02_factor_loading

December 18, 2025

1 Estimating Factor Loading

Galvao et al estimated quarterly factor loading from daily data.

1.1 Notebook setup

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sys
sys.executable
```

[1]:
```
'/Users/fanghema/Desktop/aaSTAT_5200/STAT_5200_final_project/env/bin/python'
```

For now, we load in Galvao et al's replication data.

[2]:
```
data = pd.read_csv(
    '../data/processed/data_galvao.csv',
    index_col=0,
    parse_dates=True
)
```

[3]:
```
factors = ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA']
assets = [col for col in data.columns if col != 'RF' and col not in factors]
```

1.2 Factor Loading Function

[4]:
```
def calculate_factor_loading(
    input_df: pd.DataFrame,
    factors: list[str],
    assets: list[str],
) -> tuple[pd.DataFrame, pd.DataFrame]:
    """
    Given DataFrame of (non-excess) asset returns
    and factor returns,
    returns panel data of factor loadings

    Args:
```

```python
        input_df (pd.DataFrame): DataFrame indexed on date,
            with column names corresponding to assets
        factors (list[str]): list of factors
        assets (list[str]): list of risky assets

    Returns:
        pd.DataFrame: panel data of factor loadings
        pd.DataFrame: modified returns dataframe with excess returns
    """

    assert type(input_df.index) == pd.DatetimeIndex, "input_df has wrong index"
    for factor in factors:
        assert factor in input_df.columns, f"missing factor {factor}"
    for asset in assets:
        assert asset in input_df.columns, f"missing asset {asset}"
    assert "RF" in input_df.columns, f"Missing risk free"

    input_df.sort_index(inplace=True)
    N = len(assets)
    K = len(factors)
    input_df['Quarter'] = input_df.index.to_period("Q")
    T = input_df['Quarter'].nunique()

    for col in assets:
        input_df[col] = input_df[col] - input_df["RF"]

    cols = list(assets) + list(factors)

    realized_covariance_matrices = np.zeros((N, K, T))

    quarters = sorted(input_df['Quarter'].unique())
    for i, quarter in enumerate(quarters):
        returns = (
            input_df.loc[
                input_df['Quarter'] == quarter,
                cols
            ]
            .values
        )
        Omega_hat_t = returns.T @ returns
        realized_covariance_matrices[:, :, i] = Omega_hat_t[:N, N:N+K]

    beta_loading = pd.DataFrame(
        index = pd.MultiIndex.from_product([assets, factors]),
        columns = input_df['Quarter'].unique(),
    )
```

```
    for i, asset in enumerate(assets):
        for j, factor in enumerate(factors):
            omega_i_j_series = realized_covariance_matrices[i, j, :]
            Y = omega_i_j_series[1:]
            X = (
                np.column_stack([
                    np.ones(len(Y)),
                    omega_i_j_series[:-1]
                ])
            )
            b = np.linalg.lstsq(X, Y, rcond=None)[0]
            delta0, delta1 = b
            beta_loading.loc[(asset, factor)] = delta0 + delta1 *␣
↪omega_i_j_series

    return beta_loading, input_df

beta_loading, _ = calculate_factor_loading(data, factors=factors, assets=assets)
```

[5]:
```
beta_loading
```

[5]:
```
                    1963Q3     1963Q4     1964Q1     1964Q2     1964Q3  \
Agric Mkt-RF     30.361188   29.34117  30.059214  30.543622  30.323886
      SMB         1.816013    1.79645   1.741765   1.853506   1.473105
      HML        -1.600764  -3.201627  -1.574742   0.111486  -3.858092
      RMW        -1.466798   0.215607  -2.085727  -1.616463   -0.83943
      CMA        -3.153214  -2.825331  -3.644188  -2.283014  -4.513127
…                      …          …          …          …          …
Other Mkt-RF     37.969554  45.784074  34.019282  34.391633  34.792978
      SMB        -1.998767  -2.640143   -0.85192  -0.306529  -0.633693
      HML        -0.917399   1.711179  -1.088099  -3.044638  -1.742651
      RMW        -1.124262  -2.770248  -2.015389  -2.132075  -3.193049
      CMA        -5.789944  -6.603539  -5.826818  -6.053781  -5.221453

                    1964Q4     1965Q1     1965Q2     1965Q3     1965Q4  …  \
Agric Mkt-RF     30.703206  31.228282  37.229192  32.067519  31.861254  …
      SMB          2.00402   3.045183   4.653787   1.888597   5.220041  …
      HML        -0.235879   0.086952  -1.560099  -0.679981   0.349221  …
      RMW        -2.021263  -0.083306   0.020307   0.034482  -0.414664  …
      CMA        -2.645516  -0.616308  -3.777817  -3.716449  -2.888483  …
…                      …          …          …          …          …  …
Other Mkt-RF     34.918142  35.850948  44.439513  35.915049  35.431179  …
      SMB         1.262379   0.756938   7.587363   1.695747   2.722596  …
      HML        -1.884642  -2.515519  -2.530227  -2.782289   -0.82352  …
      RMW        -2.799131  -1.728237  -1.088422   -0.85667  -2.905094  …
      CMA        -4.204414   -4.85491  -7.189725  -5.199381  -4.323135  …
```

```
                  2012Q3     2012Q4     2013Q1     2013Q2     2013Q3  \
Agric Mkt-RF   38.230127  44.228211  39.335887  48.107139  35.757627
      SMB       2.825341    2.55903   1.395667   6.672073   0.987797
      HML      -0.932763   -0.81881   0.285446   3.049976  -1.072834
      RMW      -2.202299   0.605827  -3.652465  -4.020965  -3.187997
      CMA       -4.54621  -5.343885  -2.406726  -2.818723  -1.857074
…                    …          …          …          …          …
Other Mkt-RF   45.704996   49.68986  45.765309  55.216365  43.633398
      SMB       0.412122  -0.559866   1.345408   4.362684   1.628486
      HML       0.311998   1.521048   1.230703   2.823396  -0.727235
      RMW      -2.588129  -2.488581  -5.192548  -5.965385  -5.007897
      CMA      -5.914405  -4.596196  -3.697755  -4.133301  -3.866694

                  2013Q4     2014Q1     2014Q2     2014Q3     2014Q4
Agric Mkt-RF   38.445064  40.051037  39.497776  33.855069  45.157696
      SMB       3.595121   4.930176   4.457616   2.713055   3.671996
      HML      -2.233552  -3.741736   -4.58758  -3.889497  -3.510954
      RMW      -2.908149  -2.537573  -3.156205  -1.500751  -4.065958
      CMA      -3.305422  -3.470351  -5.198915  -3.512337  -4.358914
…                    …          …          …          …          …
Other Mkt-RF   46.574096  49.724401  40.962994  42.389573  54.260984
      SMB       1.340596     2.8245   2.506789   0.791631  -1.422422
      HML        -2.7199  -2.724412   -3.61055  -4.402122  -3.550449
      RMW       -4.39195  -3.473228  -3.751173  -3.608661  -4.698762
      CMA      -5.743321  -5.302565  -5.333998  -5.560285  -5.226873

[235 rows x 206 columns]
```

1.3 Sanity Check

As a sanity check, we call our function on Galvao's original dataset, and compare the estimated beta loading against the original paper's beta loadings.

[6]:
```python
agric_betas = pd.read_csv("../gmo-files/omegareg1.txt",
                sep=r"\s+",
                header=None)
agric_betas.columns = factors
agric_betas
```

[6]:
```
       Mkt-RF      SMB      HML      RMW      CMA
0     3.60917  0.67097  0.32962 -0.66613 -1.01100
1     0.93072  0.61959 -2.20990  2.73658 -0.40266
2     2.81622  0.47597  0.37090 -1.91793 -1.92193
3     4.08822  0.76944  3.04584 -0.96883  0.60353
4     3.51122 -0.22962 -3.25128  0.60274 -3.53412
..        …        …        …        …        …
201  24.83650  5.34350 -0.67420 -3.58130 -1.29340
```

```
202  29.05360  8.84980 -3.06670 -2.83180 -1.59940
203  27.60080  7.60870 -4.40850 -4.08300 -4.80650
204  12.78370  3.02690 -3.30110 -0.73480 -1.67730
205  42.46310  5.54540 -2.70060 -5.92300 -3.24800

[206 rows x 5 columns]
```

[7]:

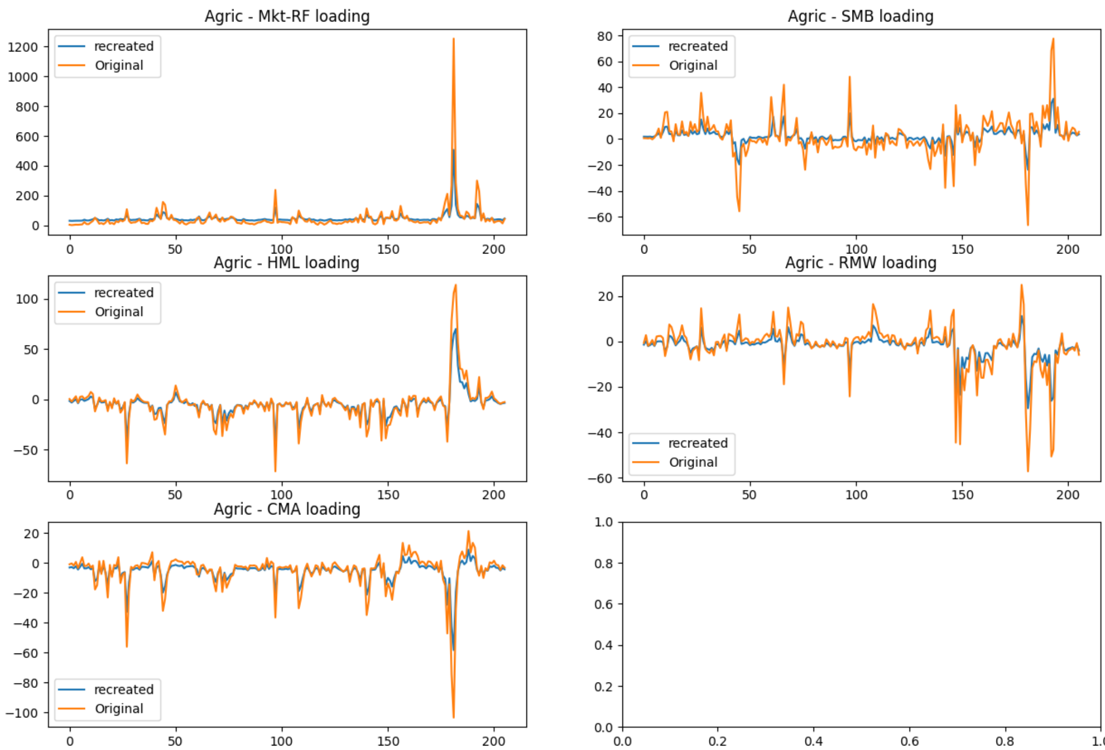
```
beta_loading.loc['Agric'].T
```

[7]:

```
           Mkt-RF       SMB       HML       RMW       CMA
1963Q3  30.361188  1.816013 -1.600764 -1.466798 -3.153214
1963Q4   29.34117   1.79645 -3.201627  0.215607 -2.825331
1964Q1  30.059214  1.741765 -1.574742 -2.085727 -3.644188
1964Q2  30.543622  1.853506  0.111486 -1.616463 -2.283014
1964Q3  30.323886  1.473105 -3.858092  -0.83943 -4.513127
…             …         …         …         …         …
2013Q4  38.445064  3.595121 -2.233552 -2.908149 -3.305422
2014Q1  40.051037  4.930176 -3.741736 -2.537573 -3.470351
2014Q2  39.497776  4.457616  -4.58758 -3.156205 -5.198915
2014Q3  33.855069  2.713055 -3.889497 -1.500751 -3.512337
2014Q4  45.157696  3.671996 -3.510954 -4.065958 -4.358914

[206 rows x 5 columns]
```

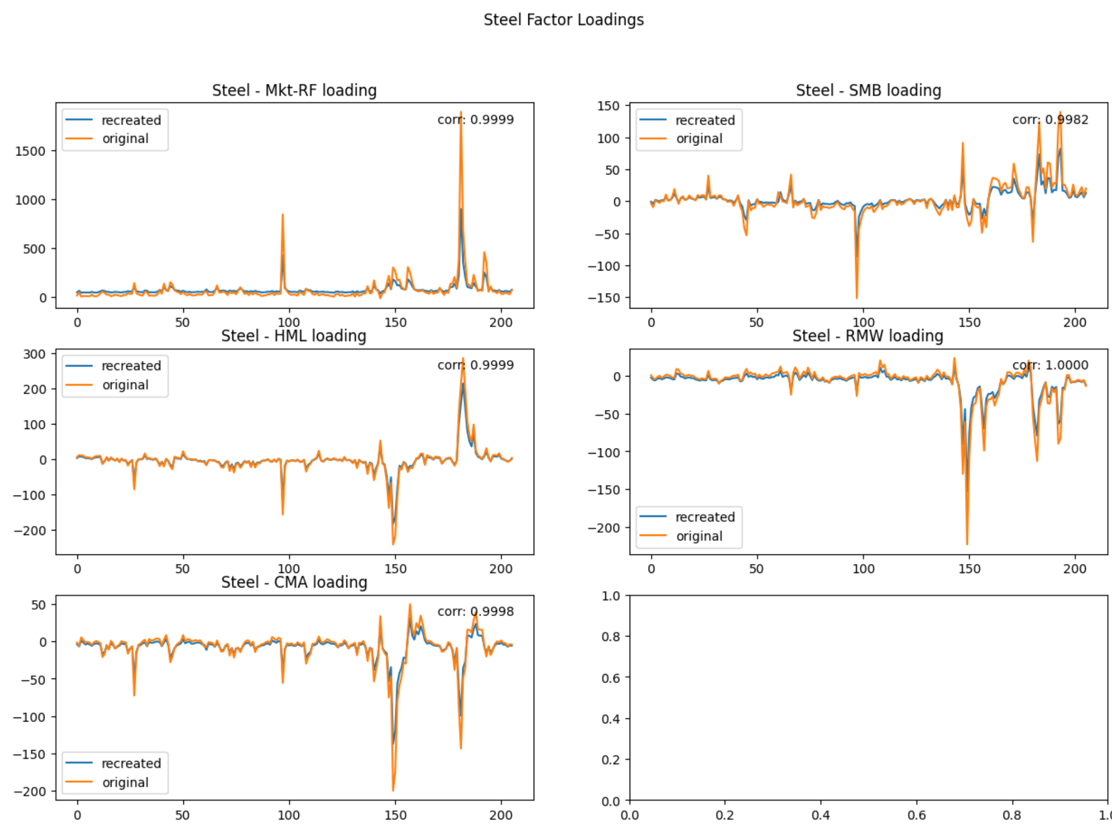
[8]:

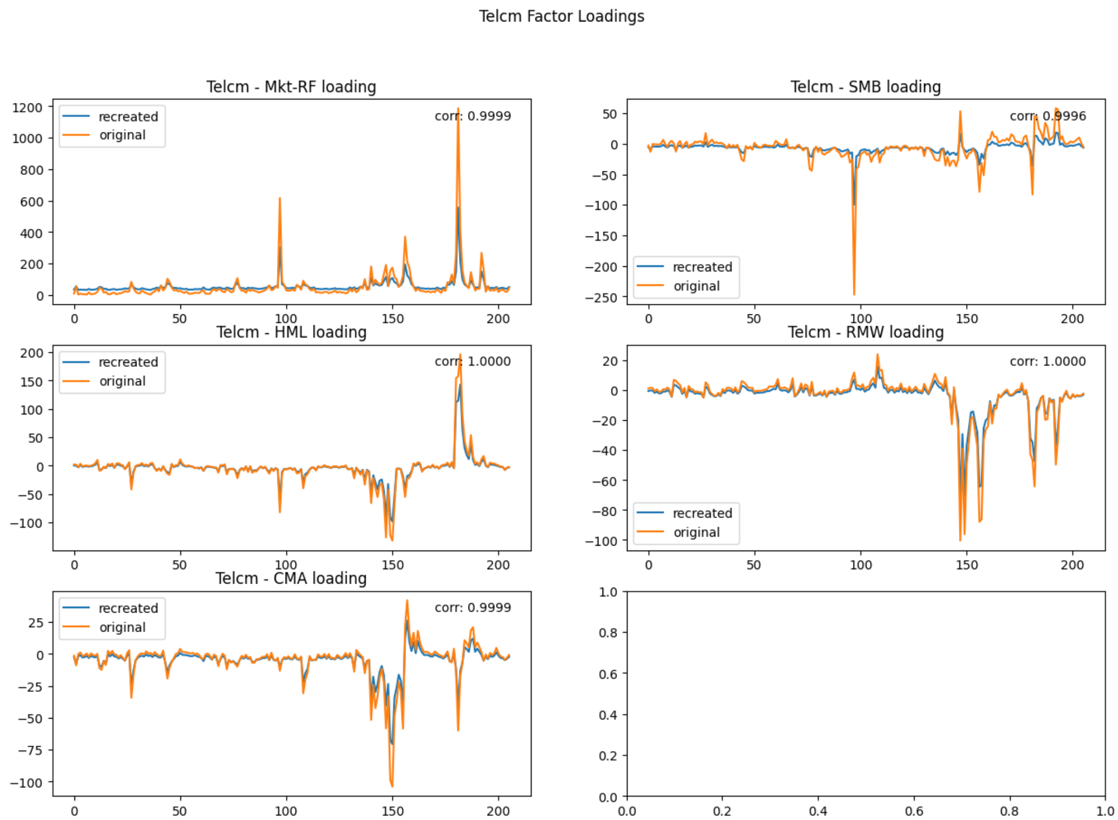
```
fig, axes = plt.subplots(3, 2, figsize = (15, 10))
axes = np.ravel(axes)
for i, factor in enumerate(factors):
    axes[i].plot(
        beta_loading.loc['Agric'].T[factor].values, label='recreated'
    )
    axes[i].plot(
        agric_betas[factor], label="Original"
    )
    axes[i].legend()
    axes[i].set_title(f"Agric - {factor} loading")
```



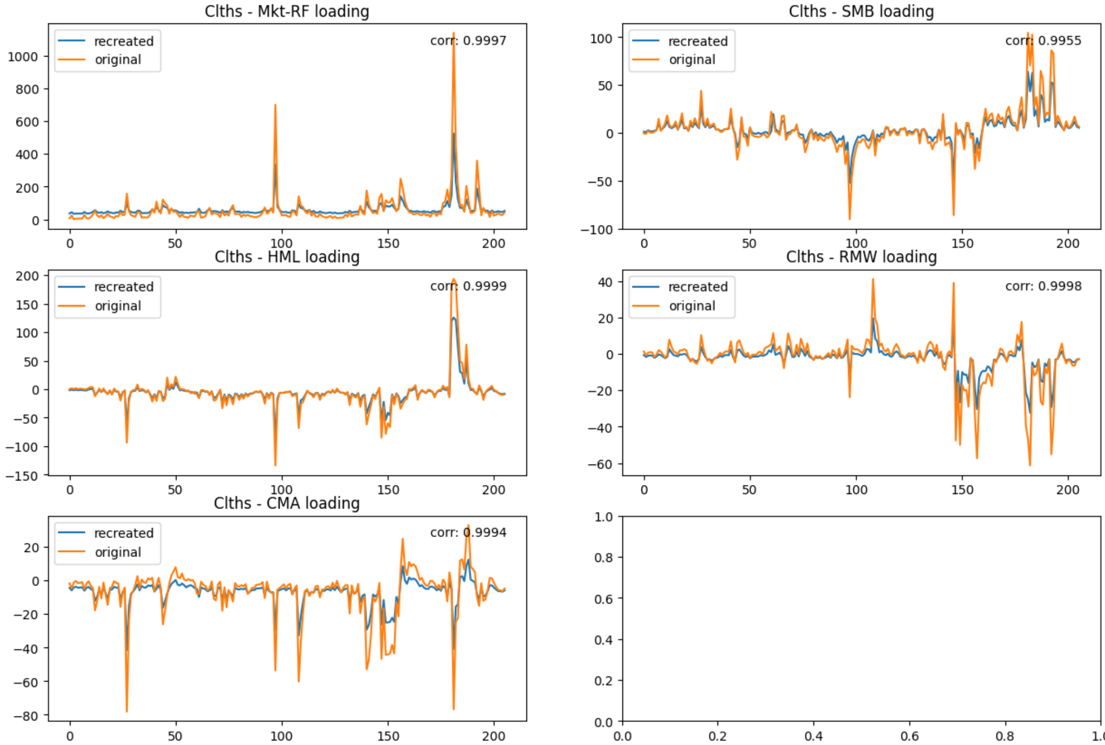
[9]:
```python
random_sample_assets = np.random.choice(47, 5, replace=False)

for rng in random_sample_assets:
    galvao_estimated_betas = pd.read_csv(f"../gmo-files/omegareg{rng + 1}.txt",
                    sep=r"\s+",
                    header=None)
    galvao_estimated_betas.columns = factors

    asset = assets[rng]

    fig, axes = plt.subplots(3, 2, figsize = (15, 10))
    axes = np.ravel(axes)
    for i, factor in enumerate(factors):
        recreated_arr = beta_loading.loc[asset].T[factor].values
        galvao_arr = galvao_estimated_betas[factor].values
        min_len = min(len(recreated_arr), len(galvao_arr))

        recreated_arr = recreated_arr[:min_len].astype(float)
        galvao_arr    = galvao_arr[:min_len].astype(float)

        axes[i].plot(recreated_arr, label='recreated')
        axes[i].plot(galvao_arr, label='original')
```

```
        recreated_arr = np.asarray(recreated_arr, dtype=float)
        galvao_arr    = np.asarray(galvao_arr, dtype=float)

        corr = np.corrcoef(recreated_arr, galvao_arr)[0, 1]
        axes[i].text(
            0.8, 0.9,
            f"corr: {corr:.4f}",
            transform = axes[i].transAxes,
        )

        axes[i].legend()
        axes[i].set_title(f"{asset} - {factor} loading")
    fig.suptitle(f"{asset} Factor Loadings")
    plt.show()
```

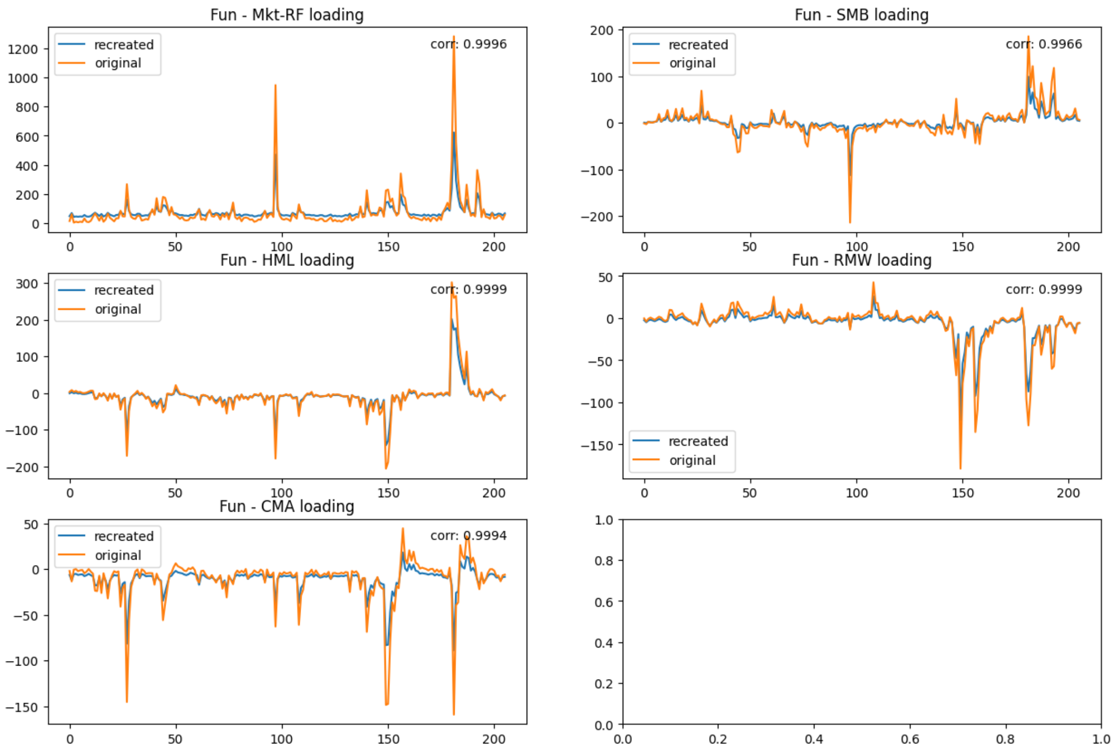


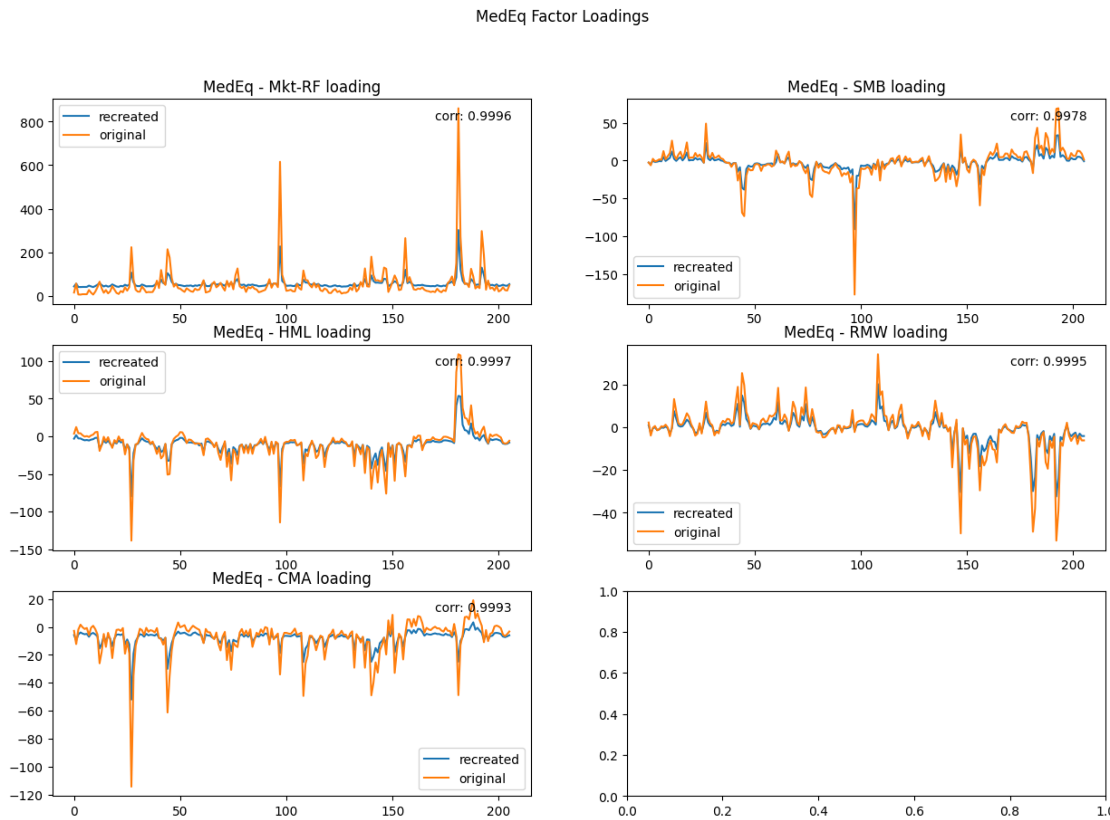

Telcm Factor Loadings
Telcm - Mkt-RF loading
recreated
original
corr: 0.9999
Telcm - SMB loading
recreated
original
corr: 0.9996
Telcm - HML loading
recreated
original
corr: 1.0000
Telcm - RMW loading
recreated
original
corr: 1.0000
Telcm - CMA loading
recreated
original
corr: 0.9999

Clths Factor Loadings



Fun Factor Loadings





As a final check, we sample all 47 industries and all 5 factors, and compute correlation for each of the 235 factor loadings.

[10]:
```
corr_df = pd.DataFrame(
    index = assets,
    columns = factors
)
for rng in range(47):
    galvao_estimated_betas = pd.read_csv(f"../gmo-files/omegareg{rng + 1}.txt",
                            sep=r"\s+",
                            header=None)
    galvao_estimated_betas.columns = factors
    asset = assets[rng]
    for i, factor in enumerate(factors):
        recreated_arr = beta_loading.loc[asset].T[factor].values
        galvao_arr = galvao_estimated_betas[factor].values
        min_len = min(len(recreated_arr), len(galvao_arr))
        recreated_arr = recreated_arr[:min_len].astype(float)
        galvao_arr    = galvao_arr[:min_len].astype(float)
        corr = np.corrcoef(recreated_arr, galvao_arr)[0, 1]
        corr_df.loc[asset, factor] = corr
```

```
corr_df
```

[10]:

	Mkt-RF	SMB	HML	RMW	CMA
Agric	0.999567	0.983181	0.999695	0.99985	0.99928
Food	0.999763	0.997571	0.999783	0.999493	0.999491
Soda	0.999783	0.999295	0.999822	0.999621	0.999576
Beer	0.999574	0.994752	0.99978	0.999852	0.99884
Smoke	0.999736	0.999549	0.999402	0.999349	0.999224
Toys	0.999617	0.995785	0.999898	0.999834	0.999364
Fun	0.999625	0.996554	0.999913	0.999914	0.999411
Books	0.999854	0.997026	0.999914	0.999753	0.99961
Hshld	0.999651	0.999656	0.999402	0.99916	0.999419
Clths	0.999723	0.995539	0.999886	0.999774	0.999352
MedEq	0.999614	0.997811	0.999664	0.999511	0.999325
Drugs	0.999699	0.999531	0.999533	0.999346	0.999181
Chems	0.99978	0.998965	0.999699	0.99958	0.999367
Rubbr	0.999662	0.992089	0.999944	0.999864	0.999184
Txtls	0.99971	0.992501	0.99996	0.999876	0.999267
BldMt	0.999812	0.995168	0.999933	0.999798	0.999365
Cnstr	0.999757	0.993609	0.999937	0.999897	0.999613
Steel	0.999887	0.998155	0.99995	0.999958	0.999834
FabPr	0.999748	0.993327	0.999954	0.999933	0.999334
Mach	0.999813	0.995988	0.999885	0.999846	0.999717
ElcEq	0.999889	0.998191	0.999959	0.999946	0.999902
Autos	0.999871	0.99897	0.999914	0.999845	0.999871
Aero	0.999621	0.999025	0.999545	0.9995	0.998883
Ships	0.999509	0.997592	0.999663	0.999658	0.998928
Guns	0.999465	0.99883	0.999349	0.998671	0.99893
Gold	0.999831	0.998684	0.999574	0.998804	0.999737
Mines	0.99991	0.998093	0.999945	0.999834	0.999785
Coal	0.99989	0.996182	0.999942	0.999888	0.999861
Oil	0.999897	0.999347	0.999777	0.999629	0.999686
Util	0.999919	0.998231	0.999909	0.999868	0.999607
Telcm	0.999922	0.999602	0.999964	0.999953	0.999907
PerSv	0.999407	0.989068	0.999864	0.999848	0.999274
BusSv	0.999636	0.988335	0.999924	0.999896	0.999398
Hardw	0.999848	0.999784	0.999915	0.999926	0.999961
Chips	0.999682	0.99777	0.99992	0.999942	0.999919
LabEq	0.999613	0.995699	0.99986	0.999913	0.999741
Paper	0.999807	0.995968	0.999781	0.999533	0.999318
Boxes	0.999828	0.999584	0.999785	0.999628	0.999747
Trans	0.999567	0.993846	0.999848	0.999749	0.999098
Whlsl	0.999585	0.987305	0.999907	0.999842	0.999364
Rtail	0.999733	0.996232	0.999763	0.999307	0.999496
Meals	0.999088	0.994654	0.999521	0.999292	0.998722
Banks	0.999883	0.998357	0.999973	0.999891	0.999817

```
Insur  0.999834   0.99638  0.999924  0.999829  0.999498
RlEst  0.999602  0.985248  0.999954  0.999904  0.999029
Fin    0.999915    0.9973  0.999977  0.999942  0.999897
Other  0.999713  0.991885  0.999973  0.999934  0.999688
```

[ ]:

```
latex_table = corr_df.round(3).to_latex(
    index=True,
    header=True,
    float_format="%.3f",
    escape=False
)

print(latex_table)
```