N=20, K=3, T=200
```

```
)

emp_var, theo_var = compare_empirical_to_avar(
    emp_cov, avars, T=200, N=20, K=3
)

Kp1 = 3 + 1
param_index = 0*Kp1 + 1  # (asset 0, slope 1)

z_vals = check_z_normality(
    eta_hats, avars, eta_true_vec, T=200, param_index=param_index
)

coverage = coverage_probability(
    eta_hats, avars, eta_true_vec, param_index=param_index, T=200
)
```

```
==== BIAS OF ETA ESTIMATOR ====
Max abs bias: 1.6593719669770914
Mean abs bias: 0.3159582355355777

==== VARIANCE COMPARISON ====
Empirical variance (first 10): [0.09689915 0.50258872 0.34710584 0.27115956
0.11882595 0.58212828
 1.82794002 0.42457894 0.1129175  0.3213619 ]
Theoretical variance (first 10): [0.00713551 0.00715691 0.00715691 0.00715691
0.00714484 0.00715691
 0.00714225 0.00715691 0.00714669 0.00715691]

==== NORMALITY CHECK ====
mean(z): -5.956843693296206
std(z): 7.766782441554891
KS test vs N(0,1): KstestResult(statistic=np.float64(0.6739886744018414),
pvalue=np.float64(4.9638594541501135e-225),
statistic_location=np.float64(-1.7278080080687428), statistic_sign=np.int8(1))

==== COVERAGE ====
Coverage probability: 0.168 (target = 0.95)
```

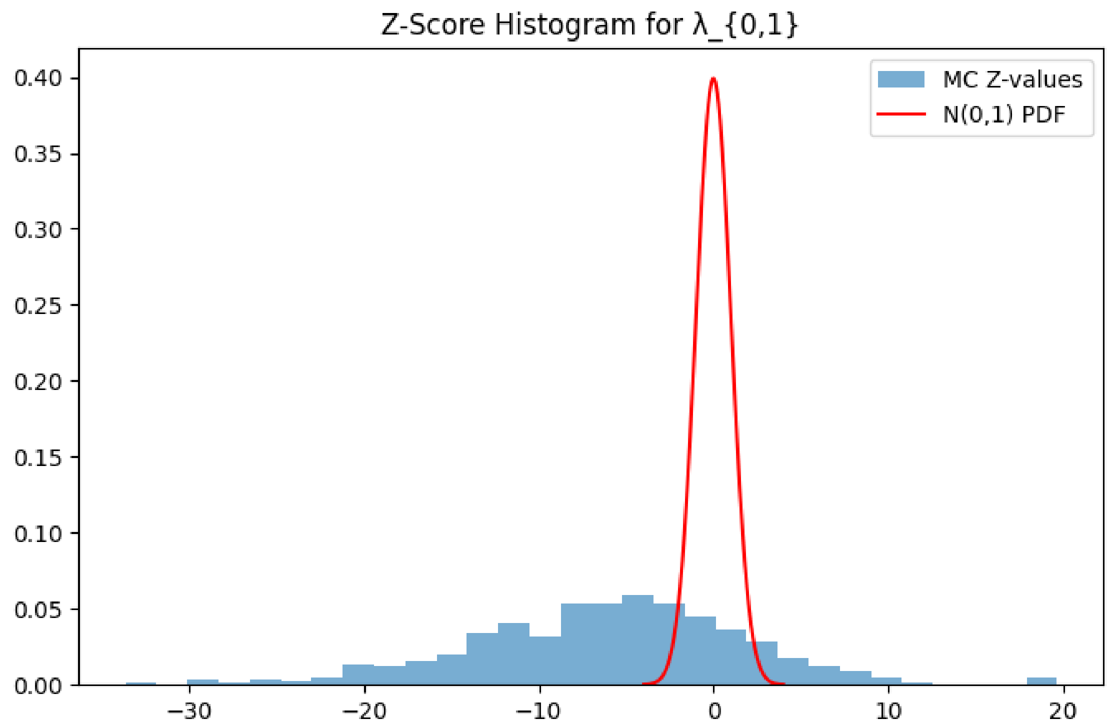
[ ]:
```
def param_labels(N, K):
    labels = []
    for i in range(N):
        labels.append(f"_{i}")
        for k in range(K):
            labels.append(f"_{i,k}")
    return labels
```

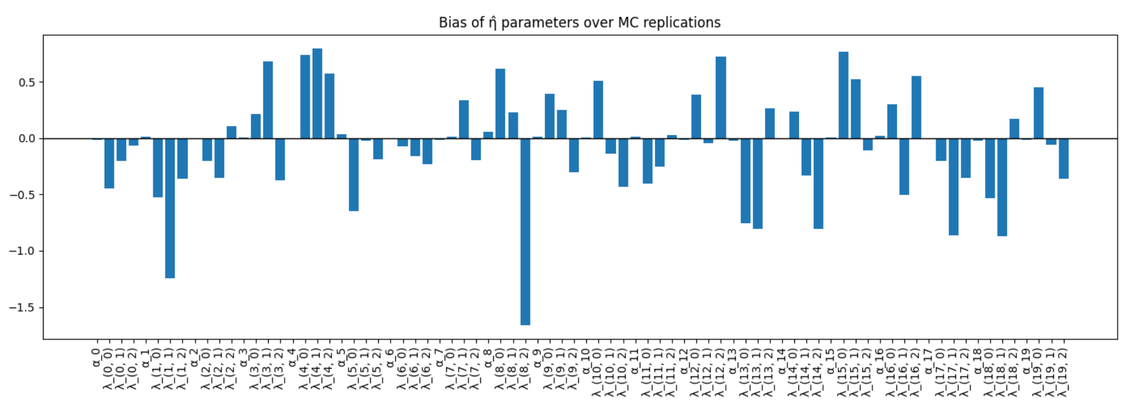
```python
def plot_eta_bias(eta_hats, eta_true, N, K):
    labels = param_labels(N, K)
    p = len(labels)

    eta_mean = eta_hats.mean(axis=0).reshape(-1)
    eta_true_vec = eta_true.reshape(-1)

    bias = eta_mean - eta_true_vec

    plt.figure(figsize=(14,5))
    plt.bar(range(p), bias)
    plt.axhline(0, color="black", linewidth=1)
    plt.xticks(range(p), labels, rotation=90)
    plt.title("Bias of ˆ parameters over MC replications")
    plt.tight_layout()
    plt.show()

def plot_variance_comparison(emp_cov, avars, T, N, K):
    labels = param_labels(N, K)
    p = len(labels)

    emp_var = np.diag(emp_cov)
    avar_mean = avars.mean(axis=0)
    theo_var = np.diag(avar_mean) / T

    plt.figure(figsize=(14,5))
    plt.plot(emp_var, label="Empirical Var(ˆ)", marker='o')
    plt.plot(theo_var, label="Theoretical Var from AVAR/T", marker='x')
    plt.xticks(range(p), labels, rotation=90)
    plt.legend()
    plt.title("Empirical vs Theoretical Variance of ˆ")
    plt.tight_layout()
    plt.show()

def plot_z_histogram(z_vals, param_label):
    plt.figure(figsize=(8,5))
    plt.hist(z_vals, bins=30, density=True, alpha=0.6, label="MC Z-values")

    x = np.linspace(-4,4,200)
    plt.plot(x, norm.pdf(x), color="red", label="N(0,1) PDF")

    plt.title(f"Z-Score Histogram for {param_label}")
    plt.legend()
    plt.show()

def plot_qq(z_vals, param_label):
```

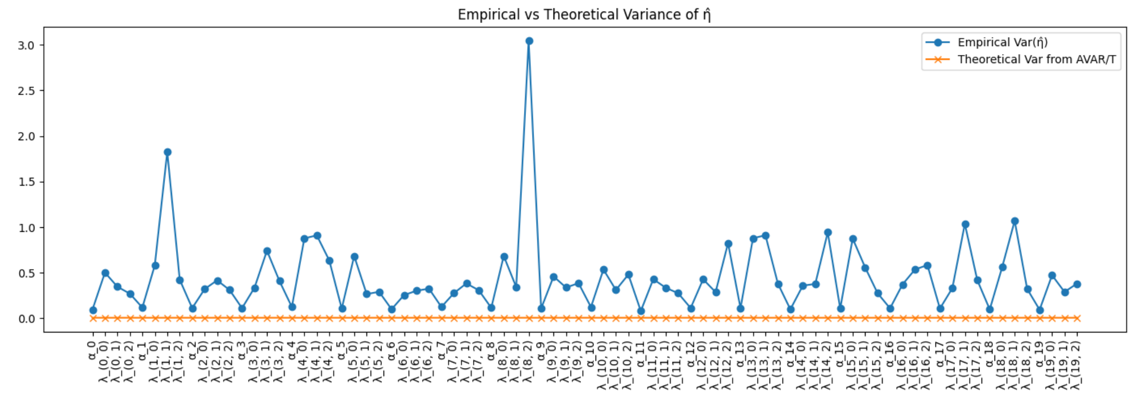
```
    plt.figure(figsize=(6,6))
    stats.probplot(z_vals, dist="norm", plot=plt)
    plt.title(f"QQ Plot for Z-scores: {param_label}")
    plt.show()

def plot_coverage(coverage, param_label):
    plt.figure(figsize=(5,4))
    plt.bar([0], [coverage], width=0.4)
    plt.axhline(0.95, color="red", linestyle="--", label="95% target")
    plt.ylim(0,1)
    plt.xticks([0], [param_label])
    plt.ylabel("Coverage probability")
    plt.legend()
    plt.title("Empirical 95% CI Coverage")
    plt.show()

def plot_eta_heatmap(eta_hats, eta_true):
    error = eta_hats.mean(axis=0) - eta_true  # (N, K+1)

    plt.figure(figsize=(10,6))
    sns.heatmap(np.abs(error), cmap="magma", annot=False)
    plt.title("Mean Absolute Error Heatmap of ^")
    plt.xlabel("Parameter ( ,  1,..., K)")
    plt.ylabel("Industry index")
    plt.show()

def plot_eta_boxplots(eta_hats, eta_true, N, K):
    labels = param_labels(N, K)
    p = len(labels)

    plt.figure(figsize=(15,6))
    plt.boxplot(eta_hats.reshape(eta_hats.shape[0], -1), labels=labels)
    plt.axhline(0, color="black")
    plt.title("Distribution of ^ Across Monte Carlo Replications")
    plt.xticks(rotation=90)
    plt.tight_layout()
    plt.show()
```

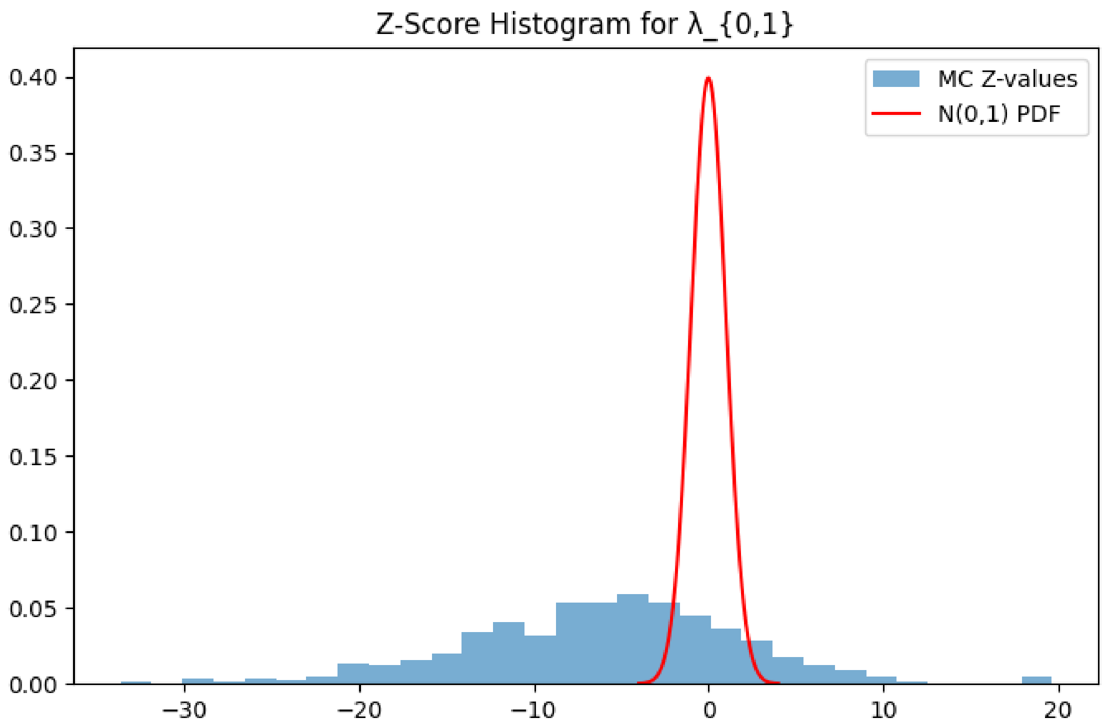
[ ]:

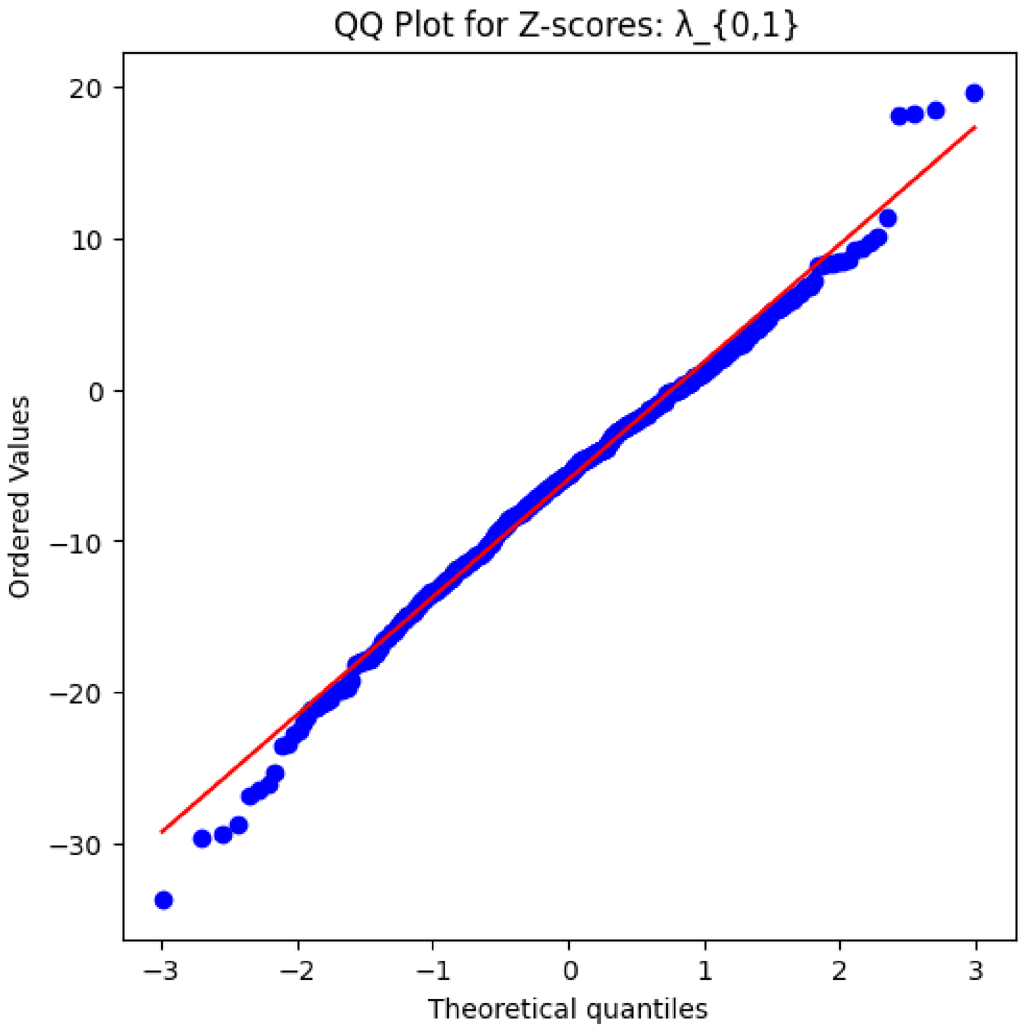
```
param_label = " _{0,1}"   # Adjust depending on param_index
plot_z_histogram(z_vals, param_label)
plot_eta_bias(eta_hats, eta_true, N=20, K=3)
plot_variance_comparison(emp_cov, avars, T=200, N=20, K=3)
plot_z_histogram(z_vals, param_label)
plot_qq(z_vals, param_label)
plot_coverage(coverage, param_label)
plot_eta_heatmap(eta_hats, eta_true)
plot_eta_boxplots(eta_hats, eta_true, N=20, K=3)
```

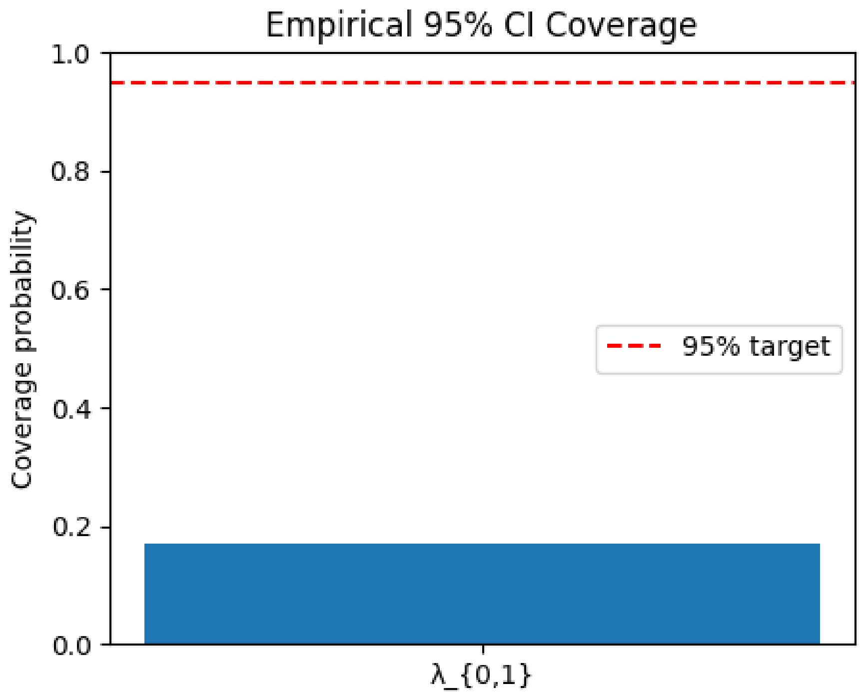

Z-Score Histogram for λ_{0,1}
MC Z-values
N(0,1) PDF
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−30
−20
−10
0
10
20


Bias of η̂ parameters over MC replications
0.5
0.0
−0.5
−1.0
−1.5

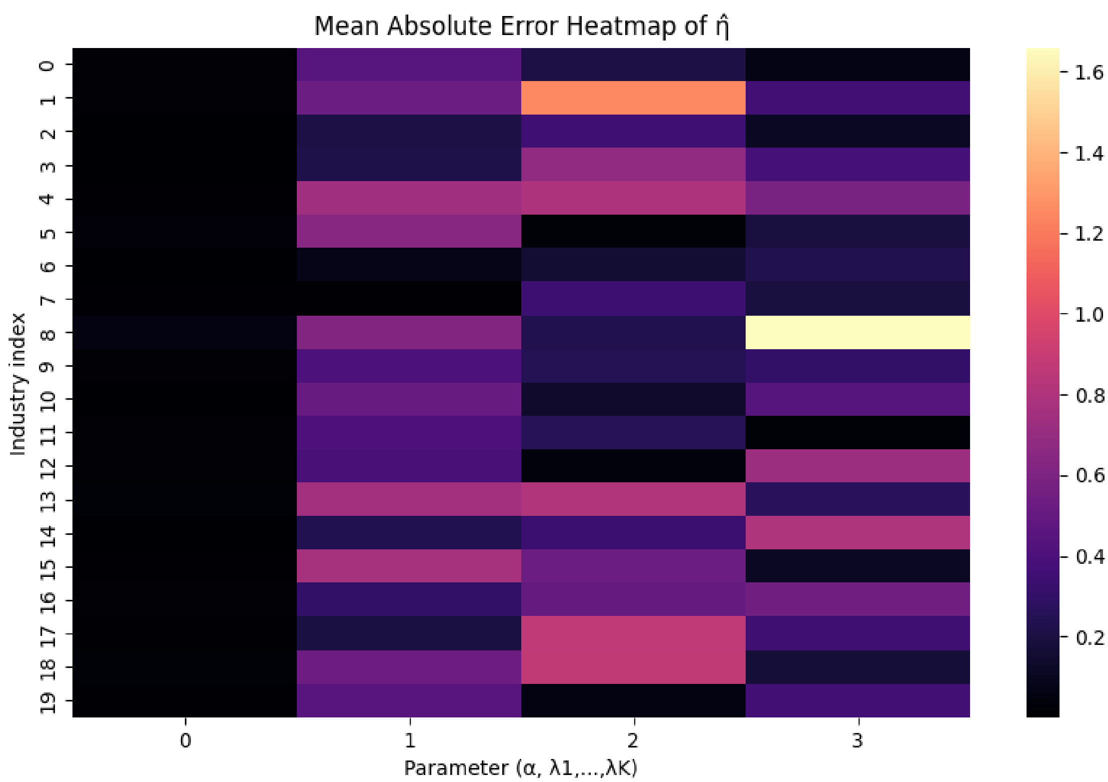

Empirical vs Theoretical Variance of η̂
Empirical Var(η̂)
Theoretical Var from AVAR/T


Z-Score Histogram for λ_{0,1}
MC Z-values
N(0,1) PDF


QQ Plot for Z-scores: λ_{0,1}
Ordered Values
Theoretical quantiles
20
10
0
−10
−20
−30
−3
−2
−1
0
1
2
3

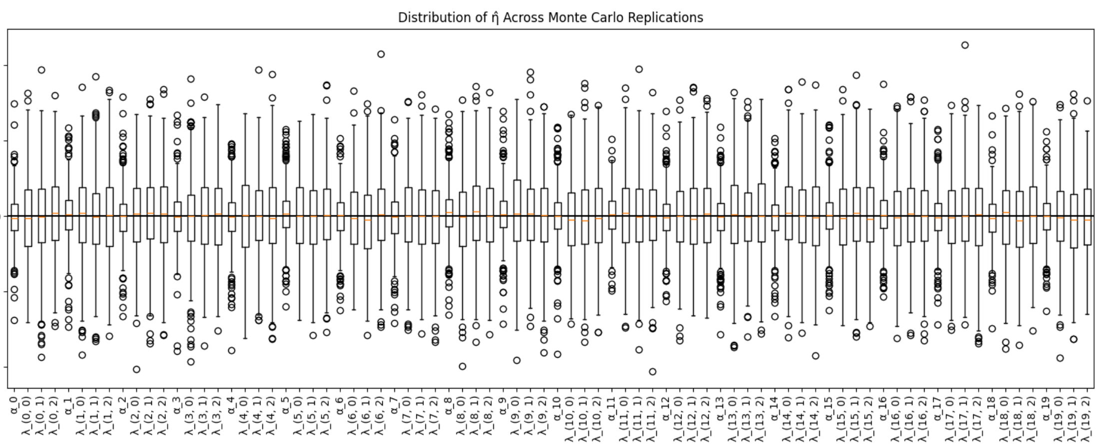


Empirical 95% CI Coverage
Coverage probability
1.0
0.8
0.6
0.4
0.2
0.0
95% target
λ_{0,1}



Mean Absolute Error Heatmap of η̂
Industry index
Parameter (α, λ1,...,λK)
0
1
2
3
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2

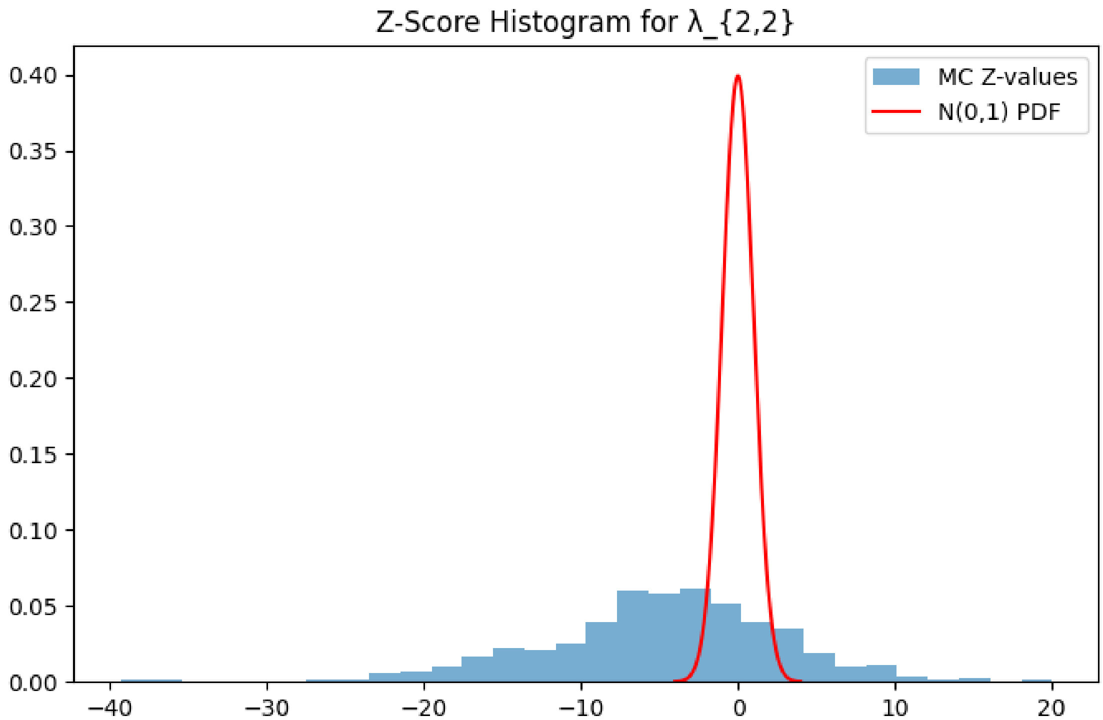
```
/var/folders/sp/cw_2m19j25xbvgpdjz48gclh0000gn/T/ipykernel_22666/4028456446.py:8
6: MatplotlibDeprecationWarning: The 'labels' parameter of boxplot() has been
renamed 'tick_labels' since Matplotlib 3.9; support for the old name will be
dropped in 3.11.
  plt.boxplot(eta_hats.reshape(eta_hats.shape[0], -1), labels=labels)
```

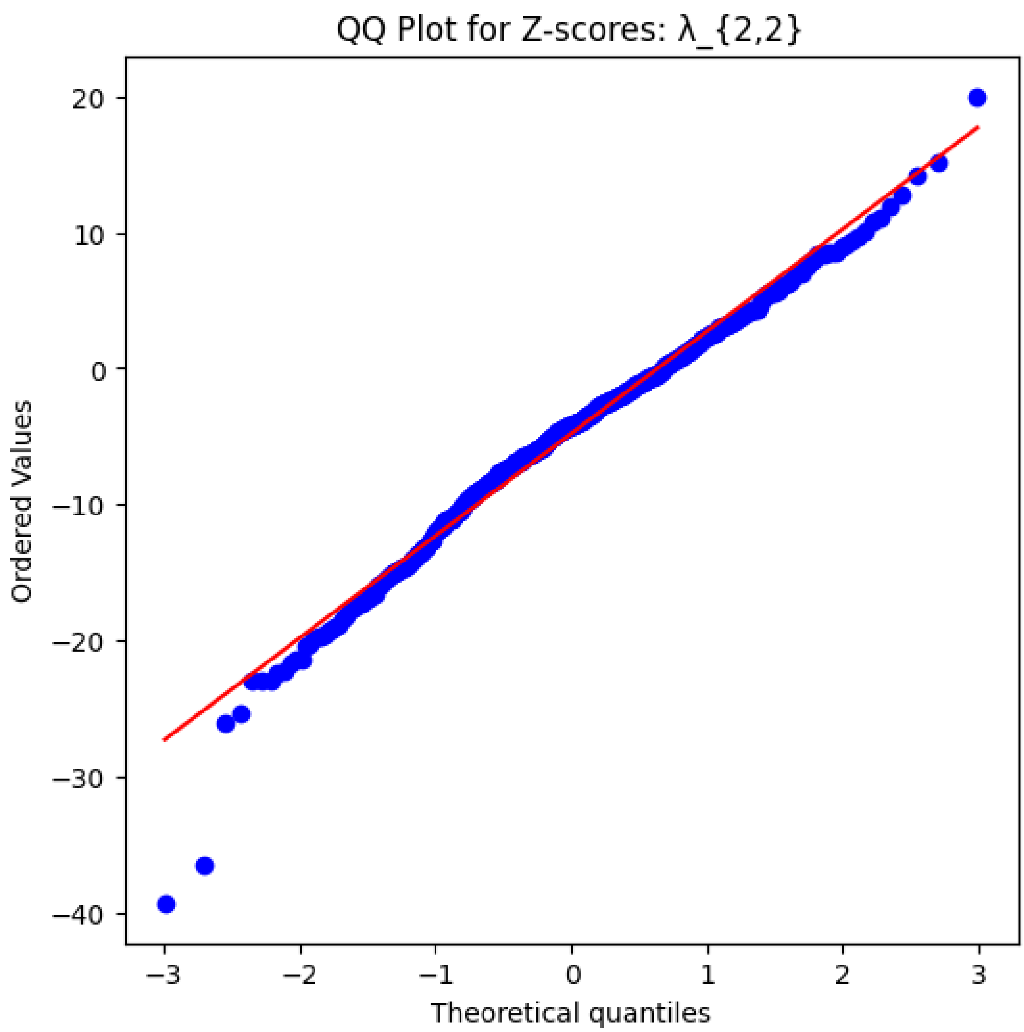


[ ]:
```python
# pick asset i and slope index j
i = 2    # asset 0
j = 2    # slope for factor 1 (assuming j=1..K)

def get_param_index(i, j, K):
    """
    i = industry index (0..N-1)
    j = 0 for intercept, 1..K for slopes
        j = 0 → intercept _i
        j >= 1 → slope _{i,j-1}

    K = number of factors
    returns index in vec(eta)
    """
    Kp1 = K + 1
    return i * Kp1 + j

param_index = get_param_index(i, j, K=3)
param_label = f" _{{{i},{j}}}"

z_vals = check_z_normality(
    eta_hats, avars, eta_true_vec, T=200, param_index=param_index
)

coverage_probability(
    eta_hats, avars, eta_true_vec, param_index=param_index, T=200
```

```
)

plot_z_histogram(z_vals, param_label)
plot_qq(z_vals, param_label)
```

```
==== NORMALITY CHECK ====
mean(z): -4.804067060910951
std(z): 7.551418289678208
KS test vs N(0,1): KstestResult(statistic=np.float64(0.6242362685243527),
pvalue=np.float64(1.0800520439165965e-188),
statistic_location=np.float64(-1.8842773985696188), statistic_sign=np.int8(1))

==== COVERAGE ====
Coverage probability: 0.184 (target = 0.95)
```



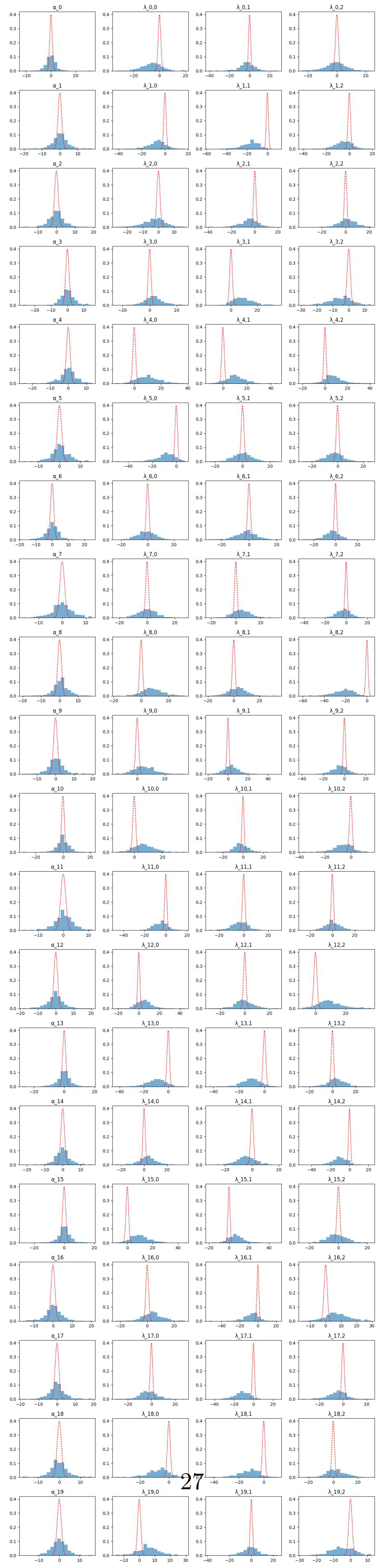


[ ]:
```python
def plot_all_z_histograms(eta_hats, avars, eta_true_vec, N, K, T, bins=20):
    """
    Creates a grid of z-score histograms:
    Shape: N rows × (K+1) columns.
    """

    p = N * (K+1)
    n_rep = eta_hats.shape[0]

    fig, axes = plt.subplots(N, K+1, figsize=(3*(K+1), 2.5*N), squeeze=False)

    for i in range(N):
        for j in range(K+1):
            ax = axes[i, j]
```

```
            param_index = get_param_index(i, j, K)

            # compute z-values for this parameter
            z_vals = []
            for rep in range(n_rep):
                eta_vec = eta_hats[rep].reshape(p)
                avar_hat = avars[rep]
                se = np.sqrt(avar_hat[param_index, param_index] / T)
                z = (eta_vec[param_index] - eta_true_vec[param_index]) / se
                z_vals.append(z)
            z_vals = np.array(z_vals)

            # histogram
            ax.hist(z_vals, bins=bins, density=True, alpha=0.6)

            # normal pdf
            x = np.linspace(-4,4,200)
            ax.plot(x, norm.pdf(x), "r--", linewidth=1)

            # label
            if j == 0:
                ax.set_title(f" _{i}")
            else:
                ax.set_title(f" _{i},{j-1}")

    plt.tight_layout()
    plt.show()

def compute_z_matrix(eta_hats, avars, eta_true_vec, N, K, T):
    """
    Returns an N x (K+1) matrix of z-score deviation:
       deviation = |mean(z)| + |std(z) - 1|
    Larger numbers = worse normal fit.
    """
    p = N * (K+1)
    n_rep = eta_hats.shape[0]

    deviation = np.zeros((N, K+1))

    for i in range(N):
        for j in range(K+1):
            param_index = get_param_index(i, j, K)

            # compute z-values
            z_vals = []
            for rep in range(n_rep):
                eta_vec = eta_hats[rep].reshape(p)
```

```
            avar_hat = avars[rep]
            se = np.sqrt(avar_hat[param_index, param_index] / T)
            z = (eta_vec[param_index] - eta_true_vec[param_index]) / se
            z_vals.append(z)
        z_vals = np.array(z_vals)

        # deviation from N(0,1)
        deviation[i,j] = abs(z_vals.mean()) + abs(z_vals.std() - 1)

    return deviation

def plot_z_heatmap(eta_hats, avars, eta_true_vec, N, K, T):
    deviation = compute_z_matrix(eta_hats, avars, eta_true_vec, N, K, T)

    plt.figure(figsize=(10,6))
    sns.heatmap(
        deviation,
        cmap="viridis",
        xticklabels=[f" "] + [f" _{j}" for j in range(K)],
        yticklabels=[f"i={i}" for i in range(N)],
        annot=True,
        fmt=".2f"
    )
    plt.title("Z-score Deviation Heatmap\n(|mean(z)| + |std(z)-1|)")
    plt.show()
```

[ ]:
```
plot_all_z_histograms(
    eta_hats, avars, eta_true_vec,
    N=20, K=3, T=200
)
```



α_0
λ_0,0
λ_0,1
λ_0,2
α_1
λ_1,0
λ_1,1
λ_1,2
α_2
λ_2,0
λ_2,1
λ_2,2
α_3
λ_3,0
λ_3,1
λ_3,2
α_4
λ_4,0
λ_4,1
λ_4,2
α_5
λ_5,0
λ_5,1
λ_5,2
α_6
λ_6,0
λ_6,1
λ_6,2
α_7
λ_7,0
λ_7,1
λ_7,2
α_8
λ_8,0
λ_8,1
λ_8,2
α_9
λ_9,0
λ_9,1
λ_9,2
α_10
λ_10,0
λ_10,1
λ_10,2
α_11
λ_11,0
λ_11,1
λ_11,2
α_12
λ_12,0
λ_12,1
λ_12,2
α_13
λ_13,0
λ_13,1
λ_13,2
α_14
λ_14,0
λ_14,1
λ_14,2
α_15
λ_15,0
λ_15,1
λ_15,2
α_16
λ_16,0
λ_16,1
λ_16,2
α_17
λ_17,0
λ_17,1
λ_17,2
α_18
λ_18,0
λ_18,1
λ_18,2
α_19
λ_19,0
λ_19,1
λ_19,2

[ ]:
```
plot_z_heatmap(
    eta_hats, avars, eta_true_vec,
    N=20, K=3, T=200,
)
```