

2026 考研数学零基础提前学课堂手迹版讲义 新浪微博: 考研数学周洋鑫

零基础提前学(5)

- 1.无穷小量
- (1) 定义
- (2) 比阶
- (3) 等价无穷小公式——8+6
- (4) 等价无穷小的替换准则——乘除、加减、推广、和取低阶
- (5) 无穷小量的阶
- (6) 高阶无穷小的运算法则
- (7) 等价无穷小的充要条件
- 2. 泰勒公式
- (1) 记住公式,了解什么时候用
- (2) 原则1: 相消不为零原则

1列1: X+0月, ex-1-1~(1+x)~ x

ilat!!

- (1) $x \sin x \sim \frac{1}{5}x^5$ 3 (4) $x \arcsin x \sim -\frac{1}{5}x^5$ (5) $x \arctan x \sim \frac{1}{5}x^5$

- (3) x-1-(1+x)~ ±x2 2 (6) ex-1-x~ ±x2

13/1: 岩*10日,f(x) 力×13 m阿大多性, 2(x)为×13n例大多小是

- (1) f(x)・な(x) 対 x 行 m+n 町えら小. ✓
- (L) f(x) + g(x) 女 x fig min(n,n) 所充分.

- (1) $f(x) f(x) \sim Ax^m \cdot Bx^n = Ap \cdot x^{m+n}$
- (4) 四首四年内 21 1 1 = ~ EB

BY. X+0 BY, X+ zlax ~ X+X = LX X+0 11, X -214X ~ 6X

In(HX4)~x4 4

【例3.19】已知当 $x \to 0$ 时, $f(x) = 3\sin x - \sin 3x$ 与 cx^k 是等价无穷小量,则().

- (A) k=1,c=4. (C) k=3,c=4. (D) k=3,c=4. (D) k=3,c=4. (E) k=3,c=4. (E) k=2,c 微博关注考研数学周洋鑫

[分析] f(x) = 3 sinx - sin3x ~ 3x =3x ×

XENIZ - XIZE = (X) , FROXE, FY $= 3 \left[x - \frac{1}{2} \left(\frac{1}{2} \right) + o(\frac{1}{2}) \right] - \left[\frac{1}{2} \left(\frac{1}{2} \right) + o(\frac{1}{2}) \right] + o(\frac{1}{2})$ =[3x-7x,+0(x,)]-[3x-7x,+0(x,)] $= 4x^{5} + o(x^{5})$ $\sim 4x^3 = cx^k \Rightarrow c=4, k=3$

(大) X+0时, Sinx = x- tx+0(x) 0 + 0 A 1. 2 1 1 0 = 0 - 6 03 + 0 (03)



【例3.19】已知当 $x \to 0$ 时, $f(x) = 3\sin x - \sin 3x$ 与 cx^k 是等价无穷小量,则().

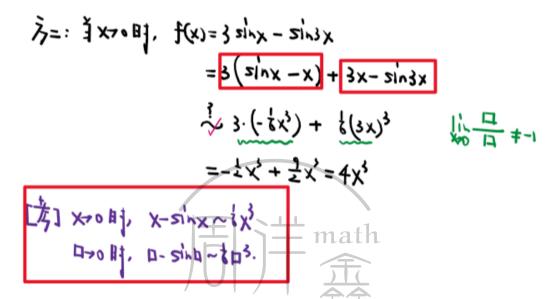
(A) k = 1, c = 4.

(B) k = 1, c = -4.

(C) k = 3, c = 4.

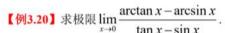
(D) k = 3, c = -4.

(E) k = 2, c = -4.



【考】原则2:上下同阶原则——经验原则

【例3.20】 求极限 $\lim_{x\to 0} \frac{\arctan x - \arcsin x}{\tan x - \sin x}$



$$\vec{\beta} =: \lim_{x \to 0} \frac{(\alpha + \alpha x - x) + (x - \alpha x + \alpha x)}{(+ \alpha x - x) + (x - \alpha x + \alpha x)} \xrightarrow{\vec{\beta}} -\frac{1}{5} x^{3} - \frac{1}{6} x^{5}$$

$$= \lim_{x \to 0} \frac{-\frac{1}{2} x^{5} + (-\frac{1}{6} x^{5})}{\frac{1}{2} x^{3} + \frac{1}{6} x^{5}}$$

$$= \lim_{x \to 0} \frac{-\frac{1}{2} x^{5}}{\frac{1}{2} x^{5}} = -1.$$

【考点4】洛必达法则

$$\lim_{N \to \infty} \frac{\frac{\partial}{\partial x}}{\frac{\partial}{\partial x}} = \lim_{N \to \infty} \frac{\partial}{\partial x} =$$



【考】如果函数好求导的时候,为何不洛必达!

【考】我们要掌握洛必达法则bug,这些bug是为了让 我们更好地学会这个内容!

[M3.21]
$$\lim_{x\to 0} \frac{e^{3x} - e^{2x} - e^{x} + 1}{\sqrt{(1-x)(1+x)} - 1} = \frac{0}{0}$$
 $\lim_{x\to 0} \frac{e^{3x} - e^{2x} - e^{x} + 1}{\sqrt{(1-x)(1+x)} - 1} = \frac{1-x^{2}}{1-1}$
 $\lim_{x\to 0} \frac{1-x^{2}}{\sqrt{(1-x)(1+x)} - 1} = \frac{1-x^{2}}{1-1}$
 $\lim_{x\to 0} \frac{e^{3x} - e^{2x} - e^{x} + 1}{\sqrt{1-x}} = \frac{1-x^{2}}{1-x}$
 $\lim_{x\to 0} \frac{1-x^{2}}{\sqrt{1-x}} = \frac{1-x^{2}}{\sqrt{1-x}}$
 $\lim_{x\to 0} \frac{1-x^{2}}{\sqrt{1-x}} = \frac{1-x^{2}}{\sqrt{1-x}} = \frac{1-x^{2}}{\sqrt{1-x}}$
 $\lim_{x\to 0} \frac{1-x^{2}}{\sqrt{1-x}} = \frac{1-x^{2}}{\sqrt{1-x}} = 0$
 $\lim_{x\to 0} \frac{1-x^{2}}{\sqrt{1-x}} = 0$



【例3.24】
$$\lim_{x \to +\infty} \frac{x^3 + x^2 + 1}{2^x + x^3} \left(\sin x + \cos x \right) =$$
A. 0

A. 0

B. 1 D. 3

C. 2

其中, |sinx+ wox| = |sinx|+100x|

刘 shat cox 方科受色.

【考点5】极限四则运算

1. 四则运算法则内容

- (1) $\lim [f(x)\pm g(x)] = \lim_{x \to \infty} f(x) \pm \lim_{x \to \infty} f(x)$
- (2) $\lim [f(x)g(x)] \stackrel{\text{def}}{=} \lim f(x) \cdot \lim f(x)$

$$\begin{aligned}
|x| &= \frac{1}{x^3} = \frac{1}{x^3} \\
&= \frac{1}{x^3} = \frac{1}{x^3} \\
&= \frac{1}{x^3} = \frac{1}{x^3} - \frac{1}{x^3} = \frac{1}{x^3} \\
&= \frac{1}{x^3} = \frac{1}{x^3} - \frac{1}{x^3} = \frac{1}{x^3} \\
&= \frac{1}{x^3} = \frac{1}{x$$





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