

# 06\_empirical\_homogeneity\_test

December 17, 2025

## 1 Empirical Homogeneity Test

### 1.1 Notebook Setup

```
[1]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import torch
import sys
sys.path.append('../')
from utils import utils
sys.executable
```

```
[1]: '/Users/fanghema/Desktop/aaSTAT_5200/STAT_5200_final_project/env/bin/python'
```

```
[2]: data = pd.read_csv(
    '../data/processed/data_galvao.csv',
    index_col=0,
    parse_dates=True
)

factors = ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA']
assets = [col for col in data.columns if col != 'RF' and col not in factors]
data['Quarter'] = data.index.to_period("Q")
```

### 1.2 Set up empirical testing parameters

```
[3]: factor_options = [
    ['Mkt-RF'],
    ['Mkt-RF', 'SMB', 'HML'],
    ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'],
]
R_options = [1, 2, 5]
sample_period_options = [
    ('1963-01-01', '2025-12-31'),
    ('1963-01-01', '1983-01-01'),
    ('1973-01-01', '1993-01-01'),
    ('1983-01-01', '2003-01-01'),
]
```

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('1993-01-01', '2013-01-01'),
('1913-01-01', '2023-01-01'),
]

results = pd.DataFrame(
    index=pd.MultiIndex.from_product([
        list(map(tuple, factor_options)),    # convert lists to tuples
        R_options,
        sample_period_options
    ]),
    columns=['gamma_a_lam', 'gamma_a', 'gamma_lam']
)

print(f"Total combinations: {results.shape[0]}")
counter = 0

for factors in factor_options:
    K = len(factors)
    for R in R_options:
        for sample_period in sample_period_options:
            print(f"Processing {counter}/{results.shape[0]}: {factors} - {R} - {sample_period}")
            data_slice = data.loc[
                (data.index > sample_period[0]) &
                (data.index < sample_period[1])
            ]
            beta_loading, returns_df, realized_covariance, residuals = utils.
            calculate_factor_loading(
                data_slice,
                factors=factors,
                assets=assets
            )

            excess_returns = returns_df.groupby("Quarter").sum()[assets].T.
            values
            industries = beta_loading.index.get_level_values(0).unique().
            tolist()
            factors_names = beta_loading.index.get_level_values(1).unique().
            tolist()

            N = len(industries)
            K = len(factors)
            T = beta_loading.shape[1]

            beta_hat_np = np.zeros((N, K, T))

```

```

for i, asset in enumerate(industries):
    for j, factor in enumerate(factors):
        beta_hat_np[i, j, :] = beta_loading.loc[(asset, factor)].
→values

eta, G, beta_star, objective = utils.iterative_convergence(
    beta_hat_np,
    excess_returns,
    N = N,
    K = K,
    R = R,
    T = T,
    n_iter=2000
)

avar = utils.estimate_avar(
    beta_hat=beta_hat_np,
    excess_returns=excess_returns,
    eta=eta,
    G=G,
    beta_star=beta_star,
    realized_covariance=realized_covariance,
    residuals=residuals,
    N = N,
    K = K,
    R = R,
    T = T,
)

gamma_a_lambda = utils.full_homogeneity_test(
    eta = eta,
    avar = avar,
    N = N,
    K = K,
    T = T
)

gamma_a = utils.intercept_homogeneity_test(
    eta = eta,
    avar = avar,
    N = N,
    K = K,
    T = T
)

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gamma_lambda = utils.slope_homogeneity_test(
    eta = eta,
    avar = avar,
    N = N,
    K = K,
    T = T
)
print(f"Test statistics")
print(f"gamma_a_lam: {gamma_a_lambda}")
print(f"gamma_a: {gamma_a}")
print(f"gamma_lam: {gamma_lambda}")

results.loc[(
    tuple(factors), R, sample_period
)] = np.asarray([
    gamma_a_lambda,
    gamma_a,
    gamma_lambda
])
counter += 1
print(f"=====")

```

Total combinations: 54

Processing 0/54: ['Mkt-RF'] - 1 - ('1963-01-01', '2025-12-31')

Iter 100/2000, frobenius\_norm(G - G\_old)=0.0005, objective=55.650212

Iter 200/2000, frobenius\_norm(G - G\_old)=0.0000, objective=55.650211

Using new avar!!

Test statistics

gamma\_a\_lam: -6.855345913521472

gamma\_a: -4.847245500126845

gamma\_lam: -4.847679857416041

=====

Processing 1/54: ['Mkt-RF'] - 1 - ('1963-01-01', '1983-01-01')

Iter 100/2000, frobenius\_norm(G - G\_old)=0.0000, objective=46.255468

Using new avar!!

Test statistics

gamma\_a\_lam: -6.847854557925456

gamma\_a: -4.838403328459578

gamma\_lam: -4.847679856687188

=====

Processing 2/54: ['Mkt-RF'] - 1 - ('1973-01-01', '1993-01-01')

Iter 100/2000, frobenius\_norm(G - G\_old)=0.0035, objective=46.566366

Iter 200/2000, frobenius\_norm(G - G\_old)=0.0000, objective=46.565911

Using new avar!!

Test statistics

gamma\_a\_lam: -6.800031008628394

gamma\_a: -4.769430449697195

gamma\_lam: -4.8476798573322455

```

=====
Processing 3/54: ['Mkt-RF'] - 1 - ('1983-01-01', '2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0001, objective=58.228455
Using new avar!!
Test statistics
gamma_a_lam: -6.8369287481724585
gamma_a: -4.821626555016538
gamma_lam: -4.847679857010494
=====
Processing 4/54: ['Mkt-RF'] - 1 - ('1993-01-01', '2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0009, objective=66.239631
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=66.239616
Using new avar!!
Test statistics
gamma_a_lam: -6.853077757548235
gamma_a: -4.844051557371137
gamma_lam: -4.847679857415792
=====
Processing 5/54: ['Mkt-RF'] - 1 - ('1913-01-01', '2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0005, objective=55.650212
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=55.650211
Using new avar!!
Test statistics
gamma_a_lam: -6.855345913521472
gamma_a: -4.847245500126845
gamma_lam: -4.847679857416041
=====
Processing 6/54: ['Mkt-RF'] - 2 - ('1963-01-01', '2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0009, objective=46.426228
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=46.426223
Using new avar!!
Test statistics
gamma_a_lam: -6.855198917794802
gamma_a: -4.847038555591234
gamma_lam: -4.847679857416099
=====
Processing 7/54: ['Mkt-RF'] - 2 - ('1963-01-01', '1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0000, objective=38.270140
Using new avar!!
Test statistics
gamma_a_lam: -6.848159497969471
gamma_a: -4.83859907591211
gamma_lam: -4.847679856771408
=====
Processing 8/54: ['Mkt-RF'] - 2 - ('1973-01-01', '1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0062, objective=35.696308
Iter 200/2000, frobenius_norm(G - G_old)=0.0001, objective=35.694865
Using new avar!!

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Test statistics
gamma_a_lam: -6.749227110586229
gamma_a: -4.6978742991665765
gamma_lam: -4.84767985730291
=====
Processing 9/54: ['Mkt-RF'] - 2 - ('1983-01-01', '2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0056, objective=47.873698
Iter 200/2000, frobenius_norm(G - G_old)=0.0001, objective=47.873319
Using new avar!!
Test statistics
gamma_a_lam: -6.834223208426526
gamma_a: -4.817540496545784
gamma_lam: -4.847679857284412
=====
Processing 10/54: ['Mkt-RF'] - 2 - ('1993-01-01', '2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0032, objective=53.475394
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=53.475352
Using new avar!!
Test statistics
gamma_a_lam: -6.838921089802824
gamma_a: -4.824126651374215
gamma_lam: -4.847679857414085
=====
Processing 11/54: ['Mkt-RF'] - 2 - ('1913-01-01', '2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0009, objective=46.426228
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=46.426223
Using new avar!!
Test statistics
gamma_a_lam: -6.855198917794802
gamma_a: -4.847038555591234
gamma_lam: -4.847679857416099
=====
Processing 12/54: ['Mkt-RF'] - 5 - ('1963-01-01', '2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0730, objective=32.648329
Iter 200/2000, frobenius_norm(G - G_old)=0.0041, objective=32.644373
Iter 300/2000, frobenius_norm(G - G_old)=0.0002, objective=32.644361
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=32.644361
Using new avar!!
Test statistics
gamma_a_lam: -6.852235054878161
gamma_a: -4.842850591751843
gamma_lam: -4.84767985741515
=====
Processing 13/54: ['Mkt-RF'] - 5 - ('1963-01-01', '1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0041, objective=24.727566
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=24.727552
Using new avar!!
Test statistics

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gamma_a_lam: -6.814811775931411
gamma_a: -4.795755029494672
gamma_lam: -4.847679854645565
=====
Processing 14/54: ['Mkt-RF'] - 5 - ('1973-01-01', '1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0479, objective=23.178482
Iter 200/2000, frobenius_norm(G - G_old)=0.0084, objective=23.159435
Iter 300/2000, frobenius_norm(G - G_old)=0.0011, objective=23.159098
Iter 400/2000, frobenius_norm(G - G_old)=0.0001, objective=23.159092
Iter 500/2000, frobenius_norm(G - G_old)=0.0000, objective=23.159092
Iter 600/2000, frobenius_norm(G - G_old)=0.0000, objective=23.159092
Using new avar!!
Test statistics
gamma_a_lam: -6.73232219492637
gamma_a: -4.6739453188607785
gamma_lam: -4.847679857322128
=====
Processing 15/54: ['Mkt-RF'] - 5 - ('1983-01-01', '2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0301, objective=29.842335
Iter 200/2000, frobenius_norm(G - G_old)=0.0006, objective=29.841906
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=29.841906
Using new avar!!
Test statistics
gamma_a_lam: -6.757567304630077
gamma_a: -4.709397823901906
gamma_lam: -4.84767985727246
=====
Processing 16/54: ['Mkt-RF'] - 5 - ('1993-01-01', '2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0235, objective=32.531576
Iter 200/2000, frobenius_norm(G - G_old)=0.0004, objective=32.531064
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=32.531063
Using new avar!!
Test statistics
gamma_a_lam: -6.624962365392575
gamma_a: -4.521731634912541
gamma_lam: -4.847679857374676
=====
Processing 17/54: ['Mkt-RF'] - 5 - ('1913-01-01', '2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0730, objective=32.648329
Iter 200/2000, frobenius_norm(G - G_old)=0.0041, objective=32.644373
Iter 300/2000, frobenius_norm(G - G_old)=0.0002, objective=32.644361
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=32.644361
Using new avar!!
Test statistics
gamma_a_lam: -6.852235054878161
gamma_a: -4.842850591751843
gamma_lam: -4.84767985741515
=====

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Processing 18/54: ['Mkt-RF', 'SMB', 'HML'] - 1 - ('1963-01-01', '2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0150, objective=53.930578
Iter 200/2000, frobenius_norm(G - G_old)=0.0006, objective=53.926880
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=53.926874
Using new avar!!
Test statistics
gamma_a_lam: -9.695024878416147
gamma_a: -4.847014648499609
gamma_lam: -8.396427811772481
=====
Processing 19/54: ['Mkt-RF', 'SMB', 'HML'] - 1 - ('1963-01-01', '1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0007, objective=42.334207
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=42.334191
Using new avar!!
Test statistics
gamma_a_lam: -9.68469376341367
gamma_a: -4.8287473386017945
gamma_lam: -8.396427117635778
=====
Processing 20/54: ['Mkt-RF', 'SMB', 'HML'] - 1 - ('1973-01-01', '1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0129, objective=43.314953
Iter 200/2000, frobenius_norm(G - G_old)=0.0011, objective=43.305352
Iter 300/2000, frobenius_norm(G - G_old)=0.0001, objective=43.305283
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=43.305283
Using new avar!!
Test statistics
gamma_a_lam: -9.54289746580538
gamma_a: -4.54407820620805
gamma_lam: -8.396427586798907
=====
Processing 21/54: ['Mkt-RF', 'SMB', 'HML'] - 1 - ('1983-01-01', '2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0036, objective=55.431078
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=55.430806
Using new avar!!
Test statistics
gamma_a_lam: -9.652916302653095
gamma_a: -4.764880517072917
gamma_lam: -8.3964273413165
=====
Processing 22/54: ['Mkt-RF', 'SMB', 'HML'] - 1 - ('1993-01-01', '2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0163, objective=61.339559
Iter 200/2000, frobenius_norm(G - G_old)=0.0011, objective=61.327757
Iter 300/2000, frobenius_norm(G - G_old)=0.0001, objective=61.327706
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=61.327706
Using new avar!!
Test statistics
gamma_a_lam: -9.691552272922197
gamma_a: -4.840104363969349

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gamma_lam: -8.396427811542317
=====
Processing 23/54: ['Mkt-RF', 'SMB', 'HML'] - 1 - ('1913-01-01', '2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0150, objective=53.930578
Iter 200/2000, frobenius_norm(G - G_old)=0.0006, objective=53.926880
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=53.926874
Using new avar!!
Test statistics
gamma_a_lam: -9.695024878416147
gamma_a: -4.847014648499609
gamma_lam: -8.396427811772481
=====
Processing 24/54: ['Mkt-RF', 'SMB', 'HML'] - 2 - ('1963-01-01', '2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0177, objective=45.107247
Iter 200/2000, frobenius_norm(G - G_old)=0.0010, objective=45.102688
Iter 300/2000, frobenius_norm(G - G_old)=0.0001, objective=45.102674
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=45.102674
Using new avar!!
Test statistics
gamma_a_lam: -9.694371614483057
gamma_a: -4.845713325518414
gamma_lam: -8.396427811725152
=====
Processing 25/54: ['Mkt-RF', 'SMB', 'HML'] - 2 - ('1963-01-01', '1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0009, objective=35.244228
Using new avar!!
Test statistics
gamma_a_lam: -9.668337198929633
gamma_a: -4.800055296275581
gamma_lam: -8.396427197427668
=====
Processing 26/54: ['Mkt-RF', 'SMB', 'HML'] - 2 - ('1973-01-01', '1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0152, objective=33.866543
Iter 200/2000, frobenius_norm(G - G_old)=0.0008, objective=33.855535
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=33.855510
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=33.855510
Using new avar!!
Test statistics
gamma_a_lam: -9.459344103808512
gamma_a: -4.377815757113633
gamma_lam: -8.396427588265896
=====
Processing 27/54: ['Mkt-RF', 'SMB', 'HML'] - 2 - ('1983-01-01', '2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0111, objective=45.376857
Iter 200/2000, frobenius_norm(G - G_old)=0.0004, objective=45.373424
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=45.373419
Using new avar!!
Test statistics

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gamma_a_lam: -9.609766336174117
gamma_a: -4.679652040370794
gamma_lam: -8.396426998029547
=====
Processing 28/54: ['Mkt-RF', 'SMB', 'HML'] - 2 - ('1993-01-01', '2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0262, objective=49.563826
Iter 200/2000, frobenius_norm(G - G_old)=0.0064, objective=49.534771
Iter 300/2000, frobenius_norm(G - G_old)=0.0011, objective=49.533840
Iter 400/2000, frobenius_norm(G - G_old)=0.0002, objective=49.533817
Iter 500/2000, frobenius_norm(G - G_old)=0.0000, objective=49.533817
Iter 600/2000, frobenius_norm(G - G_old)=0.0000, objective=49.533817
Iter 700/2000, frobenius_norm(G - G_old)=0.0000, objective=49.533817
Using new avar!!
Test statistics
gamma_a_lam: -9.689792324346381
gamma_a: -4.836595157608977
gamma_lam: -8.396427811108865
=====
Processing 29/54: ['Mkt-RF', 'SMB', 'HML'] - 2 - ('1913-01-01', '2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0177, objective=45.107247
Iter 200/2000, frobenius_norm(G - G_old)=0.0010, objective=45.102688
Iter 300/2000, frobenius_norm(G - G_old)=0.0001, objective=45.102674
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=45.102674
Using new avar!!
Test statistics
gamma_a_lam: -9.694371614483057
gamma_a: -4.845713325518414
gamma_lam: -8.396427811725152
=====
Processing 30/54: ['Mkt-RF', 'SMB', 'HML'] - 5 - ('1963-01-01', '2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.1167, objective=31.890203
Iter 200/2000, frobenius_norm(G - G_old)=0.0953, objective=31.865023
Iter 300/2000, frobenius_norm(G - G_old)=0.0638, objective=31.860826
Iter 400/2000, frobenius_norm(G - G_old)=0.0260, objective=31.859450
Iter 500/2000, frobenius_norm(G - G_old)=0.0118, objective=31.859025
Iter 600/2000, frobenius_norm(G - G_old)=0.0059, objective=31.858892
Iter 700/2000, frobenius_norm(G - G_old)=0.0031, objective=31.858850
Iter 800/2000, frobenius_norm(G - G_old)=0.0017, objective=31.858837
Iter 900/2000, frobenius_norm(G - G_old)=0.0009, objective=31.858833
Iter 1000/2000, frobenius_norm(G - G_old)=0.0005, objective=31.858832
Iter 1100/2000, frobenius_norm(G - G_old)=0.0003, objective=31.858831
Iter 1200/2000, frobenius_norm(G - G_old)=0.0002, objective=31.858831
Iter 1300/2000, frobenius_norm(G - G_old)=0.0001, objective=31.858831
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1600/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1700/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1800/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831

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Iter 1900/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 2000/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Using new avar!!
Test statistics
gamma_a_lam: -9.682738575624686
gamma_a: -4.8224523515483995
gamma_lam: -8.396427811669424
=====
Processing 31/54: ['Mkt-RF', 'SMB', 'HML'] - 5 - ('1963-01-01', '1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0290, objective=23.079201
Iter 200/2000, frobenius_norm(G - G_old)=0.0013, objective=23.077481
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=23.077479
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=23.077479
Using new avar!!
Test statistics
gamma_a_lam: -9.659392377261005
gamma_a: -4.782828638167266
gamma_lam: -8.396425914237225
=====
Processing 32/54: ['Mkt-RF', 'SMB', 'HML'] - 5 - ('1973-01-01', '1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0742, objective=22.026299
Iter 200/2000, frobenius_norm(G - G_old)=0.0676, objective=21.913718
Iter 300/2000, frobenius_norm(G - G_old)=0.0138, objective=21.903002
Iter 400/2000, frobenius_norm(G - G_old)=0.0021, objective=21.902649
Iter 500/2000, frobenius_norm(G - G_old)=0.0003, objective=21.902641
Iter 600/2000, frobenius_norm(G - G_old)=0.0000, objective=21.902641
Iter 700/2000, frobenius_norm(G - G_old)=0.0000, objective=21.902641
Iter 800/2000, frobenius_norm(G - G_old)=0.0000, objective=21.902641
Iter 900/2000, frobenius_norm(G - G_old)=0.0000, objective=21.902641
Using new avar!!
Test statistics
gamma_a_lam: -9.306729411489794
gamma_a: -4.074000332866565
gamma_lam: -8.396427611240588
=====
Processing 33/54: ['Mkt-RF', 'SMB', 'HML'] - 5 - ('1983-01-01', '2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0775, objective=28.215168
Iter 200/2000, frobenius_norm(G - G_old)=0.0411, objective=28.193347
Iter 300/2000, frobenius_norm(G - G_old)=0.0242, objective=28.188632
Iter 400/2000, frobenius_norm(G - G_old)=0.0131, objective=28.187217
Iter 500/2000, frobenius_norm(G - G_old)=0.0077, objective=28.186796
Iter 600/2000, frobenius_norm(G - G_old)=0.0046, objective=28.186676
Iter 700/2000, frobenius_norm(G - G_old)=0.0026, objective=28.186643
Iter 800/2000, frobenius_norm(G - G_old)=0.0014, objective=28.186634
Iter 900/2000, frobenius_norm(G - G_old)=0.0007, objective=28.186632
Iter 1000/2000, frobenius_norm(G - G_old)=0.0004, objective=28.186631
Iter 1100/2000, frobenius_norm(G - G_old)=0.0002, objective=28.186631
Iter 1200/2000, frobenius_norm(G - G_old)=0.0001, objective=28.186631

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Iter 1300/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 1600/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 1700/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 1800/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 1900/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Iter 2000/2000, frobenius_norm(G - G_old)=0.0000, objective=28.186631
Using new avar!!
Test statistics
gamma_a_lam: -9.54334887055355
gamma_a: -4.5466719865445135
gamma_lam: -8.396427150221054
=====
Processing 34/54: ['Mkt-RF', 'SMB', 'HML'] - 5 - ('1993-01-01', '2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0557, objective=30.778898
Iter 200/2000, frobenius_norm(G - G_old)=0.0248, objective=30.753519
Iter 300/2000, frobenius_norm(G - G_old)=0.0125, objective=30.747627
Iter 400/2000, frobenius_norm(G - G_old)=0.0063, objective=30.746017
Iter 500/2000, frobenius_norm(G - G_old)=0.0031, objective=30.745600
Iter 600/2000, frobenius_norm(G - G_old)=0.0015, objective=30.745497
Iter 700/2000, frobenius_norm(G - G_old)=0.0007, objective=30.745471
Iter 800/2000, frobenius_norm(G - G_old)=0.0004, objective=30.745465
Iter 900/2000, frobenius_norm(G - G_old)=0.0002, objective=30.745464
Iter 1000/2000, frobenius_norm(G - G_old)=0.0001, objective=30.745464
Iter 1100/2000, frobenius_norm(G - G_old)=0.0000, objective=30.745464
Iter 1200/2000, frobenius_norm(G - G_old)=0.0000, objective=30.745464
Iter 1300/2000, frobenius_norm(G - G_old)=0.0000, objective=30.745464
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=30.745464
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=30.745464
Iter 1600/2000, frobenius_norm(G - G_old)=0.0000, objective=30.745464
Using new avar!!
Test statistics
gamma_a_lam: -9.544533327439641
gamma_a: -4.546524225249478
gamma_lam: -8.396427811067099
=====
Processing 35/54: ['Mkt-RF', 'SMB', 'HML'] - 5 - ('1913-01-01', '2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.1167, objective=31.890203
Iter 200/2000, frobenius_norm(G - G_old)=0.0953, objective=31.865023
Iter 300/2000, frobenius_norm(G - G_old)=0.0638, objective=31.860826
Iter 400/2000, frobenius_norm(G - G_old)=0.0260, objective=31.859450
Iter 500/2000, frobenius_norm(G - G_old)=0.0118, objective=31.859025
Iter 600/2000, frobenius_norm(G - G_old)=0.0059, objective=31.858892
Iter 700/2000, frobenius_norm(G - G_old)=0.0031, objective=31.858850
Iter 800/2000, frobenius_norm(G - G_old)=0.0017, objective=31.858837
Iter 900/2000, frobenius_norm(G - G_old)=0.0009, objective=31.858833
Iter 1000/2000, frobenius_norm(G - G_old)=0.0005, objective=31.858832

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Iter 1100/2000, frobenius_norm(G - G_old)=0.0003, objective=31.858831
Iter 1200/2000, frobenius_norm(G - G_old)=0.0002, objective=31.858831
Iter 1300/2000, frobenius_norm(G - G_old)=0.0001, objective=31.858831
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1600/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1700/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1800/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 1900/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Iter 2000/2000, frobenius_norm(G - G_old)=0.0000, objective=31.858831
Using new avar!!
Test statistics
gamma_a_lam: -9.682738575624686
gamma_a: -4.8224523515483995
gamma_lam: -8.396427811669424
=====
Processing 36/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 1 - ('1963-01-01',
'2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0183, objective=52.486553
Iter 200/2000, frobenius_norm(G - G_old)=0.0015, objective=52.479527
Iter 300/2000, frobenius_norm(G - G_old)=0.0001, objective=52.479477
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=52.479477
Using new avar!!
Test statistics
gamma_a_lam: -11.873927943626851
gamma_a: -4.846672788164322
gamma_lam: -10.839741691820953
=====
Processing 37/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 1 - ('1963-01-01',
'1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0013, objective=40.301968
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=40.301904
Using new avar!!
Test statistics
gamma_a_lam: -11.862769677958616
gamma_a: -4.822400975787358
gamma_lam: -10.83964647515067
=====
Processing 38/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 1 - ('1973-01-01',
'1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0222, objective=41.550311
Iter 200/2000, frobenius_norm(G - G_old)=0.0035, objective=41.508127
Iter 300/2000, frobenius_norm(G - G_old)=0.0005, objective=41.507071
Iter 400/2000, frobenius_norm(G - G_old)=0.0001, objective=41.507051
Iter 500/2000, frobenius_norm(G - G_old)=0.0000, objective=41.507051
Using new avar!!
Test statistics
gamma_a_lam: -11.57503543201917

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gamma_a: -4.119074940298734
gamma_lam: -10.839740658755659
=====
Processing 39/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 1 - ('1983-01-01',
'2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0042, objective=51.728691
Iter 200/2000, frobenius_norm(G - G_old)=0.0000, objective=51.728185
Using new avar!!
Test statistics
gamma_a_lam: -11.812777983881636
gamma_a: -4.69812538381143
gamma_lam: -10.839740692131159
=====
Processing 40/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 1 - ('1993-01-01',
'2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0189, objective=57.717991
Iter 200/2000, frobenius_norm(G - G_old)=0.0034, objective=57.695290
Iter 300/2000, frobenius_norm(G - G_old)=0.0005, objective=57.694553
Iter 400/2000, frobenius_norm(G - G_old)=0.0001, objective=57.694534
Iter 500/2000, frobenius_norm(G - G_old)=0.0000, objective=57.694534
Using new avar!!
Test statistics
gamma_a_lam: -11.870530909846769
gamma_a: -4.838407866434676
gamma_lam: -10.839741688405038
=====
Processing 41/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 1 - ('1913-01-01',
'2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0183, objective=52.486553
Iter 200/2000, frobenius_norm(G - G_old)=0.0015, objective=52.479527
Iter 300/2000, frobenius_norm(G - G_old)=0.0001, objective=52.479477
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=52.479477
Using new avar!!
Test statistics
gamma_a_lam: -11.873927943626851
gamma_a: -4.846672788164322
gamma_lam: -10.839741691820953
=====
Processing 42/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 2 - ('1963-01-01',
'2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0201, objective=43.604987
Iter 200/2000, frobenius_norm(G - G_old)=0.0023, objective=43.595689
Iter 300/2000, frobenius_norm(G - G_old)=0.0003, objective=43.595564
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=43.595563
Using new avar!!
Test statistics
gamma_a_lam: -11.873484743561802
gamma_a: -4.845593810409462

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gamma_lam: -10.83974169027305
=====
Processing 43/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 2 - ('1963-01-01',
'1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0035, objective=33.417027
Iter 200/2000, frobenius_norm(G - G_old)=0.0001, objective=33.416613
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=33.416613
Using new avar!!
Test statistics
gamma_a_lam: -11.850348064722843
gamma_a: -4.795485649731687
gamma_lam: -10.839628097575883
=====
Processing 44/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 2 - ('1973-01-01',
'1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0227, objective=32.451898
Iter 200/2000, frobenius_norm(G - G_old)=0.0023, objective=32.417364
Iter 300/2000, frobenius_norm(G - G_old)=0.0002, objective=32.417060
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=32.417057
Using new avar!!
Test statistics
gamma_a_lam: -11.478248147734778
gamma_a: -3.883807397938637
gamma_lam: -10.83974055685111
=====
Processing 45/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 2 - ('1983-01-01',
'2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0183, objective=42.574626
Iter 200/2000, frobenius_norm(G - G_old)=0.0007, objective=42.564208
Iter 300/2000, frobenius_norm(G - G_old)=0.0000, objective=42.564192
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=42.564192
Using new avar!!
Test statistics
gamma_a_lam: -11.755502897276875
gamma_a: -4.558706199710262
gamma_lam: -10.839740706276315
=====
Processing 46/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 2 - ('1993-01-01',
'2013-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0250, objective=45.887640
Iter 200/2000, frobenius_norm(G - G_old)=0.0125, objective=45.840179
Iter 300/2000, frobenius_norm(G - G_old)=0.0047, objective=45.832713
Iter 400/2000, frobenius_norm(G - G_old)=0.0015, objective=45.831823
Iter 500/2000, frobenius_norm(G - G_old)=0.0005, objective=45.831733
Iter 600/2000, frobenius_norm(G - G_old)=0.0001, objective=45.831724
Iter 700/2000, frobenius_norm(G - G_old)=0.0000, objective=45.831724
Iter 800/2000, frobenius_norm(G - G_old)=0.0000, objective=45.831724
Iter 900/2000, frobenius_norm(G - G_old)=0.0000, objective=45.831724

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Using new avar!!
Test statistics
gamma_a_lam: -11.86721939448916
gamma_a: -4.83032712023006
gamma_lam: -10.839741685716284
=====
Processing 47/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 2 - ('1913-01-01',
'2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0201, objective=43.604987
Iter 200/2000, frobenius_norm(G - G_old)=0.0023, objective=43.595689
Iter 300/2000, frobenius_norm(G - G_old)=0.0003, objective=43.595564
Iter 400/2000, frobenius_norm(G - G_old)=0.0000, objective=43.595563
Using new avar!!
Test statistics
gamma_a_lam: -11.873484743561802
gamma_a: -4.845593810409462
gamma_lam: -10.83974169027305
=====
Processing 48/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 5 - ('1963-01-01',
'2025-12-31')
Iter 100/2000, frobenius_norm(G - G_old)=0.0776, objective=31.091400
Iter 200/2000, frobenius_norm(G - G_old)=0.0296, objective=31.061602
Iter 300/2000, frobenius_norm(G - G_old)=0.0116, objective=31.058271
Iter 400/2000, frobenius_norm(G - G_old)=0.0047, objective=31.057881
Iter 500/2000, frobenius_norm(G - G_old)=0.0020, objective=31.057828
Iter 600/2000, frobenius_norm(G - G_old)=0.0009, objective=31.057819
Iter 700/2000, frobenius_norm(G - G_old)=0.0004, objective=31.057818
Iter 800/2000, frobenius_norm(G - G_old)=0.0002, objective=31.057818
Iter 900/2000, frobenius_norm(G - G_old)=0.0001, objective=31.057818
Iter 1000/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1100/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1200/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1300/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Using new avar!!
Test statistics
gamma_a_lam: -11.872850450522208
gamma_a: -4.844044679107309
gamma_lam: -10.839741690893051
=====
Processing 49/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 5 - ('1963-01-01',
'1983-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.1241, objective=21.920733
Iter 200/2000, frobenius_norm(G - G_old)=0.0160, objective=21.908634
Iter 300/2000, frobenius_norm(G - G_old)=0.0033, objective=21.908323
Iter 400/2000, frobenius_norm(G - G_old)=0.0007, objective=21.908312
Iter 500/2000, frobenius_norm(G - G_old)=0.0001, objective=21.908311

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Iter 600/2000, frobenius_norm(G - G_old)=0.0000, objective=21.908311
Iter 700/2000, frobenius_norm(G - G_old)=0.0000, objective=21.908311
Iter 800/2000, frobenius_norm(G - G_old)=0.0000, objective=21.908311
Using new avar!!
Test statistics
gamma_a_lam: -11.709920693912954
gamma_a: -4.458834483346244
gamma_lam: -10.839512620394055
=====
Processing 50/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 5 - ('1973-01-01',
'1993-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0523, objective=20.669395
Iter 200/2000, frobenius_norm(G - G_old)=0.0427, objective=20.560236
Iter 300/2000, frobenius_norm(G - G_old)=0.0172, objective=20.538738
Iter 400/2000, frobenius_norm(G - G_old)=0.0044, objective=20.536775
Iter 500/2000, frobenius_norm(G - G_old)=0.0010, objective=20.536667
Iter 600/2000, frobenius_norm(G - G_old)=0.0002, objective=20.536662
Iter 700/2000, frobenius_norm(G - G_old)=0.0000, objective=20.536662
Iter 800/2000, frobenius_norm(G - G_old)=0.0000, objective=20.536662
Iter 900/2000, frobenius_norm(G - G_old)=0.0000, objective=20.536662
Using new avar!!
Test statistics
gamma_a_lam: -10.55470529804764
gamma_a: -1.6287110479303362
gamma_lam: -10.839726419047679
=====
Processing 51/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 5 - ('1983-01-01',
'2003-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0383, objective=26.718844
Iter 200/2000, frobenius_norm(G - G_old)=0.0371, objective=26.708831
Iter 300/2000, frobenius_norm(G - G_old)=0.0128, objective=26.708117
Iter 400/2000, frobenius_norm(G - G_old)=0.0043, objective=26.708036
Iter 500/2000, frobenius_norm(G - G_old)=0.0015, objective=26.708026
Iter 600/2000, frobenius_norm(G - G_old)=0.0005, objective=26.708024
Iter 700/2000, frobenius_norm(G - G_old)=0.0002, objective=26.708024
Iter 800/2000, frobenius_norm(G - G_old)=0.0001, objective=26.708024
Iter 900/2000, frobenius_norm(G - G_old)=0.0000, objective=26.708024
Iter 1000/2000, frobenius_norm(G - G_old)=0.0000, objective=26.708024
Iter 1100/2000, frobenius_norm(G - G_old)=0.0000, objective=26.708024
Iter 1200/2000, frobenius_norm(G - G_old)=0.0000, objective=26.708024
Using new avar!!
Test statistics
gamma_a_lam: -11.611155146603364
gamma_a: -4.208221341420239
gamma_lam: -10.839739274840028
=====
Processing 52/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 5 - ('1993-01-01',
'2013-01-01')

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Iter 100/2000, frobenius_norm(G - G_old)=0.0840, objective=28.745838
Iter 200/2000, frobenius_norm(G - G_old)=0.0342, objective=28.710971
Iter 300/2000, frobenius_norm(G - G_old)=0.0156, objective=28.705238
Iter 400/2000, frobenius_norm(G - G_old)=0.0073, objective=28.704031
Iter 500/2000, frobenius_norm(G - G_old)=0.0039, objective=28.703696
Iter 600/2000, frobenius_norm(G - G_old)=0.0023, objective=28.703586
Iter 700/2000, frobenius_norm(G - G_old)=0.0014, objective=28.703547
Iter 800/2000, frobenius_norm(G - G_old)=0.0008, objective=28.703533
Iter 900/2000, frobenius_norm(G - G_old)=0.0005, objective=28.703528
Iter 1000/2000, frobenius_norm(G - G_old)=0.0003, objective=28.703527
Iter 1100/2000, frobenius_norm(G - G_old)=0.0002, objective=28.703526
Iter 1200/2000, frobenius_norm(G - G_old)=0.0001, objective=28.703526
Iter 1300/2000, frobenius_norm(G - G_old)=0.0001, objective=28.703526
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Iter 1600/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Iter 1700/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Iter 1800/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Iter 1900/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Iter 2000/2000, frobenius_norm(G - G_old)=0.0000, objective=28.703526
Using new avar!!
Test statistics
gamma_a_lam: -11.745251252883248
gamma_a: -4.531985447971149
gamma_lam: -10.839741676465259
=====
Processing 53/54: ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA'] - 5 - ('1913-01-01',
'2023-01-01')
Iter 100/2000, frobenius_norm(G - G_old)=0.0776, objective=31.091400
Iter 200/2000, frobenius_norm(G - G_old)=0.0296, objective=31.061602
Iter 300/2000, frobenius_norm(G - G_old)=0.0116, objective=31.058271
Iter 400/2000, frobenius_norm(G - G_old)=0.0047, objective=31.057881
Iter 500/2000, frobenius_norm(G - G_old)=0.0020, objective=31.057828
Iter 600/2000, frobenius_norm(G - G_old)=0.0009, objective=31.057819
Iter 700/2000, frobenius_norm(G - G_old)=0.0004, objective=31.057818
Iter 800/2000, frobenius_norm(G - G_old)=0.0002, objective=31.057818
Iter 900/2000, frobenius_norm(G - G_old)=0.0001, objective=31.057818
Iter 1000/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1100/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1200/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1300/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1400/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Iter 1500/2000, frobenius_norm(G - G_old)=0.0000, objective=31.057818
Using new avar!!
Test statistics
gamma_a_lam: -11.872850450522208
gamma_a: -4.844044679107309
gamma_lam: -10.839741690893051

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[4]: results

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[4]:
(Mkt-RF,)
1 (1963-01-01, 2025-12-31)  gamma_a_lam  gamma_a  \
  (1963-01-01, 1983-01-01)  -6.855346 -4.847246
  (1973-01-01, 1993-01-01)  -6.847855 -4.838403
  (1983-01-01, 2003-01-01)  -6.800031 -4.76943
  (1993-01-01, 2013-01-01)  -6.836929 -4.821627
  (1913-01-01, 2023-01-01)  -6.853078 -4.844052
2 (1963-01-01, 2025-12-31)  -6.855346 -4.847246
  (1963-01-01, 1983-01-01)  -6.855199 -4.847039
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	5	(1963-01-01, 2025-12-31)	-11.87285	-4.844045
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		(1913-01-01, 2023-01-01)	-11.87285	-4.844045
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